Computer algebra independent integration tests

0-Independent-test-suites/Welz-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 86.21 (100)	% 13.79 (16)
Mathematica	% 81.90 (95)	% 18.10 (21)
Maple	% 66.38 (77)	% 33.62 (39)
Maxima	% 17.24 (20)	% 82.76 (96)
Fricas	% 76.72 (89)	% 23.28 (27)
Sympy	% 25.00 (29)	% 75.00 (87)
Giac	% 26.72 (31)	% 73.28 (85)
Mupad	% 31.90 (37)	% 68.10 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

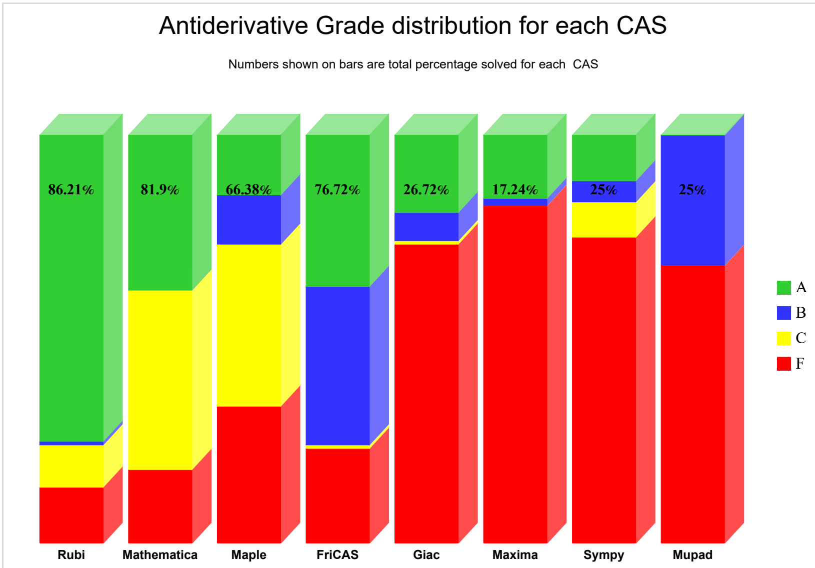
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

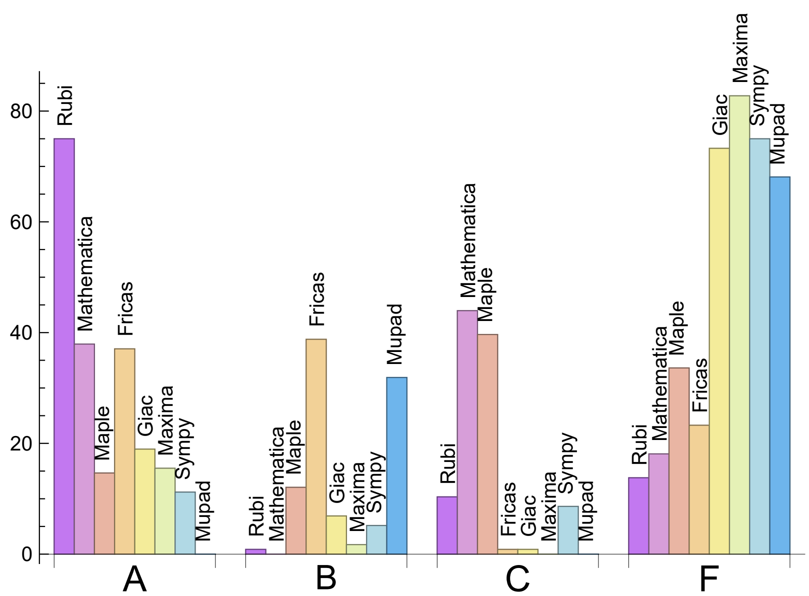
System	% A grade	% B grade	% C grade	% F grade
Rubi	75.00	0.86	10.34	13.79
Mathematica	37.93	0.00	43.97	18.10
Maple	14.66	12.07	39.66	33.62
Maxima	15.52	1.72	0.00	82.76
Fricas	37.07	38.79	0.86	23.28
Sympy	11.21	5.17	8.62	75.00
Giac	18.97	6.90	0.86	73.28
Mupad	0.00	31.90	0.00	68.10

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	16	100.00 %	0.00 %	0.00 %
Mathematica	21	100.00 %	0.00 %	0.00 %
Maple	39	92.31 %	5.13 %	2.56 %
Maxima	96	98.96 %	0.00 %	1.04 %
Fricas	27	37.04 %	33.33 %	29.63 %
Sympy	87	87.36 %	11.49 %	1.15 %
Giac	85	96.47 %	1.18 %	2.35 %
Mupad	79	98.73 %	1.27 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

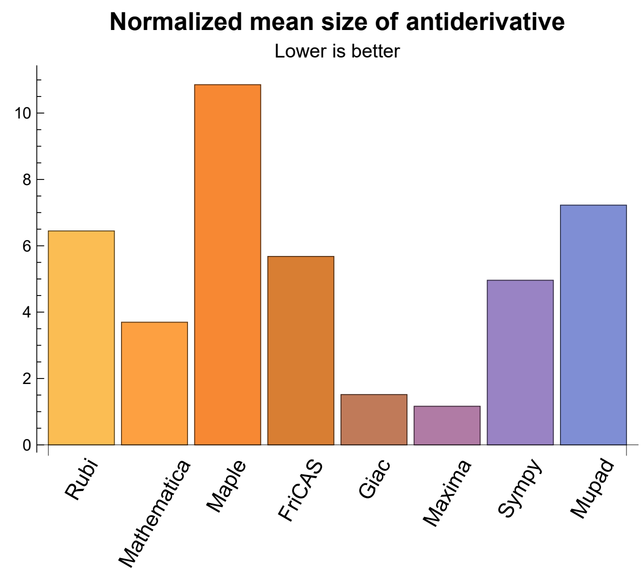
1.3 Performance

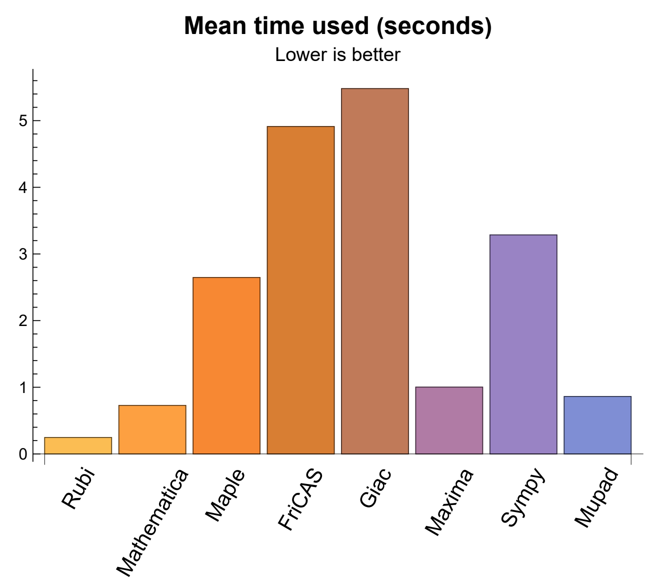
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	144.10	6.45	81.00	1.00
Mathematica	0.73	211.85	3.70	86.00	1.00
Maple	2.65	1851.86	10.85	317.00	2.31
Maxima	1.00	81.80	1.16	62.00	1.05
Fricas	4.91	802.17	5.68	171.00	1.86
Sympy	3.29	202.10	4.96	37.00	0.82
Giac	5.48	261.68	1.51	72.00	1.12
Mupad	0.86	151.70	7.22	76.00	1.09

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {9, 10, 82, 98, 113, 114, 116}

Mathematica {4, 9, 10, 39, 40, 51, 53, 54, 55, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

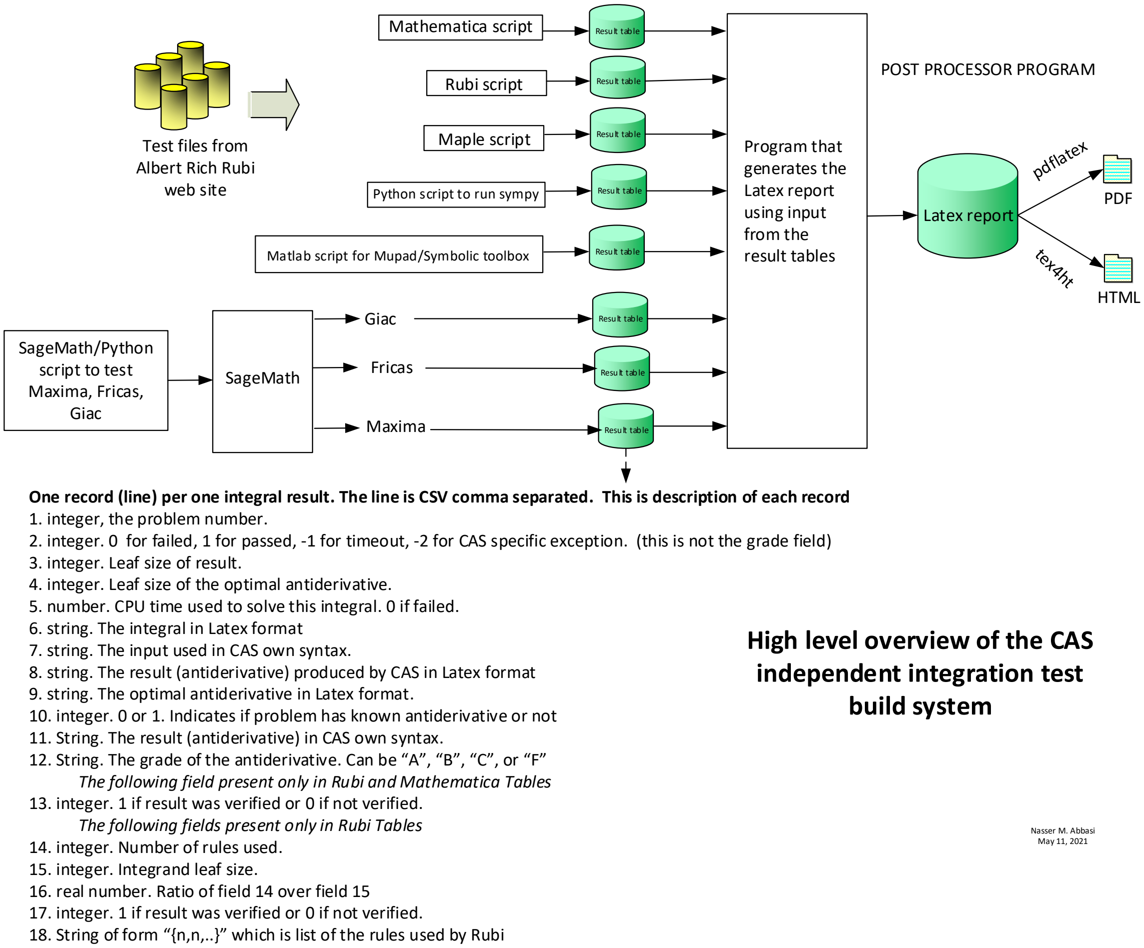
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 105, 106, 107 }

B grade: { 10 }

C grade: { 2, 46, 52, 82, 83, 98, 100, 101, 102, 113, 114, 116 }

F grade: { 43, 44, 45, 58, 59, 60, 61, 95, 103, 104, 108, 109, 110, 111, 112, 115 }

2.1.2 Mathematica

A grade: { 1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 57, 62, 85, 94, 96, 97, 99, 103, 104, 105, 107 }

B grade: { }

C grade: { 2, 4, 24, 39, 40, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116 }

F grade: { 12, 13, 37, 38, 44, 45, 46, 58, 59, 60, 61, 92, 93, 95, 100, 102, 108, 109, 110, 111, 112 }

2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 63, 64, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 37, 39, 40, 52, 55, 56, 57, 58, 59, 66, 67, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 106, 107, 116 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 38, 41, 42, 43, 44, 45, 46, 53, 54, 60, 61, 69, 70, 71, 72, 80, 94, 95, 96, 98, 108, 109, 110, 111, 112, 113, 114, 115 }

2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 47, 48, 49, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 29, 38, 44, 45, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 94, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

2.1.6 Sympy

A grade: { 1, 2, 14, 15, 20, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.7 Giac

A grade: { 1, 6, 8, 16, 21, 22, 23, 25, 26, 33, 34, 36, 41, 48, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade: { 2, 3, 4, 7, 9, 10, 24, 47 }

C grade: { 50 }

F grade: { 5, 11, 12, 13, 14, 15, 17, 18, 19, 20, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade: { }

F grade: { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.001	0.005	0.002	0.510	0.841	0.062	0.907	0.027

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	41	1	42	41	43
normalized size	1	3.47	2.47	2.27	2.73	0.07	2.80	2.73	2.87
time (sec)	N/A	0.056	0.034	0.004	0.466	0.853	59.577	1.057	0.515

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	99	370	0	100	0	177	204
normalized size	1	1.00	1.21	4.51	0.00	1.22	0.00	2.16	2.49
time (sec)	N/A	0.076	0.191	0.049	0.000	1.173	0.000	0.913	0.569

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	167	172	0	80	0	94	-1
normalized size	1	1.00	3.88	4.00	0.00	1.86	0.00	2.19	-0.02
time (sec)	N/A	0.012	3.763	0.053	0.000	1.529	0.000	0.938	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	126	115	0	105	0	0	82
normalized size	1	1.00	1.70	1.55	0.00	1.42	0.00	0.00	1.11
time (sec)	N/A	0.056	0.166	0.017	0.000	0.904	0.000	0.000	1.707

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	59	125	53	92	0	84	-1
normalized size	1	1.00	0.92	1.95	0.83	1.44	0.00	1.31	-0.02
time (sec)	N/A	0.026	0.086	0.039	1.452	0.978	0.000	1.068	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	75	45	0	83	0	101	-1
normalized size	1	1.00	1.56	0.94	0.00	1.73	0.00	2.10	-0.02
time (sec)	N/A	0.013	0.130	0.021	0.000	1.011	0.000	0.961	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	0	18	53	20	21
normalized size	1	1.00	1.00	0.70	0.00	0.60	1.77	0.67	0.70
time (sec)	N/A	0.082	0.058	0.004	0.000	0.886	0.753	0.949	0.381

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	365	340	902	0	424	0	367	-1
normalized size	1	1.66	1.55	4.10	0.00	1.93	0.00	1.67	-0.00
time (sec)	N/A	0.506	0.855	0.138	0.000	0.780	0.000	8.762	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F(-1)	B	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	541	311	1542	0	424	0	358	-1
normalized size	1	2.46	1.41	7.01	0.00	1.93	0.00	1.63	-0.00
time (sec)	N/A	0.751	0.713	0.023	0.000	0.612	0.000	9.218	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	125	278	0	161	0	0	-1
normalized size	1	1.00	0.91	2.01	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.203	0.032	0.000	1.061	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	394	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.240	0.098	0.000	4.546	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	369	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	4.56	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.173	0.088	0.000	10.157	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	60	15	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.94	0.48	0.00	-0.03
time (sec)	N/A	0.054	0.009	0.080	0.000	1.215	1.225	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	0	29	15	0	-1
normalized size	1	1.00	1.00	0.67	0.00	0.88	0.45	0.00	-0.03
time (sec)	N/A	0.063	0.011	0.100	0.000	1.621	0.860	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	28	56	15	15
normalized size	1	1.00	1.00	0.84	0.79	1.47	2.95	0.79	0.79
time (sec)	N/A	0.270	0.018	0.003	0.635	0.986	6.629	1.147	0.400

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2147	0	-1
normalized size	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.023	0.042	0.033	0.000	1.382	3.534	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.068	0.090	0.000	1.168	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	0	15
normalized size	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88
time (sec)	N/A	0.054	0.007	0.082	0.000	1.089	2.986	0.000	0.297

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	18	36	0	18
normalized size	1	1.00	1.00	0.00	0.00	0.90	1.80	0.00	0.90
time (sec)	N/A	0.060	0.007	0.092	0.000	0.625	1.598	0.000	0.296

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	48	40	52	36	47	58
normalized size	1	1.00	0.86	1.14	0.95	1.24	0.86	1.12	1.38
time (sec)	N/A	0.033	0.042	0.013	0.588	1.108	0.147	0.986	0.413

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	20	19	24
normalized size	1	1.00	1.00	0.95	0.91	0.86	0.91	0.86	1.09
time (sec)	N/A	0.024	0.021	0.003	0.604	1.247	0.120	0.776	0.385

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	51	58	51	74	47
normalized size	1	1.00	0.79	0.82	0.82	0.94	0.82	1.19	0.76
time (sec)	N/A	0.091	0.066	0.014	0.588	0.740	0.185	1.211	0.406

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	961	455	0	358	0	304	-1
normalized size	1	1.00	11.17	5.29	0.00	4.16	0.00	3.53	-0.01
time (sec)	N/A	0.084	2.520	0.043	0.000	1.013	0.000	1.061	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	15	15	15	15
normalized size	1	1.00	1.00	0.00	0.00	0.79	0.79	0.79	0.79
time (sec)	N/A	0.065	0.012	0.104	0.000	1.020	0.230	0.787	0.423

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	22	27	22	22
normalized size	1	1.00	1.00	0.00	0.00	0.85	1.04	0.85	0.85
time (sec)	N/A	0.097	0.018	180.000	0.000	0.985	1.072	0.984	0.506

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	198	46	0	-1
normalized size	1	1.00	0.89	0.00	0.00	3.14	0.73	0.00	-0.02
time (sec)	N/A	0.219	0.203	0.085	0.000	1.087	1.469	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	216	51	0	-1
normalized size	1	1.00	1.55	0.30	0.00	2.63	0.62	0.00	-0.01
time (sec)	N/A	0.072	0.048	0.013	0.000	0.771	1.316	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	679	412	0	518	0	0	0	-1
normalized size	1	1.12	0.68	0.00	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.799	0.262	0.092	0.557	1.017	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	0	15
normalized size	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88
time (sec)	N/A	0.055	0.011	0.091	0.000	0.884	2.577	0.000	0.314

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	32	2147	0	-1
normalized size	1	1.00	0.88	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.024	0.061	0.027	0.000	1.028	2.715	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	48	48	0	0	-1
normalized size	1	1.00	0.97	1.29	1.41	1.41	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.126	0.048	0.902	0.943	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	65	62	64	36	64	86
normalized size	1	1.00	0.98	1.12	1.07	1.10	0.62	1.10	1.48
time (sec)	N/A	0.036	0.015	0.105	1.092	0.948	0.944	0.916	0.543

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	48	62	64	37	64	76
normalized size	1	1.00	1.40	0.83	1.07	1.10	0.64	1.10	1.31
time (sec)	N/A	0.035	0.012	0.100	1.296	0.711	0.977	1.091	0.457

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	78	82	29	0	10
normalized size	1	1.00	1.76	0.24	1.59	1.67	0.59	0.00	0.20
time (sec)	N/A	0.005	0.040	0.092	1.361	0.896	0.893	0.000	0.334

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	62	64	32	63	80
normalized size	1	1.00	1.00	1.18	1.13	1.16	0.58	1.15	1.45
time (sec)	N/A	0.033	0.013	0.103	1.211	1.114	0.919	0.824	0.514

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	1143	0	301	0	0	-1
normalized size	1	1.00	0.00	11.78	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.080	8.055	0.000	7.448	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.080	0.616	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	176	59	1069	0	277	0	0	-1
normalized size	1	1.60	0.54	9.72	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.023	8.325	0.000	4.940	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	131	85	653	0	120	0	0	-1
normalized size	1	1.62	1.05	8.06	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.015	1.348	0.000	1.747	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	117	127	0	0	415	0	67	37
normalized size	1	1.77	1.92	0.00	0.00	6.29	0.00	1.02	0.56
time (sec)	N/A	0.056	0.079	0.141	0.000	3.859	0.000	1.049	0.394

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	145	140	0	0	665	0	0	-1
normalized size	1	1.84	1.77	0.00	0.00	8.42	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.151	0.037	0.000	3.454	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	B	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	55	0	0	1496	0	0	-1
normalized size	1	0.00	0.47	0.00	0.00	12.68	0.00	0.00	-0.01
time (sec)	N/A	21.937	0.195	0.047	0.000	52.735	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.611	1.638	0.126	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.462	0.666	0.097	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	576	0	0	0	0	0	0	-1
normalized size	1	1.17	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	0.396	0.276	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	198	584	0	957	0	767	343
normalized size	1	1.00	0.49	1.43	0.00	2.35	0.00	1.88	0.84
time (sec)	N/A	0.676	2.129	0.120	0.000	1.518	0.000	2.819	0.474

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	610	719	0	1563	0	972	567
normalized size	1	1.00	0.94	1.11	0.00	2.41	0.00	1.50	0.88
time (sec)	N/A	1.158	6.092	0.069	0.000	1.716	0.000	4.186	0.561

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	989	0	2763	0	1382	1017
normalized size	1	1.00	1.04	0.93	0.00	2.61	0.00	1.31	0.96
time (sec)	N/A	2.490	6.175	0.105	0.000	151.296	0.000	6.839	0.972

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	342	21028	0	1873	0	2509	-1
normalized size	1	1.00	0.90	55.63	0.00	4.96	0.00	6.64	-0.00
time (sec)	N/A	0.774	6.135	0.648	0.000	1.029	0.000	93.770	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	2775	0	0	-1
normalized size	1	1.00	2.24	136.04	0.00	4.35	0.00	0.00	-0.00
time (sec)	N/A	1.397	11.601	5.033	0.000	1.554	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	546	0	0	-1
normalized size	1	3.09	3.23	19.32	0.00	8.27	0.00	0.00	-0.02
time (sec)	N/A	1.234	1.150	0.119	0.000	1.128	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.234	0.122	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.238	0.114	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	2378	0	171	0	0	-1
normalized size	1	1.00	1.14	24.52	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.055	14.133	0.000	4.153	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	107	20	69	105	96	32	0	-1
normalized size	1	1.47	0.27	0.95	1.44	1.32	0.44	0.00	-0.01
time (sec)	N/A	0.044	0.003	0.104	1.252	1.033	1.022	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	90	49	71	73	37	72	83
normalized size	1	1.00	1.34	0.73	1.06	1.09	0.55	1.07	1.24
time (sec)	N/A	0.037	0.021	0.107	1.297	0.877	0.986	1.116	0.366

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	0	0	2972	0	0	0	0	-1
normalized size	1	0.00	0.00	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.353	19.957	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	0	1404	0	3085	0	0	-1
normalized size	1	0.00	0.00	5.01	0.00	11.02	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.105	33.314	0.000	16.511	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.050	0.344	0.850	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.138	4.236	0.000	18.875	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	23
normalized size	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.92
time (sec)	N/A	0.023	0.007	0.004	0.540	0.921	0.106	0.989	0.074

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	43	73	40	53
normalized size	1	1.00	1.46	0.88	0.00	0.73	1.24	0.68	0.90
time (sec)	N/A	0.032	0.018	0.031	0.000	0.847	0.172	0.848	0.335

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	66	100	64	76
normalized size	1	1.00	1.27	0.96	0.00	0.85	1.28	0.82	0.97
time (sec)	N/A	0.077	0.021	0.026	0.000	0.883	0.184	0.969	0.090

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	56	0	0	-1
normalized size	1	1.00	2.24	2.04	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.095	0.026	0.000	0.676	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	61	0	0	-1
normalized size	1	1.00	2.04	6.89	0.00	1.15	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.091	0.038	0.000	0.908	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	5727	1512	0	359	0	0	-1
normalized size	1	1.00	76.36	20.16	0.00	4.79	0.00	0.00	-0.01
time (sec)	N/A	0.094	7.076	0.092	0.000	0.750	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	2667	0	0	-1
normalized size	1	1.00	1.88	2.67	0.00	15.60	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.421	0.111	0.000	5.831	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0	-1
normalized size	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.256	0.135	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.264	0.120	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.215	0.139	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.186	0.119	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1191	0	0	653
normalized size	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14
time (sec)	N/A	0.019	0.021	0.186	0.000	1.260	0.000	0.000	0.449

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	1666	0	0	331
normalized size	1	1.00	0.34	1.53	0.00	10.61	0.00	0.00	2.11
time (sec)	N/A	0.033	0.030	0.219	0.000	1.815	0.000	0.000	15.034

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	421	0	547	0	0	533
normalized size	1	1.00	0.65	5.69	0.00	7.39	0.00	0.00	7.20
time (sec)	N/A	0.156	0.021	0.200	0.000	1.101	0.000	0.000	0.211

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	497	0	0	-1
normalized size	1	1.00	0.31	3.72	0.00	4.83	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.028	0.352	0.000	1.428	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	538	0	1792	0	0	-1
normalized size	1	1.00	1.56	6.64	0.00	22.12	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.098	10.056	0.000	3.816	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	444	0	345	0	0	-1
normalized size	1	1.00	1.56	5.48	0.00	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.095	2.458	0.000	2.760	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	704	0	1943	0	0	-1
normalized size	1	1.00	1.04	6.23	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.069	50.764	0.000	1.818	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1685	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	15.46	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.101	180.000	0.000	1.731	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	85	0	0	217
normalized size	1	1.00	1.83	2.37	0.00	0.98	0.00	0.00	2.49
time (sec)	N/A	0.874	0.934	0.041	0.000	0.702	0.000	0.000	0.169

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	C	F	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	529	127	317	0	70	0	0	207
normalized size	1	529.00	127.00	317.00	0.00	70.00	0.00	0.00	207.00
time (sec)	N/A	1.669	0.602	0.049	0.000	1.209	0.000	0.000	0.478

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	63	0	0	-1
normalized size	1	3.91	2.89	11.65	0.00	1.37	0.00	0.00	-0.02
time (sec)	N/A	1.492	1.091	0.059	0.000	0.722	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0	67
normalized size	1	1.00	10.09	8.06	0.00	0.00	0.00	0.00	2.09
time (sec)	N/A	0.096	0.463	0.122	0.000	0.000	0.000	0.000	1.686

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	44	0	0	204
normalized size	1	1.00	2.00	10.43	0.00	1.91	0.00	0.00	8.87
time (sec)	N/A	0.054	0.008	0.110	0.000	0.550	0.000	0.000	0.219

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	47	353	0	7739	0	0	-1
normalized size	1	1.00	0.22	1.62	0.00	35.50	0.00	0.00	-0.00
time (sec)	N/A	0.041	0.060	0.285	0.000	10.636	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	50	350	0	8237	0	0	-1
normalized size	1	1.00	0.24	1.67	0.00	39.22	0.00	0.00	-0.00
time (sec)	N/A	0.032	0.077	0.263	0.000	10.425	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	65	349	0	7910	0	0	-1
normalized size	1	1.00	0.29	1.57	0.00	35.63	0.00	0.00	-0.00
time (sec)	N/A	0.030	0.073	0.271	0.000	9.991	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	68	350	0	8105	0	0	-1
normalized size	1	1.00	0.32	1.64	0.00	37.87	0.00	0.00	-0.00
time (sec)	N/A	0.029	0.057	0.259	0.000	9.371	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	323	0	0	-1
normalized size	1	1.00	10.54	5.03	0.00	4.97	0.00	0.00	-0.02
time (sec)	N/A	0.129	3.211	0.161	0.000	0.710	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	112	0	0	-1
normalized size	1	1.00	13.90	4.94	0.00	1.78	0.00	0.00	-0.02
time (sec)	N/A	0.128	7.991	0.158	0.000	0.911	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	0	818	0	0	0	0	-1
normalized size	1	1.00	0.00	15.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.156	2.997	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	0	2134	0	267	0	0	-1
normalized size	1	1.00	0.00	19.76	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.036	5.820	0.000	2.696	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	135	120	0	0	0	0	0	-1
normalized size	1	1.38	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.087	0.134	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.199	0.206	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	80	0	110	387	0	113	157
normalized size	1	1.00	0.83	0.00	1.15	4.03	0.00	1.18	1.64
time (sec)	N/A	0.078	0.040	0.120	1.474	0.932	0.000	20.990	0.590

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	112	614	0	253	0	0	-1
normalized size	1	1.39	1.27	6.98	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.073	2.416	0.000	4.673	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	373	0	0	-1
normalized size	1	0.11	0.11	0.00	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.011	0.017	1.865	0.000	3.433	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	655	86	90	0	87	100
normalized size	1	1.00	0.89	7.99	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.058	0.028	3.427	1.512	0.663	0.000	0.884	0.548

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	409	0	720	0	318	0	0	-1
normalized size	1	3.03	0.00	5.33	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.138	7.804	0.000	12.832	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	409	150	676	0	318	0	0	-1
normalized size	1	3.03	1.11	5.01	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.308	7.503	0.000	12.883	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	399	0	652	0	268	0	0	-1
normalized size	1	3.35	0.00	5.48	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.128	7.938	0.000	13.536	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0	-1
normalized size	1	0.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.425	0.154	0.096	0.000	0.578	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0	-1
normalized size	1	0.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.220	0.085	0.092	0.000	0.603	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0	-1
normalized size	1	1.00	1.10	0.87	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.019	0.095	0.000	0.725	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	A	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	105	94	31	0	10
normalized size	1	1.00	1.51	0.18	1.57	1.40	0.46	0.00	0.15
time (sec)	N/A	0.010	0.120	0.102	1.300	0.622	1.021	0.000	0.342

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	66	73	75	41	74	91
normalized size	1	1.00	0.93	0.94	1.04	1.07	0.59	1.06	1.30
time (sec)	N/A	0.038	0.027	0.104	1.410	0.871	1.017	0.889	0.403

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	0.282	0.127	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.352	0.194	0.000	4.434	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F(-1)	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	1827	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	9.18	0.00	0.00	-0.01
time (sec)	N/A	0.945	0.284	180.000	0.000	5.170	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.326	0.114	0.000	5.349	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.153	0.017	0.000	4.596	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	21	111	0	0	191	0	0	-1
normalized size	1	0.16	0.84	0.00	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.008	0.096	0.603	0.000	0.631	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	26	26	0	0	0	0	0	-1
normalized size	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.012	0.014	0.116	0.000	4.310	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	383	0	138	0	0	0	0	0	-1
normalized size	1	0.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.157	0.112	0.000	11.791	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	21	109	1214	0	341	0	0	-1
normalized size	1	0.08	0.40	4.46	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.008	0.093	6.549	0.000	3.530	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.143]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	10	0.100
2	C	5	2	3.47	37	0.054
3	A	9	6	1.00	15	0.400
4	A	3	3	1.00	19	0.158
5	A	8	7	1.00	17	0.412
6	A	6	6	1.00	17	0.353
7	A	3	3	1.00	17	0.176
8	A	4	2	1.00	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.00	45	0.067
12	A	7	4	1.00	32	0.125
13	A	5	3	1.00	32	0.094
14	A	2	2	1.00	27	0.074
15	A	2	2	1.00	29	0.069
16	A	2	1	1.00	30	0.033
17	A	3	2	1.00	13	0.154
18	A	3	2	1.00	15	0.133
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	25	0.080
21	A	3	2	1.00	11	0.182
22	A	2	2	1.00	18	0.111
23	A	6	6	1.00	20	0.300
24	A	6	5	1.00	31	0.161
25	A	2	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	35	0.057
27	A	5	5	1.00	32	0.156
28	A	6	6	1.00	21	0.286
29	A	359	30	1.12	14	2.143
30	A	2	2	1.00	23	0.087
31	A	3	2	1.00	13	0.154
32	A	2	2	1.00	33	0.061
33	A	5	5	1.00	15	0.333
34	A	5	5	1.00	15	0.333
35	A	1	1	1.00	11	0.091
36	A	5	5	1.00	15	0.333
37	A	1	1	1.00	17	0.059
38	A	3	3	1.00	18	0.167
39	A	2	2	1.60	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.17	32	0.094
47	A	19	9	1.00	20	0.450
48	A	29	9	1.00	20	0.450
49	A	49	9	1.00	20	0.450
50	A	14	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	24	6	1.00	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.00	24	0.292
54	A	7	7	1.00	24	0.292
55	A	1	1	1.00	18	0.056
56	A	8	8	1.47	13	0.615
57	A	6	6	1.00	15	0.400
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	F	0	0	N/A	0	N/A
61	F	0	0	N/A	0	N/A
62	A	1	1	1.00	38	0.026
63	A	1	1	1.00	33	0.030
64	A	2	2	1.00	39	0.051
65	A	1	1	1.00	19	0.053
66	A	4	4	1.00	19	0.210
67	A	4	4	1.00	24	0.167
68	A	1	1	1.00	24	0.042
69	A	7	7	1.00	24	0.292
70	A	7	7	1.00	24	0.292
71	A	3	3	1.00	26	0.115
72	A	3	3	1.00	22	0.136
73	A	1	1	1.00	22	0.045
74	A	1	1	1.00	23	0.043
75	A	8	8	1.00	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
76	A	8	7	1.00	23	0.304
77	A	1	1	1.00	21	0.048
78	A	1	1	1.00	19	0.053
79	A	1	1	1.00	19	0.053
80	A	1	1	1.00	19	0.053
81	A	4	4	1.00	34	0.118
82	C	5	5	529.00	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.00	29	0.069
85	A	2	2	1.00	18	0.111
86	A	1	1	1.00	25	0.040
87	A	1	1	1.00	25	0.040
88	A	1	1	1.00	25	0.040
89	A	1	1	1.00	25	0.040
90	A	2	2	1.00	40	0.050
91	A	2	2	1.00	40	0.050
92	A	1	1	1.00	18	0.056
93	A	3	3	1.00	15	0.200
94	A	7	7	1.38	21	0.333
95	F	0	0	N/A	0	N/A
96	A	5	5	1.00	24	0.208
97	A	7	7	1.39	19	0.368
98	C	1	1	0.11	20	0.050
99	A	5	5	1.00	22	0.227
100	C	4	2	3.03	25	0.080
101	C	5	3	3.03	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	C	4	2	3.35	23	0.087
103	F	0	0	N/A	0	N/A
104	F	0	0	N/A	0	N/A
105	A	3	3	1.00	19	0.158
106	A	2	2	1.00	11	0.182
107	A	6	6	1.00	15	0.400
108	F	0	0	N/A	0	N/A
109	F	0	0	N/A	0	N/A
110	F	0	0	N/A	0	N/A
111	F	0	0	N/A	0	N/A
112	F	0	0	N/A	0	N/A
113	C	1	1	0.16	19	0.053
114	C	1	1	0.10	20	0.050
115	F	0	0	N/A	0	N/A
116	C	1	1	0.08	19	0.053

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.100, Rules used = {32}

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - a*x],x]
[Out] (-2*Sqrt[1 - a*x])/a
```

fricas [A] time = 0.84, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*x + 1)/a
```

giac [A] time = 0.91, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-a*x + 1)/a
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/2),x)
```

```
[Out] -2*(-a*x+1)^(1/2)/a
```

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-a*x + 1)/a
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{1-a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - a*x)^(1/2),x)
[Out] -(2*(1 - a*x)^(1/2))/a
```

sympy [A] time = 0.06, size = 12, normalized size = 0.80

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
[Out] -2*sqrt(-a*x + 1)/a
```

3.2 $\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [C] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 2295}

$$\frac{\sqrt{ax-1}\log(ax-1)}{\pi a}-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 +
a*x])/(a*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{\sqrt{-1+ax}}\,dx}{2\pi}$$
$$= \frac{\text{Subst}\left(\int \left(-2\log(-x)+\log\left(x^2\right)\right)dx, x, \sqrt{-1+ax}\right)}{a\pi}$$
$$= \frac{\text{Subst}\left(\int \log\left(x^2\right)dx, x, \sqrt{-1+ax}\right)}{a\pi} - \frac{2\,\text{Subst}\left(\int \log(-x)\,dx, x, \sqrt{-1+ax}\right)}{a\pi}$$
$$= -\frac{2\sqrt{-1+ax}\,\log\left(-\sqrt{-1+ax}\right)}{a\pi} + \frac{\sqrt{-1+ax}\,\log(-1+ax)}{a\pi}$$

Mathematica [C] time = 0.03, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1}\left(\log(ax-1)-2\log\left(-\sqrt{ax-1}\right)\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]
```

```
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

fricas [A] time = 0.85, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
```

```
[Out] 0
```

giac [B] time = 1.06, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\,\log(ax-1)-2\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="giac")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

maple [C] time = 0.00, size = 34, normalized size = 2.27

$$\frac{\sqrt{ax-1}\,\left(-2\ln\left(-\sqrt{ax-1}\right)+\ln\left(ax-1\right)\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x)
```

```
[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

maxima [B] time = 0.47, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\,\log\left(ax-1\right)-2\,\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

mupad [B] time = 0.51, size = 43, normalized size = 2.87

$$-\frac{2\,\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
```

```
[Out] -(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/(
Pi*a)
```

sympy [A] time = 59.58, size = 42, normalized size = 2.80

$$\frac{\begin{cases}\frac{-2\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\,\log\left(ax-1\right)}{a} & \text{for } a\neq 0\\ \pi x & \text{otherwise}\end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x^
2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6742, 199, 207, 444, 47, 63}

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx &= \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right)dx\\
&= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,dx,x,x^2\right)\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+3x)}\,dx,x,x^2\right)\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx,x,\sqrt{1+x^2}\right)\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 99, normalized size = 1.21

$$\frac{1}{9}\left(\frac{12x}{1-3x^2}-\frac{\frac{6x^2+6}{1-3x^2}+\sqrt{3}\sqrt{-x^2-1}\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{-x^2-1}\right)}{\sqrt{x^2+1}}-\sqrt{3}\tanh^{-1}\left(\sqrt{3}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] ((12*x)/(1 - 3*x^2) - ((6 + 6*x^2)/(1 - 3*x^2) + Sqrt[3]*Sqrt[-1 - x^2]*Arc
Tan[(Sqrt[3]*Sqrt[-1 - x^2])/2])/Sqrt[1 + x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x]
)/9
```

fricas [A] time = 1.17, size = 100, normalized size = 1.22

$$\frac{\sqrt{3}\left(3x^2-1\right)\log\left(\frac{3x^2-2\sqrt{3}x+1}{3x^2-1}\right)+\sqrt{3}\left(3x^2-1\right)\log\left(\frac{3x^2+4\sqrt{3}\sqrt{x^2+1}+7}{3x^2-1}\right)-24x+12\sqrt{x^2+1}}{18\left(3x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

giac [B] time = 0.91, size = 177, normalized size = 2.16

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-6x-8\sqrt{3}+6\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3x-4\sqrt{3}-3\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right)-\frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right.}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

maple [B] time = 0.05, size = 370, normalized size = 4.51

$$-\frac{x}{2\left(3x^2-1\right)}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\frac{\sqrt{3}\ \operatorname{arctanh}\left(\sqrt{3}\ x\right)}{9}-\sqrt{3}\left(\frac{\sqrt{\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}}\ x}{12}+\frac{\operatorname{arcsinh}(x)}{12}-\frac{\left(\left(x-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(3^(1/2)*x)*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

mupad [B] time = 0.57, size = 204, normalized size = 2.49

$$\frac{\sqrt{3}\,\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)}+\frac{\sqrt{3}\,\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{\sqrt{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$\frac{3\sqrt{x^2-1}\,x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

```
[Out] 5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 377, 207}

$$\frac{3\sqrt{x^2-1}\,x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)}\,dx\\ &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\ &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [C] time = 3.76, size = 167, normalized size = 3.88

$$-\frac{x\sqrt{x^2-1}\left(\frac{8x^2\left(x^2-1\right){}_2F_1\left(2,3;\frac{7}{2};\frac{x^2}{4-3x^2}\right)}{45x^2-60}-\frac{x^2\left(2x^2-3\right)\sqrt{\frac{x^2-1}{3x^2-4}}\left(2\sqrt{\frac{x^2-x^4}{\left(4-3x^2\right)^2}}-\sin^{-1}\left(\sqrt{\frac{x^2}{4-3x^2}}\right)\right)}{4\left(\frac{x^2}{4-3x^2}\right)^{5/2}\left(x^2-1\right)}\right)}{16\left(1-\frac{3x^2}{4}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] -1/16*(x*Sqrt[-1 + x^2]*(-1/4*(x^2*(-3 + 2*x^2)*Sqrt[(-1 + x^2)/(-4 + 3*x^2
)]*(2*Sqrt[(x^2 - x^4)/(4 - 3*x^2)^2] - ArcSin[Sqrt[x^2/(4 - 3*x^2)]]))/((x
^2/(4 - 3*x^2))^(5/2)*(-1 + x^2)) + (8*x^2*(-1 + x^2)*Hypergeometric2F1[2,
3, 7/2, x^2/(4 - 3*x^2)])/(-60 + 45*x^2)))/(1 - (3*x^2)/4)^2
```

fricas [B] time = 1.53, size = 80, normalized size = 1.86

$$-\frac{12\,x^2+5\left(3\,x^2-4\right)\log\left(3\,x^2-3\,\sqrt{x^2-1}\,x-2\right)-5\left(3\,x^2-4\right)\log\left(x^2-\sqrt{x^2-1}\,x-2\right)+12\,\sqrt{x^2-1}\,x-16}{32\left(3\,x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

giac [B] time = 0.94, size = 94, normalized size = 2.19

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)}-\frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right)+\frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

maple [B] time = 0.05, size = 172, normalized size = 4.00

$$\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}}}{16\left(x+\frac{2\sqrt{3}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)
```

```
[Out] -5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/
2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))+5/32*arctanh(3/2*(2/3+4/3*3^(1/2
)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/2))+
3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^(1/2
))+1/3)^(1/2)-1/16/(x-2/3*3^(1/2))*((x-2/3*3^(1/2))^2+4/3*3^(1/2)*(x-2/3*3^
(1/2))+1/3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-4\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(3\,x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,\left(3x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] 8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*ln
(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 97, 157, 54, 215, 93, 207}

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \left(\frac{8}{3(-1+3x)^2} - \frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2} + \frac{5}{3(-1+3x)}\right) dx$$
$$= \frac{8}{9(1-3x)} + \frac{5}{9}\log(1-3x) - 4\int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}\,dx$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\sqrt{1+x}\,(-1+3x)}\,dx$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx - \frac{10}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}\,(}$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{8}{9}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) - \frac{20}{9}\mathrm{S}$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} - \frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right) + \frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right) + \frac{5}{9}\log(1-3x)$$

Mathematica [A] time = 0.17, size = 126, normalized size = 1.70

$$\frac{12x^{3/2}+12\sqrt{x}-8\sqrt{x+1}+15\sqrt{x+1}\,x\log(1-3x)-5\sqrt{x+1}\,\log(1-3x)+10\sqrt{-x-1}\,(3x-1)\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{-x-1}}\right)}{9\sqrt{x+1}\,(3x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (12*Sqrt[x] + 12*x^(3/2) - 8*Sqrt[1 + x] - 8*Sqrt[1 + x]*(-1 + 3*x)*ArcSinh
[Sqrt[x]] + 10*Sqrt[-1 - x]*(-1 + 3*x)*ArcTan[(2*Sqrt[x])/Sqrt[-1 - x]] - 5
*Sqrt[1 + x]*Log[1 - 3*x] + 15*x*Sqrt[1 + x]*Log[1 - 3*x])/(9*Sqrt[1 + x]*(
-1 + 3*x))
```

fricas [A] time = 0.90, size = 105, normalized size = 1.42

$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\,\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x+1}\,\sqrt{x}-\right.}{9\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
```

```
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters
values [-89]Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,
1] at parameters values [63]2*((-5*(x+1)+4)/6/(3*(x+1)-4)+5/18*ln(abs(3*(x+
1)-4))+2*(1/9*ln((sqrt(x)-sqrt(x+1))^2)+5/36*ln(abs((sqrt(x)-sqrt(x+1))^2-3
))-5/36*ln(abs(3*(sqrt(x)-sqrt(x+1))^2-1))-(10*(sqrt(x)-sqrt(x+1))^2-6)/9/(
3*(sqrt(x)-sqrt(x+1))^4-10*(sqrt(x)-sqrt(x+1))^2+3)))
```

maple [B] time = 0.02, size = 115, normalized size = 1.55

$$\frac{5\ln\left(3x-1\right)}{9}-\frac{\sqrt{x+1}\,\left(-15x\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{(x+1)x}}\right)+12x\ln\left(x+\frac{1}{2}+\sqrt{(x+1)\,x}\right)+5\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{(x+1)x}}\right)-4\ln\left(x\right.\right.}{9\sqrt{(x+1)\,x}\,\left(3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(x+1)^(1/2)*(12*ln(1/2+x+((x+1)*x)^(
1/2))*x-15*arctanh(1/4*(1+5*x)/((x+1)*x)^(1/2))*x-4*ln(1/2+x+((x+1)*x)^(1/2
))+5*arctanh(1/4*(1+5*x)/((x+1)*x)^(1/2))-12*((x+1)*x)^(1/2))/((x+1)*x)^(1/
2)/(3*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\sqrt{x+1}\,+2\,\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

mupad [B] time = 1.71, size = 82, normalized size = 1.11

$$\frac{10\operatorname{atanh}\left(\frac{2662400\sqrt{x}}{81\left(\frac{665600x}{81\left(\sqrt{x+1}-1\right)^2}+\frac{665600}{81}\right)\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\left(x-\frac{1}{3}\right)}+\frac{4\sqrt{x}\sqrt{x+1}}{3\left(3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2^
(1/2)+(x^2-1)^(1/2)/(I-x)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {733, 844, 217, 206, 725, 204}

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}-i\,\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 59, normalized size = 0.92

$$
-\frac{\sqrt{x^2-1}}{x-i}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x^2]/(-I + x)) + ArcTanh[x/Sqrt[-1 + x^2]] - ArcTanh[(I + x)/(S
qrt[2]*Sqrt[-1 + x^2])]/Sqrt[2]
```

fricas [A] time = 0.98, size = 92, normalized size = 1.44

$$-\frac{\sqrt{2}\,(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}\,(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+(2x-2i)\log\left(-x+\sqrt{x^2\cdots}\right.}{2x-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x - I)
*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + (2*x - 2*I)*log(-x + sqrt(x^2 -
1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(2*x - 2*I)
```

giac [A] time = 1.07, size = 84, normalized size = 1.31

$$i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right)+\frac{2\left(ix-i\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2-2ix+2i\sqrt{x^2-1}+1}-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

```
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x^
2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(a
bs(-x + sqrt(x^2 - 1)))
```

maple [B] time = 0.04, size = 125, normalized size = 1.95

$$-\frac{\sqrt{(x-i)^2-2+2i(x-i)}\,x}{2}+\frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2-2+2i(x-i)}}\right)}{2}+\ln\left(x+\sqrt{(x-i)^2-2+2i(x-i)}\right)+\frac{\left((x-i)^2-2+\cdots\right.}{2x-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x)
```

```
[Out] 1/2/(x-I)*((x-I)^2-2+2*I*(x-I))^(3/2)+ln(x+((x-I)^2-2+2*I*(x-I))^(1/2))+1/2
*I*2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)/((x-I)^2-2+2*I*(x-I))^(1/2))-1
/2*I*((x-I)^2-2+2*I*(x-I))^(1/2)-1/2*x*((x-I)^2-2+2*I*(x-I))^(1/2)
```

maxima [A] time = 1.45, size = 53, normalized size = 0.83

$$\frac{1}{2}i\sqrt{2}\arcsin\left(\frac{ix}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {382, 377, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx = -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)} + \frac{3}{4}\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)}\,dx$$
$$= -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$
$$= -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.13, size = 75, normalized size = 1.56

$$\frac{\sqrt{x^2-1}\left(3\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\left(x^2+1\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\right)-2x^2\right)}{8\left(x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^2]*(-2*x^2 + 3*Sqrt[2]*Sqrt[x^2/(-1 + x^2)]*(1 + x^2)*ArcTanh[
Sqrt[2]*Sqrt[x^2/(-1 + x^2)]]))/(8*(x + x^3))
```

fricas [B] time = 1.01, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}\left(x^2+1\right)\log\left(\frac{9\,x^2+2\,\sqrt{2}\left(3\,x^2-1\right)+2\,\sqrt{x^2-1}\left(3\,\sqrt{2}\,x+4\,x\right)-3}{x^2+1}\right)-4\,x^2-4\,\sqrt{x^2-1}\,x-4}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

giac [B] time = 0.96, size = 101, normalized size = 2.10

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$-\frac{x}{8\sqrt{x^2-1}\,\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/8*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))
*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^2+1\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^2-1}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx$

Optimal. Leaf size=30

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \int \left(-\frac{1}{\sqrt{-1+x}} - 2\sqrt{x} + \frac{2x}{\sqrt{-1+x}}\right)dx$$
$$= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \frac{x}{\sqrt{-1+x}}\,dx$$
$$= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \left(\frac{1}{\sqrt{-1+x}} + \sqrt{-1+x}\right)dx$$
$$= 2\sqrt{-1+x} + \frac{4}{3}(-1+x)^{3/2} - \frac{4x^{3/2}}{3}$$

Mathematica [**A**] time = 0.06, size = 30, normalized size = 1.00

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

fricas [**A**] time = 0.89, size = 18, normalized size = 0.60

$$\frac{2}{3}(2x+1)\sqrt{x-1} - \frac{4}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

giac [**A**] time = 0.95, size = 20, normalized size = 0.67

$$\frac{4}{3}(x-1)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.70

$$-\frac{4x^{\frac{3}{2}}}{3}+\frac{4\left(x-1\right)^{\frac{3}{2}}}{3}+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/((x-1)^(1/2)+x^(1/2))^2,x)
```

```
[Out] 4/3*(x-1)^(3/2)-4/3*x^(3/2)+2*(x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

mupad [B] time = 0.38, size = 21, normalized size = 0.70

$$\frac{4\,x\,\sqrt{x-1}}{3}+\frac{2\,\sqrt{x-1}}{3}-\frac{4\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
```

```
[Out] (4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

sympy [B] time = 0.75, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

```
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\,\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{x^2}}{-\sqrt{5}\,x-x+}\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6742, 736, 826, 1166, 207, 203, 1018, 1034, 725, 206, 204, 985}

$$-\frac{2\sqrt{x^2-1}\,(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}\,(1-2x)}{5\left(-x^2+x+1\right)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\tan^{-1}\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int\left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}+\frac{2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{1}{\sqrt{-1+x^2}\left(-1-x+x\right.}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)+2\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int \frac{}{\sqrt{-1}}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}\,dx+\frac{2}{5}\int$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx,x,\sqrt{x}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-1}\left(\frac{2-(1}{\sqrt{2\left(-1+\right.}}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-1+}}\right.$$

Mathematica [A] time = 0.85, size = 340, normalized size = 1.55

$$\frac{2}{5}\left(\frac{\sqrt{x}(1-2x)}{-x^2+x+1}+\frac{\sqrt{x^2-1}(1-2x)}{x^2-x-1}-\frac{1}{2}\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+x-2}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\sqrt{\sqrt{5}-\frac{2}{5}}\tan^{-1}\left(\frac{(\sqrt{5}}{\sqrt{2(\sqrt{5}}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(((1 - 2*x)*Sqrt[x])/(1 + x - x^2) + ((1 - 2*x)*Sqrt[-1 + x^2])/(-1 - x
+ x^2) + Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]]
- (Sqrt[(5*(1 + Sqrt[5]))/2]*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt
[5])]*Sqrt[-1 + x^2])])/2 - Sqrt[-2/5 + Sqrt[5]]*ArcTan[(2 + (-1 + Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[(11 + 5*Sqrt[5])/10]*Ar
cTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] - Sqrt[5/(2*(1 + Sqrt[5]))]*ArcTanh[(-
2 + x + Sqrt[5]*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[2/5 + Sqr
```

```
t[5]]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])]
))/5
```

fricas [B] time = 0.78, size = 424, normalized size = 1.93

$$4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

giac [B] time = 8.76, size = 367, normalized size = 1.67

$$\frac{2}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x+22956\sqrt{5}\sqrt{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1)
 - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(-1
53040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*sqr
t(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sqrt(5
*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 7
6520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) + 76520)
) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/2)) - 1
```

```
/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) + 1/50*sq
rt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x
- sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/(
(x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 -
 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

maple [B] time = 0.14, size = 902, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)
+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*
5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*
5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+
2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arct
an(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(
x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))
+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/(2+2*5^
(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(
1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*
5^(1/2))^(1/2))-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1
/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5
^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*ar
ctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*
(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))
*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-
1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2
)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x^2-1}\,\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\,\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{x^2-}}{-\sqrt{5}x-x+2}\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [B] time = 0.75, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 736, 826, 1166, 207, 203, 975, 1034, 725, 206, 204, 1018, 1065}

$$-\frac{\sqrt{x^2-1}\,(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}\,(1-2x)}{5\left(-x^2+x+1\right)}-\frac{(3-x)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{(x+2)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right.}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
```

```
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \int\left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}-\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\right.$$

$$= -\left(2\int\frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)-\int\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int\frac{x}{\sqrt{-1+x^2}\left(-1\right.}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{25}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}$$

Mathematica [A] time = 0.71, size = 311, normalized size = 1.41

$$\frac{1}{25}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\left(5+2\sqrt{5}\right)\tanh^{-1}\left(\frac{\sqrt{5}x+x-2}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\frac{-20x^{3/2}+20\sqrt{x^2-1}\,x-10\sqrt{x^2-1}+\sqrt{50\sqrt{5}-11}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((10*Sqrt[x] - 20*x^(3/2) - 10*Sqrt[-1 + x^2] + 20*x*Sqrt[-1 + x^2] + Sqrt[
-110 + 50*Sqrt[5]]*(1 + x - x^2)*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]] + S
qrt[10*(1 + Sqrt[5])]*(1 + x - x^2)*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1
 + Sqrt[5])]*Sqrt[-1 + x^2])] + 5*Sqrt[2/(-1 + Sqrt[5])]*(-1 - x + x^2)*Arc
Tan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/(1 + x -
 x^2) - Sqrt[110 + 50*Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] + Sqr
t[2/(1 + Sqrt[5])]*(5 + 2*Sqrt[5])*ArcTanh[(-2 + x + Sqrt[5]*x)/(Sqrt[2*(1
+ Sqrt[5])]*Sqrt[-1 + x^2])])/25
```

fricas [B] time = 0.61, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

giac [B] time = 9.22, size = 358, normalized size = 1.63

$$\frac{2}{5}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1) - 1)/
sqrt(2*sqrt(5) - 2)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*
sqrt(5) - 1/2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5
) + 1/2)) - 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-520*x - 78*sqrt(5)*sqrt(
50*sqrt(5) + 110) + 260*sqrt(5) + 520*sqrt(x^2 - 1) + 130*sqrt(50*sqrt(5) +
 110) + 260)) + 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-1040*x + 156*sqrt(5)
*sqrt(50*sqrt(5) + 110) + 520*sqrt(5) + 1040*sqrt(x^2 - 1) - 260*sqrt(50*sq
rt(5) + 110) + 520)) + 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1
```

```
/2*sqrt(5) + 1/2))) + 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2
+ 3*x - 3*sqrt(x^2 - 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))
^3 - 2*(x - sqrt(x^2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2)
- sqrt(x))/(x^2 - x - 1)
```

maple [B] time = 0.02, size = 1542, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -8/25/(-2+2*5^(1/2))^(1/2)*5^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x^(1/2))-8
/25/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x^(1/2))*5^(1/2)-1/10
/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*
(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+4/25*5^(1/2)/(2+2*5^(1/2))^(1/2)
*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/
(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/
2))-1/10/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1
/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+4/25*5^(1/2)/(-2+2*5^(1/2
))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2
))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^
(1/2))^(1/2))+4/5/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x^(1/2
))+2/5/(x-1/2*5^(1/2)-1/2)*x^(1/2)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5
^(1/2))^(1/2)*x^(1/2))+1/20/(1/2-1/2*5^(1/2))*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-
5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)-2/5/(-2+2*5^(1/2))^(1/2)*
arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/
(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1
/2))+2/5/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5
^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+1/25*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5
^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/25*ln(x+((x+1/2*5^(1/2)-
1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5^(1/2)-1/2
5*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^
(1/2))^(1/2)-1/25*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/
2)+1/2+1/2*5^(1/2))^(1/2))*5^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/2*5^(1
/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))+1/20/(1/
2+1/2*5^(1/2))*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2
+2*5^(1/2))^(1/2)+1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1
/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+2/5/(x+1/2*5^(1/2)-1/2)*
x^(1/2)+1/5/(1/2-1/2*5^(1/2))*x*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*
5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/
2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^
(3/2)-1/20/(1/2-1/2*5^(1/2))*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1
)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)-1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/
```

```
2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5
^(1/2)+1/20/(1/2+1/2*5^(1/2))*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1
)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/
2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))*5^
(1/2)+1/5/(1/2+1/2*5^(1/2))*x*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(
1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*
((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(3
/2)-1/5*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/
2*5^(1/2))^(1/2))-1/5*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2)+1/2+1/2*5^(1/2))^(1/2))+1/10*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2
)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1
/2))^(1/2)-1/10*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/
2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(x^{\frac{5}{2}}-3\,x^{\frac{3}{2}}\right)}{5\left(x^{2}-x-1\right)}+\int \frac{x^{\frac{3}{2}}+\sqrt{x}}{5\left(x^{2}-x-1\right)}\,dx+\int \frac{x^{2}+x-1}{\left(x^{4}-2\,x^{3}-x^{2}+2\,x+1\right)\sqrt{x+1}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\,\left(-x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
```

```
[Out] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
[Out] Timed out
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] 1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arcta
nh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x^2
)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {731, 725, 206}

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int\left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx=\frac{\int\frac{1}{(1+x)^2\sqrt{-i+x^2}}\,dx}{\sqrt{2}}+\frac{\int\frac{1}{(1+x)^2\sqrt{i+x^2}}\,dx}{\sqrt{2}}$$
$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\int\frac{1}{(1+x)\sqrt{}}}{\sqrt{2}}$$
$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\mathrm{Subst}}{}$$
$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-}}\right)}{(1-i)^{3/2}\sqrt{2}}$$

Mathematica [A] time = 0.20, size = 125, normalized size = 0.91

$$\frac{i\left((1+i)\left(i\sqrt{x^2-i}+\sqrt{x^2+i}\right)+\sqrt{1-i}\,(x+1)\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)+\sqrt{1+i}\,(x+1)\tanh^{-1}\left(\frac{(1+i)^{3/2}(1+ix)}{2\sqrt{x^2+i}}\right)\right)}{2\sqrt{2}\,(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((I/2)*((1 + I)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2]) + Sqrt[1 - I]*(1 + x)*Ar
cTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])] + Sqrt[1 + I]*(1 + x)*ArcTanh[(
(1 + I)^(3/2)*(1 + I*x))/(2*Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

fricas [A] time = 1.06, size = 161, normalized size = 1.17

$$\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,\left(-(i-1)\,x-i+1\right)\log\left(\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,\left((i-1)\,x+i-1\right)\log\left(-\sqrt{2}\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [B] time = 0.03, size = 278, normalized size = 2.01

$$-\frac{\sqrt{2}\,\ln\left(\frac{-2x-2i+2\sqrt{1-i}\,\sqrt{-2x+(x+1)^{2}-1-i}}{x+1}\right)}{4\sqrt{1-i}}-\frac{i\sqrt{2}\,\ln\left(\frac{-2x-2i+2\sqrt{1-i}\,\sqrt{-2x+(x+1)^{2}-1-i}}{x+1}\right)}{4\sqrt{1-i}}-\frac{\sqrt{2}\,\ln\left(\frac{-2x+2i+2\sqrt{1+i}\,\sqrt{-2x+(x+1)^{2}}}{x+1}\right.}{4\sqrt{1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x+1)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(x+1)^2*2^(1/2)/(I+x^2)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/(x+1)*((x+1)^2-2*x-1-I)^(1/2)-1/4*I*2^(1/2)/(x+1)*((x+1)^2-2*x
-1-I)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((x+1)^2-2*x
-1-I)^(1/2))/(x+1))-1/4*I*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((
x+1)^2-2*x-1-I)^(1/2))/(x+1))-1/4*2^(1/2)/(x+1)*((x+1)^2-2*x-1+I)^(1/2)+1/4
*I*2^(1/2)/(x+1)*((x+1)^2-2*x-1+I)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*
x+2*(1+I)^(1/2)*((x+1)^2-2*x-1+I)^(1/2))/(x+1))+1/4*I*2^(1/2)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((x+1)^2-2*x-1+I)^(1/2))/(x+1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2}}{2\sqrt{x^2 - \mathrm{i}}\,(x+1)^2} + \frac{\sqrt{2}}{2\sqrt{x^2 + 1\mathrm{i}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2),x)
```

```
[Out] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2), x)
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3/
2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)-1
/2*(1+I*x^2)^(1/2)/(1+x)
```

Rubi [A] time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 206}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{}{(1+}\right.\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{1}}\right.\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]
```

fricas [B] time = 4.55, size = 394, normalized size = 3.15

$$\frac{4\,(x+1)\sqrt{\sqrt{2}+1}\,\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}\left(x^3+1\right)+\sqrt{x^4+1}\left(\sqrt{2}\,x-x-1\right)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{\sqrt{2}+1}+\left(2\,x^2-\sqrt{2}\left(x^2+1\right)+2\,\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right.}{2\left(x^2-2\,x+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}\,(x + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)^2\,\sqrt{x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)^2/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}\,(x + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh((
1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2133, 725, 206}

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2 - (Sqrt[1
 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx$$
$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{1-ix}{\sqrt{1+ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx\right.$$
$$= -\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]
```

fricas [B] time = 10.16, size = 369, normalized size = 4.56

$$\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(2x^2-\sqrt{2}\left(x^3-x^2+x+1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}}{2\left(x^2-2x+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
```

```
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

fricas [B] time = 1.22, size = 60, normalized size = 1.94

$$\frac{1}{4}\sqrt{2}\,\log\left(4\,x^4+4\,\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}\,x^3+\sqrt{2}\,\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^{4}+1}+x^{2}}}{\sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 1.22, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{1}{2} \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4 \sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

fricas [A] time = 1.62, size = 29, normalized size = 0.88

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [C] time = 0.10, size = 22, normalized size = 0.67

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 0.86, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6688}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right)dx\\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

fricas [A] time = 0.99, size = 28, normalized size = 1.47

$$-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}\,(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

giac [A] time = 1.15, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)^(3/2)+(x+1)^(3/2))/(x-1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] -2/(x-1)^(1/2)-2/(x+1)^(1/2)
```

maxima [A] time = 0.63, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

mupad [B] time = 0.40, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

sympy [B] time = 6.63, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(x+\sqrt{a+x^2}\right)^b\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right)dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right)dx, x, x+\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b-1}\left((b-1)x\left(\sqrt{a+x^2}+x\right)+ab\right)}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

fricas [A] time = 1.38, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

maple [B] time = 0.03, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(b+\frac{ab}{x^2}-1\right)a^{-\frac{b}{2}-\frac{1}{2}}x^{b+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{b-1}}{(b+1)(2b-2)b}+\frac{4\sqrt{\pi}\sqrt{\frac{a}{x^2}+1}\,a^{-\frac{b}{2}-\frac{1}{2}}x^{b+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{b-1}}{(b+1)b}\right)b\,a^{\frac{b}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b,x)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/
x^2*a*b+b-1)/(-2+2*b)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+
b)*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x+\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^b, x)
```

sympy [B] time = 3.53, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a
)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2)
```

```
 + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) +
a**(9/2)*a**(b/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**(7/2)*a**(b/2)*b**2*x**3*sqrt
(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
2*a**(7/2)*x**2*gamma(1 - b/2)) + a**(7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/s
qrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 -
 b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2
)) + 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(
7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**5*a*
*(b/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(
1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 -
b/2)) - 2*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2
)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*ga
mma(1 - b/2)) + 4*a**4*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**4*a**(b/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) - 2*a**4*a**(b/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2
*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 -
b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*
a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*
x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9
/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*
gamma(1 - b/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/
a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b
**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) + a**(5/2)*a**(b/2)*b*x*cos
h(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5
/2)*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2)*x*sqrt(1 + x**2/a)*sinh(b*asinh(x
```

```
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2
) - 2*a**(5/2)*gamma(1 - b/2)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b
*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)
*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
 b/2)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)
) + 2*a**2*a**(b/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)), True
))
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b\,dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{b-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{b+1}+\frac{a}{b-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

fricas [A] time = 1.17, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b,x)
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x-\sqrt{x^2+a}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b,x)
[Out] int((x - (a + x^2)^(1/2))^b, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{a+x^2}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

```
[Out] (x+(x^2+a)^(1/2))^b/b
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \text{Subst}\left(\int x^{-1+b}\,dx, x, x+\sqrt{a+x^2}\right) = \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^b/b
```

fricas [A] time = 1.09, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + a))^b/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

mupad [B] time = 0.30, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
[Out] (x + (a + x^2)^(1/2))^b/b
```

sympy [B] time = 2.99, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}\,a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b} - \frac{a^{\frac{b}{2}}x\sinh\left(-b\operatorname{asin}\right.}{\sqrt{a}} \\ -\frac{a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} - \frac{a^{\frac{b}{2}}x^2\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\right.}{\sqrt{}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/
(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*b) - a**(b/2)*x*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(b/2)*sinh(-b*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/
sqrt(a)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(-b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -(x-(x^2+a)^(1/2))^b/b
```

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\operatorname{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right) = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^b/b)
```

fricas [A] time = 0.62, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^b/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{b}}{\sqrt{x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{b}}{\sqrt{x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

mupad [B] time = 0.30, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^b/b
```

sympy [A] time = 1.60, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^{2}}\right)^{b}}{b} & \text{for}\: b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for}\: a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for}\: a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

3.21 $\int \frac{1}{(a+be^{px})^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\log(a+be^{px})}{a^2p}+\frac{x}{a^2}+\frac{1}{ap(a+be^{px})}$$

```
[Out] 1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 44}

$$-\frac{\log(a+be^{px})}{a^2p}+\frac{x}{a^2}+\frac{1}{ap(a+be^{px})}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{px}\right)}{p}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx, x, e^{px}\right)}{p}$$
$$= \frac{1}{a(a+be^{px})p} + \frac{x}{a^2} - \frac{\log(a+be^{px})}{a^2p}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.86

$$\frac{\frac{a}{a+be^{px}} - \log(a+be^{px}) + px}{a^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
```

```
[Out] (a/(a + b*E^(p*x)) + p*x - Log[a + b*E^(p*x)])/(a^2*p)
```

fricas [A] time = 1.11, size = 52, normalized size = 1.24

$$\frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

giac [A] time = 0.99, size = 47, normalized size = 1.12

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(px)}+a}+1\right|\right)}{a^2b} + \frac{1}{\left(be^{(px)}+a\right)ab}\right)}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*e^(p*x) + a) + 1))/(a^2*b) + 1/((b*e^(p*x) + a)*a*b))/p
```

maple [A] time = 0.01, size = 48, normalized size = 1.14

$$\frac{1}{(b\,\mathrm{e}^{px}+a)\,ap}-\frac{\ln\left(b\,\mathrm{e}^{px}+a\right)}{a^{2}p}+\frac{\ln\left(\mathrm{e}^{px}\right)}{a^{2}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(p*x))^2,x)
```

```
[Out] -ln(a+b*exp(p*x))/a^2/p+1/a/(a+b*exp(p*x))/p+1/p/a^2*ln(exp(p*x))
```

maxima [A] time = 0.59, size = 40, normalized size = 0.95

$$\frac{x}{a^{2}}+\frac{1}{\left(abe^{(px)}+a^{2}\right)p}-\frac{\log\left(be^{(px)}+a\right)}{a^{2}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

mupad [B] time = 0.41, size = 58, normalized size = 1.38

$$\frac{\frac{x}{a}+\frac{b\,x\,\mathrm{e}^{p\,x}}{a^{2}}-\frac{b\,\mathrm{e}^{p\,x}}{a^{2}\,p}}{a+b\,\mathrm{e}^{p\,x}}-\frac{\ln\left(a+b\,\mathrm{e}^{p\,x}\right)}{a^{2}\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(p*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
+ b*exp(p*x))/(a^2*p)
```

sympy [A] time = 0.15, size = 36, normalized size = 0.86

$$\frac{1}{a^{2}p+abpe^{px}}+\frac{x}{a^{2}}-\frac{\log\left(\frac{a}{b}+e^{px}\right)}{a^{2}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] -1/2/a/(b+a*exp(2*p*x))/p
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 261}

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\left(b+ax^2\right)^2}\,dx, x, e^{px}\right)}{p}\\ &= -\frac{1}{2a\left(b+ae^{2px}\right)p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

fricas [A] time = 1.25, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(a^2pe^{(2\,px)}+abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

giac [A] time = 0.78, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(ae^{(2\,px)}+b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$-\frac{1}{2\left(a\,\mathrm{e}^{2px}+b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

maxima [A] time = 0.60, size = 20, normalized size = 0.91

$$\frac{1}{2\left(b^2e^{(-2\,px)}+ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

mupad [B] time = 0.39, size = 24, normalized size = 1.09

$$\frac{e^{2\,p\,x}}{2\,b\,p\,\left(b+a\,e^{2\,p\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

sympy [A] time = 0.12, size = 20, normalized size = 0.91

$$\frac{1}{2abp+2b^2pe^{-2px}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] 1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] 1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
```

```
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2283

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx &= \int \frac{e^{2px}x}{\left(b+ae^{2px}\right)^2}\,dx\\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap}\\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\mathrm{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2}\\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\mathrm{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2}\\
&= \frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.79

$$
\frac{\frac{2pxe^{2px}}{ae^{2px}+b}-\frac{\log\left(ae^{2px}+b\right)}{a}}{4bp^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

fricas [A] time = 0.74, size = 58, normalized size = 0.94

$$\frac{2\,apxe^{\left(2\,px\right)}-\left(ae^{\left(2\,px\right)}+b\right)\log\left(ae^{\left(2\,px\right)}+b\right)}{4\left(a^{2}bp^{2}e^{\left(2\,px\right)}+ab^{2}p^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)
```

giac [A] time = 1.21, size = 74, normalized size = 1.19

$$\frac{2\,apxe^{\left(2\,px\right)}-ae^{\left(2\,px\right)}\log\left(-ae^{\left(2\,px\right)}-b\right)-b\log\left(-ae^{\left(2\,px\right)}-b\right)}{4\left(a^{2}bp^{2}e^{\left(2\,px\right)}+ab^{2}p^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{x\,e^{2px}}{2\left(a\,e^{2px}+b\right)bp}-\frac{\ln\left(a\,e^{2px}+b\right)}{4ab\,p^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/4/p^2/b/a*ln(a*exp(p*x)^2+b)+1/2/p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b)
```

maxima [A] time = 0.59, size = 51, normalized size = 0.82

$$\frac{xe^{\left(2\,px\right)}}{2\left(abpe^{\left(2\,px\right)}+b^{2}p\right)}-\frac{\log\left(\frac{ae^{\left(2\,px\right)}+b}{a}\right)}{4\,abp^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

mupad [B] time = 0.41, size = 47, normalized size = 0.76

$$\frac{x\,\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b+a\,\mathrm{e}^{2\,p\,x}\right)}-\frac{\ln\left(b+a\,\mathrm{e}^{2\,p\,x}\right)}{4\,a\,b\,p^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] (x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p^
2)
```

sympy [A] time = 0.18, size = 51, normalized size = 0.82

$$\frac{x}{2abp+2b^2pe^{-2px}}-\frac{x}{2abp}-\frac{\log\left(\frac{a}{b}+e^{-2px}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/(
4*a*b*p**2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

```
[Out] arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2)
/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
```

```
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,\left(1+x+x^2\right)^2}\,dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\,\left(1+x+x^2\right)}\,dx$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\,\left(1+x+x^2\right)}\,dx - \frac{1}{48}\int \frac{-48}{\sqrt{1-x+x^2}}$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24\,\text{Subst}\left(\int \frac{1}{1728-2x^2}\,dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) + 288\,\text{Su}$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

Mathematica [C] time = 2.52, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1} + \frac{\left(7-i\sqrt{3}\right)\tan^{-1}\left(\frac{3\left(\left(-21-4i\sqrt{3}\right)x^4+14\left(7-2i\sqrt{3}\right)x^3+\left(-103-36i\right.\right.}{\left(84i-113\sqrt{3}\right)x^4+2\left(52\sqrt{3-3i\sqrt{3}}\,\sqrt{x^2-x+1}+21\sqrt{3}+138i\right)x^3+\left(52\sqrt{3-3i\sqrt{3}}\,\sqrt{x^2-x+1}-59\sqrt{3}-18\right.}\right.}{4\sqrt{3-3i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + ((7 - I*Sqrt[3])*ArcTan[(3*(-17
 - (64*I)*Sqrt[3] + (94 + (32*I)*Sqrt[3])*x + (-103 - (36*I)*Sqrt[3])*x^2 +
 14*(7 - (2*I)*Sqrt[3])*x^3 + (-21 - (4*I)*Sqrt[3])*x^4))/(96*I + 67*Sqrt[3
] + (84*I - 113*Sqrt[3])*x^4 - 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]
 + 2*x*(132*I - 69*Sqrt[3] + 26*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ x^2*(-180*I - 59*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ 2*x^3*(138*I + 21*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
)])/(4*Sqrt[3 - (3*I)*Sqrt[3]]) - ((I/4)*(-7*I + Sqrt[3])*ArcTan[(3*(-17 +
(64*I)*Sqrt[3] + (94 - (32*I)*Sqrt[3])*x + (-103 + (36*I)*Sqrt[3])*x^2 + 14
*(7 + (2*I)*Sqrt[3])*x^3 + (-21 + (4*I)*Sqrt[3])*x^4))/(96*I - 67*Sqrt[3] +
 (84*I + 113*Sqrt[3])*x^4 + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] +
x^2*(-180*I + 59*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) +
x*(264*I + 138*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) - 2*
x^3*(-138*I + 21*Sqrt[3] + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]))])
/Sqrt[3 + (3*I)*Sqrt[3]] - ((-7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sq
rt[3 + (3*I)*Sqrt[3]]) - ((7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sqrt[
```

```
3 - (3*I)*Sqrt[3]]) + ((7*I + Sqrt[3])*Log[(1 + x + x^2)*(11*I + 4*Sqrt[3]
+ (11*I + 4*Sqrt[3])*x^2 + (10*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2] - x
*(17*I + 4*Sqrt[3] + (8*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(8*Sqr
t[3 - (3*I)*Sqrt[3]]) + ((-7*I + Sqrt[3])*Log[(1 + x + x^2)*(-11*I + 4*Sqrt
[3] + (-11*I + 4*Sqrt[3])*x^2 - (10*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2
] + x*(17*I - 4*Sqrt[3] + (8*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(
8*Sqrt[3 + (3*I)*Sqrt[3]])
```

fricas [B] time = 1.01, size = 358, normalized size = 4.16

$$-\frac{8\sqrt{6}\sqrt{3}\left(x^2+x+1\right)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}\,(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}\left(2x-\sqrt{6}+1\right)-\sqrt{6}\,(x+1)+4}\left(\sqrt{6}\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(2/3*sqrt(6)*sqrt(3)*(x - 1) +
 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) +
 4)*(sqrt(6)*sqrt(3) + 3*sqrt(3)) - 2/3*sqrt(x^2 - x + 1)*(sqrt(6)*sqrt(3)
+ 3*sqrt(3)) + sqrt(3)*(2*x - 1)) + 8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(
2/3*sqrt(6)*sqrt(3)*(x - 1) + 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqr
t(6) + 1) + sqrt(6)*(x + 1) + 4)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - 2/3*sqrt(x
^2 - x + 1)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - sqrt(3)*(2*x - 1)) - sqrt(6)*(x
^2 + x + 1)*log(12168*x^2 - 6084*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 60
84*sqrt(6)*(x + 1) + 24336) + sqrt(6)*(x^2 + x + 1)*log(12168*x^2 - 6084*sq
rt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 6084*sqrt(6)*(x + 1) + 24336) - 12*x^
2 - 12*sqrt(x^2 - x + 1)*(x + 1) - 12*x - 12)/(x^2 + x + 1)
```

giac [B] time = 1.06, size = 304, normalized size = 3.53

$$-\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x+\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}+\sqrt{2}}\right)+\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x-\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sqr
t(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^2
- x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3) +
3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*sqrt(
6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(x^2 -
```

```
 x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x + 1))^2
- 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x - sqrt
(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x + 1) +
3)
```

maple [B] time = 0.04, size = 455, normalized size = 5.29

$$-\frac{-\frac{6\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}\,(x+1)^2\operatorname{arctanh}\left(\frac{\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}}{4}\right)}{(-x+1)^2}-2\sqrt{6}\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}}{4}\right)+\frac{9\sqrt{2}\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,(x+1)^2 \cdots}{(-x \cdots}}{6\sqrt{\frac{\frac{(x+1)^2}{(-x+1)^2}+3}{\left(\frac{x+1}{-x+1}+1\right)^2}}\left(\frac{x+1}{-x+1}+1\right)\left(\frac{3(x+\cdots}{(-x+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)
```

```
[Out] -1/6*(9*2^(1/2)*((x+1)^2/(-x+1)^2+3)^(1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)
^2+3)^(1/2)*(x+1)/(-x+1))*(x+1)^2/(-x+1)^2-6*((x+1)^2/(-x+1)^2+3)^(1/2)*arc
tanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(1/2))*6^(1/2)*(x+1)^2/(-x+1)^2+3*2^(
1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)^2+3)^(1/2)*(x+1)/(-x+1))*((x+1)^2/(-x
+1)^2+3)^(1/2)-2*6^(1/2)*arctanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(1/2))*((
x+1)^2/(-x+1)^2+3)^(1/2)-12*(x+1)^3/(-x+1)^3-36*(x+1)/(-x+1))/(((x+1)^2/(-x
+1)^2+3)/((x+1)/(-x+1)+1)^2)^(1/2)/((x+1)/(-x+1)+1)/(3*(x+1)^2/(-x+1)^2+1)+
1/2*((x+1)^2/(-x+1)^2+3)^(1/2)*(3*2^(1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)^
2+3)^(1/2)*(x+1)/(-x+1))-6^(1/2)*arctanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(
1/2)))/((x+1)/(-x+1)+1)/(((x+1)^2/(-x+1)^2+3)/((x+1)/(-x+1)+1)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2 - x + 1}{\left(x^2 + x + 1\right)^2 \sqrt{x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2 - x + 1}{\sqrt{x^2 - x + 1}\,\left(x^2 + x + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
```

```
[Out] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 - x + 1}{\sqrt{x^2 - x + 1}\,\left(x^2 + x + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

Optimal. Leaf size=19

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2122, 30}

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}} \, dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

fricas [A] time = 1.02, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

giac [A] time = 0.79, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

mupad [B] time = 0.42, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] 2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

sympy [A] time = 0.23, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

```
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2122, 30}

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}$$
$$= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

<u>**Mathematica**</u> [**A**]   time = 0.02, size = 26, normalized size = 1.00

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

<u>**fricas**</u> [**A**]   time = 0.99, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

<u>**giac**</u> [**A**]   time = 0.98, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

mupad [B] time = 0.51, size = 22, normalized size = 0.85

$$\frac{2\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

```
[Out] (2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

sympy [A] time = 1.07, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2120, 329, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.20, size = 56, normalized size = 0.89

$$-\frac{2\left(\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

fricas [A] time = 1.09, size = 198, normalized size = 3.14

$$\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{x+\sqrt{a^2+x^2}}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

sympy [C] time = 1.47, size = 46, normalized size = 0.73

$$-\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\,\text{Subst}\left(\int \frac{1}{a+\cdots}\right.\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 127, normalized size = 1.55

$$-\frac{2\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] (-2*Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqr
t[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[x + S
qrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
```

fricas [A] time = 0.77, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

maple [C] time = 0.01, size = 25, normalized size = 0.30

$$2\sqrt{2}\,\sqrt{x}\ \mathrm{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

sympy [C] time = 1.32, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

3.29 $\int x^3 \log^3(2+x) \log(3+x) \, dx$

Optimal. Leaf size=606

$$-\frac{5609}{96} \text{PolyLog}(2,-x-2)-\frac{563}{8} \text{PolyLog}(3,-x-2)-\frac{195}{2} \text{PolyLog}(4,-x-2)-\frac{195}{4} \log^2(x+2) \text{PolyLog}(2,-x-2)+$$

```
[Out] -302177/1152*x+3/256*x^4+8029/2304*x^2-763/3456*x^3+3891/128*ln(3+x)+2069/1
44*ln(2+x)-43/12*ln(2+x)^2+377/64*(2+x)^2-71/216*(2+x)^3+3/256*(2+x)^4-5609
/96*polylog(2,-2-x)-563/8*polylog(3,-2-x)-195/2*polylog(4,-2-x)+1/2*x^3*ln(
2+x)^2*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)+1/4*x^4*ln(2+x)^3*ln(3+x)-25*x*ln
(2+x)*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-7/12*x^3*ln(2+x)*ln(3+x)+3/32*x^4*ln
(2+x)*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-3/2*x^2*ln(2+x)^2*ln(3+x)-81/4*ln(2+x)^
3*ln(3+x)+563/8*ln(2+x)*polylog(2,-2-x)-195/4*ln(2+x)^2*polylog(2,-2-x)+195
/2*ln(2+x)*polylog(3,-2-x)-187/64*x^2*ln(2+x)+83/288*x^3*ln(2+x)-3/128*x^4*
ln(2+x)+6733/32*(2+x)*ln(2+x)-377/32*(2+x)^2*ln(2+x)+71/72*(2+x)^3*ln(2+x)-
3/64*(2+x)^4*ln(2+x)-17/48*x^3*ln(2+x)^2+3/64*x^4*ln(2+x)^2-1251/16*(2+x)*l
n(2+x)^2+273/32*(2+x)^2*ln(2+x)^2-3/4*(2+x)^3*ln(2+x)^2+3/64*(2+x)^4*ln(2+x
)^2+65/4*(2+x)*ln(2+x)^3-33/8*(2+x)^2*ln(2+x)^3+3/4*(2+x)^3*ln(2+x)^3-1/16*
(2+x)^4*ln(2+x)^3-115/48*x^2*ln(3+x)+37/144*x^3*ln(3+x)-3/128*x^4*ln(3+x)+4
15/12*(3+x)*ln(3+x)-4083/32*ln(2+x)*ln(3+x)+963/16*ln(2+x)^2*ln(3+x)
```

Rubi [A] time = 4.80, antiderivative size = 679, normalized size of antiderivative = 1.12, number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.143, Rules used = {2439, 2416, 2389, 2296, 2295, 2401, 2390, 2305, 2304, 2396, 2433, 2374, 2383, 6589, 2411, 2346, 2302, 30, 2330, 2319, 43, 2334, 2301, 6742, 2430, 2393, 2391, 2394, 2395, 2398}

$$-\frac{5609}{96}\text{PolyLog}(2,-x-2)-\frac{563}{8}\text{PolyLog}(3,-x-2)-\frac{195}{2}\text{PolyLog}(4,-x-2)-\frac{195}{4}\log^2(x+2)\text{PolyLog}(2,-x-2)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6365*(2 + x)*Log[2 + x])/32 - (273*(2 + x)^2*Log[2 + x])/32 + ((2 +
x)^3*Log[2 + x])/2 - (3*(2 + x)^4*Log[2 + x])/128 + ((384*(2 + x) - 144*(2
+ x)^2 + 32*(2 + x)^3 - 3*(2 + x)^4 - 192*Log[2 + x])*Log[2 + x])/128 + (17
*(36*(2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*Log[2 + x])*Log[2 + x])/72 + (4
3*Log[2 + x]^2)/12 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (
1251*(2 + x)*Log[2 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x
)^3*Log[2 + x]^2)/4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x
```

```
]^3)/4 - (33*(2 + x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2
 + x)^4*Log[2 + x]^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48
+ (37*x^3*Log[3 + x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x
])/12 - (4083*Log[2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*
x^2*Log[2 + x]*Log[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Lo
g[2 + x]*Log[3 + x])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]
^2*Log[3 + x] - (3*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3
 + x])/2 - (3*x^4*Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x]
)/4 + (x^4*Log[2 + x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563
*Log[2 + x]*PolyLog[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4
 - (563*PolyLog[3, -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (19
5*PolyLog[4, -2 - x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
```

```
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \frac{x^4\log^3(2+x)}{3+x}\,dx - \frac{3}{4}\int \frac{x^4\log^2(2+x)\log(3}{2+x}$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int\Big(-27\log^3(2+x) + 9x\log^3(2+x) - 3x^2\log^3$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,dx -$$

$$= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\log(3$$

$$= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x)$$

$$= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) -$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{81}{4}(2+$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{765}{16}(2+$$

$$= -\frac{765x}{8} + \frac{27}{32}(2+x)^2 - \frac{1}{6}(2+x)^3 + \frac{3}{512}(2+x)^4 + \frac{765}{8}(2+x)\log(2+x) - \frac{27}{16}($$

$$= -\frac{857x}{8} + \frac{79}{32}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{83}{288}x^3$$

$$= -\frac{16463x}{96} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{83}{288}$$

$$= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+$$

Mathematica [A] time = 0.26, size = 412, normalized size = 0.68

$$\frac{-224640\,\mathrm{PolyLog}(4,-x-2) - 24\left(4680\log^2(x+2) - 6756\log(x+2) + 5609\right)\mathrm{PolyLog}(2,-x-2) + 288(780\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3 \log(x+3)\log(x+2)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log(x+3)\log(x+2)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^3 \ln(x+2)^3 \ln(x+3) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x+2)^3*ln(x+3),x)
```

```
[Out] int(x^3*ln(x+2)^3*ln(x+3),x)
```

maxima [A] time = 0.56, size = 518, normalized size = 0.85

$$\frac{3}{128}\,x^4+\frac{1}{16}\left(4\,x^4\log\left(x+3\right)-x^4+4\,x^3-18\,x^2+108\,x-324\,\log\left(x+3\right)\right)\log\left(x+2\right)^3-\frac{65}{4}\,\log\left(x+3\right)\log\left(x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\ln\left(x+2\right)^3\,\ln\left(x+3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x + 2)^3*log(x + 3),x)
```

```
[Out] int(x^3*log(x + 2)^3*log(x + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

```
[Out] (x+(x^2+b)^(1/2))^a/a
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)$$
$$= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Mathematica [**A**] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
[Out] (x + Sqrt[b + x^2])^a/a
```

fricas [**A**] time = 0.88, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + b))^a/a
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(x + \sqrt{x^2 + b}\right)^a}{\sqrt{x^2 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x + \sqrt{x^2 + b}\right)^a}{\sqrt{x^2 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

mupad [B] time = 0.31, size = 15, normalized size = 0.88

$$\frac{\left(x + \sqrt{x^2 + b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
[Out] (x + (b + x^2)^(1/2))^a/a
```

sympy [B] time = 2.58, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{b}\, b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a x \sqrt{\frac{b}{x^2} + 1}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b}} - \frac{b^{\frac{a}{2}} x \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b} \sqrt{\frac{b}{x^2} + 1}} - \frac{2 b^{\frac{a}{2}} \cosh \\ -\frac{b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{1 + \frac{x^2}{b}}} - \frac{b^{\frac{a}{2}} x^2 \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a b \sqrt{1 + \frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b}} - \frac{2 b^{\frac{a}{2}} \cosh(\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)

[Out] Piecewise((-sqrt(b)*b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(
a*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(
b)))/(a*sqrt(b)) - b**(a/2)*x*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/
(a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1
- a/2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (-b**(a/2)*sinh(-a*asinh(x/sqr
t(b)) + asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) - b**(a/2)*x**2*sinh(-a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(-
a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asin
h(x/sqrt(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

```
[Out] -1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{b+x^2}\right)^a dx = \frac{1}{2}\operatorname{Subst}\left(\int x^{-2+a}\left(b+x^2\right) dx, x, x+\sqrt{b+x^2}\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(bx^{-2+a}+x^a\right) dx, x, x+\sqrt{b+x^2}\right)$$
$$= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)} + \frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{b+x^2}+x\right)^{a-1}\left(\frac{\left(\sqrt{b+x^2}+x\right)^2}{a+1}+\frac{b}{a-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

fricas [A] time = 1.03, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+b}\,a-x\right)\left(x+\sqrt{x^2+b}\right)^a}{a^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x+\sqrt{x^2+b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

maple [B] time = 0.03, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(a+\frac{ab}{x^2}-1\right)b^{-\frac{a}{2}-\frac{1}{2}}x^{a+1}\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}}{(a+1)(2a-2)a}+\frac{4\sqrt{\pi}\,\sqrt{\frac{b}{x^2}+1}\,b^{-\frac{a}{2}-\frac{1}{2}}x^{a+1}\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}}{(a+1)a}\right)a\,b^{\frac{a}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a,x)
```

```
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(a+1)/a*x^(a+1)*b^(-1/2*a-1/2)*(a*
b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(a+1)/a*x^(a+1)*b
^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x+\sqrt{x^2+b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a,x)
```

```
[Out] int((x + (b + x^2)^(1/2))^a, x)
```

sympy [B] time = 2.71, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
```

```
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
a**2*b**(7/2)*b**(a/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(9/2
)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/s
qrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2
*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) + 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**5*
b**(a/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x
**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 -
a/2)) - 2*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + a
sinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b*
*(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*ga
mma(1 - a/2)) + 4*a*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sq
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x*
*2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a
/2)) - 2*a*b**4*b**(a/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(
7/2)*x**2*gamma(1 - a/2)) - 2*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*as
inh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(
b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*
a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*
x**2*gamma(1 - a/2)) - 2*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x/sqrt(b)) +
 asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)), Abs(x**2/b) > 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*si
nh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(
5/2)*gamma(1 - a/2)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 -
a/2) - 2*b**(5/2)*gamma(1 - a/2)) + a*b**(5/2)*b**(a/2)*x*cosh(a*asinh(x/sq
```

```
rt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 -
a/2)) + 2*a*b**3*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + 2*
a*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 -
a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*b**(5
/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))
*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)
) + 2*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(
1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)), True
))
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx$

Optimal. Leaf size=34

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

[Out] 1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1594, 1747}

$$\frac{x^{a+1}\left(3x^a+2x^{2a}+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]

[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1747

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(d*(g*x)^(
m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*g*(m + 1)), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx=\int x^a\left(1+x^a+x^{2a}\right)\left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\,dx$$
$$=\frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}$$

Mathematica [A] time = 0.13, size = 33, normalized size = 0.97

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6a+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

fricas [A] time = 0.94, size = 48, normalized size = 1.41

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a}\right)\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}\left(x^{3\,a}+x^{2\,a}+x^{a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

maple [A] time = 0.05, size = 44, normalized size = 1.29

$$\frac{\left(3x^{a}+2x^{2a}+6\right)x\,x^{a}\left(3x^{a}+2x^{2a}+6\right)^{\frac{1}{a}}}{6a+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(a+1)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

maxima [A] time = 0.90, size = 48, normalized size = 1.41

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a}\right)\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(x^{a}+x^{2\,a}+x^{3\,a}\right)\,\left(3\,x^{a}+2\,x^{2\,a}+6\right)^{1/a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
```

```
[Out] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right)\\ &= -\frac{\log(x)}{2}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\ &= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\ &= \frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{1}{2}\left(\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)+\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x] + (3*Log[1 - (1 -
 x^2)^(1/3)])/2)/2
```

fricas [A] time = 0.95, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\,\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

giac [A] time = 0.92, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [C] time = 0.10, size = 65, normalized size = 1.12

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^2\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(2\ln(x)-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3),x)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3
],[2,2],x^2)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))
```

maxima [A] time = 1.09, size = 62, normalized size = 1.07

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

mupad [B] time = 0.54, size = 86, normalized size = 1.48

$$\frac{\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 0.94, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))
```

3.34 $\int \frac{1}{x(1-x^2)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1
- (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right)-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\
&= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.40

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{4}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) + Log[1 - (1 - x^2)^
(1/3)]/2 - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/4
```

fricas [A] time = 0.71, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

giac [A] time = 1.09, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [C] time = 0.10, size = 48, normalized size = 0.83

$$\frac{\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^2\right)}{3}+\left(2\ln(x)+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\Gamma\left(\frac{2}{3}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(2/3),x)
```

```
[Out] 1/2/GAMMA(2/3)*(2/3*GAMMA(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2)+(1/6*Pi*3
^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 1.30, size = 62, normalized size = 1.07

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

mupad [B] time = 0.46, size = 76, normalized size = 1.31

$$\frac{\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(2/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*9
i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1/3
))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 0.98, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*g
amma(5/3))
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {239}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.76

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{\left(1-x^3\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

fricas [B] time = 0.90, size = 82, normalized size = 1.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

maple [C] time = 0.09, size = 12, normalized size = 0.24

$$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 1.36, size = 78, normalized size = 1.59

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

mupad [B] time = 0.33, size = 10, normalized size = 0.20

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^3)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, x^3)
```

sympy [C] time = 0.89, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= -\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

fricas [A] time = 1.11, size = 64, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.82, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.10, size = 65, normalized size = 1.18

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(3\ln(x)-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3),x)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3
],[2,2],x^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))
```

maxima [A] time = 1.21, size = 62, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 0.51, size = 80, normalized size = 1.45

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)),x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.92, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=97

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-
1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2148}

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/
3)) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]
```

fricas [B] time = 7.45, size = 301, normalized size = 3.10

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(13x^6+2x^5+19x^4-4x^3+19x^2+2x+13\right)-4\sqrt{2}\left(5x^5-5x^4+6x^3-6x^2+5\right.\right.}{6\left(3x^6-18x^5-3x^4-28x^3-3x^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

maple [C] time = 8.06, size = 1143, normalized size = 11.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+8*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+9*(-x^3+1)^(1/3)*RootOf(_Z^3-4
)^2*x-26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)-9*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+28*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2+52*(-x^3+1)^(2/3)+2
4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-18*RootOf(_Z^3-4)*x
+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3-4))/
(x+1)^2)-1/4*ln((8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)^2*RootOf(_Z^3-4)^2*x+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)^3*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x
+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Root
Of(_Z^3-4)+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2-70*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(2/3)-20*Ro
otOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-6*RootOf(_Z^3-4)*x-70*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3-4))/(x+1)
^2)*RootOf(_Z^3-4)-1/2*ln((8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-13*(-x^3+1)^(1/3)*RootOf(_
Z^3-4)^2*x+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)*RootOf(_Z^3-4)+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2-70*RootOf(RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(
2/3)-20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-6*RootOf(_Z^3
-4)*x-70*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3
-4))/(x+1)^2)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right.}{4\sqrt[3]{2}}$$

```
[Out] 1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)*
(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1
/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3
)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2152, 239, 2148}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right.}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
```

```
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right.$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(-x^3+1)^(1/3),x)
[Out] int(x/(x+1)/(-x^3+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x \sqrt[3]{2-3x+x^2}} \, dx$

Optimal. Leaf size=110

$$\frac{3 \log \left(-2^{2/3} \sqrt[3]{x^2-3x+2}-x+2\right)}{4 \sqrt[3]{2}}-\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{2} (2-x)}{\sqrt{3} \sqrt[3]{x^2-3x+2}}+\frac{1}{\sqrt{3}}\right)}{2 \sqrt[3]{2}}-\frac{\log (2-x)}{4 \sqrt[3]{2}}-\frac{\log (x)}{2 \sqrt[3]{2}}$$

```
[Out] -1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3)
)*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(1/
2))*3^(1/2)*2^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 176, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {755, 123}

$$\frac{3 \sqrt[3]{x-2} \sqrt[3]{x-1} \log \left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2} \sqrt[3]{x-1}\right)}{4 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2} \sqrt[3]{x-1} \log (x)}{2 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3} \sqrt[3]{x-2} \sqrt[3]{x-1} \tan ^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2} (x-2)^{2/3}}{\sqrt{3} \sqrt[3]{x-1}}\right)}{2 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2 + x
)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (3*
(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1 +
 x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^(1
/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 755

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[((b + q + 2*c*x)^(1/3)*(b - q
+ 2*c*x)^(1/3))/(a + b*x + c*x^2)^(1/3), Int[1/((d + e*x)*(b + q + 2*c*x)^(
```

```
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{\left(\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}$$
$$= -\frac{\sqrt{3}\,\sqrt[3]{-2+x}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(-2+x)^{2/3}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\,\sqrt[3]{2-3x+x^2}}+\frac{3\sqrt[3]{-2+x}\,\sqrt[3]{-1+x}\,\log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}}\right.}{4\sqrt[3]{2}\,\sqrt[3]{2-3x+x^2}}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.54

$$-\frac{3\sqrt[3]{1-\frac{2}{x}}\,\sqrt[3]{1-\frac{1}{x}}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 - 2/x)^(1/3)*(1 - x^(-1))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1),
 2/x])/(2*(2 - 3*x + x^2)^(1/3))
```

fricas [B] time = 4.94, size = 277, normalized size = 2.52

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+36\,x^5-612\,x^4+2880\,x^3-5760\,x^2+5184\,x-1728\right)+12\sqrt{2}\left(x^5-38\,x^4+\right.\right.}{6\left(x^6-108\,x^5+972\,x^4-3456\,x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
```

```
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - 3x + 2\right)^{\frac{1}{3}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

maple [C] time = 8.32, size = 1069, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)*RootOf(_Z^3-4)^3*x^2+112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)^2*RootOf(_Z^3-4)^2*x^2-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)^3*x-504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+306*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3+504*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-108*(x^2-3*x+2)^(1/3)*RootOf(_
Z^3-4)^2*x+258*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)*x+216*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2-516*(x^2-
3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^
3-4)+119*RootOf(_Z^3-4)*x^2+196*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*x^2-1020*RootOf(_Z^3-4)*x-1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*x-948*(x^2-3*x+2)^(2/3)+1020*RootOf(_Z^3-4)+1680*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)+1/2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2-126*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-108*(x^2-3*x+2)^(2/3)*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+126*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3+306*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-54*(x^2-3*x+2)^(1/3)*
```

```
RootOf(_Z^3-4)^2*x-237*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+108*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+4
74*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Ro
otOf(_Z^3-4)+7*RootOf(_Z^3-4)*x^2+17*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x^2+168*RootOf(_Z^3-4)*x+408*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*x+258*(x^2-3*x+2)^(2/3)-168*RootOf(_Z^3-4)-408*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - 3\,x + 2\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(x^2 - 3\,x + 2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
```

```
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-2)\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{4}\log\left(\sqrt[3]{x^3-3x^2+7x-5}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3-3x^2+7x-5}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*
(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}-\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\,\text{lo}}{4\sqrt[3]{-4(1-}}$$

Mathematica [C] time = 0.01, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix+(2-i)}\,\sqrt[3]{i(x-1)}\,(x-(1-2i))F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i(x-(1-2i)),-\frac{1}{2}i(x-(1-2i))\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*(-
5 + 7*x - 3*x^2 + x^3)^(1/3))
```

fricas [A] time = 1.75, size = 120, normalized size = 1.48

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}(x-1)+\sqrt{3}\left(20389537x^2-40779074x+53222437\right)+179}{7204617x^2-14409234x-20666867}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

maple [C] time = 1.35, size = 653, normalized size = 8.06

$$-\frac{\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-304x^2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2-320x^2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)+608x\operatorname{RootOf}\left(\_\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^2-_Z+1)*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1)
*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x+60
```

```
8*RootOf(_Z^2-_Z+1)^2*x+928*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(2/3
)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+51*(x^3-3*x^2+7*x-5)^(1/3)*
x-1856*RootOf(_Z^2-_Z+1)*x-253*x^2-51*(x^3-3*x^2+7*x-5)^(1/3)+2356*RootOf(_
Z^2-_Z+1)+506*x-713)-1/2*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z
+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x
+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^
(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+675*(x^3-3*x^2+7*x-5)^(
1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x^2+7*x-5)^(1/3)-2356*Roo
tOf(_Z^2-_Z+1)-742*x+1643)*RootOf(_Z^2-_Z+1)+1/2*ln(-304*RootOf(_Z^2-_Z+1)^
2*x^2-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(
x^3-3*x^2+7*x-5)^(1/3)*x+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^
2+675*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)
+675*(x^3-3*x^2+7*x-5)^(1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x
^2+7*x-5)^(1/3)-2356*RootOf(_Z^2-_Z+1)-742*x+1643)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3 - 3\,x^2 + 7\,x - 5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3 - 3\,x^2 + 7\,x - 5\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
```

```
[Out] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3 - 3x^2 + 7x - 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{4}\log\left(\sqrt[3]{x\left(x^2-q\right)}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x\left(x^2-q\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{\log(x)}{4}$$

```
[Out] 1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^2
-q))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx &= \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx\\
&= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}}\\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}}\\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}}\\
&= \frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\log\left(x^{2/3}-\sqrt[3]{-q+x^2}\right)}{4\sqrt[3]{-qx+x^3}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 127, normalized size = 1.92

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-q}\left(-2\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-q}}\right)+\log\left(\frac{x^{4/3}}{(x^2-q)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-q}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/
3))/Sqrt[3]] - 2*Log[1 - x^(2/3)/(-q + x^2)^(1/3)] + Log[1 + x^(4/3)/(-q +
x^2)^(2/3) + x^(2/3)/(-q + x^2)^(1/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

fricas [B] time = 3.86, size = 415, normalized size = 6.29

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(q^{12}-15q^{10}+90q^{8}-351q^{6}+810q^{4}-1215q^{2}+729\right)\left(x^{3}-qx\right)^{\frac{1}{3}}x-2\sqrt{3}\left(q^{12}+6q^{11}-15q^{...}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

giac [A] time = 1.05, size = 67, normalized size = 1.02

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\log\left(\left(-\frac{q}{x^{2}}+1\right)^{\frac{2}{3}}+\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/x
^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3) -
1))
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-q\right) x\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-q\right)x\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

mupad [B] time = 0.39, size = 37, normalized size = 0.56

$$\frac{3\,x\,{\left(1-\frac{x^2}{q}\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^2}{q}\right)}{2\,{\left(x^3-q\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(q - x^2))^(1/3),x)
```

```
[Out] (3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(1
/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x \left(-q + x^{2}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{4}\log\left(\sqrt[3]{(x-1)\left(q+x^2-2x\right)}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+2
/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-}\right.}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{-(1-q)x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+q+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{-1+q+(-1+x)^2}\ \cdots}{4\sqrt[3]{(1\cdots}}$$

Mathematica [A] time = 0.15, size = 140, normalized size = 1.77

$$\frac{\sqrt[3]{x-1}\,\sqrt[3]{q+(x-2)x}\left(-2\log\left(1-\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}\right)+\log\left(\frac{(x-1)^{4/3}}{(q+(x-2)x)^{2/3}}+\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(-1 + x)^(
2/3))/(q + (-2 + x)*x)^(1/3))/Sqrt[3]] - 2*Log[1 - (-1 + x)^(2/3)/(q + (-2
+ x)*x)^(1/3)] + Log[1 + (-1 + x)^(4/3)/(q + (-2 + x)*x)^(2/3) + (-1 + x)^(
2/3)/(q + (-2 + x)*x)^(1/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/3))
```

fricas [B] time = 3.45, size = 665, normalized size = 8.42

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(q^{12}-18q^{11}+117q^{10}-346q^{9}+414q^{8}-18q^{7}+69q^{6}-774q^{5}-234q^{4}+1058q^{3}+621q\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+q-2\,x\right)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x-1)\left(x^2+q-2x\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)*(x^2+q-2*x))^(1/3),x)
[Out] int(1/((x-1)*(x^2+q-2*x))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+q-2\,x\right)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left((x-1)\,\left(x^2-2\,x+q\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
[Out] int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

Optimal. Leaf size=118

$$-\frac{3\log\left(\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}-\sqrt[3]{q}\,(x-1)\right)}{4\sqrt[3]{q}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{q}\,(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}$$

```
[Out] 1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q*
x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x)*
(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)
```

Rubi [F] time = 21.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
 (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q
])^(2/3))/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\,d\right.$$

$$\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\ \sqrt[3]{-1+5q-4q}\right.$$

$$= \overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Mathematica [C] time = 0.20, size = 55, normalized size = 0.47

$$\frac{3\left((x-1)\left(-2qx+q+x^2\right)\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (3*((-1 + x)*(q - 2*q*x + x^2))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (q - 2
*q*x + x^2)/(q*(-1 + x)^2)])/(4*q*(-1 + x)^2)
```

fricas [B] time = 52.74, size = 1496, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
```

```
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
 ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x-1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x-1)\left(-2qx + x^2 + q\right)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((x-1)*(-2*q*x+x^2+q))^(1/3),x)
[Out] int(1/x/((x-1)*(-2*q*x+x^2+q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx-x^{2}-q\right)(x-1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left((x-1)\,\left(x^{2}-2\,q\,x+q\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Timed out
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$

Optimal. Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(k+1)x)}{2\sqrt[3]{k}}-\frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)}-\sqrt[3]{k}\,x\right)}{2\sqrt[3]{k}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{k}\,x}{\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{\sqrt[3]{k}}$$

```
[Out] 1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k*
x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*3^
(1/2))*3^(1/2)/k^(1/3)
```

Rubi [F] time = 0.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-}}}{\sqrt[3]{(1-x)x(1}}$$

$$=\frac{3\sqrt[3]{1-x}\,x\sqrt[3]{1-kx}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-x}\,\sqrt[3]{}}}{\sqrt[3]{(1-x)x(1-}}$$

Mathematica [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/((((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
[Out] Integrate[(2 - (1 + k)*x)/((((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
[Out] integrate(((k + 1)*x - 2)/((((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-(k+1)\,x+2}{((-x+1)\,(-kx+1)\,x)^{\frac{1}{3}}\,(-(k+1)\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((-x+1)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
[Out] int((2-(1+k)*x)/((-x+1)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+x-2}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,\left(kx+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=176

$$\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(kx+2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}-1\right)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}$$

```
[Out] 1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3/
4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(1/
3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/3))
*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

Rubi [F] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-kx+1}{\left((k-2)\,x+1\right)\left((-x+1)\,(-kx+1)\,x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((-x+1)*(-k*x+1)*x)^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((-x+1)*(-k*x+1)*x)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{k\,x-1}{\left(x\,\left(k-2\right)+1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx-\int\left(-\frac{}{kx\left(kx^3-kx^2-x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

3.46 $\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{(a+b)\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{(a+b)\tan^{-1}}{\sqrt[3]{2}\,\cdot}$$

```
[Out] 1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+c
)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*
(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1/3)*
x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x+2^(2
/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.85, antiderivative size = 576, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {6728, 239, 2148}

$$-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)\left(3ib-\sqrt{3}\left(2a+b-i\sqrt{3}c-c\right)\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)\left(\sqrt{3}\right.}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) - ((2*a + b - I*
Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/
(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((2*a + b + I*Sq
rt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(2 - (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 -
 x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*
(2*a + b - c - I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] +
 2*x))])/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c + I*
Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(12*2^(1
```

```
/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)])/2 - (((3*I)*b - Sqrt[3]*(
2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I + Sqrt[3])) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqr
t[3]*c))*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(
I - Sqrt[3]))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{c}{\sqrt[3]{1-x^3}} + \frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\right)dx$$
$$= c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx + \int \frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$
$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \int\left(\frac{b-\frac{i(2a+b-c)}{\sqrt{3}}+c}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{}{(-1}\right.$$
$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{1}{3}\left(3b-i\sqrt{3}(2a+b-c)+3c\right)\int \frac{}{(-}$$
$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\left(2a+b-i\sqrt{3}\,b-c-i\sqrt{3}\,c\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{(2a+b}{}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}}\left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{c\,x^2 + bx + a}{\left(x^2 - x + 1\right)\left(-x^3 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}}\left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^2 + b\,x + a}{\left(1 - x^3\right)^{1/3}\,\left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=407

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x}{1176(3-2x)^{9}}$$

```
[Out] -19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3-
2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1)^
4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-2*x
)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-3822
5/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+
2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492928512
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630
132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)-5/
6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1
/2))*(298093007954+81630132224*14^(1/2))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x}{1176(3-2x)^{9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
```

```
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
```

```
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{1}{784}\int \frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{\int \frac{325752-54\cdots}{(3-2x)^{11/2}(1+\cdots}}{4609\cdots}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{138\cdots}{32928(3-2\cdots}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{138\cdots}{32928(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+\cdots\right.}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \cdots$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{\cdots}{28(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14\cdots}{12047257\cdots}$$

Mathematica [C] time = 2.13, size = 198, normalized size = 0.49

$$\frac{45i\sqrt{14-2i\sqrt{7}}\left(146319\sqrt{7}+115739i\right)\tanh^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{7-i\sqrt{7}}}\right)-45i\sqrt{14+2i\sqrt{7}}\left(146319\sqrt{7}-115739i\right)\tanh^{-1}\left(\frac{\cdots}{\cdots}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((56*(-40289347 + 429812744*x - 135202154*x^2 + 1073855156*x^3 - 1627773523
*x^4 + 1470758860*x^5 - 2888625656*x^6 + 3106712560*x^7 - 2343370048*x^8 +
2443779648*x^9 - 1873554048*x^10 + 677249280*x^11 - 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) + (45*I)*Sqrt[14 - (2*I)*Sqrt[7]]*(115739*I +
146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7 - I*Sqrt[7]]] - (45*I)*Sqrt[14
 + (2*I)*Sqrt[7]]*(-115739*I + 146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7
 + I*Sqrt[7]]])/121436356608
```

fricas [B] time = 1.52, size = 957, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/85228286570792313424725137817 6*(2263908918780*22241759018113166^(1/4)*sqr
t(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10
+ 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
 - 1242*x^2 - 162*x - 243)*sqrt(21292357711*sqrt(14) + 81630132224)*arctan(
1/100521871569518694695269086857534372287294018150 40*22241759018113166^(3/4
)*sqrt(12577271771)*sqrt(79716926)*sqrt(-2089731384934400*22241759018113166
^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 8163013222
4)*(7645*sqrt(14) - 115739) - 4190418993502514995568679111884800*x + 209520
9496751257497784339555942400*sqrt(14) + 6285628490253772493353018667827200)
*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357711*sqrt(14) + 816
30132224) - 1/1958184534851295802906658902*22241759018113166^(3/4)*sqrt(797
16926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2*x + 3)*sqrt(21292
357711*sqrt(14) + 81630132224) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 22639089
18780*22241759018113166^(1/4)*sqrt(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2
816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^
6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(2129235771
1*sqrt(14) + 81630132224)*arctan(1/2462861907259396838466870075605045544 2*2
2241759018113166^(3/4)*sqrt(12577271771)*sqrt(22241759018113166^(1/4)*sqrt(
79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)*(7645*sqr
t(14) - 115739) - 2005242886101391892*x + 1002621443050695946*sqrt(14) + 30
```

```
07864329152087838)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357
711*sqrt(14) + 81630132224) - 1/1958184534851295802906658902*22241759018113
166^(3/4)*sqrt(79716926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2
*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224) + 2/7*sqrt(14)*sqrt(7) + s
qrt(7)) + 315*22241759018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 -
 229870452342784*x^12 + 459740904685568*x^11 - 480638218534912*x^10 + 55900
3145469952*x^9 - 734018148958208*x^8 + 498923368153088*x^7 - 34611176062976
0*x^6 + 407660880326656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 10
1384624222208*x^2 - 21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11
- 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x
^4 + 936*x^3 - 1242*x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)
*sqrt(21292357711*sqrt(14) + 81630132224)*log(2089731384934400/12577271771*
22241759018113166^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt
(14) + 81630132224)*(7645*sqrt(14) - 115739) - 3331739243453861593088800*x +
 1665869621726930796544400*sqrt(14) + 4997608865180792389632000) - 315*222417
59018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 - 229870452342784*x^1
2 + 459740904685568*x^11 - 480638218534912*x^10 + 559003145469952*x^9 - 734
018148958208*x^8 + 498923368153088*x^7 - 346111760629760*x^6 + 407660880326
656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 101384624222208*x^2 -
21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x
^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*
x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)*sqrt(21292357711*sq
rt(14) + 81630132224)*log(-2089731384934400/12577271771*22241759018113166^(
1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)
*(7645*sqrt(14) - 115739) - 3331739243453861593088800*x + 16658696217269307
9654400*sqrt(14) + 4997608865180792389632000) + 3930276056758728108832*(880704
00*x^12 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^
8 - 3106712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107
3855156*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*
x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 -
 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

giac [B] time = 2.82, size = 767, normalized size = 1.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] -5/24179327893504*sqrt(7)*(856240*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 2
568720*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 183480*14^(3/4)
*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 7645*14^(3/4)*sqrt(7)*(-8*
sqrt(14) + 32)^(3/2) + 103702144*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) + 7407
296*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(1/28*14^(3/4)*(14^(1/4)
*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)
```

```
) - 5/24179327893504*sqrt(7)*(856240*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2)
+ 2568720*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 183480*14^(3
/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 7645*14^(3/4)*sqrt(7)*(
-8*sqrt(14) + 32)^(3/2) + 103702144*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) + 7
407296*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(-1/28*14^(3/4)*(14^(
1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) +
1/2)) - 5/48358655787008*sqrt(7)*(122320*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14)
 + 4)^(3/2) + 366960*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14)
- 4) - 1284360*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 53515*14^(3
/4)*(-8*sqrt(14) + 32)^(3/2) + 14814592*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(
14) + 4) - 51851072*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqrt(1/2)
*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/48358655787008
*sqrt(7)*(122320*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 366960*14^
(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 1284360*14^(3/4)*
(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 53515*14^(3/4)*(-8*sqrt(14) + 32)^(
3/2) + 14814592*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) - 51851072*14^(
1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sq
rt(14) + 4) - 2*x + sqrt(14) + 3) + 1/5059848192*(1578405*(2*x - 3)^7*sqrt(
-2*x + 3) + 37939930*(2*x - 3)^6*sqrt(-2*x + 3) + 400127266*(2*x - 3)^5*sqr
t(-2*x + 3) + 2394090608*(2*x - 3)^4*sqrt(-2*x + 3) + 8763772549*(2*x - 3)^
3*sqrt(-2*x + 3) + 19602865030*(2*x - 3)^2*sqrt(-2*x + 3) - 24778425644*(-2
*x + 3)^(3/2) + 13623638952*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^4 + 1/
59295096*(9090*(2*x - 3)^4 - 3885*(2*x - 3)^3 + 2394*(2*x - 3)^2 - 2520*x +
 4172)/((2*x - 3)^4*sqrt(-2*x + 3))
```

maple [A] time = 0.12, size = 584, normalized size = 1.43

$$-\frac{731595\left(7+2\sqrt{14}\right)\sqrt{14}\ \arctan\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{6746464256\sqrt{-7+2\sqrt{14}}}+\frac{1424965\left(7+2\sqrt{14}\right)\arctan\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128\sqrt{-7+2\sqrt{14}}}-\frac{578}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] 1/6588344*(567651623/32*(3-2*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+98014325
15/384*(3-2*x)^(5/2)-8763772549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2
)-200063633/384*(3-2*x)^(11/2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*
x)^(15/2))/((3-2*x)^2-7+14*x)^4-731595/13492928512*ln(3-2*x+14^(1/2)-(3-2*x
)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)+1424965/6746464
256*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1
/2)-731595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14
^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/33732
32128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(
```

```
-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2
)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(
1/2)+731595/13492928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2
))*(7+2*14^(1/2))^(1/2)*14^(1/2)-1424965/6746464256*ln(3-2*x+14^(1/2)+(3-2*
x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2
*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/
2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)
*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*
14^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(
7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+1/151263/(3-2*x)^(9/2)
+5/235298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2823576/(3-2*x)^(3/2)+5
05/3294172/(3-2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^{2}+x+1\right)^{5}\left(-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

mupad [B] time = 0.47, size = 343, normalized size = 0.84

$$-\frac{\frac{272\,x}{441}-\frac{164\,{\left(2\,x-3\right)}^2}{441}+\frac{1966\,{\left(2\,x-3\right)}^3}{3087}-\frac{9091\,{\left(2\,x-3\right)}^4}{3087}-\frac{32070727\,{\left(2\,x-3\right)}^5}{5531904}-\frac{41014777\,{\left(2\,x-3\right)}^6}{11063808}-\frac{141921511\,{\left(2\,x-3\right)}^7}{154893312}+\frac{23262655\,{\left(2\,x-3\right)}^{8}}{309786624}}{38416\,{\left(3-2\,x\right)}^{9/2}-76832\,{\left(3-2\,x\right)}^{11/2}+68600\,{\left(3-2\,x\right)}^{13/2}-35672\,{\left(3-2\,x\right)}^{15/2}+11809\,{\left(3-2\,x\right)}^{17/2}-25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

```
[Out] (atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*15721589
71375i)/(3916630562536760539338506 24*((7^(1/2)*18196010718797112 5i)/1958315
28126838026966925312 - 230036728532618625/279759325895482895667036 16)) - (1
572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^
(1/2))/(3916630562536760539338506 24*((7^(1/2)*18196010718797112 5i)/19583152
8126838026966925312 - 230036728532618625/279759325895482895667036 16)))*(7^(
1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/441 - (16
4*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3087 - (3
2070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (141921511
*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (1571659*(2*x
```

```
- 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x - 3)^11)/1
20472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3 - 2*x)^(9/2)
 - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3 - 2*x)^(15/2)
 + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 - 2*x)^(21/2) -
28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)
*12577271771i - 149046503977)^(1/2)*1572158971375i)/(391663056253676053933 8
50624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 23003672
8532618625/27975932589548289566703616)) + (1572158971375*7^(1/2)*(3 - 2*x)^
(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(39166305625367605393 3
850624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 2300367
28532618625/27975932589548289566703616)))*(- 7^(1/2)*12577271771i - 1490465
03977)^(1/2)*5i)/3373232128
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
[Out] Timed out
```

3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=648

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+81}{10119696384(3-2x)^{19/2}\left(\right.}$$

```
[Out] 4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/(
3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-13515743021825/
13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)^(1
1/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/284467
3747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^(5/2)
-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19/2)/(2
*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*(8477+2
1409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2*x)^(19/
2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^5+4
1/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451/101196963
84*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/283351498752*(2896203
9+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*(14627273-35
058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/227573899787428l6768/(3
-2*x)^(1/2)+11275/1274413838809597739008*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7
+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)
-11275/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)
)^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)+11275/6372
069194047988695O4*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)-11275/6
372069194047988695O4*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+81}{10119696384(3-2x)^{19/2}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
```

```
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int \frac{2534672\cdots}{(3-2x)^{21/2}\cdots}}{27\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\cdots}{691488(3\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{5\cdots}{7056(3-2\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} + \frac{\cdots}{63(3-2x)\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{30\cdots}{15550330\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{30\cdots}{15550330\cdots}$$

Mathematica [C] time = 6.09, size = 610, normalized size = 0.94

$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}+\frac{\frac{67816x+20776}{1568(3-2x)^{19/2}\left(2x^2+x+1\right)^8}+\frac{\frac{117492592x+46521776}{1372(3-2x)^{19/2}\left(2x^2+x+1\right)^7}+\frac{\frac{164128134240x+74020332960}{1176(3-2x)^{19/2}\left(2x^2+x+1\right)^6}+\frac{\frac{184316990760000x+9420\cdots}{980(3-2x)^{19/2}\left(2x^2\cdots\right.}\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 - 2
*x)^(19/2)*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)^(
19/2)*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*x)^
(19/2)*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^5) + ((95476201213680000 + 157747397367934080*
x)/(784*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4) + ((72879297583985544960 + 8973
5798552133000960*x)/(588*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + ((3643273421
2165998389760 + 18400346379541577848320*x)/(392*(3 - 2*x)^(19/2)*(1 + x + 2
*x^2)^2) + ((6440121232839552246912000 - 15435719146659136558464000*x)/(196
*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (39479926882545221954112000/(19*(3 - 2
*x)^(19/2)) + (-908021664138480966930240000/(17*(3 - 2*x)^(17/2)) + (-19105
520493023248582746201600/(3 - 2*x)^(15/2) + (-26849557435537239465884310720
000/(13*(3 - 2*x)^(13/2)) + (-150994423858598796539274120000000/(3 - 2*x)^(
11/2) + (-8237718113587514139784976619840000/(3 - 2*x)^(9/2) + (-3383893120
36560466460044072847040000/(3 - 2*x)^(7/2) + (-1013530552857651055083639451
5648960000/(3 - 2*x)^(5/2) + (-204334375738495648812805956791073600000/(3 -
 2*x)^(3/2) + (-2230994866519889796828561036406228800000/Sqrt[3 - 2*x] + ((
Sqrt[(7 - I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 - (7175
0597240923349846054347713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 -
 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) + (Sqrt[(7 + I*Sqrt[7])/
2]*(-31233928131278457155599854509687203200000 + (7175059724092334984605434
7713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x]])/Sqrt[7 + I*Sqr
t[7]]])/(-14 - (2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)
/196)/392)/588)/784)/980)/1176)/1372)/1568)/1764
```

fricas [B] time = 1.72, size = 1563, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")

[Out] 1/1094755373086200603246995644663447631605361478665641987670016*(4732002380
085251586622550100*4787936175075825342943147314686^(1/4)*sqrt(1169607525756
986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 6468403
2*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21
+ 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 46771
2000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^
12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 2003648
4*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 +
 137781*x + 59049)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735
1936)*arctan(1/365621708519319702488553401133870353544174572418706268660249
45379489008832725311219252*4787936175075825342943147314686^(3/4)*sqrt(27763
87167632535361)*sqrt(12865682783326846)*sqrt(1169607525756986)*sqrt(4787936
175075825342943147314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14
) - 9756589235) - 71440233164918992209696826631202812*x + 28280279689505005
187146*sqrt(22335021272086100802556094) + 107160349747378488314545239946804
218)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3275718505284
62403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615767666910014
4258228441327447900096742*4787936175075825342943147314686^(3/4)*sqrt(116960
7525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(-2*x +
 3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936) + 2/7*sqr
t(14)*sqrt(7) + sqrt(7)) + 4732002380085251586622550100*4787936175075825342
943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5
505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^
23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 54
0503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720
*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 -
49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 32
76126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(32757185052846
2403199*sqrt(14) + 1226422380928157351936)*arctan(1/39296670234816303076555
330542603297083388480635973027797585697454399143598928370335464344780800*47
87936175075825342943147314686^(3/4)*sqrt(2776387167632535361)*sqrt(11696075
25756986)*sqrt(-14862107440409842545228890767360000*47879361750758253429431
47314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(32757185052846240
3199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235)
- 1061752420864956548109093061495542399038192585561809435358469816320000*x
+ 420304555190263689316852795001664341102416628348354560000*sqrt(2233502127
2086100802556094) + 1592628631297434822163639592243313598557288878342714153
037704724480000)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615
7676669100144258228441327447900096742*4787936175075825342943147314686^(3/4)
*sqrt(1169607525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))
*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735193
```

6) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 271150425*478793617507582534294314731468 6^(1/4)*sqrt(1169607525756986)*(64299853725206176173182156 8*x^28 - 6751484641146648498184126464*x^27 + 30381680885159918241828569088*x^26 - 793299445334731198536634859 52*x^25 + 146844790944939604835504750592*x^24 - 237989833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 474352077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 6323363154136437844718544486 40*x^19 + 662885025215707070319757885440*x^18 - 609018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 505075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 3046854691069420251327737364 48*x^13 + 228722407218762404519491928064*x^12 - 129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 2741193833525857491515080704*x^5 + 401791424914064043276867993 6*x^4 + 901214411022199723237834752*x^3 + 101386621239997468864256409 6*x^2 - 327571850528462403199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049) + 168977702066624481070940 16*x + 72419015171426763474468864)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936)*log(148621074404098425452288907673600 00/2776387167632535361*4787936175075825342943147314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640069460132720868272698184789257093120000*x + 151385426388014656165701481356328960000*sqrt(2233502127208610080 2556094) + 5736334794601041901990813024090472771838856396800 00) - 271150425*4787936175075825342943147314686^(1/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 675148464114664849818412646 4*x^27 + 30381680885159918241828569088*x^26 - 79329944533473119853663485952*x^25 + 146844790944939604835504750592*x^24 - 237989833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 474352077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 632336315413643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 609018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 505075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 304685469106942025132773736448*x^13 + 228722407218762404519491928064*x^12 - 129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 2741193833525857491515080704*x^5 + 4017914249140640432768679936*x^4 + 901214411022199723237834752*x^3 + 1013866212399974688642564096*x^2 - 327571850528462403199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20

```
 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9) + 1689777020666624481070940l6*x + 724190151714267634744688 64)*sqrt(32757
1850528462403199*sqrt(14) + 1226422380928157351936)*log(-148621074404098425
452288907673600000/2776387167632535361*4787936175075825342943147314686^(1/4)
*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14)
+ 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640
06946013272086827269818478925709312000 0*x + 15138542638801465616570148135 63
28960000*sqrt(22335021272086100802556094) + 5736334794601041901990813024090
47277183885639680000) + 12729350636658293157364161836105228 32*(240031204937
714427494400*x^27 - 2621948941596237063782400*x^26 + 123650450558968111054 8
4800*x^25 - 33969890064381284111155200*x^24 + 65360120291258796757811200*x^
23 - 106701725825102321939251200*x^22 + 162290307223249502039654400*x^21 -
216634228326470609547509760*x^20 + 253788172995391086570485760*x^19 - 28727
915918029130520815616 0*x^18 + 304010591010966811155955200*x^17 - 2826446645
39994827031006720*x^16 + 258819256815163249845447936*x^15 - 229408132984166
521977166336*x^14 + 172649692294614969274168896*x^13 - 1333125413772463861 1
5890240*x^12 + 102031573634317834547976132*x^11 - 597911026814941175721491 7
6*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8 +
10718131725916893151555068*x^7 - 8685973988079840377705700*x^6 + 3673303058
277822225386926*x^5 - 809990362095044210054958*x^4 + 1362587089603925431664
856*x^3 + 111926768697602999806116*x^2 + 20570245201454032279728 9*x - 48844
17100172357749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x
^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 38
6777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120
*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 +
 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*
x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 +
 826686*x^2 + 137781*x + 59049)
```

giac [A] time = 4.19, size = 972, normalized size = 1.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] 11275/228374959914679914830233 6*sqrt(7)*(240680481328*14^(3/4)*sqrt(2)*(sqr
t(14) + 4)^(3/2) + 722041443984*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(1
4) - 4) - 51574388856*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32
) + 2148932869*14^(3/4)*sqrt(7)*(-8*sqrt(14) + 32)^(3/2) + 8741903954560*14
^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) - 624421711040*14^(1/4)*sqrt(7)*sqrt(-8*s
qrt(14) + 32))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4)
```

```
+ 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) + 11275/22837495991467991483
02336*sqrt(7)*(240680481328*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 7220414
43984*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 51574388856*14^(
3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 2148932869*14^(3/4)*sq
rt(7)*(-8*sqrt(14) + 32)^(3/2) + 8741903954560*14^(1/4)*sqrt(2)*sqrt(sqrt(1
4) + 4) - 624421711040*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(-1/2
8*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(
-1/8*sqrt(14) + 1/2)) - 11275/456749919829359829660467 2*sqrt(7)*(3438292590
4*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 103148777712*14^(3/4)*sqr
t(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 361020721992*14^(3/4)*(sqr
t(14) + 4)*sqrt(-8*sqrt(14) + 32) - 15042530083*14^(3/4)*(-8*sqrt(14) + 32)
^(3/2) + 1248843422080*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 437095
1977280*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqrt(1/2)*sqrt(-2*x +
 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/45674991982935982966 04
672*sqrt(7)*(34382925904*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 10
3148777712*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 361
020721992*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 15042530083*14^(
3/4)*(-8*sqrt(14) + 32)^(3/2) + 1248843422080*14^(1/4)*sqrt(7)*sqrt(2)*sqrt
(sqrt(14) + 4) + 4370951977280*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/
4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/20
4816509808685350912*(232787883652335*(2*x - 3)^17*sqrt(-2*x + 3) + 13820106
668010555*(2*x - 3)^16*sqrt(-2*x + 3) + 389618236717151904*(2*x - 3)^15*sqr
t(-2*x + 3) + 6925854690067471092*(2*x - 3)^14*sqrt(-2*x + 3) + 86924717622
268515682*(2*x - 3)^13*sqrt(-2*x + 3) + 817308030405306394458*(2*x - 3)^12*
sqrt(-2*x + 3) + 5960699611609964201316*(2*x - 3)^11*sqrt(-2*x + 3) + 34438
539253455396724476*(2*x - 3)^10*sqrt(-2*x + 3) + 159569809573892673649239*(
2*x - 3)^9*sqrt(-2*x + 3) + 596312099501239401271299*(2*x - 3)^8*sqrt(-2*x
+ 3) + 1797250621001927736488676*(2*x - 3)^7*sqrt(-2*x + 3) + 4343978582610
098069631672*(2*x - 3)^6*sqrt(-2*x + 3) + 8317212692450176764092592*(2*x -
3)^5*sqrt(-2*x + 3) + 12350951282904546626644288*(2*x - 3)^4*sqrt(-2*x + 3)
 + 13738697725192288735303872*(2*x - 3)^3*sqrt(-2*x + 3) + 1078847966186370
2869789824*(2*x - 3)^2*sqrt(-2*x + 3) - 5340653236079401357791744*(-2*x + 3
)^(3/2) + 1255138952440667471476992*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7
)^9 + 1/3280733202692679552*(235862511885*(2*x - 3)^9 - 107316677325*(2*x -
 3)^8 + 80348352084*(2*x - 3)^7 - 64554208290*(2*x - 3)^6 + 49954696792*(2*
x - 3)^5 - 35035280280*(2*x - 3)^4 + 21058773120*(2*x - 3)^3 - 10093321056*
(2*x - 3)^2 + 6831901440*x - 10859127552)/((2*x - 3)^9*sqrt(-2*x + 3))
```

maple [A] time = 0.07, size = 719, normalized size = 1.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] 1/86812553324672*(-1352841099712333/8192*(-2*x+3)^(31/2)+4606702222670185/7
86432*(-2*x+3)^(33/2)-25865320405815/262144*(-2*x+3)^(35/2)-347698778390586
0258979/1536*(-2*x+3)^(3/2)+9364999706478908741137/2048*(-2*x+3)^(5/2)-2385
1905772903279054347/4096*(-2*x+3)^(7/2)+192983613795383541041317/36864*(-2*
x+3)^(9/2)-57758421475348449750643/16384*(-2*x+3)^(11/2)+603330358695846954
11551/32768*(-2*x+3)^(13/2)-149770885083493978040723/196608*(-2*x+3)^(15/2)
+66256899944582155696811/262144*(-2*x+3)^(17/2)-17729978841543630405471/262
144*(-2*x+3)^(19/2)+286987827121283060373/196608*(-2*x+3)^(21/2)-165574989
211387894481/65536*(-2*x+3)^(23/2)+45406001689183688581/131072*(-2*x+3)^(25
/2)-43462358811134257841/1179648*(-2*x+3)^(27/2)+192384852501874197/65536*(
-2*x+3)^(29/2)+544765170330150812273/1024*(-2*x+3)^(1/2))/(14*x+(-2*x+3)^2-
7)^9-206922416016525/1274413838809597739008*(7+2*14^(1/2))^(1/2)*14^(1/2)*l
n(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))+389615613935075/6372
06919404798869504*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*(7
+2*14^(1/2))^(1/2))-206922416016525/637206919404798869504/(-7+2*14^(1/2))^(
1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))
/(-7+2*14^(1/2))^(1/2))+389615613935075/318603459702399434752/(-7+2*14^(1/2
))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+
2*14^(1/2))^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))^(1
/2)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))
^(1/2))+206922416016525/1274413838809597739008*(7+2*14^(1/2))^(1/2)*14^(1/2
)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))-389615613935075/6
37206919404798869504*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)
*(7+2*14^(1/2))^(1/2))-206922416016525/637206919404798869504/(-7+2*14^(1/2)
)^(1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))+389615613935075/318603459702399434752/(-7+2*14^(
1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(
-7+2*14^(1/2))^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))
^(1/2)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/
2))^(1/2))+1/5367029731/(-2*x+3)^(19/2)+5/4802079233/(-2*x+3)^(17/2)+73/237
27920916/(-2*x+3)^(15/2)+165/25705247659/(-2*x+3)^(13/2)+2365/221460595216/
(-2*x+3)^(11/2)+30349/1993145356944/(-2*x+3)^(9/2)+854095/43406276662336/(-
2*x+3)^(7/2)+75933/3100448333024/(-2*x+3)^(5/2)+8519225/260437659974016/(-2
*x+3)^(3/2)+891605/12401793332096/(-2*x+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}(-2\,x+3)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

mupad [B] time = 0.56, size = 567, normalized size = 0.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

```
[Out] ((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 + (
5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (533495672
*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (78007323158*(
2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (1118693654
785651073*(2*x - 3)^10)/453254454575104 + (1624300450152249301*(2*x - 3)^11
)/97125954551808 + (35048653520674948897*(2*x - 3)^12)/906508909150208 + (9
5527511967437577915*(2*x - 3)^13)/1813017818300416 + (564066299973141561054
7*(2*x - 3)^14)/114220122552926208 + (1737142288764447500149*(2*x - 3)^15)/
50764498912411648 + (12971210667229097601055*(2*x - 3)^16)/7107029847737630
72 + (32723441206946795665235*(2*x - 3)^17)/4264217908642578432 + (10264579
7034777710681325*(2*x - 3)^18)/39799367147330732032 + (14609317874302006653
15*(2*x - 3)^19)/2094703534070038528 + (687618468821894139745*(2*x - 3)^20)
/4528256169239642112 + (39968995676603847725*(2*x - 3)^21)/1509418723079880
704 + (5940132943613849875*(2*x - 3)^22)/1625527855624486912 + (57179785036
20010375*(2*x - 3)^23)/14629750700620382208 + (178056995818325525*(2*x - 3)
^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/10159549097653043
2 + (1433237383402275*(2*x - 3)^26)/22757389978742816768 + (24229218097975*
(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(20661046784*(3 - 2*x)^(19
/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2*x)^(23/2) - 270069
397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27/2) - 187609683744*(3 -
 2*x)^(29/2) + 105782451264*(3 - 2*x)^(31/2) - 47554666992*(3 - 2*x)^(33/2)
 + 17278167438*(3 - 2*x)^(35/2) - 5111496103*(3 - 2*x)^(37/2) + 1234154817*
(3 - 2*x)^(39/2) - 242625852*(3 - 2*x)^(41/2) + 38550456*(3 - 2*x)^(43/2) -
 4883634*(3 - 2*x)^(45/2) + 482454*(3 - 2*x)^(47/2) - 35868*(3 - 2*x)^(49/2
) + 1890*(3 - 2*x)^(51/2) - 63*(3 - 2*x)^(53/2) + (3 - 2*x)^(55/2)) - (atan
(((- 7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*(3 - 2*x
)^(1/2)*43774618035829144330316520640625i)/(3300086980477615835608700826192
63806430093600589158123831296*((7^(1/2)*42709096709460747387242744942497717
8671875i)/165004349023880791780435041309631903215046800294579061915648 + 80
3365829195061345550676106938401175484375/2357204986055439882577643447280455
7602149542899225580273664)) + (43774618035829144330316520640625*7^(1/2)*(-
7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*(3 - 2*x)^(1/
2))/(330008698047761583560870082619263806430093600589158123831296*((7^(1/2)
*427090967094607473872427449424977178671875i)/16500434902388079178043504130
9631903215046800294579061915648 + 80336582919506134555067610693840117548437
5/23572049860554398825776434472804557602149542899225580273664)))*(- 7^(1/2)
*30540258843957888971i - 2293002953699236822393)^(1/2)*11275i)/318603459702
```

```
399434752 + (atan(((7^(1/2)*30540258843957888971i - 2293002953699236822393)
^(1/2)*(3 - 2*x)^(1/2)*437746180358291443303165206406 25i)/(3300086980477615
835608700826192638064300936005891581238 31296*((7^(1/2)*42709096709460747387
24274494249771786718 75i)/16500434902388079178043504130963190321504680029457
9061915648 - 80336582919506134555067610693840117548 4375/23572049860554398 82
577643447280455760214954289922558027 3664)) - (437746180358291443303165206 40
625*7^(1/2)*(7^(1/2)*30540258843957888971i - 22930029536992368223 93)^(1/2)*
(3 - 2*x)^(1/2))/(33000869804776158356087008261926380643009360058915812 3831
296*((7^(1/2)*427090967094607473872427449424977178671875i)/165004349023 8807
91780435041309631903215046800294579061915648 - 80336582919506134555067610 69
38401175484375/23572049860554398825776434472804557602149542899225580273664)
))*(7^(1/2)*30540258843957888971i - 22930029536992368223 93)^(1/2)*11275i)/3
186034597023994347 52
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx$

Optimal. Leaf size=1058

result too large to display

```
[Out] 1/33516*(113+373*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^18+1/7976808*(40657+107329*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^17+5/595601664*(751303+1831285*x)/(3-2*x)^(39/
2)/(2*x^2+x+1)^16+1/25015269888*(184959785+429411497*x)/(3-2*x)^(39/2)/(2*x
^2+x+1)^15+1/4902992898048*(41652915209+92630823167*x)/(3-2*x)^(39/2)/(2*x^
2+x+1)^14+1/297448235814912*(2871555518177+6100156355517*x)/(3-2*x)^(39/2)/
(2*x^2+x+1)^13+1/7138757659557888*(77559130805859+156274047129113*x)/(3-2*x
)^(39/2)/(2*x^2+x+1)^12+5/1099368679571914752*(2656658801194921+50208801761
34289*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^11+1/3420258114223734784*(4518792158520
8601+78752911037377255*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^10+1/43095252239219058
2784*(6063974149878048635+9477172618423641847*x)/(3-2*x)^(39/2)/(2*x^2+x+1)
^9+1/482666825079253452718O8*(691833601144925854831+919498192874055581221*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^8+23/157671162859222794554572 8*(91949819287405
5581221+908287136092467468517*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^7+115/101879828
3090362672506470 4*(908287136092467468517+2982818849445222257 47*x)/(3-2*x)^(
39/2)/(2*x^2+x+1)^6+23/2037596566180725345012940 8*(259931356880226511008 1-1
042614244862318737918 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^5-23/20018492580021161
284337664*(1042614244862318737918 7+27513723463194262383705*x)/(3-2*x)^(39/2
)/(2*x^2+x+1)^4-115/764342443964444339947438 08*(265132244281690164788 43+306
734154065537893420 19*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^3-115/125891696652967303
050166272*(88411609113007981044643-57122695362451521629 63*x)/(3-2*x)^(39/2)
/(2*x^2+x+1)^2+115/1958315281268380269669253 12*(28561347681225760814815+965
93481283901949034610 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)+1/133*x/(3-2*x)^(39/2)/
(2*x^2+x+1)^19+115/324826126509883073653212736882973136964812 8*ln(3-2*x+14^
(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(302971189122193607250286930 61-80
611109111432760539830227 87*14^(1/2))*(-14+4*14^(1/2))^(1/2)-115/3248261265 0
9883073653212736882973136964812 8*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1
/2))^(1/2))*(302971189122193607250286930 61-806111091114327605398302278 7*14^
(1/2))*(-14+4*14^(1/2))^(1/2)+115/1624130632549415368266063684414865684824 0
64*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(3
029711891221936072502869306 1+806111091114327605398302278 7*14^(1/2))*(14+4*1
4^(1/2))^(1/2)-115/162413063254941536826606368441486568482406 4*arctan((2*(3
-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(3029711891221936 0
725028693061+806111091114327605398302278 7*14^(1/2))*(14+4*14^(1/2))^(1/2)-9
270277547814767462080476205 05/580046654481934060095022744433880601722 88/(3-
2*x)^(1/2)+111551682229707742323768911 45/168516633253261656024735422401740 8
/(3-2*x)^(23/2)+140118184980910202724749563 75/101109979951956993614841253 44
104448/(3-2*x)^(21/2)-13056959628363355534285785425/10692401435725356272394
1220352/(3-2*x)^(39/2)-3948194343291401740321996415/202881463139404195937 73
```

```
4623232/(3-2*x)^(37/2)-304688229262620222736480811/53736171318004354599724
3056128/(3-2*x)^(35/2)+2124315846756567455653862925/1688851098565851144562
763890688/(3-2*x)^(33/2)+476575150745141187960959295 35/6663285243432539970365
8138959872/(3-2*x)^(31/2)+3491161999397471406217275198 5/124667917457770102 6
71360389021696/(3-2*x)^(29/2)+14906630980879476084301740482 5/16249818206564
516830956630017310 72/(3-2*x)^(27/2)+158486139641690665437343801 71/601845118
761648771516912222863360/(3-2*x)^(25/2)-101190274412779618678573275245/3963
511214116714149701777134888943616/(3-2*x)^(15/2)-460503190416958283087439 33
7135/343504305223448559640820685023708446 72/(3-2*x)^(13/2)-2211619588790911
794826342607495/40692048464931598603604911918193154457 6/(3-2*x)^(11/2)-4986
6814791877818534173165227 75/870069981722901090142534116650820902584 32/(3-2*
x)^(3/2)+1734413681498043786619358697 05/8965084889073520100515924471772610 5
6/(3-2*x)^(19/2)-227240908234699051527135195 45/16042783485710509653554812 21
264572416/(3-2*x)^(17/2)-14340146755077724762794043702 5/73985542663511997 46
1099839851260280832/(3-2*x)^(9/2)-4611053278117143010907562317585/725058318
1024175751187784305423507521536/(3-2*x)^(7/2)-405965372440630510720926890 22
7/207159519457833592891079551583528786329 6/(3-2*x)^(5/2)
```

Rubi [A] time = 2.49, antiderivative size = 1058, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -1305695962836335553428578542 5/(10692401435725356272394122035 2*(3 - 2*x)^(3
9/2)) - 3948194343291401740321996415/(2028814631394041959377346232 32*(3 - 2
*x)^(37/2)) - 304688229262620222736480811/(537361713180043545997243056128*(
3 - 2*x)^(35/2)) + 2124315846756567455653862925/(1688851098565851144562763 8
90688*(3 - 2*x)^(33/2)) + 476575150745141187960959295 35/(666328524343253997
03658138959872*(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(124667917
457770102671360389021696*(3 - 2*x)^(29/2)) + 149066309808794760843017404825
/(16249818206564516830956630017310 72*(3 - 2*x)^(27/2)) + 158486139641690665
43734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168
222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^(23/2))
 + 14011818498091020272474956375/(1011099799519569936148412534410444 8*(3 -
2*x)^(21/2)) + 173441368149804378661935869705/(896508488907352010051592447 1
77261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(160427834857105
096535548122126457241 6*(3 - 2*x)^(17/2)) - 101190274412779618678573275245/(
3963511214116714149701777134888943616*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(343504305223448559640820685023708446 72*(3 - 2*x)^(13/2)) - 2
211619588790911794826342607495/(406920484649315986036049119181931544576*(3
 - 2*x)^(11/2)) - 14340146755077724762794043702 5/(73985542663511997461099839
```

```
851260280832*(3 - 2*x)^(9/2)) - 4611053278117143010907562317585/(7250583181
0241757511877843054235075215 36*(3 - 2*x)^(7/2)) - 405965372440630510720926 8
90227/(207159519457833592891079551583528 7863296*(3 - 2*x)^(5/2)) - 49866814
79187781853417316522775/(870069981722901090142534116650820902584 32*(3 - 2*x
)^(3/2)) - 9270277547814767462080476205 05/(5800466544819340600950227444338 8
060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
+ (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
+ (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
*(919498192874055581221 + 908287136092467468517*x))/(157671162859222794554 5
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 27513723463194262383705*x))/(200184925800211612843376 64*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673
415406553789342019*x))/(7643424439644433994743808*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(88411609113007981044643 - 57122695362451521629 63*x))/(1
258916966529673030501662 72*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
1347681225760814815 + 96593481283901949034 6107*x))/(19583152812683802696692
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
971189122193607250286930 61 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
7470768413303184220743284241 2032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
1221936072502869306 1 + 8061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
8413303184220743284241 2032 + (115*(30297118912219360725028693061 - 80611109
11143276053983022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414
865684824064 - (115*(30297118912219360725028693061 - 806111091114327605398 3
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{\int \frac{3640-3164x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{19}}\,dx}{3724}$$

Mathematica [C] time = 6.18, size = 1100, normalized size = 1.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
- 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
```

x)^(39/2)(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720 + 209
24013532366815360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^13) + ((72110377354780278913835520 + 145295342948683106164016640*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((172901458108932896335179801600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((370557652515461812186329087129600 + 645802967231886306826540424448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((696175598675973438759010577554944000 + 1088028437838790621809440473088716800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 1410229454280293592108580217248432347955200*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((1283308803395067168818807997696073436639232000 + 421439161286999121770135584246204836237312000*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(359909043739097249991695788946258930146664448000 - 1443636121324398194831693460992758930913796096000*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-1152021624816869759475691381872221626869209284608000 - 3040089329780519199031170166260953381570260254720000*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-2255746282697145245681128263365627409125133109002240000 - 2609695511325529255410382651665073470845732989009920000*x)/(588*(3 - 2*x)^(39/2)*(1
 + x + 2*x^2)^3) + ((-1790251120769313069211522499042240401000172830460805120000 + 115668033214143596894295804604678509924267822733393920000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((72870860924910466043406356900947461252288728322038169600000 + 2464467090087282692969213073458776810025190662610343034880000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-530550566665897087493026465460148012491929957574880460800000/(3 - 2*x)^(39/2) + (-1708089006242241264480481073293611769771298388785813753364480000/(37*(3 - 2*x)^(37/2)
) + (-696740950089909200017539783692427216704271188038402697920512000/(3 -
2*x)^(35/2) + (757366667762147355602446006474261151597409525795681824661504000000/(3 - 2*x)^(33/2) + (6167726649054233403507372547934021941920835094010816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196645778610361943940861637274650890661068800000/(29*(3 - 2*x)^(29/2)) + (4496423323436580179825935667807239175646629240803415910250222313472000000/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352841560457216000/(3 - 2*x)^(25/2) + (42926886721523802306414887155091882259902542088067698170622802545418240000000/(3 - 2*x)^(23/2) + (2893692593980364723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (118767476492930264374166633243140666046068763101817907661320807641190359040000000/(3 - 2*x)^(19/2) + (-2313064137166228597053737241416368284722516912423159767489332810437803253760000000/(3 - 2*x)^(17/2) + (-992239519653790860422623948957964852355985846800936213338418761762097950023680000000
/(3 - 2*x)^(15/2) + (-10941518315154632243157241587901809625083601209973176

```
69014678416546026147551232000000000/(3 - 2*x)^(13/2) + (-8073268485314233063
8403379340954315600692165352258493007480189439306347456219136000000000/(3 -
2*x)^(11/2) + (-44337987226211231305207361494572283981715203938096393248399
66665118399975472138240000000000/(3 - 2*x)^(9/2) + (-18330190892216697744173
706790143700087358561576136178754174544727578117325359791923200000000/(3 -
2*x)^(7/2) + (-553541210002735957048844214716028245499086746401723523324780
66055766166841372505894912000000000/(3 - 2*x)^(5/2) + (-113323856633918397403
43974428370683887566771471384841151672642393999283182139266339840000000000/
(3 - 2*x)^(3/2) + (-1327220262908131487403839635355234271426655189754352930
643567772364100886403624675133440000000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-1858108368071384082365375489497327979997317265656094102900994881
3097412409650745451868160000000000 - (38534140062781031467679876224014966993
36335555921865837542016885265897482833115690092544000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-185810836807138408236537548949732797999731726565
60941029009948813097412409650745451868160000000000 + (3853414006278103146767
9876224014966993363355559218658375420168852658974828331156900925440000000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724
```

fricas [B] time = 151.30, size = 2763, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

```
[Out] 1/3921648664331345914522657007853836460216378489907933387059429724290507410
817719106130676765545673073138207606319477907054479185225953224252061648525
72160*(616525316537858546962128448983043227187951381815778781478549978900*5
795904991921858556653045419515717067178458593845454142080244780765852057823
32794174344701326^(1/4)*sqrt(1286846088246304897035842171743217850345005139
4)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754
688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*
x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112
*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727
296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 14912870282
33404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 27862
74020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*
x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 28615445794952
97024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 17127610
05459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 79
```

0511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^2
2 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^1
9 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16
- 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 18
2213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 265002817
27302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3
755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2
+ 19758444939*x + 3486784401)*sqrt(3781484028801678888003468129339153727662
345024772741260943*sqrt(14) + 141490223718487283855707890366841241012101616
40127797919744)*arctan(1/34885554762731597076008789349408244975617249636749
132425750095898949140452865810818124470791304767731061126710516699978714580
822916583226301682355823209315648798319267851525748818094906005095731630992
22783843446054688985482057622250395943920813921700*579590499192185855665304
541951571706717845859384545414208024478076585205782332794174344701326^(3/4)
*sqrt(1634857335323112850812492677092639503349451327418417311)*sqrt(6434230
4412315244851792108587160892517250256970)*sqrt(1286846088246304897035842171
7432178503450051394)*sqrt(5795904991921858556653045419515717067178458593845
45414208024478076585205782332794174344701326^(1/4)*sqrt(1286846088246304897
0358421717432178503450051394)*sqrt(-2*x + 3)*sqrt(3781484028801678888003468
129339153727662345024772741260943*sqrt(14) + 141490223718487283855707890366
84124101210161640127797919744)*(8061110911143276053983022787*sqrt(14) - 302
97118912219360725028693061) - 210380976680132535569563443287236823905478719
259451204168457324874865216162080856741370745650892815340*x + 9637320505996
21794425456308219340060829468062999882820661390*sqrt(1667893719659639595810
98742817586289130679764812156476721038706576007991289033281726) + 315571465
020198803354345164930855235858218078889176806252685987312297824243121285112
056118476339223010)*(30297118912219360725028693061*sqrt(14)*sqrt(7) - 11285
5552756005864755762319018*sqrt(7))*sqrt(37814840288016788880034681293391537
27662345024772741260943*sqrt(14) + 1414902237184872838557078903668412410121
0161640127797919744) - 1/33164172268077541576042406944735803543071184128057
805445740643992848947205475131833297639875732592434272266883677954804521721
584006729715127306903510*57959049919218585566530454195157170671784585938454
5414208024478076585205782332794174344701326^(3/4)*sqrt(12868460882463048970
358421717432178503450051394)*(30297118912219360725028693061*sqrt(14)*sqrt(7
) - 112855552756005864755762319018*sqrt(7))*sqrt(-2*x + 3)*sqrt(37814840288
01678888003468129339153727662345024772741260943*sqrt(14) + 1414902237184872
8385570789036684124101210161640127797919744) + 2/7*sqrt(14)*sqrt(7) + sqrt(
7)) + 616525316537858546962128448983043227187951381815778781478549978900*57
959049919218585566530454195157170671784585938454541420802447807658520578233
2794174344701326^(1/4)*sqrt(12868460882463048970358421717432178503450051394
)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 1070649447546
88*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x
^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*
x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 5170798412127272

96*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 149128702823
3404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 278627
4020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x
^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 286154457949529
7024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 171276100
5459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790
511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22
 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19
 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 -
 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182
213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 2650028172
7302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 37
55740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 +
 19758444939*x + 3486784401)*sqrt(378148402880167888800346812933915372766623
45024772741260943*sqrt(14) + 1414902237184872838557078903668412410121016164
0127797919744)*arctan(1/882212681369915578508303477421571883476414798304262
215955022242191758442824830361504884464751899654962438205380446610027450746
192342098621348232500464932063975676279800152639314467699279173080080434944
06341475991998227625530289790494302092900288913988891810201600*579590499192
185855665304541951571706717845859384545414208024478076585205782332794174344
701326^(3/4)*sqrt(1634857335323112850812492677092639503349451327418417311)*
sqrt(12868460882463048970358421717432178503450051394)*sqrt(-411483036686051
02441456509058170322829014409271501775935163876370158714880*579590499192185
855665304541951571706717845859384545414208024478076585205782332794174344701
326^(1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x +
 3)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14
) + 14149022371848728385570789036684124101210161640127797919744)*(806111091
1143276053983022787*sqrt(14) - 30297118912219360725028693061) - 86568203145
318221187283609975727790977789396744211474536443644223954841516195622280522
256097905125871171599697702873041301905017935839321574155334914762936684914
90879550259200*x + 39655939073240735699697464832307040228010334971057954913
097176452275244656121253105297648773136457461603590116807858434249634034483
200*sqrt(166789371965963959581098742817586289130679764812156476721038706576
007991289033281726) + 12985230471797733178092541496359168646668409511631721
180466546633593226227429343342078338414685768880675739954655430956195285752
690375898236123300237214440502737236319325388800)*(302971189122193607250286
93061*sqrt(14)*sqrt(7) - 112855552756005864755762319018*sqrt(7))*sqrt(37814
84028801678888003468129339153727662345024772741260943*sqrt(14) + 1414902237
1848728385570789036684124101210161640127797919744) - 1/33164172268077541576
042406944735803543071184128057805445740643992848947205475131833297639875732
592434272266883677954804521721584006729715127306903510*57959049919218585566
5304541951571706717845859384545414208024478076585205782332794174344701326^(
3/4)*sqrt(12868460882463048970358421717432178503450051394)*(302971189122193

```
60725028693061*sqrt(14)*sqrt(7) - 11285555275600586475576231901 8*sqrt(7))*s
qrt(-2*x + 3)*sqrt(37814840288016788880034681293391537276623450247727412609
43*sqrt(14) + 141490223718487283855707890366841241012101616401277979197 44)
- 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 131989413465*5795904991921858556653045 4
19515717067178458593845454142080244780765852057823327941743447013 26^(1/4)*s
qrt(12868460882463048970358421717432178503450051394)*(777850730975521785282
73174023006281340291888982044945057020560246046 72*x^58 - 159459399849981965
98296000674716287674759837241319213736689214850439577 6*x^57 + 1514864298574
82867683812006409804732910218453792532530498547541079175987 2*x^56 - 8922920
19770295427942453647511410804824823331485283075985347101722463436 8*x^55 + 3
70495164730709623341323144944955355916394035464541451115654992236935905 28*x
^54 - 1180347160935271234571702270675420597251967385236510329863225459561 12
826368*x^53 + 30915008837150181267027945667854586308468743142492823964166 23
78993516544000*x^52 - 702981321957772306733839830751157952666084707511427 57
92779434714800806 95296*x^51 + 143463306723712355468305112439226911663471236
034390984807425153131791305932 8*x^50 - 266826950559017228004904436710909000
22861104795582155581211619349600413941 76*x^49 + 457384120744655026269982119
71750106503531633604394327579162124902780779888 64*x^48 - 731617424135086661
98700167998340894258382768146404489814486698266055661322 24*x^47 + 110096075
22130108303720327150964714549103431151620256934238701471906607923200*x^46 -
 15643741584311556183830093683288060491254305358060645835780583727821171458
048*x^45 + 21100253525322245323369387355308423247443333718020271970799243 11
9655863189504*x^44 - 271651277558601625195627765823180653075779937594404074
1273107966425285892505 6*x^43 + 3343486061448879744556984802283684649070432 9
307177012620295046744354886516736*x^42 - 3942305345222015457641741476722802
06985025020670942872732323950282704053862 40*x^41 + 4472712102048365545769 71
63684573632669291499878162600868891481089937948803072*x^40 - 48867241641804
49109058887543828561168102105844573245663537517394749480868249 6*x^39 + 5141
89405125347735489480063031695833076028700966156531804416445886706986516 48*x
^38 - 5232158437337392140510238565610585273747288618769796862678103801339 29
69793536*x^37 + 5151166309751303829841801730228004104255520220166332695790 9
758403045704597504*x^36 - 48981060351917473116168661974190599957194981022 28
5273133557262743720935030784*x^35 + 451996214903394043451534985348179466091
56163695625193479550387316578561556480*x^34 - 404880582733214195930596097 36
2336238083826588763975168633651775570985940418 56*x^33 + 350560838720748004 8
706756785524906417883586889172278929164021930462584556748 8*x^32 - 295492464
301465825833445981662612329746797874010510009322612207469969182556 16*x^31 +
 24233893783873994511070788336925006580407774312144520588033549546084404035
584*x^30 - 191782567946630073729270559454181267423318375916063632871103230 7
2444760326144*x^29 + 148414880649560666743967289911504439321719723677103997
2745088667423745245184 0*x^28 - 1118495816568042648301750465576222743472727 9
7340381879341496926504615810170 88*x^27 + 808971663642646090421527072505241 0
86773775332496355715423711923880231069286 4*x^26 - 577809432215066768352460 4
05565609178817603203484066718688992476626078715084 8*x^25 + 4001981217534875
0941930164544740009926063843881646165362756970090082934456 32*x^24 - 2633329
```

69512268109981550996833745185555279818648244876840348760007175779l232*x^23
+ 1754236689732225325108990352981365760714551507499414546495918038905132154
880*x^22 - 110527023465119560828545083126526137277054522543239186566677257l
865191809024*x^21 + 657767960709374731018983428860987490965225871389829139l
668030557060721541l2*x^20 - 41868450855170421700463282020153827146482312561
5024056822995425518287847424*x^19 + 229225420425444294191147646176341699520
323501828496374466278758575222292480*x^18 - 1265190599528928078062208761554
97075319504102984981603829222251966222041088*x^17 + 78935884826693368065254
828931908211891519394127734424705049022825815343104*x^16 - 3249817795278117
5771570102356850846229225588393916533119446726744429559808*x^15 + 204918859
47010913333756876708782289162249577789257433177529423102866358272*x^14 - 10
70934193648787869612347546010978156666623212627159296329711543032532l728*x^
13 + 2578142151849856182075133918988458447112007949561957497307465546935042
048*x^12 - 3407053606551726299099037573789747390828310557439249471191823118
374010880*x^11 + 3614966874364248039306939047787007807694904874158409849416
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840476870319621019208253440*x^7 + 14699921365223365688584533151457386962024
5040110547854481309077147222016*x^6 + 5314005372292355561192939243583419827
5650668430851979724323340159025152*x^5 + 2575996506690501628749016279663913
8798257488660077636512705521601478656*x^4 + 6561500535909768299643720712351
478750499548998321662130594802672074752*x^3 + 19788652409886602808449316434
07588829515736999493834610814305567768576*x^2 - 378148402880167888800346812
9339153727662345024772741260943*sqrt(14)*(549755813888*x^58 - 1126999418470
4*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^5
4 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51
+ 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x
^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 11056411653879889
92*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 23630509399
01804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 34537
53931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37 +
3640651752731836416*x^36 - 3461798212247617536*x^35 + 3194540251789393920*
x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 20884302571277
68064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 10489408
86155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 4083
74103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 + 1
23982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20 -
29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5
578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 7568
96247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 2554923428
1774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 1
0389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^
3 + 139858796529*x^2 + 19758444939*x + 3486784401) + 2795626794748522834434
66797268108117189203842033755028120580564975616*x + 49334590495562167666494

1406943720206804477368294861814330436291133 44)*sqrt(37814840288016788880034
6812933915372766234502477274126 0943*sqrt(14) + 141490223718487283855707890 3
66841241012101616401277979197 44)*log(41148303668605102441456509058170322829
01440927150177593516387637015871 4880/16348573353231128508124926770926395033
49451327418417311*57959049919218585566530454195157170671784585938454541420 8
02447807658520578233279417434470 1326^(1/4)*sqrt(12868460882463048970358421 7
17432178503450051394)*sqrt(-2*x + 3)*sqrt(378148402880167888800346812933915
37276623450247727412609 43*sqrt(14) + 1414902237184872838557078903668412410 1
21016164012779791974 4)*(80611109111432760539830227 87*sqrt(14) - 30297118912
219360725028693061) - 5295153361391555319192290416119266357486968505075814
894398859634892573324132628969090213563223147240807581427299254272 00*x + 24
256513529606021838214197524700823604475704121457581896019753481444860834611
200*sqrt(166789371965963959581098742817586289130679764812156476721038706576
007991289033281726) + 794273004208733297878843562417889953623048027576137 22
34159828945233885998619894345363532034483472086121137214094888140800) - 131
989413465*579590499192185855665304541951571706717845859384545414208024478 07
658520578233279417434470 1326^(1/4)*sqrt(12868460882463048970358421717432178
503450051394)*(77785073097552178528273174023006281340291888982044945057020 5
6024604672*x^58 - 159459399849981965982960006747162876747598372413192137366
892148504395776*x^57 + 1514864298574828676838120064098047329102184537925325
3049854754107917598 72*x^56 - 892292019770295427942453647511410804824823331 4
852830759853471017224634368*x^55 + 370495164730709623341323144944955355916 3
940354645414511156549922369359 0528*x^54 - 1180347160935271234571702270675 42
05972519673852365103298632254595611282636 8*x^53 + 3091500883715018126702794
566785458630846874314249282396416623789935165440 00*x^52 - 70298132195777230
673383983075115795266608470751142757927794347148008069529 6*x^51 + 143463306
723712355468305112439226911663471236034390984807425153131791305932 8*x^50 -
266826950559017228004904436710909000228611047955821555812116193496004139417
6*x^49 + 457384120744655026269982119717501065035316336043943275791621249027
8077988864*x^48 - 731617424135086661987001679983408942583827681464044898144
866982660556613222 4*x^47 + 1100960752213010830372032715096471454910343115 16
20256934238701471906607923200*x^46 - 1564374158431155618383009368328806049 1
25430535806064583578058372782117145804 8*x^45 + 211002535253222453233693873 5
5308423247443333718020271970799243119655863189504*x^44 - 271651277558601625
19562776582318065307577993759440407412731079664252858925056*x^43 + 33434860
614488797445569848022836846490704329307177012620295046744354886516736*x^42
- 394230534522201545764174147672280206985025020670942872732323950282704053 8
6240*x^41 + 447271210204836554576971636845736326692914998781626008688914810
89937948803072*x^40 - 488672416418044910905888754382856116810210584457324 56
63537517394749480868249 6*x^39 + 5141894051253477354894800630316958330760287
009661565318044164458867069865164 8*x^38 - 523215843733739214051023856561058
527374728861876979686267810380133929697935 36*x^37 + 515116630975130382984 18
017302280041042555202201663326957909758403045704597504*x^36 - 4898106035191
747311616866197419059995719498102228527313355726274372093503078 4*x^35 + 451
996214903394043451534985348179466091561636956251934795503873165785615564 80*

x^34 - 404880582733214195930596097362336238083826588763975168633651775570985
94041856*x^33 + 35056083872074800487067567855249064178835868891722789291640
21930462584556748 8*x^32 - 29549246430146582583344598166261232974679787401051
00093226122074699691825561 6*x^31 + 24233893783873994511070788336925006580
4077743121445205880335495460844040355 84*x^30 - 1917825679466300737292705594
5418126742331837591606363287110323072444760326144*x^29 + 148414880649560666
7439672899115044393217197236771039972745088667423745245184 0*x^28 - 11184958
16568042648301750465576222743472727973403818793414969265046158101708 8*x^27
+ 808971663642646090421527072505241086773775332496355715423711923880231069 2
864*x^26 - 577809432215066768352460405565609178817603203484066718688992476 6
260787150848*x^25 + 400198121753487509419301645447400099260638438816461653 6
2756970090082934456 32*x^24 - 263332969512268109981550996833745185555279818 6
48244876840348760007175779123 2*x^23 + 175423668973222532510899035298136576 0
71455150749941454649591803890513215488 0*x^22 - 110527023465119560828545083 1
26526137277054522543239186566677257186519180902 4*x^21 + 657767960709374731 0
18983428860987490965225871389829139166803055706072154112*x^20 - 41868450855
17042170046328202015382714648231256150240568229954255182878474 24*x^19 + 229
2254204254442941911476461763416995203235018284963744662787585752222924 80*x^
18 - 126519059952892807806220876155497075319504102984981603829222251966222 0
41088*x^17 + 789358848266933680652548289319082118915193941277344247050490 22
825815343104*x^16 - 324981779527811757715701023568508462292255883939165331 1
9446726744429559808*x^15 + 204918859470109133337568767087822891622495777892
5743317752942310286635827 2*x^14 - 107093419364878786961234754601097815666 66
23212627159296329711543032532172 8*x^13 + 257814215184985618207513391898845 8
447112007949561957497307465546935042048*x^12 - 340705360655172629909903757 3
789747390828310557439249471191823118374010880*x^11 + 361496687436424803930 6
939047787007807694904874158409849416228071339458 56*x^10 - 37495307901989006
080062610714625460487937855057716050197265913068965068 8*x^9 + 3610929740999
72372832558605269197722032357635208049111095191908288299008*x^8 + 141002184
74776899700939259520212877910636498984047687031962101920825344 0*x^7 + 14699
92136522336568858453315145738696202450401105478544813090771472220 16*x^6 + 5
314005372292355561192939243583419827565066843085197972432334015902515 2*x^5
+ 25759965066905016287490162796639138798257488660077636512705521601478656*x
^4 + 65615005359097682996437207123514787504995489983216621305948026720747 52
*x^3 + 197886524098866028084493164340758882951573699949383461081430556776 85
76*x^2 - 378148402880167888800346812933915372766234502477274126094 3*sqrt(14
)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 6306386
38006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376
576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 1885833123
63618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 7781178
96260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 19
19929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^4
1 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 36340984671025233
92*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 34617982122
47617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 24776

32938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 -
 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x
^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 28284506959981372
8*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251
996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453
545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532
*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12
 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 255207
01332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 +
 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x +
3486784401) + 27956267947485228344346679726810811718920384203375502812058O5
64975616*x + 4933459049556216766649414069437202068044773682948618143304362９
113344)*sqrt(378148402880167888800346812933915372766234502477274126O943*sqr
t(14) + 1414902237184872838557078903668412410121016164012779791974４)*log(-4
1148303668605102441456509058170322829014409271501775935163876370158714880/1
6348573353231128508124926770926395033494513274184173１1*5795904991921858556６
530454195157170671784585938454541420802447807658520578233279417434470132６^(
1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x + 3)*s
qrt(378148402880167888800346812933915372766234502477274126０943*sqrt(14) + 1
414902237184872838557078903668412410121016164012779791974４)*(80611109111432
76053983022787*sqrt(14) - 302971189122193607250286930６1) - 5295153361391555
31919229041611926635748698685050758148943988596348925733241326289690902135６
32231472408075814272992542720０*x + 2425651352960602183821419752470082360447
57041214575818960197534814448608346112００*sqrt(1667893719659639595810987428１
7586289130679764812156476721038706576007991289033281726) + 794273004208733２
97878843562417889953623048027576137223415982894523388599861989434536353203４
48347208612113721409488814080０) + 325965782049849620329125967464809624391０9
74622517979131780050251025579633815640151882107995855730577６*(5285259508814
16658752513929485454513733769472507904０0*x^57 - 1098967795066273315162856０9
34212990594401837479100416００*x^56 + 10607209489316853390896228799650834948４
44579920210821120*x^55 - 63571167550234753994014104400074223346580880315719
352320*x^54 + 268751102085050752152483783816672599931031121283482910720*x^5
3 - 870946973219521114804962921504691759517713269107195904000*x^52 + 231375
802193244831242532164933608498102950607249760845824０*x^51 - 531660404716026
72904598563232929693457448867681610707763２0*x^50 + 1093544248800904726436６4
483912756043687543104378830743142４0*x^49 - 204765576911600011474715598862３7
05646599840563445635219456０*x^48 + 35302794239198802111604239039735944127４6
2536376667298856960*x^47 - 5671470898806852061310131397489198229777877710８3
53803878400*x^46 + 856402416640309357300397975158829414085522674588022535６1
856*x^45 - 12206325070017431655342522094916509561349432305907127654809６*x^4
4 + 16501806799621223134371667301124433392748840364433110309273６*x^43 - 212
76257974246990582022682382166446530855917594345740435456０*x^42 + 2622073258
52831458520928585736224018299226513096563188826112*x^41 - 30944053790611241
11186204458928150796845040115639697413242８8*x^40 + 351087306412578660000108
01921940535182606547313097270781542４*x^39 - 38355458210058624636216764567０8

```
9281813819144349131878694912O*x^38 + 40349260752084990899888351465254740391
5763268860927101370368*x^37 - 410091833382540310980618746942733242840005307
528588546801664*x^36 + 403232407441991792232348027512081003879684846626157З
08542976*x^35 - 382995579816527529641915302665409995875084862589265975O5024
0*x^34 + 3525872597668617131566801200521996486398163996101003388518408*x^33
- 315079971582181801347294250924732868231627903206246048727040*x^32 + 27231
663445939987053683693303500397381869550551828522131456O*x^31 - 228671395190
6710970208695645008757267975898161654211432775688*x^30 + 1868861116889859290
985661178440199186295261160425613892935688*x^29 - 147575029055999994839406288
7648843693901181887610273533861888*x^28 + 113537974641311616719165089124033
84693888843521618725100032O*x^27 - 8519641562323339617019718851297502630839
3874494506050046976*x^26 + 614907175198867437939779042891506812095480715428
12762022208*x^25 - 43499929568624033785147670292431465440609985987022819309
056*x^24 + 30015307199183492418426115232917702261364741866517547318384*x^23
 - 19714530664252367893694794632442175393727220660187813722224*x^22 + 12908
687419060491715559483506875260114803121732707547895900*x^21 - 8152620728427
620176711248504306621849196751343566681977176*x^20 + 4826566229889649998651
082918574281667310767186073269174097*x^19 - 2980031288821257171626437270731
358463613690258748044875631*x^18 + 1674381797717888336240082619136481913141
447194739865411447*x^17 - 8938932115161338699060832431287058759588041285935
29339933*x^16 + 539470558336347193822687371553759571054898242285358894340*x
^15 - 242275403875001443743419975934494764357192021279244664252*x^14 + 1307
86287070310326986845647168054788265093887227255620788*x^13 - 73538381632205
950970872198730312615396368885742113789428*x^12 + 2033263055373138660211729
3249018874668950007879116154590*x^11 - 185841889627321318186553873625864802
12623851120277665058*x^10 + 45785290434797442432221248640851770216520645234
34159250*x^9 - 1589976397316459177542751340814719678836965386418728758*x^8
+ 2136884518140645208822032972708844209401147725933248644*x^7 + 52743183825
2429406648106098496733847843023830337908772*x^6 + 5912933716464809804680808
56862103952285194702447206232*x^5 + 153671770129689537528196360895808154174
885919188027188*x^4 + 77286799075459568078148376312494588624748077088337625
*x^3 + 13203155064763141960070155528810313105199695006969241*x^2 + 41100428
98499321701713055782797445718557813264221007*x + 14248811486313979718769861
8852924003909944526763627)*sqrt(-2*x + 3))/(549755813888*x^58 - 11269994184
704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x
^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^
51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856
*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 110564116538798
8992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 236305093
9901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 345
3753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37
 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 319454025178939392
0*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 208843025712
7768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 104894
0886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 40
```

```
8374103192240192*x^25 + 2828450695998137728*x^24 - 1861138971949061 28*x^23 +
 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20
- 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 +
 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 75
6896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234
281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 +
 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*
x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)
```

giac [A] time = 6.84, size = 1382, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

```
[Out] 115/58208841870571046798655722449428786144094 45376*sqrt(7)*(902844422048046
918046098552144*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 2708533266144140754
138295656432*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 193466661
8674386252955925468888*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32
) + 806111091114327605398302278 7*14^(3/4)*sqrt(7)*(-8*sqrt(14) + 32)^(3/2)
+ 2714621854534854720962570898265 6*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) - 19
390156103820390864018363559 04*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arct
an(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))
/sqrt(-1/8*sqrt(14) + 1/2)) + 115/58208841870571046798655722449428786144094
45376*sqrt(7)*(902844422048046918046098552144*14^(3/4)*sqrt(2)*(sqrt(14) +
4)^(3/2) + 2708533266144140754138295656432*14^(3/4)*sqrt(2)*sqrt(sqrt(14) +
 4)*(sqrt(14) - 4) - 1934666618674386252955925468888*14^(3/4)*sqrt(7)*(sqrt(
14) + 4)*sqrt(-8*sqrt(14) + 32) + 8061110911143276053983022787*14^(3/4)*sqr
t(7)*(-8*sqrt(14) + 32)^(3/2) + 27146218545348547209625708982656*14^(1/4)*s
qrt(2)*sqrt(sqrt(14) + 4) - 1939015610382039086401836355904*14^(1/4)*sqrt(7
)*sqrt(-8*sqrt(14) + 32))*arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sq
rt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/1164176837
41142093597311444898857572288188907 52*sqrt(7)*(12897777457829241686372836 45
92*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 386933323734877250591185
093776*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 1354266
6330720703770691478282 16*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 5
64277637800293237788115950 9*14^(3/4)*(-8*sqrt(14) + 32)^(3/2) + 3878031220
7640781728036727118 08*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 1357310
92726742736048128544913 28*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqr
t(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 115/116417
68374114209359731144489885757228818890752*sqrt(7)*(128977774578292416863728
364592*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 38693332373487725059
1185093776*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 135
4266633072070377069147828216*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32)
```

 - 5642776378002932377881159509*14^(3/4)*(-8*sqrt(14) + 32)^(3/2) + 387803
1220764078172803672711808*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 135
73109272674273604812854491328*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/4
)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/241
12597431479447071556104988390860001680293888*(38591279629413862313248614614
4809805*(2*x - 3)^37*sqrt(-2*x + 3) + 4994416662656937088431754278268478521
5*(2*x - 3)^36*sqrt(-2*x + 3) + 3157104325190190818790417015768672100251*(2
*x - 3)^35*sqrt(-2*x + 3) + 129862663539742829727010168448772257537793*(2*x
 - 3)^34*sqrt(-2*x + 3) + 3907056032933059027385185682832433217956200*(2*x
- 3)^33*sqrt(-2*x + 3) + 91626342308240062913659469031676941328847688*(2*x
- 3)^32*sqrt(-2*x + 3) + 1743051839783716654458570168808933730174627004*(2*
x - 3)^31*sqrt(-2*x + 3) + 27638544507622729125093621837291437830917462708*
(2*x - 3)^30*sqrt(-2*x + 3) + 372498510070445411629537388290851713705080145
718*(2*x - 3)^29*sqrt(-2*x + 3) + 43299535169306873423374720142726663639696
51587314*(2*x - 3)^28*sqrt(-2*x + 3) + 438994445601123086236053311571438967
25828415934650*(2*x - 3)^27*sqrt(-2*x + 3) + 391609357365773780316151578457
972453648367489837454*(2*x - 3)^26*sqrt(-2*x + 3) + 30950317017588495750406
26937399363198202032753884252*(2*x - 3)^25*sqrt(-2*x + 3) + 217907196222246
81379416567825910093368668334676797780*(2*x - 3)^24*sqrt(-2*x + 3) + 137261
402924198725794062163116053277099106968046586092*(2*x - 3)^23*sqrt(-2*x + 3
) + 776171183055652545384871388553173912691352168500951876*(2*x - 3)^22*sqr
t(-2*x + 3) + 3950095526376994607880784338655934603802167995433166405*(2*x
- 3)^21*sqrt(-2*x + 3) + 18125803816832861597832766873339882118924015183338
007655*(2*x - 3)^20*sqrt(-2*x + 3) + 75083414508694050144426639977685085540
038804754309758915*(2*x - 3)^19*sqrt(-2*x + 3) + 28093265207334834351777609
0631271895235611343284275820345*(2*x - 3)^18*sqrt(-2*x + 3) + 9494495163668
91514866641779309597536478490489987954462580*(2*x - 3)^17*sqrt(-2*x + 3) +
2896666953760570249650513456393600983703549509654469117900*(2*x - 3)^16*sqr
t(-2*x + 3) + 7968283692957988567650795129108295704483768260379820818752*(2
*x - 3)^15*sqrt(-2*x + 3) + 19727494578812277658606009712831861626922226523
266435734336*(2*x - 3)^14*sqrt(-2*x + 3) + 43844103379423695842480030320760
116666491172278035172870400*(2*x - 3)^13*sqrt(-2*x + 3) + 87180772449453719
112409715850861698835279004734515297162496*(2*x - 3)^12*sqrt(-2*x + 3) + 15
4427451620079851403012035013949923367197814895239131529728*(2*x - 3)^11*sqr
t(-2*x + 3) + 242351725944359254347670713000225450988365795247877220072960*
(2*x - 3)^10*sqrt(-2*x + 3) + 334646091432259174045261099248092390902126268
663782608549888*(2*x - 3)^9*sqrt(-2*x + 3) + 403034519668261986708991686890
381317841470126237337802123264*(2*x - 3)^8*sqrt(-2*x + 3) + 418646794645473
329714896095169087072615863373434634780753920*(2*x - 3)^7*sqrt(-2*x + 3) +
369621715112196031007775193340564258755874521674193323966464*(2*x - 3)^6*sq
rt(-2*x + 3) + 272008032423513780299697431707644217391623176190273099661312
*(2*x - 3)^5*sqrt(-2*x + 3) + 162377109720555022535973021706211388170650620
411678744248320*(2*x - 3)^4*sqrt(-2*x + 3) + 755566667488842917662208922971
66603376040200755275694800896*(2*x - 3)^3*sqrt(-2*x + 3) + 2571521747914715
6311480451271603595696519278112265697558528*(2*x - 3)^2*sqrt(-2*x + 3) - 56

```
95058898488457914056616763522088045930624578769252515840*(-2*x + 3)^(3/2) +
 616047393270423249767303997369406352855404127230297374720*sqrt(-2*x + 3))/
((2*x - 3)^2 + 14*x - 7)^19 + 1/43768013439874312895399492130064309616640*(
991856055479912729664933375*(2*x - 3)^19 - 465215115289202563341931875*(2*x
 - 3)^18 + 376870004361848629670138100*(2*x - 3)^17 - 347816399209073565143
694750*(2*x - 3)^16 + 333480450533749292133360000*(2*x - 3)^15 - 3197782482
61094005065228000*(2*x - 3)^14 + 300292311231869293365336000*(2*x - 3)^13 -
 272225522279980529558298000*(2*x - 3)^12 + 235508819476507302437712000*(2*
x - 3)^11 - 192403914635036320216640640*(2*x - 3)^10 + 14687029154936715246
1094400*(2*x - 3)^9 - 103544963718981484751251200*(2*x - 3)^8 + 66520770217
483444975816704*(2*x - 3)^7 - 38308222816032989365145600*(2*x - 3)^6 + 1936
4536310461049463275520*(2*x - 3)^5 - 8351885944887834417868800*(2*x - 3)^4
+ 2950396963171184804659200*(2*x - 3)^3 - 800398003403553957642240*(2*x - 3
)^2 + 296499732880545408614400*x - 458814330239510651535360)/((2*x - 3)^19*
sqrt(-2*x + 3))
```

maple [A] time = 0.10, size = 989, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(41/2)/(2*x^2+x+1)^20,x)
```

```
[Out] -7192279694031133468210490184035/162413063254941536826606368441486568482406
4/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-7192279694031133468210490184035/
1624130632549415368266063684414865684824064/(-7+2*14^(1/2))^(1/2)*(7+2*14^(
1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2
))^(1/2))-7192279694031133468210490184035/324826126509883073653212736882973
1369648128*(7+2*14^(1/2))^(1/2)*14^(1/2)*ln(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*
(7+2*14^(1/2))^(1/2))+13457531633280790190212932747565/81206531627470768413
3031842207432842412032/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x
+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-34841686749052264833
78299702015/812065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2
)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(
1/2))+7192279694031133468210490184035/3248261265098830736532127368829731369
648128*(7+2*14^(1/2))^(1/2)*14^(1/2)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)*(7+2
*14^(1/2))^(1/2))+13457531633280790190212932747565/812065316274707684133031
842207432842412032/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^
(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-348416867490522648337829
9702015/812065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*14
^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)
)+683151246370725/30145677658996078082575630336/(-2*x+3)^(1/2)+769045155125
/100934188590388654294338048/(-2*x+3)^(9/2)+838467657280275/105509871806486
273289014706176/(-2*x+3)^(7/2)+9270470094105/1076631344964145645806272512/(
```

```
-2*x+3)^(5/2)+320421783064625/30145677658996078082575630336/(-2*x+3)^(3/2)+
8192823353/1863702218870150079292928/(-2*x+3)^(19/2)+8972680075/16675230379
36450070946304/(-2*x+3)^(17/2)+102495360575/16479051198430800701116416/(-2*
x+3)^(15/2)+122484655975/17852305464966700759542784/(-2*x+3)^(13/2)+1081587
8546425/1480368099325700262983624704/(-2*x+3)^(11/2)+1/3111898385606868039/
(-2*x+3)^(39/2)+10/2952313853011644037/(-2*x+3)^(37/2)+143/7819642097165976
098/(-2*x+3)^(35/2)+355/5266289575642392066/(-2*x+3)^(33/2)+52865/277038748
585308867472/(-2*x+3)^(31/2)+14333/32395660116830472406/(-2*x+3)^(29/2)+147
8345/1689042692987850837168/(-2*x+3)^(27/2)+475387/312785683886639043920/(-
2*x+3)^(25/2)+16575515/7006399319060714583808/(-2*x+3)^(23/2)+246866015/735
67192850137503129984/(-2*x+3)^(21/2)+13457531633280790190212932747565/16241
30632549415368266063684414865684824064*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1
/2)-(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))-13457531633280790190212932747565/1
6241306325494153682660636844148656848240664*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+1
4^(1/2)+(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))+1/301456776589960780825756303 3
6*(2672239984790337844292019294315182385216573077301785/117922622078976*(-2
*x+3)^(41/2)+1186323846453826237212517196312193819452761764018822545/391539
9561216*(-2*x+3)^(21/2)-17650942358963262675871173166229809316744939271143/
51904512*(-2*x+3)^(11/2)+807597736492641378942268937217995835353849465/1048
576*(-2*x+3)^(1/2)-55066091420817590167865401986871791412011888132876913/55
27622909952*(-2*x+3)^(31/2)+27374875289284393578691387749101269233637917471
41675/755914244096*(-2*x+3)^(33/2)-1166457217021587688420366823074349521448
8310113371105/9826885173248*(-2*x+3)^(35/2)+1264662933338272271690443076373
2665179119615389552413/25098715136*(-2*x+3)^(17/2)-259367320368504444169504
2001860835122939346700333136537/6199382638592*(-2*x+3)^(19/2)-7559301164046
82856570195190192032441294632160945523631/3915399561216*(-2*x+3)^(23/2)+853
5085022072145119870938660211240879080416346972440 59/7830799122432*(-2*x+3)^
(25/2)-6886173809894005543994516442461871486007042005189775/125627793408*(-
2*x+3)^(27/2)+136329987967245395141848253765147208279814148352958009/552762
2909952*(-2*x+3)^(29/2)+1808668971148992206490172102870787954874541181/3341
14095890432*(-2*x+3)^(57/2)-11968977253082880651292892111395530933265219/25
701084299264*(-2*x+3)^(59/2)+339556544641293541759958988614814460549873/982
6885173248*(-2*x+3)^(61/2)-64243396719140374998473027009027485263697/294806
55519744*(-2*x+3)^(63/2)+129886852748727110357425618672922324659/1133871366
144*(-2*x+3)^(65/2)-503502693505289734438057515605193725/103079215104*(-2*x
+3)^(67/2)+133883313322119397348791732981953297/824633720832*(-2*x+3)^(69/2
)-325485074800348342966673885017 8379/824633720832*(-2*x+3)^(71/2)+360433340
020130123942335063779145/5772436045824*(-2*x+3)^(73/2)-9283422370745767345 5
7978321305/192414534860 8*(-2*x+3)^(75/2)+1380572274182261258625859209942856
6280191230197271405/39307540692992*(-2*x+3)^(37/2)-100630472583456033324523
3940167063186576585913370455/10720238370816*(-2*x+3)^(39/2)-447963293570690
82297154473725670903546220392558695/9070970929152*(-2*x+3)^(43/2)+286072233
17693223698395672584150593863016075796143/29480655519744*(-2*x+3)^(45/2)-50
59022664167725408892162874688680417923742003781/29480655519744*(-2*x+3)^(47
/2)+73012476452577571533836489036461787385135079265/2680059592704*(-2*x+3)^
```

```
(49/2)-1939242920901534821454026903132433081580221023737/501171143835648*(-
2*x+3)^(51/2)+490738543064879423955077165987434152441563270473/100234228767
1296*(-2*x+3)^(53/2)-55011835288361289002011693179378316699033102675/100234
2287671296*(-2*x+3)^(55/2)-22397546321209486953062074374795737299957063565/
3145728*(-2*x+3)^(3/2)+404531566689883337048499233527781983599187634017/125
82912*(-2*x+3)^(5/2)-1188598027552254830082683218064697188605612952419/1258
2912*(-2*x+3)^(7/2)+3831583379166294091823572953989993625772471445345/18874
368*(-2*x+3)^(9/2)+9977850126168010187169130424774568330973123412551261/215
92276992*(-2*x+3)^(13/2)-12556967184995885809797263315720723203579692970777
45/2399141888*(-2*x+3)^(15/2))/(14*x+(-2*x+3)^2-7)^19
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{20}\left(-2\,x+3\right)^{\frac{41}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

mupad [B] time = 0.97, size = 1017, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

```
[Out] ((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3)
/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x -
3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552588
571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)/755
47389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (3250820527817
55904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^11)/272
428464995505 + (17726678744562203264*(2*x - 3)^12)/6992330601551295 - (1432
471149647610304*(2*x - 3)^13)/332968123883395 + (2043463601243388704*(2*x -
 3)^14)/241114848329355 - (96972768477343976816*(2*x - 3)^15)/4840844262612
435 + (10833870670122545927656*(2*x - 3)^16)/181389282075536535 - (44340157
049832305729324*(2*x - 3)^17)/181389282075536535 + (69150977813218626180728
2*(2*x - 3)^18)/423241658176251915 - (13577358331537082239703407*(2*x - 3)^
19)/423241658176251915 + (5094959438589599396407530394650672614981*(2*x - 3
)^20)/203594616979243053623625646080 + (47475340273724148225749886260884632
526403*(2*x - 3)^21)/203594616979243053623625646080 + (54736240672766734586
8176230754600752341499*(2*x - 3)^22)/518240843219891409223774371840 + (1363
```

```
2173991688467418032505314434961676475559*(2*x - 3)^23)/438511482724523500112
424468480 + (40035704814224807138997531075224002020138815 9*(2*x - 3)^24)/59
8568173918974577653459399475 20 + (16780353218671061875177851217431652450855
3291*(2*x - 3)^25)/1496420434797436444133648498688 0 + (5110877106069831531 9
124863093548144195799415067*(2*x - 3)^26)/33519817739462576348593726370611 2
0 + (393987083187206735082003889381221664346090053*(2*x - 3)^27)/2280259710
16752220058460723609 60 + (194509919512254900809288150922829785396777195281*
(2*x - 3)^28)/1168896208350489841899678663180288 0 + (399049412174158598496 7
811280954752578787283887 1677*(2*x - 3)^29)/288717363462570990949220629805 53
11360 + (298202908298252068565416529654031351573999658954519*(2*x - 3)^30)/
29783475388770481171603812337833738240 + (172783707178371264987902065794355
72552986029824411*(2*x - 3)^31)/27075886717064073792367102125303398 40 + (13
6589909140623157483229616961110867609087469195457*(2*x - 3)^32)/37906241403
88970330931394297542475776 0 + (121244485102821322131210667779257215167467 72
830847*(2*x - 3)^33)/66893367183334770545848134662514278 40 + (5268103225464
00392428459875677051456589568282412 9*(2*x - 3)^34)/645866993494266750097844
0588104826880 + (6171761009286202626631300590201603951028773271141 3*(2*x -
3)^35)/18730142811333735752837477705503997952 0 + (236279120368028123256783 3
349111778791410563265773 87*(2*x - 3)^36)/19722680293643728587603224387334 12
433920 + (1006918289966448819369741773577875830109223667348001*(2*x - 3)^37
)/25639484381736847163884191703534361640960 + (834315251412234134041251370 6
840068954518337251868859*(2*x - 3)^38)/717905562688631720588757367698962125
946880 + (669016452611293431036170511813057767424944839195492 3*(2*x - 3)^39
)/215371668806589516176627210309688637784064 0 + (305581065207833944849384 01
5433140408874881230574613*(2*x - 3)^40)/4074599139584125981719974249102217 4
7159040 + (7318673393711958469818414571768081348141036133 09*(2*x - 3)^41)/4
47758147207046811178019148252990930944 0 + (981565361121154923229041462906 93
53244130713267641*(2*x - 3)^42)/305594935468809448628998068682666310369280
+ (1119980151725948167868728714185939040414513261 7*(2*x - 3)^43)/1971580228
831028700832245604404298776576 + (136564747272427838170630719416707180545 54
74221*(2*x - 3)^44)/151422305976050793637586261153308213248 0 + (40305011659
0493478621865418191675419450056550 1*(2*x - 3)^45)/3146219024169055378914292
315074292875264 0 + (142800962844555649098866729552205491584243363 1*(2*x - 3
)^46)/880941326767335506096001848220802005073 92 + (160089053926633694221849
84640884245768260362 1*(2*x - 3)^47)/8809413267673355060960018482208020050 73
92 + (2100199814096720892415827167854475800682460389*(2*x - 3)^48)/11716519
646005562231076824581336666667483136 + (731521029491460764762993572365861 79
9703833*(2*x - 3)^49)/474353022105488349436308687503508771962 88 + (14527825
0114246808817452879440670605483477*(2*x - 3)^50)/12695919121058658764324732
518476264190771 2 + (305417624689119903340176820462205459591 7*(2*x - 3)^51)/
423197304035288625477491083949208806359 04 + (432262412155969602358390378764
52347793*(2*x - 3)^52)/113937735701808476090093753370940832481 28 + (1675721
416942126574649271075659765 75*(2*x - 3)^53)/10357975972891679644553977579 17
643931648 + (93575614509520833338644427364290699 9*(2*x - 3)^54)/17401399634
45802180285068233301641805168 64 + (325001551972552320039960952878829 9*(2*x
```

- 3)^55)/248591423349400311469295461900234543595552 + (3599107111994336580301763675359455*(2*x - 3)^56)/17401399634458021802850682333016418051 6864 + (927027754781476746208047620505*(2*x - 3)^57)/580046654481934060095022744433880 60172288 + 79953920/10101)/(5976303958948914397184*(3 - 2*x)^(39/2) - 56774887610014686773248*(3 - 2*x)^(41/2) + 263597692475068188590080*(3 - 2*x)^(43/2) - 796876101097706139353088*(3 - 2*x)^(45/2) + 1763207861643670399942656*(3 - 2*x)^(47/2) - 3043249843014358669590528*(3 - 2*x)^(49/2) + 4264137522753475514499072*(3 - 2*x)^(51/2) - 4984324075408572529754112*(3 - 2*x)^(53/2) + 4956568063057422401458176*(3 - 2*x)^(55/2) - 4255315771373708518529024*(3 - 2*x)^(57/2) + 3189779613484873345291264*(3 - 2*x)^(59/2) - 2106235539086912777861632*(3 - 2*x)^(61/2) + 1233708448609783150169088*(3 - 2*x)^(63/2) - 644615788666077029453568*(3 - 2*x)^(65/2) + 301787157080763250721664*(3 - 2*x)^(67/2) - 127037834354660188150464*(3 - 2*x)^(69/2) + 48214067552985728953272*(3 - 2*x)^(71/2) - 16530947936007918636468*(3 - 2*x)^(73/2) + 5127550624086495626518*(3 - 2*x)^(75/2) - 1440010379792375040419*(3 - 2*x)^(77/2) + 366253616006178259037*(3 - 2*x)^(79/2) - 84341571102081217533*(3 - 2*x)^(81/2) + 17570724326889842913*(3 - 2*x)^(83/2) - 3306899061710229804*(3 - 2*x)^(85/2) + 561126236614140036*(3 - 2*x)^(87/2) - 85611621840452988*(3 - 2*x)^(89/2) + 11703514272799272*(3 - 2*x)^(91/2) - 1427192816292922*(3 - 2*x)^(93/2) + 154386157043846*(3 - 2*x)^(95/2) - 14711313018374*(3 - 2*x)^(97/2) + 1223975378934*(3 - 2*x)^(99/2) - 87916389372*(3 - 2*x)^(101/2) + 5372380188*(3 - 2*x)^(103/2) - 273870408*(3 - 2*x)^(105/2) + 11333994*(3 - 2*x)^(107/2) - 365883*(3 - 2*x)^(109/2) + 8645*(3 - 2*x)^(111/2) - 133*(3 - 2*x)^(113/2) + (3 - 2*x)^(115/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)*8174286676615564254062463385463197516747256637092086555i - 26470388201611752216024276905374076093636415173409188826601)^(1/2)*12432068249297696284897249001366340523282983937937427139335625i)/(546445444973747744833043391094451536038531013369836763902595689946460970498681963431302609552363376731445542355 7204931772416*((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062903152597529420137587598125i)/27322272248687387241652169554722576801926550668491838195129784497323048524934098171565130477618168836572277117786024658862 08 + 777520974123765253499790235238946338576340593433713638916736753944556393731049211251145625/39031817498124838916645956506746538288466500954988340278756834996175783607048711673664472110883098337960395882551463798 3744)) + (12432068249297696284897249001366340523282983937937427139335625*7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*8174286676615564254062463385463197516747256637092086555i - 2647038820161175221602427690537407609363641517340918882 6601)^(1/2))/(546445444973747744833043391094451536038531013369836763902595689946460970498681963431302609552363376731445542355720493177 2416*((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062903152597529420137587598125i)/27322272248687387241652169554722576801926550668491838195129784497323048524934098171565130477618168836572277117786024658862 08 + 777520974123765253499790235238946338576340593433713638916736753944556393731049211251145625/390318174981248389166459565067465382884665009549883402787568349961757836070487116736644721108830983379603958825514637983744)))*(- 7^(1/2)*8

```
17428667661556425406246338546319751674725663709208655 5i - 264703882016117522160242769053740760936364151734091888266601)^(1/2)*115i)/8120653162747076841330318422074328424 12032 + (atan(((3 - 2*x)^(1/2)*(7^(1/2)*817428667661556425406246338546319751674725663709208655 5i - 264703882016117522160242769053740760936364151734091888266601)^(1/2)*1243206824929769628489724900136634052328298393793742713933 5625i)/(54644544497374774483304339109445153603853101336983676390259568994646097049868196343130260955236337673144554235572049317724 16*((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062903152597529420137587598125i)/27322272248687387241652169554722576801926550668491838195129784497323048524934098171565130477618168836572277117786024658862 08 - 7775209741237652534997902352389463385763405934337136389167367539445563937310492112511456 25/390318174981248389166459565067465382884665009549883402787568349961757836070487116736644721108830983379603958825514637983744)) - (12432068249297696284897249001366340523282983937937427139335625*7^(1/2)*(3 -
 2*x)^(1/2)*(7^(1/2)*817428667661556425406246338546319751674725663709208655 5i - 264703882016117522160242769053740760936364151734091888266601)^(1/2))/(5464454449737477448330433910944515360385310133698367639025956899464609704986819634313026095523633767314455423557204931772416*((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062903152597529420137587598125i
)/27322272248687387241652169554722576801926550668491838195129784497323048524934098171565130477618168836572277117786024658862 08 - 7775209741237652534997902352389463385763405934337136389167367539445563937310492112511456 25/3903181749812483891664595650674653828846650095498834027875683499617578360704871167366447211088309833796039588255146379837 44))*(7^(1/2)*817428667661556425406246338546319751674725663709208655 5i - 264703882016117522160242769053740760936364151734091888266601)^(1/2)*115i)/81206531627470768413303184220743284 2412032
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Timed out
```

3.50 $\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=378

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{\cdots}{117600\,(\cdots}$$

```
[Out] 1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+25
78034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3)^
(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+10*x
)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2*x^2
+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/343000*(88
22+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-3088870*x
)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587773-620
347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+1103204757
41093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-1059547098661226315O+772243330
1876572160*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+312239803*2
^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(151363871237318045+110320
475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(10595470986612263150+772243
3301876572160*2^(1/2))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{\cdots}{117600\,(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
```

```
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
```

```
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx = -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-1235+1335x-800x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^4}\,dx}{1400}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+\right.}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+\right.}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+\right.}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} +$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{280\left(3-\right.}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{3031}{686000}$$

Mathematica [**C**] time = 6.13, size = 342, normalized size = 0.90

$$-9i\sqrt{50+10i\sqrt{7}}\,\left(932587773\sqrt{7}-299844895i\right)\sqrt{x^2-2x+3}\,\left(2x^4-3x^3+5x^2+x+3\right)^4\tanh^{-1}\left(\frac{\left(-5-i\sqrt{7}\right)x+i\cdots}{\sqrt{50+10i\sqrt{7}}\,\sqrt{x\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] (560*(-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3
- 1409335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952
*x^7 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 3999656
132532*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 +
 188603773872*x^15 - 38639385552*x^16 + 4596238560*x^17) - (9*I)*Sqrt[50 +
(10*I)*Sqrt[7]]*(-299844895*I + 932587773*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 +
 x + 5*x^2 - 3*x^3 + 2*x^4)^4*ArcTanh[(13 + I*Sqrt[7] + (-5 - I*Sqrt[7])*x)
/(Sqrt[50 + (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])] + 9*Sqrt[50 - (10*I)*Sqrt
[7]]*(299844895 - (932587773*I)*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 + x + 5*x^2
 - 3*x^3 + 2*x^4)^4*ArcTanh[(-13 + I*Sqrt[7] + (5 - I*Sqrt[7])*x)/(Sqrt[50
- (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])])/(691488000000000*(3 - 2*x + x^2)^(
9/2)*(1 + x + 2*x^2)^4)
```

fricas [**B**] time = 1.03, size = 1873, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/71086524447232167580280752950240000000000*(264602060868765123019815591460
74412800*x^18 - 2116816486950120984158524731685953024 00*x^17 + 1018717934344
74572362629002712386489280 0*x^16 - 32149150395554962446907594362480411552 00
*x^15 + 768834363111805660574451677938124656920 0*x^14 - 1398091139115337718
7559506313807067863200*x^13 + 2097798213825178490941475486049712039800 0*x^1
2 - 2571270526492225082945058010019781063840 0*x^11 + 287572827277934795261 9
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 1804586055124978138995142 33
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
4096608462190931685680490685615200*x^5 - 360204287698287824 4*33780221308347
3608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16
- 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10
- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*
x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 22064095148
```

```
2187776)*arctan(1/964393622349963919677467835514205441102895152270484353118
304*sqrt(205487899)*(120712108677220094151311009251129 40*sqrt(4167294734812
9)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(205487899)*(5*337802213083473608
^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876
830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(201502465*sq
rt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) +
 24142421735444018830262201850225 88*sqrt(41672947348129)*sqrt(7)*(125*sqrt(
2) + 172))*sqrt(164483605088694913184970968*x^2 + sqrt(205487899)*(33780221
3083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 4
2834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x -
 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 22
0640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x + 3)*(4*x + 1
) - 123362703816521184888728226*x + 205604506360868641481213710*sqrt(2) + 2
87846308905216098073699194) + 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10*x - 19) +
9*x - 29) + 1/11491792746071352963204808080707518 88*sqrt(205487899)*(5*3378
02213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349) - 573381349
*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(sqrt(2)*(20
1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^(
3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*3378022130834736
08^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*x + 3))*sqr
t(151363871237318045*sqrt(2) + 220640951482187776) - 1/476*sqrt(x^2 - 2*x +
 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) + 172)) + 1/
476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) - 3602042876982878244*3378
02213083473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 +
 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 1
7389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*
x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 2
20640951482187776)*arctan(-1/9643936223499639196774678355142054411028951522
70484353118304*sqrt(205487899)*(120712108677220094151311009251129 40*sqrt(41
672947348129)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) - sqrt(205487899)*(5*3378022
13083473608^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381
349) + 2876830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(2
01502465*sqrt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 2206409514
82187776) + 24142421735444018830262201850225 88*sqrt(41672947348129)*sqrt(7)
*(125*sqrt(2) + 172))*sqrt(164483605088694913184970968*x^2 - sqrt(205487899
)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*
sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(8
9801606*x - 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*s
qrt(2) + 220640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x +
3)*(4*x + 1) - 123362703816521184888728226*x + 205604506360868641481213710*
sqrt(2) + 287846308905216098073699194) - 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10
*x - 19) + 9*x - 29) + 1/11491792746071352963204808080707518 88*sqrt(2054878
99)*(5*337802213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349)
- 573381349*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(
sqrt(2)*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213
```

```
083473608^(3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*33780
2213083473608^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*
x + 3))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) + 1/476*sqrt(
x^2 - 2*x + 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) +
 172)) - 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 9*33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(7)*(3530255223715004416*x^18 -
28242041789720035328*x^17 + 135914826113027670016*x^16 - 428926009681373036
544*x^15 + 1025759783440690970624*x^14 - 1865298603830415458304*x^13 + 2798
830469551551938560*x^12 - 3430525513645055541248*x^11 + 3836725505323763236
864*x^10 - 3640134417553133928448*x^9 + 3410447187060176453632*x^8 - 240763
4062573633011712*x^7 + 2028793548878716600320*x^6 - 969496340812733087744*x
^5 + 972364673182001528832*x^4 - 87373816786946359296*x^3 + 363395647091163
267072*x^2 - 151363871237318045*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16 - 19
44*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16
498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3
+ 1647*x^2 + 162*x + 243) + 35743834140114419712*x + 53615751210171629568)*
sqrt(151363871237318045*sqrt(2) + 220640951482187776)*log(19083512352618334
93759852130293986O992*x^2 + 2369114176935798061127434 24/2041974420058321*sq
rt(205487899)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(89801606*sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)
*(sqrt(2)*(89801606*x - 132636591) - 42834985*x + 222438197))*sqrt(15136387
1237318045*sqrt(2) + 220640951482187776) - 477087808815458373439963032573 49
65248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14312634264463751203198890977204895 74
4*x + 238543904407729186719981516286748262 40*sqrt(2) + 33396146617082086140
797412280144756736) - 9*337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*s
qrt(7)*(3530255223715004416*x^18 - 28242041789720035328*x^17 + 135914826113
027670016*x^16 - 428926009681373036544*x^15 + 1025759783440690970624*x^14 -
 1865298603830415458304*x^13 + 2798830469551551938560*x^12 - 34305255136450
55541248*x^11 + 3836725505323763236864*x^10 - 3640134417553133928448*x^9 +
3410447187060176453632*x^8 - 2407634062573633011712*x^7 + 20287935488787166
00320*x^6 - 969496340812733087744*x^5 + 972364673182001528832*x^4 - 8737381
6786946359296*x^3 + 363395647091163267072*x^2 - 151363871237318045*sqrt(2)*
(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*
x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x
^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243) + 357438341401
14419712*x + 53615751210171629568)*sqrt(151363871237318045*sqrt(2) + 220640
951482187776)*log(1908351235261833493759852130293986O992*x^2 - 236911417693
579806112743424/2041974420058321*sqrt(205487899)*(337802213083473608^(1/4)*
sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 42834985) - 337802
213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x - 132636591) - 428
34985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 220640951482187776)
 - 477087808815458373439963032573496 5248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14
31263426446375120319889097720489574 4*x + 23854390440772918671998151628674 82
6240*sqrt(2) + 33396146617082086140797412280144756736) + 728813301405404935
717704569729687107 5600*x^4 - 6548901006501936794740435888653417 16800*x^3 +
```

```
2723747464067850985085226744599034867600*x^2 + 5756926178104321961473983880
*(4596238560*x^17 - 38639385552*x^16 + 188603773872*x^15 - 606785954952*x^1
4 + 1459208021718*x^13 - 2679143870481*x^12 + 3999656132532*x^11 - 49157979
13008*x^10 + 5380603084494*x^9 - 5134334619701*x^8 + 4591320676952*x^7 - 33
59813871472*x^6 + 2503427226914*x^5 - 1409335257371*x^4 + 1002897791524*x^3
 - 266966654968*x^2 + 261702502714*x - 53205422447)*sqrt(x^2 - 2*x + 3) + 2
67909586629624687057563286354003429600*x + 40186437994443703058634492953100
5144400)/(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13
 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7
 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)
```

giac [C] time = 93.77, size = 2509, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*l
og(3136*(247430153598830145135914226638091465128017779071251327216101236181
293485559300330785024470114864584026604284622700*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
- 151363871237318045)^2 + 1443342562659842513292832988722200213246770377915
632742093923877724211999095918596245976075670043406821858326965750*sqrt(7)*
(110320475741093888*sqrt(2) - 151363871237318045)^3 + 288668512531968502658
566597744440042649354075583126548418784775544842399819183719249195215134008
6813643716653931500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 20619179466569178761326185553174288760668148255937610601341769681774
4571299416942320853725095720486688836903852250*sqrt(7722433301876572160*sqr
t(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373180
45)^3 - 1049138541122969625735220807296230414367334042656225923210840932892
59251415027575686933144355006438004151420024881229481000*sqrt(7)*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 - 10491385411229696257352
208072962304143673340426562259232108409328925925141502757568693314435500643
800415142002488122948100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 1059547
0986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 209827
708224593925147044161459246082873466808531245184642168186578518502830055151
37386628871001287600830284004976245896200*sqrt(2)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373
18045)^2 - 1223994964643464563357757608512268816761889716432263577079314421
70802459984198838301422001747507511004843323362361434394500*(11032047574109
3888*sqrt(2) - 151363871237318045)^3 + 724503996256958016683144110303659048
527226579099834730629600402015843465282345034154512602754539838193840458016
13291562600472200700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339
```

```
4099672483826624728545384123596836741810096629849015435221223887188022939 34
794271550978055578969864402015298801946834493625741 25*sqrt(7)*(110320475741
093888*sqrt(2) - 151363871237318045)^2 + 1267881993449676532082187318351088
3615083124908691112050953414593589915484319515652188210530122819093311729 52
868319415989224917443750*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 1267881993449676535381256033002156963320502179376997406802245 18
0309009244646634714302734193802952986464832554399847203010615379 07975*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 - 2126452822068698508278415644474982440028 6141
3224043395080730214418993066485220006226341300837803229114327448537040 88500
18530636804886000 2880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045) - 354408803678116418845746257791902502469699160932438355128709
627960208195159816911628734559796548815662417323599879097891861163466576081
8080*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 708817607356232837491656602 9
889536476564991204751185360922127716049447858985212646112610216597117728735
33419856888790061529145933378393 1760*sqrt(2)*sqrt(7722433301876572160*sqrt(
2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045
) - 6202154064367037342040509157792939426797311401440359382371315653626279 1
8608994017058310079485948810423450333774863531359190269690122487 10900*(1103
20475741093888*sqrt(2) - 151363871237318045)^2 + 328991188809738527103804 17
819638428242533122398715783525424459521797423442868355883980260844124987688
79366524177397299680463206375263895127106986700*sqrt(7)*sqrt(2)*sqrt(772243
3301876572160*sqrt(2) - 10595470986612263150) + 575734580417042425329673296
871504559468760385065959616403976214712902997702584706455877479060569410162
15068838775424495834136105150442722719839966690*sqrt(7)*(110320475741093888
*sqrt(2) - 151363871237318045) + 1151469160834084849934842597486051102961 31
9955824917741811909355006232593712762665385200999170511603830627304190 39816
8251483607272203321761547176248 80*sqrt(2)*(110320475741093888*sqrt(2) - 151
363871237318045) + 191911526805680809288479094590285874434409383434343159 74
6460742533445595907525751026286268767668075230467160111083651969107102211 66
349350116972946360*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) - 421908346813441135720 4
41182991157503093508759989415268192312552118941115876142980847139898672274 0
737297060377427265233396895882194727754358575990795926553 20*sqrt(7)*sqrt(2)
 - 21095417340672056867029785704555913528740324247840131983737454865741140 1
6768566407000145069097979057264203096514699933182366394905499886901205587 80
166576108*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 4219083468134411371380763977036231747195786343738197611526042097212408620
8509958755710651619462561769593203578126764710489580857304433942607045015 01
50098280*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) -
 920589952140153149540860681042500826098422635875714185943723383762178092 67
9222497201117674738063662734249105558287229015094650016923796381579964775 68
878185171912773*sqrt(7) - 8145397671270700392375835131235891700241076006749
872956875216413467916040501467073084588856054007249498459026692980650865954
```

421583924771666575361939322675559157565040950966550*sqrt(2) - 92058995214015
311115855531990633917126372266976678439922113048655699796659517564449092929
141850974541546613899009112709389970185858812421123596605881876967985540965
*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 115328821454793
719911555111506838054918092483693711591974348115163386041674344938146595482
926045968304684846611487161656745020914533707787852282815214568718566601574
0023320320)^2 + 3136*(341243148066015550413679540409950260092031930837593390
918637565727519131211355910825687206864529631615130000*sqrt(7)*sqrt(2)*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 + 19905850303850907107464639857247098505368529
298859644702538580007719493206624281314984204004308951775492500*sqrt(7)*(11
0320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149292
797144941970107370585977192894050771600154389864132485626299684080086179035
50985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2843
692900550129586780662836749585500766932756979949243219797143959927600946325
902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 76858147264
002002271476515530447061472149684267202070486664776785256332938718528406617
89826971901179214250*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 768581472640020022714765155304470614721496842672020704866664
77678525633293871852840661789826971901179214 25*sqrt(7)*sqrt(772243330187657
2160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 153716294528004004542953031060894122944299368534404140973332
95535705126658774370568132357965394380235842850*sqrt(2)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 896678384746690026500559347855215717175079649784024155677 7
557291613238842850494980772088131467218042416625*(110320475741093888*sqrt(2
) - 151363871237318045)^3 + 976510829351336985318426092720643328053917556 99
900630343142766909193115243498547681531918595187516033980577557159475001400
0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
(110320475741093888*sqrt(2) - 151363871237318045) + 854446975682419864428557
715157066658147170813177379891611604541603939915295645000738134636846495148
700981051126045418060425 00*sqrt(7)*(110320475741093888*sqrt(2) - 1513638712
37318045)^2 + 170889395136483972828841572303130724558728369619768073207870 4
280682808457268940585970707216740637793591500908112081167623 0000*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 + 170889395136483973283 83
243639115145844363442349970175333057966918507272464206160163065913782547592
370164618233826987163070 00*sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 161566291032
164298591825052515595385305284590856268393753077809569128347680816594314529
25283426109315162315474502421314543 40*sqrt(7)*sqrt(2)*(110320475741093888*s
qrt(2) - 151363871237318045) - 269277151720273830400789203609786577249938 96
167132685456256065962700766147439126263654812280972255786894984252003836063
5590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 538554303440547660947974878 1

```
986245873979284291149338262149302416677335524471916239116797369312899317505
0520555382721992288O*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 471235015510
479202304702721570436483686833835069606278309401504563239251902428980452635
3306319649781934534112633453268706200*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 4193859254705817563786545204110979417249615218275883808083
699006997908745917921159286375728564093032299349286334300105037477765458575
2870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098661226
3150) + 7339253695735180775686887281247693372124287982499010522993184507695
33849414676062708934630055035134045830109366690499889371007117918795898700*
sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 146785073914703
615416086662689820169008141192682491749201237517428277642423230140157620641
9636158097776887638820246148744285006222232316501400*sqrt(2)*(1103204757410
93888*sqrt(2) - 151363871237318045) + 2446417898578393603288255436181495977
780897934081649365296599297616538778164584609859198885592495876512187088988
51314243292243938339792092100*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 12920777533
126940193458934301499103263313648173982634148974314004813143066882043781215
9214718582829755008425909786359842144150949633159991150*sqrt(7)*sqrt(2) + 7
587629160082400034098191013754109936929303905031237281898685620725368747731
984178454740590960925712315976140313059953832332538223446495209847867715972
44159013334688*x - 64603887665634701028843734757843820568795044140595103353
004459024813467698671587411875073203121114557903957207825219859233524843054
466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 1292077753312694020423002037031005650686775286243576662380536755685487926
22685475800752550765348440250164638699943637093422466523499441710582*sqrt(2
)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 36018223914820
167914680328062277541844579926085292035680717885777055540822317688033834995
13928216083726730307366377332828337773366071083598072772386051930660*sqrt(7
) - 25665325155285930274575537408488222512257955813553854680296641781933443
840848267805576609281074561095839963944023796921540484009785711469208886126
0268773750000*sqrt(2) - 758762916008240003409819101375410993692930390503123
728189868562072536874773198417845474059096092571231597614031305995383233253
822344649520984786771597244159013334688*sqrt(x^2 - 2*x + 3) - 3601822391482
016742539674834659882392115916103273640493101259172279652681774568854688273
070958284684112917867030199254538909056956640775330160129944904845080*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150) + 189691081878191176772
044460297494206938487074845844976108591897448936648822154611593038166610328
900989958412425584079869742930256624900190943273280656848430181850997)^2) -
 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
log(3136*(24743015329451280770116633045664126608658995463429055851824680263
4639192143369324082243607312634021080788865625700*sqrt(7)*sqrt(2)*sqrt(7722
433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
 - 151363871237318045)^2 + 144334256088465804492347026099707405217177473536
6694924689773015368728620836321057146421042657031789637935049483250*sqrt(7)
```

```
*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2886685121769316089846940521994148104343549470733389849379546030737457241672642114292842085314063579275870098966500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 2061917944120940064176386087138677217388249621952421320985390021955326601194744367352030060938616842339907213547 50*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 10491385490060546359808826122527787179224366895480631937633781413602974574642526920575211541954543249127414822166994500 1000*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 1049138549006054635980882612252778717922436689548063193763378141360297457464252692057521154195454324912741482216699450 0100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 2098277098012109271961765224505557435844873379096126387526756282720594914928505384115042308390908649825482964433398900 0200*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 1223994973840397075311029714294908504242842804472740392723941164920347033708294807400441346561363379064865062586149358 34500*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 7245039955607918024832901514699923520889261707004734027805955113139290083526739387706092550205856385985554024651103233600369822 9700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339409961156928288224132378329953594189192690738560013436207699616249390969017139885443790988079276553600259078650068119581303 47875*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 1267881992231385657232442886841802706002561320719976559209591615532765613996725423586023437958666981840314039628006645155934817986250*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 1267881992231385660531511597435306808684779060141860107030791010656639727573349458714545999870257969603562137129793592899786359 66225*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 2126452792712690096116437713266837331182032949571436835653998293704475706783435782714029336413768430799117811124620902432154546997980983 9680*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045) + 3544087987854483501520832752633716588014874818830708767403800505111750511407655000601839046131539324151071812411163027198287919028630782880*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 7088175975708967001043306364303519583685211328608505691145816839655761606123161160517397220902597330097341426354794006943734919549262613360*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6202153978745346139901407055512974782136328075902825385385078928672967322202807900256688667813141891268789137333169088245654484161597453 4900*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 3289911884763015221101541114405505474753270156481065387887051759144023849028573565153578955503100992856636792237219317239230096498179223616314209700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 5757345798335276665907856 7
```

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7264130666506837186992839690009782059773522448555548409958107041560138869911
6452430421414340505030603603948624318670782547790*sqrt(7)*(11032047574109388
8*sqrt(2) - 151363871237318045) + 1151469159667055332457067358967469531237‌3
2751764286647850539889300840368314932975041454381339116609430412186908901‌25
397893446827788841283652727703008 0*sqrt(2)*(110320475741093888*sqrt(2) - 15
1363871237318045) + 191911526611175889708849887242300660213938696539031563‌9
155609757570957373975372338370836193986424378585490167158630684697779577032
6107328538508811760*sqrt(77224333018765721 60*sqrt(2) - 10595470986612263150
)*(110320475741093888*sqrt(2) - 151363871237318045) + 421908326717543641924
358820689861254773878411648930379381901809299202750782331001508374803517870
6281510163553803705731857976749294117549567614800051148210 80*sqrt(7)*sqrt(2
) + 210954163358771821772256664711669711256358377483192354207771689878568 67
08945338404996962729098703004315518036561804858541608396971651230611582280 4
6039881708*sqrt(7)*sqrt(77224333018765721 60*sqrt(2) - 10595470986612263150)
 + 421908326717543643341994015831654651625296327690259460980119715812011991
659696962885724904699692807901543935675281424621198266774916716624735720821
529103720*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 920589964087560320916761581042015283905433780645215921762515407245365322 3
479396020474916109653924902956542165396460831286616319128616458259997590278
2992478976911083*sqrt(7) + 814539727961527046694837833062145864584590634452
974565084319757683318515854195479301757363227685529077360827463499277712322
1082185639753988444491134597154930322061732374 69850*sqrt(2) - 9205899640875
602825344562588068375449965750892067890031749508881924444728233783716057581
267806102568826874405209859338973827430110551686881735838388090152238069483
5*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 11532881607348
687037747868321346713994558911627676219472353978719356293894710618931600085
322372043380298052663139565536675674564328460228800898840894611032414487006
75305025280)^2 + 3136*(341243148066015550413679540409950260092031930837593 9
09186375657275191312113559108256872068645296316151300 0*sqrt(7)*sqrt(2)*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 199058503038509071074646398572470985053685 2
929885964470253858000771949320662428131498420400430895177549250 0*sqrt(7)*(1
10320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149 29
279714494197010737058597719289405077160015438986413248562629968408008617903
550985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 284
369290055012958678066283674958550076693275697994924321979714395992760094632
590214060057204413596792750 0*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 7685814726
400200227147651553044706147214968426720207048666477678525633293871852840661
789826971901179214250*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 768581472640020022714765155304470614721496842672020704866 6
477678525633293871852840661789826971901179214 25*sqrt(7)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 153716294528004004542953031060894122944299368534404140973 3
295535705126658774370568132357965394380235842850*sqrt(2)*sqrt(7722433301876
```

572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 896678384746690026500559347855215717175079649784024155677
7557291613238842850494980772088131467218042416625*(110320475741093888*sqrt(
2) - 151363871237318045)^3 + 9765108293513369853184260927206433280539175569
990063034314276690919311524349854768153191859518751603398057755715947500140
00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) + 8544469756824198644285
771515706665814717081317737989161160454160393991529564500073813463684649514
870098105112604541806042500*sqrt(7)*(110320475741093888*sqrt(2) - 151363871
237318045)^2 + 170889395136483972828841572303130724558728369619768073207870
42806828084572689405859707072167406377935915009081120811676230000*sqrt(2)*(
110320475741093888*sqrt(2) - 151363871237318045)^2 + 1708893951364839732838
324363911514584436344234997017533305796691850727246420616016306591378254759
237016461823382698716307000*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 16156629103
216429859182505251559538530528459085626839375307780956912834768081659431452
92528342610931516231547450242131454340*sqrt(7)*sqrt(2)*(110320475741093888*
sqrt(2) - 151363871237318045) - 2692771517202738304007892036097865772499389
616713268545625606596270076614743912626365481228097225578689498425200383606
35590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110
320475741093888*sqrt(2) - 151363871237318045) - 538554303440547660947974878
198624587397928429114933826214930241667733552447191623911679736931289931750
505205553827219922880*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 47123501551
047920230470272157043648368683383506960627830940150456323925190242898045263
53306319649781934534112633453268706200*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 419385925470581756378654520411097941724961521827588380808
369900699790874591792115928637572856409303229934928633430010503747776545857
52870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150) + 733925369573518077568688728124769337212428798249901052299318450769
53384941467606270893463005503513404583010936690499889371007117918795898700
sqrt(7)(110320475741093888*sqrt(2) - 151363871237318045) + 14678507391470
361541608666268982016900814119268249174920123751742827764242323014015762064
19636158097776887638820246148744285006222232316501400*sqrt(2)*(110320475741
093888*sqrt(2) - 151363871237318045) + 244641789857839360328825543618149597
778089793408164936529659929761653877816458460985919888559249587651218708898
851314243292243938339792092100*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 1292077753
312694019345893430149910326331364817398263414897431400481314306688204378121
59214718582829755008425909786359842144150949633159991150*sqrt(7)*sqrt(2) -
758762916008240003409819101375410993692930390503123728189868562072536874773
198417845474059096092571231597614031305995383233253822344649520984786771597
244159013334688*x - 6460388766563470102884373475784382056879504414059510335
300445902481346769867158741187507320312111455790395720782521985923352484305
4466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)

```
 - 1292077753312694020423002037031005650686775286243576662380536755685487922
6226854758007525507653484402501646386999436370934224665234994417105822*sqrt(
2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 3601822391482
0167914680328062277541844579926085292035680717885777055540822317688033834999
5139282160837267303073663773328283377733660710835980727723860519306600*sqrt(
7) - 2566532515528593027457553740848822251225795581355385468029664178193344
3840848267805576609281074561095839963944023796921540484009785711469208886122
60268773750000*sqrt(2) + 7587629160082400034098191013754109936929303905031222
3728189868562072536874773198417845474059096092571231597614031305995383233252
38223446495209847867715972441590133346888*sqrt(x^2 - 2*x + 3) - 360182239148
2016742539674834659882392115916103273640493101259172279652681774568854688272
30709582846841129178670301992545389090569566407753301601299449048450800*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150) - 18969037612592882493
2865090390211289907978120405716887986342383587331788564444597329698862937711
73846258403945900689178218736966545474245695491201051417736493248163477)^2)
+ 41672947348129/28000000000*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*arctan(-1/14*(424907232964614401909498696770230156468041
018681749287786326394760620649872991113993465473093811839889694663857531357
4146106221405130037317514805920944567290474674252828*x - 424907232964614401900
949869677023015646804101868174928778632639476062064987299111399346547309381
18398896946638575313574146106221405130037317514805920944567290474674252828*sq
rt(x^2 - 2*x + 3) - 106226808241153600477374674192557539117010254670437321999
4658159869015516246824777849836636827345295997242366596438283935365265553511
28250932937870148023614182261866856322*I*sqrt(20*sqrt(2) - 25) + 10622680824
1153600477374674192557539117010254670437321946581598690155162468247778498366
63682734529599724236659643828393536526555351282509329378701480236141822618666
685632)/(24045690340022365902421207642185759487659648111880595453459789716333
550520465792352560762361627662375147331666633*(sqrt(7) + 2*sqrt(2) + sqrt(772
2433301876572160*sqrt(2) - 10595470986612263150))^7 - 1427399375677509694877
78514384982726726086177451105114601508039903300578423813275603664373381633355
28980156655785698126466*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt
(2) - 10595470986612263150))^6 + 1035005708938511449879694100445735935284666
831057025915469516609398945671350435784286059468190138006696033871436116351
184730996386*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 1059
5470986612263150))^5 - 5062982909687377389957399074025778336103290716597321
5049765624930757608699456368874374542973258344666404687384663029077837715811
6904194414*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150))^4 + 109663729603246175183765204929539035364198260073648815
3626251774799452192758375873916572220994961819156070680914794516208869024577
63393164652919228477*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2)
 - 10595470986612263150))^3 - 4219083468134411342015463100849442848230569999
0476005516661368939297975317080268800700618350130102201917477482441627065788
1217996707276477846427694382052541466*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301
876572160*sqrt(2) - 10595470986612263150))^2 - 368235980856061293812100449444
2331473355861294289510039804332924533313682318158002472147635428150206679233
```

56860508683478554304309475540771033672165453813191045495483200*sqrt(7) - 73
6471961712122587624200898846629467117225885790200796086658490666273646363 16
0049442952708563004133584713721017366957108608618951081542067344330907626 38
2090990966400*sqrt(2) - 3682359808560612938121004494233147335586129428951 00
3980433292453331368231815800247214763542815020667923568605086834785543043 09
4755407710336721654538131910454954832 00*sqrt(7722433301876572160*sqrt(2) -
10595470986612263150) + 1428352105203887288279038536932322893004029548936 48
6004185262156116702645830414445699158147702914990438794861101657539550235 07
391431740254479476659166189278113070264611396886))/(110320475741093888*sqrt
(2) - 151363871237318045) - 41672947348129/28000000000*sqrt(7)*sqrt(7722433
301876572160*sqrt(2) - 10595470986612263150)*arctan(-1/14*(4249072329646144
0190949869677023015646804101868174928778632639476062064987299111399346547 30
9381183988969466385753135741461062214051300373175148059209445672904746742 52
8*x - 4249072329646144019094986967702301564680410186817492877863263947606 20
6498729911139934654730938118398896946638575313574146106221405130037317514 80
592094456729047467425 28*sqrt(x^2 - 2*x + 3) - 1062268082411536004773746741 9
2557539117010254670437321946581598690155162468247778498366368273452959972 42
366596438283935365265555351282509329378701480236141822618668563 2*I*sqrt(20*s
qrt(2) - 25) + 106226808241153600477374674192557539117010254670437321946581
5986901551624682477784983663682734529599724236659643828393536526555351282 50
9329378701480236141822618668563 2)/(2404569031044828063179458993747728533397
3756524226487708284431742919260655332295829178193130479496703672387 33*(sqrt
(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))
^7 + 1427399386402795423103241649323508459758417264691242440494392029061629
193828915227289144427476808605323457798934284966*(sqrt(7) + 2*sqrt(2) + sqr
t(7722433301876572160*sqrt(2) - 10595470986612263150))^6 + 1035005707943988
2867370455909314910366457483004227041867621777239761173739910974366657765 66
933802707362797439106953308057309305 06*(sqrt(7) + 2*sqrt(2) + sqrt(77224333
01876572160*sqrt(2) - 10595470986612263150))^5 + 5062982839792119265762213 4
4309055178192100417667234568432139026173110004577643069326166041869687015 43
7693539183969951348395828655948414 54*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301
876572160*sqrt(2) - 10595470986612263150))^4 + 1096637294921005068525485165
0818937195218777933811989455374972683830017956420742416053632102710821823 50
2991859431883138318105242943774423785470956 77*(sqrt(7) + 2*sqrt(2) + sqrt(7
722433301876572160*sqrt(2) - 10595470986612263150))^3 + 421908326717543640 4
0546396875222547376198233034565175943578631355566494016309680203161783578 04
8298604611358451911473753977621707585096472578543279534739302 6*(sqrt(7) + 2
*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^2 - 36
8235985635024162362460774967963334004243897793303578695550410507799195844 56
7196904285900746309831059944379885387481474312223533832259969580105714665 87
5528461138380*sqrt(7) - 7364719712700483247249215499359266680084877955866 07
1573911008210155983916891343938085718014926196621198887597707749629486244 47
06766451993916021142933175105692227676 0*sqrt(2) - 3682359856350241623624607
7496796333400424389779330357869555041050779919584456719690428590074630983 10
5994437988538748147431222353383225996958010571466587552846113838 0*sqrt(7722

```
433301876572160*sqrt(2) - 1059547098661226315O) - 1428352081936135929153300
3672226045153345857894604895779030735125826227723975160464191638450688998ll
4007074862972361314506243071603362131126681307483233451817484002482889448б)
)/(110320475741093888*sqrt(2) - 151363871237318045) + 1/205800000000*(10812
1281*(x - sqrt(x^2 - 2*x + 3))^15 + 135317265*(x - sqrt(x^2 - 2*x + 3))^14
- 2309618731*(x - sqrt(x^2 - 2*x + 3))^13 - 4089866767*(x - sqrt(x^2 - 2*x
+ 3))^12 + 23951599406*(x - sqrt(x^2 - 2*x + 3))^11 + 45641347654*(x - sqrt
(x^2 - 2*x + 3))^10 - 149568395690*(x - sqrt(x^2 - 2*x + 3))^9 - 2882154309
78*(x - sqrt(x^2 - 2*x + 3))^8 + 660704292769*(x - sqrt(x^2 - 2*x + 3))^7 +
 1062639157153*(x - sqrt(x^2 - 2*x + 3))^6 - 2094971437979*(x - sqrt(x^2 -
2*x + 3))^5 - 2301192104575*(x - sqrt(x^2 - 2*x + 3))^4 + 4977175786352*(x
- sqrt(x^2 - 2*x + 3))^3 + 1302994004424*(x - sqrt(x^2 - 2*x + 3))^2 - 6052
879270032*x + 6052879270032*sqrt(x^2 - 2*x + 3) + 2841437414928)/((x - sqrt
(x^2 - 2*x + 3))^4 + (x - sqrt(x^2 - 2*x + 3))^3 - 5*(x - sqrt(x^2 - 2*x +
3))^2 - 7*x + 7*sqrt(x^2 - 2*x + 3) + 14)^4 + 1/3150000000*(3*((((((((29420
*x - 332589)*x + 1860912)*x - 6743744)*x + 17167416)*x - 31960026)*x + 4336
2368)*x - 42014736)*x + 26516604)*x - 27199867)/(x^2 - 2*x + 3)^(9/2)
```

maple [B] time = 0.65, size = 21028, normalized size = 55.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^{2}+x+1\right)^{5}\left(x^{2}-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2+x+1\right)}^5\,{\left(x^2-2\,x+3\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
[Out] Timed out
```

3.51 $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=638

$$-\frac{1210549587451867106183 3-5117656435043679338190x}{1042737204880000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)}$$

```
[Out] 1/1840124479200000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^(
19/2)+1/1042737204880000000*(476849951294984711-125181871472148210*x)/(x^
2-2*x+3)^(17/2)+1/15641058073200000000000*(785175837548333511+194216499620
4584234*x)/(x^2-2*x+3)^(15/2)-11/40666750990320000000000*(7502325106308201
089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/11470109253680000000000000*(
69053268515296359011-44840736195018286006*x)/(x^2-2*x+3)^(11/2)+1/938463484
392000000000000000*(-838519439380295335657+466189390555853643870*x)/(x^2-2*x+
3)^(9/2)+1/3128211614640000000000000000*(-1117646664729238460189+568839749685
437871554*x)/(x^2-2*x+3)^(7/2)+1/52136860244000000000000000000*(-655140551156
5449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/104273720488000000
0000000000*(-4179039782398459850819+1886993445589652402694*x)/(x^2-2*x+3)^(
3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(887+2218*x)/
(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^2-2*x+3)^(19/
2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)^(19/2)/(2*x^2
+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1
)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x+3)^(19/2)/(2*x^
2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x^2-2*x+3)^(19/2)/
(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301166*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-37857197792117-146548895467025*x
)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/1042737204880000000000000000*(-12105495
874518671061833+5117656435043679338190*x)/(x^2-2*x+3)^(1/2)-1/2259801992000
000000000000000*arctanh(1/7*(272944589523248381749+x*(656826642296538601431
-464885615909893491590*2^(1/2))-191941026386645109841*2^(1/2))*35^(1/2)/(-8
1042225921274689605478944797800854846405+5730592252300170712602636387866650
0308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-5672955814489228272383526135846
059839248350+4011414576610119498821845471506655021629440*2^(1/2))^(1/2)+1/2
259801992000000000000000000*arctan(1/7*(272944589523248381749+1919410263866
45109841*2^(1/2)+x*(656826642296538601431+464885615909893491590*2^(1/2)))*3
5^(1/2)/(81042225921274689605478944797800854846405+573059225230017071260263
63878666500308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(5672955814489228272383
526135846059839248350+4011414576610119498821845471506655021629440*2^(1/2))^
(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 638, normalized size of antiderivative

= 1.00, number of steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1210549587451867106183 3-5117656435043679338190x}{104273720488000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*
x)/(156410580732000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 7813986379726516886*x))/(4066675099032000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(3128211614640000000000000*(3 - 2*
x + x^2)^(7/2)) - (655140551156544930168 9 - 3127298559983309301910*x)/(5213
686024400000000000000*(3 - 2*x + x^2)^(5/2)) - (4179039782398459850819 -
1886993445589652402694*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
0000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(8104222592127468960547894479780085484 6405 + 57
30592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
42225921274689605478944797800854846405 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(272944589523248381749 + 191941026386645109841*Sqrt[2] +
 (656826642296538601431 + 464885615909893491590*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/32282885600000000000000000 - (Sqrt[(-81042225921274689605478944797 8
00854846405 + 57305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 573059225230017 0
71260263638786665003089 92*Sqrt[2]))]*(272944589523248381749 - 1919410263866
```

```
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/32282885600000000000000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
```

```
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{3150}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x\right.}$$

$$= \frac{37358055634422583-14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}}$$

$$= \frac{37358055634422583-14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} + \frac{47684995129498471}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} + \frac{47684995129498471}{104273720488000}$$

Mathematica [C] time = 11.60, size = 1431, normalized size = 2.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*
x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3 -
 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)^7)
 + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(-731
61291 + 43964675*x))/(90843750000000000*(3 - 2*x + x^2)^5) + (-1340879383 +
 430593031*x)/(181687500000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 11
2950205*x))/(302812500000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 152
86717673*x))/(3633750000000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277
+ 484788625685*x))/(36337500000000000000*(3 - 2*x + x^2)) + (251943 + 2217
70*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200
0000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000
0000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121010400000000
0000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/
(2964754800000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 1696303
89653846945*x)/(37059435000000000000000*(1 + x + 2*x^2)^3) + (108242210919
6374795 + 4797048907791526114*x)/(830131344000000000000000*(1 + x + 2*x^2)
^2) + (65571203144429922747 + 367152793968978953465*x)/(3631824630000000000
00000000*(1 + x + 2*x^2))) + ((2324428079549467457795*I + 216341778311919248
41*Sqrt[7])*ArcTan[(-13506373886043501689958655894873325911351 5 + (18863089
4626466690216855285995045889396405*I)*Sqrt[7] - 150624136187268800855926877
6761430483700000*x - (1057115009374721927181156513503524479386 80*I)*Sqrt[7]
*x + 491153540508443587025809789813541985707360*x^2 - (460764064177139993 39
997510087266331039942 0*I)*Sqrt[7]*x^2 - 1800849851472466891994487452649776 7
8818020*x^3 + (1978682963779138708638376809534460093968 60*I)*Sqrt[7]*x^3 -
176004816500761880926774485599831047775825*x^4 - (207342833228459577163557 0
43035558264835165*I)*Sqrt[7]*x^4 + (186244248199755548159585682605666126004
224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (114611845046003414 2
52052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]
+ (300856093245758962411638410362999126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*
x^2*Sqrt[3 - 2*x + x^2] - (1432648063075042678150659096966662507724 80*I)*Sq
rt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(23687732908388369798646 78
493023884746594823*I + 4236429402592387354739426631800259567295 05*Sqrt[7] +
 (189061348606562030176007421855674531164693 6*I)*x + 615057455931122825839 4
328777942059796320*Sqrt[7]*x + (251130025985582296234089302785223915766782 0
*I)*x^2 - 2027867550801106189867763431094227596320*Sqrt[7]*x^2 - (313421774
6230760357128318797499380812303788*I)*x^3 + 6343043160272004327919286696836
```

```
9397935660*Sqrt[7]*x^3 + (9447490648866264673283853691904607036696697*I)*x^4
 + 1638131776510726478946291722103075063483 5*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/1614144280000000000000000)*(
-2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7])*ArcTan[(35*(436249
4290663946676585186218212607628595*I + 121040840074068210135412189480007416
20843*Sqrt[7] - (4091903159661733270719609450078323740 5000*I)*x + 175730701
6946065216684093936554874227 52*Sqrt[7]*x + (2648728832926512757773396585336
4310310620*I)*x^2 - 5793907288003160542479324088840650275 2*Sqrt[7]*x^2 - (1
52388941497528256839248140210078630706 20*I)*x^3 + 181229804579200123654836 7
627667697083876*Sqrt[7]*x^3 - (7958372719599758089132442037656199635 95*I)*x
^4 + 4680376504316361368417976348865928752 81*Sqrt[7]*x^4))/(135063738860435
0168995865589487332591135 15 + (1886308946264666902168552859950458893964 05*I
)*Sqrt[7] + 1506241361872688008559268776761430483700000*x - (10571150093747
2192718115651350352447938680*I)*Sqrt[7]*x - 491153540508443587025809789813 5
41985707360*x^2 - (4607640641771399933999751008726633103994 20*I)*Sqrt[7]*x^
2 + 1800849851472466891994487452649776788180 20*x^3 + (19786829637791387086 3
8376809534460093968 60*I)*Sqrt[7]*x^3 + 17600481650076188092677448559983104 7
775825*x^4 - (2073428332284595771635570430355582648351 65*I)*Sqrt[7]*x^4 - (
1432648063075042678150659096966662507724 8*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[
3 - 2*x + x^2] - (1432648063075042678150659096966662507724 8*I)*Sqrt[70*(5 +
 I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (286529612615008535630131819393332 50
154496*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(5 +
I*Sqrt[7])] - ((-2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7])*Lo
g[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(32282885600000 00
000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/3228288560000000000000000)*(2324
4280795494674579 5*I + 2163417783119192484 1*Sqrt[7])*Log[(-I + Sqrt[7] - (4*
I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288
5600000000000000000)*(2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I
)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I
*Sqrt[70*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(-5 + I*Sqrt[7]
)] + ((-2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7])*Log[(1 + x
+ 2*x^2)*(-163*I + 15*Sqrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*S
qrt[7]*x^2 - (13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sq
rt[10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(3228288560000000000000000
*Sqrt[70*(5 + I*Sqrt[7])])
```

fricas [B] time = 1.55, size = 2775, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/14344664239923376765642165360362728636455220318608571101268133326181128 98
```

```
720320000000000000000000*(36045960776272236628083717974972055111190660172853
358396135728761934386631817942748278579200*x^38 - 5587123920322196677352976
286120668542234552326792270551401037958099829927931781125983179776 00*x^37 +
 48121357636323435898491763496587693573439531330759233458841197897182406153
4769535689519032320 0*x^36 - 28710607758300836474268681367065241896063360827
67769996252210795888073895224299139900388833280 0*x^35 + 1315091821471322213
07984934566938671566290224808133871438501689421822367827858786874250861388 8
00*x^34 - 48615412586213217241775435956082154303807288316753404802358233229
525669340249308246745496508160 0*x^33 + 15012511859003801791455877071511298 9
428464641254403988382185986540923381803861163406599333616640 0*x^32 - 396012
07680724195081933453907329150443109061726945797184770092066969689498801613 7
708626219776640 0*x^31 + 909142000002142860704234021157216421398787616081889
741815089464543532901571757526703136964242800 0*x^30 - 18424764929872158270 6
98990044243761821209838303936935404825000845297214002673057816247491027262 8
00*x^29 + 33413073756673638925333625169011170445598811516221975115590200411
293389434416479555356860509015600*x^28 - 548165325604492954597175170033826 9
96732424109361143043446298421036566229344902471080122613465864 00*x^27 + 822
45983094063518667736627604663547588572840238581597325736701493749880383650 7
494011332069990144 00*x^26 - 1137228480676396944025927358626490940938740454 4
3618754295078471234595964240139128161766283626302000*x^25 + 146086574413322
24828651419255052262409847761409409548862449358151245499125807486754431889 5
241990800*x^24 - 17502709408100102168297375299741202325173630522612714427 28
11232619626419165679723993392477178363200*x^23 + 1968872916057841594334556 5
4443374481739030277196290989156609388218395099469530751149958413044135200*x
^22 - 208068683375682167383215047521697995267539026087882795784482813901791
36043479800571072261648728200 0*x^21 + 208171444918478482519618165392015730 3
47012009814583465001141378703189206795143605224483243158516400*x^20 - 19622
755618454040835316742234157685550832000179582185155831117699557408106901596
983664287853443120 0*x^19 + 17653494167772345968142228002495257303210629952 9
48281632121958532339908697647195831098140549452320 0*x^18 - 149136255738011 3
80556954829398929258737007615204074730330565887220730783382923822619571340 7
37358000*x^17 + 121890814483587724389011961696733756253105383654426234361 5
0913799569962877883235263704480534144400*x^16 - 91983186053222129635537069 2
785885803929857457307009283885263093717767401424388346073985889921952 00*x^1
5 + 6931781413247155931639013703759255706039899683834223241488937169027139 8
0980987386433144021309544 00*x^14 - 4574307084113250024797073972709329687876
58973237085936599028628836672372493907006547585746109180 00*x^13 + 329969655
21676394929803121509049143329451789049169789455644615129199190308917673348 5
1848131157480 0*x^12 - 177700837577887379719337398920499270334848900298046 51
9382701821617409378512807078348222722743544 00*x^11 + 1354422526745145970196
03692382563743518993626839784985514837298522566552641470933373925962280288 0
0*x^10 - 481375973272848865172866855106995818624092546697867179982576756873
2599092879797201593187475517200*x^9 + 509118113363902521683262010612328032 0
347641869015804163342220634415255665812683873707564839486000*x^8 - 46421311
850305640075834899457188406077339946202653776901797199608409580382714283736
```

3184426478400*x^7 + 17712338832647821260422671418114138499869713982650322359161728898790271345427524393233724292792 00*x^6 + 23911503454316320991841103252166564975049644786785360906948778644541080475499884911645233878760 0*x^5 + 7981789112999441335336293727346445509983546 8*126493875280426512381557410511779960814905727241 8^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 59049)*sqrt(8104222592127468960547894479780085 4846405*sqrt(2) + 11461184504600341425205272775733300 0617984)*arctan(1/54206850781156887023310518673090274966005685838243268724684064391985051350175945649154733957770247431673510566373712749535014372719818364352360619 68*sqrt(795279432905272963911047)*(99395132505231928164221165932167972928150165110013784621706793019 90*sqrt(1100522448786287362112823964249088884809 8)*sqrt(2888868076710542715672947094311)*sqrt(7)*(10*sqrt(2) + 9) + sqrt(1590558865810545927822094)*(5*126493875280426512381557410511779960814905727241 8^(3/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 556695603033691074737732 9*126493875280426512381557410511779960814905727241 8^(1/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(8104222592127468960547894479780085 4846405*sqrt(2) + 11461184504600341425205272775733300 0617984) + 147461812540444568715696613114138557910359478676937042172325597372869522935182724790786*sqrt(2888868076710542715672947094311)*sqrt(7)*(125*sqrt(2) + 172))*sqrt(519179873173490157373042187501297125639064382628558151181380506 4*x^2 + sqrt(1590558865810545927822094)*(126493875280426512381557410511779960814905727241 8^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682*sqrt(2) - 62135959399493560795) - 126493875280426512381557410511779960814905727241 8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(8104222592127468960547894479780085 4846405*sqrt(2) + 11461184504600341425205272775733300 0617984) - 129794968293372539343260546875324281409766095657139537795345126 6*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117618029781640625972844229298286971418613386035379 8*x + 874869761179272589826814757400740628067 45190*sqrt(1100522448786287362112823964249088884809 8) + 908564778053607775402823828127269969868362669599976764567415886 2) + 5/35309486994022006419332*sqrt(1100522448786287362112823964249088884809 8)*sqrt(7)*(sqrt(2)*(10*x - 19) + 9*x - 29) + 1/70191822769251614708671587842329953565331111850222032074026984349485892917146977000414136*sqrt(1590558865810545927822094)*(5*126493875280426512381557410511779960814905727241 8^(3/4)*sqrt(35)*(sqrt(2)*(340613697110906370000*x + 143139522536096832703) - 483753219647003202703*x - 19747

4174574809537297) + 5566956030336910747377329*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*(sqrt(2)*(4373478266460499235 5*x + 22535043916389567877) - 6626982658099456023 2*x - 21199738748215424478) - (5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt(35)*(340613697110906370000*sqrt(2) - 4837532196470032027 03) + 5566956030336910747377329*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*(43734782664604992355*sqrt(2) - 6626982658099456023 2))*sqrt(x^2 - 2*x + 3))*sqrt(810422259212746896054789447978008548464 05*sqrt(2) + 114611845046003414252052727757333000617984) - 1/3530948699402200641 9332*sqrt(x^2 - 2*x + 3)*(5*sqrt(110052244878628736211282396424908888480 98)*sqrt(7)*(10*sqrt(2) + 9) + 74179594525256316007*sqrt(7)*(125*sqrt(2) + 172)) + 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 7981789112999441335336293727346445509983546 8*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 59049)*sqrt(8104222592127468960547894479780085484 6405*sqrt(2) + 114611845046003414252052727757333000617984)*arctan(-1/54206850781156887023310518673090274966005685838243268724684064391985051350175945649154733957770247431673510566373712749535014372719818364352360 61968*sqrt(7952794329052729639 11047)*(99395132505231928164221165932167972928150165110013784621706793019 90*sqrt(110052244878628736211282396424908888480 98)*sqrt(2888868076710542715672947094311)*sqrt(7)*(10*sqrt(2) + 9) - sqrt(1590558865810545927822094)*(5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 5566956030336910747377329*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(8104222592127468960547894479780085484 6405*sqrt(2) + 114611845046003414252052727757333000617984) + 1474618125404445687156966131141385579103594786769370421723255973728695229351827247907 86*sqrt(2888868076710542715672947094311)*sqrt(7)*(125*sqrt(2) + 172))*sqrt(51917987317349015737304218750129712563906438262855815118138050 64*x^2 - sqrt(1590558865810545927822094)*(1264938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682*sqrt(2) - 62135959399493560795) - 1264938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(8104222592127468960547894479780085484 6405*sqrt(2) + 114611845046003414252052727757333000617984) - 1297949682933725393432605468753242814097660956571395377953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117

6180297816406259728442292982869714186133860353798*x + 874869761179272589826
81475740074062806745190*sqrt(11005224487862873621128239642490888848098) + 9
085647780536077754028238281272699698683626695999767645674158862) - 5/353094
86994022006419332*sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(
sqrt(2)*(10*x - 19) + 9*x - 29) + 1/701918227692516147086715878423299535653
31111850222032074026984349485892917146977000414136*sqrt(1590558865810545927
822094)*(5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt(35)
(sqrt(2)(340613697110906370000*x + 143139522536096832703) - 4837532196470
03202703*x - 197474174574809537297) + 5566956030336910747377329*12649387528
04265123815574105117799608149057272418^(1/4)*sqrt(35)*(sqrt(2)*(43734782664
604992355*x + 22535043916389567877) - 66269826580994560232*x - 211997387482
15424478) - (5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt
(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 556695603033
6910747377329*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(
35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x +
3))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460
03414252052727757333000617984) + 1/35309486994022006419332*sqrt(x^2 - 2*x +
3)*(5*sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(10*sqrt(2)
+ 9) + 74179594525256316007*sqrt(7)*(125*sqrt(2) + 172)) - 1/476*sqrt(7)*(2
5*sqrt(2)*(5*x - 1) + 172*x - 82)) + 24453*12649387528042651238155741051177
99608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(7)*(5
8681264663553748097050996611754496316407808*x^38 - 909559602285083095504290
447482194692904321024*x^37 + 7833948832584425370956308047669225258240442368
*x^36 - 46739627304520560359301118801262456316018819072*x^35 + 214091258966
892905713578429763409810498374336512*x^34 - 7914378840963908726941828569900
21126475411881984*x^33 + 2443971981023852389183004169635504201209831489536*
x^32 - 6446905281100567635350197739288116580793540673536*x^31 + 14800438431
924516080565532176343356954693567774720*x^30 - 2999472018305304975160392093
8291975718069728182272*x^29 + 543950385039779684976755634437931462765495913
02144*x^28 - 89238943444544755685020562033988611037555993870336*x^27 + 1338
92902214827011092889528472923281328218944045056*x^26 - 18513587658740219001
1531997633716034835132689940480*x^25 + 237822622904897041561702187373073394
318812215508992*x^24 - 284936536851054039764888677994792967684286178131968*
x^23 + 320523992669231941724388481023319612454782787125248*x^22 - 338726814
722685956738928688519407226164440916295680*x^21 + 3388941060685178348864870
69250634573161464946753536*x^20 - 31944997194601654648963735003466931034597
1696140288*x^19 + 287391247503511322489973442496808422958321912250368*x^18
- 242787372161112804792074580815007335314268010577920*x^17 + 19843297253643
7767771981576557768362169992275296256*x^16 - 149744647365292015359562891324
224536622995129499648*x^15 + 1128464024652710230240544677245701140256867808
70656*x^14 - 74467740316666419201365857719494322341993062072320*x^13 + 5371
7632299767958169950489423652550531043035185152*x^12 - 289289275588053521479
65359067020100302706002886656*x^11 + 22049412762644251773309104679542809356
433843290112*x^10 - 7836592584014101531712860147940403658565697404928*x^9 +
8288222622431088813634530458617273979494874480640*x^8 - 755718873364113816

6357021202789927821668159324l6*x^7 + 288349213189327895035480258909753471729371237580 8*x^6 + 389268931244541502203228657135011133961927655424*x^5 + 77026621102026789199647241685504778793625478758 4*x^4 + 1518975997000593369833590252568040870450277908 48*x^3 + 117307057194105230541603999703274443460516511744*x^2 - 8104222592127468960547894479780085484640 5*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 15791334622283396419062076883133098158146453504*x + 6767714838121455608169461521342756353491337216)*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 114611845046003414252052727757333000617984)*log(514926300974084616887160873794732709351351010668234952341442045423193866055445590835 2*x^2 + 16517307604525632141069927349727551216675979497245715202048/16653749577489013357854121082231147111*sqrt(15905588658105459278220 94)*(12649387528042651238155741051177996081490572724 18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(4326835566238384968 2*sqrt(2) - 6213595939949356079 5) - 126493875280426512381557410511779960814905727241 8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 10540431506187741047 7) - 62135959399493560795*x + 148672670724261260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 114611845046003414252052727757333000617984) - 128731575243521154221790218448683177337837752667058738085360511355798466513861397708 8*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 38619472573056346266537065534604953201351325800117621425608153406739539954158419312 64*x + 8677020686577845807036123864705024753105175633308594321376673792 0*sqrt(110052244878628736211282396424908888480 98) + 901121026704648079552531529140782241364864268669411166597523579490589265597029783961 6) - 24453*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(159055886581054592782209 4)*sqrt(35)*sqrt(7)*(58681264663553748097050996611754496316407808*x^38 - 909559602285083095504290447482194692904321024*x^37 + 78339488325844253709563080476692252582404423 68*x^36 - 46739627304520560359301118801262456316018819072*x^35 + 21409125896689290571357842976340981049837433651 2*x^34 - 7914378840963908726941828569900211264754118819 84*x^33 + 2443971981023852389183004169635504201209831489536*x^32 - 6446905281100567635350197739288116580793540673536*x^31 + 148004384319245160805655321763433556954693567774720*x^30 - 2999472018305304975160392093829197571806972818227 2*x^29 + 543950385039779684976755634437931462765495913021 44*x^28 - 8923894344454475568502056203398861103755599387033 6*x^27 + 133892902214827011092889528472923281328218944045056*x^26 - 1851358765874021900115319976337160348351326899404 80*x^25 + 2378226229048970415617021873730733943188122155089 92*x^24 - 28493653685105403976488867799479296768428617813196 8*x^23 + 32052399266923194172438848102331961245478278712524 8*x^22 - 3387268147226859567389286885194072261644

```
40916295680*x^21 + 3388941060685178348864870692506345731614649467535536*x^20
 - 3194499719460165464896373500346693103459716961402888*x^19 + 2873912475035
113224899734424968084229583219122503688*x^18 - 2427873721611128047920745808l
5007335314268010577920*x^17 + 1984329725364377677719815765577683621699922752
296256*x^16 - 1497446473652920153595628913242245366229951294996448*x^15 + 11
284640246527102302405446772457011402568678087065656*x^14 - 744677403166664192
013658577194943223419930620723208*x^13 + 537176322997679581699504894236525550
531043035185152*x^12 - 2892892755880535214796535906702010030270600288665656*x
^11 + 22049412762644251773309104679542809356433843290112*x^10 - 78365925840
141015317128601479404036585656974049288*x^9 + 828822262243108881363453045861
7273979494874480640*x^8 - 7557188733641138166357021202789927821668159324168*
x^7 + 288349213189327895035480258909753471729371237580888*x^6 + 3892689312445
415022032286571350111339619276554248*x^5 + 770266211020267891996472416855047
787936254787584*x^4 + 151897599700059336983359025256804087045027790848*x^3
+ 117307057194105230541603999703274443460516511744*x^2 - 810422259212746896
05478944797800854846405*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808
*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 12913
5330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x
^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22
 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 -
2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 6497
38605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^
9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 +
 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 157913346222833964190620766
88313309815814645350 4*x + 676771483812145560816946152134275635349133721 6)*s
qrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460034142
520527277573330006179 84)*log(514926300974084616887160873794732709351351010 6
68234952341442045423193866055445590835 2*x^2 - 1651730760452563214106992734 9
72755121667597949724571520204 8/16653749577489013357854121082231147111*sqrt(
15905588658105459278220 94)*(126493875280426512381557410511779960814905727 24
18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(4326835566238384968 2*sqrt(2)
 - 6213595939949356079 5) - 12649387528042651238155741051177996081490572724 1
8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877
410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(81042225921
274689605478944797800854846405*sqrt(2) + 1146118450460034142520527277573330
00617984) - 128731575243521154221790218448683177337837752667058738085360511
355798466513861397708 8*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 386194725730563462 66
537065534604953201351325800117621425608153406739539954158419312 64*x + 86770
206865778458070361238647050247531051756333085943213766737920*sqrt(110052244
878628736211282396424908888480 98) + 901121026704648079552531529140782241364
864268669411166597523579490589265597029783961 6) + 473149067064481998763217 7
0955510530694351293258075604679364840163988886220998806396320543277160 0*x^4
 + 933056734920520960789163462383318633282684143000124192505535084262195413
274496474981134045752 00*x^3 + 720578468552481534074799505602958944515340229
24762066351912610467773507163772995097552926305600*x^2 + 1068899738886597 38
```

```
28268515625026705527863090230587961936087245720*(33722490019334222378242713
60*x^37 - 535022053996400313947961477l2*x^36 + 46914939408298970172949457 58
72*x^35 - 28474992209126677533830352990 72*x^34 + 1325425226110074055651238 8
253568*x^33 - 4977008005852507762806422 9832576*x^32 + 15601073493700873938 8
220889457760*x^31 - 4175163988507543971301119 19794336*x^30 + 97153817191336
5251873706873353652*x^29 - 19936532135755218378886012 04380228*x^28 + 365555
3471852957606257345414140031*x^27 - 605476999658173850375368615 5104785*x^26
 + 91554941585138692302715297463 07221*x^25 - 127401066776850481786936051030
09787*x^24 + 164427702024700763131972159368 14318*x^23 - 1977256973428874472
0189854470201506*x^22 + 222864376176219099216092066 29636086*x^21 - 23584986
64756074244318803120894 6882*x^20 + 235793972111791752401966142960 51673*x^19
 - 22218747553941794885903840542461607*x^18 + 1991229545408024658363639 1613
811979*x^17 - 16801760806053390242995145349148613*x^16 + 136134079650064752
88139078599341572*x^15 - 10279305650733178669223634020962076*x^14 + 7606288
37830344952432793897704 0824*x^13 - 50698382349927519294711904261 15248*x^12
+ 35074259705961976800160782130 30977*x^11 - 197481448306134440527585109 4534
735*x^10 + 1357002388430055881833293557852283*x^9 - 566969010759169461615 95
1049236597*x^8 + 45842600007384688243245704430 6894*x^7 - 947045576652534893
32536549937026*x^6 + 13518392042691323141520887230 3230*x^5 - 10230953189017
74638403186272874*x^4 + 293980411535249733439176017 42151*x^3 + 193395719557
0062708781629134823*x^2 + 33974623503989478480635838 43461*x - 8003871087155
5316861345369643)*sqrt(x^2 - 2*x + 3) + 97000947689757129586992241138859857
9155265693217950893198823602450797211820021087851674007 9600*x + 41571834724
18162696585381763093993910665424399505503827994958296217702336371518947935 7
45748400)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
+ 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 5.03, size = 86793, normalized size = 136.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\left(x^2-2\,x+3\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2+x+1\right)}^{10}\,{\left(x^2-2\,x+3\right)}^{21/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
```

```
[Out] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,(x-a)}{\sqrt{\left(x^2+1\right)(x-a)}}\right)$$

```
[Out] -arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^(
1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

Rubi [C] time = 1.23, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6719, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{\left(x^2+1\right)(-(a-x))}}+\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{\left(x^2+1\right)(-(a-x))}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \left(\frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}} - \frac{2\sqrt{1+a^2}}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\right.}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{1+a^2}\,\sqrt{-a+x}\,\sqrt{\right.}}{\sqrt{}}$$

$$= -\frac{\left(2\sqrt{1+a^2}\,\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-ix}\,\sqrt{1+ix}\,\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right.}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\,\sqrt{-a+x}\right.}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\,\sqrt{\frac{a-x}{i+a}}\,\sqrt{1\right.}}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{4\sqrt{1+a^2}\,\sqrt{\frac{a-x}{i+a}}\,\sqrt{1-}}{\left(1-i\left(a-\right.\right.}$$

Mathematica [C] time = 1.15, size = 213, normalized size = 3.23

$$\frac{2\sqrt{\frac{a-x}{a+i}}\left(2i\sqrt{a^2+1}\,\sqrt{1-ix}\,\sqrt{x^2+1}\,\Pi\left(\frac{2i}{a-\sqrt{a^2+1}+i};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2i}{a+i}\right)-\left(\sqrt{a^2+1}-a-i\right)\sqrt{1+ix}\,(x+i)\,\text{EllipticF}\right.}{\left(-\sqrt{a^2+1}+a+i\right)\sqrt{1-ix}\,\sqrt{\left(x^2+1\right)(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] (2*Sqrt[(a - x)/(I + a)]*(-((-I - a + Sqrt[1 + a^2])*Sqrt[1 + I*x]*(I + x)*
EllipticF[ArcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]) + (2*I)*Sqrt[1 + a
^2]*Sqrt[1 - I*x]*Sqrt[1 + x^2]*EllipticPi[(2*I)/(I + a - Sqrt[1 + a^2]), A
rcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]))/((I + a - Sqrt[1 + a^2])*Sqr
t[1 - I*x]*Sqrt[(-a + x)*(1 + x^2)])
```

fricas [A] time = 1.13, size = 546, normalized size = 8.27

$$\left[\frac{1}{4}\sqrt{-2\,a-2\,\sqrt{a^2+1}}\,\log\left(-\frac{8\,ax^7+x^8+4\left(2\,a^2+15\right)x^6-8\left(4\,a^3+15\,a\right)x^5+2\left(8\,a^4+80\,a^2+67\right)x^4+64\,a^4-\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)(a-x)}\left(a-x-\sqrt{a^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

maple [C] time = 0.12, size = 1275, normalized size = 19.32

$$\frac{2\left(-a-i\right)\sqrt{\frac{-a+x}{-a-i}}\,\sqrt{\frac{x-i}{a-i}}\,\sqrt{\frac{x+i}{a+i}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-a+x}{-a-i}},\sqrt{\frac{a+i}{a-i}}\right)}{\sqrt{-a\,x^{2}+x^{3}-a+x}}-2\sqrt{a^{2}+1}\left(-\frac{i\sqrt{-ix+1}\,\sqrt{-\frac{a}{-a-i}+\frac{x}{-a-i}}\,\sqrt{ix+1}\,a^{2}\,\mathrm{Elli}}{\sqrt{a^{2}+1}\,\sqrt{-a^{3}x^{2}+a^{2}x^{3}-a^{3}+a^{2}x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x)
```

```
[Out] 2*(-a-I)*((-a+x)/(-a-I))^(1/2)*((x-I)/(a-I))^(1/2)*((x+I)/(a+I))^(1/2)/(-a*
x^2+x^3-a+x)^(1/2)*EllipticF(((-a+x)/(-a-I))^(1/2),((a+I)/(a-I))^(1/2))-2*(
a^2+1)^(1/2)*(-I/(a^2+1)^(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2
)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2
+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)
),2^(1/2)*(-I/(-a-I))^(1/2))*a^2-I/(a^2+1)^(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*
a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x
)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(
-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))+I/(a^2+1)^(1/2)*(-I*x+1)^(1/
2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x
-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I)
)^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))*a^2+I/(a^2+1)^
(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2
+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2
*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/
2))+(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(
a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))
^(1/2),-(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))*a+I*(-1/(-a-I)*a+1/(-a-I)*
x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+
x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),-(a+I)/(a^2+1)^(1/2
),((a+I)/(a-I))^(1/2))-(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(
1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*Ellipti
cPi(((-a+x)/(-a-I))^(1/2),(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))*a-I*(-1/
(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1
/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),(
a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-\left(x^2 + 1\right)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-\left(x^2 + 1\right)\,(a - x)}\,\left(x - a + \sqrt{a^2 + 1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))),x)
```

```
[Out] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\cdots\right.}{4\,2^{2/3}}$$

```
[Out] -1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1
/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12*a
*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*
3^(1/2)*2^(1/3)
```

Rubi [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 393, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\cdots\right.}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = a\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}} +$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}} -$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}} -$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} +$$

Mathematica [C] time = 0.23, size = 145, normalized size = 0.73

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 3\right)\left(-x^2 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(-x^2 + 1\right)^{\frac{1}{3}}\left(x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 3\right)\left(-x^2 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,x}{\left(1 - x^2\right)^{1/3}\left(x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.54 $\int \frac{a+bx}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{\right.}}{4\,2^{2/3}}$$

```
[Out] -1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)
+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*ar
ctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3))*3
^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^(1/2
))*3^(1/2)*2^(1/3)
```

Rubi [A] time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 392, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{\right.}}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx = a\int \frac{1}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} +$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} +$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} +$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\,\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a}{}$$

Mathematica [C] time = 0.24, size = 153, normalized size = 0.77

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{\left(x^2-3\right)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right) - F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2};\frac{1}{3},1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3
/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2+3\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{\left(x^2+1\right)^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

```
[Out] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

```
[Out] -1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6*
arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3)*
3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {750}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.05, size = 111, normalized size = 1.14

$$-\frac{\sqrt[3]{\frac{3x+i\sqrt{3}-3}{x}}\sqrt[3]{\frac{9x-3i\sqrt{3}-9}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)}{2\sqrt[3]{3x^2-6x+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/2*(((-3 + I*Sqrt[3] + 3*x)/x)^(1/3)*((-9 - (3*I)*Sqrt[3] + 9*x)/x)^(1/3)
*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)]
)/(4 - 6*x + 3*x^2)^(1/3)
```

fricas [B] time = 4.15, size = 171, normalized size = 1.76

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}\left(3x^2-6x+4\right)^{\frac{2}{3}}(x-2)+4\left(3x^2-6x+4\right)^{\frac{1}{3}}\left(x^2-4x+4\right)\right)}{6\left(x^3-12x^2+24x-16\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(3x^2-6x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

maple [C] time = 14.13, size = 2378, normalized size = 24.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln((160*RootOf(_Z^3-2)-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+4*_Z^2)*x^3+192*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*x^2-384*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-36*(3*x
^2-6*x+4)^(2/3)+256*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-5*R
ootOf(_Z^3-2)*x^3+120*RootOf(_Z^3-2)*x^2-240*RootOf(_Z^3-2)*x-48*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-64*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3+48*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2
/3)*x+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
*(3*x^2-6*x+4)^(1/3)*x^2-72*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x-96*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)-15*RootOf(_Z^3-2
)^2*(3*x^2-6*x+4)^(1/3)*x^2+60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x+72*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x
+4)^(1/3)+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_
Z^3-2)^2*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3*x-60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)+18*(3*x^2-6*x+4)^(2/3)*x+
16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x
^3+10*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*
x^3)/x^3)-1/3*ln((-200*RootOf(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*x^3-60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+60*(3*x^2-6*
x+4)^(2/3)-80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+15*RootOf
(_Z^3-2)*x^3-150*RootOf(_Z^3-2)*x^2+300*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-16*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3-48*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x
-30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x
^2-6*x+4)^(1/3)*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)+9*RootOf(_Z^3-2)^2*(
3*x^2-6*x+4)^(1/3)*x^2-36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-120*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^
(1/3)+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-
2)^2*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3*x+36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)-30*(3*x^2-6*x+4)^(2/3)*x+4*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+10
```

```
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3)/
x^3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-1/6*ln((-200*RootO
f(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-60*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+120*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+60*(3*x^2-6*x+4)^(2/3)-80*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+15*RootOf(_Z^3-2)*x^3-150*RootOf(_Z^3
-2)*x^2+300*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*RootOf(_Z^3-2)^3-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x-30*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2+120*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4
)^(1/3)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^2*(3*x^2-6*x+4)^(2/3)+9*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x^2-36*Ro
otOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)+24*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+60*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+36*RootOf(_Z^3-2)^2*(3
*x^2-6*x+4)^(1/3)-30*(3*x^2-6*x+4)^(2/3)*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+10*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3)/x^3)*RootOf(_Z^3-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-6\,x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(3\,x^2-6\,x+4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
```

```
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{3x^2 - 6x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{1-x^3}\,x^2$$

```
[Out] 1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\sqrt[3]{1-x^3}\,x^2+\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 - Log[1 + x/(1
 - x^3)^(1/3)]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{18}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{6}\,\text{Subst}\Bigg(\int$$

$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\,\text{Subst}\Bigg(\int \frac{1}{-3-$$

$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.27

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3])/2
```

fricas [A] time = 1.03, size = 96, normalized size = 1.32

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2 - \frac{1}{9}\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,x - 2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right) - \frac{1}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) + \frac{1}{18}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^3+1\right)^{\frac{1}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

maple [C] time = 0.10, size = 69, normalized size = 0.95

$$\frac{\left(x^3-1\right)^{\frac{2}{3}} \left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}} x^2 \,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{6\left(-x^3+1\right)^{\frac{2}{3}} \mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}-\frac{\left(x^3-1\right)x^2}{3\left(-x^3+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(1/3),x)
```

```
[Out] -1/3*x^2*(x^3-1)/(-x^3+1)^(2/3)+1/6*(x^3-1)^(2/3)/signum(x^3-1)^(2/3)*(-sig
num(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)/(-x^3+1)^(2/3)
```

maxima [A] time = 1.25, size = 105, normalized size = 1.44

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(1-x^3\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1 - x^3)^(1/3),x)
```

```
[Out] int(x*(1 - x^3)^(1/3), x)
```

sympy [C] time = 1.02, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] (-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^3
+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 618, 204, 31}

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x^3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 1.34

$$\sqrt[3]{1-x^3}+\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{1}{6}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 -
 (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

fricas [A] time = 0.88, size = 73, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

giac [A] time = 1.12, size = 72, normalized size = 1.07

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

maple [C] time = 0.11, size = 49, normalized size = 0.73

$$-\frac{\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],x^3\right)-3\left(3\ln(x)+3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\Gamma\left(\frac{2}{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(1/3)/x,x)

[Out] -1/9/GAMMA(2/3)*(GAMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3)-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))

maxima [A] time = 1.30, size = 71, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)

mupad [B] time = 0.37, size = 83, normalized size = 1.24

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^3)^(1/3)/x,x)

[Out] log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)

sympy [C] time = 0.99, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)**(1/3)/x,x)

[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/3))

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

Optimal. Leaf size=482

$$\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)-$$

[Out] (-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

maple [C] time = 19.96, size = 2972, normalized size = 6.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x+1),x)
```

```
[Out] -(x^3-1)/(-x^3+1)^(2/3)+(1/2*RootOf(_Z^3-2)*ln((2*RootOf(RootOf(_Z^3-2)^2+_
Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*Root
Of(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^
3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*RootOf(_Z^3-2)^3*x+5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(Roo
tOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2-7*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4+8*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2-1
```

```
4*RootOf(_Z^3-2)*x^4-2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x-9*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)*x^3+8*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x-18*RootOf(_Z^3-2)*x^3-
2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootO
f(_Z^3-2)-16*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2+8*(x^6-2*x
^3+1)^(1/3)*RootOf(_Z^3-2)^2-32*RootOf(_Z^3-2)*x^2-9*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)*x-18*RootOf(_Z^3-2)*x+2*(x^6-2*x^3+1)^(2/3)-7*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-14*RootOf(_Z^3-2))/(x+1)^2/(x^
2+x+1))-1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(
_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3
-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2
)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*
x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+
4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x-5*(x^6
-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
*RootOf(_Z^3-2)*x^2+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4-2
*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+14*RootOf(_Z^3-2)*x^4+8*(x^6-2*x^
3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x
+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2*(x^6-2*x^3+1)^(1/
3)*RootOf(_Z^3-2)^2*x+26*RootOf(_Z^3-2)*x^3+8*(x^6-2*x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+20*RootOf(RootOf(_Z^3
-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-2*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2+40
*RootOf(_Z^3-2)*x^2+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x+26
*RootOf(_Z^3-2)*x-8*(x^6-2*x^3+1)^(2/3)+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)+14*RootOf(_Z^3-2))/(x+1)^2/(x^2+x+1))*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)-1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-
2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^
2)^2*RootOf(_Z^3-2)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
*RootOf(_Z^3-2)^3*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*R
ootOf(_Z^3-2)^2*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(
_Z^3-2)^3*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+
_Z*RootOf(_Z^3-2)+_Z^2)+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)*x^4-2*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+14*RootOf(_Z^3-
2)*x^4+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)*RootOf(_Z^3-2)*x+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2
*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+26*RootOf(_Z^3-2)*x^3+8*(x^6-2*x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+20*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-2*(x^6-2*x^3+1)^(1/3)*R
ootOf(_Z^3-2)^2+40*RootOf(_Z^3-2)*x^2+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)*x+26*RootOf(_Z^3-2)*x-8*(x^6-2*x^3+1)^(2/3)+7*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+14*RootOf(_Z^3-2))/(x+1)^2/(x^2+x+1))*Root
Of(_Z^3-2)-1/3*ln((RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf
```

```
(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_
Z^2)^2*x^3+8*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^
2)*x^6-6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*x^2-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*R
ootOf(_Z^3-2)^2*x^3+16*x^6-12*(x^6-2*x^3+1)^(1/3)*x^4+2*RootOf(_Z^3-2)^2*Ro
otOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-24*x^3+12*(x^6-2*x^3+1)^(1/3)
*x+8)/(x-1)/(x^2+x+1))+1/3*ln((RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z
^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*Root
Of(_Z^3-2)+_Z^2)^2*x^3+2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)*x^6-6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^4+6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-8*x^6+6*(x^6-2*x^3+1)^(1
/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x+12*(
x^6-2*x^3+1)^(2/3)*x^2-2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)+16*x^3-8)/(x-1)/(x^2+x+1))+1/6*ln((RootOf(RootOf(_Z^3-2)^2+_
Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootO
f(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*x^3+2*RootOf(_Z^3-2)^2*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^6-6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^4+6*(x^6-2*x^3+1)^(2/3)
*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-8*x^6
+6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Root
Of(_Z^3-2)^2*x+12*(x^6-2*x^3+1)^(2/3)*x^2-2*RootOf(_Z^3-2)^2*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+16*x^3-8)/(x-1)/(x^2+x+1))*RootOf(_Z^3-2)
^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/(-x^3+1)^(2/3)*((x^3-1)
^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx$

Optimal. Leaf size=280

$$\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)}{2\,2^{2/3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}x\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\ldots\right)}{2^{2/3}\ldots}$$

```
[Out] -1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-1/
4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*
2^(1/3)
```

Rubi [F] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] + ((2*
I)*Defer[Int][(1 - x^3)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx &= \int \left(\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)}+\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(-1+i\sqrt{3}+2x\right)}\right) dx\\ &= \frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x} \, dx}{\sqrt{3}}+\frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x} \, dx}{\sqrt{3}}\end{aligned}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

fricas [B] time = 16.51, size = 3085, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(1/3)*arctan(1/3*(26795748*sqrt(3)*2^(2/3)*(586745*x^11 - 70
6109*x^10 - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 +
 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^(1/3) + 26795748*sqr
t(3)*2^(1/3)*(340975*x^10 + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^
6 + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^(2
/3) + 7*sqrt(273426)*2^(1/6)*(6*sqrt(3)*2^(2/3)*(338078915*x^10 - 459916473
*x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448
15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^(2/3) + sqrt
(3)*2^(1/3)*(2332175065*x^12 - 3283524318*x^11 + 1882024851*x^10 - 39193009
70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11
45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*sq
rt(3)*(493920245*x^11 - 452201839*x^10 - 276972599*x^9 - 661557480*x^8 + 13
75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3
 - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^(1/3)) - 3*sqrt(3)*(2247079524645*x
^12 - 5276442179264*x^11 + 3816306322874*x^10 - 3280399521884*x^9 + 6278089
258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25
58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515
2977792))/(18230538112975*x^12 - 14115716188440*x^11 - 20854883745366*x^10
+ 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059
560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296
4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*sqrt(3)*2^(1/3)*arctan(-
1/3*(13397874*sqrt(3)*2^(2/3)*(18803*x^11 - 25367*x^10 - 203754*x^9 + 40802
1*x^8 - 139829*x^7 + 7128*x^6 - 233871*x^5 + 225275*x^4 - 47094*x^3 - 10225
*x^2 + 2921*x)*(-x^3 + 1)^(1/3) + 26795748*sqrt(3)*2^(1/3)*(10589*x^10 - 73
935*x^9 + 63883*x^8 + 142959*x^7 - 173613*x^6 - 31588*x^5 + 79410*x^4 - 437
7*x^3 - 13328*x^2 + 2921*x)*(-x^3 + 1)^(2/3) - 7*sqrt(273426)*(6*sqrt(3)*2^
(2/3)*(309683372*x^10 - 328552599*x^9 - 24698630*x^8 - 422031122*x^7 + 7021
64163*x^6 - 95703451*x^5 - 206316094*x^4 + 60985482*x^3 + 11167816*x^2 - 37
33038*x)*(-x^3 + 1)^(2/3) + sqrt(3)*2^(1/3)*(2345654785*x^12 - 2502234618*x
^11 - 252041853*x^10 - 4416416426*x^9 + 6899968311*x^8 - 1680852528*x^7 + 1
576960038*x^6 - 2990585436*x^5 + 642930363*x^4 + 528479914*x^3 - 117963261*
x^2 - 38399466*x + 8532241) - 6*sqrt(3)*(491687266*x^11 - 516958230*x^10 -
```

69305552*x^9 - 808934094*x^8 + 1418391515*x^7 - 385704187*x^6 - 112721241*x
^5 - 69510422*x^4 + 47121139*x^3 + 11465929*x^2 - 4799203*x)*(-x^3 + 1)^(1/
3))*sqrt((6*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 4
8*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 -
201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3
 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^
6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11
 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^
3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(2995162579*x^12 + 315959718008*x^11 - 8
49682072424*x^10 + 177300060912*x^9 - 508006765899*x^8 + 3583876884636*x^7
 - 3031033916540*x^6 - 1410763301208*x^5 + 2375077456341*x^4 - 546587071308*
x^3 - 175036021936*x^2 + 63861157012*x - 3114267965))/(367648430113*x^12 -
1408582980384*x^11 - 1269375810828*x^10 + 5714713216048*x^9 - 1087485936795
*x^8 - 126379999188*x^7 - 10319650860540*x^6 + 10854292018608*x^5 - 1383220
291365*x^4 - 1828745373668*x^3 + 426327416076*x^2 + 93479232396*x - 2492267
5961)) - 1/18*sqrt(3)*2^(1/3)*arctan(1/3*(13397874*sqrt(3)*2^(2/3)*(17344*x
^11 - 120304*x^10 + 110610*x^9 + 203214*x^8 - 213415*x^7 - 96387*x^6 + 3010
2*x^5 + 157561*x^4 - 101868*x^3 + 15151*x^2 + 913*x)*(-x^3 + 1)^(1/3) - 267
95748*sqrt(3)*2^(1/3)*(1277*x^10 + 57510*x^9 - 189677*x^8 + 108972*x^7 + 10
2426*x^6 - 47461*x^5 - 82155*x^4 + 56409*x^3 - 7301*x^2 - 913*x)*(-x^3 + 1)
^(2/3) + 7*sqrt(273426)*(6*sqrt(3)*2^(2/3)*(8733539*x^10 - 122586360*x^9 +
269810944*x^8 - 28009538*x^7 - 316185126*x^6 + 161786897*x^5 + 95479640*x^4
 - 80193978*x^3 + 11163982*x^2 + 1166814*x)*(-x^3 + 1)^(2/3) - sqrt(3)*2^(1
/3)*(1971824*x^12 - 78264612*x^11 + 705529692*x^10 - 1556393152*x^9 + 93384
9120*x^8 + 135726408*x^7 - 213906684*x^6 + 446158968*x^5 - 582881445*x^4 +
182390318*x^3 + 31120185*x^2 - 12999294*x - 833569) + 6*sqrt(3)*(12965988*x
^11 - 175265260*x^10 + 270273662*x^9 + 299814882*x^8 - 663644613*x^7 + 7755
3085*x^6 + 286893603*x^5 - 82332150*x^4 - 33723265*x^3 + 10863861*x^2 + 333
245*x)*(-x^3 + 1)^(1/3))*sqrt((6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x
^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1
/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x
^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^1
0 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x
)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 +
141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(67113
679084*x^12 - 61534090748*x^11 - 1006807736260*x^10 + 1996201310444*x^9 + 1
93806523788*x^8 - 2673973669800*x^7 + 775957356356*x^6 + 2110159119756*x^5
 - 1821028473882*x^4 + 377014646048*x^3 + 67410900094*x^2 - 19835743048*x -
1369553867))/(168032067092*x^12 - 2318893136652*x^11 + 4401905935020*x^10 +
 1550444734940*x^9 - 6210007783092*x^8 - 1634341806144*x^7 + 6341768478444*
x^6 - 948091553244*x^5 - 2281774840272*x^4 + 1036207535072*x^3 - 5948022808
2*x^2 - 20085678624*x - 761048497)) + 1/3*sqrt(3)*arctan((4*sqrt(3)*(-x^3 +
 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 -
1)) + 1/48*2^(1/3)*log(7717175424*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 -
54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) +

```
2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 +
33*x^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281
*x^10 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2
 - x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^
7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/48*2^(1/3)
*log(1929293856*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 -
 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12
 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 +
 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 16
2*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(
1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*
x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 1/48*2^(1/3)*log(7717175424*(6
*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x
^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 +
338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 +
 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5
+ 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10
- 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2
- 6*x + 1)) - 1/48*2^(1/3)*log(1929293856*(6*2^(2/3)*(4*x^10 - 27*x^9 + 32*
x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3
) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249
*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 -
 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)
*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 1
41*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/6*log(3*(-x^3 +
 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

maple [C] time = 33.31, size = 1404, normalized size = 5.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x)
```

```
[Out] 1/3*RootOf(_Z^2+_Z+1)*ln(-x^3*RootOf(_Z^2+_Z+1)^2-x^3*RootOf(_Z^2+_Z+1)+3*x
*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+2*x^3-RootOf(_Z^2+_Z+1)-2)-1/3*ln(x^3*
RootOf(_Z^2+_Z+1)^2+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*(-x^3+1)^(1/3)*R
ootOf(_Z^2+_Z+1)*x^2+4*x^3*RootOf(_Z^2+_Z+1)+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3
+1)^(1/3)+4*x^3-RootOf(_Z^2+_Z+1)-2)*RootOf(_Z^2+_Z+1)-1/3*ln(x^3*RootOf(_Z
^2+_Z+1)^2+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*(-x^3+1)^(1/3)*RootOf(_Z^
2+_Z+1)*x^2+4*x^3*RootOf(_Z^2+_Z+1)+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)
+4*x^3-RootOf(_Z^2+_Z+1)-2)-1/18*ln(-(36*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+
162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-5*RootOf(_Z^3+324*RootOf(_Z^2+_Z+
1)+162)^2*x^4-12*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(
-x^3+1)^(1/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^3+18*RootOf(
_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-12*RootOf(_Z^3+324*RootO
f(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+RootOf(_Z^3+324*RootOf
(_Z^2+_Z+1)+162)^2*x^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1
/3)*x^2+216*(-x^3+1)^(2/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x
-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x-108*x*(-x^3+1)^(
2/3)-RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2)/(x^2-x+1)^2)*RootOf(_Z^2+_Z+
1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)-1/18*ln(-(36*RootOf(_Z^3+324*Root
Of(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-5*RootOf(_Z^3+324*R
ootOf(_Z^2+_Z+1)+162)^2*x^4-12*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootO
f(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*
x^3+18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-12*RootOf(
_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+RootOf(_
Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+16
2)*(-x^3+1)^(1/3)*x^2+216*(-x^3+1)^(2/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+
_Z+1)+162)^2*x-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x-10
8*x*(-x^3+1)^(2/3)-RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2)/(x^2-x+1)^2)*R
ootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)+1/18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1
)+162)*ln(-(RootOf(_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^4+
2*RootOf(_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^3+6*RootOf(_
Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-RootOf(
_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^2-18*RootOf(_Z^3+324*
RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2-6*RootOf(_Z^3+3
24*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-2*RootOf(_Z^2+_Z+1)*RootOf(_Z^
3+324*RootOf(_Z^2+_Z+1)+162)^2*x+6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*R
ootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)
*(-x^3+1)^(1/3)*x^2+108*(-x^3+1)^(2/3)*x^2+RootOf(_Z^2+_Z+1)*RootOf(_Z^3+32
4*RootOf(_Z^2+_Z+1)+162)^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1
)^(1/3)*x-108*x*(-x^3+1)^(2/3))/(x^2-x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
[Out] int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$

Optimal. Leaf size=232

$$\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log$$

```
[Out] (-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(2
/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-x^
3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+1)^
(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/3))-
2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(2 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x+2),x)
```

```
[Out] int((-x^3+1)^(1/3)/(x+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 2),x)
[Out] int((1 - x^3)^(1/3)/(x + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=168

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}+\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\tan}{}$$

```
[Out] -1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1/
3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(1/3
))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] (1 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x] +
(1 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx &= \int \left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}+\frac{1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\right)dx\\ &= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x}}\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

fricas [F] time = 18.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^3+2\right)^{\frac{2}{3}}(x+2)}{x^5+x^4+x^3+2\,x^2+2\,x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

maple [F] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^2+x+1\right)\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+x+1)/(x^3+2)^(1/3),x)
[Out] int((x+2)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(x^3+2\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[3]{x^3+2}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1587}

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4} \, dx = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

fricas [A] time = 0.92, size = 23, normalized size = 0.92

$$\frac{1}{8} \log\left(320 x^4 + 80 x^3 - 12 x^2 + 24 x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

giac [A] time = 0.99, size = 23, normalized size = 0.92

$$\frac{1}{8} \log\left(320 x^4 + 80 x^3 - 12 x^2 + 24 x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="giac")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\ln\left(320x^4 + 80x^3 - 12x^2 + 24x + 9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

maxima [A] time = 0.54, size = 23, normalized size = 0.92

$$\frac{1}{8} \log\left(320 x^4 + 80 x^3 - 12 x^2 + 24 x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

mupad [B] time = 0.07, size = 23, normalized size = 0.92

$$\frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
[Out] log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

sympy [A] time = 0.11, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40*
x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2090}

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx=-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

Mathematica [C] time = 0.02, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{20\#1^2\log(x-\#1)+12\#1\log(x-\#1)+3\log(x-\#1)}{160\#1^3+30\#1^2-3\#1+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

fricas [A] time = 0.85, size = 43, normalized size = 0.73

$$\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)+\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40x-7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
```

```
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

giac [A] time = 0.85, size = 40, normalized size = 0.68

$$\frac{1}{22}\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40x-7)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac
")
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arcta
n(-1/55*sqrt(11)*(40*x - 7)))
```

maple [A] time = 0.03, size = 52, normalized size = 0.88

$$\frac{\sqrt{11}\,\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\arctan\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))+1/22*11^(1/2)*arctan(-20/33*11
^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

mupad [B] time = 0.34, size = 53, normalized size = 0.90

$$\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] (11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((5
*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x
^3)/33))/22
```

sympy [A] time = 0.17, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\left(2\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)+2\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=78

$$-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

```
[Out] 2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2
)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2100, 2090}

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4)),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rule 2100

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[(Coeff[Pm, x, m]*Log[Qn])/(n*Coeff[Qn, x, n]), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rubi steps

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\log\left(9+24x-12x^2+80x^3+320x^4\right)-\frac{\int \frac{168960+675840x+1126400x^2}{9+24x-12x^2+80x^3+320x^4}\,dx}{1280}$$
$$= 2\sqrt{11}\,\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)-2\sqrt{11}\,\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)+2$$

Mathematica [C] time = 0.02, size = 99, normalized size = 1.27

$$\frac{1}{2}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{640\#1^3\log(x-\#1)-100\#1^2\log(x-\#1)-144\#1\log(x-\#}{160\#1^3+30\#1^2-3\#1+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144
*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 +
30*#1^2 + 160*#1^3) & ]/2
```

fricas [A] time = 0.88, size = 66, normalized size = 0.85

$$-2\sqrt{11}\,\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)-2\sqrt{11}\,\arctan\left(\frac{1}{55}\sqrt{11}\,(40\,x-7)\right)+2\,\log\left(320\,x^4+80\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

giac [A] time = 0.97, size = 64, normalized size = 0.82

$$-2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}\,(40\,x-7)\right)\right)+2\,\log\left(320\,x^4+80\,x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="giac")
```

```
[Out] -2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan(
-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

maple [A] time = 0.03, size = 75, normalized size = 0.96

$$-2\sqrt{11}\ \arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)-2\sqrt{11}\ \arctan\left(\frac{400\sqrt{11}\ x^3}{33}-\frac{20\sqrt{11}\ x^2}{33}+\frac{5\sqrt{11}\ x}{11}+\frac{19\sqrt{11}}{22}\right)+2\ln\left(6400x^4+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 2*ln(6400*x^4+1600*x^3-240*x^2+480*x+180)-2*11^(1/2)*arctan(400/33*11^(1/2)
*x^3-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2))-2*11^(1/2)*arctan(1
/55*(40*x-7)*11^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\int\frac{640\,x^3-100\,x^2-144\,x-21}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

mupad [B] time = 0.09, size = 76, normalized size = 0.97

$$2\ \ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)-2\,\sqrt{11}\ \mathrm{atan}\left(\frac{8\,\sqrt{11}\ x}{11}-\frac{7\,\sqrt{11}}{55}\right)-2\,\sqrt{11}\ \mathrm{atan}\left(\frac{400\,\sqrt{11}\ x^3}{33}-\frac{20\,\sqrt{11}\ x^2}{33}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
9),x)
```

```
[Out] 2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*x
)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2))
/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

sympy [A] time = 0.18, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)-2\operatorname{atan}\left(\frac{400\sqrt{11}\,x^{3}}{33}-\frac{20\sqrt{11}\,x^{2}}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^{4}+\frac{x^{3}}{4}-\frac{3x^{2}}{80}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

```
[Out] 1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {405}

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx=\frac{1}{2}\tan^{-1}\left(\frac{x\left(1+x^2\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Mathematica [C] time = 0.10, size = 110, normalized size = 2.24

$$-\frac{5x\sqrt{1-x^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};x^4,-x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (-5*x*Sqrt[1 - x^4]*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)*(-5*
AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^4, -x^4])))
```

fricas [A] time = 0.68, size = 56, normalized size = 1.14

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}\,x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

maple [B] time = 0.03, size = 100, normalized size = 2.04

$$\frac{\arctan\left(-\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}-\frac{\arctan\left(\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}-\frac{\ln\left(\frac{-\frac{\sqrt{-x^4+1}}{x}+\frac{-x^4+1}{2x^2}+1}{\frac{\sqrt{-x^4+1}}{x}+\frac{-x^4+1}{2x^2}+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^4+1),x)
```

```
[Out] -1/4*arctan((-x^4+1)^(1/2)/x+1)+1/4*arctan(-(-x^4+1)^(1/2)/x+1)-1/8*ln((1/2
*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1)/(1/2*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^(
1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.09, size = 108, normalized size = 2.04

$$-\frac{5x\sqrt{x^4+1}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] (-5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(5*
AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))
```

fricas [A] time = 0.91, size = 61, normalized size = 1.15

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

maple [C] time = 0.04, size = 365, normalized size = 6.89

$$-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{i\sqrt{-ix^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/
2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+
1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))+1/2*I/(1/2*2^(1/2)+1/2*I*
2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(
1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1
+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-Ell
ipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^
2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\,\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

```
[Out] 1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2+
p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2071, 1093, 205, 208}

$$\frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\,\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2071

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \text{Subst}\left(\int \frac{1}{1-2px^2+\left(-4+p^2\right)x^4}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)$$
$$= \frac{1}{4}\left(-4+p^2\right)\text{Subst}\left(\int \frac{1}{-2-p+\left(-4+p^2\right)x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right) - \frac{1}{4}\left(-4+p^2\right)\text{Subst}$$
$$= \frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\,\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)$$

Mathematica [C] time = 7.08, size = 5727, normalized size = 76.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.75, size = 359, normalized size = 4.79

$$\left[\frac{1}{8}\sqrt{p-2}\,\log\left(\frac{x^4+2\left(p-1\right)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}\,x+1}{x^4+2\,x^2+1}\right)+\frac{1}{8}\sqrt{p+2}\,\log\left(\frac{x^4+2\left(p+1\right)x^2+2\sqrt{x^4+px}}{x^4-2\,x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
```

```
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

maple [C] time = 0.09, size = 1512, normalized size = 20.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x)
```

```
[Out] 1/2*(-p-1)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^
(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*Elliptic
F(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2
))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*
(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/
2))*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2)))+1/4*(p+2)*(-1/2/(p+2)^(1/2)*arctanh(1/2*(p*x^2+
2*x^2+p+2)/(p+2)^(1/2)/(x^4+p*x^2+1)^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*
x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*
x,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2
*(p^2-4)^(1/2))^(1/2)))-1/2*(p+1)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1
/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+
p*x^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1
/2*(p^2-4)^(1/2)))^(1/2))-1/4*(p+2)*(-1/2/(p+2)^(1/2)*arctanh(1/2*(p*x^2+2*
x^2+p+2)/(p+2)^(1/2)/(x^4+p*x^2+1)^(1/2))-1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2
)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^
2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,
1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(
p^2-4)^(1/2))^(1/2)))+1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(
p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)
^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-
```

```
4)^(1/2)))^(1/2))*p-1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^
2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(
1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)
^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)
/(p+(p^2-4)^(1/2))*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2
*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1
/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+
p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/
2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^
(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2
)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+
1/2*(p^2-4)^(1/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x
^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(
x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*p
+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4 + p\, x^2 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} \, dx$

Optimal. Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/
(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(p-
2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2))*
(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2072}

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p
^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rule 2072

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, -Simp[(a*Sqrt[b + q]*ArcTan[(Sqrt[b + q
]*x*(b - q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x] + Simp[(a*Sqrt[-b + q]*ArcTanh[(Sqrt[-b + q
]*x*(b + q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d +
a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = -\frac{\sqrt{p+\sqrt{4+p^2}}\ \tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+p^2}}\ \tanh^{-1}\left(\frac{\sqrt{-p+\sqrt{4+}}}{2\sqrt{}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.42, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4}-p}+2}\sqrt{1-\frac{2x^2}{\sqrt{p^2+4}+p}}\left(2i\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\,x\right),\frac{p-\sqrt{p^2+4}}{\sqrt{p^2+4}+p}\right)-(p+2i)\Pi\left(\frac{1}{2}i\left(p-\sqrt{p^2+4}\right.\right.\right.}{4\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (Sqrt[2 + (4*x^2)/(-p + Sqrt[4 + p^2])]*Sqrt[1 - (2*x^2)/(p + Sqrt[4 + p^2]
)]*((2*I)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (
p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])] - (2*I + p)*EllipticPi[(I/2)*(p - S
qrt[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - S
qrt[4 + p^2])/(p + Sqrt[4 + p^2])] + (-2*I + p)*EllipticPi[(I/2)*(-p + Sqrt
[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - Sqrt
[4 + p^2])/(p + Sqrt[4 + p^2])]))/(4*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*Sqrt[1
 + p*x^2 - x^4])
```

fricas [B] time = 5.83, size = 2667, normalized size = 15.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(1/4
*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(
3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +
4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p
^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))
*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (
p^2 + 4)*x^4)*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(
x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2
*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(
3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x
```

^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4)))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) + 8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(-1/4*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4)))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*

```
(p^2 + 4))*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*
x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(
p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x
^4 + p^2 + 4)) + (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*(p^2 + 4))*sqrt(p^2 + s
qrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x
^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 +
4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/(p^2 +
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

maple [B] time = 0.11, size = 456, normalized size = 2.67

$$-\frac{\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}\;p\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\;\sqrt{2}\;\sqrt{p+\sqrt{p^2+4}}}{x}+\frac{-x^4+p\,x^2+1}{x^2}+\sqrt{p^2+4}\right)}{32}+\frac{\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}\;p\ln\left(\frac{\sqrt{-x^4+p\,x^2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)
```

```
[Out] 1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2+(-
x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/4*2^(
1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+
2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/32*2^(1/2)*(p+(p^2+4
)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p
^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2
+4)^(1/2)*ln((-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p
*x^2+1)/x^2-(p^2+4)^(1/2))+1/4*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*
(2*(p+(p^2+4)^(1/2))^(1/2)-2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x)/(-p+(p^2+4)^(1
/2))^(1/2))+1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)^(1/2)*
2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p*x^2+1)/x^2-(p^2+4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+p\,x^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^2-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

```
[Out] -b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-1
)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = a\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{(2-x)\sqrt[4]{-1+x}}\,dx, x, \right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1+\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+x^2}\right)$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\tanh^{-1}\left(\sqrt[4]{-\right.$$

Mathematica [C] time = 0.26, size = 157, normalized size = 1.96

$$\frac{x\left(bx\sqrt[4]{1-x^2}\,(x^2-2)F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}\right)}{4(x^2-2)\sqrt[4]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx+a}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2+2\right)\left(x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{\left(x^2-1\right)^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx-\int \frac{bx}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$

Optimal. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

```
[Out] b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^2
-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1-x}\,(2+x)}\,dx, x, x$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1-x^2}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1-x^2}\right) +$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\tanh^{-1}\left(\sqrt[4]{-1\right.$$

Mathematica [C] time = 0.26, size = 162, normalized size = 1.84

$$\frac{x\left(bx\sqrt[4]{x^2+1}\,F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{(x^2+2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)\right)}\right)}{4\sqrt[4]{-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*Appe
llF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1
, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^2]
+ AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 2\right)\left(-x^2 - 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(-x^2 - 1\right)^{\frac{1}{4}} \left(x^2 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 2\right)\left(-x^2 - 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,x}{\left(-x^2 - 1\right)^{1/4} \left(x^2 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

<u>sympy</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx$

Optimal. Leaf size=149

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1)
^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1/4
)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2^(1
/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1010, 397, 439}

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx$$

$$= \frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tanh^{-1}\left(\frac{1 -}{x}\right.$$

Mathematica [C] time = 0.21, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\left(x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx+a}{\left(x^2-2\right)\left(-x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2+1\right)^{\frac{1}{4}}\left(-x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a}{\left(x^2-2\right)\left(-x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{\left(1-x^2\right)^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx-\int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*a*arctan(((x^2+1)^(1/2)+1)/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^(
1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(
1/2))*2^(1/2)-1/2*b*arctanh(1/2*((x^2+1)^(1/2)+1)/(x^2+1)^(1/4)*2^(1/2))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1010, 397, 439}

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = a\int \frac{1}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$$

$$= -\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b\tanh^{-}}{}$$

Mathematica [C] time = 0.19, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{x^2+1}\,(x^2+2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+1\right)^{\frac{1}{4}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(x^2+1\right)}^{1/4}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{x^2+1}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] time = 0.02, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

fricas [B] time = 1.26, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
```

```
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

maple [C] time = 0.19, size = 164, normalized size = 1.29

$$i\sqrt{i\left(2x+1-i\sqrt{3}\right)}\,\sqrt{\frac{x-1}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\,\left(-2\,\mathrm{RootOf}\left(\_Z^3-4\right)^2+\mathrm{RootOf}\left(\_Z^3-4\right)+1+i\sqrt{3}\,\left(-\,\mathrm{RootO}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/
2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_
alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2
)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-
4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

mupad [B] time = 0.45, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

3.74 $\int \frac{x}{\left(4-dx^3\right)\sqrt{-1+dx^3}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

```
[Out] -1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arc
tan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3^(
1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)^(1
/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {485}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - Arc
Tan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d
^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1
+ d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 485

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, -Simp[(q*ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]])/(9*2^(2/3)
*b*Rt[-c, 2]), x] + (-Simp[(q*ArcTan[(Rt[-c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c +
 d*x^3]])/(3*2^(2/3)*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[Sqrt[c + d*x^3]/(Sq
rt[3]*Rt[-c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[(S
qrt[3]*Rt[-c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*
Rt[-c, 2]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*
c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\int \frac{x}{\left(4-dx^3\right)\sqrt{-1+dx^3}}\,dx = -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\,\sqrt[3]{d}\,x}{\sqrt{-1+dx^3}}\right)}{3\,2^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\,2^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{-1+dx^3}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}} - \frac{\tanh^{-1}\left(\sqrt{\right.}{3\,2^{2/3}\sqrt{}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.34

$$\frac{x^2\sqrt{1-dx^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{8\sqrt{dx^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

fricas [B] time = 1.81, size = 1666, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*
d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt
(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1) + (2*sq
rt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))^(1/3) + sqrt(3)*(d*x^4 - x) + 3*(s
qrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1
/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d
*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
```

```
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
 + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
 64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
 - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

maple [C] time = 0.22, size = 240, normalized size = 1.53

$$-\frac{i\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\ \sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\ \sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\ \left(-2\,\mathrm{RootOf}\left(d\_Z^3-4\right)^2 d+i\sqrt{3}\ \mathrm{RootOf}\left(d\_Z^3-4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)+_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*_alpha^2*d^(2/3)*3^(1/2)-1/6*I*_alpha*d^(1/3)*3^(1/2)+1/
2*_alpha*d^(1/3)-1/6*I*3^(1/2)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

mupad [B] time = 15.03, size = 331, normalized size = 2.11

$$\frac{\sqrt{3}\ 314928^{1/3}\ \ln\left(\frac{\left(54\sqrt{d\,x^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\ d^{1/3}\,x\right)\left(\sqrt{d\,x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\ d^{1/3}\,x\right)^3}{\left(2^{2/3}-d^{1/3}\,x\right)^6}\right)}{2916\,d^{2/3}}+\frac{\sqrt{3}\ 314928^{1/3}\ \ln\left(\frac{\left(2\sqrt{d\,x^3-1}+2\sqrt{3}+2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)*
3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)*x
```

```
)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*log((
(2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*d
^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*162i +
 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(1/3)*x
)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*l
og(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(1/3)*x*16
2i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3) + 2*d^(1/
3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx$

Optimal. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

```
[Out] 1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18*
arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {486, 444, 63, 204, 2138, 203, 2145, 206}

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)}\,dx = -\left(\frac{1}{12}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right)-\frac{1}{12}\int \frac{-2-2x+x^2}{\left(4-2x+x^2\right)\sqrt{-1+x^3}}\,dx-\frac{1}{4}\int \frac{}{\left(-8-x^3\right.}$$

$$= -\left(\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}}\,dx,x,x^3\right)\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx,x,\frac{(1-x)^2}{\sqrt{-1+x^3}}\right.$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-9-x^2}\,dx,x,\sqrt{-1+x^3}\right.$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.65

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{16\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1 +
 x^3])
```

fricas [B] time = 1.10, size = 547, normalized size = 7.39

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48\,x^5+186\,x^4-56\,x^3+6\sqrt{3}\left(x^4+12\,x^3+12\,x^2-16\,x\right)\sqrt{x^3-1}-120\,x^2-96\,x+64\right)}{x^6-6\,x^5+24\,x^4-56\,x^3+96\,x^2-96\,x+64}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*(x^4 + 12*
x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24
*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
 + 1/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x +
```

```
 1)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) + (3*sqrt(3)*(x^3 +
 x^2 - 2*x) - sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 + 186*x^4
- 56*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2
 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)))/(x^3 -
 3*x^2 + 2)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) - (3*sqrt(3
)*(x^3 + x^2 - 2*x) + sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
)/(x^3 - 3*x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.20, size = 421, normalized size = 5.69

$$-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}-\frac{i\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*I*(1/2-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)*(1+I*3^(1/2))+1/3*I*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1/
2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(
1/2)*(1-I*3^(1/2))-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))-1/9*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

mupad [B] time = 0.21, size = 533, normalized size = 7.20

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\sqrt{3}\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right.}{9\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*((3^(1
/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i + 1)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3*
(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2))
/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + Ar
cTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1
+ d*x^3]/3]/(18*d^(2/3))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8-dx^3\right)\sqrt{1+dx^3}}\,dx = -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}\,x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}\,x}{\left(2-\sqrt[3]{d}\,x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d}\int \frac{x^2}{\left(8-dx^3\right)\sqrt{1+dx^3}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}\,x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx, x, x^3\right) + \frac{1}{3}d$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{1+dx^3}\right)}{6d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

Mathematica [C] time = 0.03, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

fricas [B] time = 1.43, size = 497, normalized size = 4.83

$$2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(-\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}\left(d^2x^6-40\,dx^3-32\right)\left(d^2\right)^{\frac{2}{3}}+3\sqrt{3}\left(5\,d^2x^4+8\,dx\right)\left(d^2\right)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\left(d^2\right)^{\frac{1}{6}}}{9\left(d^4x^7-7\,d^3x^4-8\,d^2x\right)}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318\,d^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
```

```
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

maple [C] time = 0.35, size = 383, normalized size = 3.72

$$-\frac{i\left(-d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-d^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3-8\right)^2d^2+i\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*3^(1/2)
```

```
*(-d^2)^(1/3)*_alpha^2*d-I*3^(1/2)*(-d^2)^(2/3)*_alpha+I*3^(1/2)*d-3*(-d^2)
^(2/3)*_alpha-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{d\,x^3+1}\,\left(d\,x^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
[Out] -int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] 1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/12
*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {395}

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right) + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(x^2-3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

fricas [B] time = 3.82, size = 1792, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11
 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(
3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^2
 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
- 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 +
 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
+ 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*x
^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 204
3*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(
x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-
3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2
 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)
*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(3
)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2 -
```

```
 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^5
 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4 +
 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660*
x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9
*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255
*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x
^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x
^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 24
5*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*x
^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(4
9*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 + 9
17*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt
((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)*
sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sqr
t(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9
*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2
106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
- 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8
 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^1
1 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^1
0 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^1
2 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/2
88*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
- 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2/
3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2
 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/288*sqrt(6)*sqrt(2)*log(1
2*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 + 1
14228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x^
4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-3
*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 43
47*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2 -
 sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x^
2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*x
^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

maple [C] time = 10.06, size = 538, normalized size = 6.64

$$-\operatorname{RootOf}\left(48\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^2-3\right)+1\right)\ln\left(\frac{-3x^2+48\left(-3x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(48\_Z^2+4\_Z\operatorname{RootOf}\left(\_\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

```
[Out] -1/12*RootOf(_Z^2-3)*ln((8*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)^2*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+192*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*RootOf(4*_Z*R
ootOf(_Z^2-3)+48*_Z^2+1)^2*x-16*RootOf(_Z^2-3)^2*RootOf(4*_Z*RootOf(_Z^2-3)
+48*_Z^2+1)*x-384*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)^2*x+
12*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)*x^2+24*(-3*x^2+1)^(
1/3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)+12*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)-4*RootOf(_Z^2-3)*x-96*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+3*x^2+3)/(x^2-3))-1/12*ln((2*(
-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*RootOf(_Z^
2-3)+48*_Z^2+1)*x+4*RootOf(_Z^2-3)*x+96*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+
1)*x+6*(-3*x^2+1)^(2/3)-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf(_Z^2-3)
-ln((2*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+4*RootOf(_Z^2-3)*x+96*RootOf(4*_Z*RootOf(_Z^2-3)+
48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf
(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2-3\right)\,{\left(1-3\,x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
```

```
[Out] -int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

```
[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

3.78 $\int \frac{1}{\left(3+x^2\right) \sqrt[3]{1+3x^2}} \, dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}} - \frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12
*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}} - \frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}} - \frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] time = 0.10, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{(x^2+3)\sqrt[3]{3x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

fricas [B] time = 2.76, size = 345, normalized size = 4.26

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(3x^4-10x^3-36x^2+18x+9\right)\left(3x^2+1\right)^{\frac{2}{3}}-4\sqrt{3}\left(x^5+15x^4-26x^3-54x^2+9x-9\right)\left(3x\right.}{x^6+126x^5-225x^4-828x^3-81x^2-162}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

maple [C] time = 2.46, size = 444, normalized size = 5.48

$$\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\ln\left(\frac{-12x^2\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)-x^2+24\left(3x^2+1\right)^{\frac{1}{3}}x\,\mathrm{RootOf}\left(48\_Z^2+1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] -1/4*ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-6*RootOf(48*_Z^2+12*
_Z+1)*x^2+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)-24*RootOf(48*_Z^2+12*_
Z+1)*x+(3*x^2+1)^(2/3)+(3*x^2+1)^(1/3)*x-x^2+6*RootOf(48*_Z^2+12*_Z+1)+(3*x
^2+1)^(1/3)-4*x+1)/(x^2+3))-ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)
*x-6*RootOf(48*_Z^2+12*_Z+1)*x^2+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)
-24*RootOf(48*_Z^2+12*_Z+1)*x+(3*x^2+1)^(2/3)+(3*x^2+1)^(1/3)*x-x^2+6*RootO
f(48*_Z^2+12*_Z+1)+(3*x^2+1)^(1/3)-4*x+1)/(x^2+3))*RootOf(48*_Z^2+12*_Z+1)+
RootOf(48*_Z^2+12*_Z+1)*ln((24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-12
*RootOf(48*_Z^2+12*_Z+1)*x^2+24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)-48*
RootOf(48*_Z^2+12*_Z+1)*x-2*(3*x^2+1)^(2/3)+4*(3*x^2+1)^(1/3)*x-x^2+12*Root
Of(48*_Z^2+12*_Z+1)+4*(3*x^2+1)^(1/3)-4*x+1)/(x^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+3\right)\left(3\,x^2+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
```

```
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.07, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

fricas [B] time = 1.82, size = 1943, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
```

```
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 50.76, size = 704, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/144*ln((-2*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-RootOf(_Z^6+108)^4*x^3-3
*RootOf(_Z^6+108)^4*x-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+216*(-x^2+1)^(
2/3)*x-126*RootOf(_Z^6+108)*x^2+54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x-
18)^2/(RootOf(_Z^6+108)^3*x+18))*RootOf(_Z^6+108)^4+1/24*ln((-2*RootOf(_Z^6
+108)^5*(-x^2+1)^(1/3)*x^2-RootOf(_Z^6+108)^4*x^3-3*RootOf(_Z^6+108)^4*x-36
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+216*(-x^2+1)^(2/3)*x-126*RootOf(_Z^6+1
08)*x^2+54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x-18)^2/(RootOf(_Z^6+108)^
3*x+18))*RootOf(_Z^6+108)-1/216*RootOf(_Z^6+108)^4*ln((-RootOf(_Z^6+108)^4*
x^6-225*RootOf(_Z^6+108)^4*x^4+72*x^5*RootOf(_Z^6+108)^4-1296*x^5*RootOf(_Z
^6+108)+486*RootOf(_Z^6+108)+189*RootOf(_Z^6+108)^4*x^2-3402*RootOf(_Z^6+10
8)*x^2+4050*RootOf(_Z^6+108)*x^4-72*RootOf(_Z^6+108)^4*x^3+1296*RootOf(_Z^6
+108)*x^3-27*RootOf(_Z^6+108)^4-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-3
24*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*
x^5+108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4-144*RootOf(_Z^6+108)^5*(-x^2+
1)^(1/3)*x^3+36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+54*RootOf(_Z^6+108)^5
*(-x^2+1)^(1/3)*x-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4+864*RootOf(_Z^6
+108)^2*(-x^2+1)^(1/3)*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+3888*(
-x^2+1)^(2/3)*x+1296*(-x^2+1)^(2/3)*x^4-9072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+
1)^(2/3)*x^2+18*RootOf(_Z^6+108)*x^6)/(x^2+3)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\tan^{-1}(x)}{6\,2^{2/3}} + \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

fricas [B] time = 1.73, size = 1685, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
```

```
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+3\right)\left(x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^{2}+1\right)^{\frac{1}{3}}\left(x^{2}-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(x^2+1\right)}^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^{2}\sqrt[3]{x^{2}+1}-3\sqrt[3]{x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

```
[Out] -2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+x
^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 6733, 1698, 205}

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\int \frac{a+x}{\sqrt{x}\,(-a+x)\sqrt{a^2-\left(1+a^2\right)x+x^2}}\,dx}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{a+x^2}{\left(-a+x^2\right)\sqrt{a^2+\left(-1-a^2\right)x^2+x^4}}\,dx, x, \cdots\right.}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{1}{-a-\left(-2a^2-a\left(-1-a^2\right)\right)x^2}\,dx, x, \frac{\cdots}{\sqrt{a^2\cdots}}\right.}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-\left(1+a^2\right)x+x^2}}\right)}{(1-a)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

Mathematica [C] time = 0.93, size = 159, normalized size = 1.83

$$-\frac{2i\left(a^2-x\right)^{3/2}\sqrt{\frac{x-1}{x-a^2}}\sqrt{\frac{x}{x-a^2}}\left((a+1)\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right),1-\frac{1}{a^2}\right)-2\Pi\left(\frac{a-1}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(a-1)\sqrt{-a^2}\sqrt{(x-1)x\left(x-a^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 +
a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticP
i[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*
Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])
```

fricas [A] time = 0.70, size = 85, normalized size = 0.98

$$\frac{\arctan\left(\frac{\sqrt{a^2x-(a^2+1)x^2+x^3}\left(a^2-2\left(a^2-a+1\right)x+x^2\right)}{2\left((a-1)x^3-\left(a^3-a^2+a-1\right)x^2+\left(a^3-a^2\right)x\right)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

maple [C] time = 0.04, size = 206, normalized size = 2.37

$$-\frac{4\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,a^3\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a^2+x}{a^2}},\frac{a^2}{a^2-a},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}\,\left(a^2-a\right)}-\frac{2\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,a^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\right.}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((x-1)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+
a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-4
*a^3*(-(-a^2+x)/a^2)^(1/2)*((x-1)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+a^
2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(a^
2/(a^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

mupad [B] time = 0.17, size = 217, normalized size = 2.49

$$\frac{4\,a\,\left(a^2-1\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{\left(a-a^2\right)\,\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}-\frac{2\,\left(a^2-1\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\cdots}}{\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

```
[Out] (4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1
))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/2)
), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*(a^
2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/a^2)
^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x - x^2
*(a^2 + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+x}{\sqrt{x\left(-a^2+x\right)(x-1)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [C] time = 1.67, antiderivative size = 529, normalized size of antiderivative = 529.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 6733, 1708, 1103, 1706}

$$\frac{2(1-a)\sqrt{x}\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}+\frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)}{a\sqrt{-}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\int \frac{-2+}{\sqrt{x}\,(-a+x)\sqrt{(2-a)a}}}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{}{\left(-a+x^2\right)}\right.}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\text{Subst}}{a\sqrt{(2-a)ax+\left(-1-2a+a^2\right)}}$$

$$= \frac{2(1-a)\sqrt{x}\sqrt{(2-a)a-\left(1+2a-a^2\right)x+x^2}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{(2-a)}}\right.}{a\sqrt{-1+2a-a^2}\sqrt{(2-a)ax-\left(1+2a-a^2\right)x^2}}$$

Mathematica [C] time = 0.60, size = 127, normalized size = 127.00

$$\frac{2\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1}+1}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right),\frac{1}{(a-1)^2}\right)-2\Pi\left(\frac{1}{1-a};\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right)|\frac{1}{(a-1)^2}\right)\right)}{\sqrt{-(a-1)^2}\sqrt{(x-1)x\left(a^2-2a+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*(El
lipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2*Elliptic
Pi[(1 - a)^(-1), ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)]))/(
Sqrt[-(-1 + a)^2]*Sqrt[(-1 + x)*x*(-2*a + a^2 + x)])
```

fricas [C] time = 1.21, size = 70, normalized size = 70.00

$$\frac{\log\left(-\frac{a^2-2\left(a^2-a\right)x-x^2+2\sqrt{\left(a^2-2\,a-1\right)x^2+x^3-\left(a^2-2\,a\right)x}\,a}{a^2-2\,ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^2-2\,a-1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

maple [C] time = 0.05, size = 317, normalized size = 317.00

$$\frac{2\left(a^2-2a\right)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{x-1}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2+2a}}\;\mathrm{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}+\frac{2\,(2a-2)\left(a^2-2a\right)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\cdots}}{\sqrt{a^2x^2-a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((x-1)/(-a^2+2*a-1))^(1/2)*(x/(-a
^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2-
2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))+2*(2*a-2)*(a^2-2*a
)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((x-1)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))^(
1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a^2-
2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^2-2\,a-1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

mupad [B] time = 0.48, size = 207, normalized size = 207.00

$$\frac{2\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,(a-1)^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\right.\right.}{a\sqrt{x^3+\left(a^2-2\,a-1\right)\,x^2+\left(2\,a-a^2\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2)),
x)
```

```
[Out] (2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x - 2
*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2 -
2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2*a
+ 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 - 2*a
+ 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+x-2}{\sqrt{x\,(x-1)\left(a^2-2a+x\right)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}\,dx$

Optimal. Leaf size=46

$$\log\left(\frac{-2\left(\sqrt{(1-x)x\left(a^2-2ax+x\right)}+x\right)-a^2+2ax+x^2}{(a-x)^2}\right)$$

```
[Out] ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

Rubi [C] time = 1.49, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {2056, 6733, 1710, 1104, 419, 1220, 537}

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\Pi\left(\frac{1}{a};\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|-\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\int \frac{\ }{\sqrt{x}\,(-a\cdots}}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1\cdots}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\cdots\right.}{\sqrt{a^2x-\left(-1+2a+a^2\right)x\cdots}}$$

$$= -\frac{\left(4(1-a)a\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\cdots\right.}{\sqrt{a^2x-\left(-1+2a+\cdots\right.}}$$

$$= -\frac{\left(4(1-a)a\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\sqrt{a^2-\left(-1+2a+\cdots\right.}\right.}{\sqrt{a^2+\left(1-2a-a^2\right)x\cdots}}$$

$$= -\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\,F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|-\cdots\right.}{\sqrt{a^2x+\left(1-2a-a^2\right)x^2-(1-2a)x^3}}$$

Mathematica [C] time = 1.09, size = 133, normalized size = 2.89

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{-\frac{a^2-2ax+x}{(2a-1)(x-1)}}\left(2a\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)\middle|-\frac{(a-1)^2}{2a-1}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right),-\frac{(a-1)^2}{2a-1}\right)\right)}{\sqrt{-\left((x-1)x\left(a^2-2ax+x\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)
*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a)
)] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*
a))]))/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

fricas [A] time = 0.72, size = 63, normalized size = 1.37

$$\log\left(-\frac{a^2-2\,(a-1)x-x^2+2\,\sqrt{(2\,a-1)x^3+a^2x-\left(a^2+2\,a-1\right)x^2}}{a^2-2\,ax+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-\left(a^2+2\,a-1\right)x^2}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

maple [C] time = 0.06, size = 536, normalized size = 11.65

$$-\frac{4\sqrt{-\frac{\left(-\frac{a^2}{2a-1}+x\right)(2a-1)}{a^2}}\,\sqrt{\frac{x-1}{\frac{a^2}{2a-1}-1}}\,\sqrt{\frac{(2a-1)x}{a^2}}\,a^3\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(-\frac{a^2}{2a-1}+x\right)(2a-1)}{a^2}},\sqrt{\frac{a^2}{(2a-1)\left(\frac{a^2}{2a-1}-1\right)}}\right)}{(2a-1)\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}-\frac{4\,(a-1)\sqrt{-\frac{\left(-\frac{a^2}{2a-1}+x\right)}{a^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x)
```

```
[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((x-1)/(a^2/(-1+2*a)-
1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(
1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-
1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((x
-1)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-
2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^
```

```
2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))
/a^2*(-1+2*a))^(1/2)*((x-1)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/
(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi(
(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(
-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-\left(a^2+2\,a-1\right)x^2}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*x
)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax-a-x}{\sqrt{x\,(x-1)\left(-a^2+2ax-x\right)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

3.84 $\int \frac{1-\sqrt[3]{2}\,x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1-\sqrt[3]{2}\,x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.46, size = 323, normalized size = 10.09

$$-\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)\mathrm{EllipticF}\left(\sin^{-1}\left(\right.\right.\right.}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*
x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4
*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I +
Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2
/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*S
qrt[3])/(3*I + Sqrt[3])]))/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

maple [C] time = 0.12, size = 258, normalized size = 8.06

$$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i}{}}{-\frac{3}{2}-\frac{i}{}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [B] time = 1.69, size = 67, normalized size = 2.09

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/2
)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{2}\,x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 2.00

$$\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)-\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] Log[3 - (1 + x)^2/Sqrt[1 + x^3]]/3 - Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

fricas [B] time = 0.55, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 + 
12*x - 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

maple [C] time = 0.11, size = 240, normalized size = 10.43

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
```

```
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 0.22, size = 204, normalized size = 8.87

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\cdots}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right.}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2
))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2
))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2))*2
^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*
(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x
^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1
/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((
2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(6*Sqrt[2]*3^(1/4))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}} - \frac{\left(2-\sqrt{3}\right.}{}$$

Mathematica [C] time = 0.06, size = 47, normalized size = 0.22

$$\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

fricas [B] time = 10.64, size = 7739, normalized size = 35.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3)
 + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(-1/1296*(6
*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*
x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 8437780
8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979
2*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x
^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*
x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x
^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 - 537864*x^14 + 1586477*
x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x^9 - 25445
5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 +
78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3538*x^15 - 310536*x^14
+ 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41002448*x^9
```

- 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*
x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13
160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273216)*(-672*sqrt(3) + 11
64)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 - 205368*x^12 - 289200*x
^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 15
58464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*sqrt(3)*(142*x^15 - 44
19*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 + 128152*x^9 -
10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 +
108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x^14 + 738528*x^13 - 186
0046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 154441
44*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2
 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^
11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 -
 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536)
+ 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x - 235008)*(-672*
sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
 24)*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 - 5976*x^16 + 5544*x^15 + 299664
*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 - 4761216*x^10 - 1046592*
x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 -
 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 + 142
55*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^9 +
415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3
+ 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x^16 - 26052*x^15 + 3328
44*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11 + 318696*x^10 + 126210
72*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 1093612
8*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^
15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610
*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154
784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17 - 96354*x^16 + 84798*x
^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 7660368
0*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 1132
69536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + sqrt(3)*(1351*x^17
 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3
3465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 -
24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 -
7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-672*sqrt(3) + 1164) + 32
61696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(97*x^17 - 523*x^16 - 217
1*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 + 26558*x^10
+ 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 9113
44*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15
 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x
^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 15478
4*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(-672*sqrt(3
) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 + 1)*((45
9*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^

11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*
x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 + sqrt(3)*(2
65*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11
 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6
 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1
383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 450359*x^13 + 4370159*x^1
2 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 146877876*x^8 - 197107784
*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 45545344*x^3 + 695175
36*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14 + 260015*x^13 + 2523
113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9 + 84799980*x^8 - 113
800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^4 - 26295616*x^3 + 4
0135968*x^2 - 7907648*x - 5562368) - 13696448*x - 9634304)*sqrt(-672*sqrt(3
) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15
- 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^11 - 324624*x^10 + 18081
6*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 1404096*x^5 + 1843584*x^4 +
135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 594
47*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 12
7784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 248
32) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 -
8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 1040
5056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 + sqrt(3)*(2855*x^15 - 21
635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 337953
6*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4
- 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt
(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqr
t(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97
) - 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*x^13 + 64440*x^12 - 128
5200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8 - 3614400*x^7 + 3895488
*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(2
5*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10
 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 37
12*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 1248*x^16 - 246120*x^15 +
 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600*x^11 + 6258120*x^10 -
21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*x^6 + 15860544*x^5 - 47
20704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(14*x^17 + 15*x^16 - 296
0*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10 - 25
6916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4 - 7
2448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17 - 35850*x^16 - 106410*
x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 591615
24*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 1005
98112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 + sqrt(3)*(1351*x^17
 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26
727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 +
26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 +
4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-672*sqrt(3) + 1164) - 203

5200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(627*x^16 - 14286*x^15 + 3
9762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x
^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*
x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^1
3 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^
7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x +
 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) - 764928*x - 239616)*
sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x
^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3
)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 221
4*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x
^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3
 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3
) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^
3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*
x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16
*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16 - 5
22*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 - 42
336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*x^3
 - 1664*x^2 + 256*x)) - 1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) +
1164) + 24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^
(3/4)*arctan(-1/1296*(6*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14
+ 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9
- 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 -
 36433344*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 +
 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 -
4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 2
1034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 -
 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10
 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864
*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3
538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623
494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 14
7990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 759
8336) + 21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273
216)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 -
 205368*x^12 - 289200*x^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992
*x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4
sqrt(3)(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 264
54*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 3
57936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x
^14 + 738528*x^13 - 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9
 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 1
55968*x^3 - 3016704*x^2 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1

073898*x^12 - 452980*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916
720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2
 + 1543936*x + 545536) + 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 66
5088*x - 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt
(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 - 5976*x^
16 + 5544*x^15 + 299664*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 -
4761216*x^10 - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016
832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 28
6*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226
888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 -
177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x
^16 - 26052*x^15 + 332844*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11
 + 318696*x^10 + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 -
 20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x
^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^
11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176
*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17
 - 96354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 +
57963744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^
7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x
^2 + sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*
x^13 + 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 8078428
0*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 1733536
0*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-6
72*sqrt(3) + 1164) + 3261696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(9
7*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 48
3752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x
^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^1
7 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11
 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x
^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 1
26592)*sqrt(-672*sqrt(3) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97
) - (sqrt(x^3 + 1)*((459*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 467
7912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 10
5313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8
256960*x^2 + sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700
776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 6080
2736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767
168*x^2 + 1207168*x + 1383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 45
0359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14
6877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4
 - 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^
14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x
^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864
*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96

34304)*sqrt(-672*sqrt(3) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 116
4)^(3/4) + 3*(984*x^15 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^1
1 - 324624*x^10 + 180816*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 14040
96*x^5 + 1843584*x^4 + 135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124
*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7527
6*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100
512*x^2 + 62400*x + 24832) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 224946
8*x^12 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*
x^7 + 768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 +
sqrt(3)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11
- 4863472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 600
7360*x^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1
738368*x - 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24
)*sqrt(-56*sqrt(3) + 97) + 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*
x^13 + 64440*x^12 - 1285200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8
- 3614400*x^7 + 3895488*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 6497
28*x^2 + 108*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12
- 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 62
40*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 12
48*x^16 - 246120*x^15 + 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600
*x^11 + 6258120*x^10 - 21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*
x^6 + 15860544*x^5 - 4720704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(
14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404
*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190
768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17
 - 35850*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 -
 46293732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^
7 + 45222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x
^2 + sqrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*
x^13 - 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 7064835
2*x^8 + 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 1709504
0*x^3 - 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-67
2*sqrt(3) + 1164) - 2035200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(62
7*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 +
 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5
 + 483816*x^4 - 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 +
 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^
9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3
- 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164
) - 764928*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656
*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 324*x^5 + 576*x^4 + 696*x^
3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (1
23*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*
x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164)

```
 + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6
 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x +
2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16
)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656
*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5
 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) + 1/5184*((7*sqrt(3) + 12)*sqr
t(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 + 72*x^7 + 1656*x^6
 + 720*x^5 + 1440*x^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 +
432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x
^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5
+ 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 1
44*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164)
 + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6
*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46
*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^
3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304
)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) -
 1/5184*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3
) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/3
6*(36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 3
24*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 1
0*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2
+ sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*
sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) +
12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 +
 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(
26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*
x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*s
qrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32
*x^3 + 64*x^2 + 32*x + 16)) + 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2
 + sqrt(3)*(x^2 - 10*x - 8) - 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3 + 6\sqrt{3} + 10\right)\sqrt{x^3 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)

maple [C] time = 0.28, size = 353, normalized size = 1.62

$$\frac{\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(2+\sqrt{3}\right)\sqrt{x^{3}+1}}-\frac{\sqrt{2}\ (}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)

[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha-2)/(-3^(1/2)+2*_alpha-1)*(-I*3^(1/2)+3)*((x+1)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-3^(1/2)*_alpha)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+6\sqrt{3}+10\right)\sqrt{x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^{3}+1}\,\left(x^{3}+6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)

[Out] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{(2+\cdots}{}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))
*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^
(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2
))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*(1+3
^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{(2+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 +
 x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/
4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4)
)])/(3*Sqrt[2]*3^(3/4))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{}}{}$$

Mathematica [C] time = 0.08, size = 50, normalized size = 0.24

$$-\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{4\left(3\sqrt{3}-5\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
3*Sqrt[3])
```

fricas [B] time = 10.43, size = 8237, normalized size = 39.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/324*(2
16*sqrt(3)*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345
761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^
7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sq
rt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12
- 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x
^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
 271808*x + 126592)*(56*sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96
354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963
744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 4
2453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 -
sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13
```

+ 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8
 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3
 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3)
+ 97) + 6*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 3457
61*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7
 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqr
t(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 -
 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^
6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
271808*x + 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + 3*sqrt(sq
rt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 -
6128*x^15 - 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84
218362*x^10 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6
- 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(21
31*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x
^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 693427
76*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242
560*x + 7598336) + 21204736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) +
 (459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972
*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 8842732
8*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)
*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x
^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*
x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x
- 1889792) - 4310784*x - 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672
sqrt(3) + 1164)^(3/4) + 6(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 -
 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15
444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704
x^2 - sqrt(3)(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 45298
0*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x
^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 5455
36) + 2674176*x + 944896)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7
653*x^14 + 41169*x^13 - 51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 -
 17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3
+ 187584*x^2 - sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41
940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 +
224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272
*x - 58752)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)
) + 108*(12*x^17 - 498*x^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^1
2 + 300000*x^11 - 396768*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 22012
8*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17
 - 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11
- 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x
^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 3148
8)*sqrt(56*sqrt(3) + 97) + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14

+ 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294
*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108
576*x^2 - 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584
*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 15564
8*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) +
128256*x + 40128)*(56*sqrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 358
50*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293
732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45
222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - s
qrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 -
987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8
+ 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3
- 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) +
97) + 6*(97*x^17 + 104*x^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 69176
2*x^12 + 584800*x^11 + 521510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7
- 1518056*x^6 + 1321712*x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt
(3)*(14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 +
84404*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6
+ 190768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 16960
0*x - 92672)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^
15 - 951114*x^14 + 450359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^
10 + 9429316*x^9 + 146877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776
*x^5 - 132260896*x^4 - 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 1
0237*x^15 - 549126*x^14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 4508
9132*x^10 + 5444020*x^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106
882416*x^5 - 76360864*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 55623
68) - 13696448*x - 9634304)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1
557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684
800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 1889
7984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 -
899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11 - 3214964
0*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 1091078
4*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) +
2090880*x + 2396160)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 11
64)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12
+ 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 +
768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3
)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863
472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x
^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368
*x - 691712)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13
485*x^13 + 102933*x^12 - 70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 +
243576*x^7 - 221784*x^6 - 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2
- sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46

860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 2
66016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*s
qrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*
x^16 - 1682*x^15 + 2496*x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^
10 + 86080*x^9 - 49176*x^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^
4 + 18944*x^3 + 18048*x^2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482
*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7
 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280)
 - 21248*x - 6656)*sqrt(56*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 1
80*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5
 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(12
3*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x
^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt
(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 +
 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)
 - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144
*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x
 + 16)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 1
1656*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256
*x^5 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) - 1/108*sqrt(3)*sqrt(sqrt(
3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)*(
56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/324*(216*sqrt(3)*(97*x^17 - 52
3*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 168068
8*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^1
6 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^1
0 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080
*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*(56*
sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96354*x^16 + 84798*x^15 +
4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 76603680*x^10
 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*
x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 - sqrt(3)*(1351*x^17 - 556
30*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3346537
6*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 245103
68*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 757158
4*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3) + 97) + 6*(97*x^17 - 523
*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688
*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^16
 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10
 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*
x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(
56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - 3*sqrt(sqrt(3)*sqrt(56*sqrt(3) +

```
97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 - 6128*x^15 - 537864*x^14
+ 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x
^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645
168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(2131*x^16 - 3538*x^15 - 31
0536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41
002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 +
 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204
736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 13425*x^15
- 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 -
49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 2
7458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)*(265*x^16 - 7751*x^15 -
 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28
730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 158
53184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) - 4310784*x -
 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) +
 6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 - 1860046*x^12 - 784596*x
^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6
 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 - sqrt(3)*(2855*x^1
5 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^11 + 4427548*x^10 -
3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 54039
04*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896
)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7653*x^14 + 41169*x^13 -
51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 - 17892*x^8 - 490248*x^7
+ 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3 + 187584*x^2 - sqrt(3)*(
142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 +
128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4
+ 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272*x - 58752)*sqrt(x^3 + 1
)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(12*x^17 - 498*x
^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^12 + 300000*x^11 - 396768
*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 220128*x^6 + 584736*x^5 - 308
256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 +
 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^
9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*
x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 31488)*sqrt(56*sqrt(3) + 97)
 + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x
^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 5392
20*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(181
*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 940
21*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 1396
64*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*(56*s
qrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 35850*x^16 - 106410*x^15 -
2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 59161524*x^10
 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 100598112*x
^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - sqrt(3)*(1351*x^17 - 2069
8*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26727704*
```

x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 + 2610894
4*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 + 4724480
*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) + 97) + 6*(97*x^17 + 104*x
^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 691762*x^12 + 584800*x^11 + 5
21510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7 - 1518056*x^6 + 1321712*
x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt(3)*(14*x^17 + 15*x^16 -
2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10
- 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4
- 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 169600*x - 92672)*sqrt(56*sqr
t(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*s
qrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^15 - 951114*x^14 + 45035
9*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14687
7876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 4
5545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14
+ 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9
+ 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^
4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96343
04)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1557*x^15 - 26415*x^14 -
1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9
+ 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4
+ 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 -
837130*x^13 + 2700776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 +
4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 2
1568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) + 2090880*x + 2396160)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(
4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 - 8423784
*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 10405056*x^
5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3)*(2855*x^15 - 21635*x^1
4 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 3379536*x^9 +
4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4 - 40441
6*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt(x^3 +
1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13485*x^13 + 102933*x^12 -
70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 + 243576*x^7 - 221784*x^6
- 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2 - sqrt(3)*(142*x^15 - 2
124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7
5276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 -
100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*sqrt(x^3 + 1)*sqrt(56*sqr
t(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*x^16 - 1682*x^15 + 2496*
x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^10 + 86080*x^9 - 49176*x
^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^4 + 18944*x^3 + 18048*x^
2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*
x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5
- 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) - 21248*x - 6656)*sqrt(5
6*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x
^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x

```
^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*
x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3
 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 2
7*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3
 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 9
7)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x
^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 +
 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16
- 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 -
 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*
x^3 - 1664*x^2 + 256*x)) - 1/1296*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqr
t(3) - 12) + 6)*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*s
qrt(3) + 1164)^(1/4)*log(1/9*(9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4
+ 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3
 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 +
12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 + 2016*x^5
+ 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1
192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*
x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 +
 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt
(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^
7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 -
4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/1296*s
qrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*sqrt(3) + 1164)^(1/4)*log(1/9*(9*x^8
 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26
*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^
6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(
3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - s
qrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36
*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*
sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 +
 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 +
32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(14*sqrt(3) + 24)*log((x^8 - 16*x^
7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 64*x^2 + 2*(5*x^6 - 54*x^5 + 96*
x^4 - 56*x^3 - 36*x^2 - 3*sqrt(3)*(x^6 - 10*x^5 + 20*x^4 - 8*x^3 - 4*x^2 +
8*x) + 24*x - 16)*sqrt(x^3 + 1)*sqrt(14*sqrt(3) + 24) + 16*sqrt(3)*(x^7 - 2
*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7
 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

maple [C] time = 0.26, size = 350, normalized size = 1.67

$$\frac{\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(-2+\sqrt{3}\right)\sqrt{x^3+1}} - \frac{\sqrt{2}\,\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alpha+2)/(-3^(1/2)-2*_alpha+1)*(-I*3^(1
/2)+3)*((x+1)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2
)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+3^(1/2
)*_alpha)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*3
^(1/2)*_alpha-1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/
2)+4))+1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1
/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3-6\,\sqrt{3}\,+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
```

```
[Out] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\,\left(x^3-6\sqrt{3}\,+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right.}{}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2)
)*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)^(
1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1/2)
)*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(1-3^
(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\right.}{}$$

Mathematica [C] time = 0.07, size = 65, normalized size = 0.29

$$-\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

fricas [B] time = 9.99, size = 7910, normalized size = 35.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3) +
 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(1/1296*(6*sq
rt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^1
3 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018320*x^9 - 25445581
2*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 784
64672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 9
15953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 -
```

146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4
 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160
704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-672*sqrt(3) + 1164)
^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*x^12 - 289200*x^11
 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1849440*x^6 + 15584
64*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)*(142*x^15 + 4419*
x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 106
92*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108
288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 738528*x^13 + 186004
6*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*
x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 +
sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11
- 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 55
19008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2
674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)
*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^1
4 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9
- 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 186
1632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x
^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 4153
84*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78
784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x
^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x
^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^
4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 -
 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9
 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*
x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15
- 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^
10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 11326953
6*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 5
5630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465
376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 2451
0368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571
584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 326169
6*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^
15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 10
51756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x
^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8
006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 +
 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^
3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) +
1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^
16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 -

55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6
+ 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x
^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 3
2149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 1
0910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 13834
24) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 -
30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7
 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x
^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*
x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 1138002
32*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135
968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sqrt(-672*sqrt(3) +
1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 14
712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 324624*x^10 + 180816*x^
9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 - 1843584*x^4 + 1359
36*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7773*x^13 - 59447*x
^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 140472*x^7 + 127784
*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 + 62400*x - 24832)
- (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8423
784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405056
*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(2855*x^15 + 21635*
x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536*x^
9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 - 40
4416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(-67
2*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*
(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) + 6
*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 64440*x^12 + 1285200*
x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*x^7 + 3895488*x^6
- 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(25*x^1
6 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^11 - 1312*x^10 - 16
624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 15552*x^4 - 3712*x^
3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 - 246120*x^15 - 5181
72*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 6258120*x^10 - 21360
336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 15860544*x^5 + 4720704
*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 - 15*x^16 - 2960*x^1
5 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^10 - 256916*
x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x^4 - 72448*
x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3) + 1164) - 2035200*

x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 14286*x^15 + 39762*
x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 -
3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 -
108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 6
2584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 1
55648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 1158
4) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928*x - 239616)*sqrt(
-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 -
2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^
6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4
- 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 +
228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)
sqrt(2(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 11
64)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*
x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 +
sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) +
20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 +
16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 - 13*x^16 - 522*x^1
5 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^
9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 166
4*x^2 + 256*x)) + 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*a
rctan(1/1296*(6*sqrt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652
*x^13 - 997302*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 821253
6*x^8 - 84377808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 3643334
4*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*
x^13 - 575818*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*
x^8 - 48715600*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*
x^3 + 7280256*x^2 + 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*
x^14 - 1586477*x^13 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018
320*x^9 - 254455812*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 1
34645168*x^4 - 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15
- 310536*x^14 - 915953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10
- 41002448*x^9 - 146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*
x^5 + 77737424*x^4 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) -
21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-6
72*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*
x^12 - 289200*x^11 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1
849440*x^6 + 1558464*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)
*(142*x^15 + 4419*x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10
+ 128152*x^9 + 10692*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^
4 + 9632*x^3 - 108288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 73
8528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 69038
64*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^
3 + 3016704*x^2 + sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x

^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^14 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9 - 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 1861632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 3261696*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) + 1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sq

rt(-672*sqrt(3) + 1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4)
+ 3*(984*x^15 + 14712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 32462
4*x^10 + 180816*x^9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 -
1843584*x^4 + 135936*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7
773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 1
40472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 +
62400*x - 24832) - (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 +
474132*x^11 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768
064*x^6 + 10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(
2855*x^15 + 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472
*x^10 + 3379536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5
+ 3826528*x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x
+ 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 11
64)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56
sqrt(3) + 97) - 6(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 644
40*x^12 + 1285200*x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*
x^7 + 3895488*x^6 - 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 1
08*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^1
1 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 1
5552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 -
246120*x^15 - 518172*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 62
58120*x^10 - 21360336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 1586
0544*x^5 + 4720704*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 -
15*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75
267*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 +
56784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x
^16 - 106410*x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*
x^11 - 59161524*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 452220
00*x^6 - 100598112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(
3)*(1351*x^17 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987
292*x^12 - 26727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46
883072*x^7 - 26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12
966272*x^2 + 4724480*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3)
+ 1164) - 2035200*x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 1
4286*x^15 + 39762*x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^
10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*
x^4 + 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^1
4 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^
8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^
2 - 37024*x + 11584) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928
*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720
*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^
2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2
016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278
*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x +

```
 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*
(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 -
5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*
x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8
+ 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 -
 13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 239
44*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^
4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-67
2*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*(
-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 +
 1440*x^4 - 2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36
*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*s
qrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*
sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3
 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 +
12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^
7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/5184*sqrt(
2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-
672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72
*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x
^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2
- 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*
x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt
(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*s
qrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x
^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6
 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 -
27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164)
+ 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 -
 32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*log((x^8 + 16*x^7 + 112*x^6 + 16*
x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^6 + 54*x^5 + 96*x^4 + 56*x^3 - 36
*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 + 8*x^3 - 4*x^2 - 8*x) - 24*x - 16)
*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*
x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^
5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.27, size = 349, normalized size = 1.57

$$\frac{\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\ \sqrt{}}{9\left(2+\sqrt{3}\right)\sqrt{x^{3}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3
^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/
(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha
+2)/(-3^(1/2)-2*_alpha-1)*(-I*3^(1/2)-3)*((x-1)/(-I*3^(1/2)-3))^(1/2)*((2*x
+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
/(x^3-1)^(1/2)*(1+2*_alpha-3^(1/2)*_alpha)*EllipticPi(((x-1)/(-3/2-1/2*I*3^
(1/2)))^(1/2),1/3*I*3^(1/2)*_alpha-1/2*3^(1/2)*_alpha-1/2*I*_alpha+_alpha+1
/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=Ro
otOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}-6\sqrt{3}-10\right)\sqrt{x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{x^{3}-1}\,\left(-x^{3}+6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
[Out] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{\right.}}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2
))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2
))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*2^(
1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4)*(1
-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {488}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x
^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4))
 + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 +
x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}} + \frac{(2}{}$$

Mathematica [C] time = 0.06, size = 68, normalized size = 0.32

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(3\sqrt{3}-5\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

fricas [B] time = 9.37, size = 8105, normalized size = 37.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*
(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/64
8*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 80158
4*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 -
2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x
^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 3214
12*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 3654
4) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17
+ 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 5
7963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7
 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^
2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x
```

^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280
*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360
*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt
(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584
*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2
sqrt(3)(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^
12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 32141
2*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544
) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(
1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816
*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 53
9220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(1
81*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 9
4021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 13
9664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*(56
*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17 - 104
*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x^11 -
521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 132171
2*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15*x^16
- 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^1
0 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x
^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt(56*s
qrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)
*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x^14 -
450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 +
146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x
^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 549126
*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 544402
0*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360
864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x -
9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26415*x
^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 6283425
6*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 4249929
6*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15249*x
^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*
x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x
^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 239616

0)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqr
t(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8
423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405
056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15 + 216
35*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536
*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 -
404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(
x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 102933*
x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 + 2217
84*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(142*x^
15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x
^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*
x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*sqrt(
56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x^15 +
2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^9 - 4
9176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3 + 18
048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 +
6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 624
0*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6656)*
sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4
- 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x
^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2
+ 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(
5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4
- 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56
sqrt(3) + 97)(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(
3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/
(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 3
*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(
3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 771817
56*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7 + 12
0105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*x^2 -
sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*x^12
+ 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 113204536*
x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 45330624
*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(56*sqr
t(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
+ 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
- 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sqrt(3)

 + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^14 + 7
38528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903
864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x
^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*
x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543
936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(
246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^10 +
221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728*x^4
+ 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13 + 296
08*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^7 - 2
67064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x + 339
20) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) +
 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 88530*x^1
3 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 549408
*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2 - sqr
t(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 +
171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^
6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) - 67
584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 1742*x^1
4 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 9136*x^8
 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 + 256*
x)) + 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan
(1/648*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x
^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 +
801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x
^2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99
812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
 36544) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*
x^17 + 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^1
2 + 57963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 10622736
0*x^7 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 264960
00*x^2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845
510*x^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 807
84280*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 173
35360*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56
sqrt(3) + 97) - 6(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^
13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 8
01584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^
2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 998
12*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
36544) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) -

sqrt(1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 2
16816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7
 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(
3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^1
1 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5
 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128
)*(56*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*
x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 591615
24*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 1005
98112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17
 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 26
727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 -
26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 +
4724480*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17
 - 104*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x
^11 - 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 1
321712*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15
*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 7526
7*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56
784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt
(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(-4*sqrt(3)*sqrt(56*sqrt(3)
 + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x
^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*
x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260
896*x^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 5
49126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5
444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 -
76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 1369644
8*x - 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26
415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62
834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42
499296*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15
249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 3627
7360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536
992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 2
396160)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6
(sqrt(3)(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^1
1 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 +
10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15
 + 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 33
79536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*
x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*
sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 10
2933*x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 +
 221784*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(1

42*x^15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26
308*x^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 1
9712*x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*
sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x
^15 + 2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^
9 - 49176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3
 + 18048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x
^12 + 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6
 - 6240*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6
656)*sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720
*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24
*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 20
16*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*
x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97)
 - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 1
0*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*
sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1
152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)
) + 3*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt
(3)*(3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 7
7181756*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7
 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*
x^2 - sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*
x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 11320
4536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 453
30624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(5
6*sqrt(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 99730
2*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377
808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609
792*x^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818
*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 4871560
0*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256
*x^2 + 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sq
rt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^1
4 + 738528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 +
 6903864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155
968*x^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 107
3898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 891672
0*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 +
 1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97)
 - 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^
10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728
*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13

```
+ 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^
7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x
+ 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt
(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 8853
0*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 5
49408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2
- sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^
12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 1256
00*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176)
 - 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 174
2*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 913
6*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 +
 256*x)) + 1/2592*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt
(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 116
4)^(1/4)*log(1/18*(18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3
 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^
2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4
) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x
^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3
+ 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27
*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3
+ 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) +
 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 -
4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/2592*(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt(-4*sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*log(1/18*(18*
x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3
)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 -
22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*
sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^
2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192
)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3
 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x
+ 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/36*sqrt(14*sqrt(3) + 24)*arctan
(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8) + 18*x - 12)*sqrt(14*sqrt(3) + 24)
/sqrt(x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.26, size = 350, normalized size = 1.64

$$\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(-2+\sqrt{3}\right)\sqrt{x^3-1}}-\frac{\sqrt{2}\ (}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alpha-2)/(-3^(1/2)+2*_alpha+1)*(-I*3^(1
/2)-3)*((x-1)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2
)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha+3^(1/2)
*_alpha)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*3
^(1/2)*_alpha+1/2*3^(1/2)*_alpha+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)
+4))+1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/
2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3-1}\,\left(x^3+6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
[Out] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 3.21, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(\left(\frac{2\left(2i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right.}\right.}{x+\sqrt{3}-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt
[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*
Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*(
(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[
3] + x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x)
)]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + S
qrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])
/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] +
 x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8
/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(
2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[
3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3]
)] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

fricas [B] time = 0.71, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\,\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^9+3708x^8-5472x^7+6432x^6+10944x^5+14832x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}\,x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

maple [C] time = 0.16, size = 327, normalized size = 5.03

$$\frac{\sqrt{-\left(-1+\frac{\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(1+\frac{\sqrt{3}}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)x, i\sqrt{1+4\sqrt{3}\ \left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+4\sqrt{3}\ x^2-4}}-2\sqrt{3}\left(-\frac{\sqrt{-\left(-1+\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1
```

```
/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 7.99, size = 876, normalized size = 13.90

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-x+\sqrt{3}+1\right)^2\left(\left(\frac{2\left(2i\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}+\sqrt{6}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}+i\sqrt{3}-i+\frac{8i(-2+\sqrt{3}}{-x+\sqrt{3}+\cdots}}\right.}{x-\sqrt{3}-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I*Sqrt
[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 +
 Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(
1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3]
- x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
+ Sqrt[3] - x))] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]] + 2*S
qrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sq
rt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 +
 Sqrt[3])*x))/(1 + Sqrt[3] - x)]))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2
- Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 +
Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + S
qrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt
[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2 -
 Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + S
qrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[
3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
))
```

fricas [B] time = 0.91, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\ldots}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

maple [C] time = 0.16, size = 311, normalized size = 4.94

$$\frac{\sqrt{-\left(-1-\frac{\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(1-\frac{\sqrt{3}}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)x, i\sqrt{1-4\sqrt{3}\,\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x^4-4\sqrt{3}\,x^2-4}}+2\sqrt{3}\left(-\frac{\sqrt{-\left(-1-\ldots\right.}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/
```

```
2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/
2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

```
[Out] ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(1
/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2151}

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 3.00, size = 818, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x+1)/(x^3+2)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3-2478*RootOf(_Z^2-_Z+1)^
2*x^2+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)
^(1/3)*x^2+3265*RootOf(_Z^2-_Z+1)*x^3-4956*RootOf(_Z^2-_Z+1)^2*x+9008*RootO
f(_Z^2-_Z+1)*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+10816*Ro
otOf(_Z^2-_Z+1)*x^2+335*(x^3+2)^(2/3)*x+335*(x^3+2)^(1/3)*x^2+1574*x^3+1801
```

```
6*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+21632*RootOf(_Z^2-_Z+1)*x+670*(x^3+2)^(2/
3)+1340*(x^3+2)^(1/3)*x+7870*x^2+17346*RootOf(_Z^2-_Z+1)+1340*(x^3+2)^(1/3)
+15740*x+11018)/(x+1)^2)-ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3-2478*RootOf(_Z^2
-_Z+1)^2*x^2-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x-4504*RootOf(_Z^2-_Z+1)*
(x^3+2)^(1/3)*x^2-5743*RootOf(_Z^2-_Z+1)*x^3-4956*RootOf(_Z^2-_Z+1)^2*x-900
8*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-5
860*RootOf(_Z^2-_Z+1)*x^2+4839*(x^3+2)^(2/3)*x+4839*(x^3+2)^(1/3)*x^2+6078*
x^3-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-11720*RootOf(_Z^2-_Z+1)*x+9678*(x
^3+2)^(2/3)+19356*(x^3+2)^(1/3)*x+16208*x^2-17346*RootOf(_Z^2-_Z+1)+19356*(
x^3+2)^(1/3)+32416*x+28364)/(x+1)^2)*RootOf(_Z^2-_Z+1)+ln(-(1239*RootOf(_Z^
2-_Z+1)^2*x^3-2478*RootOf(_Z^2-_Z+1)^2*x^2-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(
2/3)*x-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-5743*RootOf(_Z^2-_Z+1)*x^3-
4956*RootOf(_Z^2-_Z+1)^2*x-9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-18016*RootO
f(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-5860*RootOf(_Z^2-_Z+1)*x^2+4839*(x^3+2)^(2/3)*
x+4839*(x^3+2)^(1/3)*x^2+6078*x^3-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-117
20*RootOf(_Z^2-_Z+1)*x+9678*(x^3+2)^(2/3)+19356*(x^3+2)^(1/3)*x+16208*x^2-1
7346*RootOf(_Z^2-_Z+1)+19356*(x^3+2)^(1/3)+32416*x+28364)/(x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{\left(x^3+2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=108

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] -1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan(
1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3+2
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2149, 239, 2151}

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx\\ &= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right) -\end{aligned}$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

fricas [B] time = 2.70, size = 267, normalized size = 2.47

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}\left(7114781247x^4+13663058416x^3-46178206896x^2-12684255934\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
```

```
 + sqrt(3)*(93292570833559435663132301885*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 8894265631830874680155802900
48*x^3 + 888876515195959220955879945824*x^2 + 35126059825850824001997196488
0*x - 47674000995597211057816884304))/(78905434814564721745708464883*x^6 +
337746705836458222863347934450*x^5 + 15598952776058587894336070976*x^4 - 89
5430525315100108684787964824*x^3 + 361667862240477028869533375352*x^2 + 254
1802301011632510645972090336*x + 1554815286823334092314485968880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 5.82, size = 2134, normalized size = 19.76

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^3+2)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^2+_Z+1)*ln((23004340956706368*x+234055794617652*x^6+213993869
3647104*x^5+802477010117664*x^4-7356039259411920*x^3+1604954020235328*x^2-9
38391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf(_Z^2+_Z+1)^2
*x^5-1160717687406137*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*RootOf(_Z^2+_Z
+1)^2*x^4-7959206999356368*RootOf(_Z^2+_Z+1)*x^2+10391133689698608*RootOf(_
Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978360688*RootO
f(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+12498127505504256
*(x^3+2)^(1/3)+11008356238241516*RootOf(_Z^2+_Z+1)+16295099853018372*(x^3+2
)^(2/3)*x-3979603499678184*RootOf(_Z^2+_Z+1)*x^4-10197714008127436*x^3*Root
Of(_Z^2+_Z+1)-2832707206612248*RootOf(_Z^2+_Z+1)*x^5+10107087250606332*(x^3
+2)^(2/3)+6413512798877184*(x^3+2)^(1/3)*x^2+20062783627256832*(x^3+2)^(1/3
)*x+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+780469570084659*Ro
otOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+1)^2*(x
^3+2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^4-6019
04942144643*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4+1609243886794551*RootOf(_Z^
```

```
2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3
)*x^2+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3-32359551765899
22*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+5249311303832568*RootOf(_Z^2+_Z+1)*(
x^3+2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-84050
5690860402*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-1442113972435308*RootOf(_Z
^2+_Z+1)*(x^3+2)^(2/3)*x^2+4061647060486932*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)
*x^3-2161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x+7759251414704196*
RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x+3531674097632562*RootOf(_Z^2+_Z+1)*(x^3+2
)^(1/3)*x^2+11688730639030284*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-92820189036
1806*(x^3+2)^(2/3)*x^4+740020707562752*(x^3+2)^(1/3)*x^5-1959537324097146*(
x^3+2)^(2/3)*x^3+657796184500224*(x^3+2)^(1/3)*x^4+5569211342170836*(x^3+2)
^(2/3)*x^2-1644490461250560*(x^3+2)^(1/3)*x^3+7115580883942020*RootOf(_Z^2+
_Z+1)*(x^3+2)^(2/3)+9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+155591
37585059152)/(x+1)^6)-1/6*ln((8449588288647072*x+456382083491740*x^6+189724
5518515662*x^5+1564738571971680*x^4-691092869287492*x^3+3129477143943360*x^
2-938391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf(_Z^2+_Z+1
)^2*x^5-716065109657961*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*RootOf(_Z^2+
_Z+1)^2*x^4-4910160751940304*RootOf(_Z^2+_Z+1)*x^2-18718371646419984*RootOf
(_Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978360688*Roo
tOf(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+337263714759132
0*(x^3+2)^(1/3)-11008356238241516*RootOf(_Z^2+_Z+1)+5921961582988536*(x^3+2
)^(2/3)*x-2455080375970152*RootOf(_Z^2+_Z+1)*x^4+3132178772121420*x^3*RootO
f(_Z^2+_Z+1)-3318093556875132*RootOf(_Z^2+_Z+1)*x^5+2991506366664312*(x^3+2
)^(2/3)+2041333010384220*(x^3+2)^(1/3)*x^2+6212752640299800*(x^3+2)^(1/3)*x
+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+780469570084659*RootO
f(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+1)^2*(x^3+
2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^4+6745129
10348133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-48304746625233*RootOf(_Z^2+_Z+
1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^2
+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3+622068321264282*Roo
tOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1714878706153620*RootOf(_Z^2+_Z+1)*(x^3+2)
^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-84050569086
0402*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-6109114720727652*RootOf(_Z^2+_Z+
1)*(x^3+2)^(2/3)*x^2+3622976398808172*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^3-2
161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x-12987025125355476*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-5212685479353366*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/
3)*x^2-16011331334883780*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-289992964115418*
(x^3+2)^(2/3)*x^4-88753609147140*(x^3+2)^(1/3)*x^5-30525575170044*(x^3+2)^(
2/3)*x^3-1109420114339250*(x^3+2)^(1/3)*x^4+3235710968024664*(x^3+2)^(2/3)*
x^2-1863825792089940*(x^3+2)^(1/3)*x^3-7115580883942020*RootOf(_Z^2+_Z+1)*(
x^3+2)^(2/3)-9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+4550781346817
636)/(x+1)^6)*RootOf(_Z^2+_Z+1)-1/6*ln((8449588288647072*x+456382083491740*
x^6+1897245518515662*x^5+1564738571971680*x^4-691092869287492*x^3+312947714
3943360*x^2-938391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf
(_Z^2+_Z+1)^2*x^5-716065109657961*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*Ro
```

```
otOf(_Z^2+_Z+1)^2*x^4-4910160751940304*RootOf(_Z^2+_Z+1)*x^2-18718371646419
984*RootOf(_Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978
360688*RootOf(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+33726
37147591320*(x^3+2)^(1/3)-11008356238241516*RootOf(_Z^2+_Z+1)+5921961582988
536*(x^3+2)^(2/3)*x-2455080375970152*RootOf(_Z^2+_Z+1)*x^4+3132178772121420
*x^3*RootOf(_Z^2+_Z+1)-3318093556875132*RootOf(_Z^2+_Z+1)*x^5+2991506366664
312*(x^3+2)^(2/3)+2041333010384220*(x^3+2)^(1/3)*x^2+6212752640299800*(x^3+
2)^(1/3)*x+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+78046957008
4659*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+
1)^2*(x^3+2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x
^4+674512910348133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-48304746625233*RootO
f(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)
^(2/3)*x^2+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3+622068321
264282*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1714878706153620*RootOf(_Z^2+_Z+
1)*(x^3+2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-8
4050569086O402*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-6109114720727652*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2+3622976398808172*RootOf(_Z^2+_Z+1)*(x^3+2)^(
1/3)*x^3-2161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x-1298702512535
5476*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-5212685479353366*RootOf(_Z^2+_Z+1)*(
x^3+2)^(1/3)*x^2-1601133133488378O*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-289992
964115418*(x^3+2)^(2/3)*x^4-88753609147140*(x^3+2)^(1/3)*x^5-30525575170044
*(x^3+2)^(2/3)*x^3-1109420114339250*(x^3+2)^(1/3)*x^4+3235710968024664*(x^3
+2)^(2/3)*x^2-1863825792089940*(x^3+2)^(1/3)*x^3-7115580883942020*RootOf(_Z
^2+_Z+1)*(x^3+2)^(2/3)-9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+455
0781346817636)/(x+1)^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.94 $\int \frac{1}{\left(1-x^3\right) \sqrt[3]{a+bx^3}} \, dx$

Optimal. Leaf size=98

$$\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/3
)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)*3
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}}+\frac{\log\left(\frac{x^2(a+b)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) - Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*(a + b)^(1/3)) +
 Log[1 + ((a + b)^(2/3)*x^2)/(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x
^3)^(1/3)]/(6*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(1-x^3\right)\sqrt[3]{a+bx^3}}\,dx = \text{Subst}\left(\int \frac{1}{1-(a+b)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a+b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2+\sqrt[3]{a+b}\,x}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/3}x\ldots}\right.$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \ldots$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+b}\ldots}{\sqrt[3]{a+b}\ldots}\right.}{\sqrt[3]{a+b}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} - \frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}}$$

Mathematica [A] time = 0.09, size = 120, normalized size = 1.22

$$\frac{-2\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+\frac{x^2(a+b)^{2/3}}{\left(a+bx^3\right)^{2/3}}+1\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*
Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + ((a + b)^(2/3)*x^2)/
(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
[Out] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^3-1\right)\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
[Out] -int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=154

$$\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3
+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3)
)*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

Rubi [F] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3)), x]
)/3 + ((3 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3
)), x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx &= \int \left(\frac{1-\frac{i}{\sqrt{3}}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}+\frac{1+\frac{i}{\sqrt{3}}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\right)dx\\ &= \frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{}}\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+x+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
[Out] int((x+1)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(b\,x^3+a\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=96

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-
1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 55, 617, 204, 31}

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)\sqrt[3]{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\text{Subst}\left(\int \frac{\ }{\sqrt[3]{\ }}\right.}{\ }$$

$$= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}\right)}{\sqrt[3]{a+b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}} + \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.83

$$\frac{-3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)+\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]] + Log
[1 - x^3] - 3*Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

fricas [B] time = 0.93, size = 387, normalized size = 4.03

$$\left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(\left(bx^3+a\right)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3a-3\left(bx^3+a\right)^{\frac{1}{3}}(-a-b)^{\frac{2}{3}}+b}{x^3-1}\right)}{6(a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

giac [A] time = 20.99, size = 113, normalized size = 1.18

$$-\frac{(a+b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a+\sqrt{3}\,b}+\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{6\,(a+b)^{\frac{1}{3}}}-\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\right.\right.}{3\,(a+b)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] -(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
+ b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x^3
+ a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^3+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 1.47, size = 110, normalized size = 1.15

$$-\frac{\frac{2\,\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{(a+b)^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{(a+b)^{\frac{1}{3}}}+\frac{2\,b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)}{(a+b)^{\frac{1}{3}}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/
(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3) -
(a + b)^(1/3))/(a + b)^(1/3))/b
```

mupad [B] time = 0.59, size = 157, normalized size = 1.64

$$\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{9\,a+9\,b}{9\,(-a-b)^{2/3}}\right)}{3\,(-a-b)^{1/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{36\,(-a-}\right.}{6\,(-a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3)
) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a - b)
^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3) - (
(3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))/(6*
(- a - b)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)} dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.$$

$$= \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ldots}{1-\sqrt[3]{2}\,x+2\ldots}\right.}{6\ldots}$$

$$= -\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 1.27

$$\frac{2\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}+1\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 4.67, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-\ldots\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [C] time = 2.42, size = 614, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/6*RootOf(_Z^3-4)*ln((-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)^3*x^3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-3*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+2*(-x^3+1)^(1/3)*R
ootOf(_Z^3-4)^2*x^2-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*x^3-3*RootOf(_Z^3-4)*x^3+5*(-x^3+1)^(2/3)*x+9*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)+RootOf(_Z^3-4))/(x+1)/(x^2-x+1))+RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+30*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^
2+4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+12*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3+3*RootOf(_Z^3-4)*x^3-2*(-x^3+1)^(2/3)*x-12*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-3*RootOf(_Z^3-4))/(x+1)/(x^2-
x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)} dx$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)} dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

fricas [B] time = 3.43, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-33\,x\right.\right.}{6\left(x^{18}-102\,x^{15}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^{3}\right)^{1/3} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x\right.}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.89

$$\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [A] time = 0.66, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

giac [A] time = 0.88, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 3.43, size = 655, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-5*RootOf(_Z^
3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^
3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln((-12*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+1
8*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x
^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z
```

```
^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x
+1))*RootOf(_Z^3-4)-ln((-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42
*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)
```

maxima [A] time = 1.51, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 0.55, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{4}\right)\left(-1+\sqrt{3}\,1i\right)}{12}-\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\right.}{4}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.100 $\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.34, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i\sqrt{3}\right)\text{t}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}+\frac{\left(i-\sqrt{3}\right.}{}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
[Out] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

fricas [B] time = 12.83, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}\,(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}\,(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3+\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

maple [C] time = 7.80, size = 720, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*(-x^3+1)^(2/3)+RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x-2*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x+2*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)-x^2+x-1)/(x^2-x+1))-1/2*ln
(-((-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootO
f(_Z^3+4)^2+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)^3*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+(-x^3+1)^(1/3)*RootOf(
_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4)*x^2-2*(-x^3+1)^(2/3)-3*RootOf(_Z^3+4)*x+R
ootOf(_Z^3+4))/(x^2-x+1))*RootOf(_Z^3+4)-ln(-((-x^3+1)^(2/3)*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2+RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-(-x^3+1)^(1/3)*RootOf(_Z
^3+4)^2*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
```

```
^2)*RootOf(_Z^3+4)*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootO
f(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4
)*x^2-2*(-x^3+1)^(2/3)-3*RootOf(_Z^3+4)*x+RootOf(_Z^3+4))/(x^2-x+1))*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(1-x^3\right)^{1/3}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

```
[Out] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx$

Optimal. Leaf size=135

$$\frac{\log \left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\log \left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.30, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i\sqrt{3}\right)\text{t}}{2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx\\ &= \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\\ &= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}+\frac{\left(i-\sqrt{3}\right)\log}{}\end{aligned}$$

Mathematica [C] time = 0.31, size = 150, normalized size = 1.11

$$\frac{1}{3}x^3F_1\left(1;\frac{1}{3},1;2;x^3,-x^3\right)+x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}\right.}{6\sqrt[3]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] + (x^3*AppellF1[1, 1/3, 1, 2, x^3
, -x^3])/3 + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3
```

```
]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)]
 + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 12.88, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}\,(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}\,(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3+3\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^2}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [C] time = 7.50, size = 676, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)
^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(2/3)*RootOf(R
```

```
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-(-x^3+1)^(1/3)
*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+RootOf(_Z^3+4)*x^2+2*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-3*RootOf(_Z^3+4)*x-6*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*(-x^3+1)^(2/3)+Root
Of(_Z^3+4)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1)
)+1/2*RootOf(_Z^3+4)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z
^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-2*(-x^3+1)
^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x
+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2
*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-RootOf(_Z^3+4)*x^2-2*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+RootOf(_Z^3+4)*x+2*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-RootOf(_Z^3+4)-2*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.102 $\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{\left(x^3+1\right)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{2\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3
)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)
*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [C] time = 0.30, antiderivative size = 399, normalized size of antiderivative = 3.35, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6728, 2148}

$$\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}+\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}+\frac{\left(3-i\sqrt{3}\right)\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3
))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) - ((3 + I*Sqrt[3])*ArcTan[(2 - (
2^(1/3)*(1 + I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I
 - Sqrt[3])) - ((I - Sqrt[3])*Log[(1 - I*Sqrt[3] - 2*x)*(1 - I*Sqrt[3] + 2*
x)^2])/(4*2^(1/3)*(I + Sqrt[3])) - ((I + Sqrt[3])*Log[(1 + I*Sqrt[3] - 2*x)
*(1 + I*Sqrt[3] + 2*x)^2])/(4*2^(1/3)*(I - Sqrt[3])) + (3*(I - Sqrt[3])*Log
[1 - I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/(4*2^(1/3)*(I + Sqrt[3])
) + (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/
(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}} + \frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\right)dx$$

$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx + \left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}$$

$$= \frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} - \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)} - \frac{\left(i-\sqrt{3}\right)\log}{}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
[Out] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)), x]
```

fricas [B] time = 13.54, size = 268, normalized size = 2.25

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+7x^5+10x^4+7x^3+10x^2+7x+1\right)-4\sqrt{2}\left(x^5+x^4-3x^3-3x^2+x+1\right)\left(x^3+\right.\right.}{6\left(3x^6+9x^5+6x^4+x^3+6x^2+9x+3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

maple [C] time = 7.94, size = 652, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^2+x+1)/(x^3+1)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^3-4)*ln(-((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*RootOf(_Z^3-4)^3*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+2*(x^3+1)^(1/3
)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+(x^3
+1)^(1/3)*RootOf(_Z^3-4)^2+2*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+RootOf(_Z^3-4)*x^2+2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+RootOf(_Z^3-4)*x+2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+RootOf(_Z^3-4)+2*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))+RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*ln(((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)*RootOf(_Z^3-4)^3*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*Ro
otOf(_Z^3-4)^2-RootOf(_Z^3-4)*x^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*x^2+2*(x^3+1)^(2/3)-3*RootOf(_Z^3-4)*x-6*RootOf(RootOf(_Z^3-4)
```

```
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-RootOf(_Z^3-4)-2*RootOf(RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-1}{\left(x^3+1\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
[Out] -int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

3.103 $\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$x^2 \left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)
*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[
Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int]
[(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2} \, dx=\int \left(-\frac{4\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}-2x\right)^2}+\frac{4i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(-1+i\sqrt{3}-2x\right)}-\frac{4\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}+2x\right)^2}+\frac{4i\left(1-x^3\right)}{3\sqrt{3}\left(1+i\sqrt{3}\right.}\right.$$

$$=-\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}-2x}\,dx}{3\sqrt{3}}+\frac{(4i)\int}{}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 1.00

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2\,x^3+3\,x^2+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

maple [A] time = 0.10, size = 34, normalized size = 0.79

$$x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)-\frac{(x-1)\,(2x+1)}{\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x)
```

```
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=43

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [F] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((1 + I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x])
 - (1 - I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 1.00

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2\,x^3+3\,x^2+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

maple [A] time = 0.09, size = 34, normalized size = 0.79

$$x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)-\frac{(x-1)\,(2x+1)}{\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^2+x+1)/(-x^3+1)^(1/3),x)
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x-1}{\left(1-x^3\right)^{1/3}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] -int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

3.105 $\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}}\,dx$

Optimal. Leaf size=39

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

```
[Out] (1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1854, 12, 364}

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}}\,dx = \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} - \int -\frac{2x}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + 2\int \frac{x}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.10

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2\,x^3+3\,x^2+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

maple [A] time = 0.10, size = 34, normalized size = 0.87

$$x^{2}\operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)-\frac{(x-1)(2x+1)}{\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^2/(-x^3+1)^(4/3),x)
```

```
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{\left(-x^{3}+1\right)^{\frac{1}{3}}}-\int\frac{x^{2}-2\,x}{\left(x^{3}-1\right)\left(x^{2}+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
```

```
[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{(x-1)^{2}}{\left(1-x^{3}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/(1 - x^3)^(4/3),x)
```

```
[Out] int((x - 1)^2/(1 - x^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x-1)^{2}}{\left(-(x-1)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)
```

```
[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

3.106 $\int \left(1-x^3\right)^{2/3} dx$

Optimal. Leaf size=67

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 239}

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3),x]
```

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(1-x^3\right)^{2/3} dx = \frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [C] time = 0.12, size = 101, normalized size = 1.51

$$\frac{3(x-1)\left(1-x^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{x-1}{1-(-1)^{2/3}},-\frac{x-1}{1+\sqrt[3]{-1}}\right)}{5\left(\frac{x-1}{1+\sqrt[3]{-1}}+1\right)^{2/3}\left(\frac{x-1}{1-(-1)^{2/3}}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

fricas [A] time = 0.62, size = 94, normalized size = 1.40

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{2}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^3+1\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3), x)
```

maple [C] time = 0.10, size = 12, normalized size = 0.18

$$x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3),x)
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

maxima [B] time = 1.30, size = 105, normalized size = 1.57

$$-\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\,\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\,\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

mupad [B] time = 0.34, size = 10, normalized size = 0.15

$$x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3),x)
[Out] x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

sympy [C] time = 1.02, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

3.107 $\int \frac{(1-x^3)^{2/3}}{x} \, dx$

Optimal. Leaf size=70

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] 1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-x^3\right)^{2/3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log(x)}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x,\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.93

$$\frac{1}{2}\left(\left(1-x^3\right)^{2/3}+\log\left(1-\sqrt[3]{1-x^3}\right)-\log(x)\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/x,x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ((1 - x^3)^(2/3) - Log[x]
 + Log[1 - (1 - x^3)^(1/3)])/2
```

fricas [A] time = 0.87, size = 75, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.89, size = 74, normalized size = 1.06

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

maple [C] time = 0.10, size = 66, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^3\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(3\ln(x)+\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/x,x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,
1],[2,2],x^3)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))
```

maxima [A] time = 1.41, size = 73, normalized size = 1.04

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1
)^(1/3) - 1)
```

mupad [B] time = 0.40, size = 91, normalized size = 1.30

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(\frac{1}{6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

sympy [C] time = 1.02, size = 41, normalized size = 0.59

$$-\frac{x^2e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

3.108 $\int \frac{\left(1-x^3\right)^{2/3}}{a+bx} \, dx$

Optimal. Leaf size=384

$$-\frac{\left(a^3+b^3\right)^{2/3} \log \left(a^3+b^3 x^3\right)}{3 b^3}+\frac{\left(a^3+b^3\right)^{2/3} \log \left(-\frac{x \sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2 b^3}+\frac{\left(a^3+b^3\right)^{2/3} \log \left(\sqrt[3]{a^3+b^3}-b \sqrt[3]{1-x^3}\right)}{2 b^3}-$$

```
[Out] 1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3/
a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^(2
/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^3+1
)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a^3+b
^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/3)*x/a
/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1+2*b
*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+bx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(a + b*x), x]
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+bx} \, dx=\int \frac{\left(1-x^3\right)^{2/3}}{a+bx} \, dx$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+bx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(a + b*x),x)
[Out] int((1 - x^3)^(2/3)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx$

Optimal. Leaf size=234

$$\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}\,x}{3(x^3+1)}-\frac{(1-x^3)^{2/3}}{3(x^3+1)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-}{—}\right.}{3\sqrt{3}}$$

```
[Out] -1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^(
2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3+1
)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/3*(
1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [F] time = 0.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)*
Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[In
t][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int][
(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx = \int \left(-\frac{4(1-x^3)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{4(1-x^3)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}+\frac{4i(1-x^3)^{2}}{3\sqrt{3}\left(-1+i\sqrt{3}\right.}\right.$$

$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2} \, dx\right)-\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2} \, dx+\frac{(4i)\int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}-2x} \, dx}{3\sqrt{3}}+\frac{(4i)\int -}{}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

fricas [F] time = 4.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4-2\,x^3+3\,x^2-2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)

[Out] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)

[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)

3.110 $\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx$

Optimal. Leaf size=199

$$\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2}}{}$$

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x/(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3
))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

Rubi [F] time = 0.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (4*(1 + I*S
qrt[3])*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 - (4*Defe
r[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (4*(1 - I*Sqrt[3])
*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3
```

Rubi steps

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int \left(\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} - \frac{2x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\right)dx$$

$$= -\left(2\int \frac{x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx\right) + \int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

$$= -\left(2\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2} + \frac{2i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)} - \frac{2\left(1-i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}\right.\right.$$

$$= -\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right) - \frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx + \frac{1}{3}\left(4\left(1-i\sqrt{3}\right)\right)\int \frac{}{(-1}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

fricas [B] time = 5.17, size = 1827, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^(2/3)*sqrt(3)*(5
0*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^(2/3) + 7644*4^(1/3)*sqrt
(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^(1/3) - 7*
sqrt(39)*(6*4^(1/3)*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898
)*(-x^3 + 1)^(2/3) - 4^(2/3)*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 1725
4*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*
x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^(1/3))*sqrt((6*4^(1/3)*(5*x^4 + 4
*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^
```

```
4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-
x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3
)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x
- 29747))/(138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2
- 70446*x - 256859)) + 8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^(
2/3)*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^(2/3) - 76
44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
- 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
 8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3)) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
 - 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1
)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-2x+1)\left(-x^{3}+1\right)^{\frac{2}{3}}}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+1)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
[Out] int((-2*x+1)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(2\,x-1)\,\left(1-x^{3}\right)^{2/3}}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
```

```
[Out] -int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\right) dx-\int \frac{2x\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

3.111 $\int \frac{(1-x^3)^{2/3}}{1+x} dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx = \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

fricas [F] time = 5.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x+1),x)
[Out] int((-x^3+1)^(2/3)/(x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} \, dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\cdots$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} \, dx = \int \frac{(1-x^3)^{2/3}}{1+x} \, dx$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

fricas [F] time = 4.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}\left(x^2-x+1\right)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
```

```
[Out] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=132

$$-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(
-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 0.16, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.10, size = 111, normalized size = 0.84

$$-\frac{4x\left(1-x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/
3, x^3, -x^3] + 2*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

fricas [A] time = 0.63, size = 191, normalized size = 1.45

$$-\frac{1}{3} \cdot 4^{\frac{1}{3}} \sqrt{3} \arctan\left(-\frac{\sqrt{3}\, x - 4^{\frac{1}{3}} \sqrt{3} \left(-x^{3} + 1\right)^{\frac{1}{3}}}{3\, x}\right) + \frac{1}{3} \sqrt{3} \arctan\left(-\frac{\sqrt{3}\, x - 2 \sqrt{3} \left(-x^{3} + 1\right)^{\frac{1}{3}}}{3\, x}\right) + \frac{1}{3} \cdot 4^{\frac{1}{3}} \log\left(\frac{4^{\frac{2}{3}} x + 2 (-x}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3} + 1\right)^{\frac{2}{3}}}{x^{3} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3} + 1\right)^{\frac{2}{3}}}{x^{3} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^{3}\right)^{2/3}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^3 + 1),x)
[Out] int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=250

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^2}{}$$

```
[Out] -1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/6
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-
1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)*2^(2/3)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.10, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.10

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

fricas [F] time = 4.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}} x}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-x^3\right)^{2/3}}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
```

```
[Out] int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} \, dx$

Optimal. Leaf size=383

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(-\sqrt[3]{1-x^3}\right.}{\sqrt[3]{2}}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6*
ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x+2^(
2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)
```

Rubi [F] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (-2*Defer[Int][(1 - x^3)^(2/3)/(-1 - x), x])/3 - ((1 + (-1)^(2/3))*Defer[In
t][(1 - x^3)^(2/3)/(-1 + (-1)^(1/3)*x), x])/3 - ((1 - (-1)^(1/3))*Defer[Int
][(1 - x^3)^(2/3)/(-1 - (-1)^(2/3)*x), x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} \, dx &= \int \left(-\frac{2\left(1-x^3\right)^{2/3}}{3(-1-x)}+\frac{\left(-1-(-1)^{2/3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+\sqrt[3]{-1}\,x\right)}+\frac{\left(-1+\sqrt[3]{-1}\right)\left(1-x^3\right)^{2/3}}{3\left(-1-(-1)^{2/3}x\right)}\right) dx\\ &= -\left(\frac{2}{3}\int \frac{\left(1-x^3\right)^{2/3}}{-1-x}\,dx\right)+\frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{\left(1-x^3\right)^{2/3}}{-1-(-1)^{2/3}x}\,dx+\frac{1}{3}\left(-1-(-1)^{2/3}\right)\int\end{aligned}$$

Mathematica [C] time = 0.16, size = 138, normalized size = 0.36

$$-\frac{4\left(1-x^3\right)^{2/3}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*Ap
pellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1,
 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*Appell
F1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

fricas [F] time = 11.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(-x+1)\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(1-x^3\right)^{2/3}\,(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
```

```
[Out] -int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\right)dx-\int \frac{x\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3} dx$

Optimal. Leaf size=272

$$\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+$$

```
[Out] 1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^
3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)
/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.09, size = 109, normalized size = 0.40

$$-\frac{4x\sqrt[3]{1-x^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/
3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))
```

fricas [A] time = 3.53, size = 341, normalized size = 1.25

$$\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-24\sqrt{3}\,2^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x\right.}{3\left(x^{18}-102\,x^{15}+447\,x^{12}-628\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

maple [C] time = 6.55, size = 1214, normalized size = 4.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*ln(-(-6*RootOf(Root
Of(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^3-36*RootOf(Roo
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+x^6*RootOf
(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^
3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3
-2)^2*(-x^3+1)^(2/3)*x^2+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*Roo
tOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(_Z^3-2)^2*x^3-36*RootOf(RootOf(_Z^3-2)^2+3
*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3-18*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)^2+6*RootOf(RootOf
(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(x+1)^2/(x^2-x+1)^2)
-1/6*ln((6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-
2)^4*x^3+36*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z
^3-2)^3*x^3+x^6*RootOf(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-
2)+9*_Z^2)*RootOf(_Z^3-2)^2*(-x^3+1)^(2/3)*x^2+6*RootOf(_Z^3-2)*(-x^3+1)^(1
/3)*x^4+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*x^4-2*RootOf(_Z^3-2)^2*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2
)+9*_Z^2)*RootOf(_Z^3-2)*x^3+12*(-x^3+1)^(2/3)*x^2-6*RootOf(_Z^3-2)*(-x^3+1
)^(1/3)*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*x+RootOf(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*RootOf(_Z^3-2))/(x+1)^2/(x^2-x+1)^2)*RootOf(_Z^3-2)-1/2*ln((6*RootOf(Roo
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^3+36*RootOf(Ro
otOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+x^6*RootO
f(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z
^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^
3-2)^2*(-x^3+1)^(2/3)*x^2+6*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x^4+18*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-2*RootOf(_Z^3-
2)^2*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3
-2)*x^3+12*(-x^3+1)^(2/3)*x^2-6*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x-18*(-x^3+1)
^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)
^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(x
+1)^2/(x^2-x+1)^2)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^3 + 1),x)
[Out] int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```