Computer algebra independent integration tests

0-Independent-test-suites/Timofeev-Problems

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September 3, 2021 Compiled on September 3, 2021 at 1:55pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [705]. This is test number [10].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. August 23, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (705)	0.00 (0)
Mathematica	100.00 (705)	0.00 (0)
Maple	92.91 (655)	7.09 (50)
Fricas	92.48 (652)	7.52 (53)
Giac	83.26 (587)	16.74 (118)
Maxima	80.00 (564)	20.00 (141)
Mupad	76.88 (542)	23.12 (163)
Sympy	60.99 (430)	% 39.01 (275)
IntegrateAlgebraic	39.86 (281)	60.14 (424)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

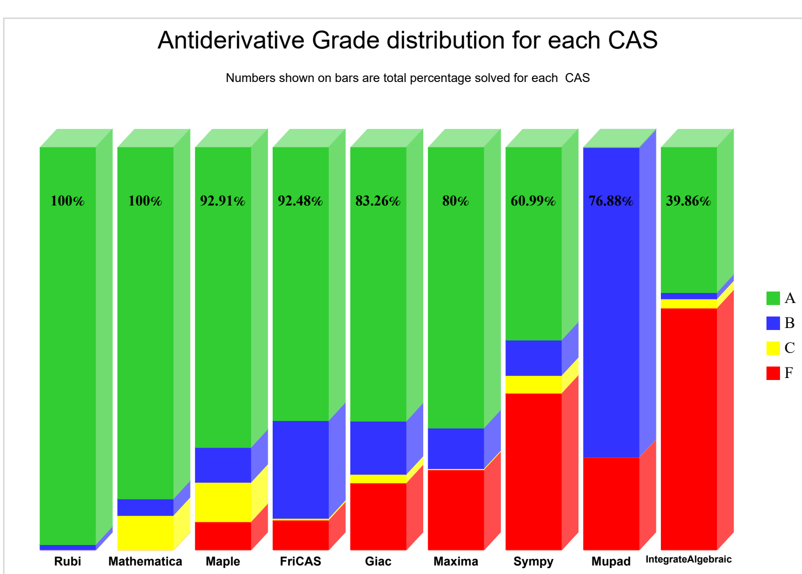
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

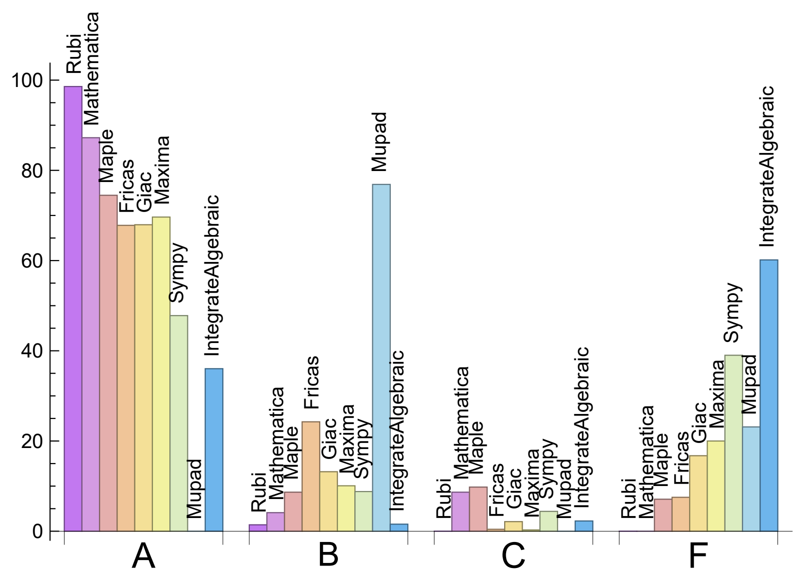
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.58	1.42	0.00	0.00
Mathematica	87.23	4.11	8.65	0.00
Maple	73.62	6.10	13.19	7.09
Maxima	69.65	10.07	0.28	20.00
Giac	67.94	13.19	2.13	16.74
Fricas	67.80	24.26	0.43	7.52
Sympy	47.80	8.79	4.40	39.01
IntegrateAlgebraic	36.03	1.56	2.27	60.14
Mupad	N/A	76.88	0.00	23.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	50	100.00 %	0.00 %	0.00 %
Fricas	53	64.15 %	28.30 %	7.55 %
IntegrateAlgebraic	424	100.00 %	0.00 %	0.00 %
Giac	118	93.22 %	4.24 %	2.54 %
Maxima	141	85.11 %	4.96 %	9.93 %
Sympy	275	74.91 %	23.27 %	1.82 %
Mupad	163	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

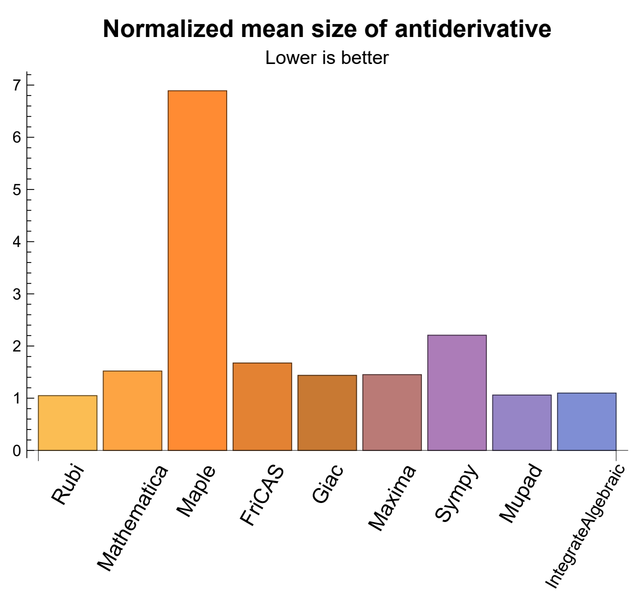
1.3 Performance

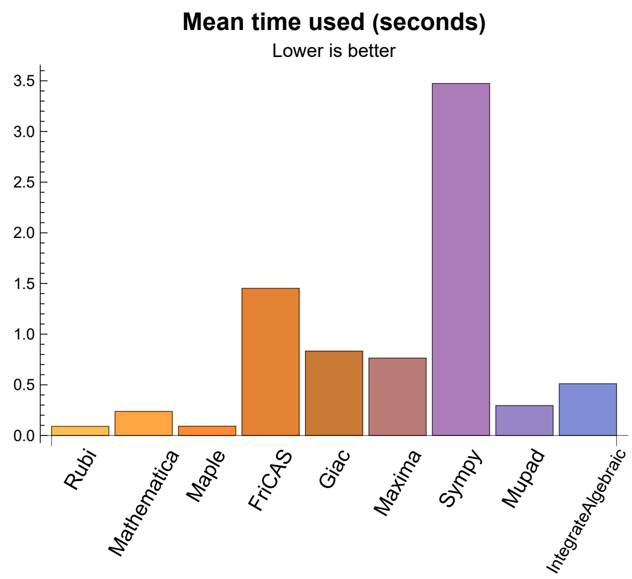
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	51.59	1.05	38.00	1.00
Mathematica	0.24	81.07	1.52	35.00	1.00
Maple	0.34	484.69	6.74	31.00	0.89
Maxima	0.76	57.61	1.45	33.00	0.91
Fricas	1.45	74.75	1.67	40.00	1.11
Sympy	3.47	90.38	2.21	36.00	1.05
Giac	0.83	62.06	1.44	34.00	0.92
Mupad	0.29	42.33	1.06	28.00	0.85
IntegrateAlgebraic	0.51	60.79	1.10	41.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {222, 417, 686, 687, 688, 689, 691}

Mathematica {113, 137, 138, 143, 144, 193, 198, 220, 222, 227, 228, 265, 317, 324, 327, 417, 435, 438, 446, 506, 511, 592, 657, 665, 674, 675, 678, 683, 689, 698, 705}

IntegrateAlgebraic {113, 177, 281, 283}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

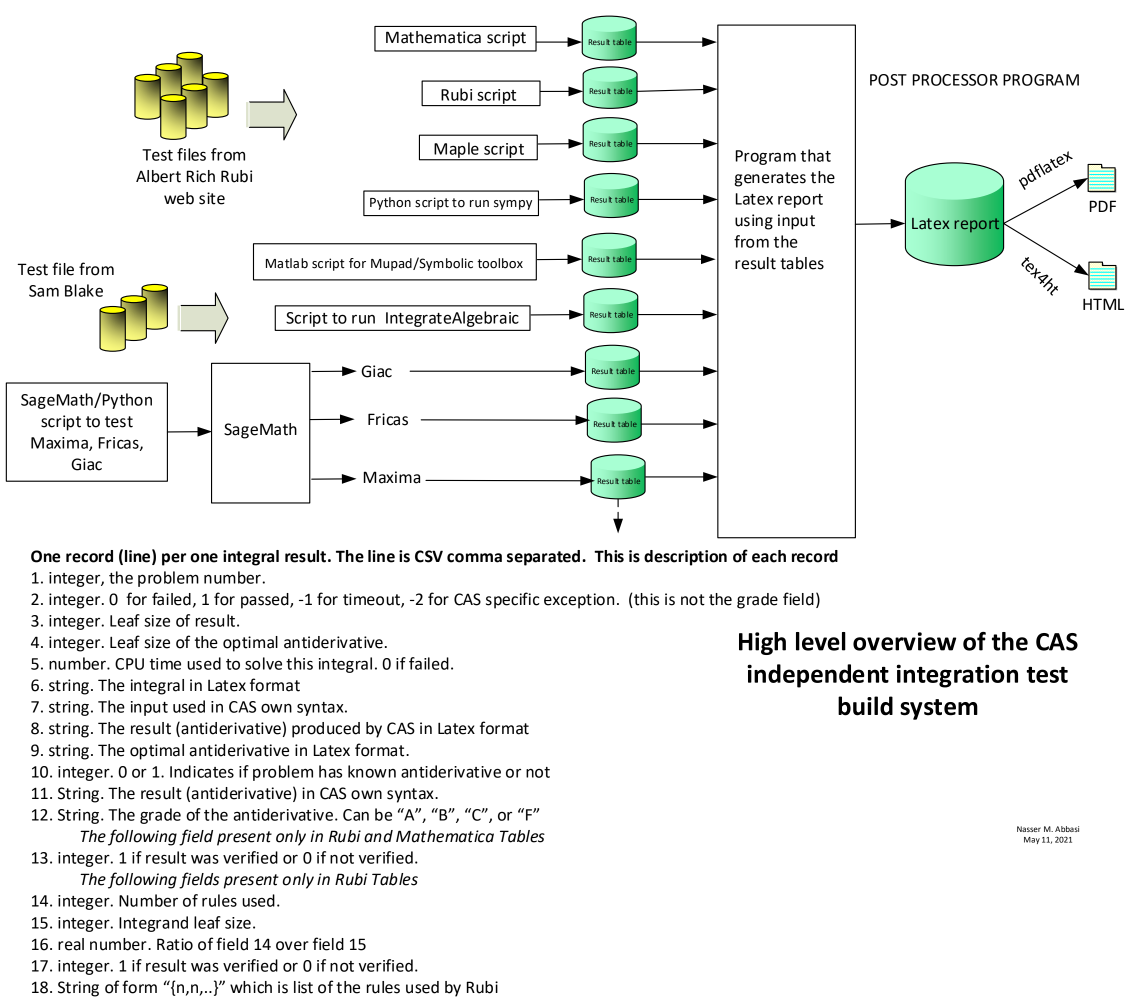
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

B grade: { 226, 228, 232, 306, 335, 377, 413, 416, 447, 695 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 223, 224, 225, 227, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 296, 297, 299, 300, 301, 307, 308, 309, 310, 313, 314, 315, 316, 318, 319, 322, 323, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 441, 442, 447, 449, 450, 453, 455, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 704 }

B grade: { 3, 4, 41, 63, 76, 99, 311, 338, 357, 361, 438, 439, 444, 445, 451, 452, 456, 488, 553, 554, 555, 557, 559, 574, 579, 622, 623, 689, 705 }

C grade: { 37, 113, 193, 198, 215, 216, 217, 221, 222, 226, 228, 231, 232, 242, 244, 245, 246, 247, 248, 249, 260, 283, 292, 294, 298, 302, 303, 304, 305, 306, 312, 317, 320, 321, 324, 325, 327, 343, 384, 389, 397, 399, 401, 411, 416, 417, 424, 426, 434, 440, 443, 446, 448, 454, 461, 476, 526, 592, 677, 678, 703 }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 218, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 250, 251, 252, 254, 255, 256, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 299, 300, 307, 309, 310, 311, 312, 318, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 353, 354, 356, 357, 358, 360, 361, 362, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 401, 412, 419, 420, 421, 422, 424, 425, 426, 428, 430, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 547, 558, 564, 565, 566, 568, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 583, 584, 585, 590, 591, 593, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 645, 646, 647, 648, 649, 650, 651, 652, 654, 655, 659, 660, 663, 664, 665, 666, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 689, 695, 696, 697, 698, 700, 701, 702 }

B grade: { 1, 13, 35, 81, 128, 219, 242, 246, 247, 248, 253, 260, 355, 359, 363, 367, 374, 383, 400, 423, 429, 431, 432, 433, 437, 438, 439, 447, 456, 488, 578, 586, 587, 588, 595, 642, 644, 657, 661, 662, 672, 674, 683 }

C grade: { 41, 118, 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 214, 217, 226, 228, 232, 240, 241, 249, 257, 294, 295, 297, 298, 301, 302, 303, 304, 305, 306, 308, 313, 314, 315, 316, 317, 319, 327, 347, 387, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 434, 435, 436, 457, 487, 492, 493, 523, 543, 544, 545, 546, 548, 556, 561, 563, 567, 569, 589, 592, 594, 641, 643, 653, 656, 658, 667, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 704 }

F grade: { 67, 126, 133, 145, 193, 198, 221, 222, 328, 329, 352, 414, 415, 418, 427, 442, 443, 444, 445, 446, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 632, 699, 703, 705 }

2.1.4 Maxima

A grade: { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 236, 237, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 704, 705 }

B grade: { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 240, 241, 257, 311, 318, 322, 323, 367, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 586, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696 }

C grade: { 421, 426 }

F grade: { 69, 126, 133, 145, 149, 193, 194, 195, 196, 197, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 427, 432, 434, 435, 436, 438, 440, 441, 442, 443, 446, 447, 452, 454, 455, 457, 473, 487, 494, 495, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 583, 587, 592, 595, 603, 621, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

2.1.5 FriCAS

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 227, 230, 231, 232, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

B grade: { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 135, 149, 159, 161, 165, 174, 180, 187, 195, 196, 197, 202, 221, 222, 223, 224, 226, 228, 229, 235, 236, 237, 239, 240, 242, 244, 245, 246, 247, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 315, 317, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 360, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 397, 399, 400, 402, 403, 404, 405, 406, 407, 408, 410, 411, 413, 416, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 456, 457, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

C grade: { 177, 401, 421 }

F grade: { 126, 133, 136, 137, 138, 139, 142, 143, 144, 145, 193, 198, 248, 329, 352, 417, 426, 442, 444, 445, 446, 449, 453, 455, 500, 506, 511, 516, 521, 529, 533, 543, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

2.1.6 Sympy

A grade: { 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 240, 241, 250, 251, 292, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 361, 364, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 470, 471, 472, 476, 479, 480, 483, 484, 485, 486, 494, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 530, 531, 535, 537, 538, 539, 541, 542, 543, 544, 545, 546, 564, 565, 566, 567, 568, 569, 570, 571, 577, 578, 583, 584, 585, 586, 587, 596, 597, 598, 599, 603, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 628, 629, 630, 631, 632, 634, 635, 637, 638, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

B grade: { 1, 4, 8, 19, 21, 30, 31, 36, 41, 62, 149, 159, 194, 195, 196, 197, 211, 214, 215, 252, 253, 254, 295, 297, 299, 312, 335, 340, 353, 354, 369, 370, 376, 378, 379, 382, 383, 388, 389, 467, 468, 475, 477, 487, 488, 489, 493, 547, 548, 572, 573, 576, 580, 588, 589, 590, 591, 602, 604, 655, 670, 697 }

C grade: { 2, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 216, 294, 296, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 536, 616 }

F grade: { 54, 55, 56, 57, 58, 59, 61, 64, 66, 69, 86, 193, 198, 213, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 360, 362, 363, 373, 374, 375, 380, 381, 384, 386, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 469, 473, 474, 478, 481, 482, 490, 491, 492, 495, 500, 505, 506, 510, 511, 516, 521, 528, 529, 532, 533, 534, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 574, 575, 579, 581, 582, 592, 593, 594, 595, 600, 601, 605, 606, 607, 622, 623, 624, 625, 626, 627, 633, 636, 639, 640, 643, 645, 649, 650, 657, 658, 663, 665, 672, 673, 674, 675, 678, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705 }

2.1.7 Giac

A grade: { 2, 6, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 235, 236, 237, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 280, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 390, 391, 392, 393, 394, 396, 397, 400, 401, 419, 420, 422, 423, 424, 425, 428, 429, 430, 433, 440, 441, 442, 443, 444, 445, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 618, 619, 621, 622, 624, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 696, 697, 700, 701, 705 }

B grade: { 1, 4, 5, 9, 13, 19, 22, 52, 55, 58, 73, 99, 128, 197, 220, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 265, 274, 279, 281, 283, 287, 308, 311, 322, 323, 338, 342, 343, 353, 357, 365, 367, 374, 383, 384, 388, 389, 395, 431, 447, 449, 456, 474, 476, 487, 488, 489, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 595, 596, 597, 617, 620, 635, 645, 649, 658, 666, 685, 690, 702, 704 }

C grade: { 79, 421, 437, 438, 457, 494, 495, 503, 504, 505, 508, 509, 510, 537, 703 }

F grade: { 3, 86, 126, 133, 145, 154, 193, 198, 221, 222, 223, 224, 225, 226, 228, 231, 232, 233, 234, 291, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 432, 434, 435, 436, 439, 446, 448, 453, 473, 490, 491, 500, 506, 511, 516, 521, 529, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 594, 608, 615, 616, 623, 625, 626, 632, 636, 648, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 695, 698, 699 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 266, 268, 269, 271, 272, 273, 274, 282, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 318, 322, 323, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 408, 409, 410, 412, 414, 415, 419, 422, 425, 430, 431, 437, 439, 440, 441, 442, 443, 444, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 647, 648, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 696, 697, 700, 704 }

C grade: { }

F grade: { 69, 86, 126, 133, 145, 193, 198, 221, 222, 226, 227, 228, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 257, 264, 265, 267, 270, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 290, 291, 304, 305, 310, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 328, 329, 394, 395, 396, 402, 403, 404, 405, 406, 407, 411, 413, 416, 417, 418, 420, 421, 423, 424, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 474, 490, 491, 492, 493, 500, 506, 511, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 592, 595, 616, 641, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 705 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 24, 25, 26, 27, 28, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 59, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 251, 252, 253, 256, 257, 258, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 320, 321, 324, 325, 326, 327, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482 }

B grade: { 56, 220, 236, 237, 249, 254, 255, 311, 319, 322, 323 }

C grade: { 50, 58, 113, 238, 245, 246, 247, 248, 250, 261, 262, 263, 281, 283, 328, 475 }

F grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 126, 133, 145, 154, 193, 198, 222, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 463, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	31	31	25	20	33	14	14
N.S.	1	1.00	1.00	2.21	2.21	1.79	1.43	2.36	1.00	1.00
time (sec)	N/A	0.008	0.004	0.225	0.420	1.170	0.144	1.012	0.058	0.006

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	14	26	14	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.86	1.00	1.00	1.00
time (sec)	N/A	0.005	0.003	0.306	0.948	0.593	0.141	0.922	0.040	0.006

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	37	18	17	26	29	0	11	0
N.S.	1	1.00	2.85	1.38	1.31	2.00	2.23	0.00	0.85	0.00
time (sec)	N/A	0.004	0.009	0.083	0.426	1.159	0.113	0.000	0.233	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	23	8	19	23	22	23	7	0
N.S.	1	1.00	2.09	0.73	1.73	2.09	2.00	2.09	0.64	0.00
time (sec)	N/A	0.003	0.007	0.082	0.428	0.956	0.111	0.845	0.070	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	21	27	29	22	29	24	0
N.S.	1	1.00	1.00	1.40	1.80	1.93	1.47	1.93	1.60	0.00
time (sec)	N/A	0.003	0.006	0.253	0.434	1.092	0.218	0.995	0.207	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	4	4	3	4	12	0
N.S.	1	1.00	1.00	1.50	2.00	2.00	1.50	2.00	6.00	0.00
time (sec)	N/A	0.006	0.002	0.067	0.421	1.147	0.068	0.925	0.255	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	5	6	6	5	6	6	0
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50	0.00
time (sec)	N/A	0.006	0.002	0.030	0.443	1.021	0.070	0.995	0.208	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	6	5	27	4	7	4	4	0
N.S.	1	1.00	1.00	0.83	4.50	0.67	1.17	0.67	0.67	0.00
time (sec)	N/A	0.024	0.014	0.053	0.426	1.155	0.793	1.009	0.180	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	6	5	9	9	3	30	4	0
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44	0.00
time (sec)	N/A	0.007	0.004	0.036	0.437	0.705	0.205	0.853	0.189	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	8	9	10	10	7	8	6	0
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50	0.00
time (sec)	N/A	0.008	0.009	0.037	0.418	1.052	0.344	1.075	0.002	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	13	12	15	14	13	0
N.S.	1	1.00	1.00	1.08	1.08	1.00	1.25	1.17	1.08	0.00
time (sec)	N/A	0.023	0.019	0.073	0.436	0.871	0.371	1.084	0.198	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	15	15	31	15	15	0
N.S.	1	1.00	1.00	1.07	1.00	1.00	2.07	1.00	1.00	0.00
time (sec)	N/A	0.026	0.010	0.130	0.987	1.062	0.707	0.885	0.047	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	33	33	26	44	35	15	0
N.S.	1	1.00	1.00	2.20	2.20	1.73	2.93	2.33	1.00	0.00
time (sec)	N/A	0.028	0.010	0.147	0.434	0.867	0.728	1.093	0.184	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	18	17	21	32	17	48	0
N.S.	1	1.00	1.00	1.06	1.00	1.24	1.88	1.00	2.82	0.00
time (sec)	N/A	0.035	0.010	0.178	0.438	1.037	2.977	1.040	0.606	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	20	20	23	34	21	48	0
N.S.	1	1.00	1.00	1.05	1.05	1.21	1.79	1.11	2.53	0.00
time (sec)	N/A	0.038	0.010	0.274	0.432	1.096	3.323	0.934	0.491	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	22	22	19	18	18	34	18	58	0
N.S.	1	1.22	1.22	1.06	1.00	1.00	1.89	1.00	3.22	0.00
time (sec)	N/A	0.040	0.015	0.156	0.422	0.797	2.997	0.919	0.406	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	22	22	19	19	19	32	20	58	0
N.S.	1	1.22	1.22	1.06	1.06	1.06	1.78	1.11	3.22	0.00
time (sec)	N/A	0.042	0.014	0.314	0.446	1.149	3.217	0.959	0.379	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	19	14	13	31	61	46	26	0
N.S.	1	1.00	0.46	0.34	0.32	0.76	1.49	1.12	0.63	0.00
time (sec)	N/A	0.017	0.026	0.069	0.947	1.196	0.680	0.833	0.232	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	6	6	15	15	15	16	15	0
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.50	0.00
time (sec)	N/A	0.018	0.003	0.033	0.429	0.878	0.108	0.794	0.004	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	3	4	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00	0.00
time (sec)	N/A	0.012	0.005	0.015	0.420	0.973	0.098	0.850	0.176	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	15	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00	0.00
time (sec)	N/A	0.019	0.011	0.025	0.958	0.775	0.141	0.934	0.336	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	7	8	7	7	7	22	7	0
N.S.	1	1.00	0.78	0.89	0.78	0.78	0.78	2.44	0.78	0.00
time (sec)	N/A	0.020	0.009	0.020	0.435	1.028	0.100	0.999	0.206	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	10	9	9	7	9	9	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00	0.00
time (sec)	N/A	0.018	0.016	0.017	0.419	1.380	0.107	0.878	0.209	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	22	24	22	23	24	25
N.S.	1	1.00	1.00	0.88	0.88	0.96	0.88	0.92	0.96	1.00
time (sec)	N/A	0.018	0.014	0.110	0.433	1.079	0.416	1.002	0.043	0.010

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	21	20	22	31	21	22	30
N.S.	1	1.00	1.00	0.70	0.67	0.73	1.03	0.70	0.73	1.00
time (sec)	N/A	0.094	0.018	0.016	0.427	1.110	7.857	0.973	0.034	0.010

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	8	11	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.10	0.80	1.00
time (sec)	N/A	0.005	0.002	0.292	0.423	1.017	0.080	0.889	0.063	0.004

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	24	36	40	42	37	47	47
N.S.	1	1.00	1.00	0.51	0.77	0.85	0.89	0.79	1.00	1.00
time (sec)	N/A	0.023	0.031	0.292	0.971	1.081	0.118	0.961	0.132	0.043

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	24	23	23	27	21	21	30
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.90	0.70	0.70	1.00
time (sec)	N/A	0.008	0.014	0.262	0.972	0.976	0.116	0.851	0.037	0.022

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	14	13	24	22	13	13	0
N.S.	1	1.00	1.00	0.67	0.62	1.14	1.05	0.62	0.62	0.00
time (sec)	N/A	0.008	0.012	0.098	0.424	0.997	0.548	1.235	0.167	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	24	26	11	11	0
N.S.	1	1.00	1.00	0.80	0.73	1.60	1.73	0.73	0.73	0.00
time (sec)	N/A	0.008	0.006	0.136	0.432	1.089	0.549	0.854	0.060	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	20	186	89	138	18	118	0
N.S.	1	1.00	1.00	0.95	8.86	4.24	6.57	0.86	5.62	0.00
time (sec)	N/A	0.034	0.081	0.125	1.029	1.119	1.538	0.952	1.334	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	10	10	10	10	10	0
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.00
time (sec)	N/A	0.005	0.002	0.044	0.427	1.059	0.065	0.943	0.028	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	10	10	10	10	10	0
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.00
time (sec)	N/A	0.006	0.002	0.031	0.433	1.139	0.062	0.826	0.026	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	12	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	1.50	0.00
time (sec)	N/A	0.012	0.001	0.040	0.431	1.333	0.064	0.945	0.030	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	32	14	22	15	14	11	0
N.S.	1	1.00	1.00	2.91	1.27	2.00	1.36	1.27	1.00	0.00
time (sec)	N/A	0.015	0.007	0.406	0.428	1.061	0.090	1.109	0.072	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	6	15	9	18	12	9	6	0
N.S.	1	1.00	0.86	2.14	1.29	2.57	1.71	1.29	0.86	0.00
time (sec)	N/A	0.024	0.007	0.286	0.430	1.336	0.074	1.018	0.201	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	28	17	12	23	19	18	10	0
N.S.	1	1.00	2.00	1.21	0.86	1.64	1.36	1.29	0.71	0.00
time (sec)	N/A	0.006	0.014	0.023	0.979	0.996	0.074	1.188	0.170	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	26	19	12	20	17	22	18	0
N.S.	1	1.00	1.62	1.19	0.75	1.25	1.06	1.38	1.12	0.00
time (sec)	N/A	0.011	0.011	0.033	0.965	0.802	0.152	1.028	0.270	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	12	19	10	12	12	0
N.S.	1	1.00	1.00	1.10	1.20	1.90	1.00	1.20	1.20	0.00
time (sec)	N/A	0.028	0.016	0.054	0.953	0.898	0.371	0.834	0.267	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	12	15	14	25	10	14	14	0
N.S.	1	1.00	0.86	1.07	1.00	1.79	0.71	1.00	1.00	0.00
time (sec)	N/A	0.063	0.007	0.313	0.965	0.999	1.377	0.785	0.275	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	25	15	28	24	29	12	12	0
N.S.	1	1.00	2.27	1.36	2.55	2.18	2.64	1.09	1.09	0.00
time (sec)	N/A	0.022	0.043	0.049	0.962	1.031	0.537	1.083	0.235	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	21	17	23	14	8	12	10	0
N.S.	1	1.00	1.31	1.06	1.44	0.88	0.50	0.75	0.62	0.00
time (sec)	N/A	0.026	0.023	0.042	0.956	0.913	0.485	1.013	0.166	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	19	18	22	19	18	18	0
N.S.	1	1.00	1.00	0.76	0.72	0.88	0.76	0.72	0.72	0.00
time (sec)	N/A	0.025	0.014	0.024	0.456	0.937	0.117	0.918	0.216	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	14	13	13	12	15	8	14
N.S.	1	1.00	1.00	0.67	0.62	0.62	0.57	0.71	0.38	0.67
time (sec)	N/A	0.004	0.003	0.314	0.420	0.700	0.104	0.846	0.257	0.006

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	10	10	10	10	10	16
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	1.14
time (sec)	N/A	0.016	0.005	0.032	0.956	1.239	0.113	0.991	0.063	0.007

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	47	29	35	45	42	44	34	47
N.S.	1	1.00	0.92	0.57	0.69	0.88	0.82	0.86	0.67	0.92
time (sec)	N/A	0.016	0.027	0.398	0.953	0.842	0.117	0.940	0.122	0.037

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	29	25	25	33
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.88	0.76	0.76	1.00
time (sec)	N/A	0.010	0.014	0.137	0.428	0.755	0.137	1.049	0.203	0.015

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	19	18	68	94	18	19	29
N.S.	1	1.00	1.00	0.66	0.62	2.34	3.24	0.62	0.66	1.00
time (sec)	N/A	0.014	0.007	0.324	0.967	1.495	1.324	0.959	0.205	0.012

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	20	17	22	19	37	19	20	23
N.S.	1	1.00	0.74	0.63	0.81	0.70	1.37	0.70	0.74	0.85
time (sec)	N/A	0.010	0.007	0.302	0.957	0.802	0.633	1.028	0.042	0.011

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	25	29	10	18	28
N.S.	1	1.00	1.00	0.86	0.82	1.14	1.32	0.45	0.82	1.27
time (sec)	N/A	0.009	0.004	0.344	0.956	0.938	1.075	1.058	0.094	0.092

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	41	12	26	22	20	24	22
N.S.	1	1.00	1.00	1.86	0.55	1.18	1.00	0.91	1.09	1.00
time (sec)	N/A	0.013	0.004	0.317	0.969	0.944	1.068	0.826	0.258	0.017

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	37	34	25	22	43	21	23
N.S.	1	1.00	1.00	1.61	1.48	1.09	0.96	1.87	0.91	1.00
time (sec)	N/A	0.014	0.004	0.315	0.427	0.880	1.091	0.853	0.445	0.017

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	35	12	40	7	37	26	21
N.S.	1	1.00	1.00	1.67	0.57	1.90	0.33	1.76	1.24	1.00
time (sec)	N/A	0.013	0.003	0.308	0.503	0.868	1.042	1.074	0.094	0.015

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	7	8	30	0	6	6	21
N.S.	1	1.00	1.00	0.58	0.67	2.50	0.00	0.50	0.50	1.75
time (sec)	N/A	0.006	0.007	0.289	0.972	0.914	0.000	1.077	0.182	0.090

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	18	15	16	38	0	33	26	33
N.S.	1	1.00	0.95	0.79	0.84	2.00	0.00	1.74	1.37	1.74
time (sec)	N/A	0.012	0.009	0.403	0.997	0.966	0.000	1.081	0.294	0.085

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	12	7	6	16	0	6	6	18
N.S.	1	1.00	1.50	0.88	0.75	2.00	0.00	0.75	0.75	2.25
time (sec)	N/A	0.003	0.009	0.325	0.971	0.935	0.000	0.863	0.156	0.077

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	43	0	21	40	36
N.S.	1	1.00	1.00	0.81	0.78	1.59	0.00	0.78	1.48	1.33
time (sec)	N/A	0.010	0.006	0.299	0.969	0.833	0.000	0.930	0.315	0.169

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	25	33	39	0	71	28	39
N.S.	1	1.00	1.00	0.78	1.03	1.22	0.00	2.22	0.88	1.22
time (sec)	N/A	0.011	0.006	0.309	0.958	0.898	0.000	1.193	0.338	0.102

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	23	21	16	17	17	0	28	19	21
N.S.	1	1.10	1.00	0.76	0.81	0.81	0.00	1.33	0.90	1.00
time (sec)	N/A	0.005	0.005	0.304	0.961	0.719	0.000	0.780	0.221	0.179

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	54	23	33	39	22	31	22	0
N.S.	1	1.00	1.93	0.82	1.18	1.39	0.79	1.11	0.79	0.00
time (sec)	N/A	0.129	0.064	0.145	0.447	0.929	0.821	1.065	0.278	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	48	17	14	13	31	0	46	26	0
N.S.	1	1.30	0.46	0.38	0.35	0.84	0.00	1.24	0.70	0.00
time (sec)	N/A	0.021	0.052	0.076	0.973	0.982	0.000	0.821	0.236	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	21	11	47	32	27	11	10	0
N.S.	1	1.00	1.50	0.79	3.36	2.29	1.93	0.79	0.71	0.00
time (sec)	N/A	0.034	0.018	0.131	0.438	1.203	1.480	0.959	0.215	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	23	4	15	45	15	17	3	0
N.S.	1	1.00	2.09	0.36	1.36	4.09	1.36	1.55	0.27	0.00
time (sec)	N/A	0.031	0.007	0.041	0.435	0.913	0.166	1.098	0.352	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	19	19	15	14	14	0	14	14	0
N.S.	1	1.06	1.06	0.83	0.78	0.78	0.00	0.78	0.78	0.00
time (sec)	N/A	0.047	0.017	0.061	0.430	1.056	0.000	0.999	0.315	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	15	17	14	14	0
N.S.	1	1.00	1.00	0.83	0.78	0.83	0.94	0.78	0.78	0.00
time (sec)	N/A	0.045	0.020	0.126	0.435	1.002	2.287	0.925	0.304	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	0	15	37	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.88	2.18	0.00
time (sec)	N/A	0.013	0.005	0.077	0.975	0.859	0.000	0.838	0.293	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	0	18	16	7	16	16	0
N.S.	1	1.00	1.00	0.00	0.90	0.80	0.35	0.80	0.80	0.00
time (sec)	N/A	0.026	0.004	0.013	0.442	0.593	0.737	0.933	0.341	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	15	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.00	0.00
time (sec)	N/A	0.023	0.005	0.372	0.431	0.951	1.787	0.951	0.232	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	38	0	38	0	15	-1	0
N.S.	1	1.00	1.00	0.90	0.00	0.90	0.00	0.36	-0.02	0.00
time (sec)	N/A	0.065	0.044	0.252	0.000	0.601	0.000	1.079	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.00
time (sec)	N/A	0.025	0.007	0.259	0.991	0.901	2.683	0.927	0.349	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	17	22	22	22	17	0
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61	0.00
time (sec)	N/A	0.010	0.001	0.014	0.437	0.918	0.105	0.876	0.034	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	11	13	11	15	13	9	0
N.S.	1	1.00	1.00	0.65	0.76	0.65	0.88	0.76	0.53	0.00
time (sec)	N/A	0.007	0.001	0.016	0.429	0.857	0.103	0.795	0.166	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	38	38	29	28	28	26	70	40	0
N.S.	1	1.06	1.06	0.81	0.78	0.78	0.72	1.94	1.11	0.00
time (sec)	N/A	0.026	0.006	0.051	0.445	0.933	0.132	1.048	0.345	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	23	17	15	18	17	15	21	0
N.S.	1	1.00	1.21	0.89	0.79	0.95	0.89	0.79	1.11	0.00
time (sec)	N/A	0.008	0.002	0.297	0.432	0.892	0.068	0.939	0.002	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	23	18	25	31	22	26	0
N.S.	1	1.00	0.88	0.68	0.53	0.74	0.91	0.65	0.76	0.00
time (sec)	N/A	0.034	0.012	0.329	0.434	0.880	0.065	0.869	0.022	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	71	26	42	69	46	38	33	0
N.S.	1	1.00	2.73	1.00	1.62	2.65	1.77	1.46	1.27	0.00
time (sec)	N/A	0.012	0.006	0.314	0.434	0.969	0.153	0.991	0.291	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	14	18	11	17	17	11	11	0
N.S.	1	1.00	0.61	0.78	0.48	0.74	0.74	0.48	0.48	0.00
time (sec)	N/A	0.008	0.019	0.044	0.440	0.875	0.458	0.935	0.023	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	22	22	19	21	26	19	19	0
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70	0.00
time (sec)	N/A	0.009	0.036	0.053	0.444	0.705	0.328	0.883	0.036	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	20	32	24	20	107	329	20	0
N.S.	1	1.00	0.65	1.03	0.77	0.65	3.45	10.61	0.65	0.00
time (sec)	N/A	0.010	0.020	0.065	0.442	1.396	1.058	1.142	0.026	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	10	13	15	13	13	0
N.S.	1	1.00	1.00	0.82	0.59	0.76	0.88	0.76	0.76	0.00
time (sec)	N/A	0.003	0.002	0.036	0.434	0.925	0.400	0.909	0.195	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	57	94	22	15	12	35	0
N.S.	1	1.00	1.00	4.75	7.83	1.83	1.25	1.00	2.92	0.00
time (sec)	N/A	0.019	0.016	0.267	0.974	1.014	20.571	0.989	0.428	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	20	107	21	19	23	13	0
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	0.87	0.00
time (sec)	N/A	0.014	0.019	0.029	0.992	0.831	0.181	1.000	0.023	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	33	39	22	38	20	0
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91	0.00
time (sec)	N/A	0.015	0.003	0.010	0.959	0.998	1.976	0.953	0.021	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	23	23	22	23	22	0
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88	0.00
time (sec)	N/A	0.032	0.007	0.052	0.973	0.954	0.203	0.961	0.049	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	24	19	19	19	19	0
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83	0.00
time (sec)	N/A	0.047	0.015	0.305	0.993	0.873	0.395	0.986	0.220	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	57	49	45	78	30	0	0	-1	0
N.S.	1	1.50	1.29	1.18	2.05	0.79	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.031	0.063	0.026	0.966	0.826	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	21	21	22	21	21	25
N.S.	1	1.00	1.00	0.84	0.84	0.84	0.88	0.84	0.84	1.00
time (sec)	N/A	0.007	0.002	0.252	0.442	0.689	0.064	0.944	0.043	0.005

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	29	29	29	34	29	29	30
N.S.	1	1.00	1.00	0.74	0.74	0.74	0.87	0.74	0.74	0.77
time (sec)	N/A	0.016	0.001	0.288	0.436	0.761	0.068	0.891	0.032	0.005

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	29	23	27	75	23	23	0
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.26	1.00	1.00	0.00
time (sec)	N/A	0.008	0.009	0.325	0.435	0.804	60.373	0.977	0.725	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	27	23	22	22	20	23	20	26
N.S.	1	1.00	0.90	0.77	0.73	0.73	0.67	0.77	0.67	0.87
time (sec)	N/A	0.011	0.007	0.275	0.429	0.760	0.079	0.986	0.035	0.010

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	35	27	26	32	34	26	32	39
N.S.	1	1.00	0.85	0.66	0.63	0.78	0.83	0.63	0.78	0.95
time (sec)	N/A	0.012	0.013	0.282	0.965	0.924	0.102	0.849	0.041	0.018

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	15	15	14	17	8	19
N.S.	1	1.00	1.00	0.76	0.71	0.71	0.67	0.81	0.38	0.90
time (sec)	N/A	0.006	0.004	0.282	0.433	0.782	0.110	0.816	0.089	0.007

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	29	28	28	36	28	30	36
N.S.	1	1.00	0.97	0.88	0.85	0.85	1.09	0.85	0.91	1.09
time (sec)	N/A	0.017	0.010	0.419	0.999	0.838	0.116	0.863	0.044	0.016

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	27
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	1.08
time (sec)	N/A	0.016	0.004	0.404	0.968	0.813	0.112	0.904	0.040	0.008

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	39	38	38	46	38	40	55
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.98	0.81	0.85	1.17
time (sec)	N/A	0.062	0.017	0.411	0.954	0.574	0.121	0.929	0.197	0.027

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	17	20	17	23
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.80	0.68	0.92
time (sec)	N/A	0.037	0.006	0.028	0.431	0.783	0.139	0.883	0.227	0.009

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	26	25	25	26	28	25	33
N.S.	1	1.00	0.88	0.79	0.76	0.76	0.79	0.85	0.76	1.00
time (sec)	N/A	0.054	0.014	0.043	0.438	0.725	0.259	0.969	0.088	0.019

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	61	37	26	25	25	29	29	29	21
N.S.	1	1.65	1.00	0.70	0.68	0.68	0.78	0.78	0.78	0.57
time (sec)	N/A	0.037	0.007	0.036	0.446	0.720	0.187	0.994	0.061	0.011

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	69	26	43	51	60	59	25	31
N.S.	1	1.00	2.23	0.84	1.39	1.65	1.94	1.90	0.81	1.00
time (sec)	N/A	0.009	0.018	0.042	0.982	0.885	0.583	0.989	0.083	0.036

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	26	25	25	26	29	15	25
N.S.	1	1.00	1.00	0.63	0.61	0.61	0.63	0.71	0.37	0.61
time (sec)	N/A	0.020	0.008	0.332	0.432	0.896	0.187	0.886	0.246	0.013

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	16	17	22	29	17	18	22	16
N.S.	1	1.00	0.64	0.68	0.88	1.16	0.68	0.72	0.88	0.64
time (sec)	N/A	0.011	0.009	0.299	0.432	0.772	0.096	0.994	0.168	0.013

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	33	32	39	31	45	32	39
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89	1.08
time (sec)	N/A	0.017	0.014	0.281	0.431	0.795	0.090	1.037	0.029	0.020

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	37	25	30	45	31	27	30	31
N.S.	1	1.00	0.90	0.61	0.73	1.10	0.76	0.66	0.73	0.76
time (sec)	N/A	0.037	0.021	0.029	0.445	0.891	0.142	0.881	0.064	0.026

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	28	29	45	26	37	29	29
N.S.	1	1.00	1.00	1.04	1.07	1.67	0.96	1.37	1.07	1.07
time (sec)	N/A	0.036	0.017	0.308	0.428	0.847	0.146	0.856	0.191	0.026

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	47	31	29	45	32	35	31	35
N.S.	1	1.00	1.21	0.79	0.74	1.15	0.82	0.90	0.79	0.90
time (sec)	N/A	0.036	0.018	0.354	0.429	0.888	0.159	1.106	0.047	0.025

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	35	40	65	37	40	40	41
N.S.	1	1.00	0.87	0.76	0.87	1.41	0.80	0.87	0.87	0.89
time (sec)	N/A	0.022	0.016	0.037	0.436	0.926	0.175	1.026	0.183	0.022

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	24	23	23	22	24	29	27
N.S.	1	1.00	1.00	0.83	0.79	0.79	0.76	0.83	1.00	0.93
time (sec)	N/A	0.028	0.008	0.029	0.956	0.909	0.128	1.057	0.049	0.012

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	21	22	23	27	27
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.81	0.85	1.00	1.00
time (sec)	N/A	0.025	0.007	0.322	0.981	0.901	0.190	0.779	0.049	0.011

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	32	26	25	25	29	27	17	24
N.S.	1	1.00	1.33	1.08	1.04	1.04	1.21	1.12	0.71	1.00
time (sec)	N/A	0.010	0.011	0.038	0.968	0.677	0.157	0.959	0.059	0.017

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	33	32	44	39	33	49	41
N.S.	1	1.00	1.00	0.80	0.78	1.07	0.95	0.80	1.20	1.00
time (sec)	N/A	0.087	0.023	0.041	0.986	0.854	0.162	0.981	0.252	0.030

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	35	36	57	37	62	38	46
N.S.	1	1.00	0.87	0.76	0.78	1.24	0.80	1.35	0.83	1.00
time (sec)	N/A	0.204	0.020	0.317	0.977	0.847	0.220	1.039	0.190	0.027

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	37	33	36	59	36	32	42	37
N.S.	1	1.00	0.79	0.70	0.77	1.26	0.77	0.68	0.89	0.79
time (sec)	N/A	0.043	0.026	0.362	0.989	0.879	0.175	0.908	0.043	0.032

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	67	73	54	53	53	70	53	47	58
N.S.	1	1.00	1.09	0.81	0.79	0.79	1.04	0.79	0.70	0.87
time (sec)	N/A	0.038	0.057	0.050	0.986	0.895	0.208	0.993	0.094	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	67	46	54	58	306	64	73	46
N.S.	1	1.00	1.40	0.96	1.12	1.21	6.38	1.33	1.52	0.96
time (sec)	N/A	0.053	0.018	0.341	0.977	0.894	0.591	1.116	0.167	0.036

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	46	38	43	66	46	39	44	48
N.S.	1	1.00	0.79	0.66	0.74	1.14	0.79	0.67	0.76	0.83
time (sec)	N/A	0.055	0.026	0.427	0.979	0.960	0.196	0.963	0.203	0.033

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	47	36	35	35	44	35	35	51
N.S.	1	1.00	0.92	0.71	0.69	0.69	0.86	0.69	0.69	1.00
time (sec)	N/A	0.284	0.023	0.318	0.995	0.907	0.435	0.859	0.242	0.035

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	34	33	33	32	33	33	29
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.78	0.80	0.80	0.71
time (sec)	N/A	0.311	0.008	0.283	0.426	0.861	0.191	0.971	0.056	0.013

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	52	41	49	45	73	50	64	59
N.S.	1	1.00	0.93	0.73	0.88	0.80	1.30	0.89	1.14	1.05
time (sec)	N/A	0.030	0.011	0.296	0.985	0.858	0.136	0.878	0.440	0.022

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	50	51	49	43	71	50	68	59
N.S.	1	1.00	0.89	0.91	0.88	0.77	1.27	0.89	1.21	1.05
time (sec)	N/A	0.028	0.006	0.280	0.960	0.845	0.135	1.110	0.118	0.015

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	11	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83	1.00
time (sec)	N/A	0.003	0.002	0.262	0.427	0.590	0.104	0.933	0.030	0.005

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	23	18	19	22	18	22
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.86	1.00	0.82	1.00
time (sec)	N/A	0.008	0.004	0.272	0.432	0.889	0.213	0.868	0.249	0.008

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	60	60	57	53	83	58	88	66
N.S.	1	1.00	0.95	0.95	0.90	0.84	1.32	0.92	1.40	1.05
time (sec)	N/A	0.037	0.014	0.278	0.961	0.919	0.174	0.928	0.249	0.026

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	68	60	57	62	80	58	86	68
N.S.	1	1.00	1.05	0.92	0.88	0.95	1.23	0.89	1.32	1.05
time (sec)	N/A	0.039	0.015	0.303	0.957	0.794	0.200	0.989	0.252	0.028

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	34	31	33	29	40	29	33
N.S.	1	1.00	1.00	1.03	0.94	1.00	0.88	1.21	0.88	1.00
time (sec)	N/A	0.020	0.005	0.270	0.426	0.764	0.267	0.904	0.073	0.010

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	74	66	66	68	90	67	99	80
N.S.	1	1.00	1.01	0.90	0.90	0.93	1.23	0.92	1.36	1.10
time (sec)	N/A	0.044	0.012	0.284	0.957	1.040	0.219	0.919	0.104	0.029

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	45	0	0	0	92	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.010	0.029	0.000	1.034	2.971	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	38	33	32	26	37	34	18	27
N.S.	1	1.00	1.41	1.22	1.19	0.96	1.37	1.26	0.67	1.00
time (sec)	N/A	0.008	0.005	0.278	0.974	1.041	0.152	0.860	0.070	0.009

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	30	29	26	24	30	13	15
N.S.	1	1.00	1.00	2.00	1.93	1.73	1.60	2.00	0.87	1.00
time (sec)	N/A	0.007	0.004	0.335	0.422	0.843	0.155	0.815	0.048	0.006

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	25	20	19	26	20	24
N.S.	1	1.00	1.00	0.96	1.04	0.83	0.79	1.08	0.83	1.00
time (sec)	N/A	0.010	0.006	0.316	0.420	0.585	0.252	0.948	0.275	0.009

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	46	41	40	36	44	42	31	35
N.S.	1	1.00	1.31	1.17	1.14	1.03	1.26	1.20	0.89	1.00
time (sec)	N/A	0.014	0.007	0.275	0.971	0.966	0.204	0.932	0.223	0.014

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	50	42	37	41	34	38	22	50
N.S.	1	1.00	1.92	1.62	1.42	1.58	1.31	1.46	0.85	1.92
time (sec)	N/A	0.012	0.007	0.270	0.426	0.879	0.223	0.967	0.200	0.018

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	48	41	40	45	48	42	31	37
N.S.	1	1.00	1.30	1.11	1.08	1.22	1.30	1.14	0.84	1.00
time (sec)	N/A	0.012	0.007	0.280	0.961	0.602	0.226	1.050	0.080	0.013

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	44	0	0	0	95	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.11	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.010	0.034	0.000	0.967	1.007	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	29	13	13	15
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.93	0.87	0.87	1.00
time (sec)	N/A	0.006	0.003	0.289	0.965	0.952	0.145	0.968	0.185	0.006

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	79	24	98	199	19	114	33	69
N.S.	1	1.00	0.72	0.22	0.90	1.83	0.17	1.05	0.30	0.63
time (sec)	N/A	0.062	0.029	0.271	0.966	0.965	0.140	0.968	0.100	0.314

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	204	55	180	0	39	177	174	260
N.S.	1	1.00	1.01	0.27	0.90	0.00	0.19	0.88	0.87	1.29
time (sec)	N/A	0.314	0.167	0.361	0.968	0.000	0.152	1.013	0.588	0.267

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	201	201	204	60	160	0	41	177	182	260
N.S.	1	1.00	1.01	0.30	0.80	0.00	0.20	0.88	0.91	1.29
time (sec)	N/A	0.271	0.074	0.346	0.976	0.000	0.139	0.888	0.591	0.154

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	201	201	204	58	160	0	41	177	202	260
N.S.	1	1.00	1.01	0.29	0.80	0.00	0.20	0.88	1.00	1.29
time (sec)	N/A	0.317	0.090	0.349	0.976	0.000	0.154	0.993	0.797	0.162

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	204	73	180	0	39	177	202	260
N.S.	1	1.00	1.01	0.36	0.90	0.00	0.19	0.88	1.00	1.29
time (sec)	N/A	0.313	0.061	0.319	0.976	0.000	0.151	0.929	0.410	0.140

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	11	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83	1.00
time (sec)	N/A	0.003	0.003	0.338	0.427	0.965	0.116	0.952	0.184	0.004

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	23	18	19	22	18	22
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.86	1.00	0.82	1.00
time (sec)	N/A	0.008	0.005	0.314	0.427	1.009	0.264	0.901	0.268	0.008

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	172	76	192	0	48	185	210	268
N.S.	1	1.00	0.82	0.36	0.92	0.00	0.23	0.89	1.00	1.28
time (sec)	N/A	0.329	0.171	0.362	0.966	0.000	0.194	1.099	0.394	0.250

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	211	211	174	78	173	0	51	185	210	270
N.S.	1	1.00	0.82	0.37	0.82	0.00	0.24	0.88	1.00	1.28
time (sec)	N/A	0.327	0.165	0.349	0.983	0.000	0.222	0.988	0.782	0.232

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-1)	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	211	211	175	81	172	0	51	185	214	270
N.S.	1	1.00	0.83	0.38	0.82	0.00	0.24	0.88	1.01	1.28
time (sec)	N/A	0.288	0.137	0.367	0.966	0.000	0.219	1.050	0.692	0.199

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	45	0	0	0	92	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.011	0.035	0.000	1.117	24.441	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	21	10	27	9	8	27	9	21
N.S.	1	1.00	0.60	0.29	0.77	0.26	0.23	0.77	0.26	0.60
time (sec)	N/A	0.425	0.007	0.283	0.984	0.600	0.129	0.883	0.034	0.011

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	51	44	54	71	63	44	45	56
N.S.	1	1.00	0.85	0.73	0.90	1.18	1.05	0.73	0.75	0.93
time (sec)	N/A	0.022	0.031	0.426	0.962	1.076	0.175	1.052	0.079	0.039

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	30	28	38	52	36	28	27	31
N.S.	1	1.00	0.70	0.65	0.88	1.21	0.84	0.65	0.63	0.72
time (sec)	N/A	0.014	0.013	0.275	0.966	0.745	0.145	0.915	0.182	0.019

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	88	103	0	448	323	92	159	113
N.S.	1	1.00	0.98	1.14	0.00	4.98	3.59	1.02	1.77	1.26
time (sec)	N/A	0.071	0.095	0.591	0.000	1.126	1.010	0.992	0.287	0.117

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	35	33	50	31	33	41	38
N.S.	1	1.00	1.00	0.92	0.87	1.32	0.82	0.87	1.08	1.00
time (sec)	N/A	0.026	0.016	0.397	0.977	0.935	0.141	0.918	0.202	0.021

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	41	42	58	53	43	60	56
N.S.	1	1.00	0.86	0.72	0.74	1.02	0.93	0.75	1.05	0.98
time (sec)	N/A	0.024	0.025	0.292	0.971	0.828	0.166	0.950	0.082	0.034

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	33	29	31	40	32	28	29	33
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.78	0.81	0.92
time (sec)	N/A	0.023	0.016	0.273	0.974	0.840	0.150	1.059	0.180	0.022

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	78	39	40	40	46	40	52	73
N.S.	1	1.00	1.59	0.80	0.82	0.82	0.94	0.82	1.06	1.49
time (sec)	N/A	0.037	0.015	0.277	0.958	1.092	0.147	0.934	0.086	0.035

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	20	20	14	14	13	14	0	14	0
N.S.	1	1.54	1.54	1.08	1.08	1.00	1.08	0.00	1.08	0.00
time (sec)	N/A	0.046	0.025	0.303	0.450	1.124	2.481	0.000	0.322	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	38	35	34	34	42	34	34	44
N.S.	1	1.00	0.93	0.85	0.83	0.83	1.02	0.83	0.83	1.07
time (sec)	N/A	0.040	0.012	0.033	0.975	1.148	0.128	1.514	0.202	0.019

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	25	24	24	24	25	22	30
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.75	0.78	0.69	0.94
time (sec)	N/A	0.025	0.005	0.026	0.423	1.091	0.121	0.908	0.086	0.009

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	15	19	16	16
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.60	0.76	0.64	0.64
time (sec)	N/A	0.017	0.005	0.029	0.425	1.399	0.112	0.910	0.584	0.008

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	24	22	32	46	29	23	22	24
N.S.	1	1.00	0.67	0.61	0.89	1.28	0.81	0.64	0.61	0.67
time (sec)	N/A	0.022	0.014	0.293	0.444	1.091	0.115	0.847	0.039	0.019

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	24	22	72	72	70	22	29	24
N.S.	1	1.00	0.53	0.49	1.60	1.60	1.56	0.49	0.64	0.53
time (sec)	N/A	0.012	0.006	0.288	0.440	1.060	0.166	0.995	0.099	0.009

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	41	34	48	67	42	55	43	37
N.S.	1	1.00	0.75	0.62	0.87	1.22	0.76	1.00	0.78	0.67
time (sec)	N/A	0.039	0.013	0.263	0.459	1.244	0.130	0.887	0.048	0.019

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	38	59	41	43	31	31
N.S.	1	1.00	1.06	0.97	1.06	1.64	1.14	1.19	0.86	0.86
time (sec)	N/A	0.015	0.024	0.286	0.447	1.113	0.142	1.063	0.039	0.030

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	28	31	48	29	40	34	36
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.83	1.14	0.97	1.03
time (sec)	N/A	0.014	0.017	0.312	0.427	0.738	0.131	0.982	0.220	0.024

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	46	40	50	85	51	42	45	44
N.S.	1	1.00	0.75	0.66	0.82	1.39	0.84	0.69	0.74	0.72
time (sec)	N/A	0.012	0.024	0.288	0.427	1.191	0.164	0.947	0.123	0.031

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	36	36	44	62	42	34	39	40
N.S.	1	1.00	0.71	0.71	0.86	1.22	0.82	0.67	0.76	0.78
time (sec)	N/A	0.014	0.017	0.366	0.955	1.033	0.169	0.851	0.203	0.023

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	60	105	58	47	55	43
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02	0.80
time (sec)	N/A	0.026	0.047	0.310	0.426	1.149	0.178	0.908	0.051	0.029

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	42	53	32	40	53	49	48	33	38
N.S.	1	1.17	1.47	0.89	1.11	1.47	1.36	1.33	0.92	1.06
time (sec)	N/A	0.015	0.038	0.378	0.971	1.319	0.115	1.081	0.252	0.046

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	31	41	59	39	31	40	40
N.S.	1	1.00	0.69	0.53	0.71	1.02	0.67	0.53	0.69	0.69
time (sec)	N/A	0.017	0.021	0.268	0.957	0.986	0.155	0.941	0.074	0.026

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	34	36	45	42	36	36	46
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84	1.07
time (sec)	N/A	0.021	0.035	0.512	0.969	1.169	0.142	0.851	0.191	0.048

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	62	37	47	68	58	51	34	48
N.S.	1	1.00	1.44	0.86	1.09	1.58	1.35	1.19	0.79	1.12
time (sec)	N/A	0.023	0.042	0.353	0.946	1.051	0.169	0.945	0.215	0.058

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	25	27	30	38	24	40	32	28
N.S.	1	1.00	0.68	0.73	0.81	1.03	0.65	1.08	0.86	0.76
time (sec)	N/A	0.013	0.012	0.269	0.955	1.373	0.169	1.012	0.037	0.016

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	39	26	29	28	38	26	32	28	32
N.S.	1	1.22	0.81	0.91	0.88	1.19	0.81	1.00	0.88	1.00
time (sec)	N/A	0.026	0.016	0.256	0.538	1.222	0.173	1.605	0.189	0.019

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	19	20	11	11	13
N.S.	1	1.00	1.00	0.92	0.85	1.46	1.54	0.85	0.85	1.00
time (sec)	N/A	0.002	0.004	0.258	0.498	1.414	0.281	0.926	0.199	0.006

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	46	45	57	81	51	56	52	49
N.S.	1	1.00	0.85	0.83	1.06	1.50	0.94	1.04	0.96	0.91
time (sec)	N/A	0.030	0.031	0.268	0.447	1.300	0.464	0.943	0.106	0.037

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	134	75	147	338	66	150	76	107
N.S.	1	1.00	0.85	0.48	0.94	2.15	0.42	0.96	0.48	0.68
time (sec)	N/A	0.101	0.118	0.282	1.297	1.461	0.443	0.927	0.114	0.140

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	75	53	60	78	78	50	53	54
N.S.	1	1.00	1.17	0.83	0.94	1.22	1.22	0.78	0.83	0.84
time (sec)	N/A	0.030	0.050	0.289	1.343	1.329	0.478	0.882	0.222	0.063

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	29	34	45	27	30	29	33
N.S.	1	1.00	0.85	0.74	0.87	1.15	0.69	0.77	0.74	0.85
time (sec)	N/A	0.025	0.014	0.262	0.603	1.320	0.180	0.975	0.050	0.019

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	319	319	293	47	335	1751	37	250	285	340
N.S.	1	1.00	0.92	0.15	1.05	5.49	0.12	0.78	0.89	1.07
time (sec)	N/A	0.584	0.312	0.273	1.208	41.316	0.301	1.828	1.604	0.700

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	65	39	56	73	58	55	41	48
N.S.	1	1.00	1.10	0.66	0.95	1.24	0.98	0.93	0.69	0.81
time (sec)	N/A	0.021	0.074	0.280	1.146	1.131	0.172	1.099	0.243	0.087

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	33	29	31	40	32	28	29	33
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.78	0.81	0.92
time (sec)	N/A	0.024	0.014	0.000	1.417	0.695	0.151	0.924	0.002	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	32	31	38	65	36	35	33	35
N.S.	1	1.00	0.84	0.82	1.00	1.71	0.95	0.92	0.87	0.92
time (sec)	N/A	0.062	0.055	0.032	0.637	0.691	0.147	0.872	0.065	0.030

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	65	54	50	73	60	60	63	74
N.S.	1	1.00	1.02	0.84	0.78	1.14	0.94	0.94	0.98	1.16
time (sec)	N/A	0.073	0.038	0.313	1.366	1.250	0.229	0.937	0.105	0.044

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	54	59	97	68	72	73	69
N.S.	1	1.00	1.00	0.86	0.94	1.54	1.08	1.14	1.16	1.10
time (sec)	N/A	0.121	0.040	0.398	1.425	1.042	0.204	0.903	0.254	0.054

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	46	32	36	55	34	33	25	33
N.S.	1	1.00	1.07	0.74	0.84	1.28	0.79	0.77	0.58	0.77
time (sec)	N/A	0.019	0.018	0.320	0.479	1.194	0.161	0.767	0.054	0.027

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	20	19	43	43	39	18	18	20
N.S.	1	1.00	0.74	0.70	1.59	1.59	1.44	0.67	0.67	0.74
time (sec)	N/A	0.019	0.008	0.289	0.645	1.239	0.188	1.104	0.112	0.011

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	44	35	39	55	36	40	34	38
N.S.	1	1.00	0.96	0.76	0.85	1.20	0.78	0.87	0.74	0.83
time (sec)	N/A	0.030	0.016	0.280	0.535	1.211	0.132	1.032	0.187	0.022

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	50	39	35	37	56	34	43	33	41
N.S.	1	1.14	0.89	0.80	0.84	1.27	0.77	0.98	0.75	0.93
time (sec)	N/A	0.017	0.023	0.282	0.464	1.324	0.151	0.789	0.048	0.029

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	60	105	58	47	55	43
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02	0.80
time (sec)	N/A	0.018	0.020	0.002	0.511	1.067	0.184	0.972	0.002	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	33	32	39	31	45	32	39
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89	1.08
time (sec)	N/A	0.015	0.015	0.000	0.511	1.276	0.094	0.879	0.002	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	41	38	38	39	39	39	37	41
N.S.	1	1.00	0.93	0.86	0.86	0.89	0.89	0.89	0.84	0.93
time (sec)	N/A	0.037	0.013	0.016	0.594	1.011	0.067	0.974	0.049	0.027

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	91	89	91	98	100	98	88	105
N.S.	1	1.00	0.95	0.93	0.95	1.02	1.04	1.02	0.92	1.09
time (sec)	N/A	0.104	0.031	0.321	0.547	1.082	0.091	1.106	0.190	0.063

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	167	165	171	188	189	188	164	195
N.S.	1	1.00	1.00	0.99	1.02	1.13	1.13	1.13	0.98	1.17
time (sec)	N/A	0.188	0.033	0.301	0.634	1.124	0.103	0.978	0.216	0.076

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	263	264	273	307	313	307	263	315
N.S.	1	1.00	1.00	1.00	1.04	1.17	1.19	1.17	1.00	1.20
time (sec)	N/A	0.337	0.064	0.302	0.553	1.304	0.128	1.030	0.258	0.145

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	159	159	267	0	0	0	0	0	-1	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.082	0.459	0.074	0.000	0.930	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	66	63	0	203	246	60	155	79
N.S.	1	1.00	1.02	0.97	0.00	3.12	3.78	0.92	2.38	1.22
time (sec)	N/A	0.042	0.045	0.347	0.000	0.755	0.757	0.771	0.271	0.059

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	88	103	0	447	323	92	159	113
N.S.	1	1.00	0.99	1.16	0.00	5.02	3.63	1.03	1.79	1.27
time (sec)	N/A	0.045	0.079	0.345	0.000	1.325	1.017	1.135	0.283	0.104

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	127	155	0	1104	622	194	360	195
N.S.	1	1.00	0.98	1.19	0.00	8.49	4.78	1.49	2.77	1.50
time (sec)	N/A	0.072	0.141	0.351	0.000	1.269	1.894	1.034	0.505	0.179

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	168	206	0	1950	1027	363	640	340
N.S.	1	1.00	0.97	1.19	0.00	11.27	5.94	2.10	3.70	1.97
time (sec)	N/A	0.110	0.228	0.375	0.000	1.338	3.059	0.854	0.706	0.286

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	169	169	264	0	0	0	0	0	-1	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.079	0.412	0.069	0.000	1.324	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	44	31	41	52	46	46	36	47
N.S.	1	1.00	0.90	0.63	0.84	1.06	0.94	0.94	0.73	0.96
time (sec)	N/A	0.016	0.021	0.293	1.381	1.248	0.117	0.958	0.171	0.028

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	70	56	67	97	76	61	53	53
N.S.	1	1.00	1.15	0.92	1.10	1.59	1.25	1.00	0.87	0.87
time (sec)	N/A	0.022	0.049	0.300	1.256	1.146	0.165	1.157	0.209	0.062

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	33	28	30	45	31	32	25	36
N.S.	1	1.00	0.92	0.78	0.83	1.25	0.86	0.89	0.69	1.00
time (sec)	N/A	0.008	0.017	0.289	0.646	1.135	0.130	0.981	0.051	0.022

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	87	60	90	165	88	62	65	58
N.S.	1	1.00	1.00	0.69	1.03	1.90	1.01	0.71	0.75	0.67
time (sec)	N/A	0.023	0.032	0.290	0.559	0.741	0.223	0.899	0.094	0.042

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	70	62	69	116	80	67	77	82
N.S.	1	1.00	0.86	0.77	0.85	1.43	0.99	0.83	0.95	1.01
time (sec)	N/A	0.056	0.037	0.429	1.314	0.918	0.233	1.163	0.109	0.049

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	87	60	90	165	90	62	90	62
N.S.	1	1.00	0.84	0.58	0.87	1.59	0.87	0.60	0.87	0.60
time (sec)	N/A	0.074	0.031	0.291	0.554	1.226	0.210	0.978	0.213	0.042

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	99	64	94	173	90	66	85	66
N.S.	1	1.00	0.97	0.63	0.92	1.70	0.88	0.65	0.83	0.65
time (sec)	N/A	0.037	0.056	0.299	0.618	0.885	0.228	1.070	0.213	0.066

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	39	39	37	47	37	37
N.S.	1	1.00	1.00	0.88	0.98	0.98	0.92	1.18	0.92	0.92
time (sec)	N/A	0.023	0.006	0.256	0.511	1.225	0.254	0.971	0.204	0.014

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	62	55	83	113	102	58	79	62
N.S.	1	1.00	0.75	0.66	1.00	1.36	1.23	0.70	0.95	0.75
time (sec)	N/A	0.039	0.023	0.293	1.113	0.768	0.685	0.965	0.146	0.034

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	32	31	37	42	31	31	49
N.S.	1	1.00	1.00	0.65	0.63	0.76	0.86	0.63	0.63	1.00
time (sec)	N/A	0.057	0.023	0.293	0.533	1.628	2.595	0.994	0.190	0.012

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	32	31	31	48	31	31	55
N.S.	1	1.00	1.00	0.58	0.56	0.56	0.87	0.56	0.56	1.00
time (sec)	N/A	0.238	0.047	0.286	0.499	1.184	2.745	1.055	0.066	0.015

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	18	19	18	18	22
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.86	0.82	0.82	1.00
time (sec)	N/A	0.009	0.008	0.083	0.718	1.107	0.138	0.835	0.134	0.008

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	19	13	12	11	22	12	11	15
N.S.	1	1.00	1.27	0.87	0.80	0.73	1.47	0.80	0.73	1.00
time (sec)	N/A	0.006	0.015	0.022	0.522	1.035	0.971	0.924	0.030	0.010

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	21	20	19	20	21	19	23
N.S.	1	1.00	0.96	0.84	0.80	0.76	0.80	0.84	0.76	0.92
time (sec)	N/A	0.017	0.011	0.273	0.478	0.911	0.161	0.867	0.219	0.011

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	0	26	25	31
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.00	0.79	0.76	0.94
time (sec)	N/A	0.019	0.021	0.045	0.518	0.717	0.000	1.178	0.245	0.013

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	24	17	19	19	134	19	16	56
N.S.	1	1.00	0.83	0.59	0.66	0.66	4.62	0.66	0.55	1.93
time (sec)	N/A	0.009	0.008	0.299	0.494	1.120	1.350	0.915	0.543	0.028

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	53	20	30	55	63	3966	49	43	37
N.S.	1	1.02	0.38	0.58	1.06	1.21	76.27	0.94	0.83	0.71
time (sec)	N/A	0.010	0.005	0.301	0.680	1.100	3.268	1.006	0.048	0.041

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	127	26	59	111	125	971	104	96	66
N.S.	1	1.08	0.22	0.50	0.94	1.06	8.23	0.88	0.81	0.56
time (sec)	N/A	0.038	0.006	0.332	0.523	0.976	20.367	0.918	0.210	0.068

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	22	85	105	86	0	82	120	97
N.S.	1	1.00	0.21	0.82	1.01	0.83	0.00	0.79	1.15	0.93
time (sec)	N/A	0.040	0.004	0.536	1.393	1.188	0.000	1.138	0.212	0.067

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	67	39	43	32	0	29	43	38
N.S.	1	1.00	1.76	1.03	1.13	0.84	0.00	0.76	1.13	1.00
time (sec)	N/A	0.013	0.026	0.164	1.381	0.923	0.000	0.920	0.194	0.024

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	95	115	195	130	73	0	103	140	80
N.S.	1	1.03	1.25	2.12	1.41	0.79	0.00	1.12	1.52	0.87
time (sec)	N/A	0.059	0.291	0.102	1.449	1.028	0.000	1.029	0.345	0.076

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	54	54	47	64	0	65	0	74	95	121
N.S.	1	1.00	0.87	1.19	0.00	1.20	0.00	1.37	1.76	2.24
time (sec)	N/A	0.096	0.148	0.402	0.000	0.897	0.000	0.958	0.609	0.519

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	319	153	0	0	577	0	0	-1	294
N.S.	1	1.05	0.50	0.00	0.00	1.90	0.00	0.00	-0.00	0.97
time (sec)	N/A	0.823	0.506	0.044	0.000	1.288	0.000	0.000	0.000	42.796

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	292	522	348	0	0	865	0	0	-1	0
N.S.	1	1.79	1.19	0.00	0.00	2.96	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.846	0.783	0.039	0.000	1.276	0.000	0.000	0.000	117.705

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	29	25	22	0	47	0	0	25	46
N.S.	1	1.16	1.00	0.88	0.00	1.88	0.00	0.00	1.00	1.84
time (sec)	N/A	0.013	0.010	0.043	0.000	0.944	0.000	0.000	0.236	0.420

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	30	25	22	0	69	0	0	25	25
N.S.	1	1.20	1.00	0.88	0.00	2.76	0.00	0.00	1.00	1.00
time (sec)	N/A	0.016	0.012	0.061	0.000	0.904	0.000	0.000	0.260	26.033

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	63	30	27	0	77	0	0	30	56
N.S.	1	1.19	0.57	0.51	0.00	1.45	0.00	0.00	0.57	1.06
time (sec)	N/A	0.020	0.015	0.049	0.000	0.908	0.000	0.000	0.227	2.521

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	188	49	370	0	128	0	0	-1	132
N.S.	1	2.81	0.73	5.52	0.00	1.91	0.00	0.00	-0.01	1.97
time (sec)	N/A	0.121	0.012	0.445	0.000	0.900	0.000	0.000	0.000	0.264

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	122	133	114	86	0	142	0	177	-1	97
N.S.	1	1.09	0.93	0.70	0.00	1.16	0.00	1.45	-0.01	0.80
time (sec)	N/A	0.346	0.148	0.236	0.000	0.787	0.000	0.835	0.000	0.370

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	150	404	112	1247	0	280	0	0	-1	247
N.S.	1	2.69	0.75	8.31	0.00	1.87	0.00	0.00	-0.01	1.65
time (sec)	N/A	0.327	0.072	4.797	0.000	0.877	0.000	0.000	0.000	8.044

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	27	26	51	64	0	23	27	39
N.S.	1	1.00	0.63	0.60	1.19	1.49	0.00	0.53	0.63	0.91
time (sec)	N/A	0.006	0.010	0.326	0.531	1.071	0.000	0.663	0.050	0.266

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	37	35	0	62	0	34	-1	26
N.S.	1	1.00	0.88	0.83	0.00	1.48	0.00	0.81	-0.02	0.62
time (sec)	N/A	0.043	0.009	0.142	0.000	0.739	0.000	0.649	0.000	0.028

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	35	71	0	138	0	0	-1	67
N.S.	1	1.00	0.25	0.51	0.00	0.99	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.127	0.007	0.113	0.000	0.560	0.000	0.000	0.000	3.621

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	188	49	672	0	128	0	0	-1	132
N.S.	1	2.51	0.65	8.96	0.00	1.71	0.00	0.00	-0.01	1.76
time (sec)	N/A	0.118	0.013	0.470	0.000	1.344	0.000	0.000	0.000	0.258

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	23	20	0	22	0	0	24	26
N.S.	1	1.00	0.79	0.69	0.00	0.76	0.00	0.00	0.83	0.90
time (sec)	N/A	0.033	0.009	0.031	0.000	0.880	0.000	0.000	0.052	3.223

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	32	29	0	44	0	0	37	37
N.S.	1	1.00	0.35	0.32	0.00	0.48	0.00	0.00	0.40	0.40
time (sec)	N/A	0.085	0.009	0.042	0.000	0.674	0.000	0.000	0.080	4.877

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	15	16	33	0	16	16	32
N.S.	1	1.00	1.00	0.79	0.84	1.74	0.00	0.84	0.84	1.68
time (sec)	N/A	0.014	0.011	0.359	1.297	0.857	0.000	0.658	0.197	0.104

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	5	8	29	0	4	4	23
N.S.	1	1.00	1.00	0.62	1.00	3.62	0.00	0.50	0.50	2.88
time (sec)	N/A	0.005	0.006	0.316	1.168	0.977	0.000	0.627	0.184	0.091

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	12	11	40	0	11	11	33
N.S.	1	1.00	1.00	1.00	0.92	3.33	0.00	0.92	0.92	2.75
time (sec)	N/A	0.004	0.007	0.292	1.097	0.632	0.000	0.633	0.222	0.098

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	29	0	23	0	51	79	55
N.S.	1	1.00	1.00	0.94	0.00	0.74	0.00	1.65	2.55	1.77
time (sec)	N/A	0.008	0.009	0.290	0.000	1.249	0.000	0.632	0.516	0.088

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	22	0	54	0	57	61	48
N.S.	1	1.00	1.00	0.71	0.00	1.74	0.00	1.84	1.97	1.55
time (sec)	N/A	0.010	0.011	0.287	0.000	0.945	0.000	0.637	0.485	0.081

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	49	112	48	61	42	78	24
N.S.	1	1.00	1.00	2.04	4.67	2.00	2.54	1.75	3.25	1.00
time (sec)	N/A	0.019	0.010	0.301	1.375	0.923	5.719	0.619	0.789	0.041

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	48	101	32	24	20	83	25
N.S.	1	1.00	1.00	1.92	4.04	1.28	0.96	0.80	3.32	1.00
time (sec)	N/A	0.019	0.009	0.303	1.372	1.149	5.537	0.589	0.760	0.025

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	96	70	0	72	0	74	107	65
N.S.	1	1.00	2.23	1.63	0.00	1.67	0.00	1.72	2.49	1.51
time (sec)	N/A	0.020	0.051	0.282	0.000	0.992	0.000	0.650	0.114	0.094

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	49	54	74	0	109	-1	80
N.S.	1	1.00	0.98	0.79	0.87	1.19	0.00	1.76	-0.02	1.29
time (sec)	N/A	0.036	0.026	0.425	1.295	1.088	0.000	0.734	0.000	0.201

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	159	69	0	151	0	164	-1	119
N.S.	1	1.00	1.94	0.84	0.00	1.84	0.00	2.00	-0.01	1.45
time (sec)	N/A	0.121	0.364	0.581	0.000	1.000	0.000	0.715	0.000	0.363

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	114	53	0	307	0	165	-1	102
N.S.	1	1.00	1.81	0.84	0.00	4.87	0.00	2.62	-0.02	1.62
time (sec)	N/A	0.054	0.113	0.750	0.000	1.172	0.000	0.712	0.000	0.238

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	80	128	0	303	0	152	-1	103
N.S.	1	1.00	1.43	2.29	0.00	5.41	0.00	2.71	-0.02	1.84
time (sec)	N/A	0.047	0.028	0.724	0.000	0.921	0.000	0.679	0.000	0.219

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	174	158	0	311	0	257	-1	123
N.S.	1	1.00	2.49	2.26	0.00	4.44	0.00	3.67	-0.01	1.76
time (sec)	N/A	0.064	0.450	0.954	0.000	1.187	0.000	0.661	0.000	0.305

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	89	94	192	0	0	0	629	-1	143
N.S.	1	1.11	1.18	2.40	0.00	0.00	0.00	7.86	-0.01	1.79
time (sec)	N/A	0.123	0.120	0.465	0.000	0.000	0.000	0.992	0.000	1.134

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	76	82	0	81	0	231	-1	91
N.S.	1	1.00	2.00	2.16	0.00	2.13	0.00	6.08	-0.03	2.39
time (sec)	N/A	0.024	0.040	0.361	0.000	0.679	0.000	1.839	0.000	0.502

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	40	40	48	42	155	36	35	54
N.S.	1	1.00	0.62	0.62	0.74	0.65	2.38	0.55	0.54	0.83
time (sec)	N/A	0.017	0.022	0.306	1.224	0.529	4.514	0.596	0.028	0.049

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	28	25	37	31	41	51	25	28
N.S.	1	1.00	0.57	0.51	0.76	0.63	0.84	1.04	0.51	0.57
time (sec)	N/A	0.011	0.006	0.325	1.092	0.637	2.494	0.626	0.027	0.033

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	28	25	37	44	139	26	187	28
N.S.	1	1.00	0.57	0.51	0.76	0.90	2.84	0.53	3.82	0.57
time (sec)	N/A	0.008	0.008	0.276	0.542	0.576	9.960	0.645	0.048	0.042

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	328	10	62	53	11	154	12
N.S.	1	1.00	1.00	27.33	0.83	5.17	4.42	0.92	12.83	1.00
time (sec)	N/A	0.047	0.027	0.069	0.458	0.586	3.000	0.641	0.270	0.012

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	37	29	22	24	26
N.S.	1	1.00	1.00	0.92	0.83	3.08	2.42	1.83	2.00	2.17
time (sec)	N/A	0.013	0.012	0.349	1.425	0.665	12.850	0.672	0.195	0.174

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	23	0	67	0	64	77	57
N.S.	1	1.00	1.00	0.85	0.00	2.48	0.00	2.37	2.85	2.11
time (sec)	N/A	0.011	0.014	0.276	0.000	0.573	0.000	0.632	0.173	0.091

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	74	38	0	83	0	101	117	64
N.S.	1	1.00	1.54	0.79	0.00	1.73	0.00	2.10	2.44	1.33
time (sec)	N/A	0.014	0.206	0.283	0.000	0.527	0.000	0.642	0.093	0.082

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	64	107	77	0	72	-1	66
N.S.	1	1.00	1.00	1.56	2.61	1.88	0.00	1.76	-0.02	1.61
time (sec)	N/A	0.022	0.030	0.313	1.464	0.672	0.000	0.657	0.000	0.118

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	85	53	0	50	0	123	115	54
N.S.	1	1.00	1.81	1.13	0.00	1.06	0.00	2.62	2.45	1.15
time (sec)	N/A	0.021	0.157	0.309	0.000	0.697	0.000	0.648	0.133	0.123

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	137	82	0	160	0	135	159	108
N.S.	1	1.00	1.56	0.93	0.00	1.82	0.00	1.53	1.81	1.23
time (sec)	N/A	0.239	0.205	0.920	0.000	0.593	0.000	0.687	0.382	0.317

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	261	654	0	170	0	176	216	117
N.S.	1	1.00	1.92	4.81	0.00	1.25	0.00	1.29	1.59	0.86
time (sec)	N/A	1.496	1.007	0.048	0.000	0.681	0.000	0.673	0.615	0.319

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	72	60	63	51	0	45	56	113
N.S.	1	1.00	1.12	0.94	0.98	0.80	0.00	0.70	0.88	1.77
time (sec)	N/A	0.017	0.120	0.306	1.175	0.660	0.000	0.646	0.100	0.183

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	80	69	81	60	0	54	75	121
N.S.	1	1.00	0.90	0.78	0.91	0.67	0.00	0.61	0.84	1.36
time (sec)	N/A	0.030	0.103	0.291	1.184	0.693	0.000	0.658	0.325	0.200

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	88	77	101	68	0	63	96	129
N.S.	1	1.00	0.78	0.68	0.89	0.60	0.00	0.56	0.85	1.14
time (sec)	N/A	0.043	0.118	0.286	1.100	0.596	0.000	0.634	0.137	0.234

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	64	42	54	62	0	71	-1	57
N.S.	1	1.00	1.31	0.86	1.10	1.27	0.00	1.45	-0.02	1.16
time (sec)	N/A	0.031	0.030	0.296	1.332	0.577	0.000	0.709	0.000	0.188

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	72	56	66	64	0	147	-1	57
N.S.	1	1.00	0.91	0.71	0.84	0.81	0.00	1.86	-0.01	0.72
time (sec)	N/A	0.044	0.088	0.290	1.330	0.844	0.000	0.666	0.000	0.312

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	10	11	18	0	18	12	20
N.S.	1	1.00	1.00	0.83	0.92	1.50	0.00	1.50	1.00	1.67
time (sec)	N/A	0.008	0.006	0.353	1.404	0.709	0.000	0.651	0.240	0.062

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	41	33	48	39	0	39	-1	47
N.S.	1	1.00	0.77	0.62	0.91	0.74	0.00	0.74	-0.02	0.89
time (sec)	N/A	0.023	0.020	0.348	1.264	0.735	0.000	0.631	0.000	0.079

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	22	34	0	15	13	19
N.S.	1	1.00	1.00	0.84	1.16	1.79	0.00	0.79	0.68	1.00
time (sec)	N/A	0.002	0.006	0.338	0.593	0.632	0.000	0.699	0.032	0.147

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	22	28	0	13	15	17
N.S.	1	1.00	1.00	0.82	1.29	1.65	0.00	0.76	0.88	1.00
time (sec)	N/A	0.004	0.039	0.348	0.431	0.666	0.000	0.654	0.024	0.131

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	48	33	47	64	0	38	-1	47
N.S.	1	1.00	0.86	0.59	0.84	1.14	0.00	0.68	-0.02	0.84
time (sec)	N/A	0.024	0.014	0.348	1.105	0.583	0.000	0.644	0.000	0.183

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	46	38	56	44	0	44	61	52
N.S.	1	1.00	0.71	0.58	0.86	0.68	0.00	0.68	0.94	0.80
time (sec)	N/A	0.024	0.022	0.352	1.162	0.705	0.000	0.646	0.131	0.087

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	46	38	56	44	0	44	43	52
N.S.	1	1.00	0.84	0.69	1.02	0.80	0.00	0.80	0.78	0.95
time (sec)	N/A	0.014	0.016	0.348	1.285	0.773	0.000	0.634	0.213	0.147

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	56	48	77	54	0	54	56	62
N.S.	1	1.00	0.76	0.65	1.04	0.73	0.00	0.73	0.76	0.84
time (sec)	N/A	0.019	0.026	0.348	1.430	0.598	0.000	0.617	0.075	0.281

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	31	37	52	0	67	31	33
N.S.	1	1.00	1.00	0.82	0.97	1.37	0.00	1.76	0.82	0.87
time (sec)	N/A	0.013	0.010	0.346	1.033	0.681	0.000	0.647	0.028	0.082

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	43	42	50	63	0	84	-1	42
N.S.	1	1.00	0.75	0.74	0.88	1.11	0.00	1.47	-0.02	0.74
time (sec)	N/A	0.024	0.016	0.355	1.235	0.713	0.000	0.668	0.000	0.141

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	50	41	58	94	0	80	-1	45
N.S.	1	1.00	0.81	0.66	0.94	1.52	0.00	1.29	-0.02	0.73
time (sec)	N/A	0.027	0.019	0.355	1.421	0.588	0.000	0.621	0.000	0.204

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	65	46	71	107	0	117	-1	52
N.S.	1	1.00	0.82	0.58	0.90	1.35	0.00	1.48	-0.01	0.66
time (sec)	N/A	0.037	0.014	0.351	1.087	0.665	0.000	0.625	0.000	0.217

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	22	25	30	0	32	-1	18
N.S.	1	1.00	1.00	1.00	1.14	1.36	0.00	1.45	-0.05	0.82
time (sec)	N/A	0.009	0.003	0.355	1.238	0.612	0.000	0.660	0.000	0.144

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	83	69	0	110	0	147	-1	105
N.S.	1	1.00	0.97	0.80	0.00	1.28	0.00	1.71	-0.01	1.22
time (sec)	N/A	0.278	0.069	0.416	0.000	0.626	0.000	0.727	0.000	0.228

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	56	61	92	0	33	-1	81
N.S.	1	1.00	0.98	0.90	0.98	1.48	0.00	0.53	-0.02	1.31
time (sec)	N/A	0.047	0.043	0.382	1.300	0.571	0.000	0.658	0.000	0.232

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	146	64	0	174	0	235	-1	244
N.S.	1	1.00	1.92	0.84	0.00	2.29	0.00	3.09	-0.01	3.21
time (sec)	N/A	0.068	0.400	0.715	0.000	0.569	0.000	0.665	0.000	1.680

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	26	21	28	22	0	30	19	26
N.S.	1	1.00	0.72	0.58	0.78	0.61	0.00	0.83	0.53	0.72
time (sec)	N/A	0.128	0.013	0.338	1.417	0.685	0.000	0.654	0.269	0.131

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	134	63	0	147	0	148	-1	254
N.S.	1	1.00	1.54	0.72	0.00	1.69	0.00	1.70	-0.01	2.92
time (sec)	N/A	0.209	0.159	2.706	0.000	0.724	0.000	0.680	0.000	1.589

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	39	38	76	98	0	33	69	39
N.S.	1	1.00	0.67	0.66	1.31	1.69	0.00	0.57	1.19	0.67
time (sec)	N/A	0.011	0.012	0.371	0.632	0.769	0.000	0.633	0.229	0.289

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	59	73	0	27	29	57
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62	1.21
time (sec)	N/A	0.007	0.018	0.307	0.437	0.645	0.000	0.648	0.055	0.302

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	59	51	0	39	29	57
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.83	0.62	1.21
time (sec)	N/A	0.008	0.013	0.389	0.681	0.533	0.000	0.711	0.204	0.313

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	30	40	0	39	0	60	-1	46
N.S.	1	1.00	1.03	1.38	0.00	1.34	0.00	2.07	-0.03	1.59
time (sec)	N/A	0.039	0.022	0.137	0.000	0.679	0.000	0.657	0.000	0.286

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	59	59	52	0	63	0	66	-1	54
N.S.	1	1.31	1.31	1.16	0.00	1.40	0.00	1.47	-0.02	1.20
time (sec)	N/A	0.032	0.035	0.122	0.000	0.782	0.000	0.652	0.000	0.117

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	71	55	0	54	0	54	71	67
N.S.	1	1.00	0.90	0.70	0.00	0.68	0.00	0.68	0.90	0.85
time (sec)	N/A	0.136	0.084	0.069	0.000	0.662	0.000	0.621	0.071	0.190

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	80	0	99	0	105	-1	72
N.S.	1	1.00	1.00	1.00	0.00	1.24	0.00	1.31	-0.01	0.90
time (sec)	N/A	0.343	0.119	0.385	0.000	0.712	0.000	0.667	0.000	0.256

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	151	140	0	155	0	0	-1	165
N.S.	1	1.00	0.96	0.89	0.00	0.98	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.526	0.342	0.027	0.000	0.730	0.000	0.000	0.000	2.985

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	22	30	38	28	131	29	29	32
N.S.	1	1.00	0.54	0.73	0.93	0.68	3.20	0.71	0.71	0.78
time (sec)	N/A	0.009	0.004	0.319	1.366	0.505	3.077	0.615	0.044	0.022

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	17	10	11	11	19
N.S.	1	1.00	1.00	0.92	0.85	1.31	0.77	0.85	0.85	1.46
time (sec)	N/A	0.004	0.007	0.034	0.521	0.521	0.981	0.648	0.460	0.009

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	26	67	76	80	541	76	90	88
N.S.	1	1.00	0.37	0.94	1.07	1.13	7.62	1.07	1.27	1.24
time (sec)	N/A	0.028	0.006	0.605	1.088	0.702	2.272	0.652	0.211	0.055

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	23	18	28	29	180	49	23	33
N.S.	1	1.00	0.58	0.45	0.70	0.72	4.50	1.22	0.58	0.82
time (sec)	N/A	0.007	0.013	0.300	0.612	0.621	1.755	0.632	0.201	0.008

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	53	52	52	51	53	36	48
N.S.	1	1.00	1.00	1.10	1.08	1.08	1.06	1.10	0.75	1.00
time (sec)	N/A	0.018	0.014	0.307	1.068	0.651	2.093	8.180	0.867	7.816

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	36	20	45	25	3305	63	45	80
N.S.	1	1.00	0.52	0.29	0.65	0.36	47.90	0.91	0.65	1.16
time (sec)	N/A	0.020	0.015	0.290	0.467	0.581	2.247	0.598	0.300	0.296

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	34	85	157	202	44	142	77	132
N.S.	1	1.00	0.18	0.44	0.81	1.05	0.23	0.74	0.40	0.68
time (sec)	N/A	0.113	0.009	0.316	1.202	0.613	5.632	0.603	1.278	0.337

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.004	0.285	0.482	0.649	0.537	0.632	0.291	0.019

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	12	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.272	0.602	0.578	0.537	0.616	0.319	0.034

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	84	74	64	66	42	65	76	86
N.S.	1	1.00	1.42	1.25	1.08	1.12	0.71	1.10	1.29	1.46
time (sec)	N/A	0.037	0.025	9.787	1.299	0.574	1.016	0.615	0.455	0.075

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	26	76	66	79	34	67	92	90
N.S.	1	1.00	0.37	1.09	0.94	1.13	0.49	0.96	1.31	1.29
time (sec)	N/A	0.038	0.007	9.886	1.350	0.652	1.417	0.636	0.367	0.090

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	27	20	83	146	41	83	18	68
N.S.	1	1.00	0.40	0.29	1.22	2.15	0.60	1.22	0.26	1.00
time (sec)	N/A	0.021	0.006	3.743	1.402	5.603	1.029	0.674	0.423	0.156

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	26	19	130	105	41	110	-1	83
N.S.	1	1.00	0.28	0.20	1.40	1.13	0.44	1.18	-0.01	0.89
time (sec)	N/A	0.029	0.006	3.406	1.241	0.697	1.570	0.653	0.000	0.176

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	43	20	129	105	39	109	-1	83
N.S.	1	1.00	0.46	0.22	1.39	1.13	0.42	1.17	-0.01	0.89
time (sec)	N/A	0.029	0.014	3.325	1.201	0.514	1.317	0.671	0.000	0.176

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	200	40	15	0	99	0	69	29	105
N.S.	1	2.15	0.43	0.16	0.00	1.06	0.00	0.74	0.31	1.13
time (sec)	N/A	0.144	0.011	3.229	0.000	0.737	0.000	0.676	0.370	0.146

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	114	83	51	0	87	0	66	27	69
N.S.	1	0.90	0.66	0.40	0.00	0.69	0.00	0.52	0.21	0.55
time (sec)	N/A	0.115	0.052	0.286	0.000	74.920	0.000	0.925	0.286	0.500

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	29	58	0	49	0	70	35	48
N.S.	1	1.00	0.85	1.71	0.00	1.44	0.00	2.06	1.03	1.41
time (sec)	N/A	0.032	0.015	0.299	0.000	0.609	0.000	0.662	0.515	0.415

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	47	47	0	47	0	49	43	55
N.S.	1	1.00	0.81	0.81	0.00	0.81	0.00	0.84	0.74	0.95
time (sec)	N/A	0.037	0.019	0.355	0.000	0.718	0.000	0.640	0.292	0.127

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	55	52	52	81	0	112	-1	60
N.S.	1	1.00	0.77	0.73	0.73	1.14	0.00	1.58	-0.01	0.85
time (sec)	N/A	0.052	0.028	0.223	1.207	0.762	0.000	0.668	0.000	0.209

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	57	27	52	59	0	69	49	52
N.S.	1	1.00	2.71	1.29	2.48	2.81	0.00	3.29	2.33	2.48
time (sec)	N/A	0.029	0.016	0.203	0.482	0.611	0.000	0.628	0.815	0.228

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	75	14	13	13	36	13	21	17
N.S.	1	1.00	4.41	0.82	0.76	0.76	2.12	0.76	1.24	1.00
time (sec)	N/A	0.010	0.057	0.310	0.599	0.603	1.310	0.628	0.279	0.021

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	116	34	35	221	34	34	43
N.S.	1	1.00	0.87	2.52	0.74	0.76	4.80	0.74	0.74	0.93
time (sec)	N/A	0.193	0.084	0.353	0.567	0.621	3.944	0.625	0.373	0.037

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	107	104	902	0	232	0	0	-1	113
N.S.	1	1.37	1.33	11.56	0.00	2.97	0.00	0.00	-0.01	1.45
time (sec)	N/A	0.071	0.079	2.738	0.000	3.667	0.000	0.000	0.000	0.240

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	120	150	0	388	0	0	-1	85
N.S.	1	1.00	0.85	1.06	0.00	2.75	0.00	0.00	-0.01	0.60
time (sec)	N/A	0.060	0.058	1.335	0.000	6.774	0.000	0.000	0.000	0.214

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	91	29	94	86	71	0	-1	94
N.S.	1	1.00	1.44	0.46	1.49	1.37	1.13	0.00	-0.02	1.49
time (sec)	N/A	0.011	0.035	1.633	1.210	0.859	2.005	0.000	0.000	0.152

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	41	228	0	242	0	0	-1	74
N.S.	1	1.00	0.55	3.08	0.00	3.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.024	0.013	2.832	0.000	11.890	0.000	0.000	0.000	0.219

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	59	26	23	73	40	0	0	23	27
N.S.	1	1.23	0.54	0.48	1.52	0.83	0.00	0.00	0.48	0.56
time (sec)	N/A	0.012	0.017	0.324	0.567	0.658	0.000	0.000	0.262	1.125

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	123	120	2501	0	458	0	0	-1	189
N.S.	1	1.37	1.33	27.79	0.00	5.09	0.00	0.00	-0.01	2.10
time (sec)	N/A	0.082	0.155	13.602	0.000	15.933	0.000	0.000	0.000	0.383

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	40	22	0	18	0	0	-1	23
N.S.	1	1.00	1.74	0.96	0.00	0.78	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.034	0.100	0.321	0.000	1.105	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	36	22	0	42	0	0	-1	23
N.S.	1	1.00	1.57	0.96	0.00	1.83	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.036	0.094	0.301	0.000	0.960	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	23	21	18	57	49	14	51	17	35
N.S.	1	1.44	1.31	1.12	3.56	3.06	0.88	3.19	1.06	2.19
time (sec)	N/A	0.024	0.018	0.297	1.173	0.750	9.513	0.642	0.152	0.188

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	23	19	18	57	47	14	51	17	41
N.S.	1	1.44	1.19	1.12	3.56	2.94	0.88	3.19	1.06	2.56
time (sec)	N/A	0.025	0.015	0.289	1.277	0.984	5.573	0.604	0.234	0.188

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	26	26	522	31	0	45	0	0	-1	26
N.S.	1	1.00	20.08	1.19	0.00	1.73	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.043	1.318	0.556	0.000	0.825	0.000	0.000	0.000	0.209

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	94	18	0	13	0	0	-1	15
N.S.	1	1.00	6.27	1.20	0.00	0.87	0.00	0.00	-0.07	1.00
time (sec)	N/A	0.039	0.131	0.487	0.000	0.722	0.000	0.000	0.000	0.170

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	22	14	0	0	14	16
N.S.	1	1.00	1.00	0.94	1.38	0.88	0.00	0.00	0.88	1.00
time (sec)	N/A	0.018	0.035	0.079	0.874	0.850	0.000	0.000	0.058	0.796

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	17955	247419	0	252	0	0	-1	67
N.S.	1	1.00	242.64	3343.50	0.00	3.41	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.204	6.466	0.386	0.000	1.660	0.000	0.000	0.000	0.672

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	24	0	0	62	0	0	-1	96
N.S.	1	1.00	1.09	0.00	0.00	2.82	0.00	0.00	-0.05	4.36
time (sec)	N/A	0.064	1.116	0.023	0.000	1.993	0.000	0.000	0.000	0.260

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	26	0	0	0	0	0	-1	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.068	0.088	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	10	10	10	10	10	0
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.00
time (sec)	N/A	0.007	0.002	0.030	0.574	0.919	0.062	0.600	0.185	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	15	11	9	10	8	9	9	0
N.S.	1	1.00	1.36	1.00	0.82	0.91	0.73	0.82	0.82	0.00
time (sec)	N/A	0.007	0.002	0.302	0.497	0.844	0.068	0.605	0.035	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	17	16	19	24	16	16	0
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67	0.00
time (sec)	N/A	0.010	0.002	0.311	0.499	0.794	0.064	0.592	0.032	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	23	24	25	36	22	22	0
N.S.	1	1.00	0.88	0.68	0.71	0.74	1.06	0.65	0.65	0.00
time (sec)	N/A	0.019	0.002	0.323	0.440	0.674	0.065	0.585	0.038	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	38	29	30	31	48	28	28	0
N.S.	1	1.00	0.86	0.66	0.68	0.70	1.09	0.64	0.64	0.00
time (sec)	N/A	0.021	0.002	0.355	0.583	0.966	0.066	0.609	0.027	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	64	21	15	23	37	99	14	20	0
N.S.	1	3.20	1.05	0.75	1.15	1.85	4.95	0.70	1.00	0.00
time (sec)	N/A	0.016	0.020	0.337	0.488	0.622	0.650	0.591	0.273	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	22	23	22	39	23	22	0
N.S.	1	1.00	1.00	0.71	0.74	0.71	1.26	0.74	0.71	0.00
time (sec)	N/A	0.011	0.018	0.540	0.572	0.778	0.341	0.744	0.254	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	27	18	20	37	32	20	27	0
N.S.	1	1.00	1.29	0.86	0.95	1.76	1.52	0.95	1.29	0.00
time (sec)	N/A	0.009	0.003	0.310	0.505	0.477	0.065	0.591	0.196	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	95	32	54	93	60	112	44	0
N.S.	1	1.00	2.64	0.89	1.50	2.58	1.67	3.11	1.22	0.00
time (sec)	N/A	0.020	0.008	0.360	0.515	1.003	0.172	0.616	0.249	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	57	37	33	40	66	33	51	0
N.S.	1	1.00	1.39	0.90	0.80	0.98	1.61	0.80	1.24	0.00
time (sec)	N/A	0.017	0.008	0.333	0.488	0.805	0.066	0.607	0.213	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	40	51	70	388	53	35	0
N.S.	1	1.00	1.00	1.00	1.28	1.75	9.70	1.32	0.88	0.00
time (sec)	N/A	0.012	0.013	0.651	0.507	0.740	1.305	0.640	0.621	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	30	19	18	18	31	18	18	0
N.S.	1	1.00	1.36	0.86	0.82	0.82	1.41	0.82	0.82	0.00
time (sec)	N/A	0.014	0.004	0.043	1.340	0.794	0.076	0.592	0.035	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	16	26	22	40	19	37	26	0
N.S.	1	1.00	0.80	1.30	1.10	2.00	0.95	1.85	1.30	0.00
time (sec)	N/A	0.015	0.004	0.058	0.614	0.938	0.108	0.828	0.270	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	40	38	30	70	20	53	24	0
N.S.	1	1.00	1.25	1.19	0.94	2.19	0.62	1.66	0.75	0.00
time (sec)	N/A	0.014	0.017	0.045	1.323	0.700	0.213	1.118	0.088	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	46	35	24	37	56	34	38	0
N.S.	1	1.00	0.82	0.62	0.43	0.66	1.00	0.61	0.68	0.00
time (sec)	N/A	0.062	0.016	0.320	0.570	0.972	0.068	0.768	0.043	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	55	38	25	25	27	25	25	0
N.S.	1	1.00	1.67	1.15	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.030	0.023	0.053	0.549	0.916	0.072	0.828	0.196	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	37	40	38	44	42	36	0
N.S.	1	1.00	1.00	0.80	0.87	0.83	0.96	0.91	0.78	0.00
time (sec)	N/A	0.028	0.011	0.067	0.530	0.950	0.099	0.820	0.036	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	53	38	37	55	44	26	27	0
N.S.	1	1.00	1.29	0.93	0.90	1.34	1.07	0.63	0.66	0.00
time (sec)	N/A	0.034	0.029	0.326	0.487	0.633	0.076	0.830	0.122	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	14	11	10	19	14	10	18	0
N.S.	1	1.00	0.58	0.46	0.42	0.79	0.58	0.42	0.75	0.00
time (sec)	N/A	0.027	0.005	0.049	0.572	0.699	0.072	1.000	0.044	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	22	17	16	31	31	16	32	0
N.S.	1	1.00	0.48	0.37	0.35	0.67	0.67	0.35	0.70	0.00
time (sec)	N/A	0.057	0.007	0.335	0.489	1.002	0.073	0.976	0.045	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	30	23	24	43	46	22	44	0
N.S.	1	1.00	0.44	0.34	0.35	0.63	0.68	0.32	0.65	0.00
time (sec)	N/A	0.085	0.011	0.309	0.471	0.791	0.075	0.885	0.028	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	38	29	30	55	61	28	56	0
N.S.	1	1.00	0.42	0.32	0.33	0.61	0.68	0.31	0.62	0.00
time (sec)	N/A	0.137	0.015	0.315	0.649	0.780	0.076	0.628	0.036	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	58	0	0	0	0	0	52	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.037	0.057	0.026	0.000	0.895	0.000	0.000	0.749	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	45	25	41	71	54	132	24	0
N.S.	1	1.00	1.41	0.78	1.28	2.22	1.69	4.12	0.75	0.00
time (sec)	N/A	0.021	0.069	0.094	0.474	0.856	1.656	0.752	0.284	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	11	6	14	17	6	6	0
N.S.	1	1.00	1.00	1.38	0.75	1.75	2.12	0.75	0.75	0.00
time (sec)	N/A	0.022	0.003	0.060	0.491	0.887	0.070	0.623	0.028	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	32	14	22	15	14	11	0
N.S.	1	1.00	1.00	2.91	1.27	2.00	1.36	1.27	1.00	0.00
time (sec)	N/A	0.016	0.007	0.310	0.555	0.839	0.098	0.602	0.317	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.00
time (sec)	N/A	0.013	0.005	0.061	0.475	0.617	0.071	0.628	0.323	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	71	36	38	68	41	47	24	0
N.S.	1	1.00	2.73	1.38	1.46	2.62	1.58	1.81	0.92	0.00
time (sec)	N/A	0.031	0.020	0.076	0.532	0.729	0.152	0.615	0.323	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	22	14	30	15	14	13	0
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.76	0.00
time (sec)	N/A	0.026	0.008	0.070	0.503	0.687	0.101	0.612	0.068	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	24	49	19	20	39	20	22	0
N.S.	1	1.00	0.77	1.58	0.61	0.65	1.26	0.65	0.71	0.00
time (sec)	N/A	0.026	0.054	0.203	0.541	1.008	178.370	0.653	1.118	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	22	26	13	27	0	13	32	0
N.S.	1	1.00	1.05	1.24	0.62	1.29	0.00	0.62	1.52	0.00
time (sec)	N/A	0.024	0.039	0.499	0.600	0.862	0.000	0.605	1.152	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	95	52	54	93	56	44	57	0
N.S.	1	1.00	2.50	1.37	1.42	2.45	1.47	1.16	1.50	0.00
time (sec)	N/A	0.053	0.020	0.094	0.438	0.678	0.162	0.615	0.280	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	74	76	74	82	0	95	75	0
N.S.	1	1.00	0.97	1.00	0.97	1.08	0.00	1.25	0.99	0.00
time (sec)	N/A	0.039	0.078	0.561	0.666	0.734	0.000	0.947	6.376	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	84	127	210	115	178	0	149	133	0
N.S.	1	0.95	1.44	2.39	1.31	2.02	0.00	1.69	1.51	0.00
time (sec)	N/A	0.551	2.019	0.565	1.353	0.949	0.000	0.636	0.381	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	74	55	54	54	148	54	94	0
N.S.	1	1.00	1.06	0.79	0.77	0.77	2.11	0.77	1.34	0.00
time (sec)	N/A	0.153	0.088	0.380	0.453	1.013	1.600	0.601	0.385	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	35	36	71	39	75	75	0
N.S.	1	1.00	0.88	1.06	1.09	2.15	1.18	2.27	2.27	0.00
time (sec)	N/A	0.062	0.025	0.064	1.271	0.856	0.585	0.674	0.518	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	20	21	14	26	24	14	26	0
N.S.	1	1.00	1.25	1.31	0.88	1.62	1.50	0.88	1.62	0.00
time (sec)	N/A	0.043	0.017	0.333	1.281	0.677	17.462	0.620	0.312	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	22	21	27	24	23	10	0
N.S.	1	1.00	1.00	1.83	1.75	2.25	2.00	1.92	0.83	0.00
time (sec)	N/A	0.023	0.010	0.327	0.445	0.799	1.698	0.604	0.269	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	18	20	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76	0.00
time (sec)	N/A	0.008	0.006	0.123	0.422	0.594	0.551	0.614	0.193	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	31	54	43	75	38	33	0
N.S.	1	1.00	1.00	1.19	2.08	1.65	2.88	1.46	1.27	0.00
time (sec)	N/A	0.030	0.050	0.364	0.448	0.913	17.154	0.592	0.234	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	29	30	31	139	28	36	0
N.S.	1	1.00	1.00	0.76	0.79	0.82	3.66	0.74	0.95	0.00
time (sec)	N/A	0.030	0.012	0.153	0.456	0.848	24.529	0.628	0.301	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	35	33	43	22	21	21	0
N.S.	1	1.00	1.00	1.75	1.65	2.15	1.10	1.05	1.05	0.00
time (sec)	N/A	0.046	0.013	0.143	0.433	0.973	24.839	0.633	0.067	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	19	19	25	14	13	35	0
N.S.	1	1.00	1.00	1.58	1.58	2.08	1.17	1.08	2.92	0.00
time (sec)	N/A	0.021	0.010	0.133	0.427	0.674	5.887	0.603	0.298	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	65	39	35	208	64	0	75	47	0
N.S.	1	1.02	0.61	0.55	3.25	1.00	0.00	1.17	0.73	0.00
time (sec)	N/A	0.078	0.039	0.264	1.071	1.124	0.000	0.637	0.342	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	17	17	27	92	17	0	24	25	0
N.S.	1	1.55	1.55	2.45	8.36	1.55	0.00	2.18	2.27	0.00
time (sec)	N/A	0.033	0.013	0.149	0.454	0.812	0.000	0.635	0.616	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	12	9	11	0	9	5	0
N.S.	1	1.00	1.00	1.71	1.29	1.57	0.00	1.29	0.71	0.00
time (sec)	N/A	0.039	0.033	0.125	0.960	1.084	0.000	0.606	0.266	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	83	33	43	27	38	107	78	23	0
N.S.	1	1.57	0.62	0.81	0.51	0.72	2.02	1.47	0.43	0.00
time (sec)	N/A	0.098	0.063	0.206	0.960	1.252	9.891	0.639	0.214	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	94	22	20	39	66	39	37	17	0
N.S.	1	2.85	0.67	0.61	1.18	2.00	1.18	1.12	0.52	0.00
time (sec)	N/A	0.074	0.047	0.125	0.973	1.042	0.591	0.678	0.081	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	9	8	7	21	219	20	16	0
N.S.	1	1.00	0.33	0.30	0.26	0.78	8.11	0.74	0.59	0.00
time (sec)	N/A	0.021	0.023	0.096	0.955	1.244	13.651	0.633	0.283	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	41	31	28	46	102	29	38	0
N.S.	1	1.00	1.46	1.11	1.00	1.64	3.64	1.04	1.36	0.00
time (sec)	N/A	0.041	0.043	0.213	0.956	0.997	0.492	0.631	0.320	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	58	39	38	61	0	78	120	0
N.S.	1	1.00	0.87	0.58	0.57	0.91	0.00	1.16	1.79	0.00
time (sec)	N/A	0.046	0.122	0.220	0.964	0.880	0.000	0.640	0.434	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	54	29	28	58	0	61	77	0
N.S.	1	1.00	0.98	0.53	0.51	1.05	0.00	1.11	1.40	0.00
time (sec)	N/A	0.032	0.090	0.138	0.971	1.242	0.000	0.633	0.386	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	70	45	44	77	252	39	48	0
N.S.	1	1.00	1.67	1.07	1.05	1.83	6.00	0.93	1.14	0.00
time (sec)	N/A	0.096	0.225	0.102	1.004	1.086	0.561	0.636	0.284	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	20	0	19	76	21	7	0
N.S.	1	1.00	1.00	2.22	0.00	2.11	8.44	2.33	0.78	0.00
time (sec)	N/A	0.022	0.007	0.149	0.000	1.030	5.464	0.646	0.362	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	174	13	129	33	0	49	12	0
N.S.	1	1.00	11.60	0.87	8.60	2.20	0.00	3.27	0.80	0.00
time (sec)	N/A	0.014	0.384	0.115	0.983	1.223	0.000	0.677	0.126	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	28	21	128	26	22	20	9	0
N.S.	1	1.00	1.65	1.24	7.53	1.53	1.29	1.18	0.53	0.00
time (sec)	N/A	0.042	0.016	0.136	1.018	0.812	1.128	0.650	0.242	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	81	19	0	24	15	0
N.S.	1	1.00	1.00	0.86	3.86	0.90	0.00	1.14	0.71	0.00
time (sec)	N/A	0.041	0.017	0.138	0.976	1.229	0.000	0.607	0.116	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	27	129	19	17	24	17	0
N.S.	1	1.00	1.00	1.29	6.14	0.90	0.81	1.14	0.81	0.00
time (sec)	N/A	0.023	0.008	0.137	1.000	1.186	1.334	0.607	0.260	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	28	171	50	294	48	27	0
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04	0.00
time (sec)	N/A	0.024	0.023	0.168	1.036	1.022	8.135	0.645	0.466	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	218	28	171	50	294	48	27	0
N.S.	1	1.00	8.38	1.08	6.58	1.92	11.31	1.85	1.04	0.00
time (sec)	N/A	0.035	0.343	0.190	1.039	1.015	20.129	0.664	0.288	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	25	22	0	34	0	17	21	0
N.S.	1	1.00	1.56	1.38	0.00	2.12	0.00	1.06	1.31	0.00
time (sec)	N/A	0.009	0.015	0.161	0.000	0.927	0.000	0.612	0.226	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	27	31	0	35	0	29	23	0
N.S.	1	1.00	1.59	1.82	0.00	2.06	0.00	1.71	1.35	0.00
time (sec)	N/A	0.012	0.015	0.148	0.000	0.789	0.000	0.653	0.223	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	47	9	41	55	0	41	13	0
N.S.	1	1.00	1.74	0.33	1.52	2.04	0.00	1.52	0.48	0.00
time (sec)	N/A	0.013	0.017	0.093	1.142	1.155	0.000	0.627	0.053	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	33	17	101	58	0	16	28	0
N.S.	1	1.00	1.10	0.57	3.37	1.93	0.00	0.53	0.93	0.00
time (sec)	N/A	0.013	0.016	0.137	1.167	0.835	0.000	0.637	0.245	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	61	52	433	107	0	59	-1	0
N.S.	1	1.00	1.15	0.98	8.17	2.02	0.00	1.11	-0.02	0.00
time (sec)	N/A	0.028	0.137	0.138	1.220	0.920	0.000	0.806	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	76	47	0	71	0	110	-1	0
N.S.	1	1.00	1.04	0.64	0.00	0.97	0.00	1.51	-0.01	0.00
time (sec)	N/A	0.033	0.173	0.153	0.000	0.792	0.000	0.752	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	36	31	43	40	0	43	-1	0
N.S.	1	1.00	0.65	0.56	0.78	0.73	0.00	0.78	-0.02	0.00
time (sec)	N/A	0.151	0.079	0.481	0.429	0.799	0.000	0.649	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	24	49	80	180	0	80	65	0
N.S.	1	1.00	0.24	0.50	0.82	1.84	0.00	0.82	0.66	0.00
time (sec)	N/A	0.069	0.013	0.119	0.964	0.916	0.000	0.596	0.127	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	69	69	53	52	54	0	0	67	0
N.S.	1	1.21	1.21	0.93	0.91	0.95	0.00	0.00	1.18	0.00
time (sec)	N/A	0.058	0.052	0.105	0.948	0.954	0.000	0.000	0.687	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	73	130	3213	541	0	0	63	0
N.S.	1	1.00	0.84	1.49	36.93	6.22	0.00	0.00	0.72	0.00
time (sec)	N/A	0.113	0.096	0.227	2.140	0.915	0.000	0.000	0.443	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	38	219	30	82	0	31	105	0
N.S.	1	1.00	0.95	5.48	0.75	2.05	0.00	0.78	2.62	0.00
time (sec)	N/A	0.147	1.255	0.770	0.430	0.982	0.000	0.633	1.421	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	C	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	133	62	94	117	603	0	111	228	0
N.S.	1	1.58	0.74	1.12	1.39	7.18	0.00	1.32	2.71	0.00
time (sec)	N/A	0.371	0.284	0.075	0.970	3.247	0.000	0.672	1.271	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	266	0	137	0	0	-1	0
N.S.	1	1.00	1.00	8.58	0.00	4.42	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.015	0.042	0.183	0.000	0.916	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	29	98	0	137	0	0	-1	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.015	0.031	0.184	0.000	0.999	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	41	171	0	151	0	0	-1	0
N.S.	1	1.00	0.91	3.80	0.00	3.36	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.030	0.040	0.170	0.000	0.944	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	43	442	0	76	0	0	-1	0
N.S.	1	1.00	0.91	9.40	0.00	1.62	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.096	0.064	0.309	0.000	0.663	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	50	510	0	181	0	0	-1	0
N.S.	1	1.00	0.82	8.36	0.00	2.97	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.080	0.092	0.237	0.000	1.014	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	56	1108	0	205	0	0	-1	0
N.S.	1	1.00	0.92	18.16	0.00	3.36	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.082	0.078	0.263	0.000	1.022	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	508	0	39	0	0	18	0
N.S.	1	1.00	1.00	31.75	0.00	2.44	0.00	0.00	1.12	0.00
time (sec)	N/A	0.022	0.034	0.267	0.000	1.001	0.000	0.000	0.562	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	20	286	0	32	0	0	20	0
N.S.	1	1.00	0.65	9.23	0.00	1.03	0.00	0.00	0.65	0.00
time (sec)	N/A	0.041	0.031	0.211	0.000	0.874	0.000	0.000	0.385	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	24	121	0	43	0	0	29	0
N.S.	1	1.00	0.83	4.17	0.00	1.48	0.00	0.00	1.00	0.00
time (sec)	N/A	0.048	0.032	0.161	0.000	0.606	0.000	0.000	0.430	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	95	150	761	0	136	0	0	-1	0
N.S.	1	1.40	2.21	11.19	0.00	2.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.856	6.382	0.606	0.000	1.051	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	29	19	16	6	15	0	0	15	0
N.S.	1	1.53	1.00	0.84	0.32	0.79	0.00	0.00	0.79	0.00
time (sec)	N/A	0.124	0.008	0.358	1.052	0.795	0.000	0.000	0.514	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	204	66	318	0	1006	0	0	-1	0
N.S.	1	2.22	0.72	3.46	0.00	10.93	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.239	0.080	0.510	0.000	114.303	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	63	0	13	29	0	0	32	0
N.S.	1	1.00	1.34	0.00	0.28	0.62	0.00	0.00	0.68	0.00
time (sec)	N/A	0.144	0.153	0.511	0.999	1.000	0.000	0.000	3.937	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	35	0	77	101	0	0	110	0
N.S.	1	1.00	0.50	0.00	1.10	1.44	0.00	0.00	1.57	0.00
time (sec)	N/A	0.196	0.057	0.441	1.010	0.879	0.000	0.000	3.552	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	234	105	247	0	611	0	0	-1	0
N.S.	1	2.17	0.97	2.29	0.00	5.66	0.00	0.00	-0.01	0.00
time (sec)	N/A	1.540	0.275	0.755	0.000	20.641	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-2)	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	364	665	385	22964	0	0	0	0	-1	0
N.S.	1	1.83	1.06	63.09	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	4.730	11.371	4.602	0.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	141	58	0	60	56	0	0	-1	0
N.S.	1	1.13	0.46	0.00	0.48	0.45	0.00	0.00	-0.01	0.00
time (sec)	N/A	1.019	0.345	1.316	1.175	1.307	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	67	61	56	55	108	0	55	43	0
N.S.	1	0.92	0.84	0.77	0.75	1.48	0.00	0.75	0.59	0.00
time (sec)	N/A	0.047	0.117	0.082	0.957	1.950	0.000	0.667	0.133	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	48	103	53	88	0	41	-1	0
N.S.	1	1.00	0.70	1.49	0.77	1.28	0.00	0.59	-0.01	0.00
time (sec)	N/A	0.053	0.070	0.227	0.976	1.491	0.000	0.679	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	61	82	36	122	0	41	-1	0
N.S.	1	1.00	1.05	1.41	0.62	2.10	0.00	0.71	-0.02	0.00
time (sec)	N/A	0.051	0.072	0.222	1.213	1.306	0.000	0.637	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	37	44	43	51	0	30	28	0
N.S.	1	1.00	0.67	0.80	0.78	0.93	0.00	0.55	0.51	0.00
time (sec)	N/A	0.054	0.085	0.134	0.423	0.980	0.000	0.674	0.658	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	58	716	100	0	38	-1	0
N.S.	1	1.00	1.00	1.49	18.36	2.56	0.00	0.97	-0.03	0.00
time (sec)	N/A	0.067	0.095	0.284	1.317	0.814	0.000	0.698	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	63	53	69	131	0	40	-1	0
N.S.	1	1.00	1.31	1.10	1.44	2.73	0.00	0.83	-0.02	0.00
time (sec)	N/A	0.072	0.292	0.230	0.996	1.518	0.000	0.676	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	28	46	192	1	0	33	28	0
N.S.	1	1.00	0.57	0.94	3.92	0.02	0.00	0.67	0.57	0.00
time (sec)	N/A	0.116	0.085	0.144	0.526	1.465	0.000	0.638	0.633	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	C	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	119	338	131	115	0	0	0	-1	0
N.S.	1	1.07	3.05	1.18	1.04	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.575	2.317	0.435	1.023	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-1)	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	156	0	0	195	0	0	-1	0
N.S.	1	1.00	1.39	0.00	0.00	1.74	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.444	0.335	180.000	0.000	1.507	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	38	25	26	0	35	-1	0
N.S.	1	1.00	0.88	1.15	0.76	0.79	0.00	1.06	-0.03	0.00
time (sec)	N/A	0.061	0.037	0.250	0.443	1.502	0.000	0.655	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	62	488	77	0	27	-1	0
N.S.	1	1.00	0.97	1.88	14.79	2.33	0.00	0.82	-0.03	0.00
time (sec)	N/A	0.022	0.017	0.157	1.150	1.325	0.000	0.692	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	49	55	790	103	0	48	29	0
N.S.	1	1.00	0.89	1.00	14.36	1.87	0.00	0.87	0.53	0.00
time (sec)	N/A	0.033	0.093	0.130	1.204	1.499	0.000	0.678	0.419	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	39	90	39	0	46	12	0
N.S.	1	1.00	1.00	2.44	5.62	2.44	0.00	2.88	0.75	0.00
time (sec)	N/A	0.013	0.033	0.184	1.020	1.335	0.000	0.966	0.347	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	100	0	118	0	0	-1	0
N.S.	1	1.00	1.00	2.04	0.00	2.41	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.057	0.088	0.206	0.000	0.965	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	91	62	180	1359	163	0	55	-1	0
N.S.	1	1.05	0.71	2.07	15.62	1.87	0.00	0.63	-0.01	0.00
time (sec)	N/A	0.208	0.199	0.407	1.614	1.373	0.000	0.727	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	115	754	0	130	0	0	-1	0
N.S.	1	1.00	1.69	11.09	0.00	1.91	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	0.200	0.700	0.000	0.797	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	40	79	473	0	115	0	0	-1	0
N.S.	1	1.00	1.98	11.82	0.00	2.88	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.030	0.118	0.686	0.000	0.927	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	131	975	0	97	0	0	-1	0
N.S.	1	1.00	1.39	10.37	0.00	1.03	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.189	0.524	1.140	0.000	1.118	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	36	64	63	33	0	340	20	0
N.S.	1	1.00	0.92	1.64	1.62	0.85	0.00	8.72	0.51	0.00
time (sec)	N/A	0.149	0.076	0.763	0.484	1.552	0.000	1.144	0.759	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	73	73	255	1615	0	257	0	171	-1	0
N.S.	1	1.00	3.49	22.12	0.00	3.52	0.00	2.34	-0.01	0.00
time (sec)	N/A	1.230	2.568	0.906	0.000	1.823	0.000	0.930	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	116	117	47	84	0	0	113	0
N.S.	1	1.00	2.04	2.05	0.82	1.47	0.00	0.00	1.98	0.00
time (sec)	N/A	0.849	0.923	0.968	0.976	1.577	0.000	0.000	1.473	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	49	61	0	50	0	52	90	0
N.S.	1	1.00	0.74	0.92	0.00	0.76	0.00	0.79	1.36	0.00
time (sec)	N/A	0.071	0.069	0.080	0.000	1.558	0.000	0.619	1.569	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	71	41	0	76	46	36	172	0
N.S.	1	1.00	1.31	0.76	0.00	1.41	0.85	0.67	3.19	0.00
time (sec)	N/A	0.060	0.411	0.087	0.000	1.361	8.112	0.610	0.511	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	105	0	0	0	0	186	250	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	1.40	1.88	0.00
time (sec)	N/A	0.156	0.136	0.049	0.000	0.000	0.000	0.761	1.447	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	42	0	0	117	0	79	101	0
N.S.	1	1.00	0.61	0.00	0.00	1.70	0.00	1.14	1.46	0.00
time (sec)	N/A	0.087	0.089	0.039	0.000	3.755	0.000	0.655	0.717	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	A	F(-1)	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	256	0	71	0	0	73	46	0
N.S.	1	1.00	4.92	0.00	1.37	0.00	0.00	1.40	0.88	0.00
time (sec)	N/A	0.089	0.301	0.046	0.981	0.000	0.000	1.348	0.414	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	A	F(-1)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	245	0	74	0	0	76	-1	0
N.S.	1	1.00	4.54	0.00	1.37	0.00	0.00	1.41	-0.02	0.00
time (sec)	N/A	0.087	0.282	0.049	0.967	0.000	0.000	1.078	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	133	174	4397	0	0	0	0	0	-1	0
N.S.	1	1.31	33.06	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	5.110	50.443	180.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	F	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	208	168	559	0	271	0	193	-1	0
N.S.	1	2.08	1.68	5.59	0.00	2.71	0.00	1.93	-0.01	0.00
time (sec)	N/A	1.226	4.832	1.023	0.000	1.486	0.000	1.874	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	47	127	136	185	0	0	-1	0
N.S.	1	1.00	0.42	1.13	1.21	1.65	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.169	0.043	0.511	0.983	1.356	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	F(-1)	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	162	154	0	145	0	0	146	-1	0
N.S.	1	1.71	1.62	0.00	1.53	0.00	0.00	1.54	-0.01	0.00
time (sec)	N/A	0.260	0.138	0.170	0.967	0.000	0.000	1.244	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	30	0	37	46	0	37	-1	0
N.S.	1	1.00	0.61	0.00	0.76	0.94	0.00	0.76	-0.02	0.00
time (sec)	N/A	0.108	0.498	0.962	0.433	1.269	0.000	0.909	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	42	26	86	35	0	25	43	0
N.S.	1	1.00	2.10	1.30	4.30	1.75	0.00	1.25	2.15	0.00
time (sec)	N/A	0.225	0.213	0.099	0.690	1.556	0.000	0.875	0.621	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	35	58	0	0	93	0	40	-1	0
N.S.	1	1.30	2.15	0.00	0.00	3.44	0.00	1.48	-0.04	0.00
time (sec)	N/A	0.967	4.300	0.778	0.000	7.060	0.000	0.843	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	126	89	0	100	0	0	0	-1	0
N.S.	1	1.25	0.88	0.00	0.99	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	1.405	0.426	1.085	0.993	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	140	0	0	26	0	25	-1	0
N.S.	1	1.00	5.60	0.00	0.00	1.04	0.00	1.00	-0.04	0.00
time (sec)	N/A	0.061	0.316	0.260	0.000	1.707	0.000	0.636	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	154	153	0	0	0	0	120	-1	0
N.S.	1	1.51	1.50	0.00	0.00	0.00	0.00	1.18	-0.01	0.00
time (sec)	N/A	0.192	0.126	0.442	0.000	0.000	0.000	0.681	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	18	45	88	259	50	0	85	-1	0
N.S.	1	1.06	2.65	5.18	15.24	2.94	0.00	5.00	-0.06	0.00
time (sec)	N/A	0.019	0.075	0.430	1.062	1.412	0.000	0.774	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	52	242	0	115	0	138	-1	0
N.S.	1	1.00	1.62	7.56	0.00	3.59	0.00	4.31	-0.03	0.00
time (sec)	N/A	0.039	0.073	0.763	0.000	1.248	0.000	0.834	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	24	27	30	24	32	24	31
N.S.	1	1.00	1.00	0.77	0.87	0.97	0.77	1.03	0.77	1.00
time (sec)	N/A	0.015	0.004	0.315	0.415	1.277	0.118	0.591	0.308	0.009

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	29	30	32	32	30	30	37
N.S.	1	1.00	1.00	0.74	0.77	0.82	0.82	0.77	0.77	0.95
time (sec)	N/A	0.014	0.016	0.322	0.948	1.364	0.133	0.576	0.035	0.022

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	34	46	73	41	42	44	40
N.S.	1	1.00	0.69	0.59	0.79	1.26	0.71	0.72	0.76	0.69
time (sec)	N/A	0.032	0.021	0.349	0.423	1.141	0.158	0.610	0.082	0.026

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	30	35	33	65	986	35	34	46
N.S.	1	1.00	0.58	0.67	0.63	1.25	18.96	0.67	0.65	0.88
time (sec)	N/A	0.023	0.008	0.334	0.960	0.740	3.827	0.614	0.471	0.043

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	47	46	42	47	167	46	46	49
N.S.	1	1.00	0.77	0.75	0.69	0.77	2.74	0.75	0.75	0.80
time (sec)	N/A	0.032	0.019	0.349	0.948	1.062	5.545	0.627	0.467	0.048

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	15	15	14	15	19	14	24	0
N.S.	1	1.00	0.94	0.94	0.88	0.94	1.19	0.88	1.50	0.00
time (sec)	N/A	0.003	0.002	0.012	0.422	0.861	0.061	0.583	0.456	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	36	30	35	49	32	36	35	39
N.S.	1	1.00	0.78	0.65	0.76	1.07	0.70	0.78	0.76	0.85
time (sec)	N/A	0.024	0.013	0.328	0.467	0.763	0.118	0.604	0.055	0.020

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	23	21	36	36	32	21	36	23
N.S.	1	1.00	0.58	0.52	0.90	0.90	0.80	0.52	0.90	0.58
time (sec)	N/A	0.035	0.008	0.335	0.426	0.945	0.135	0.584	0.066	0.011

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	20	17	19	21	41	19	20	25
N.S.	1	1.00	0.74	0.63	0.70	0.78	1.52	0.70	0.74	0.93
time (sec)	N/A	0.011	0.007	0.293	0.423	0.996	5.542	0.604	0.378	0.023

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	25	22	28	33	82	26	21	25
N.S.	1	1.00	0.66	0.58	0.74	0.87	2.16	0.68	0.55	0.66
time (sec)	N/A	0.015	0.010	0.326	0.421	1.105	5.063	0.607	0.454	0.027

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	23	20	25	34	61	19	99	23
N.S.	1	1.00	0.70	0.61	0.76	1.03	1.85	0.58	3.00	0.70
time (sec)	N/A	0.004	0.006	0.295	0.427	0.683	2.401	0.626	0.045	0.034

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	26	23	51	61	0	21	22	27
N.S.	1	1.00	0.60	0.53	1.19	1.42	0.00	0.49	0.51	0.63
time (sec)	N/A	0.006	0.015	0.389	0.422	1.150	0.000	0.662	0.283	0.278

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	28	23	35	40	151	62	24	28
N.S.	1	1.00	0.60	0.49	0.74	0.85	3.21	1.32	0.51	0.60
time (sec)	N/A	0.009	0.006	0.343	1.001	1.040	2.668	0.653	0.449	0.050

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	19	16	16	15	24	16	17	28
N.S.	1	1.00	0.68	0.57	0.57	0.54	0.86	0.57	0.61	1.00
time (sec)	N/A	0.006	0.007	0.291	0.436	0.889	7.849	0.627	0.272	0.009

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	47	53	74	49	58	51	49
N.S.	1	1.00	0.94	0.90	1.02	1.42	0.94	1.12	0.98	0.94
time (sec)	N/A	0.028	0.026	0.327	0.450	1.113	0.170	0.618	0.052	0.034

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	0	21	0	0	15	25
N.S.	1	1.00	1.00	0.88	0.00	0.84	0.00	0.00	0.60	1.00
time (sec)	N/A	0.057	0.009	0.281	0.000	1.276	0.000	0.000	0.450	8.544

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	35	28	79	69	0	98	-1	35
N.S.	1	1.00	0.70	0.56	1.58	1.38	0.00	1.96	-0.02	0.70
time (sec)	N/A	0.015	0.025	0.352	0.977	1.237	0.000	0.798	0.000	0.287

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	22	21	41	49	29	54	44
N.S.	1	1.00	1.00	0.92	0.88	1.71	2.04	1.21	2.25	1.83
time (sec)	N/A	0.005	0.017	0.314	0.959	0.844	0.535	0.655	0.303	0.057

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	24	32	45	45	32	77	33	46
N.S.	1	1.00	0.60	0.80	1.12	1.12	0.80	1.92	0.82	1.15
time (sec)	N/A	0.009	0.004	0.326	0.966	0.799	2.235	0.659	0.041	0.072

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	28	25	37	44	291	35	179	37
N.S.	1	1.00	0.57	0.51	0.76	0.90	5.94	0.71	3.65	0.76
time (sec)	N/A	0.007	0.011	0.314	0.422	1.191	12.347	0.661	0.276	0.063

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	29	28	59	47	0	35	29	55
N.S.	1	1.00	0.62	0.60	1.26	1.00	0.00	0.74	0.62	1.17
time (sec)	N/A	0.007	0.011	0.443	0.458	1.315	0.000	0.652	0.289	0.324

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	33	32	32	34	32	32	38
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.94	0.89	0.89	1.06
time (sec)	N/A	0.012	0.002	0.363	0.432	0.888	0.061	0.611	0.033	0.007

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	30	29	29	34	29	29	30
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.87	0.74	0.74	0.77
time (sec)	N/A	0.014	0.002	0.363	0.455	0.990	0.064	0.583	0.029	0.006

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	59	73	0	27	29	33
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62	0.70
time (sec)	N/A	0.009	0.049	0.447	0.468	0.782	0.000	0.700	0.365	0.365

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	33	30	76	73	0	41	29	45
N.S.	1	1.00	0.73	0.67	1.69	1.62	0.00	0.91	0.64	1.00
time (sec)	N/A	0.022	0.115	0.395	0.555	1.099	0.000	0.657	0.187	0.291

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	67	56	55	57	112	55	69	0
N.S.	1	1.00	0.81	0.67	0.66	0.69	1.35	0.66	0.83	0.00
time (sec)	N/A	0.094	0.054	0.383	0.513	1.122	3.486	0.585	0.396	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	51	50	49	52	92	49	59	0
N.S.	1	1.00	0.70	0.68	0.67	0.71	1.26	0.67	0.81	0.00
time (sec)	N/A	0.084	0.101	0.380	0.455	0.830	2.045	0.614	0.335	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	70	67	66	72	192	66	88	0
N.S.	1	1.00	0.67	0.64	0.63	0.69	1.83	0.63	0.84	0.00
time (sec)	N/A	0.110	0.092	0.415	0.564	1.255	5.898	0.592	0.398	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	39	32	35	36	53	35	40	0
N.S.	1	1.00	0.89	0.73	0.80	0.82	1.20	0.80	0.91	0.00
time (sec)	N/A	0.042	0.132	0.378	0.609	1.200	1.227	0.619	0.075	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	60	0	45	507	206	56	0
N.S.	1	1.00	1.00	1.82	0.00	1.36	15.36	6.24	1.70	0.00
time (sec)	N/A	0.054	0.033	0.073	0.000	1.093	2.012	0.704	0.492	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	104	76	619	47	551	341	35	0
N.S.	1	1.00	3.47	2.53	20.63	1.57	18.37	11.37	1.17	0.00
time (sec)	N/A	0.041	0.126	0.103	1.471	1.202	1.691	1.216	0.496	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	132	15	128	53	16	0
N.S.	1	1.00	1.00	0.81	8.25	0.94	8.00	3.31	1.00	0.00
time (sec)	N/A	0.018	0.018	0.341	0.603	0.869	1.034	0.597	0.352	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	57	57	66	113	0	0	-1	0
N.S.	1	1.00	0.92	0.92	1.06	1.82	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.072	0.017	0.086	1.467	1.411	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	54	59	213	138	0	0	-1	0
N.S.	1	1.00	0.92	1.00	3.61	2.34	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.061	0.014	0.069	1.297	1.474	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	14	44	78	13	0	10	-1	0
N.S.	1	1.00	1.17	3.67	6.50	1.08	0.00	0.83	-0.08	0.00
time (sec)	N/A	0.099	0.233	0.832	1.175	1.254	0.000	0.625	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	19	29	69	19	66	39	-1	0
N.S.	1	1.00	0.95	1.45	3.45	0.95	3.30	1.95	-0.05	0.00
time (sec)	N/A	0.034	0.245	0.253	0.704	1.153	1.474	0.628	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	23	0	22	42	325	22	0
N.S.	1	1.00	1.00	1.05	0.00	1.00	1.91	14.77	1.00	0.00
time (sec)	N/A	0.026	0.014	0.065	0.000	1.078	0.702	0.681	0.327	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-2)	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	41	46	42	0	52	0	436	34	0
N.S.	1	1.21	1.35	1.24	0.00	1.53	0.00	12.82	1.00	0.00
time (sec)	N/A	0.208	0.056	0.048	0.000	0.851	0.000	0.937	0.493	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	11	7	7	4	0
N.S.	1	1.00	1.00	0.89	0.78	1.22	0.78	0.78	0.44	0.00
time (sec)	N/A	0.003	0.003	0.028	0.554	1.105	0.086	0.601	0.051	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	21	17	24	8	0
N.S.	1	1.00	1.00	0.77	0.73	0.95	0.77	1.09	0.36	0.00
time (sec)	N/A	0.018	0.006	0.032	0.524	0.749	0.108	0.580	0.056	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	30	24	23	24	24	25	23	0
N.S.	1	1.00	0.97	0.77	0.74	0.77	0.77	0.81	0.74	0.00
time (sec)	N/A	0.019	0.014	0.038	0.542	0.874	0.128	0.618	0.307	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	34	29	28	31	31	36	28	0
N.S.	1	1.00	0.94	0.81	0.78	0.86	0.86	1.00	0.78	0.00
time (sec)	N/A	0.026	0.022	0.038	0.499	0.931	0.144	0.589	0.344	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	52	45	0	0	0	0	0	-1	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.047	0.011	0.010	0.000	0.943	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	33	44	41	39	54	46	41	0
N.S.	1	1.00	0.77	1.02	0.95	0.91	1.26	1.07	0.95	0.00
time (sec)	N/A	0.027	0.033	0.071	0.558	1.145	0.223	0.601	0.411	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	28	27	26	29	27	26	0
N.S.	1	1.00	1.00	1.04	1.00	0.96	1.07	1.00	0.96	0.00
time (sec)	N/A	0.011	0.006	0.043	0.628	0.925	0.264	0.629	0.362	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	55	51	62	250	691	68	0
N.S.	1	1.00	1.00	1.04	0.96	1.17	4.72	13.04	1.28	0.00
time (sec)	N/A	0.078	0.047	0.045	0.448	0.766	2.068	0.734	0.384	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	65	84	77	130	665	1033	81	0
N.S.	1	1.00	0.82	1.06	0.97	1.65	8.42	13.08	1.03	0.00
time (sec)	N/A	0.100	0.095	0.072	0.554	1.002	23.435	0.821	0.390	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	80	109	99	205	0	1359	106	0
N.S.	1	1.00	0.82	1.11	1.01	2.09	0.00	13.87	1.08	0.00
time (sec)	N/A	0.125	0.101	0.026	0.552	1.341	0.000	0.935	0.419	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	80	73	0	0	0	0	0	-1	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.077	0.031	0.050	0.000	1.389	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	28	29	28	27	0
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.04	1.00	0.96	0.00
time (sec)	N/A	0.008	0.007	0.033	0.454	1.659	0.267	0.584	0.310	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	55	51	64	248	691	69	0
N.S.	1	1.00	1.00	1.04	0.96	1.21	4.68	13.04	1.30	0.00
time (sec)	N/A	0.071	0.034	0.044	0.552	1.380	1.985	0.751	0.365	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	66	84	77	131	663	1033	81	0
N.S.	1	1.00	0.84	1.06	0.97	1.66	8.39	13.08	1.03	0.00
time (sec)	N/A	0.083	0.073	0.072	0.527	1.340	23.593	1.282	0.364	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	80	109	99	207	0	1359	106	0
N.S.	1	1.00	0.82	1.11	1.01	2.11	0.00	13.87	1.08	0.00
time (sec)	N/A	0.098	0.074	0.025	0.490	1.289	0.000	1.213	0.350	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	82	75	0	0	0	0	0	-1	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.062	0.027	0.052	0.000	0.976	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	15	19	15	15	15	0
N.S.	1	1.00	1.00	1.07	1.00	1.27	1.00	1.00	1.00	0.00
time (sec)	N/A	0.006	0.005	0.039	0.570	1.329	0.090	0.577	0.319	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	31	32	31	29	46	30	26	0
N.S.	1	1.00	0.94	0.97	0.94	0.88	1.39	0.91	0.79	0.00
time (sec)	N/A	0.013	0.012	0.039	0.529	1.168	0.118	0.582	0.318	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	35	41	46	39	71	40	34	0
N.S.	1	1.00	0.70	0.82	0.92	0.78	1.42	0.80	0.68	0.00
time (sec)	N/A	0.016	0.028	0.036	0.572	1.272	0.138	0.620	0.326	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	49	50	61	47	88	48	42	0
N.S.	1	1.00	0.75	0.77	0.94	0.72	1.35	0.74	0.65	0.00
time (sec)	N/A	0.019	0.022	0.039	0.510	1.379	0.158	0.582	0.330	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	0	0	0	0	0	55	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.021	0.013	0.017	0.000	0.882	0.000	0.000	0.315	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	17	16	21	19	16	16	0
N.S.	1	1.00	1.00	1.06	1.00	1.31	1.19	1.00	1.00	0.00
time (sec)	N/A	0.005	0.005	0.031	0.677	0.975	0.095	0.585	0.297	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	25	32	31	29	46	30	27	0
N.S.	1	1.00	0.76	0.97	0.94	0.88	1.39	0.91	0.82	0.00
time (sec)	N/A	0.013	0.018	0.030	0.537	1.425	0.117	0.605	0.303	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	35	41	46	39	71	40	35	0
N.S.	1	1.00	0.70	0.82	0.92	0.78	1.42	0.80	0.70	0.00
time (sec)	N/A	0.017	0.028	0.036	0.496	1.320	0.143	0.572	0.316	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	49	50	61	47	88	48	43	0
N.S.	1	1.00	0.75	0.77	0.94	0.72	1.35	0.74	0.66	0.00
time (sec)	N/A	0.019	0.018	0.036	0.506	1.176	0.162	0.613	0.311	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	0	0	0	0	0	57	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.019	0.015	0.018	0.000	1.116	0.000	0.000	0.316	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	24	23	22	15	26	22	0
N.S.	1	1.00	1.00	1.00	0.96	0.92	0.62	1.08	0.92	0.00
time (sec)	N/A	0.016	0.006	0.028	0.446	1.308	0.123	0.578	0.331	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	123	97	26	100	311	22	116	104	0
N.S.	1	1.23	0.97	0.26	1.00	3.11	0.22	1.16	1.04	0.00
time (sec)	N/A	0.094	0.055	0.058	1.366	1.344	0.175	0.626	1.512	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	12	11	11	8	11	11	0
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	0.92	0.92	0.00
time (sec)	N/A	0.022	0.009	0.023	0.484	1.006	0.081	0.573	0.049	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	44	38	37	39	22	37	39	0
N.S.	1	1.00	0.94	0.81	0.79	0.83	0.47	0.79	0.83	0.00
time (sec)	N/A	0.059	0.022	0.052	1.336	1.008	0.144	0.599	0.325	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	51	29	28	28	48	29	28	0
N.S.	1	1.00	1.31	0.74	0.72	0.72	1.23	0.74	0.72	0.00
time (sec)	N/A	0.062	0.045	0.089	1.155	1.286	0.206	0.630	0.088	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	33	32	37	94	32	29	0
N.S.	1	1.00	1.00	1.10	1.07	1.23	3.13	1.07	0.97	0.00
time (sec)	N/A	0.038	0.041	0.044	0.585	1.302	1.318	0.639	0.355	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	57	56	56	0	57	33	0
N.S.	1	1.00	1.00	1.06	1.04	1.04	0.00	1.06	0.61	0.00
time (sec)	N/A	0.022	0.039	0.077	1.310	1.277	0.000	0.593	0.347	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	59	0	0	0	0	0	75	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.031	0.023	0.015	0.000	1.109	0.000	0.000	0.404	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	7	18	31	18	14	0
N.S.	1	1.00	1.00	0.83	0.39	1.00	1.72	1.00	0.78	0.00
time (sec)	N/A	0.028	0.006	0.038	0.546	1.258	0.728	0.583	0.406	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	20	20	31	20	16	0
N.S.	1	1.00	1.00	0.85	1.00	1.00	1.55	1.00	0.80	0.00
time (sec)	N/A	0.029	0.006	0.040	0.614	1.304	0.728	0.615	0.409	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	0	39	46	0	48	-1	0
N.S.	1	1.00	1.00	0.00	0.98	1.15	0.00	1.20	-0.02	0.00
time (sec)	N/A	0.103	0.022	0.013	1.300	1.028	0.000	0.819	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	57	54	0	0	0	0	0	-1	0
N.S.	1	1.54	1.46	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.052	0.017	180.000	0.000	0.936	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	44	37	49	30	0	65	30	0
N.S.	1	1.00	0.60	0.51	0.67	0.41	0.00	0.89	0.41	0.00
time (sec)	N/A	0.047	0.021	0.034	0.565	1.254	0.000	0.656	0.106	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	23	21	19	19	20	19	21	0
N.S.	1	1.00	0.52	0.48	0.43	0.43	0.45	0.43	0.48	0.00
time (sec)	N/A	0.032	0.008	0.042	0.505	1.274	0.090	0.596	0.028	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	26	27	7	23	32	27	22	0
N.S.	1	1.00	0.67	0.69	0.18	0.59	0.82	0.69	0.56	0.00
time (sec)	N/A	0.033	0.024	0.069	0.581	1.401	1.340	0.598	0.272	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	29	28	27	27	39	826	27	0
N.S.	1	1.00	0.66	0.64	0.61	0.61	0.89	18.77	0.61	0.00
time (sec)	N/A	0.022	0.011	0.044	0.458	1.133	0.115	0.703	0.059	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	10	18	9	8	18	9	0
N.S.	1	1.00	1.00	0.83	1.50	0.75	0.67	1.50	0.75	0.00
time (sec)	N/A	0.049	0.005	0.033	0.556	1.237	0.091	0.571	0.049	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	25	25	33	22	40	26	0
N.S.	1	1.00	0.97	0.78	0.78	1.03	0.69	1.25	0.81	0.00
time (sec)	N/A	0.055	0.036	0.043	1.165	1.362	0.106	0.604	0.327	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	20	21	12	0	0	12	0
N.S.	1	1.00	1.00	1.33	1.40	0.80	0.00	0.00	0.80	0.00
time (sec)	N/A	0.057	0.026	0.305	0.647	1.049	0.000	0.000	0.448	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	22	22	19	21	26	19	19	0
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70	0.00
time (sec)	N/A	0.011	0.028	0.054	0.547	1.193	0.470	0.643	0.028	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	26	22	39	21	29	39	19	0
N.S.	1	1.00	0.74	0.63	1.11	0.60	0.83	1.11	0.54	0.00
time (sec)	N/A	0.111	0.062	0.100	0.661	1.353	0.846	0.604	0.104	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	37	37	31	0	70	39	33	0
N.S.	1	1.00	0.65	0.65	0.54	0.00	1.23	0.68	0.58	0.00
time (sec)	N/A	0.028	0.068	0.087	1.349	0.000	2.454	0.629	0.037	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	39	41	45	37	265	43	37	0
N.S.	1	1.00	0.72	0.76	0.83	0.69	4.91	0.80	0.69	0.00
time (sec)	N/A	0.023	0.035	0.079	0.620	1.037	2.813	0.597	0.052	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	64	66	73	65	638	63	47	0
N.S.	1	1.00	0.78	0.80	0.89	0.79	7.78	0.77	0.57	0.00
time (sec)	N/A	0.035	0.180	0.107	0.553	1.120	11.651	0.608	0.058	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	36	48	27	42	76	33	39	0
N.S.	1	1.00	0.46	0.61	0.34	0.53	0.96	0.42	0.49	0.00
time (sec)	N/A	0.045	0.044	0.100	0.630	1.384	2.470	0.634	0.296	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	21	28	27	40	70	24	18	0
N.S.	1	1.00	0.58	0.78	0.75	1.11	1.94	0.67	0.50	0.00
time (sec)	N/A	0.044	0.036	0.076	0.532	1.412	5.059	0.624	0.412	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	21	42	27	50	99	24	18	0
N.S.	1	1.00	0.58	1.17	0.75	1.39	2.75	0.67	0.50	0.00
time (sec)	N/A	0.043	0.045	0.080	0.524	1.374	4.866	0.647	0.394	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	85	97	0	0	0	0	0	-1	0
N.S.	1	1.47	1.67	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.077	0.263	0.009	0.000	1.244	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	90	0	0	0	0	0	-1	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.018	0.201	0.036	0.000	1.382	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	77	66	0	0	0	0	0	-1	0
N.S.	1	1.51	1.29	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.036	0.050	0.087	0.000	1.264	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.031	0.010	0.016	0.000	0.968	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	84	0	0	0	0	0	-1	0
N.S.	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.032	0.076	0.023	0.000	1.152	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	61	0	0	0	0	0	-1	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.034	0.621	0.021	0.000	1.161	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	61	0	0	0	0	0	-1	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.034	0.682	0.040	0.000	1.063	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	11	21	22	12	0	10	8	0
N.S.	1	1.00	0.73	1.40	1.47	0.80	0.00	0.67	0.53	0.00
time (sec)	N/A	0.030	0.232	0.095	0.716	1.147	0.000	0.643	0.387	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	45	100	0	0	0	0	0	-1	0
N.S.	1	1.10	2.44	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.113	0.225	0.072	0.000	1.417	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	10	8	22	11	0	7	7	0
N.S.	1	1.00	0.83	0.67	1.83	0.92	0.00	0.58	0.58	0.00
time (sec)	N/A	0.028	0.192	0.085	0.722	1.299	0.000	0.625	0.366	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	44	87	0	0	0	0	0	-1	0
N.S.	1	1.05	2.07	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.111	0.227	0.084	0.000	1.332	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	48	72	0	0	0	0	0	-1	0
N.S.	1	1.04	1.57	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.129	0.758	0.085	0.000	1.139	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	23	21	22	24	0	20	24	0
N.S.	1	1.00	1.64	1.50	1.57	1.71	0.00	1.43	1.71	0.00
time (sec)	N/A	0.024	0.075	0.125	0.738	1.032	0.000	0.612	0.374	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	47	73	0	0	0	0	0	-1	0
N.S.	1	1.09	1.70	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.127	0.202	0.067	0.000	1.117	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	23	21	22	24	0	21	20	0
N.S.	1	1.00	1.77	1.62	1.69	1.85	0.00	1.62	1.54	0.00
time (sec)	N/A	0.023	0.066	0.109	0.717	1.299	0.000	0.628	0.424	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	18	20	17	17	27	15	17	0
N.S.	1	1.00	0.60	0.67	0.57	0.57	0.90	0.50	0.57	0.00
time (sec)	N/A	0.039	0.027	0.059	0.439	1.377	0.829	0.624	0.066	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	25	27	26	26	48	25	21	0
N.S.	1	1.00	0.50	0.54	0.52	0.52	0.96	0.50	0.42	0.00
time (sec)	N/A	0.108	0.040	0.064	0.462	1.402	2.058	0.601	0.327	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	38	36	33	37	80	33	39	0
N.S.	1	1.00	0.51	0.48	0.44	0.49	1.07	0.44	0.52	0.00
time (sec)	N/A	0.140	0.039	0.083	0.460	1.108	2.284	0.573	0.107	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	253	72	106	77	72	202	73	83	0
N.S.	1	1.35	0.39	0.57	0.41	0.39	1.08	0.39	0.44	0.00
time (sec)	N/A	0.478	0.159	0.106	0.477	1.335	11.766	0.599	0.313	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	40	40	41	41	85	39	51	0
N.S.	1	1.00	0.46	0.46	0.47	0.47	0.98	0.45	0.59	0.00
time (sec)	N/A	0.157	0.084	0.072	0.446	1.268	2.111	0.574	0.353	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	76	106	77	72	202	73	83	0
N.S.	1	1.00	0.41	0.57	0.42	0.39	1.09	0.39	0.45	0.00
time (sec)	N/A	0.356	0.216	0.091	0.490	1.155	11.807	0.601	0.527	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	2	2	11	2	0
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00	0.00
time (sec)	N/A	0.003	0.003	0.052	0.423	1.196	0.126	0.627	0.016	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	2	2	11	2	0
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00	0.00
time (sec)	N/A	0.003	0.001	0.030	0.424	1.167	0.126	0.622	0.018	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	18	7	11	3	0
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00	0.00
time (sec)	N/A	0.003	0.004	0.020	0.421	1.094	0.129	0.634	0.020	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	18	12	12	3	0
N.S.	1	1.00	1.00	1.33	1.00	6.00	4.00	4.00	1.00	0.00
time (sec)	N/A	0.004	0.002	0.024	0.421	1.119	0.310	0.623	0.027	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	9	4	3	8	0	5	5	0
N.S.	1	1.00	3.00	1.33	1.00	2.67	0.00	1.67	1.67	0.00
time (sec)	N/A	0.003	0.004	0.027	0.427	1.237	0.000	0.619	0.022	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	5	5	7	6	5	17	0	14	5	0
N.S.	1	1.00	1.40	1.20	1.00	3.40	0.00	2.80	1.00	0.00
time (sec)	N/A	0.004	0.005	0.016	0.431	1.342	0.000	0.586	0.011	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	16	10	24	24	10	0
N.S.	1	1.00	1.00	0.79	1.14	0.71	1.71	1.71	0.71	0.00
time (sec)	N/A	0.007	0.002	0.030	0.430	1.118	0.196	0.607	0.026	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	23	18	35	42	29	37	15	0
N.S.	1	1.00	1.21	0.95	1.84	2.21	1.53	1.95	0.79	0.00
time (sec)	N/A	0.011	0.002	0.340	0.429	0.948	1.121	0.585	0.026	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	18	26	38	68	10	26	12	0
N.S.	1	1.00	1.29	1.86	2.71	4.86	0.71	1.86	0.86	0.00
time (sec)	N/A	0.011	0.003	0.025	0.434	1.283	0.228	0.630	0.066	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	36	11	45	211	0	45	16	0
N.S.	1	1.00	2.25	0.69	2.81	13.19	0.00	2.81	1.00	0.00
time (sec)	N/A	0.012	0.004	0.344	0.429	1.258	0.000	0.599	0.290	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	30	21	61	461	422	60	22	0
N.S.	1	1.00	1.15	0.81	2.35	17.73	16.23	2.31	0.85	0.00
time (sec)	N/A	0.017	0.004	0.347	1.014	1.267	2.830	0.636	0.078	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	35	257	0	43	35	0
N.S.	1	1.00	1.00	0.94	1.94	14.28	0.00	2.39	1.94	0.00
time (sec)	N/A	0.023	0.007	0.043	0.955	1.088	0.000	0.615	0.357	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	27	0	0	359	0	0	120	0
N.S.	1	1.00	0.87	0.00	0.00	11.58	0.00	0.00	3.87	0.00
time (sec)	N/A	0.031	0.054	0.059	0.000	0.671	0.000	0.000	0.165	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	42	41	36	0	175	126	32	43	0
N.S.	1	1.02	1.00	0.88	0.00	4.27	3.07	0.78	1.05	0.00
time (sec)	N/A	0.049	0.033	0.069	0.000	1.154	4.414	0.592	0.163	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	16	15	49	58	14	14	14	0
N.S.	1	1.00	0.64	0.60	1.96	2.32	0.56	0.56	0.56	0.00
time (sec)	N/A	0.017	0.012	0.037	0.432	1.132	0.385	0.607	0.288	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	55	41	42	146	43	35	0
N.S.	1	1.00	0.74	1.41	1.05	1.08	3.74	1.10	0.90	0.00
time (sec)	N/A	0.050	0.069	0.057	0.436	1.335	0.614	0.585	0.132	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	98	76	288	605	79	109	0
N.S.	1	1.00	1.00	3.16	2.45	9.29	19.52	2.55	3.52	0.00
time (sec)	N/A	0.035	0.068	0.170	0.981	1.400	46.112	0.603	0.652	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	74	0	388	892	50	106	0
N.S.	1	1.00	1.00	2.11	0.00	11.09	25.49	1.43	3.03	0.00
time (sec)	N/A	0.037	0.048	0.131	0.000	1.372	40.604	0.588	0.381	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	46	69	113	908	48	63	0
N.S.	1	1.00	0.96	1.84	2.76	4.52	36.32	1.92	2.52	0.00
time (sec)	N/A	0.017	0.102	0.083	0.964	1.322	7.436	0.588	0.400	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	37	44	70	74	102	22	22	0
N.S.	1	1.00	1.12	1.33	2.12	2.24	3.09	0.67	0.67	0.00
time (sec)	N/A	0.136	0.140	0.207	1.033	1.296	1.825	0.599	0.370	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	23	42	44	116	48	40	0
N.S.	1	1.00	1.00	0.77	1.40	1.47	3.87	1.60	1.33	0.00
time (sec)	N/A	0.035	0.010	0.128	0.437	1.011	13.119	0.629	0.403	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	23	42	111	138	48	40	0
N.S.	1	1.00	1.00	0.77	1.40	3.70	4.60	1.60	1.33	0.00
time (sec)	N/A	0.034	0.011	0.197	0.446	1.226	12.145	0.636	0.416	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	102	392	987	0	376	0	0	-1	0
N.S.	1	1.48	5.68	14.30	0.00	5.45	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.973	30.385	0.484	0.000	1.145	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	26	30	125	474	0	40	57	0
N.S.	1	1.00	0.70	0.81	3.38	12.81	0.00	1.08	1.54	0.00
time (sec)	N/A	0.045	0.072	0.054	0.582	0.944	0.000	0.733	0.139	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	21	28	177	161	0	0	47	0
N.S.	1	1.00	0.72	0.97	6.10	5.55	0.00	0.00	1.62	0.00
time (sec)	N/A	0.108	0.056	0.130	1.213	1.341	0.000	0.000	0.486	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	63	0	482	0	58	-1	0
N.S.	1	1.00	1.00	4.20	0.00	32.13	0.00	3.87	-0.07	0.00
time (sec)	N/A	0.019	0.011	0.175	0.000	0.970	0.000	0.660	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	49	93	17	51	21	0
N.S.	1	1.00	1.00	1.75	3.06	5.81	1.06	3.19	1.31	0.00
time (sec)	N/A	0.019	0.023	0.030	1.121	1.291	0.195	0.634	0.312	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	53	95	22	53	27	0
N.S.	1	1.00	1.00	1.75	3.31	5.94	1.38	3.31	1.69	0.00
time (sec)	N/A	0.021	0.023	0.033	0.643	1.315	0.663	0.588	0.293	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	23	14	13	20	26	11	16	0
N.S.	1	1.00	1.15	0.70	0.65	1.00	1.30	0.55	0.80	0.00
time (sec)	N/A	0.067	0.103	0.091	0.494	1.359	0.417	0.626	0.061	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	20	16	35	20	12	14	17	0
N.S.	1	1.00	1.33	1.07	2.33	1.33	0.80	0.93	1.13	0.00
time (sec)	N/A	0.130	0.086	0.060	0.449	1.398	0.465	0.613	0.286	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	25	22	75	0	24	24	0
N.S.	1	1.00	1.00	1.25	1.10	3.75	0.00	1.20	1.20	0.00
time (sec)	N/A	0.018	0.011	0.105	0.434	1.306	0.000	0.617	0.331	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	11	75	102	0	10	19	0
N.S.	1	1.00	1.00	0.85	5.77	7.85	0.00	0.77	1.46	0.00
time (sec)	N/A	0.017	0.015	0.350	0.457	0.774	0.000	0.588	0.311	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	7	6	16	12	6	6	0
N.S.	1	1.00	1.00	0.78	0.67	1.78	1.33	0.67	0.67	0.00
time (sec)	N/A	0.017	0.002	0.097	0.435	0.927	0.486	0.593	0.317	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	19	18	13	0	25	32	16	14	0
N.S.	1	1.46	1.38	1.00	0.00	1.92	2.46	1.23	1.08	0.00
time (sec)	N/A	0.029	0.030	0.142	0.000	0.975	0.561	0.577	0.109	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	10	1	1	0
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00	0.00
time (sec)	N/A	0.015	0.001	0.059	0.430	0.785	0.413	0.594	0.288	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	36	17	16	26	0	17	16	0
N.S.	1	1.00	1.64	0.77	0.73	1.18	0.00	0.77	0.73	0.00
time (sec)	N/A	0.024	0.035	0.048	0.462	0.959	0.000	0.609	0.059	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	18	12	11	21	0	11	11	0
N.S.	1	1.00	1.38	0.92	0.85	1.62	0.00	0.85	0.85	0.00
time (sec)	N/A	0.031	0.037	0.052	0.444	0.689	0.000	0.604	0.303	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	20	12	11	22	0	11	11	0
N.S.	1	1.00	1.33	0.80	0.73	1.47	0.00	0.73	0.73	0.00
time (sec)	N/A	0.033	0.037	0.067	0.442	1.708	0.000	0.587	0.044	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	19	19	26	25	56	0	32	0
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23	0.00
time (sec)	N/A	0.010	0.008	0.030	0.417	1.148	0.756	0.000	0.401	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	30	41	42	45	155	84	43	0
N.S.	1	1.00	0.71	0.98	1.00	1.07	3.69	2.00	1.02	0.00
time (sec)	N/A	0.024	0.017	0.036	0.422	0.918	1.572	0.639	0.354	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	21	23	22	17	34	22	17	0
N.S.	1	1.00	0.62	0.68	0.65	0.50	1.00	0.65	0.50	0.00
time (sec)	N/A	0.020	0.005	0.104	0.424	1.155	6.088	0.634	0.037	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	29	28	25	25	25	22	24	21	0
N.S.	1	1.04	1.00	0.89	0.89	0.89	0.79	0.86	0.75	0.00
time (sec)	N/A	0.011	0.002	0.022	0.421	1.046	0.113	0.577	0.332	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	81	58	69	69	71	71	71	0
N.S.	1	1.00	1.21	0.87	1.03	1.03	1.06	1.06	1.06	0.00
time (sec)	N/A	0.033	0.031	0.316	0.432	0.685	0.154	0.629	0.381	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	24	36	23	26	23	20	0
N.S.	1	1.00	1.00	1.04	1.57	1.00	1.13	1.00	0.87	0.00
time (sec)	N/A	0.014	0.003	0.023	0.468	0.801	0.108	0.582	0.294	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	73	60	48	66	48	51	52	51	0
N.S.	1	1.22	1.00	0.80	1.10	0.80	0.85	0.87	0.85	0.00
time (sec)	N/A	0.091	0.005	0.025	0.442	1.009	0.156	0.616	0.374	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	31	8	34	32	0	29	0
N.S.	1	1.00	1.00	0.72	0.19	0.79	0.74	0.00	0.67	0.00
time (sec)	N/A	0.059	0.018	0.030	0.537	0.956	0.716	0.000	0.279	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	30	32	25	0	151	0	-1	0
N.S.	1	1.00	1.03	1.10	0.86	0.00	5.21	0.00	-0.03	0.00
time (sec)	N/A	0.020	0.004	0.313	0.433	0.995	6.442	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	27	30	38	34	24	138	35	0
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.21	0.00
time (sec)	N/A	0.013	0.018	0.309	0.426	0.868	0.355	0.607	0.392	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	15	12	22	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.83	0.00
time (sec)	N/A	0.017	0.003	0.029	0.417	1.050	0.891	0.608	0.333	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	20	19	22	36	19	19	0
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.89	1.00	1.00	0.00
time (sec)	N/A	0.028	0.011	0.037	0.426	0.956	1.267	0.606	0.450	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	11	8	30	11	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	2.73	1.00	0.00
time (sec)	N/A	0.024	0.016	0.020	0.418	0.955	0.126	0.604	0.294	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	24	0	27	71	22	22	0
N.S.	1	1.00	1.00	1.04	0.00	1.17	3.09	0.96	0.96	0.00
time (sec)	N/A	0.032	0.017	0.039	0.000	0.688	20.648	0.629	0.427	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	46	15	7	18	0	18	14	0
N.S.	1	1.00	2.88	0.94	0.44	1.12	0.00	1.12	0.88	0.00
time (sec)	N/A	0.038	0.025	0.022	0.419	0.803	0.000	0.599	0.430	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	50	17	20	20	0	0	16	0
N.S.	1	1.00	2.78	0.94	1.11	1.11	0.00	0.00	0.89	0.00
time (sec)	N/A	0.042	0.037	0.022	0.432	0.996	0.000	0.000	0.392	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	7	25	0	10	16	0
N.S.	1	1.00	1.00	0.94	0.39	1.39	0.00	0.56	0.89	0.00
time (sec)	N/A	0.041	0.034	0.021	0.997	0.906	0.000	0.772	0.612	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	37	13	44	0	0	27	0
N.S.	1	1.00	1.00	1.68	0.59	2.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.088	0.019	0.022	0.421	0.903	0.000	0.000	0.581	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	39	37	27	0	0	22	0
N.S.	1	1.00	1.00	1.62	1.54	1.12	0.00	0.00	0.92	0.00
time (sec)	N/A	0.098	0.022	0.023	0.428	0.775	0.000	0.000	0.608	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	43	13	27	0	21	25	0
N.S.	1	1.00	1.00	1.87	0.57	1.17	0.00	0.91	1.09	0.00
time (sec)	N/A	0.099	0.021	0.022	0.954	0.869	0.000	1.626	0.524	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	11	10	11	8	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73	0.00
time (sec)	N/A	0.007	0.005	0.020	0.418	1.035	0.257	1.647	0.322	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	15	20	24	20	15	0
N.S.	1	1.00	1.00	1.05	0.75	1.00	1.20	1.00	0.75	0.00
time (sec)	N/A	0.018	0.007	0.036	0.421	0.653	0.305	1.209	0.377	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	30	22	29	36	29	29	0
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.24	1.00	1.00	0.00
time (sec)	N/A	0.022	0.011	0.039	0.420	1.032	0.349	0.790	0.361	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	39	29	38	48	38	38	0
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.26	1.00	1.00	0.00
time (sec)	N/A	0.026	0.009	0.039	0.424	0.933	0.394	0.952	0.371	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	0	29	14	24	0	24	0
N.S.	1	1.00	1.00	0.00	1.21	0.58	1.00	0.00	1.00	0.00
time (sec)	N/A	0.032	0.006	0.007	0.555	1.048	1.850	0.000	0.451	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	5	4	4	0	5	4	0
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.00	1.25	1.00	0.00
time (sec)	N/A	0.020	0.010	0.175	0.423	1.189	0.000	0.955	0.396	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	10	9	12	7	9	9	0
N.S.	1	1.00	1.00	1.43	1.29	1.71	1.00	1.29	1.29	0.00
time (sec)	N/A	0.047	0.004	0.073	0.430	1.031	1.730	0.973	0.387	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	46	10	32	8	0
N.S.	1	1.00	1.00	1.09	1.00	4.18	0.91	2.91	0.73	0.00
time (sec)	N/A	0.011	0.008	0.158	0.433	1.115	0.945	1.118	0.365	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	7	0	0	16	0
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.00	0.00	1.78	0.00
time (sec)	N/A	0.015	0.004	0.149	0.425	0.777	0.000	0.000	0.439	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	0	22	20	22	22	0
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85	0.00
time (sec)	N/A	0.005	0.020	0.014	0.000	1.059	6.261	1.127	0.078	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	27	26	25	26	26	27	25	0
N.S.	1	1.00	0.77	0.74	0.71	0.74	0.74	0.77	0.71	0.00
time (sec)	N/A	0.015	0.012	0.043	0.430	0.783	0.143	0.790	0.065	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	24	24	20	26	0	20	24	0
N.S.	1	1.00	0.75	0.75	0.62	0.81	0.00	0.62	0.75	0.00
time (sec)	N/A	0.054	0.041	0.029	0.440	1.214	0.000	0.941	0.380	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	42	43	42	42	0	43	31	0
N.S.	1	1.00	0.81	0.83	0.81	0.81	0.00	0.83	0.60	0.00
time (sec)	N/A	0.044	0.042	0.032	0.448	0.883	0.000	0.865	0.548	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	197	25	24	42	26	-1	0
N.S.	1	1.00	1.00	6.57	0.83	0.80	1.40	0.87	-0.03	0.00
time (sec)	N/A	0.035	0.012	0.375	0.422	0.931	5.236	1.077	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	55	25	39	46	26	148	0
N.S.	1	1.00	0.97	1.83	0.83	1.30	1.53	0.87	4.93	0.00
time (sec)	N/A	0.058	0.087	0.522	0.432	1.275	20.461	0.948	1.803	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	32	164	56	32	0	43	39	0
N.S.	1	1.00	1.14	5.86	2.00	1.14	0.00	1.54	1.39	0.00
time (sec)	N/A	0.035	0.101	0.247	0.440	0.904	0.000	0.921	0.576	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	106	86	53	107	36	164	0
N.S.	1	1.00	0.93	1.77	1.43	0.88	1.78	0.60	2.73	0.00
time (sec)	N/A	0.132	0.155	0.439	0.962	0.860	7.995	1.090	0.787	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	52	52	51	44	0	104	-1	0
N.S.	1	1.00	0.80	0.80	0.78	0.68	0.00	1.60	-0.02	0.00
time (sec)	N/A	0.109	0.033	0.081	0.957	1.070	0.000	0.989	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	42	37	47	36	54	57	-1	0
N.S.	1	1.00	0.69	0.61	0.77	0.59	0.89	0.93	-0.02	0.00
time (sec)	N/A	0.094	0.026	0.360	0.974	1.054	0.656	0.904	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	37	42	44	36	44	41	41	0
N.S.	1	1.00	0.70	0.79	0.83	0.68	0.83	0.77	0.77	0.00
time (sec)	N/A	0.116	0.022	0.096	0.998	1.005	0.625	1.087	0.344	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	56	48	64	53	53	0	44	0
N.S.	1	1.00	0.92	0.79	1.05	0.87	0.87	0.00	0.72	0.00
time (sec)	N/A	0.126	0.018	0.137	0.976	1.135	0.875	0.000	0.105	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	42	56	95	35	0	106	-1	0
N.S.	1	1.00	0.67	0.89	1.51	0.56	0.00	1.68	-0.02	0.00
time (sec)	N/A	0.067	0.048	0.079	1.227	0.846	0.000	0.961	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	92	165	0	77	0	137	-1	0
N.S.	1	1.00	0.62	1.11	0.00	0.52	0.00	0.93	-0.01	0.00
time (sec)	N/A	0.147	0.082	0.385	0.000	1.175	0.000	0.983	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	31	30	26	48	27	-1	0
N.S.	1	1.00	0.88	0.91	0.88	0.76	1.41	0.79	-0.03	0.00
time (sec)	N/A	0.029	0.009	0.312	0.972	1.330	24.119	0.847	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	33	30	26	48	27	-1	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	1.41	0.79	-0.03	0.00
time (sec)	N/A	0.030	0.015	0.315	0.968	1.028	24.579	1.124	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	26	134	22	27	37	22	-1	0
N.S.	1	1.00	0.87	4.47	0.73	0.90	1.23	0.73	-0.03	0.00
time (sec)	N/A	0.031	0.030	0.322	0.956	1.115	1.059	1.163	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	42	58	50	39	53	50	-1	0
N.S.	1	1.00	0.71	0.98	0.85	0.66	0.90	0.85	-0.02	0.00
time (sec)	N/A	0.050	0.037	0.372	1.021	1.214	2.078	1.046	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	37	27	37	63	34	-1	0
N.S.	1	1.00	0.95	1.00	0.73	1.00	1.70	0.92	-0.03	0.00
time (sec)	N/A	0.037	0.016	0.350	0.982	0.734	3.454	0.859	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	47	286	49	47	88	60	-1	0
N.S.	1	1.00	0.77	4.69	0.80	0.77	1.44	0.98	-0.02	0.00
time (sec)	N/A	0.075	0.055	0.487	0.960	0.970	162.011	1.178	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	95	95	119	230	0	0	0	0	-1	0
N.S.	1	1.00	1.25	2.42	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.158	0.281	0.441	0.000	1.073	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	36	201	35	44	0	135	-1	0
N.S.	1	1.00	0.88	4.90	0.85	1.07	0.00	3.29	-0.02	0.00
time (sec)	N/A	0.059	0.040	0.997	0.974	1.092	0.000	0.988	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	28	32	32	26	26	27	-1	0
N.S.	1	1.00	0.82	0.94	0.94	0.76	0.76	0.79	-0.03	0.00
time (sec)	N/A	0.060	0.013	0.318	0.972	0.645	0.372	1.014	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	43	53	52	39	53	50	-1	0
N.S.	1	1.00	0.70	0.87	0.85	0.64	0.87	0.82	-0.02	0.00
time (sec)	N/A	0.107	0.023	0.376	0.974	1.041	1.318	1.405	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	46	25	44	20	27	-1	0
N.S.	1	1.00	1.00	2.42	1.32	2.32	1.05	1.42	-0.05	0.00
time (sec)	N/A	0.038	0.013	0.315	0.958	1.138	11.288	1.370	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	32	47	25	44	20	27	-1	0
N.S.	1	1.00	1.88	2.76	1.47	2.59	1.18	1.59	-0.06	0.00
time (sec)	N/A	0.035	0.048	0.309	0.990	1.187	11.910	1.299	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	45	63	48	61	0	54	-1	0
N.S.	1	1.00	0.73	1.02	0.77	0.98	0.00	0.87	-0.02	0.00
time (sec)	N/A	0.039	0.097	0.316	0.961	1.162	0.000	1.376	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	40	61	45	57	37	40	-1	0
N.S.	1	1.00	1.11	1.69	1.25	1.58	1.03	1.11	-0.03	0.00
time (sec)	N/A	0.069	0.070	0.529	0.969	1.102	18.477	1.346	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	62	62	112	97	0	0	0	0	-1	0
N.S.	1	1.00	1.81	1.56	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.116	0.191	0.636	0.000	0.965	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	38	43	42	36	60	95	-1	0
N.S.	1	1.00	0.70	0.80	0.78	0.67	1.11	1.76	-0.02	0.00
time (sec)	N/A	0.091	0.049	0.312	0.954	1.084	139.932	1.006	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	50	158	52	41	78	53	-1	0
N.S.	1	1.00	0.76	2.39	0.79	0.62	1.18	0.80	-0.02	0.00
time (sec)	N/A	0.072	0.059	0.325	0.963	1.154	3.039	1.047	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	60	69	0	49	66	60	-1	0
N.S.	1	1.00	0.82	0.95	0.00	0.67	0.90	0.82	-0.01	0.00
time (sec)	N/A	0.155	0.025	0.325	0.000	0.953	1.315	0.973	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	21	27	26	19	31	26	21	0
N.S.	1	1.00	0.66	0.84	0.81	0.59	0.97	0.81	0.66	0.00
time (sec)	N/A	0.026	0.021	0.358	0.957	1.097	0.638	1.065	0.078	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	37	39	38	88	34	26	0
N.S.	1	1.00	0.82	0.84	0.89	0.86	2.00	0.77	0.59	0.00
time (sec)	N/A	0.029	0.018	0.363	0.957	1.095	1.073	0.886	0.319	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	24	19	19	19	19	0
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83	0.00
time (sec)	N/A	0.048	0.017	0.293	0.999	1.215	0.384	0.901	0.309	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	57	113	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.69	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.094	0.025	0.329	0.000	1.017	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	28	29	40	26	0	0	23	0
N.S.	1	1.00	0.82	0.85	1.18	0.76	0.00	0.00	0.68	0.00
time (sec)	N/A	0.047	0.028	0.391	0.976	0.735	0.000	0.000	0.065	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	64	139	0	0	0	0	-1	0
N.S.	1	1.00	0.81	1.76	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.113	0.079	0.349	0.000	0.836	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	89	70	149	0	0	0	0	-1	0
N.S.	1	1.00	0.79	1.67	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.234	0.225	0.344	0.000	1.008	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	23	21	26	19	25	22	0
N.S.	1	1.00	1.00	1.05	0.95	1.18	0.86	1.14	1.00	0.00
time (sec)	N/A	0.034	0.007	0.127	0.949	1.189	0.373	0.991	0.066	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	59	30	31	26	34	31	30	0
N.S.	1	1.00	1.90	0.97	1.00	0.84	1.10	1.00	0.97	0.00
time (sec)	N/A	0.023	0.006	0.084	0.947	1.019	0.836	0.946	0.322	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	F	F	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	63	81	79	71	0	0	0	53	0
N.S.	1	1.00	1.29	1.25	1.13	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.084	0.008	0.357	1.116	0.948	0.000	0.000	0.498	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	27	29	22	0	24	0
N.S.	1	1.00	1.00	0.89	0.96	1.04	0.79	0.00	0.86	0.00
time (sec)	N/A	0.057	0.007	0.402	0.992	0.928	0.573	0.000	0.089	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	38	34	36	38	32	0	31	0
N.S.	1	1.00	0.97	0.87	0.92	0.97	0.82	0.00	0.79	0.00
time (sec)	N/A	0.070	0.025	0.108	0.963	1.041	0.530	0.000	0.074	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	57	71	54	61	0	51	0
N.S.	1	1.00	0.93	0.95	1.18	0.90	1.02	0.00	0.85	0.00
time (sec)	N/A	0.078	0.026	0.151	0.972	0.997	0.897	0.000	0.120	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	82	47	78	94	51	0	0	56	0
N.S.	1	1.04	0.59	0.99	1.19	0.65	0.00	0.00	0.71	0.00
time (sec)	N/A	0.133	0.056	0.420	1.037	0.938	0.000	0.000	0.360	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	107	116	116	708	0	0	0	0	-1	0
N.S.	1	1.08	1.08	6.62	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.154	0.202	0.739	0.000	0.846	0.000	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	133	86	305	0	51	0	0	-1	0
N.S.	1	1.25	0.81	2.88	0.00	0.48	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.196	0.308	0.594	0.000	0.957	0.000	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	48	329	27	23	0	75	-1	0
N.S.	1	1.00	1.17	8.02	0.66	0.56	0.00	1.83	-0.02	0.00
time (sec)	N/A	0.052	0.061	0.778	0.992	1.141	0.000	1.163	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	67	67	128	48	75	0	58	-1	0
N.S.	1	1.03	1.03	1.97	0.74	1.15	0.00	0.89	-0.02	0.00
time (sec)	N/A	0.031	0.138	0.574	0.643	0.932	0.000	0.970	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	53	61	121	46	68	0	53	-1	0
N.S.	1	1.04	1.20	2.37	0.90	1.33	0.00	1.04	-0.02	0.00
time (sec)	N/A	0.064	0.122	0.619	0.645	1.192	0.000	1.129	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	84	72	197	0	69	0	64	-1	0
N.S.	1	1.02	0.88	2.40	0.00	0.84	0.00	0.78	-0.01	0.00
time (sec)	N/A	0.090	0.215	0.839	0.000	1.180	0.000	1.246	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	175	232	383	240	0	0	0	0	-1	0
N.S.	1	1.33	2.19	1.37	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.317	1.989	0.892	0.000	0.902	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	35	178	17	16	0	50	-1	0
N.S.	1	1.00	1.52	7.74	0.74	0.70	0.00	2.17	-0.04	0.00
time (sec)	N/A	0.048	0.046	0.509	1.027	1.172	0.000	1.040	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	F(-1)	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	91	79	702	123	81	0	105	-1	0
N.S.	1	1.30	1.13	10.03	1.76	1.16	0.00	1.50	-0.01	0.00
time (sec)	N/A	0.086	0.166	0.834	2.207	1.085	0.000	1.186	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	101	76	330	58	37	0	0	-1	0
N.S.	1	1.36	1.03	4.46	0.78	0.50	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.179	0.080	0.665	1.030	1.187	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	172	84	386	0	59	0	0	-1	0
N.S.	1	1.29	0.63	2.90	0.00	0.44	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.201	0.313	0.702	0.000	1.118	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	146	92	536	93	57	0	0	-1	0
N.S.	1	1.33	0.84	4.87	0.85	0.52	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.200	0.109	0.844	2.185	1.063	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	118	99	87	103	51	0	0	-1	0
N.S.	1	2.15	1.80	1.58	1.87	0.93	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.843	0.170	0.057	1.008	1.141	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	71	66	89	58	0	49	36	0
N.S.	1	1.00	1.78	1.65	2.22	1.45	0.00	1.22	0.90	0.00
time (sec)	N/A	0.043	0.061	0.056	0.982	1.123	0.000	1.191	0.364	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	35	32	31	50	153	32	31	0
N.S.	1	1.00	0.90	0.82	0.79	1.28	3.92	0.82	0.79	0.00
time (sec)	N/A	0.029	0.047	0.358	0.958	1.058	0.637	0.927	0.359	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	122	122	105	102	118	0	0	0	-1	0
N.S.	1	1.00	0.86	0.84	0.97	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.091	0.034	0.374	1.200	0.996	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	0	0	14	22	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.50	0.79	0.00	-0.04	0.00
time (sec)	N/A	0.033	0.030	0.080	0.000	1.097	2.564	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	25	25	26	25	25	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81	0.00
time (sec)	N/A	0.038	0.040	0.310	0.473	1.159	2.416	1.004	0.571	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	61	70	0	90	0	58	-1	0
N.S.	1	1.00	1.07	1.23	0.00	1.58	0.00	1.02	-0.02	0.00
time (sec)	N/A	0.032	0.150	0.319	0.000	1.143	0.000	0.972	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	140	72	76	116	125	0	228	-1	0
N.S.	1	1.71	0.88	0.93	1.41	1.52	0.00	2.78	-0.01	0.00
time (sec)	N/A	0.078	0.113	0.326	3.439	1.160	0.000	1.747	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	66	0	0	519	0	218	-1	0
N.S.	1	1.00	1.35	0.00	0.00	10.59	0.00	4.45	-0.02	0.00
time (sec)	N/A	0.140	0.271	0.048	0.000	1.147	0.000	1.336	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	40	854	54	423	0	70	103	0
N.S.	1	1.00	1.11	23.72	1.50	11.75	0.00	1.94	2.86	0.00
time (sec)	N/A	0.120	0.226	0.642	1.010	0.696	0.000	1.130	0.536	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	28	28	64	0	16	26	0	29	-1	0
N.S.	1	1.00	2.29	0.00	0.57	0.93	0.00	1.04	-0.04	0.00
time (sec)	N/A	0.077	0.972	0.037	1.001	1.042	0.000	1.021	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [397] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	14	0.071
2	A	1	1	1.00	13	0.077
3	A	1	1	1.00	5	0.200
4	A	2	2	1.00	10	0.200
5	A	1	1	1.00	12	0.083
6	A	2	2	1.00	5	0.400
7	A	2	2	1.00	5	0.400
8	A	2	1	1.00	7	0.143
9	A	1	1	1.00	6	0.167
10	A	1	1	1.00	8	0.125
11	A	2	2	1.00	12	0.167
12	A	2	2	1.00	17	0.118
13	A	2	2	1.00	18	0.111
14	A	3	2	1.00	19	0.105
15	A	3	2	1.00	20	0.100
16	A	3	2	1.22	19	0.105
17	A	3	2	1.22	20	0.100
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	13	0.154
20	A	2	2	1.00	8	0.250
21	A	2	1	1.00	12	0.083
22	A	2	2	1.00	12	0.167
23	A	2	2	1.00	14	0.143
24	A	3	2	1.00	16	0.125
25	A	4	2	1.00	22	0.091
26	A	2	1	1.00	13	0.077
27	A	3	2	1.00	15	0.133
28	A	3	3	1.00	15	0.200
29	A	1	1	1.00	9	0.111
30	A	1	1	1.00	9	0.111
31	A	4	3	1.00	10	0.300
32	A	2	2	1.00	4	0.500
33	A	2	2	1.00	4	0.500
34	A	2	2	1.00	7	0.286
35	A	2	1	1.00	7	0.143
36	A	3	2	1.00	9	0.222
37	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
38	A	2	2	1.00	8	0.250
39	A	4	2	1.00	9	0.222
40	A	4	4	1.00	9	0.444
41	A	2	2	1.00	9	0.222
42	A	2	2	1.00	11	0.182
43	A	3	2	1.00	19	0.105
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	16	0.188
46	A	3	2	1.00	14	0.143
47	A	4	3	1.00	11	0.273
48	A	2	2	1.00	13	0.154
49	A	3	2	1.00	13	0.154
50	A	3	3	1.00	15	0.200
51	A	3	3	1.00	17	0.176
52	A	3	3	1.00	17	0.176
53	A	3	3	1.00	15	0.200
54	A	2	2	1.00	12	0.167
55	A	2	2	1.00	14	0.143
56	A	2	2	1.00	11	0.182
57	A	3	3	1.00	18	0.167
58	A	2	2	1.00	16	0.125
59	A	1	1	1.10	18	0.056
60	A	7	5	1.00	17	0.294
61	A	2	2	1.30	10	0.200
62	A	3	1	1.00	11	0.091
63	A	2	1	1.00	17	0.059
64	A	3	2	1.06	19	0.105
65	A	3	2	1.00	19	0.105
66	A	3	3	1.00	11	0.273
67	A	3	3	1.00	17	0.176
68	A	1	1	1.00	12	0.083
69	A	2	2	1.00	24	0.083
70	A	1	1	1.00	16	0.062
71	A	2	2	1.00	6	0.333
72	A	1	1	1.00	6	0.167
73	A	5	4	1.06	12	0.333
74	A	2	1	1.00	4	0.250
75	A	4	3	1.00	9	0.333
76	A	3	2	1.00	4	0.500
77	A	1	1	1.00	8	0.125
78	A	1	1	1.00	10	0.100
79	A	1	1	1.00	6	0.167
80	A	1	1	1.00	3	0.333
81	A	3	4	1.00	8	0.500
82	A	3	3	1.00	6	0.500
83	A	4	4	1.00	6	0.667
84	A	3	3	1.00	4	0.750
85	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	A	6	6	1.50	12	0.500
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	16	0.062
89	A	1	1	1.00	15	0.067
90	A	2	1	1.00	11	0.091
91	A	3	2	1.00	13	0.154
92	A	3	2	1.00	12	0.167
93	A	4	4	1.00	16	0.250
94	A	5	5	1.00	14	0.357
95	A	7	6	1.00	29	0.207
96	A	3	2	1.00	19	0.105
97	A	2	1	1.00	39	0.026
98	A	12	5	1.65	21	0.238
99	A	3	2	1.00	20	0.100
100	A	2	1	1.00	21	0.048
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	9	0.111
103	A	2	1	1.00	25	0.040
104	A	2	1	1.00	24	0.042
105	A	3	2	1.00	19	0.105
106	A	2	1	1.00	19	0.053
107	A	5	4	1.00	19	0.210
108	A	5	4	1.00	16	0.250
109	A	3	3	1.00	14	0.214
110	A	6	5	1.00	33	0.152
111	A	5	4	1.00	21	0.190
112	A	5	4	1.00	21	0.190
113	A	9	5	1.00	10	0.500
114	A	10	9	1.00	17	0.529
115	A	6	5	1.00	26	0.192
116	A	6	2	1.00	29	0.069
117	A	3	2	1.00	30	0.067
118	A	6	6	1.00	9	0.667
119	A	6	6	1.00	11	0.546
120	A	1	1	1.00	13	0.077
121	A	4	4	1.00	13	0.308
122	A	7	7	1.00	13	0.538
123	A	7	7	1.00	13	0.538
124	A	3	2	1.00	13	0.154
125	A	8	7	1.00	13	0.538
126	A	1	1	1.00	15	0.067
127	A	3	3	1.00	11	0.273
128	A	2	2	1.00	13	0.154
129	A	4	4	1.00	15	0.267
130	A	4	4	1.00	15	0.267
131	A	3	3	1.00	15	0.200
132	A	4	4	1.00	15	0.267
133	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	2	2	1.00	11	0.182
135	A	9	6	1.00	13	0.462
136	A	6	6	1.00	9	0.667
137	A	6	6	1.00	11	0.546
138	A	6	6	1.00	13	0.462
139	A	6	6	1.00	13	0.462
140	A	1	1	1.00	13	0.077
141	A	4	4	1.00	13	0.308
142	A	7	7	1.00	13	0.538
143	A	7	7	1.00	13	0.538
144	A	7	7	1.00	13	0.538
145	A	1	1	1.00	15	0.067
146	A	22	8	1.00	13	0.615
147	A	4	3	1.00	10	0.300
148	A	4	4	1.00	16	0.250
149	A	3	3	1.00	19	0.158
150	A	5	5	1.00	26	0.192
151	A	7	7	1.00	7	1.000
152	A	4	4	1.00	18	0.222
153	A	7	7	1.00	9	0.778
154	A	5	4	1.54	18	0.222
155	A	5	5	1.00	18	0.278
156	A	5	4	1.00	16	0.250
157	A	4	3	1.00	16	0.188
158	A	2	1	1.00	16	0.062
159	A	2	1	1.00	9	0.111
160	A	3	2	1.00	15	0.133
161	A	3	2	1.00	11	0.182
162	A	2	1	1.00	11	0.091
163	A	5	3	1.00	10	0.300
164	A	4	3	1.00	10	0.300
165	A	2	1	1.00	11	0.091
166	A	4	3	1.17	11	0.273
167	A	5	3	1.00	11	0.273
168	A	3	3	1.00	18	0.167
169	A	3	3	1.00	18	0.167
170	A	4	4	1.00	11	0.364
171	A	3	3	1.22	21	0.143
172	A	1	1	1.00	13	0.077
173	A	3	2	1.00	13	0.154
174	A	12	8	1.00	13	0.615
175	A	5	4	1.00	13	0.308
176	A	3	2	1.00	13	0.154
177	A	8	8	1.00	13	0.615
178	A	5	4	1.00	16	0.250
179	A	4	4	1.00	18	0.222
180	A	2	1	1.00	44	0.023
181	A	7	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	7	6	1.00	22	0.273
183	A	2	1	1.00	15	0.067
184	A	3	2	1.00	13	0.154
185	A	3	2	1.00	13	0.154
186	A	2	1	1.14	11	0.091
187	A	2	1	1.00	11	0.091
188	A	2	1	1.00	9	0.111
189	A	2	1	1.00	17	0.059
190	A	2	1	1.00	19	0.053
191	A	2	1	1.00	19	0.053
192	A	2	1	1.00	19	0.053
193	A	2	2	1.00	19	0.105
194	A	4	4	1.00	19	0.210
195	A	3	3	1.00	19	0.158
196	A	4	4	1.00	19	0.210
197	A	5	4	1.00	19	0.210
198	A	2	2	1.00	21	0.095
199	A	3	2	1.00	14	0.143
200	A	4	4	1.00	18	0.222
201	A	4	3	1.00	14	0.214
202	A	7	3	1.00	10	0.300
203	A	7	6	1.00	16	0.375
204	A	8	5	1.00	14	0.357
205	A	7	5	1.00	20	0.250
206	A	3	2	1.00	14	0.143
207	A	6	4	1.00	13	0.308
208	A	9	4	1.00	24	0.167
209	A	5	3	1.00	33	0.091
210	A	4	3	1.00	11	0.273
211	A	2	1	1.00	13	0.077
212	A	4	3	1.00	21	0.143
213	A	5	4	1.00	19	0.210
214	A	3	2	1.00	17	0.118
215	A	5	3	1.02	11	0.273
216	A	9	3	1.08	13	0.231
217	A	8	5	1.00	11	0.454
218	A	3	3	1.00	15	0.200
219	A	4	4	1.03	19	0.210
220	A	6	6	1.00	27	0.222
221	A	33	16	1.05	52	0.308
222	A	46	21	1.79	56	0.375
223	A	2	2	1.16	15	0.133
224	A	2	2	1.20	15	0.133
225	A	3	3	1.19	15	0.200
226	B	3	3	2.81	13	0.231
227	A	9	9	1.09	19	0.474
228	B	6	6	2.69	17	0.353
229	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
230	A	4	4	1.00	17	0.235
231	A	7	5	1.00	17	0.294
232	B	3	3	2.51	17	0.176
233	A	3	3	1.00	17	0.176
234	A	5	4	1.00	17	0.235
235	A	2	2	1.00	14	0.143
236	A	2	2	1.00	14	0.143
237	A	2	2	1.00	14	0.143
238	A	2	2	1.00	19	0.105
239	A	2	2	1.00	19	0.105
240	A	3	3	1.00	22	0.136
241	A	3	3	1.00	22	0.136
242	A	5	4	1.00	17	0.235
243	A	5	5	1.00	21	0.238
244	A	9	7	1.00	20	0.350
245	A	5	5	1.00	24	0.208
246	A	5	4	1.00	21	0.190
247	A	5	4	1.00	30	0.133
248	A	5	4	1.11	32	0.125
249	A	2	2	1.00	30	0.067
250	A	4	3	1.00	15	0.200
251	A	3	2	1.00	13	0.154
252	A	3	2	1.00	11	0.182
253	A	3	2	1.00	20	0.100
254	A	2	2	1.00	18	0.111
255	A	4	4	1.00	17	0.235
256	A	3	3	1.00	17	0.176
257	A	5	5	1.00	20	0.250
258	A	4	4	1.00	24	0.167
259	A	15	9	1.00	33	0.273
260	A	32	14	1.00	44	0.318
261	A	4	4	1.00	16	0.250
262	A	5	5	1.00	18	0.278
263	A	6	5	1.00	18	0.278
264	A	5	5	1.00	19	0.263
265	A	4	4	1.00	24	0.167
266	A	2	2	1.00	10	0.200
267	A	4	4	1.00	14	0.286
268	A	1	1	1.00	10	0.100
269	A	1	1	1.00	12	0.083
270	A	4	4	1.00	14	0.286
271	A	5	5	1.00	14	0.357
272	A	4	3	1.00	10	0.300
273	A	5	3	1.00	10	0.300
274	A	3	3	1.00	14	0.214
275	A	4	4	1.00	14	0.286
276	A	4	4	1.00	14	0.286
277	A	5	5	1.00	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
278	A	2	2	1.00	16	0.125
279	A	10	7	1.00	22	0.318
280	A	6	6	1.00	18	0.333
281	A	6	6	1.00	28	0.214
282	A	4	4	1.00	34	0.118
283	A	14	10	1.00	24	0.417
284	A	3	2	1.00	12	0.167
285	A	2	2	1.00	14	0.143
286	A	2	2	1.00	14	0.143
287	A	5	4	1.00	16	0.250
288	A	3	2	1.31	14	0.143
289	A	7	6	1.00	23	0.261
290	A	26	8	1.00	29	0.276
291	A	36	8	1.00	31	0.258
292	A	4	3	1.00	11	0.273
293	A	1	1	1.00	15	0.067
294	A	6	6	1.00	13	0.462
295	A	2	1	1.00	13	0.077
296	A	6	6	1.00	17	0.353
297	A	3	2	1.00	15	0.133
298	A	13	9	1.00	17	0.529
299	A	1	1	1.00	13	0.077
300	A	1	1	1.00	13	0.077
301	A	5	5	1.00	15	0.333
302	A	6	6	1.00	13	0.462
303	A	5	5	1.00	15	0.333
304	A	6	5	1.00	15	0.333
305	A	6	6	1.00	15	0.400
306	B	12	12	2.15	13	0.923
307	A	8	6	0.90	13	0.462
308	A	3	3	1.00	22	0.136
309	A	5	5	1.00	16	0.312
310	A	5	5	1.00	18	0.278
311	A	3	3	1.00	23	0.130
312	A	1	1	1.00	23	0.043
313	A	9	4	1.00	39	0.103
314	A	7	7	1.37	17	0.412
315	A	10	7	1.00	17	0.412
316	A	2	2	1.00	15	0.133
317	A	5	5	1.00	17	0.294
318	A	3	2	1.23	17	0.118
319	A	9	9	1.37	32	0.281
320	A	2	2	1.00	24	0.083
321	A	2	2	1.00	24	0.083
322	A	6	6	1.44	18	0.333
323	A	6	6	1.44	18	0.333
324	A	2	2	1.00	27	0.074
325	A	2	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
326	A	1	1	1.00	21	0.048
327	A	1	1	1.00	44	0.023
328	A	2	2	1.00	27	0.074
329	A	2	2	1.00	31	0.065
330	A	2	2	1.00	4	0.500
331	A	2	1	1.00	4	0.250
332	A	3	2	1.00	4	0.500
333	A	4	2	1.00	4	0.500
334	A	5	2	1.00	4	0.500
335	B	3	2	3.20	14	0.143
336	A	2	1	1.00	14	0.071
337	A	2	1	1.00	4	0.250
338	A	4	2	1.00	4	0.500
339	A	2	1	1.00	4	0.250
340	A	2	2	1.00	12	0.167
341	A	4	2	1.00	4	0.500
342	A	3	2	1.00	4	0.500
343	A	3	2	1.00	14	0.143
344	A	6	3	1.00	9	0.333
345	A	3	2	1.00	9	0.222
346	A	4	3	1.00	7	0.429
347	A	3	2	1.00	9	0.222
348	A	3	3	1.00	9	0.333
349	A	5	3	1.00	9	0.333
350	A	7	3	1.00	9	0.333
351	A	9	3	1.00	9	0.333
352	A	1	1	1.00	13	0.077
353	A	3	2	1.00	23	0.087
354	A	2	2	1.00	9	0.222
355	A	2	1	1.00	7	0.143
356	A	2	2	1.00	7	0.286
357	A	3	3	1.00	9	0.333
358	A	3	2	1.00	9	0.222
359	A	3	2	1.00	11	0.182
360	A	3	2	1.00	11	0.182
361	A	4	3	1.00	9	0.333
362	A	3	3	1.00	29	0.103
363	A	8	5	0.95	22	0.227
364	A	15	7	1.00	17	0.412
365	A	4	3	1.00	17	0.176
366	A	4	2	1.00	9	0.222
367	A	4	3	1.00	7	0.429
368	A	1	1	1.00	7	0.143
369	A	4	4	1.00	9	0.444
370	A	6	2	1.00	9	0.222
371	A	4	3	1.00	9	0.333
372	A	3	1	1.00	9	0.111
373	A	7	6	1.02	11	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	5	5	1.55	9	0.556
375	A	5	4	1.00	16	0.250
376	A	3	3	1.57	14	0.214
377	B	3	3	2.85	12	0.250
378	A	2	1	1.00	16	0.062
379	A	6	4	1.00	10	0.400
380	A	4	3	1.00	9	0.333
381	A	3	2	1.00	9	0.222
382	A	4	4	1.00	21	0.190
383	A	2	1	1.00	9	0.111
384	A	2	2	1.00	7	0.286
385	A	4	3	1.00	9	0.333
386	A	4	3	1.00	9	0.333
387	A	5	5	1.00	7	0.714
388	A	4	2	1.00	7	0.286
389	A	4	2	1.00	9	0.222
390	A	1	1	1.00	10	0.100
391	A	1	1	1.00	12	0.083
392	A	2	2	1.00	10	0.200
393	A	2	2	1.00	12	0.167
394	A	3	3	1.00	12	0.250
395	A	3	2	1.00	14	0.143
396	A	4	2	1.00	32	0.062
397	A	11	8	1.00	6	1.333
398	A	9	9	1.21	8	1.125
399	A	6	5	1.00	12	0.417
400	A	4	2	1.00	32	0.062
401	A	19	13	1.58	13	1.000
402	A	1	1	1.00	11	0.091
403	A	1	1	1.00	11	0.091
404	A	2	2	1.00	11	0.182
405	A	6	5	1.00	16	0.312
406	A	4	3	1.00	13	0.231
407	A	4	3	1.00	13	0.231
408	A	1	1	1.00	13	0.077
409	A	2	2	1.00	13	0.154
410	A	3	3	1.00	11	0.273
411	A	6	4	1.40	35	0.114
412	A	5	4	1.53	11	0.364
413	B	13	9	2.22	11	0.818
414	A	4	2	1.00	13	0.154
415	A	4	2	1.00	13	0.154
416	B	27	11	2.17	27	0.407
417	A	66	21	1.83	41	0.512
418	A	13	4	1.13	28	0.143
419	A	5	3	0.92	15	0.200
420	A	5	3	1.00	18	0.167
421	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
422	A	4	3	1.00	20	0.150
423	A	3	3	1.00	19	0.158
424	A	4	3	1.00	22	0.136
425	A	4	3	1.00	23	0.130
426	A	18	13	1.07	33	0.394
427	A	27	6	1.00	39	0.154
428	A	5	3	1.00	17	0.176
429	A	3	3	1.00	11	0.273
430	A	5	4	1.00	11	0.364
431	A	1	1	1.00	11	0.091
432	A	6	6	1.00	13	0.462
433	A	11	9	1.05	28	0.321
434	A	7	6	1.00	12	0.500
435	A	4	4	1.00	12	0.333
436	A	10	10	1.00	22	0.454
437	A	5	4	1.00	23	0.174
438	A	16	12	1.00	31	0.387
439	A	10	7	1.00	48	0.146
440	A	7	5	1.00	15	0.333
441	A	6	5	1.00	15	0.333
442	A	6	6	1.00	19	0.316
443	A	7	7	1.00	15	0.467
444	A	6	6	1.00	19	0.316
445	A	6	6	1.00	20	0.300
446	A	29	16	1.31	61	0.262
447	B	21	10	2.08	29	0.345
448	A	7	7	1.00	20	0.350
449	A	14	10	1.71	15	0.667
450	A	4	3	1.00	19	0.158
451	A	2	2	1.00	33	0.061
452	A	15	9	1.30	31	0.290
453	A	14	10	1.25	52	0.192
454	A	4	3	1.00	15	0.200
455	A	14	10	1.51	15	0.667
456	A	3	3	1.06	11	0.273
457	A	6	5	1.00	11	0.454
458	A	3	2	1.00	11	0.182
459	A	4	2	1.00	11	0.182
460	A	3	2	1.00	11	0.182
461	A	5	4	1.00	13	0.308
462	A	6	4	1.00	13	0.308
463	A	1	1	1.00	7	0.143
464	A	3	2	1.00	11	0.182
465	A	4	3	1.00	19	0.158
466	A	3	2	1.00	13	0.154
467	A	3	2	1.00	13	0.154
468	A	2	2	1.00	11	0.182
469	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	3	2	1.00	13	0.154
471	A	2	1	1.00	13	0.077
472	A	3	2	1.00	11	0.182
473	A	3	3	1.00	23	0.130
474	A	2	2	1.00	19	0.105
475	A	2	2	1.00	15	0.133
476	A	3	2	1.00	15	0.133
477	A	3	2	1.00	11	0.182
478	A	2	2	1.00	14	0.143
479	A	2	1	1.00	12	0.083
480	A	2	1	1.00	16	0.062
481	A	2	2	1.00	20	0.100
482	A	2	2	1.00	25	0.080
483	A	9	4	1.00	8	0.500
484	A	8	4	1.00	8	0.500
485	A	13	4	1.00	8	0.500
486	A	4	4	1.00	10	0.400
487	A	6	5	1.00	10	0.500
488	A	5	4	1.00	8	0.500
489	A	3	3	1.00	8	0.375
490	A	8	8	1.00	8	1.000
491	A	7	7	1.00	6	1.167
492	A	2	2	1.00	18	0.111
493	A	3	3	1.00	15	0.200
494	A	2	2	1.00	11	0.182
495	A	9	4	1.21	22	0.182
496	A	3	1	1.00	11	0.091
497	A	4	3	1.00	13	0.231
498	A	3	2	1.00	13	0.154
499	A	4	3	1.00	13	0.231
500	A	4	4	1.08	13	0.308
501	A	4	3	1.00	15	0.200
502	A	3	1	1.00	11	0.091
503	A	6	2	1.00	13	0.154
504	A	7	2	1.00	13	0.154
505	A	8	2	1.00	13	0.154
506	A	2	2	1.11	13	0.154
507	A	3	1	1.00	13	0.077
508	A	6	2	1.00	15	0.133
509	A	7	2	1.00	15	0.133
510	A	8	2	1.00	15	0.133
511	A	2	2	1.11	15	0.133
512	A	2	1	1.00	7	0.143
513	A	3	2	1.00	9	0.222
514	A	3	2	1.00	9	0.222
515	A	3	2	1.00	9	0.222
516	A	2	2	1.00	9	0.222
517	A	2	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
518	A	3	2	1.00	11	0.182
519	A	3	2	1.00	11	0.182
520	A	3	2	1.00	11	0.182
521	A	2	2	1.00	11	0.182
522	A	4	4	1.00	11	0.364
523	A	7	7	1.23	15	0.467
524	A	3	2	1.00	13	0.154
525	A	5	5	1.00	24	0.208
526	A	6	5	1.00	29	0.172
527	A	2	2	1.00	21	0.095
528	A	6	6	1.00	15	0.400
529	A	2	2	1.00	15	0.133
530	A	3	3	1.00	17	0.176
531	A	3	3	1.00	19	0.158
532	A	3	3	1.00	39	0.077
533	A	4	4	1.54	17	0.235
534	A	3	2	1.00	21	0.095
535	A	4	2	1.00	11	0.182
536	A	3	2	1.00	11	0.182
537	A	3	2	1.00	9	0.222
538	A	5	3	1.00	12	0.250
539	A	6	6	1.00	13	0.462
540	A	1	1	1.00	25	0.040
541	A	1	1	1.00	10	0.100
542	A	6	4	1.00	21	0.190
543	A	2	2	1.00	16	0.125
544	A	2	2	1.00	10	0.200
545	A	2	2	1.00	10	0.200
546	A	3	3	1.00	16	0.188
547	A	4	3	1.00	14	0.214
548	A	4	3	1.00	22	0.136
549	A	5	3	1.47	10	0.300
550	A	1	1	1.00	10	0.100
551	A	2	2	1.51	10	0.200
552	A	2	2	1.00	10	0.200
553	A	2	2	1.00	12	0.167
554	A	2	2	1.00	10	0.200
555	A	2	2	1.00	12	0.167
556	A	1	1	1.00	18	0.056
557	A	7	6	1.10	16	0.375
558	A	1	1	1.00	14	0.071
559	A	7	6	1.05	16	0.375
560	A	7	6	1.04	18	0.333
561	A	1	1	1.00	16	0.062
562	A	7	6	1.09	14	0.429
563	A	1	1	1.00	16	0.062
564	A	4	3	1.00	7	0.429
565	A	11	5	1.00	9	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	11	5	1.00	11	0.454
567	A	31	8	1.35	15	0.533
568	A	11	5	1.00	13	0.385
569	A	24	6	1.00	17	0.353
570	A	1	1	1.00	2	0.500
571	A	1	1	1.00	2	0.500
572	A	1	1	1.00	2	0.500
573	A	1	1	1.00	2	0.500
574	A	1	1	1.00	2	0.500
575	A	1	1	1.00	2	0.500
576	A	2	2	1.00	4	0.500
577	A	2	1	1.00	4	0.250
578	A	3	2	1.00	4	0.500
579	A	2	2	1.00	4	0.500
580	A	3	2	1.00	4	0.500
581	A	4	3	1.00	7	0.429
582	A	3	2	1.00	11	0.182
583	A	2	2	1.02	8	0.250
584	A	2	2	1.00	6	0.333
585	A	2	2	1.00	8	0.250
586	A	2	2	1.00	14	0.143
587	A	2	2	1.00	15	0.133
588	A	3	3	1.00	10	0.300
589	A	5	3	1.00	23	0.130
590	A	5	2	1.00	11	0.182
591	A	5	2	1.00	15	0.133
592	A	8	4	1.48	31	0.129
593	A	3	3	1.00	15	0.200
594	A	5	3	1.00	21	0.143
595	A	2	2	1.00	11	0.182
596	A	3	3	1.00	6	0.500
597	A	3	3	1.00	6	0.500
598	A	13	7	1.00	16	0.438
599	A	8	4	1.00	13	0.308
600	A	3	3	1.00	10	0.300
601	A	3	3	1.00	10	0.300
602	A	2	2	1.00	13	0.154
603	A	3	3	1.46	13	0.231
604	A	2	2	1.00	11	0.182
605	A	4	3	1.00	12	0.250
606	A	3	2	1.00	14	0.143
607	A	3	2	1.00	18	0.111
608	A	1	1	1.00	6	0.167
609	A	2	2	1.00	8	0.250
610	A	2	2	1.00	10	0.200
611	A	2	1	1.04	8	0.125
612	A	4	4	1.00	10	0.400
613	A	6	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
614	A	13	8	1.22	16	0.500
615	A	5	3	1.00	8	0.375
616	A	2	2	1.00	10	0.200
617	A	2	2	1.00	10	0.200
618	A	2	2	1.00	8	0.250
619	A	2	2	1.00	12	0.167
620	A	2	2	1.00	12	0.167
621	A	2	2	1.00	14	0.143
622	A	3	2	1.00	16	0.125
623	A	3	2	1.00	18	0.111
624	A	3	2	1.00	18	0.111
625	A	4	3	1.00	20	0.150
626	A	4	3	1.00	22	0.136
627	A	4	3	1.00	22	0.136
628	A	1	1	1.00	7	0.143
629	A	3	2	1.00	9	0.222
630	A	4	2	1.00	9	0.222
631	A	5	2	1.00	9	0.222
632	A	3	2	1.00	9	0.222
633	A	3	3	1.00	8	0.375
634	A	3	1	1.00	8	0.125
635	A	2	2	1.00	6	0.333
636	A	2	3	1.00	6	0.500
637	A	2	2	1.00	14	0.143
638	A	3	2	1.00	8	0.250
639	A	8	6	1.00	20	0.300
640	A	6	6	1.00	14	0.429
641	A	4	4	1.00	8	0.500
642	A	4	3	1.00	8	0.375
643	A	4	5	1.00	14	0.357
644	A	6	7	1.00	12	0.583
645	A	5	5	1.00	8	0.625
646	A	5	5	1.00	8	0.625
647	A	10	7	1.00	8	0.875
648	A	13	8	1.00	8	1.000
649	A	5	5	1.00	8	0.625
650	A	10	5	1.00	8	0.625
651	A	3	3	1.00	14	0.214
652	A	3	3	1.00	14	0.214
653	A	2	1	1.00	15	0.067
654	A	6	5	1.00	14	0.357
655	A	3	2	1.00	15	0.133
656	A	4	5	1.00	17	0.294
657	A	10	7	1.00	17	0.412
658	A	4	3	1.00	17	0.176
659	A	3	3	1.00	17	0.176
660	A	5	3	1.00	17	0.176
661	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	2	2	1.00	15	0.133
663	A	4	4	1.00	14	0.286
664	A	3	5	1.00	17	0.294
665	A	8	6	1.00	17	0.353
666	A	4	4	1.00	17	0.235
667	A	5	4	1.00	17	0.235
668	A	6	4	1.00	19	0.210
669	A	3	3	1.00	11	0.273
670	A	4	3	1.00	11	0.273
671	A	4	4	1.00	13	0.308
672	A	8	8	1.00	13	0.615
673	A	2	2	1.00	13	0.154
674	A	8	8	1.00	13	0.615
675	A	17	11	1.00	13	0.846
676	A	8	8	1.00	11	0.727
677	A	3	2	1.00	11	0.182
678	A	12	6	1.00	13	0.462
679	A	7	7	1.00	13	0.538
680	A	8	7	1.00	8	0.875
681	A	11	8	1.00	13	0.615
682	A	4	4	1.04	15	0.267
683	A	9	7	1.08	15	0.467
684	A	11	8	1.25	15	0.533
685	A	4	4	1.00	15	0.267
686	A	4	6	1.03	12	0.500
687	A	4	5	1.04	15	0.333
688	A	5	6	1.02	15	0.400
689	A	16	11	1.33	15	0.733
690	A	2	3	1.00	15	0.200
691	A	5	8	1.30	15	0.533
692	A	6	4	1.36	17	0.235
693	A	11	9	1.29	17	0.529
694	A	8	5	1.33	17	0.294
695	B	8	7	2.15	16	0.438
696	A	4	4	1.00	16	0.250
697	A	5	4	1.00	8	0.500
698	A	5	5	1.00	13	0.385
699	A	2	2	1.00	24	0.083
700	A	2	3	1.00	21	0.143
701	A	5	5	1.00	12	0.417
702	A	7	6	1.71	10	0.600
703	A	5	6	1.00	8	0.750
704	A	6	6	1.00	10	0.600
705	A	5	6	1.00	7	0.857

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2-b^2x^2}\,dx = \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

fricas [A] time = 1.17, size = 25, normalized size = 1.79

$$\frac{\log(bx+a)-\log(bx-a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

giac [B] time = 1.01, size = 33, normalized size = 2.36

$$\frac{\log\left(|bx+a|\right)}{2\,ab}-\frac{\log\left(|bx-a|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

maple [B] time = 0.22, size = 31, normalized size = 2.21

method	result	size
default	$\frac{\ln(bx+a)}{2ab}-\frac{\ln(-bx+a)}{2ab}$	31
norman	$\frac{\ln(bx+a)}{2ab}-\frac{\ln(-bx+a)}{2ab}$	31
risch	$\frac{\ln(bx+a)}{2ab}-\frac{\ln(-bx+a)}{2ab}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x+a)/a/b-1/2/a/b*ln(-b*x+a)
```

maxima [B] time = 0.42, size = 31, normalized size = 2.21

$$\frac{\log\left(bx+a\right)}{2\,ab}-\frac{\log\left(bx-a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

mupad [B] time = 0.06, size = 14, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - b^2*x^2),x)
```

```
[Out] atanh((b*x)/a)/(a*b)
```

sympy [B] time = 0.14, size = 20, normalized size = 1.43

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b**2*x**2+a**2),x)
```

```
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

3.2 $\int \frac{1}{a^2+b^2x^2}\,dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

fricas [A] time = 0.59, size = 14, normalized size = 1.00

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] arctan(b*x/a)/(a*b)
```

giac [A] time = 0.92, size = 14, normalized size = 1.00

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] arctan(b*x/a)/(a*b)
```

maple [A] time = 0.31, size = 15, normalized size = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
risch	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(b*x/a)/a/b
```

maxima [A] time = 0.95, size = 14, normalized size = 1.00

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] arctan(b*x/a)/(a*b)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{b\,x}{a}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^2),x)
```

```
[Out] atan((b*x)/a)/(a*b)
```

sympy [C] time = 0.14, size = 26, normalized size = 1.86

$$\frac{-\frac{i\log\left(-\frac{ia}{b}+x\right)}{2}+\frac{i\log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

3.3 $\int \sec(2ax)\, dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*a*x],x]
```

```
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(2ax)\, dx = \frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.85

$$\frac{\log(\sin(ax)+\cos(ax))}{2a} - \frac{\log(\cos(ax)-\sin(ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*a*x],x]
```

```
[Out] -1/2*Log[Cos[a*x] - Sin[a*x]]/a + Log[Cos[a*x] + Sin[a*x]]/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(2ax)\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[2*a*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.16, size = 26, normalized size = 2.00

$$\frac{\log\left(\sin\left(2\,ax\right)+1\right)-\log\left(-\sin\left(2\,ax\right)+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (pi/x/2)>(-pi/x/2)Unable to check sign: (pi/x/2)>(-pi/x/2)
2/a*1/2*(-1/8*ln(abs(sin(2*a*x)+1/sin(2*a*x)-2))+1/8*ln(abs(sin(2*a*x)+1/si
n(2*a*x)+2)))
```

maple [A] time = 0.08, size = 18, normalized size = 1.38

method	result	size
derivativedivides	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
default	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
norman	$-\frac{\ln(\tan(ax)-1)}{2a}+\frac{\ln(\tan(ax)+1)}{2a}$	26
risch	$\frac{\ln\left(e^{2iax}+i\right)}{2a}-\frac{\ln\left(e^{2iax}-i\right)}{2a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

maxima [A] time = 0.43, size = 17, normalized size = 1.31

$$\frac{\log\left(\sec\left(2\,ax\right)+\tan\left(2\,ax\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

mupad [B] time = 0.23, size = 11, normalized size = 0.85

$$\frac{\mathrm{atanh}\left(\sin\left(2\,a\,x\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(2*a*x),x)
```

```
[Out] atanh(sin(2*a*x))/(2*a)
```

sympy [A] time = 0.11, size = 29, normalized size = 2.23

$$\begin{cases} \frac{-\frac{\log\left(\sin\left(2ax\right)-1\right)}{2}+\frac{\log\left(\sin\left(2ax\right)+1\right)}{2}}{2a} & \text{for}\: 2a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x)
```

```
[Out] Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(2*a,
0)), (x, True))
```

3.4 $\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx$

Optimal. Leaf size=11

$$-\frac{3}{4}\tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {12, 3770}

$$-\frac{3}{4}\tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x/3]/4,x]
```

```
[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4}\csc\left(\frac{x}{3}\right) dx &= \frac{1}{4}\int \csc\left(\frac{x}{3}\right) dx\\ &= -\frac{3}{4}\tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{4}\left(3\log\left(\sin\left(\frac{x}{6}\right)\right)-3\log\left(\cos\left(\frac{x}{6}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x/3]/4,x]
```

```
[Out] (-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x/3]/4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.96, size = 23, normalized size = 2.09

$$-\frac{3}{8} \log\left(\frac{1}{2} \cos\left(\frac{1}{3} x\right) + \frac{1}{2}\right) + \frac{3}{8} \log\left(-\frac{1}{2} \cos\left(\frac{1}{3} x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="fricas")
[Out] -3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

giac [B] time = 0.85, size = 23, normalized size = 2.09

$$-\frac{3}{8} \log\left(3 \cos\left(\frac{1}{3} x\right) + 3\right) + \frac{3}{8} \log\left(-3 \cos\left(\frac{1}{3} x\right) + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="giac")
[Out] -3/8*log(3*cos(1/3*x) + 3) + 3/8*log(-3*cos(1/3*x) + 3)
```

maple [A] time = 0.08, size = 8, normalized size = 0.73

method	result	size
norman	$\frac{3 \ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$	8
derivativedivides	$\frac{3 \ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
default	$\frac{3 \ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
risch	$-\frac{3 \ln\left({e}^{\frac{i x}{3}}+1\right)}{4}+\frac{3 \ln\left({e}^{\frac{i x}{3}}-1\right)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/4/sin(1/3*x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(tan(1/6*x))
```

maxima [B] time = 0.43, size = 19, normalized size = 1.73

$$-\frac{3}{8} \log\left(\cos\left(\frac{1}{3} x\right) + 1\right) + \frac{3}{8} \log\left(\cos\left(\frac{1}{3} x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="maxima")
[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

mupad [B] time = 0.07, size = 7, normalized size = 0.64

$$\frac{3 \ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*sin(x/3)),x)
[Out] (3*log(tan(x/6)))/4
```

sympy [B] time = 0.11, size = 22, normalized size = 2.00

$$\frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8}-\frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x)
```

```
[Out] 3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3770}

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[-Sec[Pi/4 + 2*x],x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{2}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sec[Pi/4 + 2*x],x]
```

```
[Out] -1/2*ArcTanh[Sin[Pi/4 + 2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-Sec[Pi/4 + 2*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.09, size = 29, normalized size = 1.93

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

giac [B] time = 1.00, size = 29, normalized size = 1.93

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="giac")
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

maple [A] time = 0.25, size = 21, normalized size = 1.40

method	result	size
derivativedivides	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
default	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
norman	$\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$	24
risch	$-\frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}+i\right)}{2}+\frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}-i\right)}{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/cos(1/4*Pi+2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))
```

maxima [B] time = 0.43, size = 27, normalized size = 1.80

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

mupad [B] time = 0.21, size = 24, normalized size = 1.60

$$-\frac{\ln\left(\frac{\sin\left(\frac{\Pi}{4}+2x\right)+1}{\cos\left(\frac{\Pi}{4}+2x\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/cos(Pi/4 + 2*x),x)
[Out] -log((sin(Pi/4 + 2*x) + 1)/cos(Pi/4 + 2*x))/2
```

sympy [A] time = 0.22, size = 22, normalized size = 1.47

$$\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x)
[Out] log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

3.6 $\int \sec(x) \tan(x) \, dx$

Optimal. Leaf size=2

$$\sec(x)$$

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \sec(x) \tan(x) \, dx &= \text{Subst}\left(\int 1 \, dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(x) \tan(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]*Tan[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

giac [A] time = 0.93, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

maple [A] time = 0.07, size = 3, normalized size = 1.50

method	result	size
derivativedivides	$\sec(x)$	3
default	$\sec(x)$	3
risch	$\frac{2\,e^{ix}}{1+e^{2ix}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] sec(x)
```

maxima [A] time = 0.42, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

mupad [B] time = 0.25, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x),x)
```

```
[Out] -2/(tan(x/2)^2 - 1)
```

sympy [A] time = 0.07, size = 3, normalized size = 1.50

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

3.7 $\int \cot(x)\csc(x)\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\ &= -\csc(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot(x)\csc(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]*Csc[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.02, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
[Out] -1/sin(x)
```

giac [A] time = 0.99, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
[Out] -1/sin(x)
```

maple [A] time = 0.03, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$-\csc(x)$	5
default	$-\csc(x)$	5
risch	$-\frac{2ie^{ix}}{e^{2ix}-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x),x,method=_RETURNVERBOSE)
[Out] -csc(x)
```

maxima [A] time = 0.44, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
[Out] -1/sin(x)
```

mupad [B] time = 0.21, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/sin(x),x)
[Out] -1/sin(x)
```

sympy [A] time = 0.07, size = 5, normalized size = 1.25

$$-\frac{1}{\sin{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x)
[Out] -1/sin(x)
```

3.8 $\int \csc(2x)\tan(x)\,dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{2}$$

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {8}

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Tan[x],x]
```

```
[Out] Tan[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)\tan(x)\,dx &= \text{Subst}\left(\int \frac{1}{2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Tan[x],x]
```

```
[Out] Tan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(2x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[2*x]*Tan[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.16, size = 4, normalized size = 0.67

$$\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)
```

giac [**A**] time = 1.01, size = 4, normalized size = 0.67

$$\frac{1}{2}\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*tan(x)
```

maple [**A**] time = 0.05, size = 5, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\tan(x)}{2}$	5
default	$\frac{\tan(x)}{2}$	5
norman	$\frac{\tan(x)}{2}$	5
risch	$\frac{i}{1+e^{2ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)
```

maxima [**B**] time = 0.43, size = 27, normalized size = 4.50

$$\frac{\sin(2\,x)}{\cos(2\,x)^2+\sin(2\,x)^2+2\,\cos(2\,x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="maxima")
```

```
[Out] sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [**B**] time = 0.18, size = 4, normalized size = 0.67

$$\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/sin(2*x),x)
```

```
[Out] tan(x)/2
```

sympy [**B**] time = 0.79, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x)
```

```
[Out] sin(x)/(2*cos(x))
```

3.9 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Cos[x])^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.71, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

giac [B] time = 0.85, size = 30, normalized size = 3.33

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

maple [A] time = 0.04, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)),x,method=_RETURNVERBOSE)
[Out] tan(1/2*x)
```

maxima [A] time = 0.44, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
[Out] sin(x)/(cos(x) + 1)
```

mupad [B] time = 0.19, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1),x)
[Out] tan(x/2)
```

sympy [A] time = 0.20, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
[Out] tan(x/2)
```

3.10 $\int \frac{1}{1-\cos(x)} \, dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)} \, dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-\cos(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - Cos[x])^(-1),x]
[Out] Could not integrate
```

fricas [A] time = 1.05, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 1.08, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.04, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)
```

maxima [A] time = 0.42, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

mupad [B] time = 0.00, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

sympy [A] time = 0.34, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\log(a-b\cos(x))}{b}$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2668, 31\}$

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a-b\cos(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, -b\cos(x)\right)}{b}\\ &= \frac{\log(a-b\cos(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 12, normalized size = 1.00

$$\frac{\log\left(-b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")
```

```
[Out] log(-b*cos(x) + a)/b
```

giac [A] time = 1.08, size = 14, normalized size = 1.17

$$\frac{\log\left(\left|b\cos\left(x\right)-a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*cos(x) - a))/b
```

maple [A] time = 0.07, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(a-b\cos(x))}{b}$	13
default	$\frac{\ln(a-b\cos(x))}{b}$	13
risch	$-\frac{ix}{b}+\frac{\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}+1\right)}{b}$	32
norman	$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+b\left(\tan^2\left(\frac{x}{2}\right)\right)+a-b\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a-b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-b*cos(x))/b
```

maxima [A] time = 0.44, size = 13, normalized size = 1.08

$$\frac{\log\left(b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")
```

```
[Out] log(b*cos(x) - a)/b
```

mupad [B] time = 0.20, size = 13, normalized size = 1.08

$$\frac{\ln\left(b\,\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a - b*cos(x)),x)
```

```
[Out] log(b*cos(x) - a)/b
```

sympy [A] time = 0.37, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log\left(-\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x)
[Out] Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

3.12 $\int \frac{\cos(x)}{a^2+b^2 \sin^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{a^2+b^2\sin^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{a^2+b^2x^2} \, dx, x, \sin(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [A]   time = 1.06, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

giac [A]   time = 0.88, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

maple [A]   time = 0.13, size = 16, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
default	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
risch	$\frac{i\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}-1\right)}{2ba}-\frac{i\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}-1\right)}{2ba}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(b*sin(x)/a)/a/b
```

maxima [A]   time = 0.99, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

mupad [B]   time = 0.05, size = 15, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{b\,\sin(x)}{a}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(b^2*sin(x)^2 + a^2),x)
```

```
[Out] atan((b*sin(x))/a)/(a*b)
```

sympy [A] time = 0.71, size = 31, normalized size = 2.07

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), Eq(a, 0)),
(sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True))
```

3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3190, 208}

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 26, normalized size = 1.73

$$\frac{\log\left(b\sin(x)+a\right)-\log\left(-b\sin(x)+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
[Out] 1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

giac [B] time = 1.09, size = 35, normalized size = 2.33

$$\frac{\log\left(|b\sin(x)+a|\right)}{2\,ab}-\frac{\log\left(|b\sin(x)-a|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(b*sin(x) + a))/(a*b) - 1/2*log(abs(b*sin(x) - a))/(a*b)
```

maple [B] time = 0.15, size = 33, normalized size = 2.20

method	result	size
derivativedivides	$-\frac{\ln(-b\sin(x)+a)}{2ab}+\frac{\ln(b\sin(x)+a)}{2ab}$	33
default	$-\frac{\ln(-b\sin(x)+a)}{2ab}+\frac{\ln(b\sin(x)+a)}{2ab}$	33
norman	$-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)+a\right)}{2ab}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{2ab}$	54
risch	$-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{b}-1\right)}{2ab}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{2ab}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/b*ln(-b*sin(x)+a)+1/2/a/b*ln(b*sin(x)+a)
```

maxima [B] time = 0.43, size = 33, normalized size = 2.20

$$\frac{\log\left(b\sin(x)+a\right)}{2\,ab}-\frac{\log\left(b\sin(x)-a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
[Out] 1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

mupad [B] time = 0.18, size = 15, normalized size = 1.00

$$\frac{\mathrm{atanh}\left(\frac{b\,\sin(x)}{a}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(b^2*sin(x)^2 - a^2),x)
```

```
[Out] atanh((b*sin(x))/a)/(a*b)
```

sympy [A] time = 0.73, size = 44, normalized size = 2.93

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{\log\left(-\frac{a}{b} + \sin(x)\right)}{2ab} + \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{2ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq(a, 0)), (
sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/(2
*a*b), True))
```

3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 260}

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 21, normalized size = 1.24

$$\frac{\log\left(-b^2\cos(x)^2 + a^2 + b^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
[Out] log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

giac [A] time = 1.04, size = 17, normalized size = 1.00

$$\frac{\log\left(b^2\sin(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

maple [A] time = 0.18, size = 18, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^2+b^2\left(\sin^2(x)\right)\right)}{b^2}$	18
default	$\frac{\ln\left(a^2+b^2\left(\sin^2(x)\right)\right)}{b^2}$	18
risch	$-\frac{2ix}{b^2}+\frac{\ln\left(e^{4ix}-\frac{2(2a^2+b^2)e^{2ix}}{b^2}+1\right)}{b^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

maxima [A] time = 0.44, size = 17, normalized size = 1.00

$$\frac{\log\left(b^2\sin(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

mupad [B] time = 0.61, size = 48, normalized size = 2.82

$$\frac{\operatorname{atan}\left(\frac{b^2\sin(x)^2}{a^2\cos(x)^2\,2\mathrm{i}+a^2\sin(x)^2\,2\mathrm{i}+b^2\sin(x)^2\,1\mathrm{i}}\right)2\mathrm{i}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(b^2*sin(x)^2 + a^2),x)
[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i + b^2*sin(x)^2*1i))
*2i)/b^2
```

sympy [A] time = 2.98, size = 32, normalized size = 1.88

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log\left(a^2+b^2\sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2)
/(2*b**2), True))
```

3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {12, 260}

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.10, size = 23, normalized size = 1.21

$$-\frac{\log\left(b^2\cos(x)^2 + a^2 - b^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
[Out] -log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

giac [A] time = 0.93, size = 21, normalized size = 1.11

$$-\frac{\log\left(\left|b^2\sin(x)^2 - a^2\right|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
[Out] -log(abs(b^2*sin(x)^2 - a^2))/b^2
```

maple [A] time = 0.27, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{\ln\left(a^2-b^2\left(\sin^2(x)\right)\right)}{b^2}$	20
default	$-\frac{\ln\left(a^2-b^2\left(\sin^2(x)\right)\right)}{b^2}$	20
risch	$\frac{2ix}{b^2}-\frac{\ln\left(e^{4ix}+\frac{2\left(2a^2-b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -ln(a^2-b^2*sin(x)^2)/b^2
```

maxima [A] time = 0.43, size = 20, normalized size = 1.05

$$-\frac{\log\left(b^2\sin(x)^2 - a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
[Out] -log(b^2*sin(x)^2 - a^2)/b^2
```

mupad [B] time = 0.49, size = 48, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{b^2\sin(x)^2}{a^2\cos(x)^2\,2\mathrm{i}+a^2\sin(x)^2\,2\mathrm{i}-b^2\sin(x)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(2*x)/(b^2*sin(x)^2 - a^2),x)
[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i - b^2*sin(x)^2*1i))
*2i)/b^2
```

sympy [A] time = 3.32, size = 34, normalized size = 1.79

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log\left(a^2 - b^2\sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**2
)/(2*b**2), True))
```

3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 260}

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.22

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 18, normalized size = 1.00

$$-\frac{\log\left(b^2\cos(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

giac [A] time = 0.92, size = 18, normalized size = 1.00

$$-\frac{\log\left(b^2\cos(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="giac")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

maple [A] time = 0.16, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
default	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
risch	$\frac{2ix}{b^2}-\frac{\ln\left(e^{4ix}+\frac{2\left(2a^2+b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -ln(a^2+b^2*cos(x)^2)/b^2
```

maxima [A] time = 0.42, size = 18, normalized size = 1.00

$$-\frac{\log\left(b^2\cos(x)^2 + a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

mupad [B] time = 0.41, size = 58, normalized size = 3.22

$$\frac{2\,\mathrm{atanh}\left(\frac{b^2}{2\,a^2+b^2\,\cos(x)^2+b^2}-\frac{b^2\,\cos(x)^2}{2\,a^2+b^2\,\cos(x)^2+b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(b^2*cos(x)^2 + a^2),x)
[Out] (2*atanh(b^2/(b^2*cos(x)^2 + 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 +
2*a^2 + b^2)))/b^2
```

sympy [A] time = 3.00, size = 34, normalized size = 1.89

$$2\left(\begin{cases} -\frac{\cos^{2}(x)}{2a^{2}} & \text{for } b^{2}=0 \\ -\frac{\log\left(a^{2}+b^{2}\cos^{2}(x)\right)}{2b^{2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**2
)/(2*b**2), True))
```

3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {12, 260}

$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2-b^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2-b^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.22

$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 19, normalized size = 1.06

$$\frac{\log\left(b^2\cos(x)^2 - a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

giac [A] time = 0.96, size = 20, normalized size = 1.11

$$\frac{\log\left(\left|b^2\cos(x)^2 - a^2\right|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="giac")
[Out] log(abs(b^2*cos(x)^2 - a^2))/b^2
```

maple [A] time = 0.31, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^2-b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
default	$\frac{\ln\left(a^2-b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
risch	$-\frac{2ix}{b^2}+\frac{\ln\left(e^{4ix}-\frac{2\left(2a^2-b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

maxima [A] time = 0.45, size = 19, normalized size = 1.06

$$\frac{\log\left(b^2\cos(x)^2 - a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

mupad [B] time = 0.38, size = 58, normalized size = 3.22

$$-\frac{2\,\mathrm{atanh}\left(\frac{b^2}{-2\,a^2+b^2\cos(x)^2+b^2}-\frac{b^2\cos(x)^2}{-2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(2*x)/(b^2*cos(x)^2 - a^2),x)
[Out] -(2*atanh(b^2/(b^2*cos(x)^2 - 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 -
 2*a^2 + b^2)))/b^2
```

sympy [**A**] time = 3.22, size = 32, normalized size = 1.78

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log\left(a^2-b^2\cos^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2)
/(2*b**2), True))
```

3.18 $\int \frac{1}{4-\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 203}

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)]/(2*Sqrt[
3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{4+3x^2}\,dx,x,\cot(x)\right)\\ &= \frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-\cos^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.20, size = 31, normalized size = 0.76

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos(x)^2-4\sqrt{3}}{12\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x)))
```

giac [A] time = 0.83, size = 46, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-2\sin(2x)}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x)
+ sqrt(3) - 2*cos(2*x) + 2)))
```

maple [A] time = 0.07, size = 14, normalized size = 0.34

method	result	size
default	$\frac{\sqrt{3}\arctan\left(\frac{2\tan(x)\sqrt{3}}{3}\right)}{6}$	14
risch	$\frac{i\sqrt{3}\ln\left(e^{2ix}-4\sqrt{3}-7\right)}{12}-\frac{i\sqrt{3}\ln\left(e^{2ix}+4\sqrt{3}-7\right)}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*arctan(2/3*tan(x)*3^(1/2))
```

maxima [A] time = 0.95, size = 13, normalized size = 0.32

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

mupad [B] time = 0.23, size = 26, normalized size = 0.63

$$\frac{\sqrt{3}\,(x-\mathrm{atan}(\tan(x)))}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\tan(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2 - 4),x)
```

```
[Out] (3^(1/2)*(x - atan(tan(x))))/6 + (3^(1/2)*atan((2*3^(1/2)*tan(x))/3))/6
```

sympy [A] time = 0.68, size = 61, normalized size = 1.49

$$\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)**2),x)
```

```
[Out] sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)*
(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

3.19 $\int \frac{e^x}{-1+e^{2x}}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 207}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{-1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right)\\ &= -\tanh^{-1}(e^x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{-1+e^{2x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(-1 + E^(2*x)),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.88, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

giac [B] time = 0.79, size = 16, normalized size = 2.67

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.03, size = 6, normalized size = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(e^{x}\right)$	6
norman	$\frac{\ln(-1+e^{x})}{2}-\frac{\ln(1+e^{x})}{2}$	16
risch	$\frac{\ln(-1+e^{x})}{2}-\frac{\ln(1+e^{x})}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-1+exp(2*x)),x,method=_RETURNVERBOSE)
[Out] -arctanh(exp(x))
```

maxima [B] time = 0.43, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

mupad [B] time = 0.00, size = 15, normalized size = 2.50

$$\frac{\ln\left(e^{x}-1\right)}{2}-\frac{\ln\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) - 1),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

sympy [B] time = 0.11, size = 15, normalized size = 2.50

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.20 $\int \frac{1}{x \log(x)} \, dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log(x)} \, dx &= \text{Subst}\left(\int \frac{1}{x} \, dx, x, \log(x)\right) \\ &= \log(\log(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
[Out] Log[Log[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Log[x]),x]
[Out] Could not integrate
```

fricas [A] time = 0.97, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
[Out] log(log(x))
```

giac [**A**] time = 0.85, size = 4, normalized size = 1.33

$$\log\left(\left|\log(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
[Out] log(abs(log(x)))
```

maple [**A**] time = 0.02, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x,method=_RETURNVERBOSE)
[Out] ln(ln(x))
```

maxima [**A**] time = 0.42, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

mupad [**B**] time = 0.18, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)),x)
[Out] log(log(x))
```

sympy [**A**] time = 0.10, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

3.21 $\int \frac{1}{x(1+\log^2(x))}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.77, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

giac [A] time = 0.93, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

maple [A] time = 0.02, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2} - \frac{i\ln(\ln(x)-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(ln(x))
```

maxima [A] time = 0.96, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

mupad [B] time = 0.34, size = 3, normalized size = 1.00

$$\mathrm{atan}\left(\ln(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
```

```
[Out] atan(log(x))
```

sympy [B] time = 0.14, size = 15, normalized size = 5.00

$$\mathrm{RootSum}\left(4z^2 + 1, \left(i \mapsto i\log\left(2i + \log\left(x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.22 $\int \frac{1}{x(1-\log(x))}\,dx$

Optimal. Leaf size=9

$$-\log(1-\log(x))$$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 29}

$$-\log(1-\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - Log[x])),x]
```

```
[Out] -Log[1 - Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-\log(x))}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, 1-\log(x)\right)\\ &= -\log(1-\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 0.78

$$-\log(\log(x)-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - Log[x])),x]
```

```
[Out] -Log[-1 + Log[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - Log[x])),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 7, normalized size = 0.78

$$-\log\left(\log(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="fricas")
```

```
[Out] -log(log(x) - 1)
```

giac [B] time = 1.00, size = 22, normalized size = 2.44

$$-\frac{1}{2}\log\left(\frac{1}{4}\pi^2\left(\mathrm{sgn}(x)-1\right)^2+\left(\log\left(|x|\right)-1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)
```

maple [A] time = 0.02, size = 8, normalized size = 0.89

method	result	size
norman	$-\ln\left(-1+\ln(x)\right)$	8
risch	$-\ln\left(-1+\ln(x)\right)$	8
derivativedivides	$-\ln\left(1-\ln(x)\right)$	10
default	$-\ln\left(1-\ln(x)\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1-ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-1+ln(x))
```

maxima [A] time = 0.44, size = 7, normalized size = 0.78

$$-\log\left(\log(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="maxima")
```

```
[Out] -log(log(x) - 1)
```

mupad [B] time = 0.21, size = 7, normalized size = 0.78

$$-\ln\left(\ln(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(log(x) - 1)),x)
```

```
[Out] -log(log(x) - 1)
```

sympy [A] time = 0.10, size = 7, normalized size = 0.78

$$-\log\left(\log\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-ln(x)),x)
```

```
[Out] -log(log(x) - 1)
```

3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$

Optimal. Leaf size=9

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 29}

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x/a])),x]
[Out] Log[1 + Log[x/a]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, 1+\log\left(\frac{x}{a}\right)\right)\\ &= \log\left(1+\log\left(\frac{x}{a}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x/a])),x]
[Out] Log[1 + Log[x/a]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + Log[x/a])),x]
[Out] Could not integrate
```

fricas [A] time = 1.38, size = 9, normalized size = 1.00

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="fricas")
[Out] log(log(x/a) + 1)
```

giac [A] time = 0.88, size = 9, normalized size = 1.00

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="giac")
[Out] log(log(x/a) + 1)
```

maple [A] time = 0.02, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
default	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
norman	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
risch	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x/a)),x,method=_RETURNVERBOSE)
[Out] ln(1+ln(x/a))
```

maxima [A] time = 0.42, size = 9, normalized size = 1.00

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="maxima")
[Out] log(log(x/a) + 1)
```

mupad [B] time = 0.21, size = 9, normalized size = 1.00

$$\ln\left(\ln\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x/a) + 1)),x)
[Out] log(log(x/a) + 1)
```

sympy [A] time = 0.11, size = 7, normalized size = 0.78

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x/a)),x)
```

```
[Out] log(log(x/a) + 1)
```

3.24 $\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1357, 698}

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{\left(1-x+x^2\right)^2}{x^3}\,dx,x,\sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-2+\frac{1}{x^3}-\frac{2}{x^2}+\frac{3}{x}+x\right)dx,x,\sqrt{x}\right)\\ &= -\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 25, normalized size = 1.00

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[x] + x)^2/x^2,x]
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

fricas [A] time = 1.08, size = 24, normalized size = 0.96

$$\frac{x^2+6\,x\log\left(\sqrt{x}\right)-4\,(x-1)\sqrt{x}-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")
[Out] (x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

giac [A] time = 1.00, size = 23, normalized size = 0.92

$$x-4\,\sqrt{x}+\frac{4\,\sqrt{x}-1}{x}+3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="giac")
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(abs(x))
```

maple [A] time = 0.11, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{1}{x}+x+3\ln(x)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
default	$-\frac{1}{x}+x+3\ln(x)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
trager	$\frac{(-1+x)(1+x)}{x}-\frac{4(-1+x)}{\sqrt{x}}-3\ln\left(\frac{1}{x}\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

maxima [A] time = 0.43, size = 22, normalized size = 0.88

$$x-4\,\sqrt{x}+\frac{4\,\sqrt{x}-1}{x}+3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.96

$$x+6\,\ln\left(\sqrt{x}\right)+\frac{4\,\sqrt{x}-1}{x}-4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^(1/2) + 1)^2/x^2,x)
```

```
[Out] x + 6*log(x^(1/2)) + (4*x^(1/2) - 1)/x - 4*x^(1/2)
```

sympy [**A**] time = 0.42, size = 22, normalized size = 0.88

$$-4\sqrt{x} + x + 3\log(x) - \frac{1}{x} + \frac{4}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x**(1/2))**2/x**2,x)
```

```
[Out] -4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Rubi [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 1820}

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx &= \int \frac{\left(1+\sqrt{x}\right)\left(2-x^{2/3}\right)}{x}\,dx\\
&= -\left(6\,\mathrm{Subst}\left(\int \frac{\left(1+x^3\right)\left(-2+x^4\right)}{x}\,dx,x,\sqrt[6]{x}\right)\right)\\
&= -\left(6\,\mathrm{Subst}\left(\int \left(-\frac{2}{x}-2x^2+x^3+x^6\right)\,dx,x,\sqrt[6]{x}\right)\right)\\
&= 4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

fricas [A] time = 1.11, size = 22, normalized size = 0.73

$$-\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 12\log\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

giac [A] time = 0.97, size = 21, normalized size = 0.70

$$-\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))
```

maple [A] time = 0.02, size = 21, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{3x^{\frac{2}{3}}}{2} - \frac{6x^{\frac{7}{6}}}{7} + 2\ln(x) + 4\sqrt{x}$	21
default	$-\frac{3x^{\frac{2}{3}}}{2} - \frac{6x^{\frac{7}{6}}}{7} + 2\ln(x) + 4\sqrt{x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

maxima [A] time = 0.43, size = 20, normalized size = 0.67

$$-\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.73

$$12\ln\left(x^{1/6}\right) + 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^(2/3) - 2)*(x + x^(1/2)))/x^(3/2),x)
[Out] 12*log(x^(1/6)) + 4*x^(1/2) - (3*x^(2/3))/2 - (6*x^(7/6))/7
```

sympy [A] time = 7.86, size = 31, normalized size = 1.03

$$-\frac{6x^{\frac{7}{6}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + 4\sqrt{x} + 4\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 4*log(sqrt(x))
```

3.26 $\int \frac{-1+2x}{3+2x} \, dx$

Optimal. Leaf size=10

$$x - 2\log(2x + 3)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$x - 2\log(2x + 3)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/(3 + 2*x),x]
[Out] x - 2*Log[3 + 2*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{-1+2x}{3+2x} \, dx &= \int \left(1 - \frac{4}{3+2x}\right) dx \\ &= x - 2\log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$x - 2\log(2x + 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)/(3 + 2*x),x]
[Out] x - 2*Log[3 + 2*x]
```

IntegrateAlgebraic [A] time = 0.00, size = 10, normalized size = 1.00

$$x - 2\log(2x + 3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/(3 + 2*x),x]
[Out] x - 2*Log[3 + 2*x]
```

fricas [A] time = 1.02, size = 10, normalized size = 1.00

$$x - 2 \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")
[Out] x - 2*log(2*x + 3)
```

giac [A] time = 0.89, size = 11, normalized size = 1.10

$$x - 2\log\left(|2x + 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="giac")
[Out] x - 2*log(abs(2*x + 3))
```

maple [A] time = 0.29, size = 11, normalized size = 1.10

method	result	size
default	$x - 2\ln(3 + 2x)$	11
norman	$x - 2\ln(3 + 2x)$	11
meijerg	$-2\ln\left(1 + \frac{2x}{3}\right) + x$	11
risch	$x - 2\ln(3 + 2x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(3+2*x),x,method=_RETURNVERBOSE)
[Out] x-2*ln(3+2*x)
```

maxima [A] time = 0.42, size = 10, normalized size = 1.00

$$x - 2\log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")
[Out] x - 2*log(2*x + 3)
```

mupad [B] time = 0.06, size = 8, normalized size = 0.80

$$x - 2\ln\left(x + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(2*x + 3),x)
[Out] x - 2*log(x + 3/2)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.80

$$x - 2\log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x)
[Out] x - 2*log(2*x + 3)
```

3.27 $\int \frac{-5+2x}{-2+3x^2} dx$

Optimal. Leaf size=47

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {633, 31}

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x}{-2+3x^2}\,dx &= \frac{1}{4}\left(4-5\sqrt{6}\right)\int \frac{1}{-\sqrt{6}+3x}\,dx+\frac{1}{4}\left(4+5\sqrt{6}\right)\int \frac{1}{\sqrt{6}+3x}\,dx\\ &= \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

IntegrateAlgebraic [A] time = 0.04, size = 47, normalized size = 1.00

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

fricas [A] time = 1.08, size = 40, normalized size = 0.85

$$\frac{5}{12}\sqrt{6}\log\left(\frac{3x^2+2\sqrt{6}x+2}{3x^2-2}\right)+\frac{1}{3}\log\left(3x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")
```

```
[Out] 5/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2)) + 1/3*log(3*x^2 - 2
)
```

giac [A] time = 0.96, size = 37, normalized size = 0.79

$$\frac{1}{12}\left(5\sqrt{6}+4\right)\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\left(5\sqrt{6}-4\right)\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="giac")
```

```
[Out] 1/12*(5*sqrt(6) + 4)*log(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) - 4)*log(a
bs(x - 1/3*sqrt(6)))
```

maple [A] time = 0.29, size = 24, normalized size = 0.51

method	result	size
default	$\frac{\ln(3x^2-2)}{3}+\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)}{6}$	24
meijerg	$\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1-\frac{3x^2}{2}\right)}{3}$	27
risch	$\frac{\ln(3x+\sqrt{6})}{3}+\frac{5\ln(3x+\sqrt{6})\sqrt{6}}{12}+\frac{\ln(3x-\sqrt{6})}{3}-\frac{5\ln(3x-\sqrt{6})\sqrt{6}}{12}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+2*x)/(3*x^2-2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

maxima [A] time = 0.97, size = 36, normalized size = 0.77

$$-\frac{5}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right)+\frac{1}{3}\log\left(3x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")
```

```
[Out] -5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^2 - 2)
```

mupad [B] time = 0.13, size = 47, normalized size = 1.00

$$\frac{\ln\left(x-\frac{\sqrt{6}}{3}\right)}{3}+\frac{\ln\left(x+\frac{\sqrt{6}}{3}\right)}{3}-\frac{5\sqrt{6}\,\ln\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{5\sqrt{6}\,\ln\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 5)/(3*x^2 - 2),x)
```

```
[Out] log(x - 6^(1/2)/3)/3 + log(x + 6^(1/2)/3)/3 - (5*6^(1/2)*log(x - 6^(1/2)/3)
)/12 + (5*6^(1/2)*log(x + 6^(1/2)/3))/12
```

sympy [**A**] time = 0.12, size = 42, normalized size = 0.89

$$\left(\frac{1}{3}-\frac{5\sqrt{6}}{12}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x**2-2),x)
```

```
[Out] (1/3 - 5*sqrt(6)/12)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*log(x + sqrt
(6)/3)
```

3.28 $\int \frac{-5+2x}{2+3x^2} \, dx$

Optimal. Leaf size=30

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {635, 203, 260}

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x)/(2 + 3*x^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x}{2+3x^2}\,dx &= 2\int \frac{x}{2+3x^2}\,dx-5\int \frac{1}{2+3x^2}\,dx\\ &= -\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 + 2*x)/(2 + 3*x^2),x]
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

fricas [A] time = 0.98, size = 23, normalized size = 0.77

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

giac [A] time = 0.85, size = 21, normalized size = 0.70

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(x^2+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="giac")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)
```

maple [A] time = 0.26, size = 24, normalized size = 0.80

method	result	size
default	$\frac{\ln\left(3x^2+2\right)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
risch	$\frac{\ln\left(9x^2+6\right)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
meijerg	$-\frac{5\sqrt{6}\arctan\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1+\frac{3x^2}{2}\right)}{3}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+2*x)/(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

maxima [A] time = 0.97, size = 23, normalized size = 0.77

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

mupad [**B**] time = 0.04, size = 21, normalized size = 0.70

$$\frac{\ln\left(x^2+\frac{2}{3}\right)}{3}-\frac{5\,\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 5)/(3*x^2 + 2),x)
[Out] log(x^2 + 2/3)/3 - (5*6^(1/2)*atan((6^(1/2)*x)/2))/6
```

sympy [**A**] time = 0.12, size = 27, normalized size = 0.90

$$\frac{\log\left(x^2+\frac{2}{3}\right)}{3}-\frac{5\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x**2+2),x)
[Out] log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

3.29 $\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x/4]*Sin[x],x]
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x/4]*Sin[x],x]
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x/4]*Sin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 24, normalized size = 1.14

$$-\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^{4}-7\cos\left(\frac{1}{4}x\right)^{2}+1\right)\sin\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
[Out] -16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

giac [A] time = 1.24, size = 13, normalized size = 0.62

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

maple [A] time = 0.10, size = 14, normalized size = 0.67

method	result	size
default	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
risch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
norman	$\frac{-\frac{8\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{8}\right)\right)}{15}+\frac{32\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{8}\right)}{15}+\frac{8\tan\left(\frac{x}{2}\right)}{15}-\frac{32\tan\left(\frac{x}{8}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{8}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.62

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

mupad [B] time = 0.17, size = 13, normalized size = 0.62

$$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x/4)*sin(x),x)
[Out] (2*sin((3*x)/4))/3 - (2*sin((5*x)/4))/5
```

sympy [A] time = 0.55, size = 22, normalized size = 1.05

$$-\frac{16\sin\left(\frac{x}{4}\right)\cos\left(x\right)}{15}+\frac{4\sin\left(x\right)\cos\left(\frac{x}{4}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x)
[Out] -16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x/4)/15
```

3.30 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(4x)\,dx=\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(3x)\cos(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.09, size = 24, normalized size = 1.60

$$\frac{1}{7}\left(32\,\cos(x)^6-40\,\cos(x)^4+12\,\cos(x)^2+3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

giac [A] time = 0.85, size = 11, normalized size = 0.73

$$\frac{1}{14}\,\sin\left(7\,x\right)+\frac{1}{2}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

maple [A] time = 0.14, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{7}+\frac{6\left(\tan^2(2x)\right)\tan\left(\frac{3x}{2}\right)}{7}+\frac{8\tan(2x)}{7}-\frac{6\tan\left(\frac{3x}{2}\right)}{7}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{1}{14}\,\sin\left(7\,x\right)+\frac{1}{2}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

mupad [B] time = 0.06, size = 11, normalized size = 0.73

$$\frac{\sin\left(7\,x\right)}{14}+\frac{\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x)
[Out] sin(7*x)/14 + sin(x)/2
```

sympy [B] time = 0.55, size = 26, normalized size = 1.73

$$-\frac{3\sin\left(3x\right)\cos\left(4x\right)}{7}+\frac{4\sin\left(4x\right)\cos\left(3x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x)
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

3.31 $\int -\tan(a-x)\tan(x)\,dx$

Optimal. Leaf size=21

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4612, 4610, 3475}

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

Antiderivative was successfully verified.

```
[In] Int[-(Tan[a - x]*Tan[x]),x]
```

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int -\tan(a-x)\tan(x)\,dx &= -x+\cos(a)\int \sec(a-x)\sec(x)\,dx\\ &= -x+\cot(a)\int \tan(a-x)\,dx+\cot(a)\int \tan(x)\,dx\\ &= -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.08, size = 21, normalized size = 1.00

$$\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Tan[a - x]*Tan[x]),x]
```

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\tan(a-x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-(Tan[a - x]*Tan[x]),x]
[Out] Could not integrate
```

fricas [B] time = 1.12, size = 89, normalized size = 4.24

$$\frac{(\cos(2a)+1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2-2\sin(2a)\tan(x)-\cos(2a)-1}{(\cos(2a)+1)\tan(x)^2+\cos(2a)+1}\right)-(\cos(2a)+1)\log\left(\frac{1}{\tan(x)^2+1}\right)-2x\sin(2a)}{2\sin(2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="fricas")
[Out] 1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*tan(x) - cos
(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) - (cos(2*a) + 1)*log(1
/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

giac [A] time = 0.95, size = 18, normalized size = 0.86

$$-x+\frac{\log(|\tan(a)\tan(x)+1|)}{\tan(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="giac")
[Out] -x + log(abs(tan(a)*tan(x) + 1))/tan(a)
```

maple [A] time = 0.12, size = 20, normalized size = 0.95

method	result	size
derivativedivides	$-\arctan(\tan(x))+\frac{\ln(1+\tan(x)\tan(a))}{\tan(a)}$	20
default	$-\arctan(\tan(x))+\frac{\ln(1+\tan(x)\tan(a))}{\tan(a)}$	20
risch	$-x+\frac{i\ln(e^{2ia}+e^{2ix})e^{2ia}}{e^{2ia}-1}+\frac{i\ln(e^{2ia}+e^{2ix})}{e^{2ia}-1}-\frac{i\ln(1+e^{2ix})e^{2ia}}{e^{2ia}-1}-\frac{i\ln(1+e^{2ix})}{e^{2ia}-1}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)*tan(a-x),x,method=_RETURNVERBOSE)
[Out] -arctan(tan(x))+1/tan(a)*ln(1+tan(x)*tan(a))
```

maxima [B] time = 1.03, size = 186, normalized size = 8.86

$$-\frac{\left(\cos(2a)^2+\sin(2a)^2-2\cos(2a)+1\right)x+\left(\cos(2a)^2+\sin(2a)^2-1\right)\arctan(\sin(2a)+\sin(2x),\cos(2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="maxima")
[Out] -((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2 -
 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin(2
*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) - log(cos(2*a)^2 + 2*cos(2*a)*co
s(2*x) + cos(2*x)^2 + sin(2*a)^2 + 2*sin(2*a)*sin(2*x) + sin(2*x)^2)*sin(2*
a) + log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)^2 +
sin(2*a)^2 - 2*cos(2*a) + 1)
```

mupad [B] time = 1.33, size = 118, normalized size = 5.62

$$-x-\frac{\frac{\sin(2\,a)\,\ln\left(\sin(2\,a+x)^2\,2\mathrm{i}+\sin(2\,a)^2\,2\mathrm{i}-\sin(x)^2\,2\mathrm{i}+\sin(4\,a)-\sin(2\,x)+\sin(4\,a+2\,x)\right)}{2}-\frac{\sin(2\,a)\,\ln\left(\sin(2\,a)\,\left(2\,\sin(a)^2-1\right)-\sin(2\,a)^2\,1\mathrm{i}+\sin(2\,a)\,\left(2\,\mathrm{s}\right.\right.}{2}}{\sin(a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(a - x)*tan(x),x)
```

```
[Out] - x - ((sin(2*a)*log(sin(4*a) - sin(2*x) + sin(4*a + 2*x) - sin(x)^2*2i + s
in(2*a + x)^2*2i + sin(2*a)^2*2i))/2 - (sin(2*a)*log(sin(2*a)*(2*sin(a)^2 -
 1) - sin(2*a)^2*1i + sin(2*a)*(2*sin(x)^2 - 1) - sin(2*a)*sin(2*x)*1i))/2)
/sin(a)^2
```

sympy [B] time = 1.54, size = 138, normalized size = 6.57

$$-\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2}-\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2}+\frac{\log\left(\tan^2(x)+1\right)}{2\tan^2(a)+2} & \text{for } a\neq 0\\ \frac{\log\left(\tan^2(x)+1\right)}{2} & \text{otherwise}\end{cases}\right)\tan(a)+\begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)}+\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)}+ \\ -x+\tan(x)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x)
```

```
[Out] -Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a))/(2*tan(
a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a, 0)), (log(tan(x)**
2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan(a)/(2*tan(a)**3 + 2*tan(a)) +
 2*log(tan(x) + 1/tan(a))/(2*tan(a)**3 + 2*tan(a)) + log(tan(x)**2 + 1)*tan
(a)**2/(2*tan(a)**3 + 2*tan(a)), Ne(a, 0)), (-x + tan(x), True))
```

3.32 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.06, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.94, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

maple [A] time = 0.04, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} - \frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

maxima [A] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

3.33 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.14, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.83, size = 10, normalized size = 0.71

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

maple [A] time = 0.03, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

maxima [A] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.34 $\int \cos^3(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{4}\cos^4(x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 30}

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x],x]
[Out] -Cos[x]^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin(x)\,dx &= -\text{Subst}\left(\int x^3\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\cos^4(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x],x]
[Out] -1/4*Cos[x]^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3(x)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^3*Sin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.33, size = 6, normalized size = 0.75

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="fricas")
[Out] -1/4*cos(x)^4
```

giac [A] time = 0.94, size = 6, normalized size = 0.75

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="giac")
[Out] -1/4*cos(x)^4
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\left(\cos^4(x)\right)}{4}$	7
default	$-\frac{\left(\cos^4(x)\right)}{4}$	7
risch	$-\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$	14
norman	$\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)+2\left(\tan^6\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	29
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{8}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(4x)}{\sqrt{\pi}}\right)}{32}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(x)^4
```

maxima [A] time = 0.43, size = 6, normalized size = 0.75

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="maxima")
[Out] -1/4*cos(x)^4
```

mupad [B] time = 0.03, size = 12, normalized size = 1.50

$$-\frac{\sin(x)^2\left(\sin(x)^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x),x)
[Out] -(sin(x)^2*(sin(x)^2 - 2))/4
```

sympy [**A**] time = 0.06, size = 7, normalized size = 0.88

$$-\frac{\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x),x)
```

```
[Out] -cos(x)**4/4
```

3.35 $\int \cot^3(x) \csc(x) \, dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2606}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3*Csc[x],x]

[Out] Csc[x] - Csc[x]^3/3

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \cot^3(x) \csc(x) \, dx &= -\text{Subst}\left(\int \left(-1 + x^2\right) \, dx, x, \csc(x)\right) \\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^3*Csc[x],x]

[Out] Csc[x] - Csc[x]^3/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^3(x) \csc(x) \, dx$$

Verification is Not applicable to the result.

[In] IntegrateAlgebraic[Cot[x]^3*Csc[x],x]

[Out] Could not integrate

fricas [A] time = 1.06, size = 22, normalized size = 2.00

$$\frac{3 \cos(x)^2 - 2}{3 \left(\cos(x)^2 - 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 1.11, size = 14, normalized size = 1.27

$$\frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

maple [B] time = 0.41, size = 32, normalized size = 2.91

method	result	size
default	$-\frac{\cos^4(x)}{3\sin(x)^3}+\frac{\cos^4(x)}{3\sin(x)}+\frac{(2+\cos^2(x))\sin(x)}{3}$	32
norman	$\frac{-\frac{1}{24}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34
risch	$\frac{2i\left(3\,e^{5ix}-2\,e^{3ix}+3\,e^{ix}\right)}{3\left(e^{2ix}-1\right)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.43, size = 14, normalized size = 1.27

$$\frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

mupad [B] time = 0.07, size = 11, normalized size = 1.00

$$\frac{\sin(x)^2-\frac{1}{3}}{\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^4,x)
[Out] (sin(x)^2 - 1/3)/sin(x)^3
```

sympy [A] time = 0.09, size = 15, normalized size = 1.36

$$-\frac{1-3\sin^2(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**4,x)
[Out] -(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

3.36 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal. Leaf size=7

$$\tan(x)-\cot(x)$$

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2620, 14}

$$\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
[Out] -Cot[x] + Tan[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)\sec^2(x)\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(x)\right)\\ &= -\cot(x)+\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
[Out] -2*Cot[2*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(x)\sec^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^2*Sec[x]^2,x]
[Out] Could not integrate
```

fricas [B] time = 1.34, size = 18, normalized size = 2.57

$$-\frac{2\,\cos(x)^2-1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

giac [A] time = 1.02, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
[Out] -1/tan(x) + tan(x)
```

maple [A] time = 0.29, size = 15, normalized size = 2.14

method	result	size
default	$\frac{1}{\sin(x)\cos(x)}-2\cot(x)$	15
risch	$-\frac{4i}{(1+e^{2ix})(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

maxima [A] time = 0.43, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/tan(x) + tan(x)
```

mupad [B] time = 0.20, size = 6, normalized size = 0.86

$$-2\cot(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*sin(x)^2),x)
[Out] -2*cot(2*x)
```

sympy [B] time = 0.07, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)

[Out] -2*cos(2*x)/sin(2*x)
```

3.37 $\int \cot^2\left(\frac{3x}{4}\right) dx$

Optimal. Leaf size=14

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[(3*x)/4]^2,x]
```

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2\left(\frac{3x}{4}\right) dx &= -\frac{4}{3}\cot\left(\frac{3x}{4}\right)-\int 1\,dx\\ &= -x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 2.00

$$-\frac{4}{3}\cot\left(\frac{3x}{4}\right) {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\frac{3x}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[(3*x)/4]^2,x]
```

```
[Out] (-4*Cot[(3*x)/4]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[(3*x)/4]^2])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^2\left(\frac{3x}{4}\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[(3*x)/4]^2,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.00, size = 23, normalized size = 1.64

$$-\frac{3\,x\sin\left(\frac{3}{2}\,x\right)+4\,\cos\left(\frac{3}{2}\,x\right)+4}{3\,\sin\left(\frac{3}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="fricas")
[Out] -1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

giac [A] time = 1.19, size = 18, normalized size = 1.29

$$-x-\frac{2}{3\,\tan\left(\frac{3}{8}\,x\right)}+\frac{2}{3}\,\tan\left(\frac{3}{8}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="giac")
[Out] -x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

maple [A] time = 0.02, size = 17, normalized size = 1.21

method	result	size
norman	$\frac{-\frac{4}{3}-x\tan\left(\frac{3x}{4}\right)}{\tan\left(\frac{3x}{4}\right)}$	17
risch	$-x-\frac{8i}{3\left(e^{\frac{3ix}{2}}-1\right)}$	17
derivativedivides	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
default	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3/4*x)^2,x,method=_RETURNVERBOSE)
[Out] (-4/3-x*tan(3/4*x))/tan(3/4*x)
```

maxima [A] time = 0.98, size = 12, normalized size = 0.86

$$-x-\frac{4}{3\,\tan\left(\frac{3}{4}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="maxima")
[Out] -x - 4/3/tan(3/4*x)
```

mupad [B] time = 0.17, size = 10, normalized size = 0.71

$$-x-\frac{4\cot\left(\frac{3\,x}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot((3*x)/4)^2,x)
```

```
[Out] - x - (4*cot((3*x)/4))/3
```

sympy **[A]** time = 0.07, size = 19, normalized size = 1.36

$$-x - \frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)**2,x)
```

```
[Out] -x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

3.38 $\int (1+\tan(2x))^2\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3477, 3475}

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[2*x])^2,x]
```

```
[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\tan(2x))^2\,dx &= \frac{1}{2}\tan(2x)+2\int \tan(2x)\,dx\\ &= -\log(\cos(2x))+\frac{1}{2}\tan(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.62

$$x-\frac{1}{2}\tan^{-1}(\tan(2x))+\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[2*x])^2,x]
```

```
[Out] x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1+\tan(2x))^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Tan[2*x])^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 20, normalized size = 1.25

$$-\frac{1}{2}\log\left(\frac{1}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="fricas")
[Out] -1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

giac [A] time = 1.03, size = 22, normalized size = 1.38

$$-\frac{1}{2}\log\left(\frac{4}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="giac")
[Out] -1/2*log(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

maple [A] time = 0.03, size = 19, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
default	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
norman	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
risch	$2ix+\frac{i}{e^{4ix}+1}-\ln\left(e^{4ix}+1\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(2*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

maxima [A] time = 0.96, size = 12, normalized size = 0.75

$$\log(\sec(2x))+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="maxima")
[Out] log(sec(2*x)) + 1/2*tan(2*x)
```

mupad [B] time = 0.27, size = 18, normalized size = 1.12

$$\frac{\tan(2x)}{2}+\frac{\ln\left(\tan(2x)^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(2*x) + 1)^2,x)
[Out] tan(2*x)/2 + log(tan(2*x)^2 + 1)/2
```

sympy [A] time = 0.15, size = 17, normalized size = 1.06

$$\frac{\log\left(\tan^2(2x)+1\right)}{2}+\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))**2,x)
```

```
[Out] log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

3.39 $\int (-\cot(x)+\tan(x))^2\,dx$

Optimal. Leaf size=10

$$-4x+\tan(x)-\cot(x)$$

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {461, 203}

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cot[x] + Tan[x])^2,x]
```

```
[Out] -4*x - Cot[x] + Tan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}
\int (-\cot(x)+\tan(x))^2\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{4}{1+x^2}\right)dx,x,\tan(x)\right)\\
&= -\cot(x)+\tan(x)-4\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= -4x-\cot(x)+\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cot[x] + Tan[x])^2,x]
```

```
[Out] -4*x - Cot[x] + Tan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cot(x)+\tan(x))^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-Cot[x] + Tan[x])^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.90, size = 19, normalized size = 1.90

$$-\frac{4\,x\tan(x)-\tan(x)^2+1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

giac [A] time = 0.83, size = 12, normalized size = 1.20

$$-4\,x-\frac{1}{\tan(x)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

maple [A] time = 0.05, size = 11, normalized size = 1.10

method	result	size
default	$-4x-\cot(x)+\tan(x)$	11
norman	$\frac{-1+\tan^2(x)-4x\tan(x)}{\tan(x)}$	17
risch	$-4x-\frac{4i}{(1+e^{2ix})(e^{2ix}-1)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cot(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*x-cot(x)+tan(x)
```

maxima [A] time = 0.95, size = 12, normalized size = 1.20

$$-4\,x-\frac{1}{\tan(x)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

mupad [B] time = 0.27, size = 12, normalized size = 1.20

$$\tan(x)-4\,x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) - tan(x))^2,x)
```

```
[Out] tan(x) - 4*x - 1/tan(x)
```

sympy [**A**] time = 0.37, size = 10, normalized size = 1.00

$$-4x + \tan(x) - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))**2,x)
```

```
[Out] -4*x + tan(x) - 1/tan(x)
```

3.40 $\int (-\sec(x)+\tan(x))^2\,dx$

Optimal. Leaf size=14

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4391, 2670, 2680, 8}

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[x] + Tan[x])^2,x]
```

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (-\sec(x)+\tan(x))^2\,dx &= \int \sec^2(x)(-1+\sin(x))^2\,dx\\
&= \int \frac{\cos^2(x)}{(-1-\sin(x))^2}\,dx\\
&= -\frac{2\cos(x)}{1+\sin(x)}-\int 1\,dx\\
&= -x-\frac{2\cos(x)}{1+\sin(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.86

$$-x+2\tan(x)-2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[x] + Tan[x])^2,x]
[Out] -x - 2*Sec[x] + 2*Tan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-\sec(x)+\tan(x))^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-Sec[x] + Tan[x])^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 25, normalized size = 1.79

$$-\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

giac [A] time = 0.79, size = 14, normalized size = 1.00

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="giac")
[Out] -x - 4/(tan(1/2*x) + 1)
```

maple [A] time = 0.31, size = 15, normalized size = 1.07

method	result	size
default	$2\tan(x)-\frac{2}{\cos(x)}-x$	15
risch	$-x-\frac{4}{e^{ix}+i}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*tan(x)-2/cos(x)-x
```

maxima [A] time = 0.96, size = 14, normalized size = 1.00

$$-x-\frac{2}{\cos(x)}+2\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/cos(x) + 2*tan(x)
```

mupad [B] time = 0.27, size = 14, normalized size = 1.00

$$-x - \frac{4}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) - 1/cos(x))^2,x)
```

```
[Out] - x - 4/(tan(x/2) + 1)
```

sympy [A] time = 1.38, size = 10, normalized size = 0.71

$$-x + 2\tan(x) - 2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))**2,x)
```

```
[Out] -x + 2*tan(x) - 2*sec(x)
```

3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

Optimal. Leaf size=11

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2735, 2648}

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Sin[x]),x]
[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{1+\sin(x)}\,dx &= x-\int \frac{1}{1+\sin(x)}\,dx\\ &= x+\frac{\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.04, size = 25, normalized size = 2.27

$$x-\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Sin[x]),x]
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/(1 + Sin[x]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.03, size = 24, normalized size = 2.18

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

giac [A] time = 1.08, size = 12, normalized size = 1.09

$$x+\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="giac")
[Out] x + 2/(tan(1/2*x) + 1)
```

maple [C] time = 0.05, size = 15, normalized size = 1.36

method	result	size
risch	$x+\frac{2}{e^{ix}+i}$	15
default	$2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{2}{\tan\left(\frac{x}{2}\right)+1}$	19
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+x\left(\tan^3\left(\frac{x}{2}\right)\right)+2\left(\tan^2\left(\frac{x}{2}\right)\right)+2}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)),x,method=_RETURNVERBOSE)
[Out] x+2/(exp(I*x)+I)
```

maxima [B] time = 0.96, size = 28, normalized size = 2.55

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.24, size = 12, normalized size = 1.09

$$x+\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sin(x) + 1),x)
[Out] x + 2/(tan(x/2) + 1)
```

sympy [B] time = 0.54, size = 29, normalized size = 2.64

$$\frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}+\frac{x}{\tan\left(\frac{x}{2}\right)+1}+\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x)
```

```
[Out] x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) + 2/(tan(x/2) + 1)
```

3.42 $\int \frac{\cos(x)}{1-\cos(x)}\,dx$

Optimal. Leaf size=16

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{1-\cos(x)}\,dx &= -x+\int \frac{1}{1-\cos(x)}\,dx\\ &= -x-\frac{\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.31

$$\frac{2x\sin^2\left(\frac{x}{2}\right)+\sin(x)}{\cos(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] (2*x*Sin[x/2]^2 + Sin[x])/(-1 + Cos[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.91, size = 14, normalized size = 0.88

$$-\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

giac [A] time = 1.01, size = 12, normalized size = 0.75

$$-x-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x)
```

maple [A] time = 0.04, size = 17, normalized size = 1.06

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x-\frac{2i}{e^{ix}-1}$	17
norman	$\frac{-1-\left(\tan^2\left(\frac{x}{2}\right)\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)-2*arctan(tan(1/2*x))
```

maxima [A] time = 0.96, size = 23, normalized size = 1.44

$$-\frac{\cos(x)+1}{\sin(x)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.17, size = 10, normalized size = 0.62

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(cos(x) - 1),x)
[Out] - x - cot(x/2)
```

sympy [A] time = 0.49, size = 8, normalized size = 0.50

$$-x - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x)
```

```
[Out] -x - 1/tan(x/2)
```

3.43 $\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx$

Optimal. Leaf size=25

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2248, 43}

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx &= 2\,\text{Subst}\left(\int\frac{(-1+x)^3}{x^2}\,dx,x,e^{x/2}\right)\\ &= 2\,\text{Subst}\left(\int\left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx,x,e^{x/2}\right)\\ &= 2e^{-x/2}-6e^{x/2}+e^x+3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.94, size = 22, normalized size = 0.88

$$\left(3\,xe^{\left(\frac{1}{2}\,x\right)} + e^{\left(\frac{3}{2}\,x\right)} - 6\,e^{x} + 2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")
```

```
[Out] (3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

giac [A] time = 0.92, size = 18, normalized size = 0.72

$$3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="giac")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

maple [A] time = 0.02, size = 19, normalized size = 0.76

method	result	size
risch	$e^{x}+3x-6\,e^{\frac{x}{2}}+2\,e^{-\frac{x}{2}}$	19
derivativedivides	$e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$	29
default	$e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$	29
norman	$\left(2+e^{\frac{3x}{2}}-6\,e^{x}+3x\,e^{\frac{x}{2}}\right)e^{-\frac{x}{2}}$	31
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(1/2*x))^3/exp(1/2*x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)+3*x-6*exp(1/2*x)+2*exp(-1/2*x)
```

maxima [A] time = 0.46, size = 18, normalized size = 0.72

$$3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="maxima")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

mupad [B] time = 0.22, size = 18, normalized size = 0.72

$$3\,x + 2\,e^{-\frac{x}{2}} - 6\,e^{x/2} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x/2)*(exp(x/2) - 1)^3,x)
```

```
[Out] 3*x + 2*exp(-x/2) - 6*exp(x/2) + exp(x)
```

sympy [**A**] time = 0.12, size = 19, normalized size = 0.76

$$3x - 6e^{\frac{x}{2}} + e^{x} + 2e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
[Out] 3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

3.44 $\int \frac{1}{5-6x+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {616, 31}

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5-6x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{-5+x}\,dx-\frac{1}{4}\int \frac{1}{-1+x}\,dx\\ &= -\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] -1/4*Log[1 - x] + Log[5 - x]/4
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 0.67

$$\frac{1}{2}\tanh^{-1}\left(\frac{3}{2}-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] ArcTanh[3/2 - x/2]/2
```

fricas [A] time = 0.70, size = 13, normalized size = 0.62

$$-\frac{1}{4}\log\left(x-1\right)+\frac{1}{4}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="fricas")
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

giac [A] time = 0.85, size = 15, normalized size = 0.71

$$-\frac{1}{4}\log\left(|x-1|\right)+\frac{1}{4}\log\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="giac")
[Out] -1/4*log(abs(x - 1)) + 1/4*log(abs(x - 5))
```

maple [A] time = 0.31, size = 14, normalized size = 0.67

method	result	size
default	$\frac{\ln(-5+x)}{4}-\frac{\ln(-1+x)}{4}$	14
norman	$\frac{\ln(-5+x)}{4}-\frac{\ln(-1+x)}{4}$	14
risch	$\frac{\ln(-5+x)}{4}-\frac{\ln(-1+x)}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-6*x+5),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(-5+x)-1/4*ln(-1+x)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.62

$$-\frac{1}{4}\log\left(x-1\right)+\frac{1}{4}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="maxima")
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

mupad [B] time = 0.26, size = 8, normalized size = 0.38

$$-\frac{\operatorname{atanh}\left(\frac{x}{2}-\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 6*x + 5),x)
[Out] -atanh(x/2 - 3/2)/2
```

sympy [A] time = 0.10, size = 12, normalized size = 0.57

$$\frac{\log\left(x-5\right)}{4}-\frac{\log\left(x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-6*x+5),x)
```

```
[Out] log(x - 5)/4 - log(x - 1)/4
```

3.45 $\int \frac{x^2}{13-6x^3+x^6}\,dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 - 6*x^3 + x^6),x]
```

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{13-6x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{13-6x+x^2}\,dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx, x, 2\left(-3+x^3\right)\right)\right)\\
&= \frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(-3+x^3\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 - 6*x^3 + x^6),x]
```

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.14

$$-\frac{1}{6}\tan^{-1}\left(\frac{3}{2}-\frac{x^3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(13 - 6*x^3 + x^6),x]
[Out] -1/6*ArcTan[3/2 - x^3/2]
```

fricas [A] time = 1.24, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

giac [A] time = 0.99, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="giac")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

maple [A] time = 0.03, size = 11, normalized size = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-6*x^3+13),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/2*x^3-3/2)
```

maxima [A] time = 0.96, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - 6*x^3 + 13),x)
```

```
[Out] atan(x^3/2 - 3/2)/6
```

sympy [**A**] time = 0.11, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2} - \frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-6*x**3+13),x)
```

```
[Out] atan(x**3/2 - 3/2)/6
```

3.46 $\int \frac{2+x}{-1-4x+x^2} \, dx$

Optimal. Leaf size=51

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5
] - x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{-1-4x+x^2}\,dx &= -\left(\frac{1}{10}\left(-5+4\sqrt{5}\right)\int \frac{1}{-2+\sqrt{5}+x}\,dx\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\int \frac{1}{-2-\sqrt{5}+x}\,dx\\ &= \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(x+\sqrt{5}-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt[
5] + x])/10
```

IntegrateAlgebraic [A] time = 0.04, size = 47, normalized size = 0.92

$$\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(x+\sqrt{5}-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt[
5] + x])/10
```

fricas [A] time = 0.84, size = 45, normalized size = 0.88

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x^2-2\sqrt{5}(x-2)-4x+9}{x^2-4x-1}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2*
log(x^2 - 4*x - 1)
```

giac [A] time = 0.94, size = 44, normalized size = 0.86

$$\frac{2}{5}\sqrt{5}\log\left(\frac{\left|2x-2\sqrt{5}-4\right|}{\left|2x+2\sqrt{5}-4\right|}\right)+\frac{1}{2}\log\left(\left|x^2-4x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(5)*log(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4)) + 1/2*lo
g(abs(x^2 - 4*x - 1))
```

maple [A] time = 0.40, size = 29, normalized size = 0.57

method	result	size
default	$\frac{\ln(x^2-4x-1)}{2}-\frac{4\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-4)\sqrt{5}}{10}\right)}{5}$	29
risch	$\frac{\ln(x-\sqrt{5}-2)}{2}+\frac{2\ln(x-\sqrt{5}-2)\sqrt{5}}{5}+\frac{\ln(x-2+\sqrt{5})}{2}-\frac{2\ln(x-2+\sqrt{5})\sqrt{5}}{5}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2-4*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(2*x-4)*5^(1/2))
```

maxima [A] time = 0.95, size = 35, normalized size = 0.69

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x -
1)
```

mupad [B] time = 0.12, size = 34, normalized size = 0.67

$$\ln\left(x-\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}+\frac{1}{2}\right)-\ln\left(x+\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2)/(4*x - x^2 + 1),x)
```

```
[Out] log(x - 5^(1/2) - 2)*((2*5^(1/2))/5 + 1/2) - log(x + 5^(1/2) - 2)*((2*5^(1/
2))/5 - 1/2)
```

sympy [A] time = 0.12, size = 42, normalized size = 0.82

$$\left(\frac{1}{2}-\frac{2\sqrt{5}}{5}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2-4*x-1),x)
```

```
[Out] (1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqrt
(5) - 2)
```

3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 190, 43}

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt[3]{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+\sqrt[3]{x}}\,dx, x, 1+x\right)\\
&= 3\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[3]{1+x}\right)\\
&= 3\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt[3]{1+x}\right)\\
&= -3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + x)^(1/3))^(-1),x]
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (1 + x)^(1/3))^(-1),x]
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

fricas [A] time = 0.76, size = 25, normalized size = 0.76

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

giac [A] time = 1.05, size = 25, normalized size = 0.76

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="giac")
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

maple [A] time = 0.14, size = 26, normalized size = 0.79

method	result	size
derivativedivides	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+3\ln\left(1+(1+x)^{\frac{1}{3}}\right)$	26
trager	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+\ln\left(-3(1+x)^{\frac{2}{3}}-3(1+x)^{\frac{1}{3}}-x-2\right)$	36
default	$\ln(2+x)+\frac{3(1+x)^{\frac{2}{3}}}{2}+2\ln\left(1+(1+x)^{\frac{1}{3}}\right)-\ln\left((1+x)^{\frac{2}{3}}-(1+x)^{\frac{1}{3}}+1\right)-3(1+x)^{\frac{1}{3}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1+x)^(1/3)),x,method=_RETURNVERBOSE)
[Out] -3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

maxima [A] time = 0.43, size = 25, normalized size = 0.76

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

mupad [B] time = 0.20, size = 25, normalized size = 0.76

$$3\,\ln\left((x+1)^{1/3}+1\right)-3\,(x+1)^{1/3}+\frac{3\,(x+1)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/3) + 1),x)
[Out] 3*log((x + 1)^(1/3) + 1) - 3*(x + 1)^(1/3) + (3*(x + 1)^(2/3))/2
```

sympy [A] time = 0.14, size = 29, normalized size = 0.88

$$\frac{3\,(x+1)^{\frac{2}{3}}}{2}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)**(1/3)),x)
[Out] 3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)
```

3.48 $\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{63, 205\}$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b + a*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b + a*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(b + a*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 1.50, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

giac [A] time = 0.96, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.32, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

maxima [A] time = 0.97, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.20, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b + a*x)),x)
```

```
[Out] (2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [A] time = 1.32, size = 94, normalized size = 3.24

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{i\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} + \frac{i\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2/
(a*sqrt(x)), Eq(b, 0)), (-I*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*
sqrt(1/a)) + I*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*sqrt(1/a)), Tr
ue))
```

3.49 $\int x^3\sqrt{1+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x^2],x]
```

```
[Out] -(1 + x^2)^(3/2)/3 + (1 + x^2)^(5/2)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{1+x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{1}{15}\left(x^2+1\right)^{3/2}\left(3x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{1}{15}\sqrt{x^2+1}\left(3x^4+x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[1 + x^2],x]
[Out] (Sqrt[1 + x^2]*(-2 + x^2 + 3*x^4))/15
```

fricas [A] time = 0.80, size = 19, normalized size = 0.70

$$\frac{1}{15}\left(3\,x^4+x^2-2\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

giac [A] time = 1.03, size = 19, normalized size = 0.70

$$\frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

maple [A] time = 0.30, size = 17, normalized size = 0.63

method	result	size
gosper	$\frac{\left(x^2+1\right)^{\frac{3}{2}}\left(3x^2-2\right)}{15}$	17
risch	$\frac{\left(3x^4+x^2-2\right)\sqrt{x^2+1}}{15}$	20
trager	$\left(\frac{1}{5}x^4+\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{x^2+1}$	21
default	$\frac{x^2\left(x^2+1\right)^{\frac{3}{2}}}{5}-\frac{2\left(x^2+1\right)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,\left(x^2+1\right)^{\frac{3}{2}}\left(-3x^2+2\right)}{15}}{4\sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

maxima [A] time = 0.96, size = 22, normalized size = 0.81

$$\frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.74

$$\sqrt{x^2+1}\,\left(\frac{x^4}{5}+\frac{x^2}{15}-\frac{2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)^(1/2),x)
```

```
[Out] (x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

sympy [A] time = 0.63, size = 37, normalized size = 1.37

$$\frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

3.50 $\int \frac{x}{\sqrt{a^4-x^4}} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2} \tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {275, 217, 203}

$$\frac{1}{2} \tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a^4-x^4}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{a^4-x^2}} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \frac{x^2}{\sqrt{a^4-x^4}}\right)\\
&= \frac{1}{2} \tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{2} \tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

IntegrateAlgebraic [C] time = 0.09, size = 28, normalized size = 1.27

$$-\frac{1}{2}i\log\left(\sqrt{a^4-x^4}+ix^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a^4 - x^4],x]
[Out] (-1/2*I)*Log[I*x^2 + Sqrt[a^4 - x^4]]
```

fricas [A] time = 0.94, size = 25, normalized size = 1.14

$$-\arctan\left(-\frac{a^2-\sqrt{a^4-x^4}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")
[Out] -arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

giac [A] time = 1.06, size = 10, normalized size = 0.45

$$\frac{1}{2}\arcsin\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(x^2/a^2)
```

maple [A] time = 0.34, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19
elliptic	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4-x^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

maxima [A] time = 0.96, size = 18, normalized size = 0.82

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{a^4-x^4}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

mupad [B] time = 0.09, size = 18, normalized size = 0.82

$$\frac{\mathrm{atan}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4 - x^4)^(1/2),x)
```

```
[Out] atan(x^2/(a^4 - x^4)^(1/2))/2
```

sympy [**A**] time = 1.07, size = 29, normalized size = 1.32

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4-x**4)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2/a**2)/2,
True))
```

3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt{-a^2+x^2}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 + x^2]),x]
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a^2 + x^2]),x]
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

fricas [A] time = 0.94, size = 26, normalized size = 1.18

$$\frac{2\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

giac [A] time = 0.83, size = 20, normalized size = 0.91

$$\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(-a^2 + x^2)/a)/a
```

maple [A] time = 0.32, size = 41, normalized size = 1.86

method	result	size
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+x^2)^(1/2))/x)
```

maxima [A] time = 0.97, size = 12, normalized size = 0.55

$$-\frac{\arcsin\left(\frac{a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(a/abs(x))/a
```

mupad [B] time = 0.26, size = 24, normalized size = 1.09

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - a^2)^(1/2)),x)
[Out] atan((x^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

sympy [A] time = 1.07, size = 22, normalized size = 1.00

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2+x**2)**(1/2),x)
[Out] Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, True))
```

3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 - x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2-x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-x}\,x}\,dx, x, x^2\right)\\
&= -\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt{a^2-x^2}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 - x^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 - x^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

fricas [A] time = 0.88, size = 25, normalized size = 1.09

$$\frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] log(-(a - sqrt(a^2 - x^2))/x)/a
```

giac [B] time = 0.85, size = 43, normalized size = 1.87

$$-\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{2\,a}+\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(a + sqrt(a^2 - x^2)))/a + 1/2*log(abs(-a + sqrt(a^2 - x^2)))/a
```

maple [A] time = 0.32, size = 37, normalized size = 1.61

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-x^2}}{x}\right)}{\sqrt{a^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)
```

maxima [A] time = 0.43, size = 34, normalized size = 1.48

$$-\frac{\log\left(\frac{2\,a^2}{|x|}+\frac{2\,\sqrt{a^2-x^2}\,a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(2*a^2/abs(x) + 2*sqrt(a^2 - x^2)*a/abs(x))/a
```

mupad [B] time = 0.44, size = 21, normalized size = 0.91

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 - x^2)^(1/2)),x)
[Out] -atanh((a^2 - x^2)^(1/2)/a)/a
```

sympy [A] time = 1.09, size = 22, normalized size = 0.96

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for}\: \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2-x**2)**(1/2),x)
[Out] Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, True))
```

3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, x^2\right)\\
&= \operatorname{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+x^2}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

fricas [B] time = 0.87, size = 40, normalized size = 1.90

$$-\frac{\log\left(a-x+\sqrt{a^2+x^2}\right)-\log\left(-a-x+\sqrt{a^2+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

giac [A] time = 1.07, size = 37, normalized size = 1.76

$$-\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2\,a}+\frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a
```

maple [A] time = 0.31, size = 35, normalized size = 1.67

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+x^2}}{x}\right)}{\sqrt{a^2}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

maxima [A] time = 0.50, size = 12, normalized size = 0.57

$$-\frac{\operatorname{arsinh}\left(\frac{a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/abs(x))/a
```

mupad [B] time = 0.09, size = 26, normalized size = 1.24

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a^2+x^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + x^2)^(1/2)),x)
```

```
[Out] atan((a^2 + x^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

sympy [A] time = 1.04, size = 7, normalized size = 0.33

$$-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2),x)
```

```
[Out] -asinh(a/x)/a
```

3.54 $\int \frac{1}{\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x - x^2],x]
[Out] -ArcSin[(1 - 2*x)/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x-x^2}}\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 1-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{3}(1-2x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x - x^2],x]
[Out] -ArcSin[(1 - 2*x)/3]
```

IntegrateAlgebraic [A] time = 0.09, size = 21, normalized size = 1.75

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2+x+2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + x - x^2],x]
[Out] -2*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

fricas [B] time = 0.91, size = 30, normalized size = 2.50

$$-\arctan\left(\frac{\sqrt{-x^2+x+2}\,(2\,x-1)}{2\left(x^2-x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))
```

giac [A] time = 1.08, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{2}{3}\,x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="giac")
[Out] arcsin(2/3*x - 1/3)
```

maple [A] time = 0.29, size = 7, normalized size = 0.58

method	result	size
default	$\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x+2}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-1/3+2/3*x)
```

maxima [A] time = 0.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-2/3*x + 1/3)
```

mupad [B] time = 0.18, size = 6, normalized size = 0.50

$$\mathrm{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^2 + 2)^(1/2),x)
[Out] asin((2*x)/3 - 1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + x + 2), x)
```

3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{619, 215\}$

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 4*x + 3*x^2],x]
```

```
[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{44}}}\,dx, x, -4+6x\right)}{2\sqrt{33}}\\ &= -\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{\sinh^{-1}\left(\frac{3x-2}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(-2 + 3*x)/Sqrt[11]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.08, size = 33, normalized size = 1.74

$$-\frac{\log\left(\sqrt{3}\sqrt{3x^2-4x+5}-3x+2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[5 - 4*x + 3*x^2],x]
```

```
[Out] -(Log[2 - 3*x + Sqrt[3]*Sqrt[5 - 4*x + 3*x^2]]/Sqrt[3])
```

fricas [B] time = 0.97, size = 38, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(-2\sqrt{3}\sqrt{3x^2-4x+5}(3x-2)-18x^2+24x-19\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-2*sqrt(3)*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 18*x^2 + 24*x
- 19)
```

giac [B] time = 1.08, size = 33, normalized size = 1.74

$$-\frac{1}{3}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-4x+5}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - 4*x + 5)) + 2)
```

maple [A] time = 0.40, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2-4x+5}\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

maxima [A] time = 1.00, size = 16, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{11}\sqrt{11}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

mupad [B] time = 0.29, size = 26, normalized size = 1.37

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x-\frac{2}{3}\right)+\sqrt{3x^2-4x+5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 4*x + 5)^(1/2),x)
```

```
[Out] (3^(1/2)*log(3^(1/2)*(x - 2/3) + (3*x^2 - 4*x + 5)^(1/2)))/3
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 - 4x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 - 4*x + 5), x)
```

3.56 $\int \frac{1}{\sqrt{x-x^2}} \, dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x - x^2],x]
[Out] -ArcSin[1 - 2*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x-x^2}} \, dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x - x^2],x]
[Out] -2*ArcSin[Sqrt[1 - x]]
```

IntegrateAlgebraic [B] time = 0.08, size = 18, normalized size = 2.25

$$-2\tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x - x^2],x]
[Out] -2*ArcTan[Sqrt[x - x^2]/x]
```

fricas [B] time = 0.93, size = 16, normalized size = 2.00

$$-2 \arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

giac [A] time = 0.86, size = 6, normalized size = 0.75

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
[Out] arcsin(2*x - 1)
```

maple [A] time = 0.32, size = 7, normalized size = 0.88

method	result	size
default	$\arcsin(-1+2x)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-1+2*x)
```

maxima [A] time = 0.97, size = 6, normalized size = 0.75

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 1)
```

mupad [B] time = 0.16, size = 6, normalized size = 0.75

$$\mathrm{asin}(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^2)^(1/2),x)
[Out] asin(2*x - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+x)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 + x), x)
```

3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=27

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {640, 619, 216}

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx &= -2\sqrt{2+x-x^2}+2\int \frac{1}{\sqrt{2+x-x^2}}\,dx\\ &= -2\sqrt{2+x-x^2}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,1-2x\right)\\ &= -2\sqrt{2+x-x^2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

IntegrateAlgebraic [A] time = 0.17, size = 36, normalized size = 1.33

$$-2\sqrt{-x^2+x+2}-4\tan^{-1}\left(\frac{\sqrt{-x^2+x+2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[2 + x - x^2],x]
[Out] -2*Sqrt[2 + x - x^2] - 4*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

fricas [B] time = 0.83, size = 43, normalized size = 1.59

$$-2\sqrt{-x^2+x+2}-2\arctan\left(\frac{\sqrt{-x^2+x+2}\,(2x-1)}{2\left(x^2-x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x
- 2))
```

giac [A] time = 0.93, size = 21, normalized size = 0.78

$$-2\sqrt{-x^2+x+2}+2\arcsin\left(\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

maple [A] time = 0.30, size = 22, normalized size = 0.81

method	result
default	$2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)-2\sqrt{-x^2+x+2}$
risch	$\frac{2x^2-2x-4}{\sqrt{-x^2+x+2}}+2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$
trager	$-2\sqrt{-x^2+x+2}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

maxima [A] time = 0.97, size = 21, normalized size = 0.78

$$-2\sqrt{-x^2+x+2}-2\arcsin\left(-\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

mupad [B] time = 0.31, size = 40, normalized size = 1.48

$$\mathrm{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)-2\,\sqrt{-x^2+x+2}-\ln\left(x\,1\mathrm{i}+\sqrt{-x^2+x+2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(x - x^2 + 2)^(1/2),x)
```

```
[Out] asin((2*x)/3 - 1/3) - log(x*1i + (x - x^2 + 2)^(1/2) - 1i/2)*1i - 2*(x - x^
2 + 2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{-\left(x-2\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/sqrt(-(x - 2)*(x + 1)), x)
```

3.58 $\int \frac{1}{x\sqrt{2+x-x^2}} \, dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+x-x^2}} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2} \, dx, x, \frac{4+x}{\sqrt{2+x-x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

IntegrateAlgebraic [C] time = 0.10, size = 39, normalized size = 1.22

$$i\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}+\frac{i\sqrt{-x^2+x+2}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] I*Sqrt[2]*ArcTan[x/Sqrt[2] + (I*Sqrt[2 + x - x^2])/Sqrt[2]]
```

fricas [A] time = 0.90, size = 39, normalized size = 1.22

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7x^2-16x-32}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32)
/x^2)
```

giac [B] time = 1.19, size = 71, normalized size = 2.22

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) -
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))
```

maple [A] time = 0.31, size = 25, normalized size = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(4+x)\sqrt{2}}{4\sqrt{-x^2+x+2}}\right)\sqrt{2}}{2}$	25
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2+x+2}-4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

maxima [A] time = 0.96, size = 33, normalized size = 1.03

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)
```

mupad [B] time = 0.34, size = 28, normalized size = 0.88

$$-\frac{\sqrt{2}\,\ln\left(\frac{x+2\,\sqrt{2}\,\sqrt{-x^2+x+2}+4}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - x^2 + 2)^(1/2)),x)
[Out] -(2^(1/2)*log((x + 2*2^(1/2)*(x - x^2 + 2)^(1/2) + 4)/x))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(x-2)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+x+2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {650}

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{2\sqrt{2+x-x^2}}{3(2-x)}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{2\sqrt{-x^2+x+2}}{6-3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*Sqrt[2 + x - x^2])/(6 - 3*x)
```

IntegrateAlgebraic [A] time = 0.18, size = 21, normalized size = 1.00

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

fricas [A] time = 0.72, size = 17, normalized size = 0.81

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

giac [A] time = 0.78, size = 28, normalized size = 1.33

$$-\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

maple [A] time = 0.30, size = 16, normalized size = 0.76

method	result	size
gosper	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
risch	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
trager	$\frac{2\sqrt{-x^2+x+2}}{3(-2+x)}$	18
default	$\frac{2\sqrt{-(-2+x)^2+6-3x}}{3(-2+x)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+x)/(-x^2+x+2)^(1/2)
```

maxima [A] time = 0.96, size = 17, normalized size = 0.81

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

mupad [B] time = 0.22, size = 19, normalized size = 0.90

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x - x^2 + 2)^(1/2)),x)
```

```
[Out] (2*(x - x^2 + 2)^(1/2))/(3*(x - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(x-2)(x+1)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

Rubi [A] time = 0.13, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4401, 2648, 2667, 44, 207}

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \int \left(-\frac{3}{-1+\cos(x)}-\frac{2\csc(x)}{-1+\cos(x)}\right)dx$$
$$= -\left(2\int \frac{\csc(x)}{-1+\cos(x)}\,dx\right)-3\int \frac{1}{-1+\cos(x)}\,dx$$
$$= -\frac{3\sin(x)}{1-\cos(x)}+2\,\text{Subst}\left(\int \frac{1}{(-1-x)(-1+x)^2}\,dx,x,\cos(x)\right)$$
$$= -\frac{3\sin(x)}{1-\cos(x)}+2\,\text{Subst}\left(\int \left(-\frac{1}{2(-1+x)^2}+\frac{1}{2\left(-1+x^2\right)}\right)dx,x,\cos(x)\right)$$
$$= -\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}+\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\cos(x)\right)$$
$$= -\tanh^{-1}(\cos(x))-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.93

$$\frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-3\sin(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) + 
Log[Sin[x/2]] - 3*Sin[x]))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 39, normalized size = 1.39

$$-\frac{(\cos(x)-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-6\sin(x)-2}{2(\cos(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) + 1
/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

giac [A] time = 1.06, size = 31, normalized size = 1.11

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2+6\tan\left(\frac{1}{2}x\right)+1}{2\tan\left(\frac{1}{2}x\right)^2}+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="giac")
[Out] -1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x))
)
```

maple [A] time = 0.14, size = 23, normalized size = 0.82

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{3}{\tan\left(\frac{x}{2}\right)}-\frac{1}{2\tan\left(\frac{x}{2}\right)^{2}}$	23
risch	$\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)\left(10\,e^{ix}-9+3i\right)}{\left(e^{ix}-1\right)^{2}}-\ln\left(e^{ix}+1\right)+\ln\left(e^{ix}-1\right)$	44
norman	$\frac{-\frac{1}{2}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-3\left(\tan^{3}\left(\frac{x}{2}\right)\right)-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^{2}}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*sin(x))/(1-cos(x))/sin(x),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))-3/tan(1/2*x)-1/2/tan(1/2*x)^2
```

maxima [A] time = 0.45, size = 33, normalized size = 1.18

$$-\frac{(\cos(x)+1)^{2}}{2\,\sin(x)^{2}}-\frac{3\,(\cos(x)+1)}{\sin(x)}+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x) +
 1))
```

mupad [B] time = 0.28, size = 22, normalized size = 0.79

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{3\tan\left(\frac{x}{2}\right)+\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*sin(x) + 2)/(sin(x)*(cos(x) - 1)),x)
[Out] log(tan(x/2)) - (3*tan(x/2) + 1/2)/tan(x/2)^2
```

sympy [A] time = 0.82, size = 22, normalized size = 0.79

$$\log\left(\tan\left(\frac{x}{2}\right)\right)-\frac{3}{\tan\left(\frac{x}{2}\right)}-\frac{1}{2\tan^{2}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
[Out] log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{3\sin(x)\cos(x)}{3\cos^2(x)+\sqrt{10}+2}\right)}{\sqrt{10}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 203}

$$\frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{\left(\sqrt{\frac{5}{2}}-1\right)\sin(x)\cos(x)}{\left(\sqrt{\frac{5}{2}}-1\right)\cos^2(x)+1}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[10] - ArcTan[((-1 + Sqrt[5/2])*Cos[x]*Sin[x])/(1 + (-1 + Sqrt[5/2])*
Cos[x]^2)]/Sqrt[10]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+3\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2+5x^2}\,dx,x,\cot(x)\right)\\ &= \frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{\left(-1+\sqrt{\frac{5}{2}}\right)\cos(x)\sin(x)}{1+\left(-1+\sqrt{\frac{5}{2}}\right)\cos^2(x)}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 17, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2 + 3\cos^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.98, size = 31, normalized size = 0.84

$$-\frac{1}{20}\,\sqrt{10}\arctan\left(\frac{7\,\sqrt{10}\,\cos(x)^2 - 2\,\sqrt{10}}{20\,\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(x
)))
```

giac [A] time = 0.82, size = 46, normalized size = 1.24

$$\frac{1}{10}\,\sqrt{10}\left(x + \arctan\left(-\frac{\sqrt{10}\,\sin(2\,x) - 2\,\sin(2\,x)}{\sqrt{10}\,\cos(2\,x) + \sqrt{10} - 2\,\cos(2\,x) + 2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(2
*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

maple [A] time = 0.08, size = 14, normalized size = 0.38

method	result	size
default	$\frac{\sqrt{10}\,\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)}{10}$	14
risch	$\frac{i\sqrt{10}\,\ln\left(e^{2ix}+\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20} - \frac{i\sqrt{10}\,\ln\left(e^{2ix}-\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

maxima [A] time = 0.97, size = 13, normalized size = 0.35

$$\frac{1}{10}\,\sqrt{10}\arctan\left(\frac{1}{5}\,\sqrt{10}\,\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

mupad [B] time = 0.24, size = 26, normalized size = 0.70

$$\frac{\sqrt{10}\ (x - \operatorname{atan}(\tan(x)))}{10} + \frac{\sqrt{10}\ \operatorname{atan}\left(\frac{\sqrt{10}\ \tan(x)}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(x)^2 + 2),x)
```

```
[Out] (10^(1/2)*(x - atan(tan(x))))/10 + (10^(1/2)*atan((10^(1/2)*tan(x))/5))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3\cos^2(x) + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)**2),x)
```

```
[Out] Integral(1/(3*cos(x)**2 + 2), x)
```

3.62 $\int \csc(2x)(1-\tan(x))\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {12}

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(1-\tan(x))\,dx &= \text{Subst}\left(\int \frac{1}{2}\left(-1+\frac{1}{x}\right)dx,x,\tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{x}\right)dx,x,\tan(x)\right)\\ &= \frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.50

$$-\frac{\tan(x)}{2}+\frac{1}{2}\log(\sin(x))-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] -1/2*Log[Cos[x]] + Log[Sin[x]]/2 - Tan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(2x)(1-\tan(x))\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.20, size = 32, normalized size = 2.29

$$\frac{1}{4}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{1}{\tan(x)^2+1}\right)-\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/2*tan(x)
```

giac [A] time = 0.96, size = 11, normalized size = 0.79

$$\frac{1}{2}\log\left(|\tan(x)|\right)-\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x))) - 1/2*tan(x)
```

maple [A] time = 0.13, size = 11, normalized size = 0.79

method	result	size
default	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
norman	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
risch	$-\frac{i}{1+e^{2ix}}-\frac{\ln\left(1+e^{2ix}\right)}{2}+\frac{\ln\left(e^{2ix}-1\right)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tan(x))/sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(tan(x))-1/2*tan(x)
```

maxima [B] time = 0.44, size = 47, normalized size = 3.36

$$-\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}-\frac{1}{4}\,\log\left(\cos\left(2\,x\right)+1\right)+\frac{1}{4}\,\log\left(\cos\left(2\,x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(cos(2*x) + 1
) + 1/4*log(cos(2*x) - 1)
```

mupad [B] time = 0.21, size = 10, normalized size = 0.71

$$\frac{\ln\left(\tan(x)\right)}{2}-\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(x) - 1)/sin(2*x),x)
```

```
[Out] log(tan(x))/2 - tan(x)/2
```

sympy [B] time = 1.48, size = 27, normalized size = 1.93

$$\frac{\log\left(\cos\left(2x\right)-1\right)}{4}-\frac{\log\left(\cos\left(2x\right)+1\right)}{4}-\frac{\sin\left(x\right)}{2\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x)
```

```
[Out] log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.91, size = 45, normalized size = 4.09

$$\frac{1}{4}\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{\tan(x)^2-2\tan(x)+1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*t
an(x) + 1)/(tan(x)^2 + 1))
```

giac [A] time = 1.10, size = 17, normalized size = 1.55

$$\frac{1}{2}\log\left(|\tan(x)+1|\right)-\frac{1}{2}\log\left(|\tan(x)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

maple [A] time = 0.04, size = 4, normalized size = 0.36

method	result	size
derivativedivides	$\operatorname{arctanh}(\tan(x))$	4
default	$\operatorname{arctanh}(\tan(x))$	4
norman	$-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(1+\tan(x))}{2}$	16
risch	$\frac{\ln\left(e^{2ix}+i\right)}{2}-\frac{\ln\left(e^{2ix}-i\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)/(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(tan(x))
```

maxima [A] time = 0.43, size = 15, normalized size = 1.36

$$\frac{1}{2}\log\left(\tan(x)+1\right)-\frac{1}{2}\log\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

mupad [B] time = 0.35, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x)
```

```
[Out] atanh(tan(x))
```

sympy [A] time = 0.17, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan\left(x\right)-1\right)}{2}+\frac{\log\left(\tan\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)/(1-tan(x)**2),x)
```

```
[Out] -log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

3.64 $\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{7}\left(a^2-4\cos^2(x)\right)^{7/4}$$

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 261}

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx &= \text{Subst}\left(\int 2x\left(-4+a^2+4x^2\right)^{3/4}\,dx,x,\sin(x)\right)\\ &= 2\,\text{Subst}\left(\int x\left(-4+a^2+4x^2\right)^{3/4}\,dx,x,\sin(x)\right)\\ &= \frac{1}{7}\left(-4+a^2+4\sin^2(x)\right)^{7/4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.06

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.06, size = 14, normalized size = 0.78

$$\frac{1}{7} \left(a^2 - 4 \cos(x)^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

giac [A] time = 1.00, size = 14, normalized size = 0.78

$$\frac{1}{7} \left(a^2 - 4 \cos(x)^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

maple [A] time = 0.06, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\left(a^2-4\left(\cos^2(x)\right)\right)^{\frac{7}{4}}}{7}$	15
default	$\frac{\left(a^2-4\left(\cos^2(x)\right)\right)^{\frac{7}{4}}}{7}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x,method=_RETURNVERBOSE)
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.78

$$\frac{1}{7} \left(a^2 - 4 \cos(x)^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

mupad [B] time = 0.31, size = 14, normalized size = 0.78

$$\frac{\left(a^2 - 4\cos(x)^2\right)^{7/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(a^2 - 4*cos(x)^2)^(3/4),x)
[Out] (a^2 - 4*cos(x)^2)^(7/4)/7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
```

```
[Out] Timed out
```

3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = $\{12, 261\}$

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right)\\ &= -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 15, normalized size = 0.83

$$-\frac{3}{8}\left(a^2+4\cos(x)^2-4\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")
[Out] -3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

giac [A] time = 0.92, size = 14, normalized size = 0.78

$$-\frac{3}{8}\left(a^2-4\sin(x)^2\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="giac")
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

maple [A] time = 0.13, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{3\left(a^2-4\left(\sin^2(x)\right)\right)^{\frac{2}{3}}}{8}$	15
default	$-\frac{3\left(a^2-4\left(\sin^2(x)\right)\right)^{\frac{2}{3}}}{8}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.78

$$-\frac{3}{8}\left(a^2-4\sin(x)^2\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

mupad [B] time = 0.30, size = 14, normalized size = 0.78

$$-\frac{3\left(a^2-4\sin(x)^2\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x)
[Out] -(3*(a^2 - 4*sin(x)^2)^(2/3))/8
```

sympy [**A**] time = 2.29, size = 17, normalized size = 0.94

$$-\frac{3\left(a^2-4\sin^2\left(x\right)\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

```
[Out] -3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2282, 63, 203}

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + a^(2*x)],x]
```

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,a^{2x}\right)}{2\log(a)}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+a^{2x}}\right)}{\log(a)}\\ &= \frac{\tan^{-1}\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + a^(2*x)],x]
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[-1 + a^(2*x)],x]
[Out] Could not integrate
```

fricas [A] time = 0.86, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

giac [A] time = 0.84, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

maple [A] time = 0.08, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16
default	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+a^(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)
```

maxima [A] time = 0.98, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

mupad [B] time = 0.29, size = 37, normalized size = 2.18

$$-\frac{a^x \operatorname{asin}\left(\frac{1}{a^x}\right) \sqrt{1-\frac{1}{a^{2x}}}}{\ln(a)\sqrt{a^{2x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(2*x) - 1)^(1/2),x)
[Out] -(a^x*asin(1/a^x)*(1 - 1/a^(2*x))^(1/2))/(log(a)*(a^(2*x) - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2x}-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a**(2*x))**(1/2),x)
[Out] Integral(1/sqrt(a**(2*x) - 1), x)
```

3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx$

Optimal. Leaf size=20

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 217, 206}

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, e^{x/2}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^{x/2}}{\sqrt{-1+e^x}}\right)\\
&= 2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(x/2)/Sqrt[-1 + E^x],x]
[Out] Could not integrate
```

fricas [A] time = 0.59, size = 16, normalized size = 0.80

$$-2 \log\left(\sqrt{e^x-1} - e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")
[Out] -2*log(sqrt(e^x - 1) - e^(1/2*x))
```

giac [A] time = 0.93, size = 16, normalized size = 0.80

$$-2 \log\left(-\sqrt{e^x-1} + e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="giac")
[Out] -2*log(-sqrt(e^x - 1) + e^(1/2*x))
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{x}{2}}}{\sqrt{-1+e^x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
[Out] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

maxima [A] time = 0.44, size = 18, normalized size = 0.90

$$2 \log\left(2\sqrt{e^x-1} + 2\,e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")
[Out] 2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

mupad [B] time = 0.34, size = 16, normalized size = 0.80

$$\ln\left(e^x + \sqrt{e^x}\,\sqrt{e^x-1} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x/2)/(exp(x) - 1)^(1/2),x)
```

```
[Out] log(exp(x) + exp(x)^(1/2)*(exp(x) - 1)^(1/2) - 1/2)
```

sympy [**A**] time = 0.74, size = 7, normalized size = 0.35

$$2\operatorname{acosh}\left(e^{\frac{x}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)
```

```
[Out] 2*acosh(exp(x/2))
```

3.68 $\int \frac{\tan^{-1}(x)^n}{1+x^2} dx$

Optimal. Leaf size=12

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4884}

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)^n}{1+x^2} dx = \frac{\tan^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(x)^n}{1+x^2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 12, normalized size = 1.00

$$\frac{\arctan(x)^n \arctan(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")
[Out] arctan(x)^n*arctan(x)/(n + 1)
```

giac [A] time = 0.95, size = 12, normalized size = 1.00

$$\frac{\arctan(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="giac")
[Out] arctan(x)^(n + 1)/(n + 1)
```

maple [A] time = 0.37, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\arctan(x)^{1+n}}{1+n}$	13
default	$\frac{\arctan(x)^{1+n}}{1+n}$	13
risch	$-\frac{i(-\ln(-i(x+i))+\ln(-i(-x+i)))\left(-\frac{i(-\ln(-i(x+i))+\ln(-i(-x+i)))}{2}\right)^{n}}{2(1+n)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)^(1+n)/(1+n)
```

maxima [A] time = 0.43, size = 12, normalized size = 1.00

$$\frac{\arctan(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")
[Out] arctan(x)^(n + 1)/(n + 1)
```

mupad [B] time = 0.23, size = 12, normalized size = 1.00

$$\frac{\mathrm{atan}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)^n/(x^2 + 1),x)
[Out] atan(x)^(n + 1)/(n + 1)
```

sympy [A] time = 1.79, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**n/(x**2+1),x)
[Out] Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))
```

3.69 $\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\,\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4643, 4641}

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\,\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4643

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSin[c*x])^n
/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx &= \frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2-x^2}}\\ &= \frac{2a\sqrt{1-\frac{x^2}{a^2}}\,\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\,\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
[Out] Could not integrate
```

fricas [A] time = 0.60, size = 38, normalized size = 0.90

$$\frac{2}{5}\sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2
```

giac [A] time = 1.08, size = 15, normalized size = 0.36

$$\frac{2\,|a|\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] 2/5*abs(a)*arcsin(x/a)^(5/2)/a
```

maple [A] time = 0.25, size = 38, normalized size = 0.90

method	result	size
default	$\frac{2\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{\frac{a^2-x^2}{a^2}}}{5\sqrt{a^2-x^2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsin(x/a)^(5/2)*a/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asin}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
[Out] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
[Out] Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

3.70 $\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} dx$

Optimal. Leaf size=8

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4642}

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} dx = \frac{1}{2 \cos^{-1}(x)^2}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.90, size = 6, normalized size = 0.75

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2/arccos(x)^2
```

giac [A] time = 0.93, size = 6, normalized size = 0.75

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2/arccos(x)^2
```

maple [A] time = 0.26, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{1}{2 \arccos(x)^2}$	7
default	$\frac{1}{2 \arccos(x)^2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/arccos(x)^2
```

maxima [A] time = 0.99, size = 6, normalized size = 0.75

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2/arccos(x)^2
```

mupad [B] time = 0.35, size = 6, normalized size = 0.75

$$\frac{1}{2\,\mathrm{acos}(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acos(x)^3*(1 - x^2)^(1/2)),x)
[Out] 1/(2*acos(x)^2)
```

sympy [A] time = 2.68, size = 7, normalized size = 0.88

$$\frac{1}{2 \operatorname{acos}^{2}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
[Out] 1/(2*acos(x)**2)
```

3.71 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x \log^2(x)\,dx &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x)\,dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Log[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.92, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^2\log(x)^2 - \frac{1}{2}\,x^2\log(x) + \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

giac [A] time = 0.88, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^2\log(x)^2 - \frac{1}{2}\,x^2\log(x) + \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

maxima [A] time = 0.44, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\,\log(x)^2 - 2\,\log(x) + 1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2\,\left(2\ln(x)^2 - 2\,\ln(x) + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.79

$$\frac{x^2\log{(x)}^2}{2} - \frac{x^2\log{(x)}}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.72 $\int \frac{\log(x)}{x^5}\, dx$

Optimal. Leaf size=17

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/x^5,x]
```

```
[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{x^5}\, dx = -\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/x^5,x]
```

```
[Out] -1/16*1/x^4 - Log[x]/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x^5}\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[x]/x^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.86, size = 11, normalized size = 0.65

$$-\frac{4\log(x)+1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(4*log(x) + 1)/x^4
```

giac [A] time = 0.79, size = 13, normalized size = 0.76

$$-\frac{\log(x)}{4\,x^4}-\frac{1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="giac")
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

maple [A] time = 0.02, size = 11, normalized size = 0.65

method	result	size
norman	$\frac{-\frac{1}{16}-\frac{\ln(x)}{4}}{x^4}$	11
default	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14
risch	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^5,x,method=_RETURNVERBOSE)
[Out] (-1/16-1/4*ln(x))/x^4
```

maxima [A] time = 0.43, size = 13, normalized size = 0.76

$$-\frac{\log(x)}{4\,x^4}-\frac{1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="maxima")
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

mupad [B] time = 0.17, size = 9, normalized size = 0.53

$$-\frac{\ln(x)+\frac{1}{4}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/x^5,x)
[Out] -(log(x) + 1/4)/(4*x^4)
```

sympy [A] time = 0.10, size = 15, normalized size = 0.88

$$-\frac{\log{(x)}}{4x^4}-\frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**5,x)
[Out] -log(x)/(4*x**4) - 1/(16*x**4)
```

3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right) dx$

Optimal. Leaf size=36

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{x^2}{6}-\frac{x}{3}-\frac{1}{3}\log(x-1)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2461, 2455, 263, 43}

$$-\frac{x^2}{6}+\frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{x}{3}-\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[(-1 + x)/x],x]
```

```
[Out] -x/3 - x^2/6 + (x^3*Log[1 - x^(-1)])/3 - Log[1 - x]/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \log\left(\frac{-1+x}{x}\right) dx &= \int x^2 \log\left(1-\frac{1}{x}\right) dx \\
&= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \frac{x}{1-\frac{1}{x}} dx \\
&= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \frac{x^2}{-1+x} dx \\
&= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \left(1+\frac{1}{-1+x}+x\right) dx \\
&= -\frac{x}{3} - \frac{x^2}{6} + \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\log(1-x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{x^2}{6} - \frac{x}{3} - \frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[(-1 + x)/x],x]
[Out] -1/3*x - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Log[(-1 + x)/x],x]
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 28, normalized size = 0.78

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="fricas")
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

giac [B] time = 1.05, size = 70, normalized size = 1.94

$$\frac{\frac{2(x-1)}{x} - 3}{6\left(\frac{x-1}{x} - 1\right)^2} - \frac{\log\left(\frac{x-1}{x}\right)}{3\left(\frac{x-1}{x} - 1\right)^3} - \frac{1}{3}\log\left(\frac{|x-1|}{|x|}\right) + \frac{1}{3}\log\left(\left|\frac{x-1}{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="giac")
[Out] 1/6*(2*(x - 1)/x - 3)/((x - 1)/x - 1)^2 - 1/3*log((x - 1)/x)/((x - 1)/x - 1
)^3 - 1/3*log(abs(x - 1)/abs(x)) + 1/3*log(abs((x - 1)/x - 1))
```

maple [A] time = 0.05, size = 29, normalized size = 0.81

method	result	size
risch	$-\frac{x}{3}-\frac{x^2}{6}-\frac{\ln(-1+x)}{3}+\frac{x^3\ln\left(\frac{-1+x}{x}\right)}{3}$	29
derivativedivides	$\frac{\ln\left(-\frac{1}{x}\right)}{3}-\frac{x}{3}-\frac{x^2}{6}+\frac{\ln\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)\left(\left(1-\frac{1}{x}\right)^2+\frac{3}{x}\right)x^3}{3}$	53
default	$\frac{\ln\left(-\frac{1}{x}\right)}{3}-\frac{x}{3}-\frac{x^2}{6}+\frac{\ln\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)\left(\left(1-\frac{1}{x}\right)^2+\frac{3}{x}\right)x^3}{3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln((-1+x)/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

maxima [A] time = 0.45, size = 28, normalized size = 0.78

$$\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{1}{6}x^2-\frac{1}{3}x-\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

mupad [B] time = 0.35, size = 40, normalized size = 1.11

$$\frac{x^3\ln\left(\frac{x-1}{x}\right)}{3}-\frac{\ln(x(x-1))}{6}-\frac{\ln\left(\frac{x-1}{x}\right)}{6}-\frac{x}{3}-\frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log((x - 1)/x),x)
```

```
[Out] (x^3*log((x - 1)/x))/3 - log(x*(x - 1))/6 - log((x - 1)/x)/6 - x/3 - x^2/6
```

sympy [A] time = 0.13, size = 26, normalized size = 0.72

$$\frac{x^3\log\left(\frac{x-1}{x}\right)}{3}-\frac{x^2}{6}-\frac{x}{3}-\frac{\log(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln((-1+x)/x),x)
```

```
[Out] x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

3.74 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(x)\right)\\ &= \sin(x)-\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5,x]
```

```
[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.89, size = 18, normalized size = 0.95

$$\frac{1}{15}\left(3\,\cos(x)^4+4\,\cos(x)^2+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="fricas")
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

giac [A] time = 0.94, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="giac")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.30, size = 17, normalized size = 0.89

method	result	size
default	$\frac{\left(\frac{8}{3}+\cos^4(x)+\frac{4(\cos^2(x))}{3}\right)\sin(x)}{5}$	17
risch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.00, size = 21, normalized size = 1.11

$$\frac{\sin(x)\cos(x)^4}{5} + \frac{4\sin(x)\cos(x)^2}{15} + \frac{8\sin(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x)
[Out] (8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

sympy [A] time = 0.07, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5,x)
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

3.75 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\sin^2(x)\,dx &= -\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{6}\int \cos^4(x)\,dx\\ &= \frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{8}\int \cos^2(x)\,dx\\ &= \frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{\int 1\,dx}{16}\\ &= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^4(x) \sin^2(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^4*Sin[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 0.88, size = 25, normalized size = 0.74

$$-\frac{1}{48} \left(8 \cos(x)^5 - 2 \cos(x)^3 - 3 \cos(x)\right) \sin(x) + \frac{1}{16} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

giac [A] time = 0.87, size = 22, normalized size = 0.65

$$\frac{1}{16} x - \frac{1}{192} \sin(6x) - \frac{1}{64} \sin(4x) + \frac{1}{64} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

maple [A] time = 0.33, size = 23, normalized size = 0.68

method	result
risch	$\frac{x}{16} - \frac{\sin(6x)}{192} - \frac{\sin(4x)}{64} + \frac{\sin(2x)}{64}$
default	$-\frac{\left(\cos^5(x)\right)\sin(x)}{6} + \frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{24} + \frac{x}{16}$
norman	$\frac{\frac{x}{16}+\frac{47\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{13\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{13\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}-\frac{47\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+\frac{15x\left(\tan^8\left(\frac{x}{2}\right)\right)}{16}+3x(\mathrm{ta}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/16*x-1/192*sin(6*x)-1/64*sin(4*x)+1/64*sin(2*x)
```

maxima [A] time = 0.43, size = 18, normalized size = 0.53

$$\frac{1}{48} \sin(2x)^3 + \frac{1}{16} x - \frac{1}{64} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

mupad [B] time = 0.02, size = 26, normalized size = 0.76

$$\left(\frac{\cos(x)^3}{6} + \frac{\cos(x)}{8}\right)\sin(x)^3 - \frac{\cos(x)\,\sin(x)}{16} + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x)
[Out] x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

sympy [A] time = 0.07, size = 31, normalized size = 0.91

$$\frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**2,x)
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

3.76 $\int \csc^5(x)\,dx$

Optimal. Leaf size=26

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5,x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(x)\,dx &= -\frac{1}{4}\cot(x)\csc^3(x)+\frac{3}{4}\int \csc^3(x)\,dx\\ &= -\frac{3}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)+\frac{3}{8}\int \csc(x)\,dx\\ &= -\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{3}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{3}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{3}{32}\sec^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5,x]
```

```
[Out] (-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]]
)/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.97, size = 69, normalized size = 2.65

$$\frac{6 \cos(x)^3 - 3 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 1}{16 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3*
(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4 -
 2*cos(x)^2 + 1)
```

giac [A] time = 0.99, size = 38, normalized size = 1.46

$$\frac{3 \cos(x)^3 - 5 \cos(x)}{8 \left(\cos(x)^2 - 1\right)^2} - \frac{3}{16} \log(\cos(x) + 1) + \frac{3}{16} \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*log(cos(x) + 1) + 3/16*
log(-cos(x) + 1)
```

maple [A] time = 0.31, size = 26, normalized size = 1.00

method	result	size
default	$\left(-\frac{\left(\csc^3(x)\right)}{4} - \frac{3 \csc(x)}{8}\right) \cot(x) + \frac{3 \ln(\csc(x) - \cot(x))}{8}$	26
norman	$\frac{-\frac{1}{64} - \frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{8} + \frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{8} + \frac{\left(\tan^8\left(\frac{x}{2}\right)\right)}{64}}{\tan\left(\frac{x}{2}\right)^4} + \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$	42
risch	$\frac{3 e^{7ix} - 11 e^{5ix} - 11 e^{3ix} + 3 e^{ix}}{4\left(e^{2ix} - 1\right)^4} + \frac{3 \ln\left(e^{ix} - 1\right)}{8} - \frac{3 \ln\left(e^{ix} + 1\right)}{8}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

maxima [B] time = 0.43, size = 42, normalized size = 1.62

$$\frac{3 \cos(x)^3 - 5 \cos(x)}{8 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right)} - \frac{3}{16} \log(\cos(x) + 1) + \frac{3}{16} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*log(cos(x) +
 1) + 3/16*log(cos(x) - 1)
```

mupad [B] time = 0.29, size = 33, normalized size = 1.27

$$-\frac{3\,\mathrm{atanh}\left(\cos(x)\right)}{8}-\frac{\frac{5\,\cos(x)}{8}-\frac{3\cos(x)^3}{8}}{\cos(x)^4-2\cos(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^5,x)
```

```
[Out] - (3*atanh(cos(x)))/8 - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(cos(x)^4 - 2*cos(x
)^2 + 1)
```

sympy [A] time = 0.15, size = 46, normalized size = 1.77

$$\frac{3\cos^3(x)-5\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}+\frac{3\log(\cos(x)-1)}{16}-\frac{3\log(\cos(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**5,x)
```

```
[Out] (3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log(cos(x) -
1)/16 - 3*log(cos(x) + 1)/16
```

3.77 $\int e^{-x}\sin(x)\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4432}

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/E^x,x]
```

```
[Out] -Cos[x]/(2*E^x) - Sin[x]/(2*E^x)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-x}\sin(x)\,dx=-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.61

$$-\frac{1}{2}e^{-x}(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/E^x,x]
```

```
[Out] -1/2*(Cos[x] + Sin[x])/E^x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-x}\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/E^x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.88, size = 17, normalized size = 0.74

$$-\frac{1}{2}\cos(x)e^{(-x)}-\frac{1}{2}e^{(-x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="fricas")
```

[Out] -1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)

giac [A] time = 0.94, size = 11, normalized size = 0.48

$$-\frac{1}{2}\left(\cos(x)+\sin(x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="giac")
```

[Out] -1/2*(cos(x) + sin(x))*e^(-x)

maple [A] time = 0.04, size = 18, normalized size = 0.78

method	result	size
default	$-\frac{e^{-x}\cos(x)}{2}-\frac{e^{-x}\sin(x)}{2}$	18
norman	$\frac{\left(-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)\right)e^{-x}}{1+\tan^2\left(\frac{x}{2}\right)}$	32
risch	$-\frac{e^{(-1+i)x}}{4}+\frac{ie^{(-1+i)x}}{4}-\frac{e^{(-1-i)x}}{4}-\frac{ie^{(-1-i)x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/exp(x),x,method=_RETURNVERBOSE)
```

[Out] -1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)

maxima [A] time = 0.44, size = 11, normalized size = 0.48

$$-\frac{1}{2}\left(\cos(x)+\sin(x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="maxima")
```

[Out] -1/2*(cos(x) + sin(x))*e^(-x)

mupad [B] time = 0.02, size = 11, normalized size = 0.48

$$-\frac{e^{-x}\left(\cos(x)+\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*sin(x),x)
```

[Out] -(exp(-x)*(cos(x) + sin(x)))/2

sympy [A] time = 0.46, size = 17, normalized size = 0.74

$$-\frac{e^{-x}\sin\left(x\right)}{2}-\frac{e^{-x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x)
```

[Out] -exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2

3.78 $\int e^{2x} \sin(3x) \, dx$

Optimal. Leaf size=27

$$\frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4432}

$$\frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Sin[3*x],x]
```

```
[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{2x} \sin(3x) \, dx = -\frac{3}{13} e^{2x} \cos(3x) + \frac{2}{13} e^{2x} \sin(3x)$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.81

$$\frac{1}{13} e^{2x} (2 \sin(3x) - 3 \cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Sin[3*x],x]
```

```
[Out] (E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{2x} \sin(3x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(2*x)*Sin[3*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.71, size = 21, normalized size = 0.78

$$-\frac{3}{13} \cos(3x) e^{(2x)} + \frac{2}{13} e^{(2x)} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

giac [A] time = 0.88, size = 19, normalized size = 0.70

$$-\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="giac")
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

maple [A] time = 0.05, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{3\,\mathrm{e}^{2x}\cos(3x)}{13}+\frac{2\,\mathrm{e}^{2x}\sin(3x)}{13}$	22
risch	$-\frac{3\,\mathrm{e}^{(2+3i)x}}{26}-\frac{i\mathrm{e}^{(2+3i)x}}{13}-\frac{3\,\mathrm{e}^{(2-3i)x}}{26}+\frac{i\mathrm{e}^{(2-3i)x}}{13}$	36
norman	$\frac{\frac{4\,\mathrm{e}^{2x}\tan\left(\frac{3x}{2}\right)}{13}+\frac{3\,\mathrm{e}^{2x}\left(\tan^{2}\left(\frac{3x}{2}\right)\right)}{13}-\frac{3\,\mathrm{e}^{2x}}{13}}{1+\tan^{2}\left(\frac{3x}{2}\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

maxima [A] time = 0.44, size = 19, normalized size = 0.70

$$-\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{2\,x}\,\left(3\,\cos\left(3\,x\right)-2\,\sin\left(3\,x\right)\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*exp(2*x),x)
[Out] -(exp(2*x)*(3*cos(3*x) - 2*sin(3*x)))/13
```

sympy [A] time = 0.33, size = 26, normalized size = 0.96

$$\frac{2e^{2x}\sin\left(3x\right)}{13} - \frac{3e^{2x}\cos\left(3x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x)
[Out] 2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

3.79 $\int a^x \cos(x)\,dx$

Optimal. Leaf size=31

$$\frac{a^x\sin(x)}{\log^2(a)+1}+\frac{a^x\log(a)\cos(x)}{\log^2(a)+1}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{a^x\sin(x)}{\log^2(a)+1}+\frac{a^x\log(a)\cos(x)}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In] Int[a^x*Cos[x],x]
[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int a^x\cos(x)\,dx=\frac{a^x\cos(x)\log(a)}{1+\log^2(a)}+\frac{a^x\sin(x)}{1+\log^2(a)}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.65

$$\frac{a^x(\log(a)\cos(x)+\sin(x))}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x*Cos[x],x]
[Out] (a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a^x\cos(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[a^x*Cos[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.40, size = 20, normalized size = 0.65

$$\frac{\left(\cos(x)\log(a)+\sin(x)\right)a^x}{\log(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

giac [C] time = 1.14, size = 329, normalized size = 10.61

$$|a|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(a)-\frac{1}{2}\pi x+x\right)\log(|a|)}{\left(\pi-\pi\mathrm{sgn}(a)-2\right)^2+4\log(|a|)^2}-\frac{\left(\pi-\pi\mathrm{sgn}(a)-2\right)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(a)-\frac{1}{2}\pi x+x\right)}{\left(\pi-\pi\mathrm{sgn}(a)-2\right)^2+4\log(|a|)^2}\right)+|a|^x\left(\frac{2\cos\left(\frac{1}{2}\right.}{\left(\pi-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(a
) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) - 1/
2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*cos(1
/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*log(
abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/((pi
- pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) - 1/2*I*abs(a)^x*(-2*I*e^(1/2*I*pi*x
*sgn(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I)
 + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) +
4*log(abs(a)) - 4*I)) - 1/2*I*abs(a)^x*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*p
i*x - I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) + 2*I*e^(-1/2*I*
pi*x*sgn(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4
*I))
```

maple [A] time = 0.06, size = 32, normalized size = 1.03

method	result	size
risch	$\frac{a^x\cos(x)\ln(a)}{1+\ln(a)^2}+\frac{a^x\sin(x)}{1+\ln(a)^2}$	32
norman	$\frac{\frac{\ln(a)e^{x\ln(a)}}{1+\ln(a)^2}+\frac{2e^{x\ln(a)}\tan\left(\frac{x}{2}\right)}{1+\ln(a)^2}-\frac{\ln(a)e^{x\ln(a)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(a)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] a^x*cos(x)*ln(a)/(1+ln(a)^2)+a^x*sin(x)/(1+ln(a)^2)
```

maxima [A] time = 0.44, size = 24, normalized size = 0.77

$$\frac{a^x\cos(x)\log(a)+a^x\sin(x)}{\log(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="maxima")
```

```
[Out] (a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.65

$$\frac{a^x\ (\sin(x)+\ln(a)\ \cos(x))}{\ln(a)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*cos(x),x)
```

```
[Out] (a^x*(sin(x) + log(a)*cos(x)))/(log(a)^2 + 1)
```

sympy [A] time = 1.06, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } a = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } a = e^{i} \\ \frac{a^x\log(a)\cos(x)}{\log(a)^2+1} + \frac{a^x\sin(x)}{\log(a)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(
x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*ex
p(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a**x
*sin(x)/(log(a)**2 + 1), True))
```

3.80 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx=\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[Log[x]],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

giac [A] time = 0.91, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

method	result	size
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

maxima [A] time = 0.43, size = 10, normalized size = 0.59

$$\frac{1}{2}\,x\left(\cos\left(\log(x)\right)+\sin\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

mupad [B] time = 0.20, size = 13, normalized size = 0.76

$$\frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(x)),x)
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

sympy [A] time = 0.40, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

3.81 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8, 2554, 3473}

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \log(\cos(x)) \sec^2(x) \, dx &= \log(\cos(x)) \tan(x) + \int \tan^2(x) \, dx \\ &= \tan(x) + \log(\cos(x)) \tan(x) - \int 1 \, dx \\ &= -x + \tan(x) + \log(\cos(x)) \tan(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\cos(x)) \sec^2(x)\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 22, normalized size = 1.83

$$-\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

giac [A] time = 0.99, size = 12, normalized size = 1.00

$$\log(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

maple [B] time = 0.27, size = 57, normalized size = 4.75

method	result
norman	$\frac{x - x\left(\tan^2\left(\frac{x}{2}\right)\right) - 2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right) - 2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)}{2} - \frac{1}{2}}{1+e^{2ix}} - \frac{\ln\left(1+e^{2ix}\right)}{4} + \frac{\ln(2)}{2+2\,e^{2ix}}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{1+e^{2ix}} + \frac{\pi\,\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(i\cos(x)) - \pi\,\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)\mathrm{csgn}(i\cos(x))^2 - \pi\,\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(i\cos(x))^2 + \pi\mathrm{csgn}(i}{1+e^{2ix}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (x-x*tan(1/2*x)^2-2*tan(1/2*x)*ln((1-tan(1/2*x)^2)/(1+tan(1/2*x)^2))-2*tan(
1/2*x))/(tan(1/2*x)^2-1)
```

maxima [B] time = 0.97, size = 94, normalized size = 7.83

$$-\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x) + 1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x) + 1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.43, size = 35, normalized size = 2.92

$$\tan(x) - 2\,x + \ln\left(\cos(x)\right)\,\tan(x) + \ln\left(\cos(x)\right)\,1\mathrm{i} - \ln\left(\cos\left(2\,x\right) + 1 + \sin\left(2\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))/cos(x)^2,x)
```

```
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

sympy [A] time = 20.57, size = 15, normalized size = 1.25

$$-x + \log\left(\cos\left(x\right)\right)\tan\left(x\right) + \frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

3.82 $\int x \tan^2(x)\, dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3475, 30}

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tan^2(x)\,dx &= x\tan(x) - \int x\,dx - \int \tan(x)\,dx\\ &= -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Tan[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 0.83, size = 21, normalized size = 1.40

$$-\frac{1}{2}\,x^2+x\tan(x)+\frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 1.00, size = 23, normalized size = 1.53

$$-\frac{1}{2}\,x^2+x\tan(x)+\frac{1}{2}\,\log\left(\frac{4}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

maple [A] time = 0.03, size = 20, normalized size = 1.33

method	result	size
norman	$x\tan(x)-\frac{x^2}{2}-\frac{\ln\left(1+\tan^2(x)\right)}{2}$	20
risch	$-\frac{x^2}{2}-2ix+\frac{2ix}{1+e^{2ix}}+\ln\left(1+e^{2ix}\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

maxima [B] time = 0.99, size = 107, normalized size = 7.13

$$-\frac{x^2\cos(2x)^2+x^2\sin(2x)^2+2x^2\cos(2x)+x^2-\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\log\left(\cos(2x)^2+\right.}{2\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(\cos(x))+x\tan(x)-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
```

```
[Out] log(cos(x)) + x*tan(x) - x^2/2
```

sympy [**A**] time = 0.18, size = 19, normalized size = 1.27

$$-\frac{x^2}{2}+x\tan{(x)}-\frac{\log{\left(\tan^2{(x)}+1\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

3.83 $\int \frac{\sin^{-1}(x)}{x^2} \, dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4627, 266, 63, 206}

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{x^2} \, dx &= -\frac{\sin^{-1}(x)}{x}+\int \frac{1}{x\sqrt{1-x^2}} \, dx\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x} \, dx,x,x^2\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(x)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x]/x^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 39, normalized size = 1.77

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

giac [A] time = 0.95, size = 38, normalized size = 1.73

$$-\frac{\arcsin(x)}{x}-\frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right)+\frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 33, normalized size = 1.50

$$-\frac{\arcsin(x)}{x}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.02, size = 20, normalized size = 0.91

$$-\operatorname{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right)-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/x^2,x)
```

```
[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

sympy [A] time = 1.98, size = 22, normalized size = 1.00

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2,x)
```

```
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

3.84 $\int \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4619, 4677, 8}

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)^2 \, dx &= x\sin^{-1}(x)^2-2\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\ &= 2\sqrt{1-x^2}\sin^{-1}(x)+x\sin^{-1}(x)^2-2\int 1 \, dx\\ &= -2x+2\sqrt{1-x^2}\sin^{-1}(x)+x\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-1}(x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

giac [A] time = 0.96, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

maple [A] time = 0.05, size = 24, normalized size = 0.96

method	result	size
default	$-2x + x\arcsin(x)^2 + 2\arcsin(x)\sqrt{-x^2+1}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

mupad [B] time = 0.05, size = 22, normalized size = 0.88

$$2\,\mathrm{asin}(x)\sqrt{1-x^2} + x\left(\mathrm{asin}(x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)^2,x)
```

```
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

sympy [A] time = 0.20, size = 22, normalized size = 0.88

$$x\,\mathrm{asin}^2(x) - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

3.85 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4916, 4846, 260, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(x)}{1+x^2}\,dx &= \int \tan^{-1}(x)\,dx-\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\ &= x\tan^{-1}(x)-\frac{1}{2}\tan^{-1}(x)^2-\int \frac{x}{1+x^2}\,dx\\ &= x\tan^{-1}(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

giac [A] time = 0.99, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

maple [A] time = 0.30, size = 20, normalized size = 0.87

method	result	size
default	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8} + \frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2} + \frac{\ln(-ix+1)^2}{8} + \frac{i\ln(-ix+1)x}{2} - \frac{\ln(x^2+1)}{2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

maxima [A] time = 0.99, size = 24, normalized size = 1.04

$$(x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.22, size = 19, normalized size = 0.83

$$-\frac{\operatorname{atan}(x)^2}{2} + x\operatorname{atan}(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(x))/(x^2 + 1),x)
```

```
[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

sympy [A] time = 0.39, size = 19, normalized size = 0.83

$$x\operatorname{atan}(x) - \frac{\log\left(x^2+1\right)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

3.86 $\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx$

Optimal. Leaf size=38

$$(x+1)\left(\sqrt{\frac{1}{x+1}}\sqrt{\frac{x}{x+1}}+\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)\right)$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4841, 12, 6719, 50, 63, 203}

$$\sqrt{\frac{x}{(x+1)^2}}(x+1)+x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] Sqrt[x/(1 + x)^2]*(1 + x) + x*ArcCos[Sqrt[x/(1 + x)]] - (Sqrt[x/(1 + x)^2]*
(1 + x)*ArcTan[Sqrt[x]])/Sqrt[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx &= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \int \frac{1}{2}\sqrt{\frac{x}{(1+x)^2}}\, dx \\
&= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \frac{1}{2}\int \sqrt{\frac{x}{(1+x)^2}}\, dx \\
&= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{\sqrt{x}}{1+x}\, dx}{2\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{1}{\sqrt{x}(1+x)}\, dx}{2\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \sqrt{x}\right)}{\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.29

$$x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right) + \frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(\sqrt{x} - \tan^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[Sq
rt[x]]))/Sqrt[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.83, size = 30, normalized size = 0.79

$$(x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right) + \sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x + 1)*sqrt(x/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x+1
)]Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep+1)]Warning, choosing ro
ot of [1,0,%%%{2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{-4,[0,2]%%%}+%%%{-6,[0,1]%%%
}+%%%{-2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{2,[4,3]%%%}+%%%{1,[4,2]%%%}+%%%{-2
,[2,3]%%%}+%%%{-4,[2,2]%%%}+%%%{-2,[2,1]%%%}+%%%{1,[0,2]%%%}+%%%{2,[0,1]%%%
}+%%%{1,[0,0]%%%}] at parameters values [86,-97]Warning, choosing root of [
1,0,%%%{2,[2,2]%%%}+%%%{-2,[2,1]%%%}+%%%{-4,[0,2]%%%}+%%%{6,[0,1]%%%}+%%%{-
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-2,[4,3]%%%}+%%%{1,[4,2]%%%}+%%%{2,[2,3]%
%%}+%%%{-4,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[0,2]%%%}+%%%{-2,[0,1]%%%}+%%%{1
,[0,0]%%%}] at parameters values [-82,7]Undef/Unsigned Inf encountered in l
imit
```

maple [A] time = 0.03, size = 45, normalized size = 1.18

method	result	size
default	$x \arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,\left(-\sqrt{x}+\arctan\left(\sqrt{x}\right)\right)}{\sqrt{\frac{x}{1+x}}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos((x/(1+x))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccos((x/(1+x))^(1/2))-1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(-x^(1/
2)+arctan(x^(1/2)))
```

maxima [B] time = 0.97, size = 78, normalized size = 2.05

$$-\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1}-\frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)}-\frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] -arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x
/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x + 1)) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos((x/(x + 1))^(1/2)),x)
```

```
[Out] int(acos((x/(x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos((x/(1+x))**(1/2)),x)
[Out] Integral(acos(sqrt(x/(x + 1))), x)
```

3.87 $\int \left(2x+3x^2\right)^3\,dx$

Optimal. Leaf size=25

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {611}

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^2)^3,x]
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(2x+3x^2\right)^3\,dx &= \int \left(8x^3+36x^4+54x^5+27x^6\right)dx\\ &= 2x^4+\frac{36x^5}{5}+9x^6+\frac{27x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^2)^3,x]
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

IntegrateAlgebraic [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{35}\left(135x^7+315x^6+252x^5+70x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x + 3*x^2)^3,x]
[Out] (70*x^4 + 252*x^5 + 315*x^6 + 135*x^7)/35
```

fricas [A] time = 0.69, size = 21, normalized size = 0.84

$$\frac{27}{7}x^7+9x^6+\frac{36}{5}x^5+2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="fricas")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

giac [A] time = 0.94, size = 21, normalized size = 0.84

$$\frac{27}{7}\,x^{7}+9\,x^{6}+\frac{36}{5}\,x^{5}+2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="giac")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

maple [A] time = 0.25, size = 21, normalized size = 0.84

method	result	size
gosper	$\frac{x^{4}\left(135x^{3}+315x^{2}+252x+70\right)}{35}$	21
default	$2x^{4}+\frac{36}{5}x^{5}+9x^{6}+\frac{27}{7}x^{7}$	22
norman	$2x^{4}+\frac{36}{5}x^{5}+9x^{6}+\frac{27}{7}x^{7}$	22
risch	$2x^{4}+\frac{36}{5}x^{5}+9x^{6}+\frac{27}{7}x^{7}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/35*x^4*(135*x^3+315*x^2+252*x+70)
```

maxima [A] time = 0.44, size = 21, normalized size = 0.84

$$\frac{27}{7}\,x^{7}+9\,x^{6}+\frac{36}{5}\,x^{5}+2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="maxima")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$\frac{27\,x^{7}}{7}+9\,x^{6}+\frac{36\,x^{5}}{5}+2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2)^3,x)
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$\frac{27x^{7}}{7}+9x^{6}+\frac{36x^{5}}{5}+2x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x)**3,x)
[Out] 27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

3.88 $\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {631}

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx &= \int \left(-1+5x-2x^2-14x^3+3x^4+9x^5\right)dx\\ &= -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

IntegrateAlgebraic [A] time = 0.01, size = 30, normalized size = 0.77

$$\frac{1}{30}x\left(45x^5+18x^4-105x^3-20x^2+75x-30\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] (x*(-30 + 75*x - 20*x^2 - 105*x^3 + 18*x^4 + 45*x^5))/30
```

fricas [A] time = 0.76, size = 29, normalized size = 0.74

$$\frac{3}{2}x^6+\frac{3}{5}x^5-\frac{7}{2}x^4-\frac{2}{3}x^3+\frac{5}{2}x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

giac [A] time = 0.89, size = 29, normalized size = 0.74

$$\frac{3}{2}\,x^{6} + \frac{3}{5}\,x^{5} - \frac{7}{2}\,x^{4} - \frac{2}{3}\,x^{3} + \frac{5}{2}\,x^{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="giac")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

maple [A] time = 0.29, size = 29, normalized size = 0.74

method	result	size
gosper	$\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$	29
default	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
norman	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
risch	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(3*x^2+2*x-1)^2,x,method=_RETURNVERBOSE)
[Out] 1/30*x*(45*x^5+18*x^4-105*x^3-20*x^2+75*x-30)
```

maxima [A] time = 0.44, size = 29, normalized size = 0.74

$$\frac{3}{2}\,x^{6} + \frac{3}{5}\,x^{5} - \frac{7}{2}\,x^{4} - \frac{2}{3}\,x^{3} + \frac{5}{2}\,x^{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

mupad [B] time = 0.03, size = 29, normalized size = 0.74

$$\frac{3\,x^6}{2}+\frac{3\,x^5}{5}-\frac{7\,x^4}{2}-\frac{2\,x^3}{3}+\frac{5\,x^2}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)*(2*x + 3*x^2 - 1)^2,x)
[Out] (5*x^2)/2 - x - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

sympy [A] time = 0.07, size = 34, normalized size = 0.87

$$\frac{3x^{6}}{2} + \frac{3x^{5}}{5} - \frac{7x^{4}}{2} - \frac{2x^{3}}{3} + \frac{5x^{2}}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x**2+2*x-1)**2,x)
[Out] 3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

3.89 $\int x^{-1+k} \left(a+bx^k\right)^n \,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+k} \left(a+bx^k\right)^n \,dx = \frac{\left(a+bx^k\right)^{1+n}}{bk(1+n)}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-1+k} \left(a+bx^k\right)^n \,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 27, normalized size = 1.17

$$\frac{\left(bx^k+a\right)\left(bx^k+a\right)^n}{bkn+bk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")
```

```
[Out] (b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

giac [A] time = 0.98, size = 23, normalized size = 1.00

$$\frac{\left(bx^k+a\right)^{n+1}}{bk(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="giac")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

maple [A] time = 0.32, size = 29, normalized size = 1.26

method	result	size
risch	$\frac{\left(a+b\,x^k\right)\left(a+b\,x^k\right)^n}{b(1+n)k}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+k)*(a+b*x^k)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

maxima [A] time = 0.43, size = 23, normalized size = 1.00

$$\frac{\left(bx^k+a\right)^{n+1}}{bk(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

mupad [B] time = 0.73, size = 23, normalized size = 1.00

$$\frac{\left(a+b\,x^k\right)^{n+1}}{b\,k\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(k - 1)*(a + b*x^k)^n,x)
```

```
[Out] (a + b*x^k)^(n + 1)/(b*k*(n + 1))
```

sympy [A] time = 60.37, size = 75, normalized size = 3.26

$$\begin{cases} \frac{\log\left(x\right)}{a} & \text{for}\: b = 0 \wedge k = 0 \wedge n = -1 \\ \frac{a^n x^k}{k} & \text{for}\: b = 0 \\ \left(a+b\right)^n \log\left(x\right) & \text{for}\: k = 0 \\ \frac{\log\left(\frac{a}{b}+x^k\right)}{bk} & \text{for}\: n = -1 \\ \frac{a\left(a+bx^k\right)^n}{bkn+bk} + \frac{bx^k\left(a+bx^k\right)^n}{bkn+bk} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x**k/k, Eq(b,
0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n, -1)), (a*
(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n + b*k), True)
)
```

3.90 $\int \frac{x^3}{1+2x} dx$

Optimal. Leaf size=30

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x),x]
```

```
[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x}\,dx &= \int \left(\frac{1}{8}-\frac{x}{4}+\frac{x^2}{2}-\frac{1}{8(1+2x)}\right) dx\\ &= \frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{1}{16}\log(1+2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.90

$$\frac{1}{96}\left(16x^3-12x^2+12x-6\log(2x+1)+11\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x),x]
```

```
[Out] (11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

IntegrateAlgebraic [A] time = 0.01, size = 26, normalized size = 0.87

$$\frac{1}{24}x\left(4x^2-3x+3\right)-\frac{1}{16}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 + 2*x),x]
```

```
[Out] (x*(3 - 3*x + 4*x^2))/24 - Log[1 + 2*x]/16
```

fricas [A] time = 0.76, size = 22, normalized size = 0.73

$$\frac{1}{6}x^3-\frac{1}{8}x^2+\frac{1}{8}x-\frac{1}{16}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

giac [A] time = 0.99, size = 23, normalized size = 0.77

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(|2x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="giac")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))
```

maple [A] time = 0.28, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
norman	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
meijerg	$\frac{x(16x^2-12x+12)}{96}-\frac{\ln(1+2x)}{16}$	23
risch	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+2*x),x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

maxima [A] time = 0.43, size = 22, normalized size = 0.73

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="maxima")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.67

$$\frac{x}{8} - \frac{\ln\left(x+\frac{1}{2}\right)}{16} - \frac{x^2}{8} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x + 1),x)
[Out] x/8 - log(x + 1/2)/16 - x^2/8 + x^3/6
```

sympy [A] time = 0.08, size = 20, normalized size = 0.67

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log(2x+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+2*x),x)
[Out] x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

3.91 $\int \frac{x^6}{2+3x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 203}

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2),x]
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{2+3x^2} \, dx &= \int \left(\frac{4}{27}-\frac{2x^2}{9}+\frac{x^4}{3}-\frac{8}{27\left(2+3x^2\right)}\right) dx\\
&= \frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{8}{27}\int \frac{1}{2+3x^2} \, dx\\
&= \frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{1}{405}\left(27x^5-30x^3+60x-20\sqrt{6}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2),x]
[Out] (60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.95

$$\frac{1}{135}x\left(9x^4-10x^2+20\right)-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(2 + 3*x^2),x]
[Out] (x*(20 - 10*x^2 + 9*x^4))/135 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

fricas [A] time = 0.92, size = 32, normalized size = 0.78

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="fricas")
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) +
4/27*x
```

giac [A] time = 0.85, size = 26, normalized size = 0.63

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="giac")
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

maple [A] time = 0.28, size = 27, normalized size = 0.66

method	result	size
default	$\frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
risch	$\frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
meijerg	$\frac{2\sqrt{2}\sqrt{3}\left(\frac{x\sqrt{2}\sqrt{3}\left(\frac{189}{4}x^4-\frac{105}{2}x^2+105\right)}{105}-2\arctan\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)\right)}{81}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

maxima [A] time = 0.97, size = 26, normalized size = 0.63

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="maxima")
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.78

$$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4\sqrt{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2),x)
```

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*2^(1/2)*3^(1/2)*atan((2^(1/2)*3^(1/2)*x
)/2))/81
```

sympy [**A**] time = 0.10, size = 34, normalized size = 0.83

$$\frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2),x)
```

```
[Out] x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

3.92 $\int \frac{1}{2-7x+3x^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + 3*x^2)^(-1),x]
[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-7x+3x^2}\,dx &= \frac{3}{5}\int \frac{1}{-6+3x}\,dx-\frac{3}{5}\int \frac{1}{-1+3x}\,dx\\ &= -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + 3*x^2)^(-1),x]
[Out] -1/5*Log[1 - 3*x] + Log[2 - x]/5
```

IntegrateAlgebraic [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{1}{5}\log(x-2)-\frac{1}{5}\log(3x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 7*x + 3*x^2)^(-1),x]
[Out] Log[-2 + x]/5 - Log[-1 + 3*x]/5
```

fricas [A] time = 0.78, size = 15, normalized size = 0.71

$$-\frac{1}{5}\log\left(3x-1\right)+\frac{1}{5}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

giac [A] time = 0.82, size = 17, normalized size = 0.81

$$-\frac{1}{5}\log\left(|3x-1|\right)+\frac{1}{5}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="giac")
[Out] -1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))
```

maple [A] time = 0.28, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
norman	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
risch	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-7*x+2),x,method=_RETURNVERBOSE)
[Out] -1/5*ln(-1+3*x)+1/5*ln(-2+x)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.71

$$-\frac{1}{5}\log\left(3x-1\right)+\frac{1}{5}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

mupad [B] time = 0.09, size = 8, normalized size = 0.38

$$-\frac{2\,\mathrm{atanh}\left(\frac{6x}{5}-\frac{7}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 7*x + 2),x)
[Out] -(2*atanh((6*x)/5 - 7/5))/5
```

sympy [A] time = 0.11, size = 14, normalized size = 0.67

$$\frac{\log\left(x-2\right)}{5}-\frac{\log\left(x-\frac{1}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-7*x+2),x)
```

```
[Out] log(x - 2)/5 - log(x - 1/3)/5
```

3.93 $\int \frac{-1+3x}{1-x+x^2} \, dx$

Optimal. Leaf size=33

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x)/(1 - x + x^2),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+3x}{1-x+x^2} \, dx &= \frac{1}{2}\int \frac{1}{1-x+x^2} \, dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2} \, dx\\ &= \frac{3}{2}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x)/(1 - x + x^2),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

IntegrateAlgebraic [A] time = 0.02, size = 36, normalized size = 1.09

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x)/(1 - x + x^2),x]
[Out] -(ArcTan[1/Sqrt[3] - (2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

fricas [A] time = 0.84, size = 28, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

giac [A] time = 0.86, size = 28, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

maple [A] time = 0.42, size = 29, normalized size = 0.88

method	result	size
default	$\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+3*x)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

maxima [A] time = 1.00, size = 28, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.91

$$\frac{3\,\ln\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 1)/(x^2 - x + 1),x)
```

```
[Out] (3*log(x^2 - x + 1))/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.09

$$\frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x**2-x+1),x)
```

```
[Out] 3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$

Optimal. Leaf size=25

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {703, 634, 618, 204, 628}

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2}{5+2x+x^2}\,dx = x + \int \frac{-5-2x}{5+2x+x^2}\,dx$$
$$= x - 3\int \frac{1}{5+2x+x^2}\,dx - \int \frac{2+2x}{5+2x+x^2}\,dx$$
$$= x - \log\left(5+2x+x^2\right) + 6\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx, x, 2+2x\right)$$
$$= x - \frac{3}{2}\tan^{-1}\left(\frac{1+x}{2}\right) - \log\left(5+2x+x^2\right)$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\log\left(x^2+2x+5\right) + x - \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(5 + 2*x + x^2),x]
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 1.08

$$-\log\left(x^2+2x+5\right) + x - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(5 + 2*x + x^2),x]
[Out] x - (3*ArcTan[1/2 + x/2])/2 - Log[5 + 2*x + x^2]
```

fricas [A] time = 0.81, size = 21, normalized size = 0.84

$$x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

giac [A] time = 0.90, size = 21, normalized size = 0.84

$$x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="giac")
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

maple [A] time = 0.40, size = 22, normalized size = 0.88

method	result	size

default	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^2+2x+5\right)$	22
risch	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^2+2x+5\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+5),x,method=_RETURNVERBOSE)
```

```
[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

maxima [A] time = 0.97, size = 21, normalized size = 0.84

$$x-\frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)-\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$x-\ln\left(x^2+2x+5\right)-\frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x + x^2 + 5),x)
```

```
[Out] x - log(2*x + x^2 + 5) - (3*atan(x/2 + 1/2))/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.88

$$x-\log\left(x^2+2x+5\right)-\frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+5),x)
```

```
[Out] x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx$

Optimal. Leaf size=47

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1594, 1628, 634, 618, 204, 628}

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -x^2/2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx &= \int \frac{x^2\left(4-5x+6x^2\right)}{1-x+2x^2}\,dx\\
&= \int \left(-x+3x^2+\frac{x}{1-x+2x^2}\right)dx\\
&= -\frac{x^2}{2}+x^3+\int \frac{x}{1-x+2x^2}\,dx\\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\int \frac{1}{1-x+2x^2}\,dx+\frac{1}{4}\int \frac{-1+4x}{1-x+2x^2}\,dx\\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\log\left(1-x+2x^2\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,-1+4x\right)\\
&= -\frac{x^2}{2}+x^3-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{4x-1}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -1/2*x^2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2
]/4
```

IntegrateAlgebraic [A] time = 0.03, size = 55, normalized size = 1.17

$$\frac{1}{4}\log\left(2x^2-x+1\right)+\frac{1}{2}\left(2x^3-x^2\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{7}}-\frac{4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] (-x^2 + 2*x^3)/2 - ArcTan[1/Sqrt[7] - (4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 -
x + 2*x^2]/4
```

fricas [A] time = 0.57, size = 38, normalized size = 0.81

$$x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

giac [A] time = 0.93, size = 38, normalized size = 0.81

$$x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="giac")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

maple [A] time = 0.41, size = 39, normalized size = 0.83

method	result	size
default	$x^3-\frac{x^2}{2}+\frac{\ln(2x^2-x+1)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{14}$	39
risch	$x^3-\frac{x^2}{2}+\frac{\ln(16x^2-8x+8)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{14}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(4*x-1)*7^(1/2))
```

maxima [A] time = 0.95, size = 38, normalized size = 0.81

$$x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

mupad [B] time = 0.20, size = 40, normalized size = 0.85

$$\frac{\ln\left(2\,x^2-x+1\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\,\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{14}-\frac{x^2}{2}+x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 5*x^3 + 6*x^4)/(2*x^2 - x + 1),x)
```

```
[Out] log(2*x^2 - x + 1)/4 + (7^(1/2)*atan((4*7^(1/2)*x)/7 - 7^(1/2)/7))/14 - x^2
/2 + x^3
```

sympy [A] time = 0.12, size = 46, normalized size = 0.98

$$x^3-\frac{x^2}{2}+\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

```
[Out] x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*x/7 - sqrt
(7)/7)/14
```

3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1594, 1628}

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx &= \int \frac{-1+x+x^2}{x\left(-6+x+x^2\right)} \, dx\\ &= \int \left(\frac{1}{2(-2+x)}+\frac{1}{6x}+\frac{1}{3(3+x)}\right) dx\\ &= \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{1}{2}\log(x-2)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
[Out] Log[-2 + x]/2 + Log[x]/6 + Log[3 + x]/3
```

fricas [A] time = 0.78, size = 17, normalized size = 0.68

$$\frac{1}{3}\log\left(x+3\right)+\frac{1}{2}\log\left(x-2\right)+\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

giac [A] time = 0.88, size = 20, normalized size = 0.80

$$\frac{1}{3}\log\left(|x+3|\right)+\frac{1}{2}\log\left(|x-2|\right)+\frac{1}{6}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] 1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))
```

maple [A] time = 0.03, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$	18
norman	$\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$	18
risch	$\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(3+x)+1/6*ln(x)+1/2*ln(-2+x)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.68

$$\frac{1}{3}\log\left(x+3\right)+\frac{1}{2}\log\left(x-2\right)+\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

mupad [B] time = 0.23, size = 17, normalized size = 0.68

$$\frac{\ln\left(x-2\right)}{2}+\frac{\ln\left(x+3\right)}{3}+\frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/(x^2 - 6*x + x^3),x)
[Out] log(x - 2)/2 + log(x + 3)/3 + log(x)/6
```

sympy [A] time = 0.14, size = 17, normalized size = 0.68

$$\frac{\log\left(x\right)}{6}+\frac{\log\left(x-2\right)}{2}+\frac{\log\left(x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(x**3+x**2-6*x),x)
```

```
[Out] log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$

Optimal. Leaf size=33

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2074}

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

Antiderivative was successfully verified.

```
[In] Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx &= \int \left(-\frac{9}{2(a-x)}+\frac{17}{2a-x}-\frac{35}{2(3a-x)}\right)dx\\ &= \frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$\frac{35}{2}\log(x-3a)-17\log(x-2a)+\frac{9}{2}\log(x-a)$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
[Out] (35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 +
x^3),x]
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

fricas [A] time = 0.73, size = 25, normalized size = 0.76

$$\frac{9}{2}\log\left(-a+x\right)-17\log\left(-2a+x\right)+\frac{35}{2}\log\left(-3a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
fricas")
```

```
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

giac [A] time = 0.97, size = 28, normalized size = 0.85

$$\frac{9}{2} \log\left(|-a+x|\right) - 17 \log\left(|-2\,a+x|\right) + \frac{35}{2} \log\left(|-3\,a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
giac")
```

```
[Out] 9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))
```

maple [A] time = 0.04, size = 26, normalized size = 0.79

method	result	size
risch	$-17\ln\left(-2a+x\right)+\frac{9\ln(-a+x)}{2}+\frac{35\ln(-3a+x)}{2}$	26
default	$\frac{9\ln(a-x)}{2}-17\ln\left(2a-x\right)+\frac{35\ln(3a-x)}{2}$	30
norman	$\frac{9\ln(a-x)}{2}-17\ln\left(2a-x\right)+\frac{35\ln(3a-x)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x,method=_RETURNVERB
OSE)
```

```
[Out] -17*ln(-2*a+x)+9/2*ln(-a+x)+35/2*ln(-3*a+x)
```

maxima [A] time = 0.44, size = 25, normalized size = 0.76

$$\frac{9}{2} \log\left(-a+x\right) - 17 \log\left(-2\,a+x\right) + \frac{35}{2} \log\left(-3\,a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
maxima")
```

```
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

mupad [B] time = 0.09, size = 25, normalized size = 0.76

$$\frac{9\,\ln\left(x-a\right)}{2} - 17\,\ln\left(x-2\,a\right) + \frac{35\,\ln\left(x-3\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(11*a^2 - 7*a*x + 5*x^2)/(6*a*x^2 - 11*a^2*x + 6*a^3 - x^3),x)
```

```
[Out] (9*log(x - a))/2 - 17*log(x - 2*a) + (35*log(x - 3*a))/2
```

sympy [A] time = 0.26, size = 26, normalized size = 0.79

$$\frac{35\log\left(-3a+x\right)}{2} - 17\log\left(-2a+x\right) + \frac{9\log\left(-a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

```
[Out] 35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1673, 1161, 616, 31, 1107}

$$\frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(2-x)+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/2 + Log[1 + x]/2 - Log[2 + x]/2 + Log[1 - x^2]/6
 - Log[4 - x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx &= -\int \frac{x}{4-5x^2+x^4}\,dx + \int \frac{2+x^2}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\int \frac{1}{2-3x+x^2}\,dx + \frac{1}{2}\int \frac{1}{2+3x+x^2}\,dx - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right)\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{1}{2}\int \frac{1}{-2+x}\,dx - \frac{1}{2} \\
&= -\frac{1}{2}\log(1-x) + \frac{1}{2}\log(2-x) + \frac{1}{2}\log(1+x) - \frac{1}{2}\log(2+x) + \frac{1}{6}\log\left(1-x^2\right) - \frac{1}{6}\log\left(4\right.
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$
-\frac{1}{3}\log(1-x) + \frac{1}{3}\log(2-x) + \frac{2}{3}\log(x+1) - \frac{2}{3}\log(x+2)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
[Out] -1/3*Log[1 - x] + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 0.57

$$
\frac{2}{3}\tanh^{-1}(3-2x) - \frac{4}{3}\tanh^{-1}(2x+3)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
[Out] (2*ArcTanh[3 - 2*x])/3 - (4*ArcTanh[3 + 2*x])/3
```

fricas [A] time = 0.72, size = 25, normalized size = 0.68

$$
-\frac{2}{3}\log(x+2) + \frac{2}{3}\log(x+1) - \frac{1}{3}\log(x-1) + \frac{1}{3}\log(x-2)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

giac [A] time = 0.99, size = 29, normalized size = 0.78

$$
-\frac{2}{3}\log(|x+2|) + \frac{2}{3}\log(|x+1|) - \frac{1}{3}\log(|x-1|) + \frac{1}{3}\log(|x-2|)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] -2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log(
abs(x - 2))
```

maple [A] time = 0.04, size = 26, normalized size = 0.70

method	result	size

default	$\frac{\ln(-2+x)}{3} - \frac{2\ln(2+x)}{3} - \frac{\ln(-1+x)}{3} + \frac{2\ln(1+x)}{3}$	26
norman	$\frac{\ln(-2+x)}{3} - \frac{2\ln(2+x)}{3} - \frac{\ln(-1+x)}{3} + \frac{2\ln(1+x)}{3}$	26
risch	$\frac{\ln(-2+x)}{3} - \frac{2\ln(2+x)}{3} - \frac{\ln(-1+x)}{3} + \frac{2\ln(1+x)}{3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-2+x)-2/3*ln(2+x)-1/3*ln(-1+x)+2/3*ln(1+x)
```

maxima [A] time = 0.45, size = 25, normalized size = 0.68

$$-\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

mupad [B] time = 0.06, size = 29, normalized size = 0.78

$$\frac{2\,\mathrm{atanh}\left(\frac{64}{3\,(24\,x-16)}-\frac{5}{3}\right)}{3}+\frac{4\,\mathrm{atanh}\left(\frac{128}{3\,(48\,x+32)}+\frac{5}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] (2*atanh(64/(3*(24*x - 16)) - 5/3))/3 + (4*atanh(128/(3*(48*x + 32)) + 5/3)
)/3
```

sympy [A] time = 0.19, size = 29, normalized size = 0.78

$$\frac{\log(x-2)}{3}-\frac{\log(x-1)}{3}+\frac{2\log(x+1)}{3}-\frac{2\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1166, 207}

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx &= \int \frac{1}{-3+x^2} \, dx + \int \frac{1}{-2+x^2} \, dx\\ &= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [B] time = 0.02, size = 69, normalized size = 2.23

$$\frac{1}{12}\left(3\sqrt{2}\log\left(\sqrt{2}-x\right)+2\sqrt{3}\log\left(\sqrt{3}-x\right)-3\sqrt{2}\log\left(x+\sqrt{2}\right)-2\sqrt{3}\log\left(x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] (3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[Sq
rt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

IntegrateAlgebraic [A] time = 0.04, size = 31, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

fricas [B] time = 0.88, size = 51, normalized size = 1.65

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2 -
 2*sqrt(3)*x + 3)/(x^2 - 3))
```

giac [B] time = 0.99, size = 59, normalized size = 1.90

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}\right|}{\left|2x+2\sqrt{2}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*sqrt(2)*lo
g(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

maple [A] time = 0.04, size = 26, normalized size = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26
risch	$\frac{\sqrt{3}\ \ln\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\ \ln\left(x+\sqrt{3}\right)}{6}+\frac{\sqrt{2}\ \ln\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\ \ln\left(x+\sqrt{2}\right)}{4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5)/(x^4-5*x^2+6),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 0.98, size = 43, normalized size = 1.39

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2)
)/(x + sqrt(2)))
```

mupad [B] time = 0.08, size = 25, normalized size = 0.81

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x)
```

```
[Out] - (2^(1/2)*atanh((2^(1/2)*x)/2))/2 - (3^(1/2)*atanh((3^(1/2)*x)/3))/3
```

sympy **[A]** time = 0.58, size = 60, normalized size = 1.94

$$\frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

```
[Out] sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x - s
qrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {180}

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(\frac{1}{6(-4+x)}-\frac{1}{2(-3+x)}+\frac{1}{2(-2+x)}-\frac{1}{6(-1+x)}\right)dx\\ &= -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

IntegrateAlgebraic [A] time = 0.01, size = 25, normalized size = 0.61

$$\frac{1}{6}\log(x-4)-\frac{1}{6}\log(x-1)-\tanh^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -ArcTanh[5 - 2*x] + Log[-4 + x]/6 - Log[-1 + x]/6
```

fricas [A] time = 0.90, size = 25, normalized size = 0.61

$$-\frac{1}{6}\log(x-1)+\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x-3)+\frac{1}{6}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

giac [A] time = 0.89, size = 29, normalized size = 0.71

$$-\frac{1}{6}\,\log\left(|x-1|\right)+\frac{1}{2}\,\log\left(|x-2|\right)-\frac{1}{2}\,\log\left(|x-3|\right)+\frac{1}{6}\,\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(x - 1)) + 1/2*log(abs(x - 2)) - 1/2*log(abs(x - 3)) + 1/6*log(
abs(x - 4))
```

maple [A] time = 0.33, size = 26, normalized size = 0.63

method	result	size
default	$\frac{\ln(-2+x)}{2}+\frac{\ln(-4+x)}{6}-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}$	26
norman	$\frac{\ln(-2+x)}{2}+\frac{\ln(-4+x)}{6}-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}$	26
risch	$\frac{\ln(-2+x)}{2}+\frac{\ln(-4+x)}{6}-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-2+x)+1/6*ln(-4+x)-1/2*ln(-3+x)-1/6*ln(-1+x)
```

maxima [A] time = 0.43, size = 25, normalized size = 0.61

$$-\frac{1}{6}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(x-2\right)-\frac{1}{2}\,\log\left(x-3\right)+\frac{1}{6}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

mupad [B] time = 0.25, size = 15, normalized size = 0.37

$$\mathrm{atanh}\left(2\,x-5\right)-\frac{\mathrm{atanh}\left(\frac{2\,x}{3}-\frac{5}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x)
```

```
[Out] atanh(2*x - 5) - atanh((2*x)/3 - 5/3)/3
```

sympy [A] time = 0.19, size = 26, normalized size = 0.63

$$\frac{\log\left(x-4\right)}{6}-\frac{\log\left(x-3\right)}{2}+\frac{\log\left(x-2\right)}{2}-\frac{\log\left(x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

3.101 $\int \frac{1+x^2}{(-1+x)^3} \, dx$

Optimal. Leaf size=25

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(-1+x)^3} \, dx &= \int \left(\frac{2}{(-1+x)^3} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} \right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.64

$$\frac{1-2x}{(x-1)^2} + \log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] (1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 0.64

$$\frac{1-2x}{(x-1)^2} + \log(x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] (1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

fricas [A] time = 0.77, size = 29, normalized size = 1.16

$$\frac{\left(x^2 - 2x + 1\right)\log(x-1) - 2x + 1}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.99, size = 18, normalized size = 0.72

$$-\frac{2\,x-1}{(x-1)^2}+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -(2*x - 1)/(x - 1)^2 + log(abs(x - 1))
```

maple [A] time = 0.30, size = 17, normalized size = 0.68

method	result	size
norman	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
risch	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
default	$-\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\ln(-1+x)$	20
meijerg	$-\frac{x(2-x)}{2(1-x)^2}+\frac{x(-9x+6)}{6(1-x)^2}+\ln(1-x)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-1+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1-2*x)/(-1+x)^2+ln(-1+x)
```

maxima [A] time = 0.43, size = 22, normalized size = 0.88

$$-\frac{2\,x-1}{x^2-2\,x+1}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

mupad [B] time = 0.17, size = 22, normalized size = 0.88

$$\ln\left(x-1\right)-\frac{2\,x-1}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x - 1)^3,x)
```

```
[Out] log(x - 1) - (2*x - 1)/(x^2 - 2*x + 1)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.68

$$\frac{1-2x}{x^2-2x+1}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-1+x)**3,x)
```

```
[Out] (1 - 2*x)/(x**2 - 2*x + 1) + log(x - 1)
```

3.102 $\int \frac{x^5}{(3+x)^2}\,dx$

Optimal. Leaf size=36

$$\frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+\frac{243}{x+3}+405\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+\frac{243}{x+3}+405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + x)^2,x]
```

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(3+x)^2}\,dx &= \int \left(-108+27x-6x^2+x^3-\frac{243}{(3+x)^2}+\frac{405}{3+x}\right)dx\\ &= -108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4}\left(x^4-8x^3+54x^2-432x+\frac{972}{x+3}-2079\right)+405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 1.08

$$\frac{x^5-5x^4+30x^3-270x^2-1296x+972}{4(x+3)}+405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(3 + x)^2,x]
```

```
[Out] (972 - 1296*x - 270*x^2 + 30*x^3 - 5*x^4 + x^5)/(4*(3 + x)) + 405*Log[3 + x
]
```

fricas [A] time = 0.79, size = 39, normalized size = 1.08

$$\frac{x^5 - 5\,x^4 + 30\,x^3 - 270\,x^2 + 1620\,(x+3)\log(x+3) - 1296\,x + 972}{4\,(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)
```

giac [A] time = 1.04, size = 45, normalized size = 1.25

$$-\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

maple [A] time = 0.28, size = 33, normalized size = 0.92

method	result	size
default	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\ln(3+x)$	33
risch	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\ln(3+x)$	33
norman	$\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x} + 405\ln(3+x)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)} + 405\ln\left(1+\frac{x}{3}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

maxima [A] time = 0.43, size = 32, normalized size = 0.89

$$\frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.89

$$405\,\ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x + 3)^2,x)
```

```
[Out] 405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

sympy [A] time = 0.09, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3+x)**2,x)
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$

Optimal. Leaf size=41

$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2074}

$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x]
)/64
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx &= \int \left(\frac{133}{4(-3+x)^3}+\frac{407}{16(-3+x)^2}+\frac{313}{64(-3+x)}+\frac{7}{64(1+x)}\right)dx\\ &= -\frac{133}{8(3-x)^2}+\frac{407}{16(3-x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.90

$$-\frac{407}{16(x-3)}-\frac{133}{8(x-3)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 +
x])/64
```

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 0.76

$$\frac{955-407x}{16(x-3)^2}+\frac{313}{64}\log(x-3)+\frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] (955 - 407*x)/(16*(-3 + x)^2) + (313*Log[-3 + x])/64 + (7*Log[1 + x])/64
```

fricas [A] time = 0.89, size = 45, normalized size = 1.10

$$\frac{7\left(x^{2}-6x+9\right)\log\left(x+1\right)+313\left(x^{2}-6x+9\right)\log\left(x-3\right)-1628x+3820}{64\left(x^{2}-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")
```

```
[Out] 1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628*
x + 3820)/(x^2 - 6*x + 9)
```

giac [A] time = 0.88, size = 27, normalized size = 0.66

$$-\frac{407x-955}{16\left(x-3\right)^{2}}+\frac{7}{64}\log\left(|x+1|\right)+\frac{313}{64}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="giac")
```

```
[Out] -1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x - 3
))
```

maple [A] time = 0.03, size = 25, normalized size = 0.61

method	result	size
norman	$\frac{-\frac{407x}{16}+\frac{955}{16}}{(-3+x)^2}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	25
default	$-\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	28
risch	$\frac{-\frac{407x}{16}+\frac{955}{16}}{x^2-6x+9}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x,method=_RETURNVERBOSE)
```

```
[Out] (-407/16*x+955/16)/(-3+x)^2+313/64*ln(-3+x)+7/64*ln(1+x)
```

maxima [A] time = 0.45, size = 30, normalized size = 0.73

$$-\frac{407x-955}{16\left(x^{2}-6x+9\right)}+\frac{7}{64}\log\left(x+1\right)+\frac{313}{64}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")
```

```
[Out] -1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)
```

mupad [B] time = 0.06, size = 30, normalized size = 0.73

$$\frac{7\ln\left(x+1\right)}{64}+\frac{313\ln\left(x-3\right)}{64}-\frac{\frac{407x}{16}-\frac{955}{16}}{x^{2}-6x+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3 - 2)/(18*x^2 - 8*x^3 + x^4 - 27),x)
```

```
[Out] (7*log(x + 1))/64 + (313*log(x - 3))/64 - ((407*x)/16 - 955/16)/(x^2 - 6*x
+ 9)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.76

$$\frac{955-407x}{16x^2-96x+144}+\frac{313\log(x-3)}{64}+\frac{7\log(x+1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

```
[Out] (955 - 407*x)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64
```

3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx$

Optimal. Leaf size=27

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1620}

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx &= \int \left(-\frac{99}{(3+x)^2}+\frac{264}{3+x}-\frac{181}{(4+x)^2}-\frac{263}{4+x}\right)dx\\ &= -\frac{99}{3+x}-\frac{181}{4+x}+264\log(3+x)-263\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 1.07

$$\frac{280x+939}{(x+3)(x+4)}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] (939 + 280*x)/((3 + x)*(4 + x)) + 264*Log[3 + x] - 263*Log[4 + x]
```

fricas [A] time = 0.85, size = 45, normalized size = 1.67

$$-\frac{263\left(x^2+7x+12\right)\log\left(x+4\right)-264\left(x^2+7x+12\right)\log\left(x+3\right)-280\,x-939}{x^2+7\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")
```

```
[Out] -(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x + 3) - 280*x
 - 939)/(x^2 + 7*x + 12)
```

giac [A] time = 0.86, size = 37, normalized size = 1.37

$$\frac{181}{x+4} - \frac{99}{\frac{1}{x+4}-1} + \log\left(|x+4|\right) + 264\,\log\left(\left|-\frac{1}{x+4}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="giac")
```

```
[Out] 181/(x + 4) - 99/(1/(x + 4) - 1) + log(abs(x + 4)) + 264*log(abs(-1/(x + 4)
 + 1))
```

maple [A] time = 0.31, size = 28, normalized size = 1.04

method	result	size
default	$\frac{99}{3+x}+\frac{181}{4+x}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	28
norman	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30
risch	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

maxima [A] time = 0.43, size = 29, normalized size = 1.07

$$\frac{280\,x+939}{x^2+7\,x+12} - 263\,\log\left(x+4\right) + 264\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")
```

```
[Out] (280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

mupad [B] time = 0.19, size = 29, normalized size = 1.07

$$264\,\ln\left(x+3\right) - 263\,\ln\left(x+4\right) + \frac{280\,x+939}{x^2+7\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 6*x^2 + x^3 - 9)/((x + 3)^2*(x + 4)^2),x)
```

```
[Out] 264*log(x + 3) - 263*log(x + 4) + (280*x + 939)/(7*x + x^2 + 12)
```

sympy [A] time = 0.15, size = 26, normalized size = 0.96

$$\frac{280x+939}{x^2+7x+12} + 264\log\left(x+3\right) - 263\log\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
```

```
[Out] (280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

3.105 $\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{x+3}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1805, 801}

$$\frac{x+3}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 + x])/4
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx &= \frac{3+x}{2\left(1-x^2\right)}+\frac{1}{2}\int \frac{-4+x}{x\left(-1+x^2\right)}\,dx\\
&= \frac{3+x}{2\left(1-x^2\right)}+\frac{1}{2}\int \left(-\frac{3}{2(-1+x)}+\frac{4}{x}-\frac{5}{2(1+x)}\right)dx\\
&= \frac{3+x}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.21

$$\frac{1}{4}\left(-\frac{4}{x^2-1}-4\log\left(1-x^2\right)-\frac{2}{x-1}+\log(1-x)+8\log(x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1
- x^2])/4
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.90

$$\frac{-x-3}{2\left(x^2-1\right)}-\log\left(x^2-1\right)+2\log(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
[Out] (-3 - x)/(2*(-1 + x^2)) - ArcTanh[x]/2 + 2*Log[x] - Log[-1 + x^2]
```

fricas [A] time = 0.89, size = 45, normalized size = 1.15

$$-\frac{5\left(x^2-1\right)\log(x+1)+3\left(x^2-1\right)\log(x-1)-8\left(x^2-1\right)\log(x)+2x+6}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")
[Out] -1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x)
+ 2*x + 6)/(x^2 - 1)
```

giac [A] time = 1.11, size = 35, normalized size = 0.90

$$-\frac{x+3}{2(x+1)(x-1)}-\frac{5}{4}\log(|x+1|)-\frac{3}{4}\log(|x-1|)+2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1))
+ 2*log(abs(x))
```

maple [A] time = 0.35, size = 31, normalized size = 0.79

method	result	size
norman	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
risch	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
default	$2\ln(x)-\frac{1}{-1+x}-\frac{3\ln(-1+x)}{4}+\frac{1}{2x+2}-\frac{5\ln(1+x)}{4}$	32
meijerg	$\frac{i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}(x)\right)}{2}+\frac{3x^2}{-2x^2+2}-\ln\left(-x^2+1\right)+1+2\ln(x)+i\pi$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] (-1/2*x-3/2)/(x^2-1)+2*ln(x)-3/4*ln(-1+x)-5/4*ln(1+x)
```

maxima [A] time = 0.43, size = 29, normalized size = 0.74

$$-\frac{x+3}{2\left(x^2-1\right)}-\frac{5}{4}\log(x+1)-\frac{3}{4}\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)
```

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$2\ \ln(x) - \frac{5\ \ln(x+1)}{4} - \frac{3\ \ln(x-1)}{4} - \frac{\frac{x}{2}+\frac{3}{2}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 2)/(x*(x^2 - 1)^2),x)
[Out] 2*log(x) - (5*log(x + 1))/4 - (3*log(x - 1))/4 - (x/2 + 3/2)/(x^2 - 1)
```

sympy [A] time = 0.16, size = 32, normalized size = 0.82

$$\frac{-x-3}{2x^2-2} + 2\log(x) - \frac{3\log(x-1)}{4} - \frac{5\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
[Out] (-x - 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4
```

3.106 $\int \frac{1}{x^3-x^4-x^5+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2058}

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 + x]/4
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3-x^4-x^5+x^6}\,dx &= \int \left(\frac{1}{2(-1+x)^2}-\frac{7}{4(-1+x)}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-\frac{1}{4(1+x)}\right)dx\\ &= \frac{1}{2(1-x)}-\frac{1}{2x^2}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{1}{4}\left(-\frac{2}{x^2}-\frac{2}{x-1}-\frac{4}{x}-7\log(1-x)+8\log(x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] (-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x])/4
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 0.89

$$\frac{-3x^2+x+1}{2(x-1)x^2}-\frac{7}{4}\log(x-1)+2\log(x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] (1 + x - 3*x^2)/(2*(-1 + x)*x^2) - (7*Log[-1 + x])/4 + 2*Log[x] - Log[1 + x]/4
```

fricas [A] time = 0.93, size = 65, normalized size = 1.41

$$-\frac{6\,x^2+\left(x^3-x^2\right)\log\left(x+1\right)+7\left(x^3-x^2\right)\log\left(x-1\right)-8\left(x^3-x^2\right)\log(x)-2\,x-2}{4\left(x^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) - 8*(x^3 -
x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

giac [A] time = 1.03, size = 40, normalized size = 0.87

$$-\frac{3\,x^2-x-1}{2\,(x-1)x^2}-\frac{1}{4}\,\log\left(|x+1|\right)-\frac{7}{4}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*log(abs(x + 1)) - 7/4*log(abs(x -
1)) + 2*log(abs(x))
```

maple [A] time = 0.04, size = 35, normalized size = 0.76

method	result	size
default	$-\frac{1}{2x^2}-\frac{1}{x}+2\ln(x)-\frac{1}{2(-1+x)}-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	35
norman	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37
risch	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^5-x^4+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/x+2*ln(x)-1/2/(-1+x)-7/4*ln(-1+x)-1/4*ln(1+x)
```

maxima [A] time = 0.44, size = 40, normalized size = 0.87

$$-\frac{3\,x^2-x-1}{2\,\left(x^3-x^2\right)}-\frac{1}{4}\,\log\left(x+1\right)-\frac{7}{4}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1) + 2*log(
x)
```

mupad [B] time = 0.18, size = 40, normalized size = 0.87

$$2\,\ln(x)-\frac{\ln\left(x+1\right)}{4}-\frac{7\,\ln\left(x-1\right)}{4}-\frac{-\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{2}}{x^2-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - x^4 - x^5 + x^6),x)
```

```
[Out] 2*log(x) - log(x + 1)/4 - (7*log(x - 1))/4 - (x/2 - (3*x^2)/2 + 1/2)/(x^2 -
 x^3)
```

sympy [A] time = 0.17, size = 37, normalized size = 0.80

$$2\log\left(x\right)-\frac{7\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{4}+\frac{-3x^2+x+1}{2x^3-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**5-x**4+x**3),x)
```

```
[Out] 2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 + (-3*x**2 + x + 1)/(2*x**3 - 2*x*
*2)
```

3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2074, 635, 203, 260}

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx &= \int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right) \, dx \\
&= x+\frac{x^2}{2}+\log(1-x)+\int \frac{-1-x}{1+x^2} \, dx \\
&= x+\frac{x^2}{2}+\log(1-x)-\int \frac{1}{1+x^2} \, dx-\int \frac{x}{1+x^2} \, dx \\
&= x+\frac{x^2}{2}-\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 0.93

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x(x+2)+\log(x-1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(-1 + x - x^2 + x^3),x]
[Out] (x*(2 + x))/2 - ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

fricas [A] time = 0.91, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2+x-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

giac [A] time = 1.06, size = 24, normalized size = 0.83

$$\frac{1}{2}x^2+x-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="giac")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x - 1))
```

maple [A] time = 0.03, size = 24, normalized size = 0.83

method	result	size
default	$x+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}-\arctan(x)+\ln(-1+x)$	24
risch	$x+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}-\arctan(x)+\ln(-1+x)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^3-x^2+x-1),x,method=_RETURNVERBOSE)
[Out] x+1/2*x^2-1/2*ln(x^2+1)-arctan(x)+ln(-1+x)
```

maxima [A] time = 0.96, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2+x-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

mupad [B] time = 0.05, size = 29, normalized size = 1.00

$$x+\ln (x-1)+\frac{x^2}{2}+\ln (x-\mathrm{i}) \left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln (x+1\mathrm{i}) \left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x - x^2 + x^3 - 1),x)
```

```
[Out] x + log(x - 1) - log(x - 1i)*(1/2 - 1i/2) - log(x + 1i)*(1/2 + 1i/2) + x^2/
2
```

sympy [A] time = 0.13, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+x+\log (x-1)-\frac{\log \left(x^2+1\right)}{2}-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**3-x**2+x-1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

3.108 $\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {894, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{1}{x}-\frac{1}{2(1+x)}+\frac{-1-x}{2\left(1+x^2\right)}\right)dx\\
&= \log(x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{-1-x}{1+x^2}\,dx\\
&= \log(x)-\frac{1}{2}\log(1+x)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{2}\tan^{-1}(x)+\log(x)-\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)*(1 + x^2)),x]
[Out] -1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x)*(1 + x^2)),x]
[Out] -1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

fricas [A] time = 0.90, size = 21, normalized size = 0.78

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

giac [A] time = 0.78, size = 23, normalized size = 0.85

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.32, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$	22
risch	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

maxima [A] time = 0.98, size = 21, normalized size = 0.78

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

mupad [B] time = 0.05, size = 27, normalized size = 1.00

$$\ln(x) - \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4} + \frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4} - \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)*(x + 1)),x)
[Out] log(x) - log(x - 1i)*(1/4 - 1i/4) - log(x + 1i)*(1/4 + 1i/4) - log(x + 1)/2
```

sympy [A] time = 0.19, size = 22, normalized size = 0.81

$$\log(x) - \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x**2+1),x)
[Out] log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

3.109 $\int \frac{x^2}{-2+x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1130, 203, 207}

$$\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + x^2 + x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-2+x^2+x^4} \, dx &= \frac{1}{3}\int \frac{1}{-1+x^2} \, dx + \frac{2}{3}\int \frac{1}{2+x^2} \, dx \\ &= \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.33

$$\frac{1}{6}\left(\log(1-x) - \log(x+1) + 2\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-2 + x^2 + x^4),x]
[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

fricas [A] time = 0.68, size = 25, normalized size = 1.04

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

giac [A] time = 0.96, size = 27, normalized size = 1.12

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log\left(|x+1|\right)+\frac{1}{6}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="giac")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1
))
```

maple [A] time = 0.04, size = 26, normalized size = 1.08

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(-1+x)}{6}-\frac{\ln(1+x)}{6}$	26
risch	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(-1+x)}{6}-\frac{\ln(1+x)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+x^2-2),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ln(-1+x)-1/6*ln(1+x)
```

maxima [A] time = 0.97, size = 25, normalized size = 1.04

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.71

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}-\frac{\mathrm{atanh}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + x^4 - 2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x)/2))/3 - atanh(x)/3
```

sympy [**A**] time = 0.16, size = 29, normalized size = 1.21

$$\frac{\log\left(x-1\right)}{6}-\frac{\log\left(x+1\right)}{6}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+x**2-2),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx$

Optimal. Leaf size=41

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1594, 2075, 635, 203, 260}

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 + x^2])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx &= \int \frac{x\left(6+4x+x^2\right)}{2+4x+3x^2+2x^3+x^4}\,dx\\ &= \int \left(-\frac{1}{(1+x)^2}-\frac{1}{3(1+x)}+\frac{4(2+x)}{3\left(2+x^2\right)}\right)dx\\ &= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{2+x}{2+x^2}\,dx\\ &= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{x}{2+x^2}\,dx+\frac{8}{3}\int \frac{1}{2+x^2}\,dx\\ &= \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 41, normalized size = 1.00

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 + x^2])/3
```

IntegrateAlgebraic [**A**] time = 0.03, size = 41, normalized size = 1.00

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 + x^2])/3
```

fricas [**A**] time = 0.85, size = 44, normalized size = 1.07

$$\frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2(x+1)\log\left(x^2+2\right)-(x+1)\log(x+1)+3}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x + 1)*log(x + 1) + 3)/(x + 1)
```

giac [**A**] time = 0.98, size = 33, normalized size = 0.80

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1}+\frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
abs(x + 1))
```

maple [A] time = 0.04, size = 33, normalized size = 0.80

method	result	size
default	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33
risch	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

maxima [A] time = 0.99, size = 32, normalized size = 0.78

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1}+\frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
x + 1)
```

mupad [B] time = 0.25, size = 49, normalized size = 1.20

$$\frac{1}{x+1}-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)+\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x + 4*x^2 + x^3)/(4*x + 3*x^2 + 2*x^3 + x^4 + 2),x)
```

```
[Out] 1/(x + 1) - log(x + 1)/3 - log(x - 2^(1/2)*1i)*((2^(1/2)*2i)/3 - 2/3) + log
(x + 2^(1/2)*1i)*((2^(1/2)*2i)/3 + 2/3)
```

sympy [A] time = 0.16, size = 39, normalized size = 0.95

$$-\frac{\log\left(x+1\right)}{3}+\frac{2\log\left(x^2+2\right)}{3}+\frac{4\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

```
[Out] -log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3 + 1/(x +
1)
```

3.111 $\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)} \, dx$

Optimal. Leaf size=46

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

Rubi [A] time = 0.20, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6725, 635, 203, 260}

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*L
og[1 + x^2])/100
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)} \, dx &= \int \left(-\frac{1}{2(1+x)}-\frac{4}{5(1+2x)^2}+\frac{32}{25(1+2x)}+\frac{1-7x}{50\left(1+x^2\right)}\right) dx \\
&= \frac{2}{5(1+2x)}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)+\frac{1}{50}\int \frac{1-7x}{1+x^2} \, dx \\
&= \frac{2}{5(1+2x)}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)+\frac{1}{50}\int \frac{1}{1+x^2} \, dx-\frac{7}{50}\int \frac{x}{1+x^2} \, dx \\
&= \frac{2}{5(1+2x)}+\frac{1}{50}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)-\frac{7}{100}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{1}{100}\left(-7\log\left(x^2+1\right)+\frac{40}{2x+1}-50\log(x+1)+64\log(2x+1)+2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] (40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7*Log[1 + x
^2])/100
```

IntegrateAlgebraic [A] time = 0.03, size = 46, normalized size = 1.00

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*L
og[1 + x^2])/100
```

fricas [A] time = 0.85, size = 57, normalized size = 1.24

$$\frac{2\,(2\,x+1)\arctan(x)-7\,(2\,x+1)\log\left(x^2+1\right)+64\,(2\,x+1)\log\left(2\,x+1\right)-50\,(2\,x+1)\log\left(x+1\right)+40}{100\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x + 1)*log(
2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

giac [A] time = 1.04, size = 62, normalized size = 1.35

$$\frac{2}{5\,(2\,x+1)}+\frac{1}{50}\arctan\left(-\frac{5}{2\,(2\,x+1)}+\frac{1}{2}\right)-\frac{7}{100}\log\left(-\frac{2}{2\,x+1}+\frac{5}{(2\,x+1)^2}+1\right)-\frac{1}{2}\log\left(\left|-\frac{1}{2\,x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*log(-2/(2*x + 1)
+ 5/(2*x + 1)^2 + 1) - 1/2*log(abs(-1/(2*x + 1) - 1))
```

maple [A] time = 0.32, size = 35, normalized size = 0.76

method	result	size
risch	$\frac{1}{\frac{5}{2}+5x}-\frac{\ln(1+x)}{2}+\frac{16\ln(1+2x)}{25}-\frac{7\ln\left(x^2+1\right)}{100}+\frac{\arctan(x)}{50}$	35
default	$\frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{\ln(1+x)}{2}+\frac{16\ln(1+2x)}{25}-\frac{7\ln\left(x^2+1\right)}{100}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(1+2*x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(1/2+x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)+1/50*arctan(x)
```

maxima [A] time = 0.98, size = 36, normalized size = 0.78

$$\frac{2}{5\,(2\,x+1)}+\frac{1}{50}\,\arctan(x)-\frac{7}{100}\,\log\left(x^2+1\right)+\frac{16}{25}\,\log\left(2\,x+1\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2*x + 1) -
1/2*log(x + 1)
```

mupad [B] time = 0.19, size = 38, normalized size = 0.83

$$\frac{16\,\ln\left(x+\frac{1}{2}\right)}{25}-\frac{\ln\left(x+1\right)}{2}+\frac{1}{5\,\left(x+\frac{1}{2}\right)}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{7}{100}-\frac{1}{100}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{7}{100}+\frac{1}{100}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((2*x + 1)^2*(x^2 + 1)*(x + 1)),x)
```

```
[Out] (16*log(x + 1/2))/25 - log(x + 1)/2 - log(x - 1i)*(7/100 + 1i/100) - log(x
+ 1i)*(7/100 - 1i/100) + 1/(5*(x + 1/2))
```

sympy [A] time = 0.22, size = 37, normalized size = 0.80

$$\frac{16\log\left(x+\frac{1}{2}\right)}{25}-\frac{\log\left(x+1\right)}{2}-\frac{7\log\left(x^2+1\right)}{100}+\frac{\operatorname{atan}\left(x\right)}{50}+\frac{2}{10x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

```
[Out] 16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)/50 + 2/(1
0*x + 5)
```

3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3\left(1+x^2\right)} \, dx$

Optimal. Leaf size=47

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -1/(2*(1 - x)^2) + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
+ x^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x+3x^2}{(-1+x)^3\left(1+x^2\right)} \, dx &= \int \left(\frac{1}{(-1+x)^3}+\frac{5}{2(-1+x)^2}-\frac{3}{2(-1+x)}+\frac{-2+3x}{2\left(1+x^2\right)}\right) \, dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{1}{2}\int \frac{-2+3x}{1+x^2} \, dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2} \, dx-\int \frac{1}{1+x^2} \, dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\tan^{-1}(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.79

$$\frac{1}{4}\left(3\log\left(x^2+1\right)-\frac{10}{x-1}-\frac{2}{(x-1)^2}-6\log(x-1)-4\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2]
)/4
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.79

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{4-5x}{2(x-1)^2}-\frac{3}{2}\log(x-1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (4 - 5*x)/(2*(-1 + x)^2) - ArcTan[x] - (3*Log[-1 + x])/2 + (3*Log[1 + x^2])
/4
```

fricas [A] time = 0.88, size = 59, normalized size = 1.26

$$-\frac{4\left(x^2-2x+1\right)\arctan(x)-3\left(x^2-2x+1\right)\log\left(x^2+1\right)+6\left(x^2-2x+1\right)\log\left(x-1\right)+10x-8}{4\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1) + 6*(x^2
 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

giac [A] time = 0.91, size = 32, normalized size = 0.68

$$-\frac{5x-4}{2(x-1)^2}-\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)-\frac{3}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x - 1
))
```

maple [A] time = 0.36, size = 33, normalized size = 0.70

method	result	size
risch	$\frac{-\frac{5x}{2}+2}{(-1+x)^2}-\frac{3\ln(-1+x)}{2}+\frac{3\ln\left(4x^2+4\right)}{4}-\arctan(x)$	33
default	$\frac{3\ln\left(x^2+1\right)}{4}-\arctan(x)-\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}-\frac{3\ln(-1+x)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-5/2*x+2)/(-1+x)^2-3/2*ln(-1+x)+3/4*ln(4*x^2+4)-arctan(x)
```

maxima [A] time = 0.99, size = 36, normalized size = 0.77

$$-\frac{5\,x-4}{2\left(x^2-2\,x+1\right)}-\arctan(x)+\frac{3}{4}\,\log\left(x^2+1\right)-\frac{3}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(x -
 1)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.89

$$-\frac{3\,\ln\left(x-1\right)}{2}-\frac{\frac{5\,x}{2}-2}{x^2-2\,x+1}+\ln\left(x-\mathrm{i}\right)\,\left(\frac{3}{4}+\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{3}{4}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 2)/((x^2 + 1)*(x - 1)^3),x)
```

```
[Out] log(x - 1i)*(3/4 + 1i/2) - (3*log(x - 1))/2 + log(x + 1i)*(3/4 - 1i/2) - ((
5*x)/2 - 2)/(x^2 - 2*x + 1)
```

sympy [A] time = 0.18, size = 36, normalized size = 0.77

$$\frac{4-5x}{2x^2-4x+2}-\frac{3\log\left(x-1\right)}{2}+\frac{3\log\left(x^2+1\right)}{4}-\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

```
[Out] (4 - 5*x)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/4 - atan(x)
```

3.113 $\int \frac{1}{1+x^2+x^4} dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx$$
$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subs}$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)$$

Mathematica [C] time = 0.06, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right) - \sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

IntegrateAlgebraic [C] time = 0.00, size = 58, normalized size = 0.87

$$\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}\,\tan^{-1}\left(\sqrt[6]{-1}\,x\right) - \sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,\tan^{-1}\left((-1)^{5/6}x\right)$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] Sqrt[(-1 - I*Sqrt[3])/6]*ArcTan[(-1)^(1/6)*x] - Sqrt[(-1 + I*Sqrt[3])/6]*Ar
cTan[(-1)^(5/6)*x]
```

fricas [A] time = 0.89, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

giac [A] time = 0.99, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

maple [A] time = 0.05, size = 54, normalized size = 0.81

method	result	size
default	$\frac{\ln\left(x^{2}+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(x^{2}-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{6}$	54
risch	$\frac{\ln\left(4x^{2}+4x+4\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(4x^{2}-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{6}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^2-x+1)+1/6
*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

maxima [A] time = 0.99, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{4}\log\left(x^{2}+x+1\right)-\frac{1}{4}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

mupad [B] time = 0.09, size = 47, normalized size = 0.70

$$\operatorname{atanh}\left(\frac{2x}{-1+\sqrt{3}\,1i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{6}\right)+\operatorname{atanh}\left(\frac{2x}{1+\sqrt{3}\,1i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^4 + 1),x)
```

```
[Out] atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

sympy [A] time = 0.21, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^{2}-x+1\right)}{4}+\frac{\log\left(x^{2}+x+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.114 $\int \frac{3+2x^3}{-9x+x^5} \, dx$

Optimal. Leaf size=48

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1593, 1831, 266, 36, 31, 29, 298, 203, 206}

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9 -
 x^4]/12
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+2x^3}{-9x+x^5}\,dx &= \int \frac{3+2x^3}{x\left(-9+x^4\right)}\,dx \\
&= \int \left(\frac{3}{x\left(-9+x^4\right)}+\frac{2x^2}{-9+x^4}\right)dx \\
&= 2\int \frac{x^2}{-9+x^4}\,dx+3\int \frac{1}{x\left(-9+x^4\right)}\,dx \\
&= \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{(-9+x)x}\,dx,x,x^4\right)-\int \frac{1}{3-x^2}\,dx+\int \frac{1}{3+x^2}\,dx \\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{-9+x}\,dx,x,x^4\right)-\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right) \\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{12}\log\left(9-x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.40

$$\frac{1}{12}\left(\log\left(9-x^4\right)-4\log(x)+2\sqrt{3}\log\left(3-\sqrt{3}x\right)-2\sqrt{3}\log\left(\sqrt{3}x+3\right)+4\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2*
Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.96

$$\frac{1}{12}\log\left(x^4-9\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[-9
+ x^4]/12
```

fricas [A] time = 0.89, size = 58, normalized size = 1.21

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)
/(x^2 - 3)) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

giac [A] time = 1.12, size = 64, normalized size = 1.33

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(\left|x^2-3\right|\right)-\frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/ab
s(2*x + 2*sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(abs(x^2 - 3)) - 1/3*log(
abs(x))
```

maple [A] time = 0.34, size = 46, normalized size = 0.96

method	result	size
default	$\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln(x)}{3}+\frac{\ln(x^2-3)}{12}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	46
meijerg	$\frac{\ln\left(1-\frac{x^4}{9}\right)}{12}-\frac{\ln(x)}{3}+\frac{\ln(3)}{6}-\frac{i\pi}{12}+\frac{x^3\sqrt{3}\left(\ln\left(1-\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)-\ln\left(1+\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)+2\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)\right)}{6\left(x^4\right)^{\frac{3}{4}}}$	79
risch	$-\frac{\ln(x)}{3}+\frac{\ln(4x^2+12)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(2x-2\sqrt{3})}{12}+\frac{\ln(2x-2\sqrt{3})\sqrt{3}}{6}+\frac{\ln(2x+2\sqrt{3})}{12}-\frac{\ln(2x+2\sqrt{3})\sqrt{3}}{6}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3)/(x^5-9*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*ln(x)+1/12*ln(x^2-3)-1
/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 0.98, size = 54, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt
(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

mupad [B] time = 0.17, size = 73, normalized size = 1.52

$$\ln\left(x-\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{12}\right)-\ln\left(x+\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{12}\right)-\frac{\ln(x)}{3}-\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{12}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^3 + 3)/(9*x - x^5),x)
```

```
[Out] log(x - 3^(1/2))*(3^(1/2)/6 + 1/12) - log(x + 3^(1/2))*(3^(1/2)/6 - 1/12) -
 log(x)/3 - log(x - 3^(1/2)*1i)*((3^(1/2)*1i)/6 - 1/12) + log(x + 3^(1/2)*1
i)*((3^(1/2)*1i)/6 + 1/12)
```

sympy [C] time = 0.59, size = 306, normalized size = 6.38

$$-\frac{\log(x)}{3}+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{6}\right)\log\left(x+\frac{17413}{11544}-\frac{943\sqrt{3}\,i}{5772}+\frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{6}\right)^{3}}{481}+\frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{6}\right)^{2}}{481}-\frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{6}\right)^{4}}{481}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3)/(x**5-9*x),x)
```

```
[Out] -log(x)/3 + (1/12 + sqrt(3)*I/6)*log(x + 17413/11544 - 943*sqrt(3)*I/5772 +
 1368*(1/12 + sqrt(3)*I/6)**3/481 + 4158*(1/12 + sqrt(3)*I/6)**2/481 - 1080
00*(1/12 + sqrt(3)*I/6)**4/481) + (1/12 - sqrt(3)*I/6)*log(x + 17413/11544
- 108000*(1/12 - sqrt(3)*I/6)**4/481 + 4158*(1/12 - sqrt(3)*I/6)**2/481 + 1
368*(1/12 - sqrt(3)*I/6)**3/481 + 943*sqrt(3)*I/5772) + (1/12 - sqrt(3)/6)*
log(x - 108000*(1/12 - sqrt(3)/6)**4/481 + 1368*(1/12 - sqrt(3)/6)**3/481 +
 943*sqrt(3)/5772 + 4158*(1/12 - sqrt(3)/6)**2/481 + 17413/11544) + (1/12 +
 sqrt(3)/6)*log(x - 108000*(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/5772 + 1
368*(1/12 + sqrt(3)/6)**3/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 17413/1154
4)
```

3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3\left(8-4x+x^2\right)} \, dx$

Optimal. Leaf size=58

$$\frac{45}{32} \log \left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16} \log(4-x)-\frac{3}{16} \tan^{-1}\left(1-\frac{x}{2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1628, 634, 617, 204, 628}

$$\frac{45}{32} \log \left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16} \log(4-x)-\frac{3}{16} \tan^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 - x
])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\left(8-4x+x^2\right)}\,dx = \int \left(\frac{83}{2(-4+x)^3}+\frac{41}{4(-4+x)^2}-\frac{45}{16(-4+x)}+\frac{3(-28+15x)}{16\left(8-4x+x^2\right)}\right)dx$$

$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{16}\int \frac{-28+15x}{8-4x+x^2}\,dx$$

$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{8}\int \frac{1}{8-4x+x^2}\,dx+\frac{45}{32}\int \frac{-4+2x}{8-4x+}$$

$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)+\frac{3}{16}\text{Subst}\left(\int -\right.$$

$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right.$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.79

$$\frac{1}{32}\left(45\log\left(x^2-4x+8\right)-\frac{328}{x-4}-\frac{664}{(x-4)^2}-90\log(x-4)+6\tan^{-1}\left(\frac{x-2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] + 4
5*Log[8 - 4*x + x^2])/32
```

IntegrateAlgebraic [A] time = 0.03, size = 48, normalized size = 0.83

$$\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{81-41x}{4(x-4)^2}-\frac{45}{16}\log(x-4)-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (81 - 41*x)/(4*(-4 + x)^2) - (3*ArcTan[1 - x/2])/16 - (45*Log[-4 + x])/16 +
 (45*Log[8 - 4*x + x^2])/32
```

fricas [A] time = 0.96, size = 66, normalized size = 1.14

$$\frac{6\left(x^2-8x+16\right)\arctan\left(\frac{1}{2}x-1\right)+45\left(x^2-8x+16\right)\log\left(x^2-4x+8\right)-90\left(x^2-8x+16\right)\log(x-4)-328x}{32\left(x^2-8x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")
```

```
[Out] 1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*log(x^2 -
4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)
```

giac [A] time = 0.96, size = 39, normalized size = 0.67

$$-\frac{41x-81}{4(x-4)^2}+\frac{3}{16}\arctan\left(\frac{1}{2}x-1\right)+\frac{45}{32}\log\left(x^2-4x+8\right)-\frac{45}{16}\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="giac")
```

```
[Out] -1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x +
 8) - 45/16*log(abs(x - 4))
```

maple [A] time = 0.43, size = 38, normalized size = 0.66

method	result	size
risch	$\frac{-\frac{41x}{4}+\frac{81}{4}}{(-4+x)^2}+\frac{45\ln\left(x^2-4x+8\right)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{45\ln(-4+x)}{16}$	38
default	$\frac{45\ln\left(x^2-4x+8\right)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{83}{4(-4+x)^2}-\frac{41}{4(-4+x)}-\frac{45\ln(-4+x)}{16}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x,method=_RETURNVERBOSE)
```

```
[Out] (-41/4*x+81/4)/(-4+x)^2+45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1/2*x)-45/16*ln(
-4+x)
```

maxima [A] time = 0.98, size = 43, normalized size = 0.74

$$-\frac{41\,x-81}{4\left(x^2-8\,x+16\right)}+\frac{3}{16}\,\arctan\left(\frac{1}{2}\,x-1\right)+\frac{45}{32}\,\log\left(x^2-4\,x+8\right)-\frac{45}{16}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")
```

```
[Out] -1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2
- 4*x + 8) - 45/16*log(x - 4)
```

mupad [B] time = 0.20, size = 44, normalized size = 0.76

$$-\frac{45\,\ln\left(x-4\right)}{16}-\frac{\frac{41\,x}{4}-\frac{81}{4}}{x^2-8\,x+16}+\ln\left(x-2-2{}\mathrm{i}\right)\,\left(\frac{45}{32}-\frac{3}{32}{}\mathrm{i}\right)+\ln\left(x-2+2{}\mathrm{i}\right)\,\left(\frac{45}{32}+\frac{3}{32}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 5*x^3 - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x)
```

```
[Out] log(x - (2 + 2i))*(45/32 - 3i/32) - (45*log(x - 4))/16 + log(x - (2 - 2i))*
(45/32 + 3i/32) - ((41*x)/4 - 81/4)/(x^2 - 8*x + 16)
```

sympy [A] time = 0.20, size = 46, normalized size = 0.79

$$\frac{81-41x}{4x^2-32x+64}-\frac{45\log\left(x-4\right)}{16}+\frac{45\log\left(x^2-4x+8\right)}{32}+\frac{3\operatorname{atan}\left(\frac{x}{2}-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

```
[Out] (81 - 41*x)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x + 8
)/32 + 3*atan(x/2 - 1)/16
```

3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.28, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {6725, 203}

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/12 + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/Sq
rt[3]]/(2*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)\left(3+x^2\right)\left(4+x^2\right)} \, dx &= \int \left(\frac{1}{6\left(1+x^2\right)}-\frac{1}{2\left(2+x^2\right)}+\frac{1}{2\left(3+x^2\right)}-\frac{1}{6\left(4+x^2\right)}\right) dx\\
&= \frac{1}{6}\int \frac{1}{1+x^2}\,dx-\frac{1}{6}\int \frac{1}{4+x^2}\,dx-\frac{1}{2}\int \frac{1}{2+x^2}\,dx+\frac{1}{2}\int \frac{1}{3+x^2}\,dx\\
&= -\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{1}{12}\left(-\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] (-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcTa
n[x/Sqrt[3]])/12
```

IntegrateAlgebraic [A] time = 0.03, size = 51, normalized size = 1.00

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/12*ArcTan[x/2] + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/
Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.91, size = 35, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

giac [A] time = 0.86, size = 35, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

maple [A] time = 0.32, size = 36, normalized size = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*arc
tan(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 0.99, size = 35, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

mupad [B] time = 0.24, size = 35, normalized size = 0.69

$$\frac{\operatorname{atan}(x)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

```
[Out] atan(x)/6 - atan(x/2)/12 - (2^(1/2)*atan((2^(1/2)*x)/2))/4 + (3^(1/2)*atan(
(3^(1/2)*x)/3))/6
```

sympy [A] time = 0.44, size = 44, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}+\frac{\operatorname{atan}(x)}{6}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] -atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)*atan(sqrt
(3)*x/3)/6
```

3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

Rubi [A] time = 0.31, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6694, 180}

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 6694

```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((e_.) + (f_.)*(w_))^(p_.)*((a_.) + (b_
.)*(y_))^(m_.)*((g_.) + (h_.)*(z_))^(q_.), x_Symbol] :> With[{r = Derivativ
eDivides[y, u, x]}, Dist[r, Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(
g + h*x)^q, x], x, y], x] /;  !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1+x^2\right)\left(2+x^2\right)\left(3+x^2\right)\left(4+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(2+x)(3+x)(4+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{6(1+x)}-\frac{1}{2(2+x)}+\frac{1}{2(3+x)}-\frac{1}{6(4+x)}\right)dx,x,x^2\right)\\
&= \frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\log\left(2+x^2\right)+\frac{1}{4}\log\left(3+x^2\right)-\frac{1}{12}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{1}{12}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{12}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

IntegrateAlgebraic [A] time = 0.01, size = 29, normalized size = 0.71

$$\frac{1}{2}\tanh^{-1}\left(2x^2+5\right)-\frac{1}{6}\tanh^{-1}\left(\frac{2x^2}{3}+\frac{5}{3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/6*ArcTanh[5/3 + (2*x^2)/3] + ArcTanh[5 + 2*x^2]/2
```

fricas [A] time = 0.86, size = 33, normalized size = 0.80

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

giac [A] time = 0.97, size = 33, normalized size = 0.80

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

maple [A] time = 0.28, size = 34, normalized size = 0.83

method	result	size
default	$\frac{\ln\left(x^2+1\right)}{12}-\frac{\ln\left(x^2+2\right)}{4}+\frac{\ln\left(x^2+3\right)}{4}-\frac{\ln\left(x^2+4\right)}{12}$	34
norman	$\frac{\ln\left(x^2+1\right)}{12}-\frac{\ln\left(x^2+2\right)}{4}+\frac{\ln\left(x^2+3\right)}{4}-\frac{\ln\left(x^2+4\right)}{12}$	34
risch	$\frac{\ln\left(x^2+1\right)}{12}-\frac{\ln\left(x^2+2\right)}{4}+\frac{\ln\left(x^2+3\right)}{4}-\frac{\ln\left(x^2+4\right)}{12}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

maxima [A] time = 0.43, size = 33, normalized size = 0.80

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

mupad [B] time = 0.06, size = 33, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{3072}{5\,\left(1280\,x^2+3072\right)}-\frac{1}{5}\right)}{2}-\frac{\operatorname{atanh}\left(\frac{1024}{405\,\left(\frac{640\,x^2}{243}+\frac{1024}{243}\right)}-\frac{3}{5}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

```
[Out] atanh(3072/(5*(1280*x^2 + 3072)) - 1/5)/2 - atanh(1024/(405*((640*x^2)/243
+ 1024/243)) - 3/5)/6
```

sympy [**A**] time = 0.19, size = 32, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{12}-\frac{\log\left(x^2+2\right)}{4}+\frac{\log\left(x^2+3\right)}{4}-\frac{\log\left(x^2+4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 + 4)/12
```

3.118 $\int \frac{1}{a^3+x^3} \, dx$

Optimal. Leaf size=56

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^2}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2) - Log[a
^2 - a*x + x^2]/(6*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a^3+x^3}\,dx &= \frac{\int \frac{1}{a+x}\,dx}{3a^2}+\frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^2}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^2}+\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^2}\\
&= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.93

$$\frac{-\log\left(a^2-ax+x^2\right)+2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}\,a}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + x^3)^(-1),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x +
x^2])/(6*a^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 59, normalized size = 1.05

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^3 + x^3)^(-1),x]
[Out] -(ArcTan[1/Sqrt[3] - (2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2)
 - Log[a^2 - a*x + x^2]/(6*a^2)
```

fricas [A] time = 0.86, size = 45, normalized size = 0.80

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(a^2-ax+x^2\right)+2\log\left(a+x\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a^2 - a*x + x^2) + 2*
log(a + x))/a^2
```

giac [A] time = 0.88, size = 50, normalized size = 0.89

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log\left(|a+x|\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(abs(a + x))/a^2
```

maple [C] time = 0.30, size = 41, normalized size = 0.73

method	result	size
risch	$\frac{\ln(a+x)}{3a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4\_Z^2+a^2\_Z+1\right)}\_R\ln\left(\_R\,a^3+x\right)\right)}{3}$	41
default	$\frac{-\frac{\ln(a^2-ax+x^2)}{2}+\sqrt{3}\ \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^2}+\frac{\ln(a+x)}{3a^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(a+x)/a^2+1/3*sum(_R*ln(_R*a^3+x),_R=RootOf(_Z^2*a^4+_Z*a^2+1))
```

maxima [A] time = 0.98, size = 49, normalized size = 0.88

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)}{3\,a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6\,a^2}+\frac{\log\left(a+x\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(a + x)/a^2
```

mupad [B] time = 0.44, size = 64, normalized size = 1.14

$$\frac{\ln\left(a+x\right)}{3\,a^2}+\frac{\ln\left(x+\frac{a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2}-\frac{\ln\left(x-\frac{a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + x^3),x)
```

```
[Out] log(a + x)/(3*a^2) + (log(x + (a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*
a^2) - (log(x - (a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^2)
```

sympy [C] time = 0.14, size = 73, normalized size = 1.30

$$\frac{\frac{\log{(a+x)}}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)\log{\left(3a\left(-\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)+x\right)}+\left(-\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)\log{\left(3a\left(-\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)+x\right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x**3),x)
```

```
[Out] (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6) + x) + (-
1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a**2
```

3.119 $\int \frac{x}{a^3+x^3} \, dx$

Optimal. Leaf size=56

$$\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^3 + x^3),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) + Log[a^2 -
 a*x + x^2]/(6*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{a^3+x^3}\,dx &= -\frac{\int \frac{1}{a+x}\,dx}{3a}+\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a}\\
&= -\frac{\log(a+x)}{3a}+\frac{1}{2}\int \frac{1}{a^2-ax+x^2}\,dx+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a}\\
&= -\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a}\\
&= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a}-\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.89

$$\frac{\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\,\tan^{-1}\left(\frac{2x-a}{\sqrt{3}\,a}\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2])/(6*a)
```

IntegrateAlgebraic [A] time = 0.02, size = 59, normalized size = 1.05

$$\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^3 + x^3),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) + L
og[a^2 - a*x + x^2]/(6*a)
```

fricas [A] time = 0.84, size = 43, normalized size = 0.77

$$\frac{2\,\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)+\log\left(a^2-ax+x^2\right)-2\,\log\left(a+x\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + log(a^2 - a*x + x^2) - 2*
log(a + x))/a
```

giac [A] time = 1.11, size = 50, normalized size = 0.89

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)}{3\,a}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a}-\frac{\log\left(|a+x|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(abs(a + x))/a
```

maple [A] time = 0.28, size = 51, normalized size = 0.91

method	result	size
default	$\frac{\frac{\ln(a^2-ax+x^2)}{2}+\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a}-\frac{\ln(a+x)}{3a}$	51
risch	$\frac{\ln(4a^2-4ax+4x^2)}{6a}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a}-\frac{\ln(a+x)}{3a}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/3*ln(a+
x)/a
```

maxima [A] time = 0.96, size = 49, normalized size = 0.88

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)}{3\,a}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a}-\frac{\log\left(a+x\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(a + x)/a
```

mupad [B] time = 0.12, size = 68, normalized size = 1.21

$$-\frac{\ln\left(a+x\right)}{3\,a}-\frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a}+\frac{\ln\left(x+\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^3 + x^3),x)
```

```
[Out] (log(x + (a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a) - (log(x + (a*(3
^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a) - log(a + x)/(3*a)
```

sympy [C] time = 0.13, size = 71, normalized size = 1.27

$$\frac{-\frac{\log\left(a+x\right)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)^2+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**3+x**3),x)
```

```
[Out] (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) +
(1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a
```

3.120 $\int \frac{x^2}{a^3+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {260}

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{a^3+x^3} \, dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

IntegrateAlgebraic [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

fricas [A] time = 0.59, size = 10, normalized size = 0.83

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(a^3 + x^3)
```

giac [A] time = 0.93, size = 11, normalized size = 0.92

$$\frac{1}{3}\log\left(\left|a^3 + x^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="giac")
[Out] 1/3*log(abs(a^3 + x^3))
```

maple [A] time = 0.26, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\ln(a^3+x^3)}{3}$	11
default	$\frac{\ln(a^3+x^3)}{3}$	11
risch	$\frac{\ln(a^3+x^3)}{3}$	11
norman	$\frac{\ln(a+x)}{3}+\frac{\ln(a^2-ax+x^2)}{3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(a^3+x^3)
```

maxima [A] time = 0.43, size = 10, normalized size = 0.83

$$\frac{1}{3}\log\left(a^3 + x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="maxima")
[Out] 1/3*log(a^3 + x^3)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.83

$$\frac{\ln\left(a^3 + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^3 + x^3),x)
[Out] log(a^3 + x^3)/3
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$\frac{\log\left(a^3 + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**3+x**3),x)
[Out] log(a**3 + x**3)/3
```

3.121 $\int \frac{1}{x(a^3+x^3)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^3+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(a^3+x\right)}\,dx,x,x^3\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a^3}-\frac{\operatorname{Subst}\left(\int \frac{1}{a^3+x}\,dx,x,x^3\right)}{3a^3}\\ &= \frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + x^3)),x]
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^3} - \frac{\log\left(a^3 + x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 + x^3)),x]
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

fricas [A] time = 0.89, size = 18, normalized size = 0.82

$$-\frac{\log\left(a^3 + x^3\right) - 3\log(x)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="fricas")
[Out] -1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

giac [A] time = 0.87, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|a^3 + x^3\right|\right)}{3a^3} + \frac{\log(|x|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="giac")
[Out] -1/3*log(abs(a^3 + x^3))/a^3 + log(abs(x))/a^3
```

maple [A] time = 0.27, size = 21, normalized size = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^3} - \frac{\ln(a^3+x^3)}{3a^3}$	21
default	$-\frac{\ln(a^2-ax+x^2)}{3a^3} + \frac{\ln(x)}{a^3} - \frac{\ln(a+x)}{3a^3}$	34
norman	$-\frac{\ln(a^2-ax+x^2)}{3a^3} + \frac{\ln(x)}{a^3} - \frac{\ln(a+x)}{3a^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

maxima [A] time = 0.43, size = 23, normalized size = 1.05

$$-\frac{\log\left(a^3 + x^3\right)}{3a^3} + \frac{\log\left(x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

mupad [B] time = 0.25, size = 18, normalized size = 0.82

$$-\frac{\ln\left(a^3+x^3\right)-3\ \ln(x)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 + x^3)),x)
[Out] -(log(a^3 + x^3) - 3*log(x))/(3*a^3)
```

sympy [A] time = 0.21, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**3+x**3),x)
[Out] log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

3.122 $\int \frac{1}{x^2(a^3+x^3)}\,dx$

Optimal. Leaf size=63

$$\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^4}-\frac{1}{a^3x}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}-\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[a + x]/(3*
a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx &= -\frac{1}{a^3x}-\frac{\int \frac{x}{a^3+x^3}\,dx}{a^3}\\
&= -\frac{1}{a^3x}+\frac{\int \frac{1}{a+x}\,dx}{3a^4}-\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a^4}\\
&= -\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}-\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^4}-\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^3}\\
&= -\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}-\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^4}\\
&= -\frac{1}{a^3x}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^4}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 60, normalized size = 0.95

$$
-\frac{x\log\left(a^2-ax+x^2\right)-2x\log(a+x)+2\sqrt{3}\,x\tan^{-1}\left(\frac{2x-a}{\sqrt{3}\,a}\right)+6a}{6a^4x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -1/6*(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a + x] + x
*Log[a^2 - a*x + x^2])/(a^4*x)
```

IntegrateAlgebraic [A] time = 0.03, size = 66, normalized size = 1.05

$$
\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^4}-\frac{1}{a^3x}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(1/(a^3*x)) + ArcTan[1/Sqrt[3] - (2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[a
+ x]/(3*a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

fricas [A] time = 0.92, size = 53, normalized size = 0.84

$$
-\frac{2\,\sqrt{3}\,x\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)+x\log\left(a^2-ax+x^2\right)-2\,x\log\left(a+x\right)+6\,a}{6\,a^4x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + x*log(a^2 - a*x + x^2)
 - 2*x*log(a + x) + 6*a)/(a^4*x)
```

giac [A] time = 0.93, size = 58, normalized size = 0.92

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}+\frac{\log\left(|a+x|\right)}{3a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(abs(a + x))/a^4 - 1/(a^3*x)
```

maple [A] time = 0.28, size = 60, normalized size = 0.95

method	result	size
default	$\frac{-\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^4}-\frac{1}{a^3x}+\frac{\ln(a+x)}{3a^4}$	60
risch	$-\frac{1}{a^3x}+\frac{\ln(-a-x)}{3a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^2+a^4\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{12}+3\right)x-a^9\_R^2\right)\right)}{3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^4*(-1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/a^3/
x+1/3*ln(a+x)/a^4
```

maxima [A] time = 0.96, size = 57, normalized size = 0.90

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}+\frac{\log\left(a+x\right)}{3a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

mupad [B] time = 0.25, size = 88, normalized size = 1.40

$$\frac{\ln\left(a+x\right)}{3a^4}-\frac{1}{a^3x}+\frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2a^4}{4}+x\,a^3\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6a^4}-\frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2a^4}{4}+x\,a^3\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^3 + x^3)),x)
```

```
[Out] log(a + x)/(3*a^4) - 1/(a^3*x) + (log(a^3*x + (a^4*(3^(1/2)*1i - 1)^2)/4)*(
3^(1/2)*1i - 1))/(6*a^4) - (log(a^3*x + (a^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2
)*1i + 1))/(6*a^4)
```

<u>**sympy**</u> **[C]** time = 0.17, size = 83, normalized size = 1.32

$$-\frac{1}{a^3x}+\frac{\frac{\log(a+x)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(-\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)^2+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(-\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)^2+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**3+x**3),x)
```

```
[Out] -1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6 - sqrt(3)*
I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqrt(3)*I/6)**2 + x))/a
**4
```

3.123 $\int \frac{1}{x^3(a^3+x^3)} \, dx$

Optimal. Leaf size=65

$$-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/(2*a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]/(
3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx &= -\frac{1}{2a^3x^2}-\frac{\int \frac{1}{a^3+x^3}\,dx}{a^3}\\
&= -\frac{1}{2a^3x^2}-\frac{\int \frac{1}{a+x}\,dx}{3a^5}-\frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^5}\\
&= -\frac{1}{2a^3x^2}-\frac{\log(a+x)}{3a^5}+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^5}-\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^4}\\
&= -\frac{1}{2a^3x^2}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^5}\\
&= -\frac{1}{2a^3x^2}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.05

$$
-\frac{\log(a+x)}{3a^5}-\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/2*1/(a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x
]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

IntegrateAlgebraic [A] time = 0.03, size = 68, normalized size = 1.05

$$
-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}\,a}\right)}{\sqrt{3}\,a^5}-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/2*1/(a^3*x^2) + ArcTan[1/Sqrt[3] - (2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Lo
g[a + x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

fricas [A] time = 0.79, size = 62, normalized size = 0.95

$$
-\frac{2\,\sqrt{3}\,x^2\arctan\left(-\frac{\sqrt{3}\,(a-2\,x)}{3\,a}\right)-x^2\log\left(a^2-ax+x^2\right)+2\,x^2\log\left(a+x\right)+3\,a^2}{6\,a^5x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - x^2*log(a^2 - a*x +
x^2) + 2*x^2*log(a + x) + 3*a^2)/(a^5*x^2)
```

giac [A] time = 0.99, size = 58, normalized size = 0.89

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log\left(|a+x|\right)}{3a^5}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

maple [A] time = 0.30, size = 60, normalized size = 0.92

method	result	size
default	$\frac{\frac{\ln(a^2-ax+x^2)}{2}-\sqrt{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^5}-\frac{\ln(a+x)}{3a^5}-\frac{1}{2a^3x^2}$	60
risch	$-\frac{1}{2a^3x^2}+\frac{\ln(4a^2-4ax+4x^2)}{6a^5}-\frac{\sqrt{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^5}-\frac{\ln(a+x)}{3a^5}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^5*(1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/3*ln(
a+x)/a^5-1/2/a^3/x^2
```

maxima [A] time = 0.96, size = 57, normalized size = 0.88

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log\left(a+x\right)}{3a^5}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

mupad [B] time = 0.25, size = 86, normalized size = 1.32

$$-\frac{\ln\left(a+x\right)}{3a^5}-\frac{1}{2a^3x^2}-\frac{\ln\left(\frac{3a^7\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3a^6x\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6a^5}+\frac{\ln\left(\frac{3a^7\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}-3a^6x\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^3 + x^3)),x)
```

```
[Out] (log((3*a^7*(3^(1/2)*1i + 1))/2 - 3*a^6*x)*(3^(1/2)*1i + 1))/(6*a^5) - 1/(2
*a^3*x^2) - (log((3*a^7*(3^(1/2)*1i - 1))/2 + 3*a^6*x)*(3^(1/2)*1i - 1))/(6
*a^5) - log(a + x)/(3*a^5)
```

sympy [C] time = 0.20, size = 80, normalized size = 1.23

$$-\frac{1}{2a^3x^2}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**3+x**3),x)
```

```
[Out] -1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*(1/6 - sqr
t(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sqrt(3)*I/6) + x))/a**
5
```

3.124 $\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx$

Optimal. Leaf size=33

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}-\frac{\log(x)}{a^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a^3+x\right)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{1}{a^6x}+\frac{1}{a^6\left(a^3+x\right)}\right)dx,x,x^3\right)\\ &= -\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

IntegrateAlgebraic [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{\log(x)}{a^6}-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

fricas [A] time = 0.76, size = 33, normalized size = 1.00

$$\frac{x^3\log\left(a^3+x^3\right)-3\,x^3\log(x)-a^3}{3\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

giac [A] time = 0.90, size = 40, normalized size = 1.21

$$\frac{\log\left(\left|a^3+x^3\right|\right)}{3\,a^6}-\frac{\log\left(|x|\right)}{a^6}-\frac{a^3-x^3}{3\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)
```

maple [A] time = 0.27, size = 34, normalized size = 1.03

method	result	size
risch	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln\left(-a^3-x^3\right)}{3a^6}$	34
default	$\frac{\ln\left(a^2-ax+x^2\right)}{3a^6}-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(a+x)}{3a^6}$	43
norman	$\frac{\ln\left(a^2-ax+x^2\right)}{3a^6}-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(a+x)}{3a^6}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(-a^3-x^3)
```

maxima [A] time = 0.43, size = 31, normalized size = 0.94

$$\frac{\log\left(a^3+x^3\right)}{3\,a^6}-\frac{\log\left(x^3\right)}{3\,a^6}-\frac{1}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

mupad [B] time = 0.07, size = 29, normalized size = 0.88

$$\frac{\ln\left(a^3+x^3\right)}{3\,a^6}-\frac{\ln(x)}{a^6}-\frac{1}{3\,a^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^3 + x^3)),x)
```

```
[Out] log(a^3 + x^3)/(3*a^6) - log(x)/a^6 - 1/(3*a^3*x^3)
```

sympy [A] time = 0.27, size = 29, normalized size = 0.88

$$-\frac{1}{3a^3x^3} - \frac{\log(x)}{a^6} + \frac{\log\left(a^3 + x^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**3+x**3),x)
[Out] -1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

3.125 $\int \frac{1}{x^5(a^3+x^3)}\,dx$

Optimal. Leaf size=73

$$-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{1}{a^6x}-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7) -
Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx &= -\frac{1}{4a^3x^4}-\frac{\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx}{a^3}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}+\frac{\int \frac{x}{a^3+x^3}\,dx}{a^6}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\int \frac{1}{a+x}\,dx}{3a^7}+\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a^7}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^7}+\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^6}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^7}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.01

$$-\frac{\log(a+x)}{3a^7}+\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{1}{a^6x}-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/4*1/(a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7)
 - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

IntegrateAlgebraic [A] time = 0.03, size = 80, normalized size = 1.10

$$-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{4x^3-a^3}{4a^6x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] (-a^3 + 4*x^3)/(4*a^6*x^4) - ArcTan[1/Sqrt[3] - (2*x)/(Sqrt[3]*a)]/(Sqrt[3]
*a^7) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

fricas [A] time = 1.04, size = 68, normalized size = 0.93

$$\frac{4\sqrt{3}x^4\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+2x^4\log\left(a^2-ax+x^2\right)-4x^4\log(a+x)-3a^4+12ax^3}{12a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x
+ x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)
```

giac [A] time = 0.92, size = 67, normalized size = 0.92

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(|a+x|\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

maple [A] time = 0.28, size = 66, normalized size = 0.90

method	result	size
default	$\frac{\frac{\ln(a^2-ax+x^2)}{2}+\sqrt{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^7}-\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\ln(a+x)}{3a^7}$	66
risch	$\frac{\frac{x^3}{a^6}-\frac{1}{4a^3}}{x^4}-\frac{\ln(a+x)}{3a^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{21}-3\right)x+a^{15}\_R^2\right)\right)}{3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^7*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/4/a^3
/x^4+1/a^6/x-1/3*ln(a+x)/a^7
```

maxima [A] time = 0.96, size = 66, normalized size = 0.90

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(a+x\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

mupad [B] time = 0.10, size = 99, normalized size = 1.36

$$-\frac{\frac{1}{4a^3}-\frac{x^3}{a^6}}{x^4}-\frac{\ln\left(a+x\right)}{3a^7}-\frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1i\right)^2a^7}{4}+x\,a^6\right)\left(-1+\sqrt{3}\,1i\right)}{6a^7}+\frac{\ln\left(\frac{\left(1+\sqrt{3}\,1i\right)^2a^7}{4}+x\,a^6\right)\left(1+\sqrt{3}\,1i\right)}{6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^3 + x^3)),x)
```

```
[Out] (log(a^6*x + (a^7*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^7) - log(a
+ x)/(3*a^7) - (log(a^6*x + (a^7*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(
6*a^7) - (1/(4*a^3) - x^3/a^6)/x^4
```

sympy [C] time = 0.22, size = 90, normalized size = 1.23

$$\frac{-a^3+4x^3}{4a^6x^4}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}\,i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}\,i}{6}\right)^2+x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a**3+x**3),x)
```

```
[Out] (-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9
*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)
*I/6)**2 + x))/a**7
```

3.126 $\int \frac{x^{-m}}{a^3+x^3}\,dx$

Optimal. Leaf size=46

$$\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{3};\frac{4-m}{3};-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {364}

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
- m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^3+x^3}\,dx = \frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{3};\frac{4-m}{3};-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.98

$$-\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1}{3}-\frac{m}{3};\frac{4}{3}-\frac{m}{3};-\frac{x^3}{a^3}\right)}{a^3(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*(
-1 + m)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^3+x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^{3}+x^{3}\right)x^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")
[Out] integral(1/((a^3 + x^3)*x^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{3}+x^{3}\right)x^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="giac")
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^{3}+x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^3+x^3),x)
[Out] int(1/(x^m)/(a^3+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{3}+x^{3}\right)x^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{m}\,\left(a^{3}+x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m*(a^3 + x^3)),x)
[Out] int(1/(x^m*(a^3 + x^3)), x)
```

sympy [C] time = 2.97, size = 92, normalized size = 2.00

$$-\frac{mxx^{-m}\Phi\left(\frac{x^{3}e^{i\pi}}{a^{3}},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^{3}\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}+\frac{xx^{-m}\Phi\left(\frac{x^{3}e^{i\pi}}{a^{3}},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^{3}\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**3+x**3),x)

[Out] -m*x*x**(-m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3 -
m/3)/(9*a**3*gamma(4/3 - m/3)) + x*x**(-m)*lerchphi(x**3*exp_polar(I*pi)/a*
*3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*gamma(4/3 - m/3))
```

3.127 $\int \frac{1}{a^4-x^4} \, dx$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^4-x^4} \, dx &= \frac{\int \frac{1}{a^2-x^2} \, dx}{2a^2}+\frac{\int \frac{1}{a^2+x^2} \, dx}{2a^2}\\ &= \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.41

$$-\frac{\log(a-x)}{4a^3}+\frac{\log(a+x)}{4a^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^4 - x^4)^(-1),x]
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

fricas [A] time = 1.04, size = 26, normalized size = 0.96

$$\frac{2\arctan\left(\frac{x}{a}\right)+\log\left(a+x\right)-\log\left(-a+x\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="fricas")
[Out] 1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

giac [A] time = 0.86, size = 34, normalized size = 1.26

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3}+\frac{\log\left(|a+x|\right)}{4\,a^3}-\frac{\log\left(|-a+x|\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="giac")
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(abs(a + x))/a^3 - 1/4*log(abs(-a + x))/a^3
```

maple [A] time = 0.28, size = 33, normalized size = 1.22

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\ln(a+x)}{4a^3}-\frac{\ln(a-x)}{4a^3}$	33
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}-\frac{\ln(-a+x)}{4a^3}+\frac{\ln(a+x)}{4a^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x/a)/a^3+1/4*ln(a+x)/a^3-1/4/a^3*ln(a-x)
```

maxima [A] time = 0.97, size = 32, normalized size = 1.19

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3}+\frac{\log\left(a+x\right)}{4\,a^3}-\frac{\log\left(-a+x\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="maxima")
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

mupad [B] time = 0.07, size = 18, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{x}{a}\right)+\mathrm{atanh}\left(\frac{x}{a}\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^4 - x^4),x)
```

```
[Out] (atan(x/a) + atanh(x/a))/(2*a^3)
```

sympy [C] time = 0.15, size = 37, normalized size = 1.37

$$-\frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i\log(-ia+x)}{4} - \frac{i\log(ia+x)}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**4-x**4),x)
```

```
[Out] -(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a + x)/4)/a**3
```

3.128 $\int \frac{x}{a^4-x^4} \, dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 206}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^4-x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^4-x^2} \, dx, x, x^2\right)\\ &= \frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^4 - x^4),x]
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

fricas [A] time = 0.84, size = 26, normalized size = 1.73

$$\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="fricas")
[Out] 1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

giac [B] time = 0.81, size = 30, normalized size = 2.00

$$\frac{\log\left(a^2+x^2\right)}{4\,a^2}-\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="giac")
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2
```

maple [B] time = 0.34, size = 30, normalized size = 2.00

method	result	size
default	$\frac{\ln\left(a^2+x^2\right)}{4a^2}-\frac{\ln\left(a^2-x^2\right)}{4a^2}$	30
risch	$-\frac{\ln\left(-a^2+x^2\right)}{4a^2}+\frac{\ln\left(a^2+x^2\right)}{4a^2}$	30
norman	$-\frac{\ln(a-x)}{4a^2}-\frac{\ln(a+x)}{4a^2}+\frac{\ln\left(a^2+x^2\right)}{4a^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] 1/4/a^2*ln(a^2+x^2)-1/4/a^2*ln(a^2-x^2)
```

maxima [B] time = 0.42, size = 29, normalized size = 1.93

$$\frac{\log\left(a^2+x^2\right)}{4\,a^2}-\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="maxima")
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\mathrm{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4 - x^4),x)
[Out] atanh(x^2/a^2)/(2*a^2)
```

sympy [A] time = 0.15, size = 24, normalized size = 1.60

$$-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4-x**4),x)
```

```
[Out] -(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

3.129 $\int \frac{1}{x(a^4-x^4)}\,dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 36, 31, 29}

$$\frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^4-x^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(a^4-x\right)x}\,dx,x,x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{a^4-x}\,dx,x,x^4\right)}{4a^4}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)}{4a^4}\\ &= \frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\log(x)}{a^4}-\frac{\log\left(x^4-a^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\log(x)}{a^4} - \frac{\log\left(a^4 - x^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

fricas [A] time = 0.59, size = 20, normalized size = 0.83

$$-\frac{\log\left(-a^4 + x^4\right) - 4\log(x)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

giac [A] time = 0.95, size = 26, normalized size = 1.08

$$\frac{\log\left(x^4\right)}{4a^4} - \frac{\log\left(\left|-a^4 + x^4\right|\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/4*log(x^4)/a^4 - 1/4*log(abs(-a^4 + x^4))/a^4
```

maple [A] time = 0.32, size = 23, normalized size = 0.96

method	result	size
risch	$\frac{\ln(x)}{a^4} - \frac{\ln\left(-a^4+x^4\right)}{4a^4}$	23
default	$-\frac{\ln\left(a^2+x^2\right)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4} - \frac{\ln(a-x)}{4a^4}$	41
norman	$-\frac{\ln\left(a^2+x^2\right)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4} - \frac{\ln(a-x)}{4a^4}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^4-1/4/a^4*ln(-a^4+x^4)
```

maxima [A] time = 0.42, size = 25, normalized size = 1.04

$$-\frac{\log\left(-a^4 + x^4\right)}{4a^4} + \frac{\log\left(x^4\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] -1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

mupad [B] time = 0.28, size = 20, normalized size = 0.83

$$-\frac{\ln\left(x^4 - a^4\right) - 4\ \ln(x)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^4 - x^4)),x)
```

```
[Out] -(log(x^4 - a^4) - 4*log(x))/(4*a^4)
```

sympy [A] time = 0.25, size = 19, normalized size = 0.79

$$\frac{\log\left(x\right)}{a^4} - \frac{\log\left(-a^4 + x^4\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**4-x**4),x)
```

```
[Out] log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

3.130 $\int \frac{1}{x^2\left(a^4-x^4\right)} \, dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 298, 203, 206}

$$-\frac{1}{a^4x}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a^4-x^4\right)} \, dx &= -\frac{1}{a^4x}+\frac{\int \frac{x^2}{a^4-x^4} \, dx}{a^4}\\
&= -\frac{1}{a^4x}+\frac{\int \frac{1}{a^2-x^2} \, dx}{2a^4}-\frac{\int \frac{1}{a^2+x^2} \, dx}{2a^4}\\
&= -\frac{1}{a^4x}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.31

$$-\frac{\log(a-x)}{4a^5}+\frac{\log(a+x)}{4a^5}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^4 - x^4)),x]
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^5
)
```

IntegrateAlgebraic [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^4 - x^4)),x]
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

fricas [A] time = 0.97, size = 36, normalized size = 1.03

$$-\frac{2\,x\arctan\left(\frac{x}{a}\right)-x\log\left(a+x\right)+x\log\left(-a+x\right)+4\,a}{4\,a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")
[Out] -1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*x)
```

giac [A] time = 0.93, size = 42, normalized size = 1.20

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\log\left(|a+x|\right)}{4\,a^5}-\frac{\log\left(|-a+x|\right)}{4\,a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="giac")
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(abs(a + x))/a^5 - 1/4*log(abs(-a + x))/a^5 -
 1/(a^4*x)
```

maple [A] time = 0.28, size = 41, normalized size = 1.17

method	result	size
default	$-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}+\frac{\ln(a+x)}{4a^5}-\frac{\ln(a-x)}{4a^5}$	41
risch	$-\frac{1}{a^4x}-\frac{\ln(-a+x)}{4a^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^2+1\right)}\_R\ln\left(\left(5\_R^4a^{20}-4\right)x+a^{16}\_R^3\right)\right)}{4}+\frac{\ln(a+x)}{4a^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x/a)/a^5-1/a^4/x+1/4*ln(a+x)/a^5-1/4/a^5*ln(a-x)
```

maxima [A] time = 0.97, size = 40, normalized size = 1.14

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\log\left(a+x\right)}{4\,a^5}-\frac{\log\left(-a+x\right)}{4\,a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 - 1/(a^4*x)
```

mupad [B] time = 0.22, size = 31, normalized size = 0.89

$$\frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2\,a^5}-\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2\,a^5}-\frac{1}{a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^4 - x^4)),x)
[Out] atanh(x/a)/(2*a^5) - atan(x/a)/(2*a^5) - 1/(a^4*x)
```

sympy [C] time = 0.20, size = 44, normalized size = 1.26

$$-\frac{1}{a^4x}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}-\frac{i\log\left(-ia+x\right)}{4}+\frac{i\log\left(ia+x\right)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**4-x**4),x)
[Out] -1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 + I*log(I*a
 + x)/4)/a**5
```

3.131 $\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {275, 325, 206}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/(2*a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a^4-x^2\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{2a^4x^2}+\frac{\text{Subst}\left(\int \frac{1}{a^4-x^2}\,dx,x,x^2\right)}{2a^4}\\ &= -\frac{1}{2a^4x^2}+\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.92

$$-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}-\frac{1}{2a^4x^2}+\frac{\log\left(a^2+x^2\right)}{4a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2]
/(4*a^6)
```

IntegrateAlgebraic [A] time = 0.02, size = 50, normalized size = 1.92

$$-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}-\frac{1}{2a^4x^2}+\frac{\log\left(a^2+x^2\right)}{4a^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2]
/(4*a^6)
```

fricas [A] time = 0.88, size = 41, normalized size = 1.58

$$\frac{x^2\log\left(a^2+x^2\right)-x^2\log\left(-a^2+x^2\right)-2\,a^2}{4\,a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

giac [A] time = 0.97, size = 38, normalized size = 1.46

$$\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^2)
```

maple [A] time = 0.27, size = 42, normalized size = 1.62

method	result	size
risch	$-\frac{1}{2a^4x^2}-\frac{\ln\left(a^2-x^2\right)}{4a^6}+\frac{\ln\left(-a^2-x^2\right)}{4a^6}$	42
default	$\frac{\ln\left(a^2+x^2\right)}{4a^6}-\frac{\ln(a+x)}{4a^6}-\frac{\ln(a-x)}{4a^6}-\frac{1}{2a^4x^2}$	43
norman	$\frac{\ln\left(a^2+x^2\right)}{4a^6}-\frac{\ln(a+x)}{4a^6}-\frac{\ln(a-x)}{4a^6}-\frac{1}{2a^4x^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4/x^2-1/4/a^6*ln(a^2-x^2)+1/4/a^6*ln(-a^2-x^2)
```

maxima [A] time = 0.43, size = 37, normalized size = 1.42

$$\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(-a^2+x^2\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

mupad [B] time = 0.20, size = 22, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^6}-\frac{1}{2\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^4 - x^4)),x)
```

```
[Out] atanh(x^2/a^2)/(2*a^6) - 1/(2*a^4*x^2)
```

sympy [A] time = 0.22, size = 34, normalized size = 1.31

$$-\frac{1}{2a^4x^2}-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**4-x**4),x)
```

```
[Out] -1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**6
```

3.132 $\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 212, 206, 203}

$$-\frac{1}{3a^4x^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx &= -\frac{1}{3a^4x^3}+\frac{\int \frac{1}{a^4-x^4}\,dx}{a^4}\\ &= -\frac{1}{3a^4x^3}+\frac{\int \frac{1}{a^2-x^2}\,dx}{2a^6}+\frac{\int \frac{1}{a^2+x^2}\,dx}{2a^6}\\ &= -\frac{1}{3a^4x^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.30

$$-\frac{\log(a-x)}{4a^7}+\frac{\log(a+x)}{4a^7}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(4
*a^7)
```

IntegrateAlgebraic [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

fricas [A] time = 0.60, size = 45, normalized size = 1.22

$$\frac{6\,x^3\arctan\left(\frac{x}{a}\right)+3\,x^3\log\left(a+x\right)-3\,x^3\log\left(-a+x\right)-4\,a^3}{12\,a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) - 4*a^3)/(a^
7*x^3)
```

giac [A] time = 1.05, size = 42, normalized size = 1.14

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\log\left(|a+x|\right)}{4\,a^7}-\frac{\log\left(|-a+x|\right)}{4\,a^7}-\frac{1}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 -
1/3/(a^4*x^3)
```

maple [A] time = 0.28, size = 41, normalized size = 1.11

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}+\frac{\ln(a+x)}{4a^7}-\frac{\ln(a-x)}{4a^7}$	41
risch	$-\frac{1}{3a^4x^3}-\frac{\ln(a-x)}{4a^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^2+1\right)}\_R\ln\left(\left(-5\_R^4a^{28}+4\right)x-a^8\_R\right)\right)}{4}+\frac{\ln(-a-x)}{4a^7}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x/a)/a^7-1/3/a^4/x^3+1/4*ln(a+x)/a^7-1/4/a^7*ln(a-x)
```

maxima [A] time = 0.96, size = 40, normalized size = 1.08

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\log\left(a+x\right)}{4\,a^7}-\frac{\log\left(-a+x\right)}{4\,a^7}-\frac{1}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 - 1/3/(a^4*x
^3)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.84

$$\frac{\mathrm{atan}\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\mathrm{atanh}\left(\frac{x}{a}\right)}{2\,a^7}-\frac{1}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^4 - x^4)),x)
```

```
[Out] atan(x/a)/(2*a^7) + atanh(x/a)/(2*a^7) - 1/(3*a^4*x^3)
```

sympy [C] time = 0.23, size = 48, normalized size = 1.30

$$-\frac{1}{3a^4x^3}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}+\frac{i\log\left(-ia+x\right)}{4}-\frac{i\log\left(ia+x\right)}{4}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**4-x**4),x)
```

```
[Out] -1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*lo
g(I*a + x)/4)/a**7
```

3.133 $\int \frac{x^{-m}}{a^4-x^4}\,dx$

Optimal. Leaf size=45

$$\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 - m
))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^4-x^4}\,dx = \frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.98

$$-\frac{x^{1-m}\,{}_2F_1\left(1,\frac{1}{4}-\frac{m}{4};\frac{5}{4}-\frac{m}{4};\frac{x^4}{a^4}\right)}{a^4(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1
+ m)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^4-x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4 - x^4\right)x^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")
[Out] integral(1/((a^4 - x^4)*x^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^4 - x^4\right)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="giac")
[Out] integrate(1/((a^4 - x^4)*x^m), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^4 - x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^4-x^4),x)
[Out] int(1/(x^m)/(a^4-x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^4 - x^4\right)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")
[Out] integrate(1/((a^4 - x^4)*x^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^m\,\left(a^4 - x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m*(a^4 - x^4)),x)
[Out] int(1/(x^m*(a^4 - x^4)), x)
```

sympy [C] time = 1.01, size = 95, normalized size = 2.11

$$-\frac{mxx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right)\Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4} - \frac{m}{4}\right)} + \frac{xx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right)\Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4} - \frac{m}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**4-x**4),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4
- m/4)/(16*a**4*gamma(5/4 - m/4)) + x*x**(-m)*lerchphi(x**4*exp_polar(2*I*p
i)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*gamma(5/4 - m/4))
```

3.134 $\int \frac{x}{a^4+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 203}

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^4+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

fricas [A] time = 0.95, size = 13, normalized size = 0.87

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

giac [A] time = 0.97, size = 13, normalized size = 0.87

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

maple [A] time = 0.29, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
risch	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

maxima [A] time = 0.97, size = 13, normalized size = 0.87

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

mupad [B] time = 0.19, size = 13, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4 + x^4),x)
```

```
[Out] atan(x^2/a^2)/(2*a^2)
```

sympy [C] time = 0.15, size = 29, normalized size = 1.93

$$\frac{-\frac{i\log\left(-ia^2+x^2\right)}{4}+\frac{i\log\left(ia^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4+x**4),x)
```

```
[Out] (-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

3.135 $\int \frac{x^2}{a^4+x^4}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(a^2-\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\log\left(a^2+\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{a}\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{a}+1\right)}{2\sqrt{2}\,a}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(a^2-\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\log\left(a^2+\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{a}\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{a}+1\right)}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^4 + x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/a]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)/a]/(2*Sqr
t[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) - Log[a^2 + Sqrt[2]*a*
x + x^2]/(4*Sqrt[2]*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a^4+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}\,dx\right)+\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{a^2-\sqrt{2}\,ax+x^2}\,dx+\frac{1}{4}\int \frac{1}{a^2+\sqrt{2}\,ax+x^2}\,dx+\frac{\int \frac{\sqrt{2}\,a+2x}{-a^2-\sqrt{2}\,ax-x^2}\,dx}{4\sqrt{2}\,a}+\frac{\int \frac{\sqrt{2}\,a-2x}{-a^2+\sqrt{2}\,ax-x^2}\,dx}{4\sqrt{2}\,a}\\ &= \frac{\log\left(a^2-\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\log\left(a^2+\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\,x}{a}\right)}{2\sqrt{2}\,a}-\frac{\text{Subst}\left(\int \frac{1}{-1-\dots}\right.}{2\dots}\\ &= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{a}\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,x}{a}\right)}{2\sqrt{2}\,a}+\frac{\log\left(a^2-\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}-\frac{\log\left(a^2+\sqrt{2}\,ax+x^2\right)}{4\sqrt{2}\,a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.72

$$\frac{\log\left(a^2-\sqrt{2}\,ax+x^2\right)-\log\left(a^2+\sqrt{2}\,ax+x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{a}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,x}{a}+1\right)}{4\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^4 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log[a^2 - Sqr
t[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt[2]*a)
```

IntegrateAlgebraic [A] time = 0.31, size = 69, normalized size = 0.63

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,ax}{a^2+x^2}\right)}{2\sqrt{2}\,a}-\frac{\tan^{-1}\left(\frac{\frac{a}{\sqrt{2}}-\frac{x^2}{\sqrt{2}\,a}}{x}\right)}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^4 + x^4),x]
```

```
[Out] -1/2*ArcTan[(a/Sqrt[2] - x^2/(Sqrt[2]*a))/x]/(Sqrt[2]*a) - ArcTanh[(Sqrt[2]
*a*x)/(a^2 + x^2)]/(2*Sqrt[2]*a)
```

fricas [B] time = 0.97, size = 199, normalized size = 1.83

$$-\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{\sqrt{2}\,a^4\left(\frac{1}{a^4}\right)^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}x+\sqrt{2}\dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(a^(-4))^(1/4)*arctan(-sqrt(2)*(a^(-4))^(1/4)*x + sqrt(2)*sqrt
(sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2)*(a^(-4))^(1/4) - 1)
```

```
 - 1/2*sqrt(2)*(a^(-4))^(1/4)*arctan(-sqrt(2)*(a^(-4))^(1/4)*x + sqrt(2)*sq
rt(-sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2)*(a^(-4))^(1/4) +
 1) - 1/8*sqrt(2)*(a^(-4))^(1/4)*log(sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqr
t(a^(-4)) + x^2) + 1/8*sqrt(2)*(a^(-4))^(1/4)*log(-sqrt(2)*a^4*(a^(-4))^(3/
4)*x + a^4*sqrt(a^(-4)) + x^2)
```

giac [A] time = 0.97, size = 114, normalized size = 1.05

$$\frac{\sqrt{2}\,|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,|a|+2\,x\right)}{2\,|a|}\right)}{4\,a^2}+\frac{\sqrt{2}\,|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,|a|-2\,x\right)}{2\,|a|}\right)}{4\,a^2}-\frac{\sqrt{2}\,|a|\log\left(\sqrt{2}\,x|a|+x^2+|a|^2\right)}{8\,a^2}+\frac{\sqrt{2}\,|a|\log\left(-\sqrt{2}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^2 +
1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a))/a^2 -
 1/8*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2 + 1/8*sqrt(2
)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2
```

maple [C] time = 0.27, size = 24, normalized size = 0.22

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+a^4\right)}\frac{\ln(x-\_R)}{\_R}\right)}{4}$	24
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(a^4\right)^{\frac{1}{4}}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^4+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+a^4))
```

maxima [A] time = 0.97, size = 98, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a+2\,x\right)}{2\,a}\right)}{4\,a}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a-2\,x\right)}{2\,a}\right)}{4\,a}-\frac{\sqrt{2}\log\left(\sqrt{2}\,ax+a^2+x^2\right)}{8\,a}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,ax+a^2+x^2\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - 1/8*sqrt(2)*log(sqrt(2)*a*x + a^2 + x
^2)/a + 1/8*sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a
```

mupad [B] time = 0.10, size = 33, normalized size = 0.30

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,x}{a}\right)-(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^4 + x^4),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*x)/a) - (-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(2
*a)
```

sympy [A] time = 0.14, size = 19, normalized size = 0.17

$$\frac{\text{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**4+x**4),x)
```

```
[Out] RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

3.136 $\int \frac{1}{a^5+x^5} \, dx$

Optimal. Leaf size=201

$$\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4}-\frac{\left(1-\sqrt{5}\right)\log\left(a\right.}{}$$

Rubi [A] time = 0.31, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {201, 634, 618, 204, 628, 31}

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}+\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + x^5)^(-1),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a + x]/(5*a^4) - ((1 - Sqrt[5])
*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2
- ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a^5+x^5}\,dx = \frac{2\int \frac{a-\frac{1}{4}(1-\sqrt{5})x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{5a^4} + \frac{2\int \frac{a-\frac{1}{4}(1+\sqrt{5})x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{5a^4} + \frac{\int \frac{1}{a+x}\,dx}{5a^4}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}(1-\sqrt{5})a+2x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{20a^4} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}(1+\sqrt{5})a+2x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{20a^4} + \frac{\left(5-\sqrt{5}\right)\int \frac{}{a^2-}}{20}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}\,ax+2x^2\right)}{20a^4}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}\,a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}\left((1+\sqrt{5})a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4}$$

Mathematica [A] time = 0.17, size = 204, normalized size = 1.01

$$-\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + x^5)^(-1),x]
```

```
[Out] -1/20*(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/
(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2
 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 +
 Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/a
^4
```

IntegrateAlgebraic [A] time = 0.27, size = 260, normalized size = 1.29

$$\frac{\log(a+x)}{5a^4} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2-\frac{2}{\sqrt{5}}}\,x}{a} - \frac{1}{2}\sqrt{\frac{1}{10}\left(5-\sqrt{5}\right)} + \frac{1}{2}\sqrt{\frac{1}{10}\left(25-5\sqrt{5}\right)}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(-\frac{\sqrt{2+}}{}\right)}{}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^5 + x^5)^(-1),x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/2 - Sqrt[(5 - Sqrt[
5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*Ar
cTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10]/2 - (Sqrt[2 + 2/S
qrt[5]]*x)/a])/(5*a^4) + Log[a + x]/(5*a^4) + ((-1 - Sqrt[5])*Log[2*a^2 - a
*x - Sqrt[5]*a*x + 2*x^2])/(20*a^4) + ((-1 + Sqrt[5])*Log[2*a^2 - a*x + Sqr
t[5]*a*x + 2*x^2])/(20*a^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.01, size = 177, normalized size = 0.88

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^4}+\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^4}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^4 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^4 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*log(abs(a + x))/a
^4
```

maple [C] time = 0.36, size = 55, normalized size = 0.27

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^4+a^{12}\_Z^3+a^8\_Z^2+a^4\_Z+1\right)}\_R\ln\left(\_R\,a^5+x\right)\right)}{5}+\frac{\ln(a+x)}{5a^4}$	55
default	$\frac{\ln(a+x)}{5a^4}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\,\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_R\,a^2+4a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}}{5a^4}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum(_R*ln(_R*a^5+x),_R=RootOf(_Z^4*a^16+_Z^3*a^12+_Z^2*a^8+_Z*a^4+1))+1
/5*ln(a+x)/a^4
```

maxima [A] time = 0.97, size = 180, normalized size = 0.90

$$\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^4\sqrt{2\sqrt{5}+10}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^4\sqrt{-2\sqrt{5}+10}}-\frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2a^2-\right.}{10a^4\left(\sqrt{5}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/(a^4*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(
sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^4*sqrt(-2*sqrt(5) + 10))
- 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt(5)
```

```
+ 1)) - 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^4*(sqr
t(5) - 1)) + 1/5*log(a + x)/a^4
```

mupad [B] time = 0.59, size = 174, normalized size = 0.87

$$\frac{\ln(a+x)}{5\,a^4}-\frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^4}-\frac{\ln\left(x-\frac{a\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\,\sqrt{5}-10}-\right.}{20\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + x^5),x)
```

```
[Out] log(a + x)/(5*a^4) - (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)
*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4) - (log(x - (a*((- 2*5^(1/
2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))
/(20*a^4) + (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/
2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^4) - (log(x - (a*(5^(1/2) + (2*5^
(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4)
```

sympy [A] time = 0.15, size = 39, normalized size = 0.19

$$\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log(5ta+x)\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(5*_t*a + x))))/a**4
```

3.137 $\int \frac{x}{a^5+x^5} \, dx$

Optimal. Leaf size=201

$$-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{5a^3}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3}+\frac{\left(1+\sqrt{5}\right)\log}{}$$

Rubi [A] time = 0.27, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a + x]/(5*a^3) + ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^3} + \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^3} - \frac{\int \frac{1}{a+x}\,dx}{5a^3}$$

$$= -\frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^3} + \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^3} - \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)a\ldots}}{2\sqrt{5}\,a^2\ldots}$$

$$= -\frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}\,ax+2x^2\right)}{20a^3}\ldots$$

$$= \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} - \frac{\log(a+x)}{5a^3} + \ldots$$

Mathematica [A] time = 0.07, size = 204, normalized size = 1.01

$$\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2\ldots\right.}{20a\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a^5 + x^5),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[
10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2
] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[
5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3
)
```

IntegrateAlgebraic [A] time = 0.15, size = 260, normalized size = 1.29

$$-\frac{\log(a+x)}{5a^3} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2-\frac{2}{\sqrt{5}}}\,x}{a} - \frac{1}{2}\sqrt{\frac{1}{10}\left(5-\sqrt{5}\right)} + \frac{1}{2}\sqrt{\frac{1}{10}\left(25-5\sqrt{5}\right)}\right)}{5a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^5 + x^5),x]
```

```
[Out] -1/5*(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/2 - Sqrt[(5 -
Sqrt[5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/a^3 - (Sqrt[(5 + Sqrt[5])/2]*A
rcTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10]/2 - (Sqrt[2 + 2/
Sqrt[5]]*x)/a])/(5*a^3) - Log[a + x]/(5*a^3) + ((1 - Sqrt[5])*Log[2*a^2 - a
```

```
*x - Sqrt[5]*a*x + 2*x^2])/(20*a^3) + ((1 + Sqrt[5])*Log[2*a^2 - a*x + Sqrt
[5]*a*x + 2*x^2])/(20*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.89, size = 177, normalized size = 0.88

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^3}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^3}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}(\sqrt{5}a+a)x+x^2\right)}{20a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^3 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^3 + 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^3 - 1/5*log(abs(a + x))/
a^3
```

maple [C] time = 0.35, size = 60, normalized size = 0.30

method	result	size
risch	$-\frac{\ln(a+x)}{5a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}\_Z^4-a^9\_Z^3+a^6\_Z^2-a^3\_Z+1\right)}\_R\ln\left(-a^{10}\_R^3+x\right)\right)}{5}$	60
default	$-\frac{\ln(a+x)}{5a^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a+3\_R a^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R a^2-a^3}}{5a^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(a+x)/a^3+1/5*sum(_R*ln(-_R^3*a^10+x),_R=RootOf(_Z^4*a^12-_Z^3*a^9+_
Z^2*a^6-_Z*a^3+1))
```

maxima [A] time = 0.98, size = 160, normalized size = 0.80

$$-\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^3\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^3\sqrt{-2\sqrt{5}+10}}-\frac{\log(a+x)}{5a^3}-\frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{5a^3(\sqrt{5}+1)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^3*
sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
```

```
(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) - 1/5*log(a + x)/a^3 - 1/5*
log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) + 1/5*log(a*x*(
sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1))
```

mupad [B] time = 0.59, size = 182, normalized size = 0.91

$$\frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}-\frac{\ln\left(a+x\right)}{5\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}\right.}{20\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^5 + x^5),x)
```

```
[Out] (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(
1/2) - 10)^(1/2) + 1))/(20*a^3) - log(a + x)/(5*a^3) + (log(x - (a*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2)
 + 1))/(20*a^3) - (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/6
4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^3) + (log(x - (a*(5^(1/2
) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) +
1))/(20*a^3)
```

sympy [A] time = 0.14, size = 41, normalized size = 0.20

$$\frac{-\frac{\log\left(a+x\right)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(-125t^3a+x\right)\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambd
a(_t, _t*log(-125*_t**3*a + x))))/a**3
```

3.138 $\int \frac{x^2}{a^5+x^5} dx$

Optimal. Leaf size=201

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5\right.}}}{}$$

Rubi [A] time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {293, 634, 618, 204, 628, 31}

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^2} + \frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^2} + \frac{\int \frac{1}{a+x}\,dx}{5a^2}$$

$$= \frac{\log(a+x)}{5a^2} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^2} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^2} - \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+\cdots}}{2\sqrt{5}\,a}$$

$$= \frac{\log(a+x)}{5a^2} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^2} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}\,ax+2x^2\right)}{20a^2} \cdots$$

$$= \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{a^2} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \cdots$$

Mathematica [A] time = 0.09, size = 204, normalized size = 1.01

$$-\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(\cdots\right.}{20\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a^5 + x^5),x]
```

```
[Out] -1/20*(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 +
Sqrt[5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(
Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2
+ x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/a^
2
```

IntegrateAlgebraic [A] time = 0.16, size = 260, normalized size = 1.29

$$\frac{\left(\sqrt{5}-1\right)\log\left(2a^2-\sqrt{5}\,ax-ax+2x^2\right)}{20a^2}+\frac{\left(-1-\sqrt{5}\right)\log\left(2a^2+\sqrt{5}\,ax-ax+2x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^5 + x^5),x]
```

```
[Out] -1/5*(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/2 - Sqrt[(5 -
Sqrt[5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/a^2 - (Sqrt[(5 + Sqrt[5])/2]*A
rcTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10]/2 - (Sqrt[2 + 2/
Sqrt[5]]*x)/a])/(5*a^2) + Log[a + x]/(5*a^2) + ((-1 + Sqrt[5])*Log[2*a^2 -
```

```
a*x - Sqrt[5]*a*x + 2*x^2])/(20*a^2) + ((-1 - Sqrt[5])*Log[2*a^2 - a*x + Sq
rt[5]*a*x + 2*x^2])/(20*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.99, size = 177, normalized size = 0.88

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}(\sqrt{5}a+a)x+x^2\right)}{20a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^2 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^2 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^2 + 1/5*log(abs(a + x))/
a^2
```

maple [C] time = 0.35, size = 58, normalized size = 0.29

method	result	size
risch	$\frac{\ln(a+x)}{5a^2}+\frac{\sum_{\_R=\mathrm{RootOf}(a^8\_Z^4+a^6\_Z^3+a^4\_Z^2+a^2\_Z+1)}\_R\ln(x\_R^3a^5+1)}{5}$	58
default	$\frac{\ln(a+x)}{5a^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4)}\frac{(-\_R^3+2\_R^2a+2\_Ra^2-a^3)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln(a+x)/a^2+1/5*sum(_R*ln(_R^3*a^5*x+1),_R=RootOf(_Z^4*a^8+_Z^3*a^6+_Z^
2*a^4+_Z*a^2+1))
```

maxima [A] time = 0.98, size = 160, normalized size = 0.80

$$-\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^2\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^2\sqrt{-2\sqrt{5}+10}}+\frac{\log(a+x)}{5a^2}+\frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{5a^2(\sqrt{5}+1)}-\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^2*
sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
```

```
(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) + 1/5*log(a + x)/a^2 + 1/5*
log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) - 1/5*log(a*x*(
sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1))
```

mupad [B] time = 0.80, size = 202, normalized size = 1.00

$$\frac{\ln(a+x)}{5a^2}+\frac{\ln\left(a^5+\frac{x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3a^4}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^2}-\frac{\ln\left(a^5-\frac{a^4x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{\right.}{20a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^5 + x^5),x)
```

```
[Out] log(a + x)/(5*a^2) + (log(a^5 + (a^4*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2)
- 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^2) - (log(a^5 -
 (a^4*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2)
 - 10)^(1/2) + 1))/(20*a^2) - (log(a^5 - (a^4*x*(5^(1/2) - (2*5^(1/2) - 10)
^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^2) - (log(
a^5 - (a^4*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2)
- 10)^(1/2) - 5^(1/2) + 1))/(20*a^2)
```

sympy [A] time = 0.15, size = 41, normalized size = 0.20

$$\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(25*_t**2*a + x))))/a**2
```

3.139 $\int \frac{x^3}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}}{}$$

Rubi [A] time = 0.31, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^5 + x^5),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 +
 Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x]/(5*a) + ((1 - Sqrt[5])*Log[a
^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a}+\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a}-\frac{\int \frac{1}{a+x}\,dx}{5a}$$

$$= -\frac{\log(a+x)}{5a}+\frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx+\frac{1}{20}\left(5+\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax$$

$$= -\frac{\log(a+x)}{5a}+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}\,ax+2x^2\right)}{20a}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a}-\frac{\log(a+x)}{5a}$$

Mathematica [A] time = 0.06, size = 204, normalized size = 1.01

$$\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(\right.}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^5 + x^5),x]
```

```
[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[1
0 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2]
 - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

IntegrateAlgebraic [A] time = 0.14, size = 260, normalized size = 1.29

$$\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-\sqrt{5}\,ax-ax+2x^2\right)}{20a}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2+\sqrt{5}\,ax-ax+2x^2\right)}{20a}-\frac{\log(a+x)}{5a}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan}{}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/2 - Sqrt[(5 - Sqrt[
5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcT
an[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10]/2 - (Sqrt[2 + 2/Sqr
t[5]]*x)/a])/(5*a) - Log[a + x]/(5*a) + ((1 + Sqrt[5])*Log[2*a^2 - a*x - Sq
rt[5]*a*x + 2*x^2])/(20*a) + ((1 - Sqrt[5])*Log[2*a^2 - a*x + Sqrt[5]*a*x +
 2*x^2])/(20*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.93, size = 177, normalized size = 0.88

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a}+\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*s
qrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x
^2)/a - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a + 1/20*log(ab
s(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*log(abs(a + x))/a
```

maple [C] time = 0.32, size = 73, normalized size = 0.36

method	result	size
risch	$-\frac{\ln(a+x)}{5a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4\_Z^4-a^3\_Z^3+\_Z^2a^2-a\_Z+1\right)}\_R\ln\left(\_R^3a^4-\_R^2a^3+\_R\,a^2-a+x\right)\right)}{5}$	73
default	$-\frac{\ln(a+x)}{5a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3+3\_R^2a-2\_R\,a^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}}{5a}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(a+x)/a+1/5*sum(_R*ln(_R^3*a^4-_R^2*a^3+_R*a^2-a+x),_R=RootOf(_Z^4*a
^4-_Z^3*a^3+_Z^2*a^2-_Z*a+1))
```

maxima [A] time = 0.98, size = 180, normalized size = 0.90

$$\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a\sqrt{2\sqrt{5}+10}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a\sqrt{-2\sqrt{5}+10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2a^2\right.}{10a\left(\sqrt{5}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/(a*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(sq
rt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) + 1/
10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1))
+ 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1
)) - 1/5*log(a + x)/a
```

mupad [B] time = 0.41, size = 202, normalized size = 1.00

$$\frac{\ln\left(5a^{10}-\frac{5a^9x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20a}-\frac{\ln\left(5a^{10}+\frac{5x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)a^9}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}\right.}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^5 + x^5),x)
```

```
[Out] (log(5*a^10 - (5*a^9*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2)
+ (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) - (log(5*a^10 + (5*a^9*x*(5^(1/2) + (
- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/
(20*a) - log(a + x)/(5*a) + (log(5*a^10 - (5*a^9*x*(5^(1/2) - (2*5^(1/2) -
10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) + (log(5*
a^10 - (5*a^9*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2)
- 10)^(1/2) - 5^(1/2) + 1))/(20*a)
```

sympy [A] time = 0.15, size = 39, normalized size = 0.19

$$\frac{-\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambd
a(_t, _t*log(625*_t**4*a + x))))/a
```

3.140 $\int \frac{x^4}{a^5+x^5} \, dx$

Optimal. Leaf size=12

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {260}

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^5 + x^5),x]
[Out] Log[a^5 + x^5]/5
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{a^5+x^5} \, dx = \frac{1}{5}\log\left(a^5+x^5\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^5 + x^5),x]
[Out] Log[a^5 + x^5]/5
```

IntegrateAlgebraic [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a^5 + x^5),x]
[Out] Log[a^5 + x^5]/5
```

fricas [A] time = 0.97, size = 10, normalized size = 0.83

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] 1/5*log(a^5 + x^5)
```

giac [A] time = 0.95, size = 11, normalized size = 0.92

$$\frac{1}{5} \log \left(\left| a^5 + x^5 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="giac")
[Out] 1/5*log(abs(a^5 + x^5))
```

maple [A] time = 0.34, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\ln(a^5+x^5)}{5}$	11
default	$\frac{\ln(a^5+x^5)}{5}$	11
risch	$\frac{\ln(a^5+x^5)}{5}$	11
norman	$\frac{\ln(a+x)}{5} + \frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(a^5+x^5)
```

maxima [A] time = 0.43, size = 10, normalized size = 0.83

$$\frac{1}{5} \log \left(a^5 + x^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="maxima")
[Out] 1/5*log(a^5 + x^5)
```

mupad [B] time = 0.18, size = 10, normalized size = 0.83

$$\frac{\ln \left(a^5 + x^5 \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^5 + x^5),x)
[Out] log(a^5 + x^5)/5
```

sympy [A] time = 0.12, size = 8, normalized size = 0.67

$$\frac{\log \left(a^5 + x^5 \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**5+x**5),x)
[Out] log(a**5 + x**5)/5
```

3.141 $\int \frac{1}{x\left(a^5+x^5\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^5+x^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(a^5+x\right)}\,dx,x,x^5\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)}{5a^5}-\frac{\operatorname{Subst}\left(\int \frac{1}{a^5+x}\,dx,x,x^5\right)}{5a^5}\\ &= \frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^5 + x^5)),x]
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^5} - \frac{\log\left(a^5 + x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^5 + x^5)),x]
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

fricas [A] time = 1.01, size = 18, normalized size = 0.82

$$-\frac{\log\left(a^5 + x^5\right) - 5\log(x)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="fricas")
[Out] -1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

giac [A] time = 0.90, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|a^5 + x^5\right|\right)}{5\,a^5} + \frac{\log\left(|x|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="giac")
[Out] -1/5*log(abs(a^5 + x^5))/a^5 + log(abs(x))/a^5
```

maple [A] time = 0.31, size = 21, normalized size = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^5} - \frac{\ln\left(a^5+x^5\right)}{5a^5}$	21
default	$\frac{\ln(x)}{a^5} - \frac{\ln(a+x)}{5a^5} - \frac{\ln\left(a^4-a^3x+a^2x^2-a\,x^3+x^4\right)}{5a^5}$	49
norman	$\frac{\ln(x)}{a^5} - \frac{\ln(a+x)}{5a^5} - \frac{\ln\left(a^4-a^3x+a^2x^2-a\,x^3+x^4\right)}{5a^5}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

maxima [A] time = 0.43, size = 23, normalized size = 1.05

$$-\frac{\log\left(a^5 + x^5\right)}{5\,a^5} + \frac{\log\left(x^5\right)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

mupad [B] time = 0.27, size = 18, normalized size = 0.82

$$-\frac{\ln\left(a^5+x^5\right)-5\,\ln(x)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^5 + x^5)),x)
```

```
[Out] -(log(a^5 + x^5) - 5*log(x))/(5*a^5)
```

sympy [A] time = 0.26, size = 19, normalized size = 0.86

$$\frac{\log\left(x\right)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**5+x**5),x)
```

```
[Out] log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

3.142 $\int \frac{1}{x^2(a^5+x^5)}\,dx$

Optimal. Leaf size=209

$$\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}-\frac{1}{a^5x}-\frac{\left(1-\sqrt{5}\right)\log}{}$$

Rubi [A] time = 0.33, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sq
rt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^6) + Log[a + x]/(5*a^6) -
((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqr
t[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx &= -\frac{1}{a^5x}-\frac{\int \frac{x^3}{a^5+x^5}\,dx}{a^5}\\
&= -\frac{1}{a^5x}+\frac{\int \frac{1}{a+x}\,dx}{5a^6}-\frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^6}-\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^6}\\
&= -\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}-\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^6}-\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^6}-\\
&= -\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^6}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax\right.}{20a^6}\\
&= -\frac{1}{a^5x}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right.}{5a^6}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 172, normalized size = 0.82

$$
-\frac{-\left(\sqrt{5}-1\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\frac{20a}{x}-4\log(a+x)+\dots}{20a^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -1/20*((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(
Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*
a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[a^2
 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a
*x)/2 + x^2])/a^6
```

IntegrateAlgebraic [A] time = 0.25, size = 268, normalized size = 1.28

$$
\frac{\log(a+x)}{5a^6}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2-\frac{2}{\sqrt{5}}}\,x}{a}-\frac{1}{2}\sqrt{\frac{1}{10}\left(5-\sqrt{5}\right)}+\frac{1}{2}\sqrt{\frac{1}{10}\left(25-5\sqrt{5}\right)}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(-\dots\right.}{}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -(1/(a^5*x)) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/2 -
Sqrt[(5 - Sqrt[5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a]))/(5*a^6) + (Sqrt[(5 -
 Sqrt[5])/2]*ArcTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10]/2
- (Sqrt[2 + 2/Sqrt[5]]*x)/a]))/(5*a^6) + Log[a + x]/(5*a^6) + ((-1 - Sqrt[5]
)*Log[2*a^2 - a*x - Sqrt[5]*a*x + 2*x^2])/(20*a^6) + ((-1 + Sqrt[5])*Log[2*
a^2 - a*x + Sqrt[5]*a*x + 2*x^2])/(20*a^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.10, size = 185, normalized size = 0.89

$$-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^6}-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^6}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^6}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a + x))/
a^6 - 1/(a^5*x)
```

maple [C] time = 0.36, size = 76, normalized size = 0.36

method	result	size
risch	$-\frac{1}{a^5x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{24}\_Z^4+a^{18}\_Z^3+a^{12}\_Z^2+a^6\_Z+1\right)}\_R\ln\left(\left(6\_R^5a^{30}-5\right)x+a^{25}\_R^4\right)\right)}{5}+\frac{\ln(a+x)}{5a^6}$	76
default	$-\frac{1}{a^5x}+\frac{\ln(a+x)}{5a^6}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3-3\_R^2a+2\_R a^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R a^2-a^3}}{5a^6}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5/x+1/5*sum(_R*ln((6*_R^5*a^30-5)*x+a^25*_R^4),_R=RootOf(_Z^4*a^24+_Z^
3*a^18+_Z^2*a^12+_Z*a^6+1))+1/5*ln(a+x)/a^6
```

maxima [A] time = 0.97, size = 192, normalized size = 0.92

$$-\frac{\frac{2\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a\sqrt{-2\sqrt{5}+10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a\left(\sqrt{5}+1\right)}+\frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+\cdots\right)}{a\left(\sqrt{5}-1\right)}\cdots}{10a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/10*(2*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqr
t(5) + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 2*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*
(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) +
 (sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1)) +
(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1)) - 2*
log(a + x)/a)/a^5 - 1/(a^5*x)
```

mupad [B] time = 0.39, size = 210, normalized size = 1.00

$$\frac{\ln(a+x)}{5a^6} - \frac{1}{a^5 x} + \frac{\ln\left(5a^{30} + \frac{5x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)a^{29}}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}+\sqrt{2\sqrt{5}-\cdots}\right.}{4}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^5 + x^5)),x)
```

```
[Out] log(a + x)/(5*a^6) - 1/(a^5*x) + (log(5*a^30 + (5*a^29*x*(5^(1/2) + (- 2*5^
(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^
6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^
(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*(5^
(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) +
 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2)
 + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^6)
```

sympy [A] time = 0.19, size = 48, normalized size = 0.23

$$-\frac{1}{a^5x} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t\log\left(625t^4a + x\right)\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**5+x**5),x)
```

```
[Out] -1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*
_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

3.143 $\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx$

Optimal. Leaf size=211

$$-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{5a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7}-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right.}{}$$

Rubi [A] time = 0.33, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/(2*a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^7) - Log[a + x]/(5*a^7)
+ ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx = -\frac{1}{2a^5x^2}-\frac{\int \frac{x^2}{a^5+x^5}\,dx}{a^5}$$

$$= -\frac{1}{2a^5x^2}-\frac{\int \frac{1}{a+x}\,dx}{5a^7}-\frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^7}-\frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^7}$$

$$= -\frac{1}{2a^5x^2}-\frac{\log(a+x)}{5a^7}+\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^7}+\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^7}$$

$$= -\frac{1}{2a^5x^2}-\frac{\log(a+x)}{5a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^7}+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-a\right.}{20a^7}$$

$$= -\frac{1}{2a^5x^2}-\frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7}$$

Mathematica [A] time = 0.17, size = 174, normalized size = 0.82

$$-\frac{\frac{10a^2}{x^2}-\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+4\log(a+x)-2}{20a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/20*((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x
)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[
5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a + x] - (1 + Sqrt[5])*Log[
a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/a^7
```

IntegrateAlgebraic [A] time = 0.23, size = 270, normalized size = 1.28

$$-\frac{\log(a+x)}{5a^7}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2-\frac{2}{\sqrt{5}}}\,x}{a}-\frac{1}{2}\sqrt{\frac{1}{10}\left(5-\sqrt{5}\right)}+\frac{1}{2}\sqrt{\frac{1}{10}\left(25-5\sqrt{5}\right)}\right)}{5a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/2*1/(a^5*x^2) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/
2 - Sqrt[(5 - Sqrt[5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/(5*a^7) + (Sqrt[
(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10
]/2 - (Sqrt[2 + 2/Sqrt[5]]*x)/a])/(5*a^7) - Log[a + x]/(5*a^7) + ((1 - Sqrt
[5])*Log[2*a^2 - a*x - Sqrt[5]*a*x + 2*x^2])/(20*a^7) + ((1 + Sqrt[5])*Log[
2*a^2 - a*x + Sqrt[5]*a*x + 2*x^2])/(20*a^7)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.99, size = 185, normalized size = 0.88

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^7} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^7} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^7} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 + 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a + x))/a
^7 - 1/2/(a^5*x^2)
```

maple [C] time = 0.35, size = 78, normalized size = 0.37

method	result	size
risch	$-\frac{1}{2a^5x^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^{28}\_Z^4-a^{21}\_Z^3+a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{35}-5\right)x+a^{15}\_R^2\right)}{5}-\frac{\ln(a+x)}{5a^7}$	78
default	$-\frac{\ln(a+x)}{5a^7}-\frac{1}{2a^5x^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a-2\_Ra^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^7}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^5/x^2+1/5*sum(_R*ln((-6*_R^5*a^35-5)*x+a^15*_R^2),_R=RootOf(_Z^4*a^2
8-_Z^3*a^21+_Z^2*a^14-_Z*a^7+1))-1/5*ln(a+x)/a^7
```

maxima [A] time = 0.98, size = 173, normalized size = 0.82

$$\frac{\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^2\sqrt{2\sqrt{5}+10}}-\frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^2\sqrt{-2\sqrt{5}+10}}-\frac{\log(a+x)}{a^2}-\frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}+1\right)}+\frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}-1\right)}}{5a^5}-\frac{1}{2a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^
2*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) - log(a + x)/a^2 - log(-a*x
*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) + log(a*x*(sqrt(5) - 1)
 + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1)))/a^5 - 1/2/(a^5*x^2)
```

mupad [B] time = 0.78, size = 210, normalized size = 1.00

$$\frac{\ln\left(a^{20}-\frac{a^{19}\,x\,{\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}^3}{64}\right)\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^7}-\frac{1}{2\,a^5\,x^2}-\frac{\ln\left(a^{20}+\frac{x\,{\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}^3\,a^{19}}{64}\right)\,\left(\sqrt{5}+\right.}{20\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^5 + x^5)),x)
```

```
[Out] (log(a^20 - (a^19*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2)
+ (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^7) - 1/(2*a^5*x^2) - (log(a^20 + (a^19
*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2)
- 10)^(1/2) - 1))/(20*a^7) - log(a + x)/(5*a^7) + (log(a^20 - (a^19*x*(5^(1
/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2)
+ 1))/(20*a^7) + (log(a^20 - (a^19*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) +
1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^7)
```

sympy [A] time = 0.22, size = 51, normalized size = 0.24

$$-\frac{1}{2a^5x^2}+\frac{-\frac{\log{(a+x)}}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**5+x**5),x)
```

```
[Out] -1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**
2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

3.144 $\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx$

Optimal. Leaf size=211

$$\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)l\ldots}{}$$

Rubi [A] time = 0.29, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}+\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\ldots\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] -1/(3*a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^8) + Log[a + x]/(5*a^8)
- ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx = -\frac{1}{3a^5x^3} - \frac{\int \frac{x}{a^5+x^5}\,dx}{a^5}$$

$$= -\frac{1}{3a^5x^3} + \frac{\int \frac{1}{a+x}\,dx}{5a^8} - \frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^8} - \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^8}$$

$$= -\frac{1}{3a^5x^3} + \frac{\log(a+x)}{5a^8} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^8} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^8}$$

$$= -\frac{1}{3a^5x^3} + \frac{\log(a+x)}{5a^8} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}\,ax+2x^2\right)}{20a^8} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-a\right.}{20a^8}$$

$$= -\frac{1}{3a^5x^3} - \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}\,a}\right)}{a^8} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}$$

Mathematica [A] time = 0.14, size = 175, normalized size = 0.83

$$\frac{-\frac{20a^3}{x^3} - 3\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right) + 3\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right) + 12\log(a+x)}{60a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] ((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sq
rt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a
) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log[a^
2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/(60*a^8)
```

IntegrateAlgebraic [A] time = 0.20, size = 270, normalized size = 1.28

$$\frac{\log(a+x)}{5a^8} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2-\frac{2}{\sqrt{5}}}\,x}{a} - \frac{1}{2}\sqrt{\frac{1}{10}\left(5-\sqrt{5}\right)} + \frac{1}{2}\sqrt{\frac{1}{10}\left(25-5\sqrt{5}\right)}\right)}{5a^8} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] -1/3*1/(a^5*x^3) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(25 - 5*Sqrt[5])/10]/
2 - Sqrt[(5 - Sqrt[5])/10]/2 + (Sqrt[2 - 2/Sqrt[5]]*x)/a])/(5*a^8) + (Sqrt[
(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + Sqrt[5])/10]/2 + Sqrt[(25 + 5*Sqrt[5])/10
]/2 - (Sqrt[2 + 2/Sqrt[5]]*x)/a])/(5*a^8) + Log[a + x]/(5*a^8) + ((-1 + Sqr
t[5])*Log[2*a^2 - a*x - Sqrt[5]*a*x + 2*x^2])/(20*a^8) + ((-1 - Sqrt[5])*Lo
g[2*a^2 - a*x + Sqrt[5]*a*x + 2*x^2])/(20*a^8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.05, size = 185, normalized size = 0.88

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^8}-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^8}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^8}-\frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a + x))/a
^8 - 1/3/(a^5*x^3)
```

maple [C] time = 0.37, size = 81, normalized size = 0.38

method	result	size
risch	$-\frac{1}{3a^5x^3}+\frac{\ln(-a-x)}{5a^8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{32}\_Z^4+a^{24}\_Z^3+a^{16}\_Z^2+a^8\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{40}+5\right)x-a^{25}\_R^3\right)\right)}{5}$	81
default	$-\frac{1}{3a^5x^3}+\frac{\ln(a+x)}{5a^8}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_Ra^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^8}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^5/x^3+1/5/a^8*ln(-a-x)+1/5*sum(_R*ln((-6*_R^5*a^40+5)*x-a^25*_R^3),_
R=RootOf(_Z^4*a^32+_Z^3*a^24+_Z^2*a^16+_Z*a^8+1))
```

maxima [A] time = 0.97, size = 172, normalized size = 0.82

$$\frac{\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^3\sqrt{2\sqrt{5}+10}}-\frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^3\sqrt{-2\sqrt{5}+10}}+\frac{\log(a+x)}{a^3}+\frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{a^3(\sqrt{5}+1)}-\frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{a^3(\sqrt{5}-1)}}{5a^5}-\frac{1}{3a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^
3*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) + log(a + x)/a^3 + log(-a*x
*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) - log(a*x*(sqrt(5) - 1)
 + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1)))/a^5 - 1/3/(a^5*x^3)
```

mupad [B] time = 0.69, size = 214, normalized size = 1.01

$$\frac{\ln(a+x)}{5\,a^8}-\frac{1}{3\,a^5\,x^3}-\frac{\ln\left(a^{15}\,x-\frac{a^{16}\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^8}-\frac{\ln\left(a^{15}\,x-\frac{a^{16}\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}\cdots\right.}{64}\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^5 + x^5)),x)
```

```
[Out] log(a + x)/(5*a^8) - 1/(3*a^5*x^3) - (log(a^15*x - (a^16*(5^(1/2) - (2*5^(1
/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8
) - (log(a^15*x - (a^16*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((-
 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^8) + (log(a^15*x + (a^16*(5^(1
/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1
/2) - 1))/(20*a^8) - (log(a^15*x - (a^16*(5^(1/2) + (2*5^(1/2) - 10)^(1/2)
+ 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8)
```

sympy [A] time = 0.22, size = 51, normalized size = 0.24

$$-\frac{1}{3a^5x^3}+\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3a+x\right)\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**5+x**5),x)
```

```
[Out] -1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2
 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

3.145 $\int \frac{x^{-m}}{a^5+x^5} \, dx$

Optimal. Leaf size=46

$$\frac{x^{1-m} {}_2F_1\left(1,\frac{1-m}{5};\frac{6-m}{5};-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{364\}$

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
 - m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^5+x^5} \, dx = \frac{x^{1-m} {}_2F_1\left(1,\frac{1-m}{5};\frac{6-m}{5};-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.98

$$-\frac{x^{1-m} {}_2F_1\left(1,\frac{1}{5}-\frac{m}{5};\frac{6}{5}-\frac{m}{5};-\frac{x^5}{a^5}\right)}{a^5(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*(
-1 + m)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^5+x^5} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^5+x^5\right)x^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")
[Out] integral(1/((a^5 + x^5)*x^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^5+x^5\right)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="giac")
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^5+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^5+x^5),x)
[Out] int(1/(x^m)/(a^5+x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^5+x^5\right)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^m\,\left(a^5+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m*(a^5 + x^5)),x)
[Out] int(1/(x^m*(a^5 + x^5)), x)
```

sympy [C] time = 24.44, size = 92, normalized size = 2.00

$$-\frac{mxx^{-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}+\frac{xx^{-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**5+x**5),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5 -
m/5)/(25*a**5*gamma(6/5 - m/5)) + x*x**(-m)*lerchphi(x**5*exp_polar(I*pi)/a
**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*gamma(6/5 - m/5))
```

3.146 $\int \frac{1+x^4}{1+x^6} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Rubi [A] time = 0.43, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1876, 209, 634, 618, 204, 628, 203, 295}

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^6),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/3 + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{1+x^6}\,dx &= \int \left(\frac{1}{1+x^6}+\frac{x^4}{1+x^6}\right)dx\\
&= \int \frac{1}{1+x^6}\,dx+\int \frac{x^4}{1+x^6}\,dx\\
&= \frac{1}{3}\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx\\
&= \frac{2}{3}\tan^{-1}(x)+2\left(\frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx\right)+2\left(\frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx\right)\\
&= \frac{2}{3}\tan^{-1}(x)-2\left(\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)\right)-2\left(\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\right)\\
&= -\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}+2x\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.60

$$\frac{2}{3}\tan^{-1}(x)-\frac{1}{3}\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + x^6),x]
[Out] (2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 0.60

$$\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{x}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + x^6),x]
[Out] (2*ArcTan[x])/3 + ArcTan[(-1 + x^2)/x]/3
```

fricas [A] time = 0.60, size = 9, normalized size = 0.26

$$\frac{1}{3} \arctan\left(x^{3}\right) + \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="fricas")
[Out] 1/3*arctan(x^3) + arctan(x)
```

giac [A] time = 0.88, size = 27, normalized size = 0.77

$$\frac{1}{3} \arctan\left(2 x + \sqrt{3}\right) + \frac{1}{3} \arctan\left(2 x - \sqrt{3}\right) + \frac{2}{3} \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="giac")
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

maple [A] time = 0.28, size = 10, normalized size = 0.29

method	result
risch	$\arctan(x) + \frac{\arctan(x^3)}{3}$
default	$\frac{2\arctan(x)}{3} + \frac{\arctan(2x-\sqrt{3})}{3} + \frac{\arctan(2x+\sqrt{3})}{3}$
meijerg	$\frac{x^5\sqrt{3}\,\ln\left(1-\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}} + \frac{x^5\arctan\left(\frac{(x^6)^{\frac{1}{6}}}{2-\sqrt{3}\,(x^6)^{\frac{1}{6}}}\right)}{6(x^6)^{\frac{5}{6}}} + \frac{x^5\arctan\left((x^6)^{\frac{1}{6}}\right)}{3(x^6)^{\frac{5}{6}}} - \frac{x^5\sqrt{3}\,\ln\left(1+\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}} + \frac{x^5 \text{arctar}\cdots}{6\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^6+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)+1/3*arctan(x^3)
```

maxima [A] time = 0.98, size = 27, normalized size = 0.77

$$\frac{1}{3} \arctan\left(2 x + \sqrt{3}\right) + \frac{1}{3} \arctan\left(2 x - \sqrt{3}\right) + \frac{2}{3} \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="maxima")
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.26

$$\frac{\mathrm{atan}\left(x^3\right)}{3} + \mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^6 + 1),x)
[Out] atan(x^3)/3 + atan(x)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.23

$$\operatorname{atan}(x) + \frac{\operatorname{atan}(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**6+1),x)
```

```
[Out] atan(x) + atan(x**3)/3
```

3.147 $\int \frac{1}{\left(5+3x+x^2\right)^3} \, dx$

Optimal. Leaf size=60

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {614, 618, 204}

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x + x^2)) + (1
2*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(5+3x+x^2\right)^3} \, dx &= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3}{11}\int \frac{1}{\left(5+3x+x^2\right)^2} \, dx\\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}+\frac{6}{121}\int \frac{1}{5+3x+x^2} \, dx\\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}-\frac{12}{121}\text{Subst}\left(\int \frac{1}{-11-x^2} \, dx,x,3+2x\right)\\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}+\frac{12\tan^{-1}\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.85

$$\frac{\frac{11(2x+3)\left(6x^2+18x+41\right)}{\left(x^2+3x+5\right)^2}+24\sqrt{11}\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{2662}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] ((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[11]*ArcTan[
(3 + 2*x)/Sqrt[11]])/2662
```

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.93

$$\frac{(2x+3)\left(6x^2+18x+41\right)}{242\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x}{\sqrt{11}}+\frac{3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] ((3 + 2*x)*(41 + 18*x + 6*x^2))/(242*(5 + 3*x + x^2)^2) + (12*ArcTan[3/Sqrt
[11] + (2*x)/Sqrt[11]])/(121*Sqrt[11])
```

fricas [A] time = 1.08, size = 71, normalized size = 1.18

$$\frac{132\,x^3+24\,\sqrt{11}\left(x^4+6\,x^3+19\,x^2+30\,x+25\right)\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x+3)\right)+594\,x^2+1496\,x+1353}{2662\left(x^4+6\,x^3+19\,x^2+30\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")
```

```
[Out] 1/2662*(132*x^3 + 24*sqrt(11)*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/1
1*sqrt(11)*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 + 30
*x + 25)
```

giac [A] time = 1.05, size = 44, normalized size = 0.73

$$\frac{12}{1331}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x+3)\right)+\frac{12\,x^3+54\,x^2+136\,x+123}{242\left(x^2+3\,x+5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="giac")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^2 + 3*x + 5)^2
```

maple [A] time = 0.43, size = 44, normalized size = 0.73

method	result	size
risch	$\frac{\frac{6}{121}x^3+\frac{27}{121}x^2+\frac{68}{121}x+\frac{123}{242}}{\left(x^2+3x+5\right)^2}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	44

default	$\frac{3+2x}{22(x^2+3x+5)^2}+\frac{\frac{9}{121}+\frac{6x}{121}}{x^2+3x+5}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+5)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (6/121*x^3+27/121*x^2+68/121*x+123/242)/(x^2+3*x+5)^2+12/1331*arctan(1/11*(
3+2*x)*11^(1/2))*11^(1/2)
```

maxima [A] time = 0.96, size = 54, normalized size = 0.90

$$\frac{12}{1331}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+3)\right)+\frac{12x^3+54x^2+136x+123}{242\left(x^4+6x^3+19x^2+30x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

mupad [B] time = 0.08, size = 45, normalized size = 0.75

$$6\left(x+\frac{3}{2}\right)\left(\frac{1}{121\left(x^2+3x+5\right)}+\frac{1}{66\left(x^2+3x+5\right)^2}\right)+\frac{12\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}\left(x+\frac{3}{2}\right)}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + x^2 + 5)^3,x)
```

```
[Out] 6*(x + 3/2)*(1/(121*(3*x + x^2 + 5)) + 1/(66*(3*x + x^2 + 5)^2)) + (12*11^(
1/2)*atan((2*11^(1/2)*(x + 3/2))/11))/1331
```

sympy [A] time = 0.18, size = 63, normalized size = 1.05

$$\frac{12x^3+54x^2+136x+123}{242x^4+1452x^3+4598x^2+7260x+6050}+\frac{12\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{3\sqrt{11}}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+5)**3,x)
```

```
[Out] (12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260*
x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331
```

3.148 $\int \frac{1+x^2+x^4}{(1+x^2)^4} \, dx$

Optimal. Leaf size=43

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7}{16}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1157, 385, 199, 203}

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7}{16}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (7*ArcTan[x
])/16
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4}\,dx &= \frac{x}{6\left(1+x^2\right)^3} - \frac{1}{6}\int \frac{-5-6x^2}{\left(1+x^2\right)^3}\,dx \\
&= \frac{x}{6\left(1+x^2\right)^3} - \frac{x}{24\left(1+x^2\right)^2} + \frac{7}{8}\int \frac{1}{\left(1+x^2\right)^2}\,dx \\
&= \frac{x}{6\left(1+x^2\right)^3} - \frac{x}{24\left(1+x^2\right)^2} + \frac{7x}{16\left(1+x^2\right)} + \frac{7}{16}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{6\left(1+x^2\right)^3} - \frac{x}{24\left(1+x^2\right)^2} + \frac{7x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.70

$$
\frac{1}{48}\left(\frac{x\left(21x^4+40x^2+27\right)}{\left(x^2+1\right)^3} + 21\tan^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
[Out] ((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

IntegrateAlgebraic [A] time = 0.02, size = 31, normalized size = 0.72

$$
\frac{x\left(21x^4+40x^2+27\right)}{48\left(x^2+1\right)^3} + \frac{7}{16}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)/(1 + x^2)^4,x]
[Out] (x*(27 + 40*x^2 + 21*x^4))/(48*(1 + x^2)^3) + (7*ArcTan[x])/16
```

fricas [A] time = 0.75, size = 52, normalized size = 1.21

$$
\frac{21\,x^5 + 40\,x^3 + 21\left(x^6 + 3\,x^4 + 3\,x^2 + 1\right)\arctan(x) + 27\,x}{48\left(x^6 + 3\,x^4 + 3\,x^2 + 1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")
[Out] 1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^6
 + 3*x^4 + 3*x^2 + 1)
```

giac [A] time = 0.91, size = 28, normalized size = 0.65

$$
\frac{21\,x^5 + 40\,x^3 + 27\,x}{48\left(x^2+1\right)^3} + \frac{7}{16}\arctan(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="giac")
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

maple [A] time = 0.28, size = 28, normalized size = 0.65

method	result	size
default	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{\left(x^2+1\right)^3}+\frac{7\arctan\left(x\right)}{16}$	28
risch	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{\left(x^2+1\right)^3}+\frac{7\arctan\left(x\right)}{16}$	28
meijerg	$\frac{x\left(15x^4+40x^2+33\right)}{48\left(x^2+1\right)^3}+\frac{7\arctan\left(x\right)}{16}-\frac{x\left(-15x^4+40x^2+15\right)}{240\left(x^2+1\right)^3}-\frac{x\left(-3x^4-8x^2+3\right)}{48\left(x^2+1\right)^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)^4,x,method=_RETURNVERBOSE)
[Out] (7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

maxima [A] time = 0.97, size = 38, normalized size = 0.88

$$\frac{21\,x^5+40\,x^3+27\,x}{48\,\left(x^6+3\,x^4+3\,x^2+1\right)}+\frac{7}{16}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)
```

mupad [B] time = 0.18, size = 27, normalized size = 0.63

$$\frac{7\,\mathrm{atan}\left(x\right)}{16}+\frac{\frac{7\,x^5}{16}+\frac{5\,x^3}{6}+\frac{9\,x}{16}}{{\left(x^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/(x^2 + 1)^4,x)
[Out] (7*atan(x))/16 + ((9*x)/16 + (5*x^3)/6 + (7*x^5)/16)/(x^2 + 1)^3
```

sympy [A] time = 0.15, size = 36, normalized size = 0.84

$$\frac{21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}+\frac{7\,\mathrm{atan}\,(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)**4,x)
[Out] (21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x)
/16
```

3.149 $\int \frac{B+Ax}{\left(c+2bx+ax^2\right)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{-x(Ab-aB)-Ac+bB}{2\left(b^2-ac\right)\left(ax^2+2bx+c\right)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {638, 618, 206}

$$-\frac{-x(Ab-aB)-Ac+bB}{2\left(b^2-ac\right)\left(ax^2+2bx+c\right)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] -(b*B - A*c - (A*b - a*B)*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b - a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{B+Ax}{\left(c+2bx+ax^2\right)^2}\,dx &= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}+\frac{(Ab-aB)\int \frac{1}{c+2bx+ax^2}\,dx}{2\left(b^2-ac\right)}\\
&= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}-\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{4\left(b^2-ac\right)-x^2}\,dx,x,2b+2ax\right)}{b^2-ac}\\
&= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.98

$$\frac{\frac{(Ab-aB)\tan^{-1}\left(\frac{ax+b}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{-aBx+Abx+Ac-bB}{x(ax+2b)+c}}{2\left(b^2-ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] ((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[(
b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

IntegrateAlgebraic [A] time = 0.12, size = 113, normalized size = 1.26

$$\frac{-aBx+Abx+Ac-bB}{2\left(b^2-ac\right)\left(ax^2+2bx+c\right)}+\frac{(Ab-aB)\tan^{-1}\left(\frac{ax}{\sqrt{ac-b^2}}+\frac{b}{\sqrt{ac-b^2}}\right)}{2\left(b^2-ac\right)\sqrt{ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] (-(b*B) + A*c + A*b*x - a*B*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2)) + ((A*b
 - a*B)*ArcTan[b/Sqrt[-b^2 + a*c] + (a*x)/Sqrt[-b^2 + a*c]])/(2*(b^2 - a*c)*
Sqrt[-b^2 + a*c])
```

fricas [B] time = 1.13, size = 448, normalized size = 4.98

$$\left[-\frac{2\,Bb^3+2\,Aac^2-\left(\left(Ba^2-Aab\right)x^2+(Ba-Ab)c+2\left(Bab-Ab^2\right)x\right)\sqrt{b^2-ac}\,\log\left(\frac{a^2x^2+2\,abx+2\,b^2-ac+2\,\sqrt{b^2-ac}\,(ax+b)}{ax^2+2\,bx+c}\right.}{4\left(b^4c-2\,ab^2c^2+a^2c^3+\left(ab^4-2\,a^2b^2c+a^3c^2\right)x^2+2\left(b^5-2\,ab^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*
b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sqrt
(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B*a*
b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a*b^4
 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1/2*(B
*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x
)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(a*x + b)/(b^2 - a*c)) - (B*a*b
 + A*b^2)*c + (B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2
 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2
*b*c^2)*x)]
```

giac [A] time = 0.99, size = 92, normalized size = 1.02

$$-\frac{(Ba-Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\left(b^2-ac\right)\sqrt{-b^2+ac}}-\frac{Bax-Abx+Bb-Ac}{2\left(ax^2+2\,bx+c\right)\left(b^2-ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2
+ a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b*x + c)*(b^2 - a*c))
```

maple [A] time = 0.59, size = 103, normalized size = 1.14

method	result
default	$\frac{(-2Ab+2Ba)x+2bB-2Ac}{(4ac-4b^2)(a x^2+2bx+c)}+\frac{(-2Ab+2Ba)\arctan\left(\frac{2ax+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(Ab-Ba)x}{2(ac-b^2)}-\frac{Ac-bB}{2(ac-b^2)}}{a x^2+2bx+c}+\frac{\ln\left((-a^2c+a b^2)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)Ab}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-a^2c+a b^2)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)Ba}{4(-ac+b^2)^{\frac{3}{2}}}-\ln\left((a^2c-a b^2)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x+B)/(a*x^2+2*b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)+(-2*A*b+2*B*a)
/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more details)Is 4*b^2-4*a*c positive or negative?
```

mupad [B] time = 0.29, size = 159, normalized size = 1.77

$$\frac{\operatorname{atan}\left(\frac{2\left(a\,c-b^2\right)\left(\frac{\left(4\,b^3-4\,a\,b\,c\right)\left(A\,b-B\,a\right)}{8\,\left(a\,c-b^2\right)^{5/2}}-\frac{a\,x\,\left(A\,b-B\,a\right)}{2\,\left(a\,c-b^2\right)^{3/2}}\right)}{A\,b-B\,a}\right)\left(A\,b-B\,a\right)}{2\,\left(a\,c-b^2\right)^{3/2}}-\frac{\frac{A\,c-B\,b}{2\,\left(a\,c-b^2\right)}+\frac{x\,\left(A\,b-B\,a\right)}{2\,\left(a\,c-b^2\right)}}{a\,x^2+2\,b\,x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B + A*x)/(c + 2*b*x + a*x^2)^2,x)
```

```
[Out] (atan((2*(a*c - b^2)*(((4*b^3 - 4*a*b*c)*(A*b - B*a))/(8*(a*c - b^2)^(5/2))
 - (a*x*(A*b - B*a))/(2*(a*c - b^2)^(3/2))))/(A*b - B*a))*(A*b - B*a))/(2*(
a*c - b^2)^(3/2)) - ((A*c - B*b)/(2*(a*c - b^2)) + (x*(A*b - B*a))/(2*(a*c
- b^2)))/(c + 2*b*x + a*x^2)
```

sympy [B] time = 1.01, size = 323, normalized size = 3.59

$$-\frac{\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}\,(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)}{-Aab+Ba^2}\right)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

```
[Out] -sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**2
*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3
```

```
)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B*a*
*2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b + a
**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a*c -
b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*
b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c + x**2*
(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

3.150 $\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1660, 634, 618, 204, 628}

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx = \frac{1-x}{5-4x+x^2}+\frac{1}{4}\int \frac{-32+20x}{5-4x+x^2}\,dx$$
$$= \frac{1-x}{5-4x+x^2}+2\int \frac{1}{5-4x+x^2}\,dx+\frac{5}{2}\int \frac{-4+2x}{5-4x+x^2}\,dx$$
$$= \frac{1-x}{5-4x+x^2}+\frac{5}{2}\log\left(5-4x+x^2\right)-4\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-4+2x\right)$$
$$= \frac{1-x}{5-4x+x^2}-2\tan^{-1}(2-x)+\frac{5}{2}\log\left(5-4x+x^2\right)$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

fricas [A] time = 0.93, size = 50, normalized size = 1.32

$$\frac{4\left(x^2-4x+5\right)\arctan\left(x-2\right)+5\left(x^2-4x+5\right)\log\left(x^2-4x+5\right)-2x+2}{2\left(x^2-4x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")
[Out] 1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2 - 4*x + 5)
 - 2*x + 2)/(x^2 - 4*x + 5)
```

giac [A] time = 0.92, size = 33, normalized size = 0.87

$$-\frac{x-1}{x^2-4x+5}+2\arctan\left(x-2\right)+\frac{5}{2}\log\left(x^2-4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="giac")
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

maple [A] time = 0.40, size = 35, normalized size = 0.92

method	result	size

default	$\frac{1-x}{x^2-4x+5}+2\arctan(-2+x)+\frac{5\ln(x^2-4x+5)}{2}$	35
risch	$\frac{1-x}{x^2-4x+5}+2\arctan(-2+x)+\frac{5\ln(x^2-4x+5)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

maxima [A] time = 0.98, size = 33, normalized size = 0.87

$$-\frac{x-1}{x^2-4\,x+5}+2\,\arctan\left(x-2\right)+\frac{5}{2}\,\log\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")
```

```
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

mupad [B] time = 0.20, size = 41, normalized size = 1.08

$$2\,\mathrm{atan}\left(x-2\right)+\frac{5\,\ln\left(x^2-4\,x+5\right)}{2}-\frac{x}{x^2-4\,x+5}+\frac{1}{x^2-4\,x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((55*x - 27*x^2 + 5*x^3 - 41)/(x^2 - 4*x + 5)^2,x)
```

```
[Out] 2*atan(x - 2) + (5*log(x^2 - 4*x + 5))/2 - x/(x^2 - 4*x + 5) + 1/(x^2 - 4*x
 + 5)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.82

$$\frac{1-x}{x^2-4x+5}+\frac{5\log\left(x^2-4x+5\right)}{2}+2\,\mathrm{atan}\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

```
[Out] (1 - x)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)
```

3.151 $\int \frac{1}{\left(-1+x^3\right)^2} dx$

Optimal. Leaf size=57

$$\frac{x}{3\left(1-x^3\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log(1-x)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {199, 200, 31, 634, 618, 204, 628}

$$\frac{x}{3\left(1-x^3\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log(1-x)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(-2),x]
```

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*Log[1 - x]
)/9 + Log[1 + x + x^2]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+x^3\right)^2}\,dx &= \frac{x}{3\left(1-x^3\right)}-\frac{2}{3}\int \frac{1}{-1+x^3}\,dx\\
&= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\int \frac{1}{-1+x}\,dx-\frac{2}{9}\int \frac{-2-x}{1+x+x^2}\,dx\\
&= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\log(1-x)+\frac{1}{9}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\log(1-x)+\frac{1}{9}\log\left(1+x+x^2\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{x}{3\left(1-x^3\right)}+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\log(1-x)+\frac{1}{9}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{1}{9}\left(-\frac{3x}{x^3-1}+\log\left(x^2+x+1\right)-2\log(1-x)+2\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(-2),x]
[Out] ((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + L
og[1 + x + x^2])/9
```

IntegrateAlgebraic [A] time = 0.03, size = 56, normalized size = 0.98

$$-\frac{x}{3\left(x^3-1\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log(x-1)+\frac{2\tan^{-1}\left(\frac{2x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(-2),x]
[Out] -1/3*x/(-1 + x^3) + (2*ArcTan[1/Sqrt[3] + (2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*
Log[-1 + x])/9 + Log[1 + x + x^2]/9
```

fricas [A] time = 0.83, size = 58, normalized size = 1.02

$$\frac{2\sqrt{3}\left(x^3-1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\left(x^3-1\right)\log\left(x^2+x+1\right)-2\left(x^3-1\right)\log\left(x-1\right)-3x}{9\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + (x^3 - 1)*log(x^2
+ x + 1) - 2*(x^3 - 1)*log(x - 1) - 3*x)/(x^3 - 1)
```

giac [A] time = 0.95, size = 43, normalized size = 0.75

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3(x^3-1)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(abs(x - 1))
```

maple [A] time = 0.29, size = 41, normalized size = 0.72

method	result	size
risch	$-\frac{x}{3(x^3-1)}-\frac{2\ln(-1+x)}{9}+\frac{\ln(x^2+x+1)}{9}+\frac{2\sqrt{3}\arctan\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{9}$	41
default	$\frac{-1+x}{9x^2+9x+9}+\frac{\ln(x^2+x+1)}{9}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{1}{9(-1+x)}-\frac{2\ln(-1+x)}{9}$	53
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3x(-1)^{\frac{1}{3}}}{-3x^3+3}-\frac{2x(-1)^{\frac{1}{3}}\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}\right)}{3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x/(x^3-1)-2/9*ln(-1+x)+1/9*ln(x^2+x+1)+2/9*3^(1/2)*arctan(2/3*(1/2+x)*
3^(1/2))
```

maxima [A] time = 0.97, size = 42, normalized size = 0.74

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3(x^3-1)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(x - 1)
```

mupad [B] time = 0.08, size = 60, normalized size = 1.05

$$-\frac{2\ln(x-1)}{9}-\frac{x}{3\left(x^3-1\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(2x+1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 1)^2,x)
```

```
[Out] log(2*x + 3^(1/2)*1i + 1)*((3^(1/2)*1i)/9 + 1/9) - x/(3*(x^3 - 1)) - log(x
- (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1/9) - (2*log(x - 1))/9
```

sympy [**A**] time = 0.17, size = 53, normalized size = 0.93

$$-\frac{x}{3x^3-3}-\frac{2\log(x-1)}{9}+\frac{\log\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)**2,x)
```

```
[Out] -x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3)*atan(2*s
qrt(3)*x/3 + sqrt(3)/3)/9
```

3.152 $\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1260, 456, 453, 203}

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1260

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^
(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(
q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x
^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && I
LtQ[q, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-16+9x^2}{x^2\left(1+x^2\right)^2}\,dx\\ &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}+\frac{1}{8}\int \frac{32-25x^2}{x^2\left(1+x^2\right)}\,dx\\ &= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.92

$$-\frac{57x^4+103x^2+32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 0.92

$$\frac{-57x^4-103x^2-32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] (-32 - 103*x^2 - 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

fricas [A] time = 0.84, size = 40, normalized size = 1.11

$$-\frac{57\,x^4+103\,x^2+57\left(x^5+2\,x^3+x\right)\arctan(x)+32}{8\left(x^5+2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

giac [A] time = 1.06, size = 28, normalized size = 0.78

$$-\frac{25\,x^3+39\,x}{8\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

maple [A] time = 0.27, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{4}{x}-\frac{\frac{25}{8}x^{3}+\frac{39}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}$	29
risch	$\frac{-\frac{57}{8}x^{4}-\frac{103}{8}x^{2}-4}{\left(x^{2}+1\right)^{2}x}-\frac{57\arctan\left(x\right)}{8}$	29
meijerg	$-\frac{15x^{4}+25x^{2}+8}{2x\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{x\left(-3x^{2}+3\right)}{8\left(x^{2}+1\right)^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/x-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)
```

maxima [A] time = 0.97, size = 31, normalized size = 0.86

$$-\frac{57\,x^{4}+103\,x^{2}+32}{8\left(x^{5}+2\,x^{3}+x\right)}-\frac{57}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

mupad [B] time = 0.18, size = 29, normalized size = 0.81

$$-\frac{57\,\mathrm{atan}\left(x\right)}{8}-\frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,{\left(x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
```

```
[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

sympy [A] time = 0.15, size = 32, normalized size = 0.89

$$\frac{-57x^{4}-103x^{2}-32}{8x^{5}+16x^{3}+8x}-\frac{57\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

3.153 $\int \frac{x}{1+x^6}\, dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^4-x^2+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {275, 200, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1 - x^2 + x
^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, x^2\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.59

$$\frac{1}{12}\left(2\log\left(x^2+1\right) - \log\left(x^2-\sqrt{3}x+1\right) - \log\left(x^2+\sqrt{3}x+1\right) - 2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2x\right) - 2\sqrt{3}\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Log
[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

IntegrateAlgebraic [A] time = 0.03, size = 73, normalized size = 1.49

$$-\frac{1}{12}\log\left(-x^2+\sqrt{3}x-1\right) + \frac{1}{6}\log\left(x^2+1\right) - \frac{1}{12}\log\left(x^2+\sqrt{3}x+1\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x^6),x]
```

```
[Out] -1/2*ArcTan[1/Sqrt[3] - (2*x^2)/Sqrt[3]]/Sqrt[3] - Log[-1 + Sqrt[3]*x - x^2
]/12 + Log[1 + x^2]/6 - Log[1 + Sqrt[3]*x + x^2]/12
```

fricas [A] time = 1.09, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - \frac{1}{12}\log\left(x^4-x^2+1\right) + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

giac [A] time = 0.93, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - \frac{1}{12}\log\left(x^4-x^2+1\right) + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

maple [A] time = 0.28, size = 39, normalized size = 0.80

method	result	size
risch	$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	39
default	$-\frac{\ln\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x^2+1\right)}{6}$	41
meijerg	$\frac{x^2\ln\left(1+\left(x^6\right)^{\frac{1}{3}}\right)}{6\left(x^6\right)^{\frac{1}{3}}}-\frac{x^2\ln\left(1-\left(x^6\right)^{\frac{1}{3}}+\left(x^6\right)^{\frac{2}{3}}\right)}{12\left(x^6\right)^{\frac{1}{3}}}+\frac{x^2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{3}}}{2-\left(x^6\right)^{\frac{1}{3}}}\right)}{6\left(x^6\right)^{\frac{1}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

maxima [A] time = 0.96, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

mupad [B] time = 0.09, size = 52, normalized size = 1.06

$$\frac{\ln\left(x^2+1\right)}{6}-\ln\left(x^2-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 + 1),x)
```

```
[Out] log(x^2 + 1)/6 - log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) +
 log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.15, size = 46, normalized size = 0.94

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 -
 sqrt(3)/3)/6
```

3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx$

Optimal. Leaf size=13

$$\frac{\log(x^n - nx)}{n}$$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 514, 446, 72}

$$\frac{\log\left(1 - nx^{1-n}\right)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

```
[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx &= \int \frac{x^{-n}\left(-1+x^{-1+n}\right)}{1-nx^{1-n}}\,dx \\
&= \int \frac{1-x^{1-n}}{x\left(1-nx^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{1-x}{x(1-nx)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{1-n}{-1+nx}\right)dx, x, x^{1-n}\right)}{1-n} \\
&= \log(x)+\frac{\log\left(1-nx^{1-n}\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.54

$$
\frac{\log\left(1-nx^{1-n}\right)}{n}+\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 13, normalized size = 1.00

$$
\frac{\log\left(-nx+x^n\right)}{n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
[Out] log(-n*x + x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int -\frac{x^{n-1}-1}{nx-x^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="giac")
[Out] integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

maple [A] time = 0.30, size = 14, normalized size = 1.08

method	result	size
risch	$\frac{\ln(-nx+x^n)}{n}$	14
norman	$\frac{\ln\left(nx-e^{n\ln(x)}\right)}{n}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(-1+n))/(-n*x+x^n),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-n*x+x^n)/n
```

maxima [A] time = 0.45, size = 14, normalized size = 1.08

$$\frac{\log(nx - x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")
```

```
[Out] log(n*x - x^n)/n
```

mupad [B] time = 0.32, size = 14, normalized size = 1.08

$$\frac{\ln(n\,x - x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(n - 1) - 1)/(n*x - x^n),x)
```

```
[Out] log(n*x - x^n)/n
```

sympy [A] time = 2.48, size = 14, normalized size = 1.08

$$\begin{cases} \frac{\log\left(x - \frac{x^n}{n}\right)}{n} & \text{for } n \neq 0 \\ -x + \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

```
[Out] Piecewise((log(x - x**n/n)/n, Ne(n, 0)), (-x + log(x), True))
```

3.155 $\int \frac{x^3}{1-2x^2+3x^4} \, dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 634, 618, 204, 628}

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] -ArcTan[(1 - 3*x^2)/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{1-2x^2+3x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx, x, x^2\right) \\
&= \frac{1}{12}\,\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, x^2\right) \\
&= \frac{1}{12}\log\left(1-2x^2+3x^4\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(-1+3x^2\right)\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12}\log\left(1-2x^2+3x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.93

$$\frac{1}{12}\left(\sqrt{2}\tan^{-1}\left(\frac{3x^2-1}{\sqrt{2}}\right) + \log\left(3x^4-2x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

IntegrateAlgebraic [A] time = 0.02, size = 44, normalized size = 1.07

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{2}} - \frac{3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] -1/6*ArcTan[1/Sqrt[2] - (3*x^2)/Sqrt[2]]/Sqrt[2] + Log[1 - 2*x^2 + 3*x^4]/1
2
```

fricas [A] time = 1.15, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right) + \frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

giac [A] time = 1.51, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right) + \frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

maple [A] time = 0.03, size = 35, normalized size = 0.85

method	result	size
default	$\frac{\ln(3x^4-2x^2+1)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(6x^2-2)\sqrt{2}}{4}\right)}{12}$	35
risch	$\frac{\ln(9x^4-6x^2+3)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(3x^2-1)\sqrt{2}}{2}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

maxima [A] time = 0.97, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

mupad [B] time = 0.20, size = 34, normalized size = 0.83

$$\frac{\ln\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\sqrt{2}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 - 2*x^2 + 1),x)
```

```
[Out] log(x^4 - (2*x^2)/3 + 1/3)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*x^2)/2
))/12
```

sympy [A] time = 0.13, size = 42, normalized size = 1.02

$$\frac{\log\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x^2}{2}-\frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**4-2*x**2+1),x)
```

```
[Out] log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)/
12
```

3.156 $\int \frac{x^5}{-4+x^2+3x^4} \, dx$

Optimal. Leaf size=32

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 703, 632, 31}

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{-4+x^2+3x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-4+x+3x^2} \, dx,x,x^2\right)\\
&= \frac{x^2}{6}+\frac{1}{6}\text{Subst}\left(\int \frac{4-x}{-4+x+3x^2} \, dx,x,x^2\right)\\
&= \frac{x^2}{6}+\frac{3}{14}\text{Subst}\left(\int \frac{1}{-3+3x} \, dx,x,x^2\right)-\frac{8}{21}\text{Subst}\left(\int \frac{1}{4+3x} \, dx,x,x^2\right)\\
&= \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(-4 + x^2 + 3*x^4),x]
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

IntegrateAlgebraic [A] time = 0.01, size = 30, normalized size = 0.94

$$\frac{x^2}{6}+\frac{1}{14}\log\left(x^2-1\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(-4 + x^2 + 3*x^4),x]
[Out] x^2/6 + Log[-1 + x^2]/14 - (8*Log[4 + 3*x^2])/63
```

fricas [A] time = 1.09, size = 24, normalized size = 0.75

$$\frac{1}{6}x^2-\frac{8}{63}\log\left(3x^2+4\right)+\frac{1}{14}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

giac [A] time = 0.91, size = 25, normalized size = 0.78

$$\frac{1}{6}x^2-\frac{8}{63}\log\left(3x^2+4\right)+\frac{1}{14}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="giac")
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))
```

maple [A] time = 0.03, size = 25, normalized size = 0.78

method	result	size
default	$\frac{x^2}{6}-\frac{8\ln\left(3x^2+4\right)}{63}+\frac{\ln\left(x^2-1\right)}{14}$	25
risch	$\frac{x^2}{6}-\frac{8\ln\left(3x^2+4\right)}{63}+\frac{\ln\left(x^2-1\right)}{14}$	25
norman	$\frac{x^2}{6}+\frac{\ln(-1+x)}{14}+\frac{\ln(1+x)}{14}-\frac{8\ln\left(3x^2+4\right)}{63}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4+x^2-4),x,method=_RETURNVERBOSE)
[Out] 1/6*x^2-8/63*ln(3*x^2+4)+1/14*ln(x^2-1)
```

maxima [A] time = 0.42, size = 24, normalized size = 0.75

$$\frac{1}{6}x^2-\frac{8}{63}\log\left(3x^2+4\right)+\frac{1}{14}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

mupad [B] time = 0.09, size = 22, normalized size = 0.69

$$\frac{\ln\left(x^2-1\right)}{14}-\frac{8\,\ln\left(x^2+\frac{4}{3}\right)}{63}+\frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2 + 3*x^4 - 4),x)
```

```
[Out] log(x^2 - 1)/14 - (8*log(x^2 + 4/3))/63 + x^2/6
```

sympy [A] time = 0.12, size = 24, normalized size = 0.75

$$\frac{x^2}{6}+\frac{\log\left(x^2-1\right)}{14}-\frac{8\log\left(x^2+\frac{4}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**4+x**2-4),x)
```

```
[Out] x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

3.157 $\int \frac{x^2}{9-10x^3+x^6}\,dx$

Optimal. Leaf size=25

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{9-10x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx,x,x^3\right)\\ &= \frac{1}{24}\text{Subst}\left(\int \frac{1}{-9+x}\,dx,x,x^3\right)-\frac{1}{24}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^3\right)\\ &= -\frac{1}{24}\log\left(1-x^3\right)+\frac{1}{24}\log\left(9-x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -1/24*Log[1 - x^3] + Log[9 - x^3]/24
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 0.64

$$\frac{1}{12}\tanh^{-1}\left(\frac{5}{4}-\frac{x^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(9 - 10*x^3 + x^6),x]
[Out] ArcTanh[5/4 - x^3/4]/12
```

fricas [A] time = 1.40, size = 17, normalized size = 0.68

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

giac [A] time = 0.91, size = 19, normalized size = 0.76

$$-\frac{1}{24}\log\left(\left|x^3-1\right|\right)+\frac{1}{24}\log\left(\left|x^3-9\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="giac")
[Out] -1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))
```

maple [A] time = 0.03, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln\left(x^3-9\right)}{24}-\frac{\ln\left(x^3-1\right)}{24}$	18
risch	$\frac{\ln\left(x^3-9\right)}{24}-\frac{\ln\left(x^3-1\right)}{24}$	18
norman	$-\frac{\ln(-1+x)}{24}+\frac{\ln\left(x^3-9\right)}{24}-\frac{\ln\left(x^2+x+1\right)}{24}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-10*x^3+9),x,method=_RETURNVERBOSE)
[Out] 1/24*ln(x^3-9)-1/24*ln(x^3-1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.68

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

mupad [B] time = 0.58, size = 16, normalized size = 0.64

$$\frac{\operatorname{atanh}\left(\frac{81}{320\left(\frac{5x^3}{4}-\frac{9}{8}\right)}-\frac{41}{40}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - 10*x^3 + 9),x)
```

```
[Out] atanh(81/(320*((5*x^3)/4 - 9/8)) - 41/40)/12
```

sympy [A] time = 0.11, size = 15, normalized size = 0.60

$$\frac{\log\left(x^3-9\right)}{24}-\frac{\log\left(x^3-1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-10*x**3+9),x)
```

```
[Out] log(x**3 - 9)/24 - log(x**3 - 1)/24
```

3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx$

Optimal. Leaf size=36

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1850}

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx &= \int \left(-\frac{7}{(-2+x)^4}-\frac{4}{(-2+x)^3}+\frac{2}{(-2+x)^2}+\frac{1}{-2+x}\right)dx\\ &= -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.67

$$\frac{-6x^2+30x-29}{3(x-2)^3}+\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
[Out] (-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.67

$$\frac{-6x^2+30x-29}{3(x-2)^3}+\log(x-2)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
[Out] (-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

fricas [A] time = 1.09, size = 46, normalized size = 1.28

$$-\frac{6\,x^2-3\left(x^3-6\,x^2+12\,x-8\right)\log\left(x-2\right)-30\,x+29}{3\left(x^3-6\,x^2+12\,x-8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*x
^2 + 12*x - 8)
```

giac [A] time = 0.85, size = 23, normalized size = 0.64

$$-\frac{6\,x^2 - 30\,x + 29}{3\,(x-2)^3} + \log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="giac")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))
```

maple [A] time = 0.29, size = 22, normalized size = 0.61

method	result	size
norman	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
risch	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
default	$\frac{2}{(-2+x)^2}+\frac{7}{3(-2+x)^3}+\ln(-2+x)-\frac{2}{-2+x}$	27
meijerg	$\frac{x\left(\frac{1}{4}x^2-\frac{3}{2}x+3\right)}{48\left(1-\frac{x}{2}\right)^3}+\frac{x\left(\frac{11}{2}x^2-15x+12\right)}{24\left(1-\frac{x}{2}\right)^3}+\ln\left(1-\frac{x}{2}\right)-\frac{x^3}{12\left(1-\frac{x}{2}\right)^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+1)/(-2+x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*x^2+10*x-29/3)/(-2+x)^3+ln(-2+x)
```

maxima [A] time = 0.44, size = 32, normalized size = 0.89

$$-\frac{6\,x^2 - 30\,x + 29}{3\,\left(x^3 - 6\,x^2 + 12\,x - 8\right)} + \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.61

$$\ln\left(x-2\right) - \frac{2\,x^2 - 10\,x + \frac{29}{3}}{(x-2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4*x^2 + 1)/(x - 2)^4,x)
```

```
[Out] log(x - 2) - (2*x^2 - 10*x + 29/3)/(x - 2)^3
```

sympy [A] time = 0.12, size = 29, normalized size = 0.81

$$\frac{-6x^2 + 30x - 29}{3x^3 - 18x^2 + 36x - 24} + \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

```
[Out] (-6*x**2 + 30*x - 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

3.159 $\int \frac{x^3}{(-1+x)^{12}}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-1 + x)^12,x]
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(-1+x)^{12}}\,dx &= \int \left(\frac{1}{(-1+x)^{12}}+\frac{3}{(-1+x)^{11}}+\frac{3}{(-1+x)^{10}}+\frac{1}{(-1+x)^9}\right)dx\\ &= \frac{1}{11(1-x)^{11}}-\frac{3}{10(1-x)^{10}}+\frac{1}{3(1-x)^9}-\frac{1}{8(1-x)^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.53

$$\frac{-165x^3+55x^2-11x+1}{1320(x-1)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-1 + x)^12,x]
[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.53

$$\frac{-165x^3+55x^2-11x+1}{1320(x-1)^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-1 + x)^12,x]
[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

fricas [B] time = 1.06, size = 72, normalized size = 1.60

$$-\frac{165\,x^{3}-55\,x^{2}+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^{9}-165\,x^{8}+330\,x^{7}-462\,x^{6}+462\,x^{5}-330\,x^{4}+165\,x^{3}-55\,x^{2}+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

giac [A] time = 1.00, size = 22, normalized size = 0.49

$$-\frac{165\,x^{3}-55\,x^{2}+11\,x-1}{1320\,(x-1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

maple [A] time = 0.29, size = 22, normalized size = 0.49

method	result	size
norman	$\frac{-\frac{1}{8}x^{3}+\frac{1}{24}x^{2}-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
risch	$\frac{-\frac{1}{8}x^{3}+\frac{1}{24}x^{2}-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
gosper	$-\frac{165x^{3}-55x^{2}+11x-1}{1320(-1+x)^{11}}$	23
default	$-\frac{1}{8(-1+x)^{8}}-\frac{3}{10(-1+x)^{10}}-\frac{1}{11(-1+x)^{11}}-\frac{1}{3(-1+x)^{9}}$	30
meijerg	$\frac{x^{4}\left(-x^{7}+11x^{6}-55x^{5}+165x^{4}-330x^{3}+462x^{2}-462x+330\right)}{1320(1-x)^{11}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-1+x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1+x)^11*(-1/8*x^3+1/24*x^2-1/120*x+1/1320)
```

maxima [B] time = 0.44, size = 72, normalized size = 1.60

$$-\frac{165\,x^{3}-55\,x^{2}+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^{9}-165\,x^{8}+330\,x^{7}-462\,x^{6}+462\,x^{5}-330\,x^{4}+165\,x^{3}-55\,x^{2}+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

mupad [B] time = 0.10, size = 29, normalized size = 0.64

$$-\frac{1}{8\,(x-1)^{8}}-\frac{1}{3\,(x-1)^{9}}-\frac{3}{10\,(x-1)^{10}}-\frac{1}{11\,(x-1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x - 1)^12,x)
```

```
[Out] - 1/(8*(x - 1)^8) - 1/(3*(x - 1)^9) - 3/(10*(x - 1)^10) - 1/(11*(x - 1)^11)
```

sympy [B] time = 0.17, size = 70, normalized size = 1.56

$$\frac{-165x^3 + 55x^2 - 11x + 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x^3 - 7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-1+x)**12,x)
```

```
[Out] (-165*x**3 + 55*x**2 - 11*x + 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 - 2
17800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 217800
*x**3 - 72600*x**2 + 14520*x - 1320)
```

3.160 $\int \frac{-3x+x^4}{(1+2x)^5} dx$

Optimal. Leaf size=55

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32}\log(2x+1)$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 1620}

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1 +
 2*x)) + Log[1 + 2*x]/32
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3x+x^4}{(1+2x)^5}\,dx &= \int \frac{x\left(-3+x^3\right)}{(1+2x)^5}\,dx \\
&= \int \left(\frac{25}{16(1+2x)^5} - \frac{7}{4(1+2x)^4} + \frac{3}{8(1+2x)^3} - \frac{1}{4(1+2x)^2} + \frac{1}{16(1+2x)}\right)dx \\
&= -\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32}\log(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.75

$$\frac{384x^3 + 432x^2 + 368x + 12(2x+1)^4\log(2x+1) + 49}{384(2x+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] (49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2*
x)^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 37, normalized size = 0.67

$$\frac{384x^3+432x^2+368x+49}{384(2x+1)^4}+\frac{1}{32}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*x + x^4)/(1 + 2*x)^5,x]
[Out] (49 + 368*x + 432*x^2 + 384*x^3)/(384*(1 + 2*x)^4) + Log[1 + 2*x]/32
```

fricas [A] time = 1.24, size = 67, normalized size = 1.22

$$\frac{384\,x^3+432\,x^2+12\left(16\,x^4+32\,x^3+24\,x^2+8\,x+1\right)\log\left(2\,x+1\right)+368\,x+49}{384\left(16\,x^4+32\,x^3+24\,x^2+8\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")
[Out] 1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x
+ 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

giac [A] time = 0.89, size = 55, normalized size = 1.00

$$\frac{1}{8\left(2\,x+1\right)}-\frac{3}{32\left(2\,x+1\right)^2}+\frac{7}{24\left(2\,x+1\right)^3}-\frac{25}{128\left(2\,x+1\right)^4}-\frac{1}{32}\log\left(\frac{|2\,x+1|}{2\left(2\,x+1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="giac")
[Out] 1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x + 1)^4 -
1/32*log(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

maple [A] time = 0.26, size = 34, normalized size = 0.62

method	result	size
risch	$\frac{x^3+\frac{9}{8}x^2+\frac{23}{24}x+\frac{49}{384}}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	34
norman	$\frac{-\frac{37}{12}x^3-\frac{31}{16}x^2-\frac{1}{16}x-\frac{49}{24}x^4}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	37
default	$-\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8+16x}+\frac{\ln(1+2x)}{32}$	46
meijerg	$-\frac{x\left(1000x^3+1040x^2+420x+60\right)}{960(1+2x)^4}+\frac{\ln(1+2x)}{32}-\frac{x^2\left(4x^2+8x+6\right)}{4(1+2x)^4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3*x)/(1+2*x)^5,x,method=_RETURNVERBOSE)
[Out] 16*(1/16*x^3+9/128*x^2+23/384*x+49/6144)/(1+2*x)^4+1/32*ln(1+2*x)
```

maxima [A] time = 0.46, size = 48, normalized size = 0.87

$$\frac{384\,x^3+432\,x^2+368\,x+49}{384\left(16\,x^4+32\,x^3+24\,x^2+8\,x+1\right)}+\frac{1}{32}\log\left(2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
 + 1/32*log(2*x + 1)
```

mupad [B] time = 0.05, size = 43, normalized size = 0.78

$$\frac{\ln\left(x+\frac{1}{2}\right)}{32}+\frac{\frac{x^3}{16}+\frac{9\,x^2}{128}+\frac{23\,x}{384}+\frac{49}{6144}}{x^4+2\,x^3+\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^4)/(2*x + 1)^5,x)
```

```
[Out] log(x + 1/2)/32 + ((23*x)/384 + (9*x^2)/128 + x^3/16 + 49/6144)/(x/2 + (3*x
^2)/2 + 2*x^3 + x^4 + 1/16)
```

sympy [A] time = 0.13, size = 42, normalized size = 0.76

$$\frac{384x^3+432x^2+368x+49}{6144x^4+12288x^3+9216x^2+3072x+384}+\frac{\log(2x+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3*x)/(1+2*x)**5,x)
```

```
[Out] (384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 30
72*x + 384) + log(2*x + 1)/32
```

3.161 $\int \frac{1}{(-1+x)^2(1+x)^3}\,dx$

Optimal. Leaf size=36

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {44, 207}

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^2(1+x)^3}\,dx &= \int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right)dx\\ &= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}-\frac{3}{8}\int \frac{1}{-1+x^2}\,dx\\ &= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.06

$$\frac{1}{16}\left(\frac{-6x^2-6x+4}{(x-1)(x+1)^2}-3\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] ((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16
```

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 0.86

$$\frac{-3x^2-3x+2}{8(x-1)(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (2 - 3*x - 3*x^2)/(8*(-1 + x)*(1 + x)^2) + (3*ArcTanh[x])/8
```

fricas [B] time = 1.11, size = 59, normalized size = 1.64

$$-\frac{6\,x^2-3\left(x^3+x^2-x-1\right)\log\left(x+1\right)+3\left(x^3+x^2-x-1\right)\log\left(x-1\right)+6\,x-4}{16\left(x^3+x^2-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 - x - 1)*log
(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

giac [A] time = 1.06, size = 43, normalized size = 1.19

$$-\frac{1}{8\,(x-1)}+\frac{\frac{12}{x-1}+5}{32\left(\frac{2}{x-1}+1\right)^2}+\frac{3}{16}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*log(abs(-2/(x
 - 1) - 1))
```

maple [A] time = 0.29, size = 35, normalized size = 0.97

method	result	size
default	$-\frac{1}{8(-1+x)}-\frac{3\ln(-1+x)}{16}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3\ln(1+x)}{16}$	35
norman	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$	35
risch	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(-1+x)-3/16*ln(-1+x)-1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)
```

maxima [A] time = 0.45, size = 38, normalized size = 1.06

$$-\frac{3\,x^2+3\,x-2}{8\left(x^3+x^2-x-1\right)}+\frac{3}{16}\log\left(x+1\right)-\frac{3}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/16*log(x -
 1)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.86

$$\frac{3\,\mathrm{atanh}(x)}{8}+\frac{\frac{3\,x^2}{8}+\frac{3\,x}{8}-\frac{1}{4}}{-x^3-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^2*(x + 1)^3),x)
```

```
[Out] (3*atanh(x))/8 + ((3*x)/8 + (3*x^2)/8 - 1/4)/(x - x^2 - x^3 + 1)
```

sympy [A] time = 0.14, size = 41, normalized size = 1.14

$$\frac{-3x^2 - 3x + 2}{8x^3 + 8x^2 - 8x - 8} - \frac{3\log(x-1)}{16} + \frac{3\log(x+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(1+x)**3,x)
```

```
[Out] (-3*x**2 - 3*x + 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16 + 3*log(x
 + 1)/16
```

3.162 $\int \frac{1}{(5-6x)^2x^2}\,dx$

Optimal. Leaf size=35

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Int[1/((5 - 6*x)^2*x^2),x]
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5-6x)^2x^2}\,dx &= \int \left(\frac{1}{25x^2}+\frac{12}{125x}+\frac{36}{25(-5+6x)^2}-\frac{72}{125(-5+6x)}\right)dx\\ &= \frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.89

$$\frac{1}{125}\left(\frac{30}{5-6x}-\frac{5}{x}-12\log(5-6x)+12\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((5 - 6*x)^2*x^2),x]
[Out] (30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

IntegrateAlgebraic [A] time = 0.02, size = 36, normalized size = 1.03

$$\frac{5-12x}{25x(6x-5)}+\frac{12\log(x)}{125}-\frac{12}{125}\log(6x-5)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((5 - 6*x)^2*x^2),x]
[Out] (5 - 12*x)/(25*x*(-5 + 6*x)) + (12*Log[x])/125 - (12*Log[-5 + 6*x])/125
```

fricas [A] time = 0.74, size = 48, normalized size = 1.37

$$-\frac{12\,\left(6\,x^{2}-5\,x\right)\log\left(6\,x-5\right)-12\,\left(6\,x^{2}-5\,x\right)\log\left(x\right)+60\,x-25}{125\,\left(6\,x^{2}-5\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) + 60*x - 25
)/(6*x^2 - 5*x)
```

giac [A] time = 0.98, size = 40, normalized size = 1.14

$$-\frac{6}{25\,(6\,x-5)}+\frac{6}{125\,\left(\frac{5}{6\,x-5}+1\right)}+\frac{12}{125}\,\log\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="giac")
```

```
[Out] -6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*log(abs(-5/(6*x - 5) - 1
))
```

maple [A] time = 0.31, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{1}{25x}+\frac{12\ln(x)}{125}-\frac{6}{25(-5+6x)}-\frac{12\ln(-5+6x)}{125}$	28
risch	$\frac{-\frac{12x}{25}+\frac{1}{5}}{x(-5+6x)}+\frac{12\ln(x)}{125}-\frac{12\ln(-5+6x)}{125}$	31
norman	$\frac{\frac{1}{5}-\frac{72x^2}{125}}{x(-5+6x)}+\frac{12\ln(x)}{125}-\frac{12\ln(-5+6x)}{125}$	32
meijerg	$\frac{108x}{625\left(3-\frac{18x}{5}\right)}-\frac{12\ln\left(1-\frac{6x}{5}\right)}{125}+\frac{6}{125}+\frac{12\ln(x)}{125}+\frac{12\ln(2)}{125}+\frac{12\ln(3)}{125}-\frac{12\ln(5)}{125}+\frac{12i\pi}{125}-\frac{1}{25x}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5-6*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/25/x+12/125*ln(x)-6/25/(-5+6*x)-12/125*ln(-5+6*x)
```

maxima [A] time = 0.43, size = 31, normalized size = 0.89

$$-\frac{12\,x-5}{25\,\left(6\,x^{2}-5\,x\right)}-\frac{12}{125}\,\log\left(6\,x-5\right)+\frac{12}{125}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)
```

mupad [B] time = 0.22, size = 34, normalized size = 0.97

$$\frac{1}{5\,x\,\left(6\,x-5\right)}-\frac{12}{25\,\left(6\,x-5\right)}-\frac{12\,\ln\left(\frac{6\,x-5}{x}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x - 5)^2),x)
```

```
[Out] 1/(5*x*(6*x - 5)) - 12/(25*(6*x - 5)) - (12*log((6*x - 5)/x))/125
```

sympy [A] time = 0.13, size = 29, normalized size = 0.83

$$\frac{5-12x}{150x^2-125x}+\frac{12\log(x)}{125}-\frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)**2/x**2,x)
```

```
[Out] (5 - 12*x)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/125
```

3.163 $\int \frac{1}{\left(-3-2x+x^2\right)^3}\,dx$

Optimal. Leaf size=61

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {614, 616, 31}

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)) + (3*Log
[3 - x])/512 - (3*Log[1 + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-3-2x+x^2\right)^3}\,dx &= \frac{1-x}{16\left(3+2x-x^2\right)^2}-\frac{3}{16}\int \frac{1}{\left(-3-2x+x^2\right)^2}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{128}\int \frac{1}{-3-2x+x^2}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{512}\int \frac{1}{-3+x}\,dx-\frac{3}{512}\int \frac{1}{1+x}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.75

$$\frac{1}{512}\left(\frac{4\left(3x^3-9x^2-11x+17\right)}{\left(x^2-2x-3\right)^2}+3\log(3-x)-3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] ((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log[
1 + x])/512
```

IntegrateAlgebraic [A] time = 0.03, size = 44, normalized size = 0.72

$$\frac{3x^3-9x^2-11x+17}{128\left(x^2-2x-3\right)^2}+\frac{3}{256}\tanh^{-1}\left(\frac{1}{2}-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] (17 - 11*x - 9*x^2 + 3*x^3)/(128*(-3 - 2*x + x^2)^2) + (3*ArcTanh[1/2 - x/2
])/256
```

fricas [A] time = 1.19, size = 85, normalized size = 1.39

$$\frac{12\,x^3-36\,x^2-3\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)\log\left(x+1\right)+3\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)\log\left(x-3\right)-44\,x+68}{512\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x + 1) + 3*
(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2*x
^2 + 12*x + 9)
```

giac [A] time = 0.95, size = 42, normalized size = 0.69

$$\frac{3\,x^3-9\,x^2-11\,x+17}{128\left(x^2-2\,x-3\right)^2}-\frac{3}{512}\,\log\left(|x+1|\right)+\frac{3}{512}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="giac")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1))
 + 3/512*log(abs(x - 3))
```

maple [A] time = 0.29, size = 40, normalized size = 0.66

method	result	size
norman	$\frac{\frac{3}{128}x^3-\frac{9}{128}x^2-\frac{11}{128}x+\frac{17}{128}}{\left(x^2-2x-3\right)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$	40
risch	$\frac{\frac{3}{128}x^3-\frac{9}{128}x^2-\frac{11}{128}x+\frac{17}{128}}{\left(x^2-2x-3\right)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$	40

default	$-\frac{1}{128(-3+x)^{2}}+\frac{3}{256(-3+x)}+\frac{3\ln(-3+x)}{512}+\frac{1}{128(1+x)^{2}}+\frac{3}{256(1+x)}-\frac{3\ln(1+x)}{512}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/128*x^3-9/128*x^2-11/128*x+17/128)/(x^2-2*x-3)^2+3/512*ln(-3+x)-3/512*ln
(1+x)
```

maxima [A] time = 0.43, size = 50, normalized size = 0.82

$$\frac{3\,x^{3}-9\,x^{2}-11\,x+17}{128\left(x^{4}-4\,x^{3}-2\,x^{2}+12\,x+9\right)}-\frac{3}{512}\,\log\left(x+1\right)+\frac{3}{512}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512*
log(x + 1) + 3/512*log(x - 3)
```

mupad [B] time = 0.12, size = 45, normalized size = 0.74

$$-\frac{3\,\ln\left(\frac{x+1}{x-3}\right)}{512}-6\left(\frac{1}{256\,\left(-x^{2}+2\,x+3\right)}+\frac{1}{96\left(-x^{2}+2\,x+3\right)^{2}}\right)\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(2*x - x^2 + 3)^3,x)
```

```
[Out] - (3*log((x + 1)/(x - 3)))/512 - 6*(1/(256*(2*x - x^2 + 3)) + 1/(96*(2*x -
x^2 + 3)^2))*(x - 1)
```

sympy [A] time = 0.16, size = 51, normalized size = 0.84

$$\frac{3x^{3}-9x^{2}-11x+17}{128x^{4}-512x^{3}-256x^{2}+1536x+1152}+\frac{3\log\left(x-3\right)}{512}-\frac{3\log\left(x+1\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-3)**3,x)
```

```
[Out] (3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 11
52) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

3.164 $\int \frac{1}{(13-4x+x^2)^3}\,dx$

Optimal. Leaf size=51

$$-\frac{2-x}{216\left(x^2-4x+13\right)}-\frac{2-x}{36\left(x^2-4x+13\right)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {614, 618, 204}

$$-\frac{2-x}{216\left(x^2-4x+13\right)}-\frac{2-x}{36\left(x^2-4x+13\right)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] -(2 - x)/(36*(13 - 4*x + x^2)^2) - (2 - x)/(216*(13 - 4*x + x^2)) + ArcTan[
(-2 + x)/3]/648
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(13-4x+x^2\right)^3}\,dx &= -\frac{2-x}{36\left(13-4x+x^2\right)^2}+\frac{1}{12}\int \frac{1}{\left(13-4x+x^2\right)^2}\,dx\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{216}\int \frac{1}{13-4x+x^2}\,dx\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}-\frac{1}{108}\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,-4+2x\right)\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{648}\tan^{-1}\left(\frac{1}{3}(-2+x)\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.71

$$\frac{1}{648}\left(\frac{3(x-2)\left(x^2-4x+19\right)}{\left(x^2-4x+13\right)^2}+\tan^{-1}\left(\frac{x-2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] ((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/648
```

IntegrateAlgebraic [A] time = 0.02, size = 40, normalized size = 0.78

$$\frac{(x-2)\left(x^2-4x+19\right)}{216\left(x^2-4x+13\right)^2}-\frac{1}{648}\tan^{-1}\left(\frac{2}{3}-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] ((-2 + x)*(19 - 4*x + x^2))/(216*(13 - 4*x + x^2)^2) - ArcTan[2/3 - x/3]/64
8
```

fricas [A] time = 1.03, size = 62, normalized size = 1.22

$$\frac{3\,x^3-18\,x^2+\left(x^4-8\,x^3+42\,x^2-104\,x+169\right)\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)+81\,x-114}{648\left(x^4-8\,x^3+42\,x^2-104\,x+169\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arctan(1/3*x -
 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)
```

giac [A] time = 0.85, size = 34, normalized size = 0.67

$$\frac{x^3-6\,x^2+27\,x-38}{216\left(x^2-4\,x+13\right)^2}+\frac{1}{648}\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="giac")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x - 2
/3)
```

maple [A] time = 0.37, size = 36, normalized size = 0.71

method	result	size
risch	$\frac{\frac{1}{216}x^3-\frac{1}{36}x^2+\frac{1}{8}x-\frac{19}{108}}{\left(x^2-4x+13\right)^2}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$	36
default	$\frac{2x-4}{72\left(x^2-4x+13\right)^2}+\frac{2x-4}{432x^2-1728x+5616}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+13)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/216*x^3-1/36*x^2+1/8*x-19/108)/(x^2-4*x+13)^2+1/648*arctan(-2/3+1/3*x)
```

maxima [A] time = 0.96, size = 44, normalized size = 0.86

$$\frac{x^3 - 6\,x^2 + 27\,x - 38}{216\,\left(x^4 - 8\,x^3 + 42\,x^2 - 104\,x + 169\right)} + \frac{1}{648}\,\arctan\left(\frac{1}{3}\,x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/64
8*arctan(1/3*x - 2/3)
```

mupad [B] time = 0.20, size = 39, normalized size = 0.76

$$\frac{\mathrm{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648} + 6\,\left(x-2\right)\,\left(\frac{1}{1296\,\left(x^2-4\,x+13\right)} + \frac{1}{216\,{\left(x^2-4\,x+13\right)}^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 4*x + 13)^3,x)
```

```
[Out] atan(x/3 - 2/3)/648 + 6*(x - 2)*(1/(1296*(x^2 - 4*x + 13)) + 1/(216*(x^2 -
4*x + 13)^2))
```

sympy [A] time = 0.17, size = 42, normalized size = 0.82

$$\frac{x^3 - 6x^2 + 27x - 38}{216x^4 - 1728x^3 + 9072x^2 - 22464x + 36504} + \frac{\mathrm{atan}\left(\frac{x}{3} - \frac{2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+13)**3,x)
```

```
[Out] (x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x + 3
6504) + atan(x/3 - 2/3)/648
```

3.165 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)^3(3+x)^4}\,dx &= \int \left(\frac{1}{(2+x)^3}-\frac{4}{(2+x)^2}+\frac{10}{2+x}-\frac{1}{(3+x)^4}-\frac{3}{(3+x)^3}-\frac{6}{(3+x)^2}-\frac{10}{3+x}\right)dx\\ &= -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 0.80

$$\frac{60x^4+630x^3+2450x^2+4175x+2627}{6(x+2)^2(x+3)^3}-20\tanh^{-1}(2x+5)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] (2627 + 4175*x + 2450*x^2 + 630*x^3 + 60*x^4)/(6*(2 + x)^2*(3 + x)^3) - 20*
ArcTanh[5 + 2*x]
```

fricas [B] time = 1.15, size = 105, normalized size = 1.94

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 - 60\left(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108\right)\log\left(x + 3\right) + 60\left(x^5 + 13\,x^4 + 67\,x^3 + \right.}{6\left(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

giac [A] time = 0.91, size = 47, normalized size = 0.87

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x + 3)^3(x + 2)^2} - 10\,\log\left(|x + 3|\right) + 10\,\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

maple [A] time = 0.31, size = 45, normalized size = 0.83

method	result	size
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3} + 10\ln(2+x) - 10\ln(3+x)$	45
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3} + 10\ln(2+x) - 10\ln(3+x)$	45
default	$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\ln(2+x) - 10\ln(3+x)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*l
n(3+x)
```

maxima [A] time = 0.43, size = 60, normalized size = 1.11

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\left(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108\right)} - 10\,\log\left(x + 3\right) + 10\,\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

mupad [B] time = 0.05, size = 55, normalized size = 1.02

$$\frac{10\,x^4 + 105\,x^3 + \frac{1225\,x^2}{3} + \frac{4175\,x}{6} + \frac{2627}{6}}{x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108} - 20\,\mathrm{atanh}\left(2\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2)^3*(x + 3)^4),x)
```

```
[Out] ((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

sympy [**A**] time = 0.18, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x+2) - 10\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

3.166 $\int \frac{x^6}{(-2+x^2)^2} \, dx$

Optimal. Leaf size=36

$$\frac{x^3}{3} - \frac{2x}{x^2-2} + 4x - 5\sqrt{2} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {288, 302, 207}

$$\frac{x^5}{2(2-x^2)} + \frac{5x^3}{6} + 5x - 5\sqrt{2} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + x^2)^2,x]
```

```
[Out] 5*x + (5*x^3)/6 + x^5/(2*(2 - x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(-2+x^2)^2} \, dx &= \frac{x^5}{2(2-x^2)} + \frac{5}{2} \int \frac{x^4}{-2+x^2} \, dx \\
&= \frac{x^5}{2(2-x^2)} + \frac{5}{2} \int \left(2+x^2+\frac{4}{-2+x^2}\right) dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} + 10 \int \frac{1}{-2+x^2} \, dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} - 5\sqrt{2} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.47

$$\frac{x^3}{3}-\frac{2x}{x^2-2}+4x+\frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + x^2)^2,x]
```

```
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqrt
[2] + x])/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 1.06

$$\frac{x\left(x^4+10x^2-30\right)}{3\left(x^2-2\right)}-5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(-2 + x^2)^2,x]
```

```
[Out] (x*(-30 + 10*x^2 + x^4))/(3*(-2 + x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

fricas [A] time = 1.32, size = 53, normalized size = 1.47

$$\frac{2\,x^5+20\,x^3+15\,\sqrt{2}\left(x^2-2\right)\log\left(\frac{x^2-2\,\sqrt{2}\,x+2}{x^2-2}\right)-60\,x}{6\left(x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2
 - 2)) - 60*x)/(x^2 - 2)
```

giac [A] time = 1.08, size = 48, normalized size = 1.33

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}\right|}{\left|2\,x+2\,\sqrt{2}\right|}\right)+4\,x-\frac{2\,x}{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))) + 4*x
- 2*x/(x^2 - 2)
```

maple [A] time = 0.38, size = 32, normalized size = 0.89

method	result	size
default	$4x+\frac{x^3}{3}-\frac{2x}{x^2-2}-5\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$\frac{x^3}{3}+4x-\frac{2x}{x^2-2}+\frac{5\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{2}-\frac{5\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{2}$	44
meijerg	$i\sqrt{2}\left(-\frac{ix\sqrt{2}\left(-\frac{7}{2}x^4-35x^2+105\right)}{42\left(-\frac{x^2}{2}+1\right)}+5i\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\right)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^2-2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

maxima [A] time = 0.97, size = 40, normalized size = 1.11

$$\frac{1}{3}\,x^{3} + \frac{5}{2}\,\sqrt{2}\,\log\left(\frac{x - \sqrt{2}}{x + \sqrt{2}}\right) + 4\,x - \frac{2\,x}{x^{2} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 - 2
)
```

mupad [B] time = 0.25, size = 33, normalized size = 0.92

$$4\,x - \frac{2\,x}{x^{2} - 2} + \frac{x^{3}}{3} + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,1\mathrm{i}}{2}\right)\,5\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^2 - 2)^2,x)
```

```
[Out] 4*x + 2^(1/2)*atan((2^(1/2)*x*1i)/2)*5i - (2*x)/(x^2 - 2) + x^3/3
```

sympy [A] time = 0.12, size = 49, normalized size = 1.36

$$\frac{x^{3}}{3} + 4x - \frac{2x}{x^{2} - 2} + \frac{5\sqrt{2}\log\left(x - \sqrt{2}\right)}{2} - \frac{5\sqrt{2}\log\left(x + \sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**2-2)**2,x)
```

```
[Out] x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*sqrt(2)*lo
g(x + sqrt(2))/2
```

3.167 $\int \frac{x^8}{\left(4+x^2\right)^4} \, dx$

Optimal. Leaf size=58

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {288, 321, 203}

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(4 + x^2)^4,x]
```

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*x^3)/(48*(
4 + x^2)) - (35*ArcTan[x/2])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(4+x^2)^4}\,dx &= -\frac{x^7}{6(4+x^2)^3}+\frac{7}{6}\int \frac{x^6}{(4+x^2)^3}\,dx\\ &= -\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}+\frac{35}{24}\int \frac{x^4}{(4+x^2)^2}\,dx\\ &= -\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}+\frac{35}{16}\int \frac{x^2}{4+x^2}\,dx\\ &= \frac{35x}{16}-\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}-\frac{35}{4}\int \frac{1}{4+x^2}\,dx\\ &= \frac{35x}{16}-\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.69

$$\frac{x(12x^6+231x^4+1120x^2+1680)}{12(x^2+4)^3}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(4 + x^2)^4,x]
```

```
[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2]
)/8
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 0.69

$$\frac{x(12x^6+231x^4+1120x^2+1680)}{12(x^2+4)^3}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(4 + x^2)^4,x]
```

```
[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2]
)/8
```

fricas [A] time = 0.99, size = 59, normalized size = 1.02

$$\frac{24\,x^7+462\,x^5+2240\,x^3-105\,(x^6+12\,x^4+48\,x^2+64)\arctan\left(\frac{1}{2}\,x\right)+3360\,x}{24\,(x^6+12\,x^4+48\,x^2+64)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*arctan
(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

giac [A] time = 0.94, size = 31, normalized size = 0.53

$$x+\frac{87\,x^5+544\,x^3+912\,x}{12\,(x^2+4)^3}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="giac")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2*x)
```

maple [A] time = 0.27, size = 31, normalized size = 0.53

method	result	size
risch	$x+\frac{\frac{29}{4}x^5+\frac{136}{3}x^3+76x}{\left(x^2+4\right)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$	31
default	$x-\frac{16\left(-\frac{29}{64}x^5-\frac{17}{6}x^3-\frac{19}{4}x\right)}{\left(x^2+4\right)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$	32
meijerg	$\frac{x\left(\frac{9}{4}x^6+\frac{693}{16}x^4+210x^2+315\right)}{144\left(\frac{x^2}{4}+1\right)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^2+4)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x+(29/4*x^5+136/3*x^3+76*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

maxima [A] time = 0.96, size = 41, normalized size = 0.71

$$x+\frac{87\,x^5+544\,x^3+912\,x}{12\left(x^6+12\,x^4+48\,x^2+64\right)}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="maxima")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*arc
tan(1/2*x)
```

mupad [B] time = 0.07, size = 40, normalized size = 0.69

$$x-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}+\frac{\frac{29\,x^5}{4}+\frac{136\,x^3}{3}+76\,x}{x^6+12\,x^4+48\,x^2+64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^2 + 4)^4,x)
```

```
[Out] x - (35*atan(x/2))/8 + (76*x + (136*x^3)/3 + (29*x^5)/4)/(48*x^2 + 12*x^4 +
 x^6 + 64)
```

sympy [A] time = 0.15, size = 39, normalized size = 0.67

$$x+\frac{87x^5+544x^3+912x}{12x^6+144x^4+576x^2+768}-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**2+4)**4,x)
```

```
[Out] x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 + 768) - 35
*atan(x/2)/8
```

3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{19x+39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {638, 618, 204}

$$-\frac{19x+39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] -(39 + 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+7x}{\left(5+2x+3x^2\right)^2}\,dx &= -\frac{39+19x}{28\left(5+2x+3x^2\right)}-\frac{19}{28}\int \frac{1}{5+2x+3x^2}\,dx \\
&= -\frac{39+19x}{28\left(5+2x+3x^2\right)}+\frac{19}{14}\operatorname{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+6x\right) \\
&= -\frac{39+19x}{28\left(5+2x+3x^2\right)}-\frac{19\tan^{-1}\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.00

$$\frac{-19x-39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

IntegrateAlgebraic [A] time = 0.05, size = 46, normalized size = 1.07

$$\frac{-19x-39}{28\left(3x^2+2x+5\right)}-\frac{19\tan^{-1}\left(\frac{3x}{\sqrt{14}}+\frac{1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[1/Sqrt[14] + (3*x)/Sqrt[14
]])/(28*Sqrt[14])
```

fricas [A] time = 1.17, size = 45, normalized size = 1.05

$$-\frac{19\sqrt{14}\left(3x^2+2x+5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(3x+1)\right)+266x+546}{392\left(3x^2+2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")
```

```
[Out] -1/392*(19*sqrt(14)*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x + 1)) + 266
*x + 546)/(3*x^2 + 2*x + 5)
```

giac [A] time = 0.85, size = 36, normalized size = 0.84

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}\,(3x+1)\right)-\frac{19x+39}{28\left(3x^2+2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="giac")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

maple [A] time = 0.51, size = 34, normalized size = 0.79

method	result	size
risch	$\frac{-\frac{19x}{84}-\frac{13}{28}}{x^2+\frac{2}{3}x+\frac{5}{3}}-\frac{19\arctan\left(\frac{(1+3x)\sqrt{14}}{14}\right)\sqrt{14}}{392}$	34
default	$\frac{-38x-78}{168x^2+112x+280}-\frac{19\sqrt{14}\ \arctan\left(\frac{(6x+2)\sqrt{14}}{28}\right)}{392}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+7*x)/(3*x^2+2*x+5)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-19/84*x-13/28)/(x^2+2/3*x+5/3)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1
/2)
```

maxima [A] time = 0.97, size = 36, normalized size = 0.84

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}\,(3\,x+1)\right)-\frac{19\,x+39}{28\left(3\,x^2+2\,x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

mupad [B] time = 0.19, size = 36, normalized size = 0.84

$$-\frac{\frac{19\,x}{84}+\frac{13}{28}}{x^2+\frac{2\,x}{3}+\frac{5}{3}}-\frac{19\,\sqrt{14}\,\mathrm{atan}\left(\frac{3\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x - 4)/(2*x + 3*x^2 + 5)^2,x)
```

```
[Out] - ((19*x)/84 + 13/28)/((2*x)/3 + x^2 + 5/3) - (19*14^(1/2)*atan((3*14^(1/2)
*x)/14 + 14^(1/2)/14))/392
```

sympy [A] time = 0.14, size = 42, normalized size = 0.98

$$\frac{-19x-39}{84x^2+56x+140}-\frac{19\sqrt{14}\,\mathrm{atan}\left(\frac{3\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

```
[Out] (-19*x - 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)*x/14 + sq
rt(14)/14)/392
```

3.169 $\int \frac{5-4x}{\left(-2-4x+3x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {638, 618, 206}

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]

[Out] -(18 - 7*x)/(20*(2 + 4*x - 3*x^2)) - (7*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sq
rt[10])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5-4x}{\left(-2-4x+3x^2\right)^2} \, dx &= -\frac{18-7x}{20\left(2+4x-3x^2\right)}-\frac{7}{20}\int \frac{1}{-2-4x+3x^2} \, dx\\
&= -\frac{18-7x}{20\left(2+4x-3x^2\right)}+\frac{7}{10}\text{Subst}\left(\int \frac{1}{40-x^2} \, dx, x, -4+6x\right)\\
&= -\frac{18-7x}{20\left(2+4x-3x^2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.44

$$\frac{18-7x}{20\left(3x^2-4x-2\right)}-\frac{7\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}}+\frac{7\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[1
0]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

IntegrateAlgebraic [A] time = 0.06, size = 48, normalized size = 1.12

$$\frac{18-7x}{20\left(3x^2-4x-2\right)}-\frac{7\tanh^{-1}\left(\sqrt{\frac{2}{5}}-\frac{3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*ArcTanh[Sqrt[2/5] - (3*x)/Sqrt[10]]
)/(20*Sqrt[10])
```

fricas [A] time = 1.05, size = 68, normalized size = 1.58

$$\frac{7\sqrt{10}\left(3x^2-4x-2\right)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-140x+360}{400\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(7*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)
```

giac [A] time = 0.95, size = 51, normalized size = 1.19

$$-\frac{7}{400}\sqrt{10}\log\left(\frac{\left|6x-2\sqrt{10}-4\right|}{\left|6x+2\sqrt{10}-4\right|}\right)-\frac{7x-18}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="giac")
```

```
[Out] -7/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

maple [A] time = 0.35, size = 37, normalized size = 0.86

method	result	size
default	$-\frac{14x-36}{40(3x^2-4x-2)}+\frac{7\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{7x}{60}+\frac{3}{10}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{7\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{7\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5-4*x)/(3*x^2-4*x-2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

maxima [A] time = 0.95, size = 47, normalized size = 1.09

$$-\frac{7}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)-\frac{7x-18}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")
```

```
[Out] -7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(7*x
- 18)/(3*x^2 - 4*x - 2)
```

mupad [B] time = 0.21, size = 34, normalized size = 0.79

$$\frac{7\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\,\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200}+\frac{\frac{7x}{60}-\frac{3}{10}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 5)/(4*x - 3*x^2 + 2)^2,x)
```

```
[Out] (7*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + ((7*x)/60 - 3/10)/((4*x
)/3 - x^2 + 2/3)
```

sympy [A] time = 0.17, size = 58, normalized size = 1.35

$$-\frac{7x-18}{60x^2-80x-40}+\frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

```
[Out] -(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/40
0 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

3.170 $\int \frac{x^5}{\left(1+x^4\right)^3} \, dx$

Optimal. Leaf size=37

$$\frac{1}{16}\tan^{-1}\left(x^2\right)+\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {275, 288, 199, 203}

$$\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}+\frac{1}{16}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4)^3,x]
```

```
[Out] -x^2/(8*(1 + x^4)^2) + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(1+x^4\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^3}\,dx,x,x^2\right)\\
&= -\frac{x^2}{8\left(1+x^4\right)^2}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{x^2}{8\left(1+x^4\right)^2}+\frac{x^2}{16\left(1+x^4\right)}+\frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\
&= -\frac{x^2}{8\left(1+x^4\right)^2}+\frac{x^2}{16\left(1+x^4\right)}+\frac{1}{16}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$\frac{1}{16}\left(\tan^{-1}\left(x^2\right)+\frac{\left(x^4-1\right)x^2}{\left(x^4+1\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4)^3,x]
[Out] ((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.76

$$\frac{1}{16}\tan^{-1}\left(x^2\right)+\frac{\left(x^4-1\right)x^2}{16\left(x^4+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(1 + x^4)^3,x]
[Out] (x^2*(-1 + x^4))/(16*(1 + x^4)^2) + ArcTan[x^2]/16
```

fricas [A] time = 1.37, size = 38, normalized size = 1.03

$$\frac{x^6-x^2+\left(x^8+2\,x^4+1\right)\arctan\left(x^2\right)}{16\left(x^8+2\,x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="fricas")
[Out] 1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1)
```

giac [A] time = 1.01, size = 40, normalized size = 1.08

$$\frac{x^2-\frac{1}{x^2}}{16\left(\left(x^2-\frac{1}{x^2}\right)^2+4\right)}+\frac{1}{32}\arctan\left(\frac{x^4-1}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)
```

maple [A] time = 0.27, size = 27, normalized size = 0.73

method	result	size
meijerg	$-\frac{x^2(-3x^4+3)}{48(x^4+1)^2}+\frac{\arctan(x^2)}{16}$	27
risch	$\frac{\frac{1}{16}x^6-\frac{1}{16}x^2}{(x^4+1)^2}+\frac{\arctan(x^2)}{16}$	27
default	$\frac{\frac{1}{8}x^6-\frac{1}{8}x^2}{2(x^4+1)^2}+\frac{\arctan(x^2)}{16}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/48*x^2*(-3*x^4+3)/(x^4+1)^2+1/16*arctan(x^2)
```

maxima [A] time = 0.95, size = 30, normalized size = 0.81

$$\frac{x^6-x^2}{16\left(x^8+2\,x^4+1\right)}+\frac{1}{16}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="maxima")
[Out] 1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.86

$$\frac{\operatorname{atan}\left(x^2\right)}{16}-\frac{\frac{x^2}{16}-\frac{x^6}{16}}{x^8+2\,x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + 1)^3,x)
[Out] atan(x^2)/16 - (x^2/16 - x^6/16)/(2*x^4 + x^8 + 1)
```

sympy [A] time = 0.17, size = 24, normalized size = 0.65

$$\frac{x^6-x^2}{16x^8+32x^4+16}+\frac{\operatorname{atan}\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1)**3,x)
[Out] (x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

3.171 $\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{4(x^4+2x^2+2)}+\frac{1}{4}\log\left(x^4+2x^2+2\right)$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1247, 686, 628}

$$\frac{1}{4}\log\left(x^4+2x^2+2\right)-\frac{(x^2+1)^2}{4(x^4+2x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

```
[Out] -(1 + x^2)^2/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^3}{(2+2x+x^2)^2}\,dx,x,x^2\right)\\
&= -\frac{(1+x^2)^2}{4(2+2x^2+x^4)}+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{2+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{(1+x^2)^2}{4(2+2x^2+x^4)}+\frac{1}{4}\log\left(2+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.81

$$\frac{1}{4}\left(\frac{1}{\left(x^2+1\right)^2+1}+\log\left(\left(x^2+1\right)^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
[Out] ((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

IntegrateAlgebraic [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{1}{4\left(x^4+2x^2+2\right)}+\frac{1}{4}\log\left(x^4+2x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
[Out] 1/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

fricas [A] time = 1.22, size = 38, normalized size = 1.19

$$\frac{\left(x^4+2\,x^2+2\right)\log\left(x^4+2\,x^2+2\right)+1}{4\left(x^4+2\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")
[Out] 1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2)
```

giac [A] time = 1.61, size = 32, normalized size = 1.00

$$\frac{1}{4\left(x^4+2\,x^2+2\right)}-\frac{1}{4}\log\left(\frac{1}{2\left(x^4+2\,x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="giac")
[Out] 1/4/(x^4 + 2*x^2 + 2) - 1/4*log(1/2/(x^4 + 2*x^2 + 2))
```

maple [A] time = 0.26, size = 29, normalized size = 0.91

method	result	size
default	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29
norman	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29
risch	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

maxima [A] time = 0.54, size = 28, normalized size = 0.88

$$\frac{1}{4\left(x^4+2\,x^2+2\right)}+\frac{1}{4}\,\log\left(x^4+2\,x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

mupad [B] time = 0.19, size = 28, normalized size = 0.88

$$\frac{\ln\left(x^4+2\,x^2+2\right)}{4}+\frac{1}{4\,\left(x^4+2\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 1)^3)/(2*x^2 + x^4 + 2)^2,x)
[Out] log(2*x^2 + x^4 + 2)/4 + 1/(4*(2*x^2 + x^4 + 2))
```

sympy [A] time = 0.17, size = 26, normalized size = 0.81

$$\frac{\log\left(x^4+2x^2+2\right)}{4}+\frac{1}{4x^4+8x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
[Out] log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

3.172 $\int \frac{x^3}{(a^4+x^4)^3} dx$

Optimal. Leaf size=13

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^4 + x^4)^3,x]
```

```
[Out] -1/(8*(a^4 + x^4)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a^4+x^4\right)^3} dx = -\frac{1}{8\left(a^4+x^4\right)^2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^4 + x^4)^3,x]
```

```
[Out] -1/8*1/(a^4 + x^4)^2
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^4 + x^4)^3,x]
```

```
[Out] -1/8*1/(a^4 + x^4)^2
```

fricas [A] time = 1.41, size = 19, normalized size = 1.46

$$-\frac{1}{8\left(a^8+2\,a^4x^4+x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(a^8 + 2*a^4*x^4 + x^8)
```

giac [A] time = 0.93, size = 11, normalized size = 0.85

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

maple [A] time = 0.26, size = 12, normalized size = 0.92

method	result	size
gosper	$-\frac{1}{8\left(a^4+x^4\right)^2}$	12
derivativedivides	$-\frac{1}{8\left(a^4+x^4\right)^2}$	12
default	$-\frac{1}{8\left(a^4+x^4\right)^2}$	12
norman	$-\frac{1}{8\left(a^4+x^4\right)^2}$	12
risch	$-\frac{1}{8\left(a^4+x^4\right)^2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a^4+x^4)^2
```

maxima [A] time = 0.50, size = 11, normalized size = 0.85

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

mupad [B] time = 0.20, size = 11, normalized size = 0.85

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^4 + x^4)^3,x)
```

```
[Out] -1/(8*(a^4 + x^4)^2)
```

sympy [**A**] time = 0.28, size = 20, normalized size = 1.54

$$-\frac{1}{8a^8 + 16a^4x^4 + 8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**4+x**4)**3,x)
```

```
[Out] -1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

3.173 $\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx$

Optimal. Leaf size=54

$$\frac{\log(x)}{a^{12}}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{1}{4a^8\left(a^4+x^4\right)}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{\log(x)}{a^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - Log[a^4 + x
^4]/(4*a^12)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(a^4+x\right)^3}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{a^{12}x}-\frac{1}{a^4\left(a^4+x\right)^3}-\frac{1}{a^8\left(a^4+x\right)^2}-\frac{1}{a^{12}\left(a^4+x\right)}\right)dx,x,x^4\right)\\
&= \frac{1}{8a^4\left(a^4+x^4\right)^2}+\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{\log(x)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.85

$$\frac{-2\log\left(a^4+x^4\right)+\frac{3a^8+2a^4x^4}{\left(a^4+x^4\right)^2}+8\log(x)}{8a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] ((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)
```

IntegrateAlgebraic [A] time = 0.04, size = 49, normalized size = 0.91

$$\frac{\log(x)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{3a^4+2x^4}{8a^8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] (3*a^4 + 2*x^4)/(8*a^8*(a^4 + x^4)^2) + Log[x]/a^12 - Log[a^4 + x^4]/(4*a^1
2)
```

fricas [A] time = 1.30, size = 81, normalized size = 1.50

$$\frac{3\,a^8+2\,a^4x^4-2\left(a^8+2\,a^4x^4+x^8\right)\log\left(a^4+x^4\right)+8\left(a^8+2\,a^4x^4+x^8\right)\log(x)}{8\left(a^{20}+2\,a^{16}x^4+a^{12}x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4) + 8*(a^8
+ 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^8)
```

giac [A] time = 0.94, size = 56, normalized size = 1.04

$$-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}+\frac{6\,a^8+8\,a^4x^4+3\,x^8}{8\left(a^4+x^4\right)^2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(a^4 + x^4)/a^12 + 1/4*log(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x^4 + 3*x
^8)/((a^4 + x^4)^2*a^12)
```

maple [A] time = 0.27, size = 45, normalized size = 0.83

method	result	size
norman	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{\left(a^4+x^4\right)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln\left(a^4+x^4\right)}{4a^{12}}$	45
risch	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{\left(a^4+x^4\right)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln\left(a^4+x^4\right)}{4a^{12}}$	45
default	$-\frac{-\frac{a^4}{2\left(a^4+x^4\right)}+\frac{\ln\left(a^4+x^4\right)}{2}-\frac{a^8}{4\left(a^4+x^4\right)^2}}{2a^{12}}+\frac{\ln(x)}{a^{12}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/8/a^4+1/4/a^8*x^4)/(a^4+x^4)^2+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

maxima [A] time = 0.45, size = 57, normalized size = 1.06

$$\frac{3\,a^4 + 2\,x^4}{8\left(a^{16} + 2\,a^{12}x^4 + a^8x^8\right)} - \frac{\log\left(a^4 + x^4\right)}{4\,a^{12}} + \frac{\log\left(x^4\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 + x^4)/a^12
 + 1/4*log(x^4)/a^12
```

mupad [B] time = 0.11, size = 52, normalized size = 0.96

$$\frac{\ln(x)}{a^{12}} + \frac{\frac{3}{8\,a^4} + \frac{x^4}{4\,a^8}}{a^8 + 2\,a^4\,x^4 + x^8} - \frac{\ln\left(a^4 + x^4\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^4 + x^4)^3),x)
```

```
[Out] log(x)/a^12 + (3/(8*a^4) + x^4/(4*a^8))/(a^8 + x^8 + 2*a^4*x^4) - log(a^4 +
 x^4)/(4*a^12)
```

sympy [A] time = 0.46, size = 51, normalized size = 0.94

$$\frac{3a^4 + 2x^4}{8a^{16} + 16a^{12}x^4 + 8a^8x^8} + \frac{\log\left(x\right)}{a^{12}} - \frac{\log\left(a^4 + x^4\right)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**4+x**4)**3,x)
```

```
[Out] (3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x)/a**12 -
log(a**4 + x**4)/(4*a**12)
```

3.174 $\int \frac{1}{x^2(a^4+x^4)^3}\,dx$

Optimal. Leaf size=157

$$\frac{45\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{64\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}a\right.}{128\sqrt{2}a^{13}}$$

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{45\tan^{-1}\left(1-\right.}{64\sqrt{2}a^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 + x^4)) + (4
5*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*ArcTan[1 + (Sqrt[2]*x)
/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^1
3) + (45*Log[a^2 + Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^13)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9\int \frac{1}{x^2\left(a^4+x^4\right)^2}\,dx}{8a^4}$$

$$= \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{1}{x^2\left(a^4+x^4\right)}\,dx}{32a^8}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{x^2}{a^4+x^4}\,dx}{32a^{12}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{a^2-x^2}{a^4+x^4}\,dx}{64a^{12}}-\frac{45\int \frac{a^2+x^2}{a^4+x^4}\,dx}{64a^{12}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{\sqrt{2}\,a+2x}{-a^2-\sqrt{2}\,ax-x^2}\,dx}{128\sqrt{2}\,a^{13}}-\frac{45\int \frac{\sqrt{2}\,a-2x}{-a^2+\sqrt{2}\,ax-x^2}\,dx}{128\sqrt{2}\,a^{13}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\log\left(a^2-\sqrt{2}\,ax+x^2\right)}{128\sqrt{2}\,a^{13}}+\frac{45\log\left(a^2 \ldots\right.}{128 \ldots}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{a}\right)}{64\sqrt{2}\,a^{13}}-\frac{45\tan^{-1}\left(1+\frac{\sqrt{2} \ldots}{a}\right.}{64\sqrt{2}\,a^{13}}$$

Mathematica [A] time = 0.12, size = 134, normalized size = 0.85

$$-\frac{\frac{104ax^3}{a^4+x^4}+45\sqrt{2}\,\log\left(a^2-\sqrt{2}\,ax+x^2\right)-45\sqrt{2}\,\log\left(a^2+\sqrt{2}\,ax+x^2\right)+\frac{32a^5x^3}{\left(a^4+x^4\right)^2}+\frac{256a}{x}-90\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2} \ldots}{a}\right. \ldots}{256a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -1/256*((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) -
90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/a]
 + 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2]*a
*x + x^2])/a^13
```

IntegrateAlgebraic [A] time = 0.14, size = 107, normalized size = 0.68

$$\frac{45\tan^{-1}\left(\frac{\frac{a}{\sqrt{2}}-\frac{x^2}{\sqrt{2}a}}{x}\right)}{64\sqrt{2}\,a^{13}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}\,ax}{a^2+x^2}\right)}{64\sqrt{2}\,a^{13}}+\frac{-32a^8-81a^4x^4-45x^8}{32a^{12}x\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] (-32*a^8 - 81*a^4*x^4 - 45*x^8)/(32*a^12*x*(a^4 + x^4)^2) + (45*ArcTan[(a/S
qrt[2] - x^2/(Sqrt[2]*a))/x])/(64*Sqrt[2]*a^13) + (45*ArcTanh[(Sqrt[2]*a*x)
/(a^2 + x^2)])/(64*Sqrt[2]*a^13)
```

fricas [B] time = 1.46, size = 338, normalized size = 2.15

$$-\frac{256\,a^8+648\,a^4x^4+360\,x^8-180\,\sqrt{2}\left(a^{20}x+2\,a^{16}x^5+a^{12}x^9\right)\left(\frac{1}{a^{52}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a^{12}\left(\frac{1}{a^{52}}\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{\sqrt{2}\,a^{40}\left(\frac{1}{a^{52}}\right)^{\frac{3}{4}}x+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(256*a^8 + 648*a^4*x^4 + 360*x^8 - 180*sqrt(2)*(a^20*x + 2*a^16*x^5
+ a^12*x^9)*(a^(-52))^(1/4)*arctan(-sqrt(2)*a^12*(a^(-52))^(1/4)*x + sqrt(2
)*sqrt(sqrt(2)*a^40*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2)*a^12*(a^(
-52))^(1/4) - 1) - 180*sqrt(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(
1/4)*arctan(-sqrt(2)*a^12*(a^(-52))^(1/4)*x + sqrt(2)*sqrt(-sqrt(2)*a^40*(a
^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2)*a^12*(a^(-52))^(1/4) + 1) - 45*
sqrt(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(sqrt(2)*a^40*(
a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2) + 45*sqrt(2)*(a^20*x + 2*a^16*
x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(-sqrt(2)*a^40*(a^(-52))^(3/4)*x + a^28*
sqrt(a^(-52)) + x^2))/(a^20*x + 2*a^16*x^5 + a^12*x^9)
```

giac [A] time = 0.93, size = 150, normalized size = 0.96

$$-\frac{45\,\sqrt{2}\,|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,|a|+2\,x\right)}{2\,|a|}\right)}{128\,a^{14}}-\frac{45\,\sqrt{2}\,|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,|a|-2\,x\right)}{2\,|a|}\right)}{128\,a^{14}}+\frac{45\,\sqrt{2}\,|a|\log\left(\sqrt{2}\,x|a|+x^2+|a|^2\right)}{256\,a^{14}}-\frac{45\,\sqrt{2}\,|a|\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^
14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a
))/a^14 + 45/256*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14
 - 45/256*sqrt(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14 - 1/3
2*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^12) - 1/(a^12*x)
```

maple [C] time = 0.28, size = 75, normalized size = 0.48

method	result	size
risch	$\frac{-\frac{45x^8}{32a^{12}}-\frac{81x^4}{32a^8}-\frac{1}{a^4}}{x(a^4+x^4)^2}+\frac{45\left(\sum_{\_R=\mathrm{RootOf}(a^{52}\_Z^4+1)}\_R\ln\left((5\_R^4a^{52}+4)x+\_R^3a^{40}\right)\right)}{128}$	75
default	$-\frac{\frac{\frac{17}{32}a^4x^3+\frac{13}{32}x^7}{(a^4+x^4)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x^2-(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(a^4)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(a^4)^{\frac{1}{4}}}-1\right)\right)}{256(a^4)^{\frac{1}{4}}}}{a^{12}}-\frac{1}{a^{12}x}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-45/32/a^12*x^8-81/32/a^8*x^4-1/a^4)/x/(a^4+x^4)^2+45/128*sum(_R*ln((5*_R^
4*a^52+4)*x+_R^3*a^40),_R=RootOf(_Z^4*a^52+1))
```

maxima [A] time = 1.30, size = 147, normalized size = 0.94

$$-\frac{32\,a^8+81\,a^4x^4+45\,x^8}{32\left(a^{20}x+2\,a^{16}x^5+a^{12}x^9\right)}-\frac{45\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(\sqrt{2}a+2x)}{2a}\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}(\sqrt{2}a-2x)}{2a}\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{a}+\frac{\sqrt{2}\log}{}\right.}{256\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^8 + 81*a^4*x^4 + 45*x^8)/(a^20*x + 2*a^16*x^5 + a^12*x^9) - 45/
256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - sqrt(2)*log(sqrt(2)*a*x + a^2 + x^2)
/a + sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a)/a^12
```

mupad [B] time = 0.11, size = 76, normalized size = 0.48

$$\frac{45\,(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{64\,a^{13}}-\frac{45\,(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{64\,a^{13}}-\frac{\frac{1}{a^4}+\frac{81\,x^4}{32\,a^8}+\frac{45\,x^8}{32\,a^{12}}}{a^8\,x+2\,a^4\,x^5+x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^4 + x^4)^3),x)
```

```
[Out] (45*(-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(64*a^13) - (45*(-1)^(1/4)*atan(((-
1)^(1/4)*x)/a))/(64*a^13) - (1/a^4 + (81*x^4)/(32*a^8) + (45*x^8)/(32*a^12)
)/(a^8*x + x^9 + 2*a^4*x^5)
```

sympy [A] time = 0.44, size = 66, normalized size = 0.42

$$\frac{-32a^8-81a^4x^4-45x^8}{32a^{20}x+64a^{16}x^5+32a^{12}x^9}+\frac{\mathrm{RootSum}\left(268435456t^4+4100625,\left(t\mapsto t\log\left(-\frac{2097152t^3a}{91125}+x\right)\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**4+x**4)**3,x)
```

```
[Out] (-32*a**8 - 81*a**4*x**4 - 45*x**8)/(32*a**20*x + 64*a**16*x**5 + 32*a**12*
x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _t*log(-2097152*_t**3
*a/91125 + x)))/a**13
```

3.175 $\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 290, 325, 203}

$$\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(a^4 + x^4)
) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^2}\,dx, x, x^2\right)}{8a^4}$$
$$= \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)}\,dx, x, x^2\right)}{16a^8}$$
$$= -\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} - \frac{15\,\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)}{16a^{12}}$$
$$= -\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} - \frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.17

$$\frac{-\frac{a^2\left(8a^8+25a^4x^4+15x^8\right)}{x^2\left(a^4+x^4\right)^2} + 15\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right) + 15\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] (-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1 -
 (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^14)
```

IntegrateAlgebraic [A] time = 0.06, size = 54, normalized size = 0.84

$$\frac{-8a^8-25a^4x^4-15x^8}{16a^{12}x^2\left(a^4+x^4\right)^2} - \frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] (-8*a^8 - 25*a^4*x^4 - 15*x^8)/(16*a^12*x^2*(a^4 + x^4)^2) - (15*ArcTan[x^2
/a^2])/(16*a^14)
```

fricas [A] time = 1.33, size = 78, normalized size = 1.22

$$-\frac{8\,a^{10} + 25\,a^6x^4 + 15\,a^2x^8 + 15\left(a^8x^2 + 2\,a^4x^6 + x^{10}\right)\arctan\left(\frac{x^2}{a^2}\right)}{16\left(a^{22}x^2 + 2\,a^{18}x^6 + a^{14}x^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^10)*a
rctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

giac [A] time = 0.88, size = 50, normalized size = 0.78

$$-\frac{9\,a^{4}x^{2}+7\,x^{6}}{16\,\left(a^{4}+x^{4}\right)^{2}a^{12}}-\frac{15\,\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,a^{14}}-\frac{1}{2\,a^{12}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2/a^2)/a^14
 - 1/2/(a^12*x^2)
```

maple [A] time = 0.29, size = 53, normalized size = 0.83

method	result	size
default	$-\frac{\frac{\frac{9}{8}a^{4}x^{2}+\frac{7}{8}x^{6}}{\left(a^{4}+x^{4}\right)^{2}}+\frac{15\arctan\left(\frac{x^{2}}{a^{2}}\right)}{8a^{2}}}{2a^{12}}-\frac{1}{2a^{12}x^{2}}$	53
risch	$\frac{-\frac{15x^{8}}{16a^{12}}-\frac{25x^{4}}{16a^{8}}-\frac{1}{2a^{4}}}{x^{2}\left(a^{4}+x^{4}\right)^{2}}+\frac{15\left(\sum_{\_R=\mathrm{RootOf}\left(a^{28}\_Z^{2}+1\right)}\_R\ln\left(\left(-5\_R^{2}a^{28}-4\right)x^{2}-a^{16}\_R\right)\right)}{32}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^12*((9/8*a^4*x^2+7/8*x^6)/(a^4+x^4)^2+15/8*arctan(x^2/a^2)/a^2)-1/2/
a^12/x^2
```

maxima [A] time = 1.34, size = 60, normalized size = 0.94

$$-\frac{8\,a^{8}+25\,a^{4}x^{4}+15\,x^{8}}{16\,\left(a^{20}x^{2}+2\,a^{16}x^{6}+a^{12}x^{10}\right)}-\frac{15\,\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12*x^10) - 1
5/16*arctan(x^2/a^2)/a^14
```

mupad [B] time = 0.22, size = 53, normalized size = 0.83

$$-\frac{15\,\mathrm{atan}\left(\frac{x^{2}}{a^{2}}\right)}{16\,a^{14}}-\frac{\frac{a^{10}}{2}+\frac{25\,a^{6}\,x^{4}}{16}+\frac{15\,a^{2}\,x^{8}}{16}}{a^{14}\,x^{2}\,\left(a^{4}+x^{4}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^4 + x^4)^3),x)
```

```
[Out] - (15*atan(x^2/a^2))/(16*a^14) - (a^10/2 + (15*a^2*x^8)/16 + (25*a^6*x^4)/1
6)/(a^14*x^2*(a^4 + x^4)^2)
```

sympy [C] time = 0.48, size = 78, normalized size = 1.22

$$\frac{-8a^{8}-25a^{4}x^{4}-15x^{8}}{16a^{20}x^{2}+32a^{16}x^{6}+16a^{12}x^{10}}+\frac{\frac{15i\log\left(-ia^{2}+x^{2}\right)}{32}-\frac{15i\log\left(ia^{2}+x^{2}\right)}{32}}{a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**4+x**4)**3,x)
```

```
[Out] (-8*a**8 - 25*a**4*x**4 - 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6 + 16*a**1
2*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a**2 + x**2)/32)/a**14
```

3.176 $\int \frac{x^{14}}{(3+2x^5)^3} \, dx$

Optimal. Leaf size=39

$$\frac{3}{20(2x^5+3)} - \frac{9}{80(2x^5+3)^2} + \frac{1}{40}\log(2x^5+3)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{20(2x^5+3)} - \frac{9}{80(2x^5+3)^2} + \frac{1}{40}\log(2x^5+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(3+2x^5)^3} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{x^2}{(3+2x)^3} \, dx, x, x^5\right) \\
&= \frac{1}{5}\text{Subst}\left(\int \left(\frac{9}{4(3+2x)^3} - \frac{3}{2(3+2x)^2} + \frac{1}{4(3+2x)}\right) dx, x, x^5\right) \\
&= -\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40}\log(3+2x^5)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{1}{80}\left(\frac{3(8x^5+9)}{(2x^5+3)^2} + 2\log(2x^5+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] ((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{3\left(8x^5+9\right)}{80\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/(3 + 2*x^5)^3,x]
[Out] (3*(9 + 8*x^5))/(80*(3 + 2*x^5)^2) + Log[3 + 2*x^5]/40
```

fricas [A] time = 1.32, size = 45, normalized size = 1.15

$$\frac{24\,x^5+2\left(4\,x^{10}+12\,x^5+9\right)\log\left(2\,x^5+3\right)+27}{80\left(4\,x^{10}+12\,x^5+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="fricas")
[Out] 1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^10 + 12*x^
5 + 9)
```

giac [A] time = 0.97, size = 30, normalized size = 0.77

$$-\frac{3\left(x^{10}+x^5\right)}{20\left(2\,x^5+3\right)^2}+\frac{1}{40}\log\left(\left|2\,x^5+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="giac")
[Out] -3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*log(abs(2*x^5 + 3))
```

maple [A] time = 0.26, size = 29, normalized size = 0.74

method	result	size
norman	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{\left(2x^5+3\right)^2}+\frac{\ln\left(2x^5+3\right)}{40}$	29
risch	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{\left(2x^5+3\right)^2}+\frac{\ln\left(2x^5+3\right)}{40}$	30
meijerg	$-\frac{x^5\left(6x^5+6\right)}{360\left(1+\frac{2x^5}{3}\right)^2}+\frac{\ln\left(1+\frac{2x^5}{3}\right)}{40}$	33
default	$-\frac{9}{80\left(2x^5+3\right)^2}+\frac{3}{20\left(2x^5+3\right)}+\frac{\ln\left(2x^5+3\right)}{40}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
[Out] (3/10*x^5+27/80)/(2*x^5+3)^2+1/40*ln(2*x^5+3)
```

maxima [A] time = 0.60, size = 34, normalized size = 0.87

$$\frac{3\left(8\,x^5+9\right)}{80\left(4\,x^{10}+12\,x^5+9\right)}+\frac{1}{40}\log\left(2\,x^5+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="maxima")
```

```
[Out] 3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

mupad [B] time = 0.05, size = 29, normalized size = 0.74

$$\frac{\ln\left(x^5+\frac{3}{2}\right)}{40}+\frac{\frac{3\,x^5}{40}+\frac{27}{320}}{x^{10}+3\,x^5+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(2*x^5 + 3)^3,x)
```

```
[Out] log(x^5 + 3/2)/40 + ((3*x^5)/40 + 27/320)/(3*x^5 + x^10 + 9/4)
```

sympy [A] time = 0.18, size = 27, normalized size = 0.69

$$\frac{24x^5+27}{320x^{10}+960x^5+720}+\frac{\log\left(2x^5+3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(2*x**5+3)**3,x)
```

```
[Out] (24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

3.177 $\int \frac{x^6}{(3+2x^5)^3} \, dx$

Optimal. Leaf size=319

$$\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{x^2}{150\left(2x^5+3\right)}-\frac{x}{20\left(2x^5\right.}$$

Rubi [A] time = 0.58, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {288, 290, 293, 634, 618, 204, 628, 31}

$$\frac{x^2}{150\left(2x^5+3\right)}-\frac{x^2}{20\left(2x^5+3\right)^2}+\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}\right.}{1000\ 2^{2/5}3^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] -x^2/(20*(3 + 2*x^5)^2) + x^2/(150*(3 + 2*x^5)) - (Sqrt[5 + Sqrt[5]]*ArcTan
[Sqrt[(5 + 2*Sqrt[5])/5] - (2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 - Sqrt[5]])])/(25
0*2^(9/10)*3^(3/5)) - (Sqrt[5 - Sqrt[5]]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (
2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 + Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^
(1/5) + 2^(1/5)*x]/(250*2^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^
(1/5)*(1 - Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)) + ((1
 - Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])
/(1000*2^(2/5)*3^(3/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\left(3+2x^5\right)^3}\,dx = -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{1}{10}\int \frac{x}{\left(3+2x^5\right)^2}\,dx$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{x^2}{150\left(3+2x^5\right)}+\frac{1}{50}\int \frac{x}{3+2x^5}\,dx$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{x^2}{150\left(3+2x^5\right)}-\frac{\int \frac{1}{\sqrt[5]{3}+\sqrt[5]{2}x}\,dx}{250\sqrt[5]{2}\,3^{3/5}}+\frac{\int \frac{\frac{1}{4}\sqrt[5]{3}\left(1-\sqrt{5}\right)-\frac{\left(-1-\sqrt{5}\right)x}{2\,2^{4/5}}}{3^{2/5}-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2}\,dx}{125\sqrt[5]{2}\,3^{3/5}}+\frac{\int \frac{\frac{1}{4}\sqrt[5]{3}\left(1+\sqrt{5}\right)-}{3^{2/5}-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right.}{2^{4/5}}}}{125\sqrt[5]{2}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{x^2}{150\left(3+2x^5\right)}-\frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\,2^{2/5}3^{3/5}}-\frac{\int \frac{1}{3^{2/5}-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2}\,dx}{100\sqrt[5]{2}\,3^{2/5}\sqrt{5}}+\frac{\int \frac{}{3^{2/5}-\frac{\sqrt[5]{3}\left(\right.}{2}}}{100\sqrt[5]{}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{x^2}{150\left(3+2x^5\right)}-\frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\,2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2\,3^{2/5}-\sqrt[5]{6}x-\sqrt{5}\sqrt[5]{6}\right.}{1000\,2^{2/5}3^{3/5}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2}+\frac{x^2}{150\left(3+2x^5\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)-4\sqrt[5]{2}x}{\sqrt[5]{3}\sqrt{2\left(5-\sqrt{5}\right)}}\right)}{25\,2^{9/10}3^{3/5}\sqrt{5\left(5-\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt[5]{3}\sqrt{3-\sqrt{5}}+2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{25\,2^{9/10}3^{3/5}\sqrt{5\left(5+\sqrt{5}\right)}}-$$

Mathematica [A] time = 0.31, size = 293, normalized size = 0.92

$$2^{3/5}3^{2/5}\left(1+\sqrt{5}\right)\log\left(2^{2/5}3^{3/5}x^2+\left(\frac{3}{2}\right)^{4/5}\left(\sqrt{5}-1\right)x+3\right)-2^{3/5}3^{2/5}\left(\sqrt{5}-1\right)\log\left(2^{2/5}3^{3/5}x^2-\left(\frac{3}{2}\right)^{4/5}\left(1+\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] ((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt[
5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 + S
qrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5])
+ 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Log[3
+ 2^(1/5)*3^(4/5)*x] + 2^(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/5)*(-
1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5])*Log[
3 - (3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

IntegrateAlgebraic [A] time = 0.70, size = 340, normalized size = 1.07

$$-\frac{\sqrt[5]{5\sqrt{5}-11}\log\left(2\,2^{2/5}3^{3/5}x^2-\sqrt[5]{2}\,3^{4/5}\sqrt{5}\,x-\sqrt[5]{2}\,3^{4/5}x+6\right)}{500\,6^{3/5}}+\frac{\sqrt[5]{11+5\sqrt{5}}\log\left(2\,2^{2/5}3^{3/5}x^2+\sqrt[5]{2}\,3^{4/5}\sqrt{5}\,x-\right.}{500\,6^{3/5}}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] (x^2*(-9 + 4*x^5))/(300*(3 + 2*x^5)^2) - ((25 - 11*Sqrt[5])^(1/10)*ArcTan[S
qrt[1 - 2/Sqrt[5]] + (2*(50 - 22*Sqrt[5])^(1/10)*x)/(3^(1/5)*5^(3/10))])/(5
0*Sqrt[2]*3^(3/5)*5^(4/5)) - ((25 + 11*Sqrt[5])^(1/10)*ArcTan[Sqrt[1 + 2/Sq
rt[5]] - (2*(50 + 22*Sqrt[5])^(1/10)*x)/(3^(1/5)*5^(3/10))])/(50*Sqrt[2]*3^
(3/5)*5^(4/5)) - Log[3 + 2^(1/5)*3^(4/5)*x]/(250*2^(2/5)*3^(3/5)) - ((-11 +
 5*Sqrt[5])^(1/5)*Log[6 - 2^(1/5)*3^(4/5)*x - 2^(1/5)*3^(4/5)*Sqrt[5]*x + 2
*2^(2/5)*3^(3/5)*x^2])/(500*6^(3/5)) + ((11 + 5*Sqrt[5])^(1/5)*Log[6 - 2^(1
/5)*3^(4/5)*x + 2^(1/5)*3^(4/5)*Sqrt[5]*x + 2*2^(2/5)*3^(3/5)*x^2])/(500*6^
(3/5))
```

fricas [C] time = 41.32, size = 1751, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")
```

```
[Out] 1/216000*(2880*x^7 - 2*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5)
+ I*sqrt(-2*sqrt(5) + 10) + 1)*log(-1/6912*108^(3/5)*(-1)^(2/5)*(108^(4/5)*
(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)
)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/64*108^(2/5)*(-1)^(3/5)*(sq
rt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 - 1/6912*108^(4/5)*(-1)^(1/5)*(108^(
3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1
)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*
(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 1/16*108^(2/5)*(-1)^(3/5)*(sqrt(5
) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*
sqrt(-2*sqrt(5) + 10) + 1) + 108^(2/5)*(-1)^(3/5) + 6*x) - 2*108^(4/5)*(-1)
^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)*log(1/
384*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 + x) + 8
*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*log(-108^(2/5)*(-1)^(3/5) + 6*x
) - 6480*x^2 + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) + I*sqr
```

```
t(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5
 + 9) - 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(
108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*
(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(
2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*
(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)))*log(1/768*108^
(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1
/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5
) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10)
+ 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 9
/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 9/4*10
8^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 1/3456*(108^(4
/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) +
10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-
2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(
5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x) + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*
x^5 + 9)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*
x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-
1)^(1/5)*(4*x^10 + 12*x^5 + 9) + 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/8
64*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5)
+ 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1
) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3
/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*10
8^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-
1)^(2/5)))*log(1/768*108^(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2
*sqrt(5) + 10) + 1)^2 + 1/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(s
qrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(
-2*sqrt(5) + 10) + 1) - 9/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt
(5) + 10) + 1)^2 + 9/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1) - 1/3456*(108^(4/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt
(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(
5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4
/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(
1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(
sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I
*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x))/(4*x^10 + 1
2*x^5 + 9)
```

giac [A] time = 1.83, size = 250, normalized size = 0.78

$$-\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}-\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}\right)\arctan\left(\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x\right)}{3\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="giac")
```

```
[Out] -1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(2*sqrt(5) + 10) - (3/2)^(2/5)*sqrt(2*sqrt
(5) + 10))*arctan(2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) - 1) + 4*x)/sqrt(2*
sqrt(5) + 10)) + 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(-2*sqrt(5) + 10) + (3/2)^
(2/5)*sqrt(-2*sqrt(5) + 10))*arctan(-2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5)
+ 1) - 4*x)/sqrt(-2*sqrt(5) + 10)) - 1/6000*((3/2)^(2/5)*(sqrt(5) - 5) + sq
rt(5)*(3/2)^(2/5) + 3*(3/2)^(2/5))*log(x^2 - 1/2*x*(sqrt(5)*(3/2)^(1/5) + (
3/2)^(1/5)) + (3/2)^(2/5)) + 1/6000*((3/2)^(2/5)*(sqrt(5) + 5) + sqrt(5)*(3
/2)^(2/5) - 3*(3/2)^(2/5))*log(x^2 + 1/2*x*(sqrt(5)*(3/2)^(1/5) - (3/2)^(1/
5)) + (3/2)^(2/5)) - 1/750*(3/2)^(2/5)*log(abs(x + (3/2)^(1/5))) + 1/300*(4
*x^7 - 9*x^2)/(2*x^5 + 3)^2
```

maple [C] time = 0.27, size = 47, normalized size = 0.15

method	result
risch	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{\left(2x^5+3\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^5+3\right)}\frac{\ln(x-\_R)}{\_R^3}\right)}{500}$
meijerg	$\frac{108^{\frac{4}{5}}\left(-\frac{x^2 2^{\frac{2}{5}}3^{\frac{3}{5}}\left(-\frac{28x^5}{3}+21\right)}{105\left(1+\frac{2x^5}{3}\right)^2}+\frac{2\,108^{\frac{1}{5}}x^2\left(-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\ln\left(1+\frac{2^{\frac{1}{5}}3^{\frac{4}{5}}\left(x^5\right)^{\frac{1}{5}}}{3}\right)}{2\left(x^5\right)^{\frac{2}{5}}}-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\cos\left(\frac{2\pi}{5}\right)\ln\left(1-\frac{2\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}3^{\frac{4}{5}}\left(x^5\right)^{\frac{1}{5}}}{3}+\frac{2^{\frac{2}{5}}3^{\frac{3}{5}}\left(x^5\right)^{\frac{2}{5}}}{3}\right)}{2\left(x^5\right)^{\frac{2}{5}}}+\frac{72^{\frac{1}{5}}\arctan\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{6480\cdots}$
default	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{\left(2x^5+3\right)^2}+\frac{48^{\frac{2}{5}}\ln\left(48^{\frac{1}{5}}+2x\right)}{150\left(5+\sqrt{5}\right)\left(\sqrt{5}-5\right)}+\frac{48^{\frac{2}{5}}\ln\left(x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)\sqrt{5}}{12000}+\frac{48^{\frac{2}{5}}\ln\left(x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)}{12000}-\frac{48^{\frac{3}{5}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/500*sum(1/_R^3*ln(x-_R),_R=RootOf(2*_
Z^5+3))
```

maxima [A] time = 1.21, size = 335, normalized size = 1.05

$$\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}-5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x+\sqrt{5}\,3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}\,3^{\frac{2}{5}}2^{\frac{1}{5}}-3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{2\sqrt{5}+10}}+\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}+5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x-\sqrt{5}\,3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{-2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}\,3^{\frac{2}{5}}2^{\frac{1}{5}}+3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{-2\sqrt{5}+10}}-\frac{1}{1500}\cdot 3\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")
```

```
[Out] 1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x
 + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqrt(
5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4/5)
```

```
*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5)*3^
(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2
^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(3/5)*l
og(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9) - 1/2
50*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)) + 2*3^
(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2^(2/5)*x^
2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(
3/5)*2^(2/5) - 3^(3/5)*2^(2/5))
```

mupad [B] time = 1.60, size = 285, normalized size = 0.89

$$\frac{3^{2/5}\,\ln\left(x-\frac{3^{1/5}\left(2\,2^{1/10}\,\sqrt{-\sqrt{5}-5}-2^{3/5}\,\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\,\sqrt{-\sqrt{5}-5}-2^{3/5}\,\left(\sqrt{5}-1\right)\right)}{6000}-\frac{\frac{3\,x^2}{400}-\frac{x^7}{300}}{x^{10}+3\,x^5+\frac{9}{4}}-\frac{3^{2/5}\,\ln\left(x+\frac{3^{1}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^5 + 3)^3,x)
```

```
[Out] (3^(2/5)*log(x - (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1
/2) - 1))^3)/256)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1/2) - 1)
))/6000 - ((3*x^2)/400 - x^7/300)/(3*x^5 + x^10 + 9/4) - (3^(2/5)*log(x + (
3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1))^3)/256)*
(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1)))/6000 - (72^(1/5
)*log(x + 72^(3/5)/12))/1500 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^(1/2)
+ 1) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1) - 2*2
^(1/10)*(5^(1/2) - 5)^(1/2)))/6000 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^
(1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1)
 + 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000
```

sympy [A] time = 0.30, size = 37, normalized size = 0.12

$$\frac{4x^7-9x^2}{1200x^{10}+3600x^5+2700}+\mathrm{RootSum}\left(105468750000000t^5+1,\left(t\mapsto t\log\left(-281250000t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(2*x**5+3)**3,x)
```

```
[Out] (4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(105468750000000
*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

3.178 $\int \frac{9}{5x^2(3-2x^2)^3} \, dx$

Optimal. Leaf size=59

$$\frac{1}{8x(3-2x^2)} + \frac{3}{20x(3-2x^2)^2} - \frac{1}{8x} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {12, 290, 325, 206}

$$\frac{1}{8x(3-2x^2)} + \frac{3}{20x(3-2x^2)^2} - \frac{1}{8x} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] -1/(8*x) + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh[Sqrt[2/3]
*x]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx &= \frac{9}{5}\int \frac{1}{x^2\left(3-2x^2\right)^3}\,dx\\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{3}{4}\int \frac{1}{x^2\left(3-2x^2\right)^2}\,dx\\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{3}{8}\int \frac{1}{x^2\left(3-2x^2\right)}\,dx\\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{1}{4}\int \frac{1}{3-2x^2}\,dx\\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.10

$$\frac{1}{240}\left(-\frac{12\left(10x^4-25x^2+12\right)}{x\left(3-2x^2\right)^2}-5\sqrt{6}\log\left(\sqrt{6}-2x\right)+5\sqrt{6}\log\left(2x+\sqrt{6}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] ((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] - 2
*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

IntegrateAlgebraic [A] time = 0.09, size = 48, normalized size = 0.81

$$\frac{-10x^4+25x^2-12}{20x\left(2x^2-3\right)^2}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] (-12 + 25*x^2 - 10*x^4)/(20*x*(-3 + 2*x^2)^2) + ArcTanh[Sqrt[2/3]*x]/(4*Sqr
t[6])
```

fricas [A] time = 1.13, size = 73, normalized size = 1.24

$$-\frac{120\,x^4-5\,\sqrt{6}\left(4\,x^5-12\,x^3+9\,x\right)\log\left(\frac{2\,x^2+2\,\sqrt{6}\,x+3}{2\,x^2-3}\right)-300\,x^2+144}{240\left(4\,x^5-12\,x^3+9\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*x
 + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)
```

giac [A] time = 1.10, size = 55, normalized size = 0.93

$$-\frac{1}{48}\sqrt{6}\log\left(\frac{\left|4\,x-2\,\sqrt{6}\right|}{\left|4\,x+2\,\sqrt{6}\right|}\right)-\frac{14\,x^3-27\,x}{60\left(2\,x^2-3\right)^2}-\frac{1}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*log(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/60*(14*x^3
 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

maple [A] time = 0.28, size = 39, normalized size = 0.66

method	result	size
default	$-\frac{8\left(\frac{7}{16}x^3-\frac{27}{32}x\right)}{15\left(2x^2-3\right)^2}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{3}\right)\sqrt{6}}{24}-\frac{1}{15x}$	39
meijerg	$\frac{i\sqrt{6}\left(\frac{i\sqrt{6}\left(\frac{20}{3}x^4-\frac{50}{3}x^2+8\right)}{4x\left(-\frac{2x^2}{3}+1\right)^2}-\frac{15i\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{3}\right)}{2}\right)}{180}$	51
risch	$\frac{-\frac{1}{2}x^4+\frac{5}{4}x^2-\frac{3}{5}}{\left(2x^2-3\right)^2x}+\frac{\sqrt{6}\,\ln\left(x+\frac{\sqrt{6}}{2}\right)}{48}-\frac{\sqrt{6}\,\ln\left(x-\frac{\sqrt{6}}{2}\right)}{48}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(9/5/x^2/(-2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^(1/2))*6^(1/2)-1/
15/x
```

maxima [A] time = 1.15, size = 56, normalized size = 0.95

$$-\frac{1}{48}\sqrt{6}\log\left(\frac{2x-\sqrt{6}}{2x+\sqrt{6}}\right)-\frac{10x^4-25x^2+12}{20\left(4x^5-12x^3+9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4 - 25*x^2
+ 12)/(4*x^5 - 12*x^3 + 9*x)
```

mupad [B] time = 0.24, size = 41, normalized size = 0.69

$$\frac{\sqrt{6}\operatorname{atanh}\left(\frac{\sqrt{6}\,x}{3}\right)}{24}-\frac{\frac{x^4}{8}-\frac{5x^2}{16}+\frac{3}{20}}{x^5-3x^3+\frac{9x}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-9/(5*x^2*(2*x^2 - 3)^3),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/3))/24 - (x^4/8 - (5*x^2)/16 + 3/20)/((9*x)/4 -
3*x^3 + x^5)
```

sympy [A] time = 0.17, size = 58, normalized size = 0.98

$$-\frac{9\left(10x^4-25x^2+12\right)}{720x^5-2160x^3+1620x}-\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{2}\right)}{48}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

```
[Out] -9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(x
 - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

3.179 $\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1260, 456, 453, 203}

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1260

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^
(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(
q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x
^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && I
LtQ[q, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-16+9x^2}{x^2\left(1+x^2\right)^2}\,dx\\
&= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}+\frac{1}{8}\int \frac{32-25x^2}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.92

$$
-\frac{57x^4+103x^2+32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

IntegrateAlgebraic [A] time = 0.00, size = 33, normalized size = 0.92

$$
\frac{-57x^4-103x^2-32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
[Out] (-32 - 103*x^2 - 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

fricas [A] time = 0.70, size = 40, normalized size = 1.11

$$
-\frac{57\,x^4+103\,x^2+57\left(x^5+2\,x^3+x\right)\arctan(x)+32}{8\left(x^5+2\,x^3+x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

giac [A] time = 0.92, size = 28, normalized size = 0.78

$$
-\frac{25\,x^3+39\,x}{8\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\arctan(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{4}{x}-\frac{\frac{25}{8}x^{3}+\frac{39}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}$	29
risch	$\frac{-\frac{57}{8}x^{4}-\frac{103}{8}x^{2}-4}{\left(x^{2}+1\right)^{2}x}-\frac{57\arctan\left(x\right)}{8}$	29
meijerg	$-\frac{15x^{4}+25x^{2}+8}{2x\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{x\left(-3x^{2}+3\right)}{8\left(x^{2}+1\right)^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -4/x-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)
```

maxima [A] time = 1.42, size = 31, normalized size = 0.86

$$-\frac{57\,x^{4}+103\,x^{2}+32}{8\,\left(x^{5}+2\,x^{3}+x\right)}-\frac{57}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

mupad [B] time = 0.00, size = 29, normalized size = 0.81

$$-\frac{57\,\mathrm{atan}\left(x\right)}{8}-\frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,{\left(x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

sympy [A] time = 0.15, size = 32, normalized size = 0.89

$$\frac{-57x^{4}-103x^{2}-32}{8x^{5}+16x^{3}+8x}-\frac{57\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx$

Optimal. Leaf size=38

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2074}

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}-\frac{1}{(1+x)^2}-\frac{2}{1+x}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x}+\log(1-x)-2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.84

$$-\frac{2}{x-1}+\frac{1}{x+1}-\frac{3}{2(x-1)^2}+\log(x-1)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x
^5),x]
```

```
[Out] -3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*Log[1 + x]
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.92

$$\frac{-2x^2-7x+3}{2(x-1)^2(x+1)}+\log(x-1)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3
- x^4 + x^5),x]
```

```
[Out] (3 - 7*x - 2*x^2)/(2*(-1 + x)^2*(1 + x)) + Log[-1 + x] - 2*Log[1 + x]
```

fricas [B] time = 0.69, size = 65, normalized size = 1.71

$$-\frac{2\,x^2+4\left(x^3-x^2-x+1\right)\log\left(x+1\right)-2\left(x^3-x^2-x+1\right)\log\left(x-1\right)+7\,x-3}{2\left(x^3-x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log(
x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

giac [A] time = 0.87, size = 35, normalized size = 0.92

$$-\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2}-2\,\log\left(|x+1|\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
giac")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x
- 1))
```

maple [A] time = 0.03, size = 31, normalized size = 0.82

method	result	size
default	$\ln(-1+x)-\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x}-2\ln(1+x)$	31
norman	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{(-1+x)^2(1+x)}-2\ln(1+x)+\ln(-1+x)$	33
risch	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{x^3-x^2-x+1}-2\ln(1+x)+\ln(-1+x)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x,method=_RETURNVERB
OSE)
```

```
[Out] ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)+1/(1+x)-2*ln(1+x)
```

maxima [A] time = 0.64, size = 38, normalized size = 1.00

$$-\frac{2\,x^2+7\,x-3}{2\left(x^3-x^2-x+1\right)}-2\,\log\left(x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
maxima")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x - 1)
```

mupad [B] time = 0.07, size = 33, normalized size = 0.87

$$\ln\left(x-1\right)-2\,\ln\left(x+1\right)+\frac{x^2+\frac{7\,x}{2}-\frac{3}{2}}{-x^3+x^2+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2 - 3*x + 5*x^3 - x^4 + 5)/(x + 2*x^2 - 2*x^3 - x^4 + x^5 - 1),x)
[Out] log(x - 1) - 2*log(x + 1) + ((7*x)/2 + x^2 - 3/2)/(x + x^2 - x^3 - 1)
```

sympy [**A**] time = 0.15, size = 36, normalized size = 0.95

$$-\frac{2x^2 + 7x - 3}{2x^3 - 2x^2 - 2x + 2} + \log(x - 1) - 2\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)
[Out] -(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1)
```

3.181 $\int \frac{1+x^2}{x(1+x^3)^2} \, dx$

Optimal. Leaf size=64

$$-\frac{5}{18}\log\left(x^2-x+1\right)+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}+\log(x)-\frac{4}{9}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1829, 1834, 634, 618, 204, 628}

$$\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}-\frac{5}{18}\log\left(x^2-x+1\right)+\log(x)-\frac{4}{9}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3]) + Log[x
] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx &= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{1}{3}\int \frac{-3-x^2}{x\left(1+x^3\right)}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{1}{3}\int \left(-\frac{3}{x}+\frac{4}{3(1+x)}+\frac{-4+5x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)-\frac{1}{9}\int \frac{-4+5x}{1-x+x^2}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)+\frac{1}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{5}{18}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)-\frac{5}{18}\log\left(1-x+x^2\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1
\right.\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{4}{9}\log(1+x)-\frac{5}{18}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{1}{18}\left(-6\log\left(x^3+1\right)+\log\left(x^2-x+1\right)+\frac{6\left(x^2+1\right)}{x^3+1}+18\log(x)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] ((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 18*Log[x]
 - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

IntegrateAlgebraic [A] time = 0.04, size = 74, normalized size = 1.16

$$-\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^2-x+1\right)+\frac{x^2+1}{3\left(x^3+1\right)}+\log(x)-\frac{1}{9}\log(x+1)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] (1 + x^2)/(3*(1 + x^3)) - ArcTan[1/Sqrt[3] - (2*x)/Sqrt[3]]/(3*Sqrt[3]) + L
og[x] - Log[1 + x]/9 + Log[1 - x + x^2]/18 - Log[1 + x^3]/3
```

fricas [A] time = 1.25, size = 73, normalized size = 1.14

$$\frac{2\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+6x^2-5\left(x^3+1\right)\log\left(x^2-x+1\right)-8\left(x^3+1\right)\log\left(x+1\right)+18\left(x^3+1\right)}{18\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2 - 5*(x^3 +
1)*log(x^2 - x + 1) - 8*(x^3 + 1)*log(x + 1) + 18*(x^3 + 1)*log(x) + 6)/(x^
3 + 1)
```

giac [A] time = 0.94, size = 60, normalized size = 0.94

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{x^2+1}{3\left(x^2-x+1\right)(x+1)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 - x + 1)*(x
 + 1)) - 5/18*log(x^2 - x + 1) - 4/9*log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.31, size = 54, normalized size = 0.84

method	result	siz
risch	$\frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1}-\frac{5\ln\left(4x^2-4x+4\right)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{4\ln(1+x)}{9}+\ln(x)$	54
default	$\ln(x)-\frac{-1-x}{9\left(x^2-x+1\right)}-\frac{5\ln\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}+\frac{2}{9(1+x)}-\frac{4\ln(1+x)}{9}$	61
meijerg	$\frac{x^2}{3x^3+3}-\frac{x^2\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{9\left(x^3\right)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{18\left(x^3\right)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{9\left(x^3\right)^{\frac{2}{3}}}-\frac{2x^3}{3\left(2x^3+2\right)}-\frac{\ln\left(x^3+1\right)}{3}+\frac{1}{3}+\ln(x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*x^2+1/3)/(x^3+1)-5/18*ln(4*x^2-4*x+4)+1/9*3^(1/2)*arctan(1/3*(-1+2*x)*
3^(1/2))-4/9*ln(1+x)+ln(x)
```

maxima [A] time = 1.37, size = 50, normalized size = 0.78

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{x^2+1}{3\left(x^3+1\right)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 + 1) - 5/18*
log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

mupad [B] time = 0.10, size = 63, normalized size = 0.98

$$\ln(x)-\frac{4\ln\left(x+1\right)}{9}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{5}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-\frac{5}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x*(x^3 + 1)^2),x)
```

```
[Out] log(x) - (4*log(x + 1))/9 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18
+ 5/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18 - 5/18) + (x^2/3 +
 1/3)/(x^3 + 1)
```

sympy [A] time = 0.23, size = 60, normalized size = 0.94

$$\frac{x^2+1}{3x^3+3}+\log(x)-\frac{4\log(x+1)}{9}-\frac{5\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x/(x**3+1)**2,x)
```

```
[Out] (x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 - x + 1)/18
+ sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.182 $\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=63

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

<u>**Rubi**</u> **[A]** time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 628</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

<u>Rule 634</u>

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

<u>Rule 1628</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

<u>Rule 1646</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
```

```
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx &= -\frac{7+5x}{3\left(1+x+x^2\right)}+\frac{1}{3}\int \frac{-8-19x-5x^2}{(1+x)^2\left(1+x+x^2\right)}\,dx\\ &= -\frac{7+5x}{3\left(1+x+x^2\right)}+\frac{1}{3}\int \left(\frac{6}{(1+x)^2}-\frac{3}{1+x}+\frac{-11+3x}{1+x+x^2}\right)dx\\ &= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{3}\int \frac{-11+3x}{1+x+x^2}\,dx\\ &= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{6}\int \frac{1}{1+x+x^2}\,d\\ &= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)+\frac{25}{3}\text{Subst}\left(\int \frac{1}{-3-}\right.\\ &= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\frac{25\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

IntegrateAlgebraic [A] time = 0.05, size = 69, normalized size = 1.10

$$\frac{-11x^2-18x-13}{3(x+1)\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] (-13 - 18*x - 11*x^2)/(3*(1 + x)*(1 + x + x^2)) - (25*ArcTan[1/Sqrt[3] + (2
*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

fricas [A] time = 1.04, size = 97, normalized size = 1.54

$$-\frac{50\sqrt{3}\left(x^3+2x^2+2x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+66x^2-9\left(x^3+2x^2+2x+1\right)\log\left(x^2+x+1\right)+18\left(x^3\right.}{18\left(x^3+2x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + 6
6*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*x
+ 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)
```

giac [A] time = 0.90, size = 72, normalized size = 1.14

$$-\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right)+\frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)}-\frac{2}{x+1}+\frac{1}{2}\log\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] -25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x + 1) - 2)/(1
/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*log(-1/(x + 1) + 1/(x + 1)^2
+ 1)
```

maple [A] time = 0.40, size = 54, normalized size = 0.86

method	result	size
default	$\frac{-\frac{5x}{3}-\frac{7}{3}}{x^2+x+1}+\frac{\ln(x^2+x+1)}{2}-\frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{2}{1+x}-\ln(1+x)$	54
risch	$\frac{-\frac{11}{3}x^2-6x-\frac{13}{3}}{(x^2+x+1)(1+x)}-\ln(1+x)+\frac{\ln(625x^2+625x+625)}{2}-\frac{25\sqrt{3}\arctan\left(\frac{2\left(25x+\frac{25}{2}\right)\sqrt{3}}{75}\right)}{9}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-2/(1+x)-ln(1+x)
```

maxima [A] time = 1.43, size = 59, normalized size = 0.94

$$-\frac{25}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{11x^2+18x+13}{3\left(x^3+2x^2+2x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^3
 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

mupad [B] time = 0.25, size = 73, normalized size = 1.16

$$-\ln(x+1)-\frac{\frac{11x^2}{3}+6x+\frac{13}{3}}{x^3+2x^2+2x+1}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^2 + 2)/((x + 1)^2*(x + x^2 + 1)^2),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*25i)/18 + 1/2) - (6*x + (11*x^2)/3
+ 13/3)/(2*x + 2*x^2 + x^3 + 1) - log(x + 1) - log(x + (3^(1/2)*1i)/2 + 1/2
)*((3^(1/2)*25i)/18 - 1/2)
```

sympy [A] time = 0.20, size = 68, normalized size = 1.08

$$\frac{-11x^2-18x-13}{3x^3+6x^2+6x+3}-\log(x+1)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{25\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

```
[Out] (-11*x**2 - 18*x - 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2
+ x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.183 $\int \frac{1}{(1-4x)^3(2-3x)} \, dx$

Optimal. Leaf size=43

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 - 3
*x])/125
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-4x)^3(2-3x)} \, dx &= \int \left(\frac{27}{125(-2+3x)}-\frac{4}{5(-1+4x)^3}-\frac{12}{25(-1+4x)^2}-\frac{36}{125(-1+4x)}\right) dx\\ &= \frac{1}{10(1-4x)^2}-\frac{3}{25(1-4x)}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.07

$$\frac{120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(4x-1)-5}{250(1-4x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] (-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-1 + 4*x])/
(250*(1 - 4*x)^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.77

$$\frac{24x-1}{50(4x-1)^2}-\frac{18}{125}\tanh^{-1}\left(\frac{x+1}{7x-3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] (-1 + 24*x)/(50*(-1 + 4*x)^2) - (18*ArcTanh[(1 + x)/(-3 + 7*x)])/125
```

fricas [A] time = 1.19, size = 55, normalized size = 1.28

$$-\frac{18\left(16\,x^{2}-8\,x+1\right)\log\left(4\,x-1\right)-18\left(16\,x^{2}-8\,x+1\right)\log\left(3\,x-2\right)-120\,x+5}{250\left(16\,x^{2}-8\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="fricas")
```

```
[Out] -1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x
- 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

giac [A] time = 0.77, size = 33, normalized size = 0.77

$$\frac{24\,x-1}{50\,(4\,x-1)^{2}}-\frac{9}{125}\,\log\left(|4\,x-1|\right)+\frac{9}{125}\,\log\left(|3\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="giac")
```

```
[Out] 1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x -
 2))
```

maple [A] time = 0.32, size = 32, normalized size = 0.74

method	result	size
risch	$\frac{\frac{12x}{25}-\frac{1}{50}}{(4x-1)^{2}}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(4x-1)}{125}$	32
norman	$\frac{\frac{8}{25}x+\frac{8}{25}x^{2}}{(4x-1)^{2}}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(4x-1)}{125}$	35
default	$\frac{9\ln(-2+3x)}{125}+\frac{1}{10(4x-1)^{2}}+\frac{3}{25(4x-1)}-\frac{9\ln(4x-1)}{125}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-4*x)^3/(2-3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 16*(3/100*x-1/800)/(4*x-1)^2+9/125*ln(-2+3*x)-9/125*ln(4*x-1)
```

maxima [A] time = 0.48, size = 36, normalized size = 0.84

$$\frac{24\,x-1}{50\left(16\,x^{2}-8\,x+1\right)}-\frac{9}{125}\,\log\left(4\,x-1\right)+\frac{9}{125}\,\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="maxima")
```

```
[Out] 1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*log(3*x - 2
)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.58

$$\frac{\frac{3\,x}{100}-\frac{1}{800}}{x^{2}-\frac{x}{2}+\frac{1}{16}}-\frac{18\,\mathrm{atanh}\left(\frac{24\,x}{5}-\frac{11}{5}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x - 2)*(4*x - 1)^3),x)
```

```
[Out] ((3*x)/100 - 1/800)/(x^2 - x/2 + 1/16) - (18*atanh((24*x)/5 - 11/5))/125
```

sympy [A] time = 0.16, size = 34, normalized size = 0.79

$$\frac{24x-1}{800x^2-400x+50}+\frac{9\log\left(x-\frac{2}{3}\right)}{125}-\frac{9\log\left(x-\frac{1}{4}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)**3/(2-3*x),x)
```

```
[Out] (24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/12
5
```

3.184 $\int \frac{x^3}{(2-5x^2)^7} dx$

Optimal. Leaf size=27

$$\frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(2-5x^2)^7} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(2-5x)^7} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2}{5(-2+5x)^7} - \frac{1}{5(-2+5x)^6}\right) dx, x, x^2\right) \\
&= \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{15x^2-1}{750(2-5x^2)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] (-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

IntegrateAlgebraic [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{15x^2-1}{750\left(5x^2-2\right)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] (-1 + 15*x^2)/(750*(-2 + 5*x^2)^6)
```

fricas [A] time = 1.24, size = 43, normalized size = 1.59

$$\frac{15\,x^2-1}{750\left(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="fricas")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

giac [A] time = 1.10, size = 18, normalized size = 0.67

$$\frac{15\,x^2-1}{750\left(5\,x^2-2\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="giac")
```

```
[Out] 1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

maple [A] time = 0.29, size = 19, normalized size = 0.70

method	result	size
gosper	$\frac{15x^2-1}{750\left(5x^2-2\right)^6}$	19
risch	$\frac{\frac{x^2}{50}-\frac{1}{750}}{\left(5x^2-2\right)^6}$	19
default	$\frac{1}{150\left(5x^2-2\right)^6}+\frac{1}{250\left(5x^2-2\right)^5}$	24
norman	$\frac{-\frac{25}{32}x^{10}-\frac{5}{12}x^6+\frac{1}{8}x^4+\frac{25}{32}x^8+\frac{125}{384}x^{12}}{\left(5x^2-2\right)^6}$	37
meijerg	$\frac{x^4\left(\frac{625}{16}x^8-\frac{375}{4}x^6+\frac{375}{4}x^4-50x^2+15\right)}{7680\left(1-\frac{5x^2}{2}\right)^6}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-5*x^2+2)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/750*(15*x^2-1)/(5*x^2-2)^6
```

maxima [A] time = 0.64, size = 43, normalized size = 1.59

$$\frac{15\,x^2-1}{750\left(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="maxima")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

mupad [B] time = 0.11, size = 18, normalized size = 0.67

$$\frac{15\,x^2-1}{750\,\left(5\,x^2-2\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(5*x^2 - 2)^7,x)
```

```
[Out] (15*x^2 - 1)/(750*(5*x^2 - 2)^6)
```

sympy [A] time = 0.19, size = 39, normalized size = 1.44

$$-\frac{1-15x^2}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-5*x**2+2)**7,x)
```

```
[Out] -(1 - 15*x**2)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000*
x**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

3.185 $\int \frac{x^7}{\left(2-5x^2\right)^3} \, dx$

Optimal. Leaf size=46

$$-\frac{x^2}{250}-\frac{6}{625\left(2-5x^2\right)}+\frac{2}{625\left(2-5x^2\right)^2}-\frac{3}{625}\log\left(2-5x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{x^2}{250}-\frac{6}{625\left(2-5x^2\right)}+\frac{2}{625\left(2-5x^2\right)^2}-\frac{3}{625}\log\left(2-5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -x^2/250 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2 - 5*x^2])
/625
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(2-5x^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(2-5x)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{125}-\frac{8}{125(-2+5x)^3}-\frac{12}{125(-2+5x)^2}-\frac{6}{125(-2+5x)}\right)dx,x,x^2\right)\\
&= -\frac{x^2}{250}+\frac{2}{625\left(2-5x^2\right)^2}-\frac{6}{625\left(2-5x^2\right)}-\frac{3}{625}\log\left(2-5x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$-\frac{125x^6-150x^4+6\left(2-5x^2\right)^2\log\left(5x^2-2\right)+12}{1250\left(2-5x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -1/1250*(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(2 - 5*x
^2)^2
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 0.83

$$\frac{-125x^6+150x^4-12}{1250\left(5x^2-2\right)^2}-\frac{3}{625}\log\left(5x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] (-12 + 150*x^4 - 125*x^6)/(1250*(-2 + 5*x^2)^2) - (3*Log[-2 + 5*x^2])/625
```

fricas [A] time = 1.21, size = 55, normalized size = 1.20

$$-\frac{125\,x^6-100\,x^4-40\,x^2+6\left(25\,x^4-20\,x^2+4\right)\log\left(5\,x^2-2\right)+20}{1250\left(25\,x^4-20\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 - 2
) + 20)/(25*x^4 - 20*x^2 + 4)
```

giac [A] time = 1.03, size = 40, normalized size = 0.87

$$-\frac{1}{250}\,x^2+\frac{225\,x^4-120\,x^2+16}{1250\left(5\,x^2-2\right)^2}-\frac{3}{625}\,\log\left(\left|5\,x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs(
5*x^2 - 2))
```

maple [A] time = 0.28, size = 35, normalized size = 0.76

method	result	size
risch	$-\frac{x^2}{250}+\frac{\frac{6x^2}{125}-\frac{2}{125}}{\left(5x^2-2\right)^2}-\frac{3\ln\left(5x^2-2\right)}{625}$	35
norman	$\frac{-\frac{6}{125}x^2+\frac{9}{50}x^4-\frac{1}{10}x^6}{\left(5x^2-2\right)^2}-\frac{3\ln\left(5x^2-2\right)}{625}$	38
meijerg	$-\frac{x^2\left(25x^4-45x^2+12\right)}{1000\left(1-\frac{5x^2}{2}\right)^2}-\frac{3\ln\left(1-\frac{5x^2}{2}\right)}{625}$	38
default	$-\frac{x^2}{250}+\frac{6}{625\left(5x^2-2\right)}+\frac{2}{625\left(5x^2-2\right)^2}-\frac{3\ln\left(5x^2-2\right)}{625}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-5*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/250*x^2+25*(6/3125*x^2-2/3125)/(5*x^2-2)^2-3/625*ln(5*x^2-2)
```

maxima [A] time = 0.53, size = 39, normalized size = 0.85

$$-\frac{1}{250}\,x^2 + \frac{2\left(3\,x^2 - 1\right)}{125\left(25\,x^4 - 20\,x^2 + 4\right)} - \frac{3}{625}\,\log\left(5\,x^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2)
```

mupad [B] time = 0.19, size = 34, normalized size = 0.74

$$\frac{\frac{6\,x^2}{3125} - \frac{2}{3125}}{x^4 - \frac{4\,x^2}{5} + \frac{4}{25}} - \frac{3\,\ln\left(x^2 - \frac{2}{5}\right)}{625} - \frac{x^2}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/(5*x^2 - 2)^3,x)
```

```
[Out] ((6*x^2)/3125 - 2/3125)/(x^4 - (4*x^2)/5 + 4/25) - (3*log(x^2 - 2/5))/625 -
 x^2/250
```

sympy [A] time = 0.13, size = 36, normalized size = 0.78

$$-\frac{x^2}{250} - \frac{2 - 6x^2}{3125x^4 - 2500x^2 + 500} - \frac{3\log\left(5x^2 - 2\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-5*x**2+2)**3,x)
```

```
[Out] -x**2/250 - (2 - 6*x**2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)/
625
```

3.186 $\int \frac{1}{(-2+x)^3(1+x)^2}\,dx$

Optimal. Leaf size=44

$$\frac{2}{27(x-2)}+\frac{1}{27(x+1)}-\frac{1}{18(x-2)^2}+\frac{1}{27}\log(x-2)-\frac{1}{27}\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{2}{27(2-x)}+\frac{1}{27(x+1)}-\frac{1}{18(2-x)^2}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] -1/(18*(2 - x)^2) - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[1
 + x]/27
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)^3(1+x)^2}\,dx &= \int \left(\frac{1}{9(-2+x)^3}-\frac{2}{27(-2+x)^2}+\frac{1}{27(-2+x)}-\frac{1}{27(1+x)^2}-\frac{1}{27(1+x)}\right)dx\\ &= -\frac{1}{18(2-x)^2}-\frac{2}{27(2-x)}+\frac{1}{27(1+x)}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\frac{3\left(2x^2-5x-1\right)}{(x-2)^2(x+1)}+2\log(x-2)-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] ((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x]
)/54
```

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 0.93

$$\frac{2x^2-5x-1}{18(x-2)^2(x+1)}+\frac{1}{27}\log(x-2)-\frac{1}{27}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] (-1 - 5*x + 2*x^2)/(18*(-2 + x)^2*(1 + x)) + Log[-2 + x]/27 - Log[1 + x]/27
```

fricas [A] time = 1.32, size = 56, normalized size = 1.27

$$\frac{6\,x^2 - 2\left(x^3 - 3\,x^2 + 4\right)\log\left(x + 1\right) + 2\left(x^3 - 3\,x^2 + 4\right)\log\left(x - 2\right) - 15\,x - 3}{54\left(x^3 - 3\,x^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x -
2) - 15*x - 3)/(x^3 - 3*x^2 + 4)
```

giac [A] time = 0.79, size = 43, normalized size = 0.98

$$\frac{1}{27\left(x + 1\right)} - \frac{\frac{18}{x+1} - 5}{162\left(\frac{3}{x+1} - 1\right)^2} + \frac{1}{27}\log\left(\left|-\frac{3}{x + 1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*log(abs(-3/(
x + 1) + 1))
```

maple [A] time = 0.28, size = 35, normalized size = 0.80

method	result	size
default	$-\frac{1}{18(-2+x)^2}+\frac{2}{27(-2+x)}+\frac{1}{27+27x}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
norman	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
risch	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)^3/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

maxima [A] time = 0.46, size = 37, normalized size = 0.84

$$\frac{2\,x^2 - 5\,x - 1}{18\left(x^3 - 3\,x^2 + 4\right)} - \frac{1}{27}\log\left(x + 1\right) + \frac{1}{27}\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x - 2
)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.75

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{3} - \frac{1}{3}\right)}{27} - \frac{-\frac{x^2}{9} + \frac{5\,x}{18} + \frac{1}{18}}{x^3 - 3\,x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^2*(x - 2)^3),x)
```

```
[Out] - (2*atanh((2*x)/3 - 1/3))/27 - ((5*x)/18 - x^2/9 + 1/18)/(x^3 - 3*x^2 + 4)
```

sympy [A] time = 0.15, size = 34, normalized size = 0.77

$$\frac{2x^2 - 5x - 1}{18x^3 - 54x^2 + 72} + \frac{\log(x - 2)}{27} - \frac{\log(x + 1)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)**3/(1+x)**2,x)
```

```
[Out] (2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/27
```

3.187 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)^3(3+x)^4}\,dx &= \int \left(\frac{1}{(2+x)^3}-\frac{4}{(2+x)^2}+\frac{10}{2+x}-\frac{1}{(3+x)^4}-\frac{3}{(3+x)^3}-\frac{6}{(3+x)^2}-\frac{10}{3+x}\right)dx\\ &= -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.00

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

IntegrateAlgebraic [A] time = 0.00, size = 43, normalized size = 0.80

$$\frac{60x^4+630x^3+2450x^2+4175x+2627}{6(x+2)^2(x+3)^3}-20\tanh^{-1}(2x+5)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] (2627 + 4175*x + 2450*x^2 + 630*x^3 + 60*x^4)/(6*(2 + x)^2*(3 + x)^3) - 20*
ArcTanh[5 + 2*x]
```

fricas [B] time = 1.07, size = 105, normalized size = 1.94

$$\frac{60\,x^4+630\,x^3+2450\,x^2-60\left(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108\right)\log\left(x+3\right)+60\left(x^5+13\,x^4+67\,x^3\right.}{6\left(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

giac [A] time = 0.97, size = 47, normalized size = 0.87

$$\frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x+3)^3(x+2)^2}-10\,\log\left(|x+3|\right)+10\,\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

method	result	size
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$	45
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$	45
default	$-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*l
n(3+x)
```

maxima [A] time = 0.51, size = 60, normalized size = 1.11

$$\frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\left(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108\right)}-10\,\log\left(x+3\right)+10\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

mupad [B] time = 0.00, size = 55, normalized size = 1.02

$$\frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108}-20\,\mathrm{atanh}\left(2\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2)^3*(x + 3)^4),x)
```

```
[Out] ((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

sympy [A] time = 0.18, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x+2) - 10\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

3.188 $\int \frac{x^5}{(3+x)^2} \, dx$

Optimal. Leaf size=36

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + x)^2,x]
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(3+x)^2} \, dx &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x}\right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4}\left(x^4 - 8x^3 + 54x^2 - 432x + \frac{972}{x+3} - 2079\right) + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + x)^2,x]
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

IntegrateAlgebraic [A] time = 0.00, size = 39, normalized size = 1.08

$$\frac{x^5 - 5x^4 + 30x^3 - 270x^2 - 1296x + 972}{4(x+3)} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(3 + x)^2,x]
[Out] (972 - 1296*x - 270*x^2 + 30*x^3 - 5*x^4 + x^5)/(4*(3 + x)) + 405*Log[3 + x
]
```

fricas [A] time = 1.28, size = 39, normalized size = 1.08

$$\frac{x^5 - 5\,x^4 + 30\,x^3 - 270\,x^2 + 1620\,(x+3)\log(x+3) - 1296\,x + 972}{4\,(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)
```

giac [A] time = 0.88, size = 45, normalized size = 1.25

$$-\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

method	result	size
default	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\ln(3+x)$	33
risch	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\ln(3+x)$	33
norman	$\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x} + 405\ln(3+x)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)} + 405\ln\left(1+\frac{x}{3}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

maxima [A] time = 0.51, size = 32, normalized size = 0.89

$$\frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

mupad [B] time = 0.00, size = 32, normalized size = 0.89

$$405\,\ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x + 3)^2,x)
```

```
[Out] 405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

sympy [A] time = 0.09, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

3.189 $\int (\mathbf{b1} + \mathbf{c1}x) \left(a + 2bx + cx^2\right) dx$

Optimal. Leaf size=44

$$\frac{1}{2}x^2(a\text{c1} + 2b\text{b1}) + a\text{b1}x + \frac{1}{3}x^3(2b\text{c1} + \text{b1}c) + \frac{1}{4}c\text{c1}x^4$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{1}{2}x^2(a\text{c1} + 2b\text{b1}) + a\text{b1}x + \frac{1}{3}x^3(2b\text{c1} + \text{b1}c) + \frac{1}{4}c\text{c1}x^4$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right) dx &= \int \left(a\text{b1} + (2b\text{b1} + a\text{c1})x + (\text{b1}c + 2b\text{c1})x^2 + c\text{c1}x^3\right) dx\\ &= a\text{b1}x + \frac{1}{2}(2b\text{b1} + a\text{c1})x^2 + \frac{1}{3}(\text{b1}c + 2b\text{c1})x^3 + \frac{1}{4}c\text{c1}x^4\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{1}{12}x(6a(2\text{b1} + \text{c1}x) + x(4b(3\text{b1} + 2\text{c1}x) + cx(4\text{b1} + 3\text{c1}x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
[Out] (x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12
```

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 0.93

$$\frac{1}{12}x\left(12a\text{b1} + 6a\text{c1}x + 12b\text{b1}x + 8b\text{c1}x^2 + 4\text{b1}cx^2 + 3c\text{c1}x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
[Out] (x*(12*a*b1 + 12*b*b1*x + 6*a*c1*x + 4*b1*c*x^2 + 8*b*c1*x^2 + 3*c*c1*x^3))
/12
```

fricas [A] time = 1.01, size = 39, normalized size = 0.89

$$\frac{1}{4}x^4c_1c + \frac{1}{3}x^3cb_1 + \frac{2}{3}x^3c_1b + x^2b_1b + \frac{1}{2}x^2c_1a + xb_1a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c1*c + 1/3*x^3*c*b1 + 2/3*x^3*c1*b + x^2*b1*b + 1/2*x^2*c1*a + x*b1
*a
```

giac [A] time = 0.97, size = 39, normalized size = 0.89

$$\frac{1}{4}\,cc_1x^4 + \frac{1}{3}\,b_1cx^3 + \frac{2}{3}\,bc_1x^3 + bb_1x^2 + \frac{1}{2}\,ac_1x^2 + ab_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b1
*x
```

maple [A] time = 0.02, size = 38, normalized size = 0.86

method	result	size
norman	$\frac{cc1\,x^4}{4} + \left(\frac{2bc1}{3} + \frac{b1c}{3}\right)x^3 + \left(\frac{ac1}{2} + bb1\right)x^2 + ab1x$	38
default	$ab1x + \frac{(ac1+2bb1)x^2}{2} + \frac{(2bc1+b1c)x^3}{3} + \frac{cc1\,x^4}{4}$	39
gosper	$\frac{1}{4}cc1\,x^4 + \frac{2}{3}x^3bc1 + \frac{1}{3}x^3b1c + \frac{1}{2}x^2ac1 + x^2bb1 + ab1x$	40
risch	$\frac{1}{4}cc1\,x^4 + \frac{2}{3}x^3bc1 + \frac{1}{3}x^3b1c + \frac{1}{2}x^2ac1 + x^2bb1 + ab1x$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*c1*x^4+(2/3*b*c1+1/3*b1*c)*x^3+(1/2*a*c1+b*b1)*x^2+a*b1*x
```

maxima [A] time = 0.59, size = 38, normalized size = 0.86

$$\frac{1}{4}\,cc_1x^4 + \frac{1}{3}\,(b_1c + 2\,bc_1)x^3 + ab_1x + \frac{1}{2}\,(2\,bb_1 + ac_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

mupad [B] time = 0.05, size = 37, normalized size = 0.84

$$\frac{c\,c_1\,x^4}{4} + \left(\frac{2\,b\,c_1}{3} + \frac{b_1\,c}{3}\right)\,x^3 + \left(\frac{a\,c_1}{2} + b\,b_1\right)\,x^2 + a\,b_1\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2),x)
```

```
[Out] x^2*((a*c1)/2 + b*b1) + x^3*((2*b*c1)/3 + (b1*c)/3) + a*b1*x + (c*c1*x^4)/4
```

sympy [A] time = 0.07, size = 39, normalized size = 0.89

$$ab_1x + \frac{cc_1x^4}{4} + x^3\left(\frac{2bc_1}{3} + \frac{b_1c}{3}\right) + x^2\left(\frac{ac_1}{2} + bb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)

[Out] a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)
```

3.190 $\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2\,dx$

Optimal. Leaf size=96

$$a^2\mathrm{b1}x+\frac{1}{2}x^4\left(ac\mathrm{c1}+2b^2\mathrm{c1}+2b\mathrm{b1}c\right)+\frac{2}{3}x^3\left(2ab\mathrm{c1}+a\mathrm{b1}c+2b^2\mathrm{b1}\right)+\frac{1}{2}ax^2(a\mathrm{c1}+4b\mathrm{b1})+\frac{1}{5}cx^5(4b\mathrm{c1}+\mathrm{b1}c)+\frac{1}{6}c^2c$$

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$a^2\mathrm{b1}x+\frac{1}{2}x^4\left(ac\mathrm{c1}+2b^2\mathrm{c1}+2b\mathrm{b1}c\right)+\frac{2}{3}x^3\left(2ab\mathrm{c1}+a\mathrm{b1}c+2b^2\mathrm{b1}\right)+\frac{1}{2}ax^2(a\mathrm{c1}+4b\mathrm{b1})+\frac{1}{5}cx^5(4b\mathrm{c1}+\mathrm{b1}c)+\frac{1}{6}c^2c$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^
3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5 +
 (c^2*c1*x^6)/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx = \int \left(a^2\mathrm{b1}+a(4b\mathrm{b1}+a\mathrm{c1})x+2\left(2b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^2+2\left(2b\mathrm{b1}c+2\right.\right.$$
$$= a^2\mathrm{b1}x+\frac{1}{2}a(4b\mathrm{b1}+a\mathrm{c1})x^2+\frac{2}{3}\left(2b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3+\frac{1}{2}\left(2b\mathrm{b1}c+\right.$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.95

$$\frac{1}{30}x\left(15a^2(2\mathrm{b1}+\mathrm{c1}x)+5ax(4b(3\mathrm{b1}+2\mathrm{c1}x)+cx(4\mathrm{b1}+3\mathrm{c1}x))+x^2\left(10b^2(4\mathrm{b1}+3\mathrm{c1}x)+6bcx(5\mathrm{b1}+4\mathrm{c1}x)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))
) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1 +
 5*c1*x))))/30
```

IntegrateAlgebraic [A] time = 0.06, size = 105, normalized size = 1.09

$$\frac{1}{30}\left(30a^2\mathrm{b1}x+15a^2\mathrm{c1}x^2+60ab\mathrm{b1}x^2+40ab\mathrm{c1}x^3+20a\mathrm{b1}cx^3+15ac\mathrm{c1}x^4+40b^2\mathrm{b1}x^3+30b^2\mathrm{c1}x^4+30b\mathrm{b1}cx\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (30*a^2*b1*x + 60*a*b*b1*x^2 + 15*a^2*c1*x^2 + 40*b^2*b1*x^3 + 20*a*b1*c*x^
3 + 40*a*b*c1*x^3 + 30*b*b1*c*x^4 + 30*b^2*c1*x^4 + 15*a*c*c1*x^4 + 6*b1*c^
2*x^5 + 24*b*c*c1*x^5 + 5*c^2*c1*x^6)/30
```

fricas [A] time = 1.08, size = 98, normalized size = 1.02

$$\frac{1}{6}x^6c_1c^2+\frac{1}{5}x^5c^2b_1+\frac{4}{5}x^5c_1cb+x^4cb_1b+x^4c_1b^2+\frac{1}{2}x^4c_1ca+\frac{4}{3}x^3b_1b^2+\frac{2}{3}x^3cb_1a+\frac{4}{3}x^3c_1ba+2x^2b_1ba+\frac{1}{2}x^2c_1a^2+xb_1a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c1*c^2 + 1/5*x^5*c^2*b1 + 4/5*x^5*c1*c*b + x^4*c*b1*b + x^4*c1*b^2
+ 1/2*x^4*c1*c*a + 4/3*x^3*b1*b^2 + 2/3*x^3*c*b1*a + 4/3*x^3*c1*b*a + 2*x^2
*b1*b*a + 1/2*x^2*c1*a^2 + x*b1*a^2
```

giac [A] time = 1.11, size = 98, normalized size = 1.02

$$\frac{1}{6}c^2c_1x^6+\frac{1}{5}b_1c^2x^5+\frac{4}{5}bcc_1x^5+bb_1cx^4+b^2c_1x^4+\frac{1}{2}acc_1x^4+\frac{4}{3}b^2b_1x^3+\frac{2}{3}ab_1cx^3+\frac{4}{3}abc_1x^3+2abb_1x^2+\frac{1}{2}a^2c_1x^2+a^2b_1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4
+ 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a*b
*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

maple [A] time = 0.32, size = 89, normalized size = 0.93

method	result
norman	$\frac{c^2c1\,x^6}{6}+\left(\frac{4}{5}c1bc+\frac{1}{5}b1\,c^2\right)x^5+\left(\frac{1}{2}acc1+b^2c1+bb1c\right)x^4+\left(\frac{4}{3}abc1+\frac{2}{3}ab1c+\frac{4}{3}b^2b1\right)x^3+\left(\frac{1}{2}c1\,a^2+2\right.$
default	$\frac{c^2c1\,x^6}{6}+\frac{\left(4c1bc+b1\,c^2\right)x^5}{5}+\frac{\left(4bb1c+c1\left(2ac+4b^2\right)\right)x^4}{4}+\frac{\left(b1\left(2ac+4b^2\right)+4abc1\right)x^3}{3}+\frac{\left(c1\,a^2+4b1ab\right)x^2}{2}+a^2b1x$
gosper	$\frac{1}{6}c^2c1\,x^6+\frac{4}{5}x^5c1bc+\frac{1}{5}x^5b1\,c^2+\frac{1}{2}x^4acc1+x^4b^2c1+x^4bb1c+\frac{4}{3}x^3abc1+\frac{2}{3}x^3ab1c+\frac{4}{3}x^3b^2b1+\frac{1}{2}x^2c$
risch	$\frac{1}{6}c^2c1\,x^6+\frac{4}{5}x^5c1bc+\frac{1}{5}x^5b1\,c^2+\frac{1}{2}x^4acc1+x^4b^2c1+x^4bb1c+\frac{4}{3}x^3abc1+\frac{2}{3}x^3ab1c+\frac{4}{3}x^3b^2b1+\frac{1}{2}x^2c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*c1*x^6+(4/5*c1*b*c+1/5*b1*c^2)*x^5+(1/2*a*c*c1+b^2*c1+b*b1*c)*x^4+(
4/3*a*b*c1+2/3*a*b1*c+4/3*b^2*b1)*x^3+(1/2*c1*a^2+2*b1*a*b)*x^2+a^2*b1*x
```

maxima [A] time = 0.55, size = 91, normalized size = 0.95

$$\frac{1}{6}c^2c_1x^6+\frac{1}{5}\left(b_1c^2+4\,bcc_1\right)x^5+\frac{1}{2}\left(2\,bb_1c+\left(2\,b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\left(2\,b^2b_1+ab_1c+2\,abc_1\right)x^3+\frac{1}{2}\left(4\,abb_1+a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c
)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*
b1 + a^2*c1)*x^2
```

mupad [B] time = 0.19, size = 88, normalized size = 0.92

$$x^3\left(\frac{4\,b_1\,b^2}{3}+\frac{4\,a\,c_1\,b}{3}+\frac{2\,a\,b_1\,c}{3}\right)+x^4\left(c_1\,b^2+b_1\,c\,b+\frac{a\,c\,c_1}{2}\right)+x^2\left(\frac{c_1\,a^2}{2}+2\,b\,b_1\,a\right)+x^5\left(\frac{b_1\,c^2}{5}+\frac{4\,b\,c_1\,c}{5}\right)+\frac{c^2\,c_1\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x)
```

```
[Out] x^3*((4*b^2*b1)/3 + (4*a*b*c1)/3 + (2*a*b1*c)/3) + x^4*(b^2*c1 + (a*c*c1)/2
 + b*b1*c) + x^2*((a^2*c1)/2 + 2*a*b*b1) + x^5*((b1*c^2)/5 + (4*b*c*c1)/5)
+ (c^2*c1*x^6)/6 + a^2*b1*x
```

sympy [A] time = 0.09, size = 100, normalized size = 1.04

$$a^2b_1x+\frac{c^2c_1x^6}{6}+x^5\left(\frac{4bcc_1}{5}+\frac{b_1c^2}{5}\right)+x^4\left(\frac{acc_1}{2}+b^2c_1+bb_1c\right)+x^3\left(\frac{4abc_1}{3}+\frac{2ab_1c}{3}+\frac{4b^2b_1}{3}\right)+x^2\left(\frac{a^2c_1}{2}+2abb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

```
[Out] a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/2
 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**2*
(a**2*c1/2 + 2*a*b*b1)
```

3.191 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^3 dx$

Optimal. Leaf size=167

$$a^3\text{b1}x+\frac{1}{4}x^4\left(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{2}a^2x^2(a\text{c1}+6b\text{b1})+\frac{1}{2}cx^6\left(a c\text{c1}+4b^2\text{c1}+2b\text{b1}c\right)+ax^3\left(2ab\right.$$

Rubi [A] time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$\frac{1}{4}x^4\left(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{2}a^2x^2(a\text{c1}+6b\text{b1})+a^3\text{b1}x+\frac{1}{5}x^5\left(12abc\text{c1}+3a\text{b1}c^2+12b^2\text{b1}c+8b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^3 dx &= \int \left(a^3\text{b1}+a^2(6b\text{b1}+a\text{c1})x+3a\left(4b^2\text{b1}+a\text{b1}c+2ab\text{c1}\right)x^2+\left(8b^3\text{b1}+12a\right.\right.\\ &= a^3\text{b1}x+\frac{1}{2}a^2(6b\text{b1}+a\text{c1})x^2+a\left(4b^2\text{b1}+a\text{b1}c+2ab\text{c1}\right)x^3+\frac{1}{4}\left(8b^3\text{b1}+12\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 167, normalized size = 1.00

$$a^3\text{b1}x+\frac{1}{4}x^4\left(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{2}a^2x^2(a\text{c1}+6b\text{b1})+\frac{1}{2}cx^6\left(a c\text{c1}+4b^2\text{c1}+2b\text{b1}c\right)+ax^3\left(2ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

IntegrateAlgebraic [A] time = 0.08, size = 195, normalized size = 1.17

$$\frac{1}{280}\left(280a^3\text{b1}x+140a^3\text{c1}x^2+840a^2b\text{b1}x^2+560a^2bc1x^3+280a^2\text{b1}cx^3+210a^2c\text{c1}x^4+1120ab^2\text{b1}x^3+840ab^2\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] (280*a^3*b1*x + 840*a^2*b*b1*x^2 + 140*a^3*c1*x^2 + 1120*a*b^2*b1*x^3 + 280
*a^2*b1*c*x^3 + 560*a^2*b*c1*x^3 + 560*b^3*b1*x^4 + 840*a*b*b1*c*x^4 + 840*
a*b^2*c1*x^4 + 210*a^2*c*c1*x^4 + 672*b^2*b1*c*x^5 + 168*a*b1*c^2*x^5 + 448
*b^3*c1*x^5 + 672*a*b*c*c1*x^5 + 280*b*b1*c^2*x^6 + 560*b^2*c*c1*x^6 + 140*
a*c^2*c1*x^6 + 40*b1*c^3*x^7 + 240*b*c^2*c1*x^7 + 35*c^3*c1*x^8)/280
```

fricas [A] time = 1.12, size = 188, normalized size = 1.13

$$\frac{1}{8}x^8c_1c^3+\frac{1}{7}x^7c^3b_1+\frac{6}{7}x^7c_1c^2b+x^6c^2b_1b+2x^6c_1cb^2+\frac{1}{2}x^6c_1c^2a+\frac{12}{5}x^5cb_1b^2+\frac{8}{5}x^5c_1b^3+\frac{3}{5}x^5c^2b_1a+\frac{12}{5}x^5c_1cba+2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c1*c^3 + 1/7*x^7*c^3*b1 + 6/7*x^7*c1*c^2*b + x^6*c^2*b1*b + 2*x^6*c
1*c*b^2 + 1/2*x^6*c1*c^2*a + 12/5*x^5*c*b1*b^2 + 8/5*x^5*c1*b^3 + 3/5*x^5*c
^2*b1*a + 12/5*x^5*c1*c*b*a + 2*x^4*b1*b^3 + 3*x^4*c*b1*b*a + 3*x^4*c1*b^2*
a + 3/4*x^4*c1*c*a^2 + 4*x^3*b1*b^2*a + x^3*c*b1*a^2 + 2*x^3*c1*b*a^2 + 3*x
^2*b1*b*a^2 + 1/2*x^2*c1*a^3 + x*b1*a^3
```

giac [A] time = 0.98, size = 188, normalized size = 1.13

$$\frac{1}{8}c^3c_1x^8+\frac{1}{7}b_1c^3x^7+\frac{6}{7}bc^2c_1x^7+bb_1c^2x^6+2b^2cc_1x^6+\frac{1}{2}ac^2c_1x^6+\frac{12}{5}b^2b_1cx^5+\frac{3}{5}ab_1c^2x^5+\frac{8}{5}b^3c_1x^5+\frac{12}{5}abcc_1x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6 + 2*b^2*c
*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*b1*c^2*x^5 + 8/5*b^3
*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 + 3*a*b*b1*c*x^4 + 3*a*b^2*c1*x^
4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 + 3*a
^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x
```

maple [A] time = 0.30, size = 165, normalized size = 0.99

method	result
norman	$\frac{c^3c1\,x^8}{8}+\left(\frac{6}{7}c1b\,c^2+\frac{1}{7}b1\,c^3\right)x^7+\left(\frac{1}{2}a\,c^2c1+2b^2cc1+b1b\,c^2\right)x^6+\left(\frac{12}{5}abcc1+\frac{3}{5}ab1\,c^2+\frac{8}{5}b^3c1+\frac{12}{5}\right.$
gosper	$\frac{1}{8}c^3c1\,x^8+\frac{6}{7}x^7c1b\,c^2+\frac{1}{7}x^7b1\,c^3+\frac{1}{2}x^6a\,c^2c1+2x^6b^2cc1+x^6b1b\,c^2+\frac{12}{5}x^5abcc1+\frac{3}{5}x^5ab1\,c^2+\frac{8}{5}x^5$
risch	$\frac{1}{8}c^3c1\,x^8+\frac{6}{7}x^7c1b\,c^2+\frac{1}{7}x^7b1\,c^3+\frac{1}{2}x^6a\,c^2c1+2x^6b^2cc1+x^6b1b\,c^2+\frac{12}{5}x^5abcc1+\frac{3}{5}x^5ab1\,c^2+\frac{8}{5}x^5$
default	$\frac{c^3c1\,x^8}{8}+\frac{\left(6c1b\,c^2+b1\,c^3\right)x^7}{7}+\frac{\left(6b1b\,c^2+c1\left(c^2a+8b^2c+c\left(2ac+4b^2\right)\right)\right)x^6}{6}+\frac{\left(b1\left(c^2a+8b^2c+c\left(2ac+4b^2\right)\right)+c1\left(8abc+2b\left(2ac+4b^2\right)\right)\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^3*c1*x^8+(6/7*c1*b*c^2+1/7*b1*c^3)*x^7+(1/2*a*c^2*c1+2*b^2*c*c1+b1*b*
c^2)*x^6+(12/5*a*b*c*c1+3/5*a*b1*c^2+8/5*b^3*c1+12/5*b^2*b1*c)*x^5+(3/4*a^2
*c*c1+3*a*b^2*c1+3*a*b*b1*c+2*b^3*b1)*x^4+(2*a^2*b*c1+a^2*b1*c+4*a*b^2*b1)*
x^3+(1/2*c1*a^3+3*b1*a^2*b)*x^2+a^3*b1*x
```

maxima [A] time = 0.63, size = 171, normalized size = 1.02

$$\frac{1}{8}c^3c_1x^8+\frac{1}{7}\left(b_1c^3+6\,bc^2c_1\right)x^7+\frac{1}{2}\left(2\,bb_1c^2+\left(4\,b^2c+ac^2\right)c_1\right)x^6+\frac{1}{5}\left(12\,b^2b_1c+3\,ab_1c^2+4\left(2\,b^3+3\,abc\right)c_1\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*c
 + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c1)
*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*x^4
 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)*x^2
```

mupad [B] time = 0.22, size = 164, normalized size = 0.98

$$x^7\left(\frac{b_1\,c^3}{7}+\frac{6\,b\,c_1\,c^2}{7}\right)+x^3\,\left(2\,c_1\,a^2\,b+b_1\,c\,a^2+4\,b_1\,a\,b^2\right)+x^6\,\left(2\,c_1\,b^2\,c+b_1\,b\,c^2+\frac{a\,c_1\,c^2}{2}\right)+x^4\,\left(\frac{3\,c\,c_1\,a^2}{4}+3\,c_1\,a\,b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x)
```

```
[Out] x^7*((b1*c^3)/7 + (6*b*c^2*c1)/7) + x^3*(4*a*b^2*b1 + 2*a^2*b*c1 + a^2*b1*c
) + x^6*((a*c^2*c1)/2 + b*b1*c^2 + 2*b^2*c*c1) + x^4*(2*b^3*b1 + 3*a*b^2*c1
 + (3*a^2*c*c1)/4 + 3*a*b*b1*c) + x^5*((8*b^3*c1)/5 + (3*a*b1*c^2)/5 + (12*
b^2*b1*c)/5 + (12*a*b*c*c1)/5) + x^2*((a^3*c1)/2 + 3*a^2*b*b1) + (c^3*c1*x^
8)/8 + a^3*b1*x
```

sympy [A] time = 0.10, size = 189, normalized size = 1.13

$$a^3b_1x+\frac{c^3c_1x^8}{8}+x^7\left(\frac{6bc^2c_1}{7}+\frac{b_1c^3}{7}\right)+x^6\left(\frac{ac^2c_1}{2}+2b^2cc_1+bb_1c^2\right)+x^5\left(\frac{12abcc_1}{5}+\frac{3ab_1c^2}{5}+\frac{8b^3c_1}{5}+\frac{12b^2b_1c}{5}\right)+x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

```
[Out] a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x**6*(a*c**
2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 + 8
*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b*b1
*c + 2*b**3*b1) + x**3*(2*a**2*b*c1 + a**2*b1*c + 4*a*b**2*b1) + x**2*(a**3
*c1/2 + 3*a**2*b*b1)
```

3.192 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^4\, dx$

Optimal. Leaf size=263

$$a^4\text{b1}x+\frac{1}{2}a^3x^2(a\text{c1}+8b\text{b1})+\frac{4}{3}a^2x^3\left(2abc\text{c1}+a\text{b1}c+6b^2\text{b1}\right)+ax^4\left(a^2c\text{c1}+6ab^2\text{c1}+6ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{3}x^6\left(3a\right.$$

Rubi [A] time = 0.34, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+16b^3\text{b1}c+8b^4\text{c1}\right)+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+24ab^2\text{b1}c+16ab^3\text{c1}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^4\,dx = \int \left(a^4\text{b1}+a^3(8b\text{b1}+a\text{c1})x+4a^2\left(6b^2\text{b1}+a\text{b1}c+2abc\text{c1}\right)x^2+4a\left(8b^3\text{b}\right.\right.$$

$$= a^4\text{b1}x+\frac{1}{2}a^3(8b\text{b1}+a\text{c1})x^2+\frac{4}{3}a^2\left(6b^2\text{b1}+a\text{b1}c+2abc\text{c1}\right)x^3+a\left(8b^3\text{b1}\right.$$

Mathematica [A] time = 0.06, size = 263, normalized size = 1.00

$$a^4\text{b1}x+\frac{1}{2}a^3x^2(a\text{c1}+8b\text{b1})+\frac{4}{3}a^2x^3\left(2abc\text{c1}+a\text{b1}c+6b^2\text{b1}\right)+ax^4\left(a^2c\text{c1}+6ab^2\text{c1}+6ab\text{b1}c+8b^3\text{b1}\right)+\frac{1}{3}x^6\left(3a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

IntegrateAlgebraic [A] time = 0.15, size = 315, normalized size = 1.20

$$\frac{1}{630}\left(630a^4\text{b1}x + 315a^4\text{c1}x^2 + 2520a^3b\text{b1}x^2 + 1680a^3b\text{c1}x^3 + 840a^3\text{b1}cx^3 + 630a^3c\text{c1}x^4 + 5040a^2b^2\text{b1}x^3 + 378\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] (630*a^4*b1*x + 2520*a^3*b*b1*x^2 + 315*a^4*c1*x^2 + 5040*a^2*b^2*b1*x^3 +
840*a^3*b1*c*x^3 + 1680*a^3*b*c1*x^3 + 5040*a*b^3*b1*x^4 + 3780*a^2*b*b1*c*
x^4 + 3780*a^2*b^2*c1*x^4 + 630*a^3*c*c1*x^4 + 2016*b^4*b1*x^5 + 6048*a*b^2
*b1*c*x^5 + 756*a^2*b1*c^2*x^5 + 4032*a*b^3*c1*x^5 + 3024*a^2*b*c*c1*x^5 +
3360*b^3*b1*c*x^6 + 2520*a*b*b1*c^2*x^6 + 1680*b^4*c1*x^6 + 5040*a*b^2*c*c1
*x^6 + 630*a^2*c^2*c1*x^6 + 2160*b^2*b1*c^2*x^7 + 360*a*b1*c^3*x^7 + 2880*b
^3*c*c1*x^7 + 2160*a*b*c^2*c1*x^7 + 630*b*b1*c^3*x^8 + 1890*b^2*c^2*c1*x^8
+ 315*a*c^3*c1*x^8 + 70*b1*c^4*x^9 + 560*b*c^3*c1*x^9 + 63*c^4*c1*x^10)/630
```

fricas [A] time = 1.30, size = 307, normalized size = 1.17

$$\frac{1}{10}x^{10}c_1c^4+\frac{1}{9}x^9c^4b_1+\frac{8}{9}x^9c_1c^3b+x^8c^3b_1b+3x^8c_1c^2b^2+\frac{1}{2}x^8c_1c^3a+\frac{24}{7}x^7c^2b_1b^2+\frac{32}{7}x^7c_1cb^3+\frac{4}{7}x^7c^3b_1a+\frac{24}{7}x^7c_1c^2ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c1*c^4 + 1/9*x^9*c^4*b1 + 8/9*x^9*c1*c^3*b + x^8*c^3*b1*b + 3*x^8
*c1*c^2*b^2 + 1/2*x^8*c1*c^3*a + 24/7*x^7*c^2*b1*b^2 + 32/7*x^7*c1*c*b^3 +
4/7*x^7*c^3*b1*a + 24/7*x^7*c1*c^2*b*a + 16/3*x^6*c*b1*b^3 + 8/3*x^6*c1*b^4
 + 4*x^6*c^2*b1*b*a + 8*x^6*c1*c*b^2*a + x^6*c1*c^2*a^2 + 16/5*x^5*b1*b^4 +
 48/5*x^5*c*b1*b^2*a + 32/5*x^5*c1*b^3*a + 6/5*x^5*c^2*b1*a^2 + 24/5*x^5*c1
*c*b*a^2 + 8*x^4*b1*b^3*a + 6*x^4*c*b1*b*a^2 + 6*x^4*c1*b^2*a^2 + x^4*c1*c*
a^3 + 8*x^3*b1*b^2*a^2 + 4/3*x^3*c*b1*a^3 + 8/3*x^3*c1*b*a^3 + 4*x^2*b1*b*a
^3 + 1/2*x^2*c1*a^4 + x*b1*a^4
```

giac [A] time = 1.03, size = 307, normalized size = 1.17

$$\frac{1}{10}c^4c_1x^{10}+\frac{1}{9}b_1c^4x^9+\frac{8}{9}bc^3c_1x^9+bb_1c^3x^8+3b^2c^2c_1x^8+\frac{1}{2}ac^3c_1x^8+\frac{24}{7}b^2b_1c^2x^7+\frac{4}{7}ab_1c^3x^7+\frac{32}{7}b^3cc_1x^7+\frac{24}{7}abc^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^2
*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 + 3
2/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^2*x
^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x^5 +
 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a^2*b*
c*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3*c*c1*
x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3*b*b1*x
^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

maple [A] time = 0.30, size = 264, normalized size = 1.00

method	result
norman	$\frac{c^4\text{c1}\,x^{10}}{10}+\left(\frac{8}{9}\text{c1}b\,c^3+\frac{1}{9}\text{b1}\,c^4\right)x^9+\left(\frac{1}{2}a\,c^3\text{c1}+3b^2c^2\text{c1}+\text{b1}b\,c^3\right)x^8+\left(\frac{24}{7}ab\,c^2\text{c1}+\frac{4}{7}a\text{b1}\,c^3+\frac{32}{7}b^3c\text{c1}+\right.$
gosper	$\frac{1}{2}x^2\text{c1}\,a^4+\frac{8}{3}x^6b^4\text{c1}+\frac{16}{5}x^5b^4\text{b1}+\frac{1}{9}x^9\text{b1}\,c^4+\frac{1}{10}c^4\text{c1}\,x^{10}+\frac{4}{3}x^3a^3\text{b1}c+8x^3a^2b^2\text{b1}+4x^2\text{b1}\,a^3b+a^3c\text{c1}$
risch	$\frac{1}{2}x^2\text{c1}\,a^4+\frac{8}{3}x^6b^4\text{c1}+\frac{16}{5}x^5b^4\text{b1}+\frac{1}{9}x^9\text{b1}\,c^4+\frac{1}{10}c^4\text{c1}\,x^{10}+\frac{4}{3}x^3a^3\text{b1}c+8x^3a^2b^2\text{b1}+4x^2\text{b1}\,a^3b+a^3c\text{c1}$

default	$\frac{c^4 c1\, x^{10}}{10}+\frac{\left(8c1b\,c^3+b1\,c^4\right)x^9}{9}+\frac{\left(8b1b\,c^3+c1\left(2\left(2ac+4b^2\right)c^2+16b^2c^2\right)\right)x^8}{8}+\frac{\left(b1\left(2\left(2ac+4b^2\right)c^2+16b^2c^2\right)+c1\left(8ab\,c^2+8\left(2ac+4b^2\right)\right.\right.}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^4*c1*x^10+(8/9*c1*b*c^3+1/9*b1*c^4)*x^9+(1/2*a*c^3*c1+3*b^2*c^2*c1+b
1*b*c^3)*x^8+(24/7*a*b*c^2*c1+4/7*a*b1*c^3+32/7*b^3*c*c1+24/7*b^2*b1*c^2)*x
^7+(a^2*c^2*c1+8*a*b^2*c*c1+4*a*b*b1*c^2+8/3*b^4*c1+16/3*b^3*b1*c)*x^6+(24/
5*a^2*b*c*c1+6/5*a^2*b1*c^2+32/5*a*b^3*c1+48/5*a*b^2*b1*c+16/5*b^4*b1)*x^5+
(a^3*c*c1+6*a^2*b^2*c1+6*a^2*b*b1*c+8*a*b^3*b1)*x^4+(8/3*c1*a^3*b+4/3*a^3*b
1*c+8*a^2*b^2*b1)*x^3+(1/2*c1*a^4+4*b1*a^3*b)*x^2+a^4*b1*x
```

maxima [A] time = 0.55, size = 273, normalized size = 1.04

$$\frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^2
*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b*c
^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3*a^
2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 +
4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*b^2 +
 a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/2*(8*a
^3*b*b1 + a^4*c1)*x^2
```

mupad [B] time = 0.26, size = 263, normalized size = 1.00

$$x^9\left(\frac{b_1\,c^4}{9}+\frac{8\,b\,c_1\,c^3}{9}\right)+x^3\left(\frac{8\,c_1\,a^3\,b}{3}+\frac{4\,b_1\,c\,a^3}{3}+8\,b_1\,a^2\,b^2\right)+x^8\left(3\,c_1\,b^2\,c^2+b_1\,b\,c^3+\frac{a\,c_1\,c^3}{2}\right)+x^5\left(\frac{24\,c_1\,a^2\,b\,c}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x)
```

```
[Out] x^9*((b1*c^4)/9 + (8*b*c^3*c1)/9) + x^3*(8*a^2*b^2*b1 + (8*a^3*b*c1)/3 + (4
*a^3*b1*c)/3) + x^8*(3*b^2*c^2*c1 + (a*c^3*c1)/2 + b*b1*c^3) + x^5*((16*b^4
*b1)/5 + (6*a^2*b1*c^2)/5 + (32*a*b^3*c1)/5 + (48*a*b^2*b1*c)/5 + (24*a^2*b
*c*c1)/5) + x^6*((8*b^4*c1)/3 + a^2*c^2*c1 + (16*b^3*b1*c)/3 + 4*a*b*b1*c^2
 + 8*a*b^2*c*c1) + x^4*(6*a^2*b^2*c1 + 8*a*b^3*b1 + a^3*c*c1 + 6*a^2*b*b1*c
) + x^7*((24*b^2*b1*c^2)/7 + (4*a*b1*c^3)/7 + (32*b^3*c*c1)/7 + (24*a*b*c^2
*c1)/7) + x^2*((a^4*c1)/2 + 4*a^3*b*b1) + (c^4*c1*x^10)/10 + a^4*b1*x
```

sympy [A] time = 0.13, size = 313, normalized size = 1.19

$$a^4b_1x+\frac{c^4c_1x^{10}}{10}+x^9\left(\frac{8bc^3c_1}{9}+\frac{b_1c^4}{9}\right)+x^8\left(\frac{ac^3c_1}{2}+3b^2c^2c_1+bb_1c^3\right)+x^7\left(\frac{24abc^2c_1}{7}+\frac{4ab_1c^3}{7}+\frac{32b^3cc_1}{7}+\frac{24b^2}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

```
[Out] a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*c
**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*c*
*3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b**2*
c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1
/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5) +
 x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x**3*(8*
a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*a**3*b*
b1)
```

3.193 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^n \, dx$

Optimal. Leaf size=159

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}{}_2F_1\left(-n,n+1;n+2;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right.}{c(n+1)\sqrt{b^2-ac}}$$

Rubi [A] time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {640, 624}

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}\text{Hypergeometric2F1}\left(-n,n+1,\right.}{c(n+1)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b -
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1 +
 n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt
[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^n\,dx=\frac{\text{c1}\left(a+2bx+cx^2\right)^{1+n}}{2c(1+n)}+\frac{(2\text{b1}c-2b\text{c1})\int\left(a+2bx+cx^2\right)^n\,dx}{2c}$$

$$=\frac{\text{c1}\left(a+2bx+cx^2\right)^{1+n}}{2c(1+n)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}\left(a+2bx+cx^2\right)^{1}}{c\sqrt{b^2-ac}\,(1+}$$

Mathematica [C] time = 0.46, size = 267, normalized size = 1.68

$$\frac{1}{2}(a+x(2b+cx))^n\left(\text{c1}x^2\left(\frac{-\sqrt{b^2-ac}+b+cx}{b-\sqrt{b^2-ac}}\right)^{-n}\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}+b}\right)^{-n}F_1\left(2;-n,-n;3;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-a}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] ((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b^
2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])])/(((b - Sqrt[b^2 - a*c] + c*x)/(b
 - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^
n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1 + n,
 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*((b +
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (c1x + b1)\left(c\,x^2 + 2bx + a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

```
[Out] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b_1 + c_1\,x)\,\left(c\,x^2 + 2\,b\,x + a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x)
[Out] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b_1 + c_1x)\left(a + 2bx + cx^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
[Out] Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)
```

3.194 $\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {634, 618, 206, 628}

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2 - a*c])) +
 (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx &= \frac{\text{c1}\int \frac{2b+2cx}{a+2bx+cx^2}\,dx}{2c}+\frac{(2\text{b1}c-2b\text{c1})\int \frac{1}{a+2bx+cx^2}\,dx}{2c}\\
&= \frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(2\text{b1}c-2b\text{c1})\,\text{Subst}\left(\int \frac{1}{4\left(b^2-ac\right)-x^2}\,dx,x,2b+2cx\right)}{c}\\
&= -\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}+\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.02

$$\frac{(\text{b1}c-b\text{c1})\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{c\sqrt{ac-b^2}}+\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] ((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) + (
c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

IntegrateAlgebraic [A] time = 0.06, size = 79, normalized size = 1.22

$$\frac{(\text{b1}c-b\text{c1})\tan^{-1}\left(\frac{cx}{\sqrt{ac-b^2}}+\frac{b}{\sqrt{ac-b^2}}\right)}{c\sqrt{ac-b^2}}+\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] ((b1*c - b*c1)*ArcTan[b/Sqrt[-b^2 + a*c] + (c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt
[-b^2 + a*c]) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

fricas [A] time = 0.75, size = 203, normalized size = 3.12

$$\left[\frac{\left(b^2-ac\right)c_1\log\left(cx^2+2\,bx+a\right)-\sqrt{b^2-ac}\,(b_1c-bc_1)\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{b^2-ac}\,(cx+b)}{cx^2+2\,bx+a}\right)}{2\left(b^2c-ac^2\right)},\frac{\left(b^2-ac\right)c_1\log\left(cx^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1)
*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2
 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a)
 - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2
 - a*c)))/(b^2*c - a*c^2)]
```

giac [A] time = 0.77, size = 60, normalized size = 0.92

$$\frac{c_1\log\left(cx^2+2\,bx+a\right)}{2\,c}+\frac{(b_1c-bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2
+ a*c))/(sqrt(-b^2 + a*c)*c)
```

maple [A] time = 0.35, size = 63, normalized size = 0.97

method	result

default	$\frac{c1\ln\left(c\,x^{2}+2bx+a\right)}{2c}+\frac{\left(b1-\frac{c1b}{c}\right)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^{2}}}\right)}{\sqrt{ac-b^{2}}}$
risch	$\frac{\ln\left(-abcc1+ab1\,c^{2}+b^{3}c1-b^{2}b1c-\sqrt{-(bc1-b1c)^{2}\left(ac-b^{2}\right)}\,cx-\sqrt{-(bc1-b1c)^{2}\left(ac-b^{2}\right)}\,b\right)ac1}{2ac-2b^{2}}-\frac{\ln\left(-abcc1+ab1\,c^{2}+b^{3}c1-b^{2}b1c-\sqrt{-(b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c+(b1-c1*b/c)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*
b)/(a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more details)Is 4*b^2-4*a*c positive or negative?
```

mupad [B] time = 0.27, size = 155, normalized size = 2.38

$$\frac{b_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}}+\frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{\sqrt{a\,c-b^2}}-\frac{2\,b^2\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c}+\frac{2\,a\,c\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c}-\frac{b\,c_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}}+\frac{c}{\sqrt{a}}\right.}{c\,\sqrt{a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2),x)
```

```
[Out] (b1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(a*c - b^2)^(1/2)
- (2*b^2*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) + (2*a*c*c1*log(a +
 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) - (b*c1*atan(b/(a*c - b^2)^(1/2) + (c*
x)/(a*c - b^2)^(1/2)))/(c*(a*c - b^2)^(1/2))
```

sympy [B] time = 0.76, size = 246, normalized size = 3.78

$$\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\left(ac-b^2\right)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\left(ac-b^2\right)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\left(ac-b^2\right)}\right)-bb_1}{bc_1-b_1c}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

```
[Out] (c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2
*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*c1
 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) -
 b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a
*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2
*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*
c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

3.195 $\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {638, 618, 206}

$$\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^2}\,dx &= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}-\frac{(\text{b1}c-b\text{c1})\int \frac{1}{a+2bx+cx^2}\,dx}{2\left(b^2-ac\right)}\\
&= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}+\frac{(\text{b1}c-b\text{c1})\,\text{Subst}\left(\int \frac{1}{4\left(b^2-ac\right)-x^2}\,dx,x,2b+2cx\right)}{b^2-ac}\\
&= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}+\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.99

$$\frac{\frac{(bc1-b1c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{ac1-bb1+bc1x-b1cx}{a+x(2b+cx)}}{2\left(b^2-ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] ((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1)
*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

IntegrateAlgebraic [A] time = 0.10, size = 113, normalized size = 1.27

$$\frac{ac1-bb1+bc1x-b1cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}+\frac{(bc1-b1c)\tan^{-1}\left(\frac{cx}{\sqrt{ac-b^2}}+\frac{b}{\sqrt{ac-b^2}}\right)}{2\left(b^2-ac\right)\sqrt{ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((
-(b1*c) + b*c1)*ArcTan[b/Sqrt[-b^2 + a*c] + (c*x)/Sqrt[-b^2 + a*c]])/(2*(b^
2 - a*c)*Sqrt[-b^2 + a*c])
```

fricas [B] time = 1.33, size = 447, normalized size = 5.02

$$\left[-\frac{2\,b^3b_1-2\,abb_1c-\left(ab_1c-abc_1+\left(b_1c^2-bcc_1\right)x^2+2\left(bb_1c-b^2c_1\right)x\right)\sqrt{b^2-ac}\,\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{b^2-ac}}{cx^2+2\,bx+a}\right.}{4\left(ab^4-2\,a^2b^2c+a^3c^2+\left(b^4c-2\,ab^2c^2+a^2c^3\right)x^2+2\left(b^5-2\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c
+ 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*c1
+ 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*
c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)
*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(
b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)
*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 +
2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x)]
```

giac [A] time = 1.14, size = 92, normalized size = 1.03

$$-\frac{\left(b_1c-bc_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\left(b^2-ac\right)\sqrt{-b^2+ac}}-\frac{b_1cx-bc_1x+bb_1-ac_1}{2\left(cx^2+2\,bx+a\right)\left(b^2-ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^
2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2 -
 a*c))
```

maple [A] time = 0.34, size = 103, normalized size = 1.16

method	result
default	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{(4ac-4b^2)(cx^2+2bx+a)}+\frac{(-2bc1+2b1c)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(bc1-b1c)x}{2(ac-b^2)}-\frac{ac1-bb1}{2(ac-b^2)}}{cx^2+2bx+a}+\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)bc1}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)b1c}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((c^2a-b^2c)x-(\right.}{4(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+(-2*b*c1+2
*b1*c)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more details)Is 4*b^2-4*a*c positive or negative?
```

mupad [B] time = 0.28, size = 159, normalized size = 1.79

$$\frac{\operatorname{atan}\left(\frac{2\left(\frac{(4b^3-4abc)(bc_1-b_1c)}{8(ac-b^2)^{5/2}}-\frac{cx(bc_1-b_1c)}{2(ac-b^2)^{3/2}}\right)(ac-b^2)}{bc_1-b_1c}\right)(bc_1-b_1c)}{2(ac-b^2)^{3/2}}-\frac{\frac{ac_1-bb_1}{2(ac-b^2)}+\frac{x(bc_1-b_1c)}{2(ac-b^2)}}{cx^2+2bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x)
```

```
[Out] (atan((2*(((4*b^3 - 4*a*b*c)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - (c*x*(b
*c1 - b1*c))/(2*(a*c - b^2)^(3/2)))*(a*c - b^2))/(b*c1 - b1*c))*(b*c1 - b1*
c))/(2*(a*c - b^2)^(3/2)) - ((a*c1 - b*b1)/(2*(a*c - b^2)) + (x*(b*c1 - b1*
c))/(2*(a*c - b^2)))/(a + 2*b*x + c*x^2)
```

sympy [B] time = 1.02, size = 323, normalized size = 3.63

$$\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)\log\left(x+\frac{-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4}-\frac{\sqrt{-\frac{1}{(a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

```
[Out] sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-1/(a*c - b
**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)
- b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c1 -
 b1*c**2))/4 - sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (a**2*c**2*sq
rt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*
(b*c1 - b1*c) + b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b
1*c)/(b*c*c1 - b1*c**2))/4 + (-a*c1 + b*b1 + x*(-b*c1 + b1*c))/(2*a**2*c -
2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

3.196 $\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{3(b+cx)(\text{b1}c-b\text{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {638, 614, 618, 206}

$$\frac{3(b+cx)(\text{b1}c-b\text{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(4*(b^2 - a*c)*(a + 2*b*x + c*x^2)^2) + (3
*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)) - (3*c*(b1*
c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(8*(b^2 - a*c)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\text{b1} + \text{c1}x}{\left(a + 2bx + cx^2\right)^3}\, dx = -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} - \frac{(3(\text{b1}c - b\text{c1}))\int \frac{1}{\left(a+2bx+cx^2\right)^2}dx}{4\left(b^2 - ac\right)}$$

$$= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} + \frac{(3c(\text{b1}c - b\text{c1}))\int \frac{}{a+}}{8\left(b^2 - ac\right)}$$

$$= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} - \frac{(3c(\text{b1}c - b\text{c1}))\,\text{Sub}}{}$$

$$= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} - \frac{3c(\text{b1}c - b\text{c1})\tanh^{-}}{8\left(b^2 - ac\right)^5}$$

Mathematica [A] time = 0.14, size = 127, normalized size = 0.98

$$\frac{\frac{2\left(b^2-ac\right)(a\text{c1}-b\text{b1}+b\text{c1}x-\text{b1}cx)}{(a+x(2b+cx))^2} + \frac{3c(\text{b1}c-b\text{c1})\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}} + \frac{3(b+cx)(\text{b1}c-b\text{c1})}{a+x(2b+cx)}}{8\left(b^2-ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] ((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^2 +
 (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c*(b1*c - b*c1)*ArcTa
n[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(8*(b^2 - a*c)^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 195, normalized size = 1.50

$$\frac{-2a^2c\text{c1} - ab^2\text{c1} + 5ab\text{b1}c - 5abc\text{c1}x + 5a\text{b1}c^2x - 2b^3\text{b1} - 4b^3\text{c1}x + 4b^2\text{b1}cx - 9b^2c\text{c1}x^2 + 9b\text{b1}c^2x^2 - 3bc^2\text{c}}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] (-2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1 + 4*b^2*b1*c*x + 5*a*b1*c^2
*x - 4*b^3*c1*x - 5*a*b*c*c1*x + 9*b*b1*c^2*x^2 - 9*b^2*c*c1*x^2 + 3*b1*c^3
*x^3 - 3*b*c^2*c1*x^3)/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (3*(-(b1*c
^2) + b*c*c1)*ArcTan[b/Sqrt[-b^2 + a*c] + (c*x)/Sqrt[-b^2 + a*c]])/(8*(b^2
- a*c)^2*Sqrt[-b^2 + a*c])
```

fricas [B] time = 1.27, size = 1104, normalized size = 8.49

$$\left[-\frac{4b^5b_1 - 14ab^3b_1c + 10a^2bb_1c^2 - 6\left(b^2b_1c^3 - ab_1c^4 - \left(b^3c^2 - abc^3\right)c_1\right)x^3 - 18\left(b^3b_1c^2 - abb_1c^3 - \left(b^4c - ab^2c\right.\right.}{16\left(a^2b^6 - 3a^3b^4c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*c
^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a
*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 +
4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b
*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2
 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)
) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*c^2 - 5
*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c
 + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c
^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^
8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a
^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*b^3*b1*c + 5
*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 9*
(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^
2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b
^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*
c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2*b1*c^2 - 5*a^2*b1
*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c + 3*a^
4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c^5)*x^4
 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^8 - 5*a
*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a^2*b^5*
c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x)]
```

giac [A] time = 1.03, size = 194, normalized size = 1.49

$$\frac{3\left(b_1c^2 - bcc_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\left(b^4 - 2\,ab^2c + a^2c^2\right)\sqrt{-b^2+ac}} + \frac{3\,b_1c^3x^3 - 3\,bc^2c_1x^3 + 9\,bb_1c^2x^2 - 9\,b^2cc_1x^2 + 4\,b^2b_1cx + 5\,ab_1c^2x - 4\,b^3c_1x -}{8\left(b^4 - 2\,ab^2c + a^2c^2\right)\left(cx^2 + 2\,bx + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 - 2*a*b^2*c
+ a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b1*
c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*a*b
*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^2*c
+ a^2*c^2)*(c*x^2 + 2*b*x + a)^2)
```

maple [A] time = 0.35, size = 155, normalized size = 1.19

method	result
default	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{2\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)^2}+\frac{3(-2bc1+2b1c)\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right)}{2\left(4ac-4b^2\right)}$
risch	$\frac{-\frac{3c^2(bc1-b1c)x^3}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{9bc(bc1-b1c)x^2}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{\left(5ac+4b^2\right)(bc1-b1c)x}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{2a^2cc1+a\,b^2c1-5abb1c+2b^3b1}{8\left(a^2c^2-2a\,b^2c+b^4\right)}}{\left(c\,x^2+2bx+a\right)^2}-\frac{3c\ln\left(\left(a^2c^3-2a\,b^2c^2+b^4c\right)x-\left(-ac+b^2\right)^{\frac{5}{2}}+a^2b\,c\right.}{16\left(-ac+b^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2+3/2*
(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2
*c/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more details)Is 4*b^2-4*a*c positive or negative?
```

mupad [B] time = 0.50, size = 360, normalized size = 2.77

$$\frac{3c\,\mathrm{atan}\left(\frac{8\left(\frac{3c^2x\,(b\,c_1-b_1\,c)}{8\,(a\,c-b^2)^{5/2}}+\frac{3c\,(b\,c_1-b_1\,c)\,(16\,a^2\,b\,c^2-32\,a\,b^3\,c+16\,b^5)}{128\,(a\,c-b^2)^{5/2}\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}\right)(a^2\,c^2-2\,a\,b^2\,c+b^4)}{3\,b_1\,c^2-3\,b\,c\,c_1}\right)(b\,c_1-b_1\,c)}{8\,(a\,c-b^2)^{5/2}}-\frac{\frac{2\,c\,c_1\,a^2+c_1\,a\,b^2-5\,b_1\,c\,a\,b+2\,b_1\,b^3}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+}{a^2+x^2\,(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x)
```

```
[Out] (3*c*atan((8*((3*c^2*x*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) + (3*c*(b*c1 -
b1*c)*(16*b^5 + 16*a^2*b*c^2 - 32*a*b^3*c))/(128*(a*c - b^2)^(5/2)*(b^4 + a
^2*c^2 - 2*a*b^2*c)))*(b^4 + a^2*c^2 - 2*a*b^2*c))/(3*b1*c^2 - 3*b*c*c1))*(
b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - ((2*b^3*b1 + a*b^2*c1 + 2*a^2*c*c1 -
5*a*b*b1*c)/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (x*(5*a*c + 4*b^2)*(b*c1 - b1
*c))/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (3*c^2*x^3*(b*c1 - b1*c))/(8*(b^4 +
a^2*c^2 - 2*a*b^2*c)) + (9*b*c*x^2*(b*c1 - b1*c))/(8*(b^4 + a^2*c^2 - 2*a*b
^2*c)))/(a^2 + x^2*(2*a*c + 4*b^2) + c^2*x^4 + 4*a*b*x + 4*b*c*x^3)
```

sympy [B] time = 1.89, size = 622, normalized size = 4.78

$$\frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)\log\left(x+\frac{-3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)+9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)-9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}\,(bc_1-b_1c)+3b^6c\sqrt{-}}{3bc^2c_1-3b_1c^3}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*sqrt(-1/(a
*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1/(a*c - b**2)**5)*(b
*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**6
*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b
*c**2*c1 - 3*b1*c**3))/16 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(
x + (3*a**3*c**4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2*c**3*
sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)
**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b*
*2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 + (-2*a**2*c*c1 - a*b*
*2*c1 + 5*a*b*b1*c - 2*b**3*b1 + x**3*(-3*b*c**2*c1 + 3*b1*c**3) + x**2*(-9
*b**2*c*c1 + 9*b*b1*c**2) + x*(-5*a*b*c*c1 + 5*a*b1*c**2 - 4*b**3*c1 + 4*b*
*2*b1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8*a**2*c**4 -
 16*a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b**3*c**2 + 32
*b**5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(32*a**3*b*c**2
- 64*a**2*b**3*c + 32*a*b**5))
```

3.197 $\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=173

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-\ldots}{6\left(b^2-ac\right)\left(a+\ldots\right.}$$

Rubi [A] time = 0.11, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {638, 614, 618, 206}

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-\ldots}{6\left(b^2-ac\right)\left(a+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(6*(b^2 - a*c)*(a + 2*b*x + c*x^2)^3) + (5
*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (5*c*(
b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a + 2*b*x + c*x^2)) + (5*c^2*(b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{\left(a+2bx+cx^2\right)^4}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3} - \frac{(5(\mathrm{b1}c-b\mathrm{c1}))\int \frac{1}{\left(a+2bx+cx^2\right)^3}dx}{6\left(b^2-ac\right)}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3} + \frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2} + \frac{(5c(\mathrm{b1}c-b\mathrm{c1}))\int}{8\left(b^2-\right.}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3} + \frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2} - \frac{5c(\mathrm{b1}c-b\mathrm{c1})(}{16\left(b^2-ac\right)^3\left(a+\right.}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3} + \frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2} - \frac{5c(\mathrm{b1}c-b\mathrm{c1})(}{16\left(b^2-ac\right)^3\left(a+\right.}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3} + \frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2} - \frac{5c(\mathrm{b1}c-b\mathrm{c1})(}{16\left(b^2-ac\right)^3\left(a+\right.}$$

Mathematica [A] time = 0.23, size = 168, normalized size = 0.97

$$\frac{\frac{15c^2(b\mathrm{c1}-\mathrm{b1}c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}} - \frac{10\left(b^2-ac\right)(b+cx)(b\mathrm{c1}-\mathrm{b1}c)}{(a+x(2b+cx))^2} + \frac{8\left(b^2-ac\right)^2(a\mathrm{c1}-b\mathrm{b1}+b\mathrm{c1}x-\mathrm{b1}cx)}{(a+x(2b+cx))^3} + \frac{15c(b+cx)(b\mathrm{c1}-\mathrm{b1}c)}{a+x(2b+cx)}}{48\left(b^2-ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] ((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^3
 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (15*
c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b*c1
)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)^3)
```

IntegrateAlgebraic [A] time = 0.29, size = 340, normalized size = 1.97

$$8a^3c^2\mathrm{c1} + 9a^2b^2c\mathrm{c1} - 33a^2b\mathrm{b1}c^2 + 33a^2bc^2\mathrm{c1}x - 33a^2\mathrm{b1}c^3x - 2ab^4\mathrm{c1} + 26ab^3\mathrm{b1}c + 54ab^3c\mathrm{c1}x - 54ab^2\mathrm{b1}c^2x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] (-8*b^5*b1 + 26*a*b^3*b1*c - 33*a^2*b*b1*c^2 - 2*a*b^4*c1 + 9*a^2*b^2*c*c1
+ 8*a^3*c^2*c1 + 12*b^4*b1*c*x - 54*a*b^2*b1*c^2*x - 33*a^2*b1*c^3*x - 12*b
^5*c1*x + 54*a*b^3*c*c1*x + 33*a^2*b*c^2*c1*x - 30*b^3*b1*c^2*x^2 - 120*a*b
*b1*c^3*x^2 + 30*b^4*c*c1*x^2 + 120*a*b^2*c^2*c1*x^2 - 110*b^2*b1*c^3*x^3 -
 40*a*b1*c^4*x^3 + 110*b^3*c^2*c1*x^3 + 40*a*b*c^3*c1*x^3 - 75*b*b1*c^4*x^4
 + 75*b^2*c^3*c1*x^4 - 15*b1*c^5*x^5 + 15*b*c^4*c1*x^5)/(48*(b^2 - a*c)^3*(
a + 2*b*x + c*x^2)^3) + (5*(-(b1*c^3) + b*c^2*c1)*ArcTan[b/Sqrt[-b^2 + a*c]
 + (c*x)/Sqrt[-b^2 + a*c]])/(16*(b^2 - a*c)^3*Sqrt[-b^2 + a*c])
```

fricas [B] time = 1.34, size = 1950, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 +
30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4 -
 a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^2*b
1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 6
0*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*
a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*
x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c
^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b
^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3
)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x
^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x +
a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 6*(4*b^6*b1
*c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5
*c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^
4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 -
4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4
*a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3
 - 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 -
 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^
6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^
5*c^2 - 4*a^5*b^3*c^3 + a^6*b*c^4)*x), -1/48*(8*b^7*b1 - 34*a*b^5*b1*c + 59
*a^2*b^3*b1*c^2 - 33*a^3*b*b1*c^3 + 15*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 -
a*b*c^5)*c1)*x^5 + 75*(b^3*b1*c^4 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*
x^4 + 10*(11*b^4*b1*c^3 - 7*a*b^2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a
*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 30*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b
*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 -
a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*
(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3
+ 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^
2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^
2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 3*(4*b^6*b1*c - 22
*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5*c + 7*
a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^4*c^2 -
 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 - 4*a^3*b
^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4*a^3*b^
3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3 - 10*a
^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 - 10*a^4
*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^6*c^2 -
 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^5*c^2 -
 4*a^5*b^3*c^3 + a^6*b*c^4)*x)]
```

giac [B] time = 0.85, size = 363, normalized size = 2.10

$$-\frac{5\left(b_1c^3-bc^2c_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\left(b^6-3\,ab^4c+3\,a^2b^2c^2-a^3c^3\right)\sqrt{-b^2+ac}}-\frac{15\,b_1c^5x^5-15\,bc^4c_1x^5+75\,bb_1c^4x^4-75\,b^2c^3c_1x^4+110\,b^2b_1c^3x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] -5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^6 - 3*a*b^
4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48*(15*b1*c^5*x^5 - 15
*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3 +
40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^2*x^
2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b^4*b1
```

```
*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c*c1*x
- 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2*a*b^4*
c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3
*c^3)*(c*x^2 + 2*b*x + a)^3)
```

maple [A] time = 0.38, size = 206, normalized size = 1.19

method	result
default	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{3(4ac-4b^2)(cx^2+2bx+a)^3}+\frac{5(-2bc1+2b1c)\left(\frac{2cx+2b}{2(4ac-4b^2)(cx^2+2bx+a)^2}+\frac{3c\left(\frac{2cx+2b}{(4ac-4b^2)(cx^2+2bx+a)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}\right)}{4ac-4b^2}\right)}{3(4ac-4b^2)}$
risch	$\frac{-\frac{5c^4(bc1-b1c)x^5}{16(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)}-\frac{25c^3(bc1-b1c)bx^4}{16(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)}-\frac{5(4ac+11b^2)c^2(bc1-b1c)x^3}{24(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)}-\frac{5b(4ac+b^2)c(bc1-b1c)x^2}{8(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)}-\frac{(11a^2bc^2c1-11a^2b1c^3+}{16(c^3a}}{(cx^2+2bx+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3+5/3*
(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*(1/2*(2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+
a)^2+3*c/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2*c/(4*a*
c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more details)Is 4*b^2-4*a*c positive or negative?
```

mupad [B] time = 0.71, size = 640, normalized size = 3.70

$$\frac{\frac{5\,c^4\,x^5\,(b\,c_1-b_1\,c)}{16\,\left(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6\right)}-\frac{-8\,c_1\,a^3\,c^2-9\,c_1\,a^2\,b^2\,c+33\,b_1\,a^2\,b\,c^2+2\,c_1\,a\,b^4-26\,b_1\,a\,b^3\,c+8\,b_1\,b^5}{48\,\left(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6\right)}+\frac{x\,(b\,c_1-b_1\,c)\,\left(11\,a^2\,c^2+18\,a\,b^2\,c-4\,b^4\right)}{16\,\left(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6\right)}\ \cdots}{x^3\,\left(8\,b^3+12\,a\,c\,b\right)+x^2\,\left(3\,c\,a^2+12\,a\,b^2\right)+x^4\,\left(12\,b^2\,c+3\,a\,c\ \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x)
```

```
[Out] ((5*c^4*x^5*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c))
 - (8*b^5*b1 - 8*a^3*c^2*c1 + 2*a*b^4*c1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2
- 9*a^2*b^2*c*c1)/(48*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (x*(b*
c1 - b1*c)*(11*a^2*c^2 - 4*b^4 + 18*a*b^2*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^
2*c^2 - 3*a*b^4*c)) + (5*c*x^3*(4*a*c^2 + 11*b^2*c)*(b*c1 - b1*c))/(24*(b^6
 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^2*(b^3 + 4*a*b*c)*(b*c1 -
 b1*c))/(8*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (25*b*c^3*x^4*(b*
```

```
c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))/(x^3*(8*b^3 +
 12*a*b*c) + x^2*(12*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 12*b^2*c) + a^3 + c^
3*x^6 + 6*b*c^2*x^5 + 6*a^2*b*x) - (5*c^2*atan((16*((5*c^3*x*(b*c1 - b1*c))
/(16*(a*c - b^2)^(7/2)) + (5*c^2*(b*c1 - b1*c)*(32*b^7 - 32*a^3*b*c^3 + 96*
a^2*b^3*c^2 - 96*a*b^5*c))/(512*(a*c - b^2)^(7/2)*(b^6 - a^3*c^3 + 3*a^2*b^
2*c^2 - 3*a*b^4*c)))*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c))/(5*b1*c^3
 - 5*b*c^2*c1))*(b*c1 - b1*c))/(16*(a*c - b^2)^(7/2))
```

sympy [B] time = 3.06, size = 1027, normalized size = 5.94

$$\frac{5c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^7}}\,(bc_1-b_1c)\log\left(x+\frac{-5a^4c^6\sqrt{-\frac{1}{\left(ac-b^2\right)^7}}(bc_1-b_1c)+20a^3b^2c^5\sqrt{-\frac{1}{\left(ac-b^2\right)^7}}(bc_1-b_1c)-30a^2b^4c^4\sqrt{-\frac{1}{\left(ac-b^2\right)^7}}(bc_1-b_1c)+20ab^6c}{5bc^3c_1-5b_1c^4}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

```
[Out] 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c**6*sqrt(-1
/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqrt(-1/(a*c - b**2)**7
)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)
+ 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*sqrt(
-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3
*c1 - 5*b1*c**4))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x
+ (5*a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2*c**5*s
qrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sqrt(-1/(a*c - b*
*2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*
c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 -
5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 + (-8*a**3*c**2*c1 - 9*a**2*b**2
*c*c1 + 33*a**2*b*b1*c**2 + 2*a*b**4*c1 - 26*a*b**3*b1*c + 8*b**5*b1 + x**5
*(-15*b*c**4*c1 + 15*b1*c**5) + x**4*(-75*b**2*c**3*c1 + 75*b*b1*c**4) + x*
*3*(-40*a*b*c**3*c1 + 40*a*b1*c**4 - 110*b**3*c**2*c1 + 110*b**2*b1*c**3) +
 x**2*(-120*a*b**2*c**2*c1 + 120*a*b*b1*c**3 - 30*b**4*c*c1 + 30*b**3*b1*c*
*2) + x*(-33*a**2*b*c**2*c1 + 33*a**2*b1*c**3 - 54*a*b**3*c*c1 + 54*a*b**2*
b1*c**2 + 12*b**5*c1 - 12*b**4*b1*c))/(48*a**6*c**3 - 144*a**5*b**2*c**2 +
144*a**4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144*a**2*b**2*c**5 +
144*a*b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5 - 864*a**2*b**3*c**
4 + 864*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**4*c**5 + 144*a**3*b**2*
c**4 - 1296*a**2*b**4*c**3 + 1584*a*b**6*c**2 - 576*b**8*c) + x**3*(576*a**
4*b*c**4 - 1344*a**3*b**3*c**3 + 576*a**2*b**5*c**2 + 576*a*b**7*c - 384*b*
*9) + x**2*(144*a**5*c**4 + 144*a**4*b**2*c**3 - 1296*a**3*b**4*c**2 + 1584
*a**2*b**6*c - 576*a*b**8) + x*(288*a**5*b*c**3 - 864*a**4*b**3*c**2 + 864*
a**3*b**5*c - 288*a**2*b**7))
```

3.198 $\int (\mathbf{b1} + \mathbf{c1}x) \left(a + 2bx + cx^2\right)^{-n} dx$

Optimal. Leaf size=169

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}{}_2F_1\left(1-n,n;2-n;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right.}{c(1-n)\sqrt{b^2-ac}}$$

Rubi [A] time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {640, 624}

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\text{Hypergeometric2F1}\left(1-n,n,\right.}{c(1-n)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sqr
t[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n)*
Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2
- a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^{-n}dx &= \frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}+\frac{(2\text{b1}c-2b\text{c1})\int\left(a+2bx+cx^2\right)^{-n}dx}{2c}\\ &= \frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}\left(a+2bx+c\right.}{c\sqrt{b^2-ac}\,(1}\end{aligned}$$

Mathematica [C] time = 0.41, size = 264, normalized size = 1.56

$$\frac{1}{2}(a+x(2b+cx))^{-n}\left(\text{c1}x^2\left(\frac{-\sqrt{b^2-ac}+b+cx}{b-\sqrt{b^2-ac}}\right)^n\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}+b}\right)^n F_1\left(2;n,n;3;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-ac}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^
2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b +
Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sqrt[
b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hypergeom
etric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])]
)/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^{-n}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{c_1x + b_1}{\left(cx^2 + 2\,bx + a\right)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c_1x + b_1}{\left(cx^2 + 2\,bx + a\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (c1x + b1)\left(c\,x^2 + 2bx + a\right)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

```
[Out] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c_1x + b_1}{\left(cx^2 + 2\,bx + a\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b_1 + c_1 x}{\left(c x^2 + 2 b x + a\right)^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x)
[Out] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
[Out] Timed out
```

3.199 $\int \frac{x}{3+6x+2x^2} dx$

Optimal. Leaf size=49

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 31}

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3] +
 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{3+6x+2x^2}\,dx &= \frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{3-\sqrt{3}+2x}\,dx+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{3+\sqrt{3}+2x}\,dx\\ &= \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{1}{4}\left(\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)-\left(\sqrt{3}-1\right)\log\left(-2x+\sqrt{3}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] (-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3 + Sqrt[3]
+ 2*x])/4
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.96

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(-2x+\sqrt{3}-3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] ((1 - Sqrt[3])*Log[-3 + Sqrt[3] - 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3]
+ 2*x])/4
```

fricas [A] time = 1.25, size = 52, normalized size = 1.06

$$\frac{1}{4}\sqrt{3}\log\left(\frac{2x^2+\sqrt{3}(2x+3)+6x+6}{2x^2+6x+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
1/4*log(2*x^2 + 6*x + 3)
```

giac [A] time = 0.96, size = 46, normalized size = 0.94

$$-\frac{1}{4}\sqrt{3}\log\left(\frac{\left|4x-2\sqrt{3}+6\right|}{\left|4x+2\sqrt{3}+6\right|}\right)+\frac{1}{4}\log\left(\left|2x^2+6x+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) + 1/4*l
og(abs(2*x^2 + 6*x + 3))
```

maple [A] time = 0.29, size = 31, normalized size = 0.63

method	result	size
default	$\frac{\ln(2x^2+6x+3)}{4}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{2}$	31
risch	$\frac{\ln(3+2x+\sqrt{3})}{4}+\frac{\ln(3+2x+\sqrt{3})\sqrt{3}}{4}+\frac{\ln(3+2x-\sqrt{3})}{4}-\frac{\ln(3+2x-\sqrt{3})\sqrt{3}}{4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2+6*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(4*x+6)*3^(1/2))
```

maxima [A] time = 1.38, size = 41, normalized size = 0.84

$$-\frac{1}{4}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*log(2*x^2 +
 6*x + 3)
```

mupad [B] time = 0.17, size = 36, normalized size = 0.73

$$\ln\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\right)-\ln\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(6*x + 2*x^2 + 3),x)
```

```
[Out] log(x + 3^(1/2)/2 + 3/2)*(3^(1/2)/4 + 1/4) - log(x - 3^(1/2)/2 + 3/2)*(3^(1
/2)/4 - 1/4)
```

sympy [A] time = 0.12, size = 46, normalized size = 0.94

$$\left(\frac{1}{4}-\frac{\sqrt{3}}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x**2+6*x+3),x)
```

```
[Out] (1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqrt
(3)/2 + 3/2)
```

3.200 $\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3} \, dx$

Optimal. Leaf size=61

$$-\frac{2x+3}{2\left(2x^2+6x+3\right)}+\frac{4x+5}{4\left(2x^2+6x+3\right)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {638, 614, 618, 206}

$$-\frac{2x+3}{2\left(2x^2+6x+3\right)}+\frac{4x+5}{4\left(2x^2+6x+3\right)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2)) + ArcTa
nh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}+3\int \frac{1}{\left(3+6x+2x^2\right)^2}\,dx\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}-\int \frac{1}{3+6x+2x^2}\,dx\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,6+4x\right)\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+\frac{\tanh^{-1}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 1.15

$$\frac{1}{12}\left(-\frac{3\left(8x^3+36x^2+44x+13\right)}{\left(2x^2+6x+3\right)^2}-2\sqrt{3}\log\left(-2x+\sqrt{3}-3\right)+2\sqrt{3}\log\left(2x+\sqrt{3}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] ((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3 +
 Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

IntegrateAlgebraic [A] time = 0.06, size = 53, normalized size = 0.87

$$\frac{-8x^3-36x^2-44x-13}{4\left(2x^2+6x+3\right)^2}+\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}+\sqrt{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] (-13 - 44*x - 36*x^2 - 8*x^3)/(4*(3 + 6*x + 2*x^2)^2) + ArcTanh[Sqrt[3] + (
2*x)/Sqrt[3]]/Sqrt[3]
```

fricas [A] time = 1.15, size = 97, normalized size = 1.59

$$-\frac{24\,x^3-2\,\sqrt{3}\left(4\,x^4+24\,x^3+48\,x^2+36\,x+9\right)\log\left(\frac{2\,x^2+\sqrt{3}\,(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+108\,x^2+132\,x+39}{12\left(4\,x^4+24\,x^3+48\,x^2+36\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(24*x^3 - 2*sqrt(3)*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*x^2 +
 sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/(4
*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

giac [A] time = 1.16, size = 61, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|4\,x-2\,\sqrt{3}+6\right|}{\left|4\,x+2\,\sqrt{3}+6\right|}\right)-\frac{8\,x^3+36\,x^2+44\,x+13}{4\left(2\,x^2+6\,x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) - 1/4*(
8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

maple [A] time = 0.30, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{-24x-30}{24(2x^2+6x+3)^2}-\frac{4x+6}{4(2x^2+6x+3)}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{3}$	56
risch	$\frac{-2x^3-9x^2-11x-\frac{13}{4}}{(2x^2+6x+3)^2}+\frac{\ln(3+2x+\sqrt{3})\sqrt{3}}{6}-\frac{\ln(3+2x-\sqrt{3})\sqrt{3}}{6}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/(2*x^2+6*x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(4*x+6)/(2*x^2+6*x+3)+1/3*3^(1/2)*arct
anh(1/6*(4*x+6)*3^(1/2))
```

maxima [A] time = 1.26, size = 67, normalized size = 1.10

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right)-\frac{8x^3+36x^2+44x+13}{4\left(4x^4+24x^3+48x^2+36x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(8*x^3 + 36
*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

mupad [B] time = 0.21, size = 53, normalized size = 0.87

$$\frac{\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\left(\frac{2x}{3}+1\right)\right)}{3}-\frac{\frac{x^3}{2}+\frac{9x^2}{4}+\frac{11x}{4}+\frac{13}{16}}{x^4+6x^3+12x^2+9x+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)/(6*x + 2*x^2 + 3)^3,x)
```

```
[Out] (3^(1/2)*atanh(3^(1/2)*((2*x)/3 + 1)))/3 - ((11*x)/4 + (9*x^2)/4 + x^3/2 +
13/16)/(9*x + 12*x^2 + 6*x^3 + x^4 + 9/4)
```

sympy [A] time = 0.17, size = 76, normalized size = 1.25

$$\frac{-8x^3-36x^2-44x-13}{16x^4+96x^3+192x^2+144x+36}-\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

```
[Out] (-8*x**3 - 36*x**2 - 44*x - 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36)
 - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/6
```

3.201 $\int \frac{-1+x}{(4+5x+x^2)^2} \, dx$

Optimal. Leaf size=36

$$\frac{7x+13}{9(x^2+5x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {638, 616, 31}

$$\frac{7x+13}{9(x^2+5x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{(4+5x+x^2)^2} \, dx &= \frac{13+7x}{9(4+5x+x^2)}+\frac{7}{9}\int \frac{1}{4+5x+x^2} \, dx \\
&= \frac{13+7x}{9(4+5x+x^2)}+\frac{7}{27}\int \frac{1}{1+x} \, dx-\frac{7}{27}\int \frac{1}{4+x} \, dx \\
&= \frac{13+7x}{9(4+5x+x^2)}+\frac{7}{27}\log(1+x)-\frac{7}{27}\log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.92

$$\frac{1}{27}\left(\frac{21x+39}{x^2+5x+4}+7\log(x+1)-7\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] ((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

IntegrateAlgebraic [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{7x+13}{9\left(x^2+5x+4\right)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

fricas [A] time = 1.13, size = 45, normalized size = 1.25

$$-\frac{7\left(x^2+5\,x+4\right)\log\left(x+4\right)-7\left(x^2+5\,x+4\right)\log\left(x+1\right)-21\,x-39}{27\left(x^2+5\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1) - 21*x -
 39)/(x^2 + 5*x + 4)
```

giac [A] time = 0.98, size = 32, normalized size = 0.89

$$\frac{7\,x+13}{9\left(x^2+5\,x+4\right)}-\frac{7}{27}\,\log\left(|x+4|\right)+\frac{7}{27}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="giac")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(abs(x + 4)) + 7/27*log(abs(x + 1)
)
```

maple [A] time = 0.29, size = 28, normalized size = 0.78

method	result	size
default	$\frac{5}{9(4+x)}-\frac{7\ln(4+x)}{27}+\frac{2}{9(1+x)}+\frac{7\ln(1+x)}{27}$	28
norman	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30
risch	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2+5*x+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/9/(4+x)-7/27*ln(4+x)+2/9/(1+x)+7/27*ln(1+x)
```

maxima [A] time = 0.65, size = 30, normalized size = 0.83

$$\frac{7\,x+13}{9\left(x^2+5\,x+4\right)}-\frac{7}{27}\,\log\left(x+4\right)+\frac{7}{27}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.69

$$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}-\frac{14\,\mathrm{atanh}\left(\frac{2x}{3}+\frac{5}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(5*x + x^2 + 4)^2,x)
```

```
[Out] ((7*x)/9 + 13/9)/(5*x + x^2 + 4) - (14*atanh((2*x)/3 + 5/3))/27
```

sympy [A] time = 0.13, size = 31, normalized size = 0.86

$$\frac{7x+13}{9x^2+45x+36}+\frac{7\log\left(x+1\right)}{27}-\frac{7\log\left(x+4\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2+5*x+4)**2,x)
```

```
[Out] (7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/27
```

3.202 $\int \frac{1}{(2+3x+x^2)^5} \, dx$

Optimal. Leaf size=87

$$\frac{-2x-3}{4\left(x^2+3x+2\right)^4}+\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}+70\log(x+1)-70\log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {614, 616, 31}

$$\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] -(3 + 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(2+3x+x^2\right)^5}\,dx = -\frac{3+2x}{4\left(2+3x+x^2\right)^4} - \frac{7}{2}\int \frac{1}{\left(2+3x+x^2\right)^4}\,dx$$
$$= -\frac{3+2x}{4\left(2+3x+x^2\right)^4} + \frac{7(3+2x)}{6\left(2+3x+x^2\right)^3} + \frac{35}{3}\int \frac{1}{\left(2+3x+x^2\right)^3}\,dx$$
$$= -\frac{3+2x}{4\left(2+3x+x^2\right)^4} + \frac{7(3+2x)}{6\left(2+3x+x^2\right)^3} - \frac{35(3+2x)}{6\left(2+3x+x^2\right)^2} - 35\int \frac{1}{\left(2+3x+x^2\right)^2}\,dx$$
$$= -\frac{3+2x}{4\left(2+3x+x^2\right)^4} + \frac{7(3+2x)}{6\left(2+3x+x^2\right)^3} - \frac{35(3+2x)}{6\left(2+3x+x^2\right)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\int \frac{1}{2+3x+}$$
$$= -\frac{3+2x}{4\left(2+3x+x^2\right)^4} + \frac{7(3+2x)}{6\left(2+3x+x^2\right)^3} - \frac{35(3+2x)}{6\left(2+3x+x^2\right)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\int \frac{1}{1+x}\,dx$$
$$= -\frac{3+2x}{4\left(2+3x+x^2\right)^4} + \frac{7(3+2x)}{6\left(2+3x+x^2\right)^3} - \frac{35(3+2x)}{6\left(2+3x+x^2\right)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x)$$

Mathematica [A] time = 0.03, size = 87, normalized size = 1.00

$$\frac{-2x-3}{4\left(x^2+3x+2\right)^4}+\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] (-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

IntegrateAlgebraic [A] time = 0.04, size = 58, normalized size = 0.67

$$\frac{(2x+3)\left(420x^6+3780x^5+13790x^4+26040x^3+26824x^2+14322x+3105\right)}{12\left(x^2+3x+2\right)^4}-140\tanh^{-1}(2x+3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] ((3 + 2*x)*(3105 + 14322*x + 26824*x^2 + 26040*x^3 + 13790*x^4 + 3780*x^5 +
 420*x^6))/(12*(2 + 3*x + x^2)^4) - 140*ArcTanh[3 + 2*x]
```

fricas [B] time = 0.74, size = 165, normalized size = 1.90

$$\frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2-840\left(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+\right.}{12\left(x^8+12\,x^7+62\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
- 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x
 + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^
3 + 248*x^2 + 96*x + 16)*log(x + 1) + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^
6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

giac [A] time = 0.90, size = 62, normalized size = 0.71

$$\frac{840\,x^7 + 8820\,x^6 + 38920\,x^5 + 93450\,x^4 + 131768\,x^3 + 109116\,x^2 + 49176\,x + 9315}{12\left(x^2 + 3\,x + 2\right)^4} - 70\,\log\left(|x + 2|\right) + 70\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x + 1
))
```

maple [A] time = 0.29, size = 60, normalized size = 0.69

method	result
norman	$\frac{4098x+\frac{9730}{3}x^5+\frac{32942}{3}x^3+70x^7+735x^6+9093x^2+\frac{15575}{2}x^4+\frac{3105}{4}}{(x^2+3x+2)^4} + 70\ln(1+x) - 70\ln(2+x)$
risch	$\frac{4098x+\frac{9730}{3}x^5+\frac{32942}{3}x^3+70x^7+735x^6+9093x^2+\frac{15575}{2}x^4+\frac{3105}{4}}{(x^2+3x+2)^4} + 70\ln(1+x) - 70\ln(2+x)$
default	$\frac{1}{4(2+x)^4} + \frac{5}{3(2+x)^3} + \frac{15}{2(2+x)^2} + \frac{35}{2+x} - 70\ln(2+x) - \frac{1}{4(1+x)^4} + \frac{5}{3(1+x)^3} - \frac{15}{2(1+x)^2} + \frac{35}{1+x} + 70\ln(1+x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (4098*x+9730/3*x^5+32942/3*x^3+70*x^7+735*x^6+9093*x^2+15575/2*x^4+3105/4)/
(x^2+3*x+2)^4+70*ln(1+x)-70*ln(2+x)
```

maxima [A] time = 0.56, size = 90, normalized size = 1.03

$$\frac{840\,x^7 + 8820\,x^6 + 38920\,x^5 + 93450\,x^4 + 131768\,x^3 + 109116\,x^2 + 49176\,x + 9315}{12\left(x^8 + 12\,x^7 + 62\,x^6 + 180\,x^5 + 321\,x^4 + 360\,x^3 + 248\,x^2 + 96\,x + 16\right)} - 70\,\log\left(x + 2\right) + 70\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x + 1)
```

mupad [B] time = 0.09, size = 65, normalized size = 0.75

$$70\,\ln\left(\frac{x+1}{x+2}\right) + 70\,\left(x + \frac{3}{2}\right)\,\left(\frac{1}{x^2 + 3\,x + 2} - \frac{1}{6\,\left(x^2 + 3\,x + 2\right)^2} + \frac{1}{30\,\left(x^2 + 3\,x + 2\right)^3} - \frac{1}{140\,\left(x^2 + 3\,x + 2\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + x^2 + 2)^5,x)
```

```
[Out] 70*log((x + 1)/(x + 2)) + 70*(x + 3/2)*(1/(3*x + x^2 + 2) - 1/(6*(3*x + x^2
 + 2)^2) + 1/(30*(3*x + x^2 + 2)^3) - 1/(140*(3*x + x^2 + 2)^4))
```

sympy [A] time = 0.22, size = 88, normalized size = 1.01

$$\frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} + 70\log\left(x + 1\right) - 70\log\left(x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+2)**5,x)
```

```
[Out] (840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**2
 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4 +
 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)
```

3.203 $\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{2-3x}{35x^2\left(2x^2-6x+7\right)}-\frac{1}{490x^2}-\frac{40\log\left(2x^2-6x+7\right)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {740, 800, 634, 618, 204, 628}

$$-\frac{2-3x}{35x^2\left(2x^2-6x+7\right)}-\frac{1}{490x^2}-\frac{40\log\left(2x^2-6x+7\right)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] -1/(490*x^2) - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2)) - (234*Ar
cTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/2401 - (40*Log[7 - 6
*x + 2*x^2])/2401
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx = -\frac{2-3x}{35x^2(7-6x+2x^2)}+\frac{1}{140}\int \frac{4+36x}{x^3(7-6x+2x^2)}\,dx$$

$$= -\frac{2-3x}{35x^2(7-6x+2x^2)}+\frac{1}{140}\int \left(\frac{4}{7x^3}+\frac{276}{49x^2}+\frac{1600}{343x}-\frac{8(-717+400x)}{343(7-6x+2x^2)}\right)dx$$

$$= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)}+\frac{80\log(x)}{2401}-\frac{2\int \frac{-717+400x}{7-6x+2x^2}\,dx}{12005}$$

$$= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)}+\frac{80\log(x)}{2401}-\frac{40\int \frac{-6+4x}{7-6x+2x^2}\,dx}{2401}+\frac{234\int \frac{}{7-6x}}{1200}$$

$$= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)}+\frac{80\log(x)}{2401}-\frac{40\log(7-6x+2x^2)}{2401}-\frac{468\,\mathrm{S}}{}$$

$$= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}+\frac{80\log(x)}{2401}-\frac{40\log(7}{24}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.86

$$\frac{-\frac{140(9x-41)}{2x^2-6x+7}-\frac{1225}{x^2}-2000\log\left(2x^2-6x+7\right)-\frac{4200}{x}+4000\log(x)+468\sqrt{5}\tan^{-1}\left(\frac{2x-3}{\sqrt{5}}\right)}{120050}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] (-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*Sqrt[5]*Arc
Tan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050
```

IntegrateAlgebraic [A] time = 0.05, size = 82, normalized size = 1.01

$$-\frac{40\log\left(2x^2-6x+7\right)}{2401}+\frac{-276x^3+814x^2-630x-245}{3430x^2\left(2x^2-6x+7\right)}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3}{\sqrt{5}}-\frac{2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] (-245 - 630*x + 814*x^2 - 276*x^3)/(3430*x^2*(7 - 6*x + 2*x^2)) - (234*ArcT
an[3/Sqrt[5] - (2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/2401 - (40*Log
[7 - 6*x + 2*x^2])/2401
```

fricas [A] time = 0.92, size = 116, normalized size = 1.43

$$-\frac{9660\,x^3-468\,\sqrt{5}\left(2\,x^4-6\,x^3+7\,x^2\right)\arctan\left(\frac{1}{5}\,\sqrt{5}\,(2\,x-3)\right)-28490\,x^2+2000\left(2\,x^4-6\,x^3+7\,x^2\right)\log\left(2\,x^2\right.}{120050\left(2\,x^4-6\,x^3+7\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")
```

```
[Out] -1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(5
)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x + 7
) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3 +
7*x^2)
```

giac [A] time = 1.16, size = 67, normalized size = 0.83

$$\frac{234}{60025}\,\sqrt{5}\,\arctan\left(\frac{1}{5}\,\sqrt{5}\,(2\,x-3)\right)-\frac{276\,x^3-814\,x^2+630\,x+245}{3430\left(2\,x^2-6\,x+7\right)x^2}-\frac{40}{2401}\,\log\left(2\,x^2-6\,x+7\right)+\frac{80}{2401}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="giac")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(abs(x))
```

maple [A] time = 0.43, size = 62, normalized size = 0.77

method	result	size
default	$-\frac{4\left(\frac{63x}{20}-\frac{287}{20}\right)}{2401\left(x^2-3x+\frac{7}{2}\right)}-\frac{40\ln\left(2x^2-6x+7\right)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(4x-6)\sqrt{5}}{10}\right)}{60025}-\frac{1}{98x^2}-\frac{12}{343x}+\frac{80\ln(x)}{2401}$	62
risch	$\frac{-\frac{138}{1715}x^3+\frac{407}{1715}x^2-\frac{9}{49}x-\frac{1}{14}}{x^2\left(2x^2-6x+7\right)}-\frac{40\ln\left(4x^2-12x+14\right)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(-3+2x)\sqrt{5}}{5}\right)}{60025}+\frac{80\ln(x)}{2401}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(2*x^2-6*x+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/2401*ln(2*x^2-6*x+7)+234/60025*5^
(1/2)*arctan(1/10*(4*x-6)*5^(1/2))-1/98/x^2-12/343/x+80/2401*ln(x)
```

maxima [A] time = 1.31, size = 69, normalized size = 0.85

$$\frac{234}{60025}\,\sqrt{5}\,\arctan\left(\frac{1}{5}\,\sqrt{5}\,(2\,x-3)\right)-\frac{276\,x^3-814\,x^2+630\,x+245}{3430\left(2\,x^4-6\,x^3+7\,x^2\right)}-\frac{40}{2401}\,\log\left(2\,x^2-6\,x+7\right)+\frac{80}{2401}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(x)
```

mupad [B] time = 0.11, size = 77, normalized size = 0.95

$$\frac{80\,\ln(x)}{2401}-\frac{\frac{69\,x^3}{1715}-\frac{407\,x^2}{3430}+\frac{9\,x}{98}+\frac{1}{28}}{x^4-3\,x^3+\frac{7\,x^2}{2}}-\ln\left(x-\frac{3}{2}-\frac{\sqrt{5}\,1i}{2}\right)\left(\frac{40}{2401}+\frac{\sqrt{5}\,117i}{60025}\right)+\ln\left(x-\frac{3}{2}+\frac{\sqrt{5}\,1i}{2}\right)\left(-\frac{40}{2401}+\frac{\sqrt{5}}{60\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(2*x^2 - 6*x + 7)^2),x)
```

```
[Out] (80*log(x))/2401 - ((9*x)/98 - (407*x^2)/3430 + (69*x^3)/1715 + 1/28)/((7*x
^2)/2 - 3*x^3 + x^4) - log(x - (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025
+ 40/2401) + log(x + (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 - 40/2401)
```

sympy [**A**] time = 0.23, size = 80, normalized size = 0.99

$$\frac{80\log(x)}{2401} - \frac{40\log\left(x^2 - 3x + \frac{7}{2}\right)}{2401} + \frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x}{5} - \frac{3\sqrt{5}}{5}\right)}{60025} + \frac{-276x^3 + 814x^2 - 630x - 245}{6860x^4 - 20580x^3 + 24010x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

```
[Out] 80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5)
*x/5 - 3*sqrt(5)/5)/60025 + (-276*x**3 + 814*x**2 - 630*x - 245)/(6860*x**4
 - 20580*x**3 + 24010*x**2)
```

3.204 $\int \frac{x^9}{\left(2+3x+x^2\right)^5} \, dx$

Optimal. Leaf size=104

$$-\frac{(1593x+2206)x^2}{2\left(x^2+3x+2\right)}+\frac{(3x+4)x^8}{4\left(x^2+3x+2\right)^4}-\frac{(81x+110)x^6}{12\left(x^2+3x+2\right)^3}+\frac{(135x+184)x^4}{2\left(x^2+3x+2\right)^2}+735x-1471\log(x+1)+1472\log($$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {738, 818, 773, 632, 31}

$$\frac{(3x+4)x^8}{4\left(x^2+3x+2\right)^4}-\frac{(81x+110)x^6}{12\left(x^2+3x+2\right)^3}+\frac{(135x+184)x^4}{2\left(x^2+3x+2\right)^2}-\frac{(1593x+2206)x^2}{2\left(x^2+3x+2\right)}+735x-1471\log(x+1)+1472\log(x$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x))/(12*(2 +
 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)^2) - (x^2*(2206 + 1
593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(2+3x+x^2\right)^5}\,dx &= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{1}{4}\int \frac{x^7(32+3x)}{\left(2+3x+x^2\right)^4}\,dx\\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}-\frac{1}{12}\int \frac{(-660-72x)x^5}{\left(2+3x+x^2\right)^3}\,dx\\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{1}{24}\int \frac{x^3(8832+1476x)}{\left(2+3x+x^2\right)^2}\,dx\\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-\frac{1}{24}\int \frac{(-\\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-\frac{}{2}\\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-1\\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-1
\end{aligned}$$

Mathematica [A] time = 0.03, size = 87, normalized size = 0.84

$$\frac{3(456x+451)}{4\left(x^2+3x+2\right)^2}-\frac{2(729x+1114)}{x^2+3x+2}+\frac{1998x+415}{12\left(x^2+3x+2\right)^3}+\frac{513x+514}{4\left(x^2+3x+2\right)^4}-1471\log(x+1)+1472\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] (514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3)
 + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x +
x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

IntegrateAlgebraic [A] time = 0.04, size = 62, normalized size = 0.60

$$\frac{-17496x^7-184200x^6-813888x^5-1955853x^4-2759400x^3-2286008x^2-1030560x-195280}{12\left(x^2+3x+2\right)^4}-1471\log(x+1$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] (-195280 - 1030560*x - 2286008*x^2 - 2759400*x^3 - 1955853*x^4 - 813888*x^5
 - 184200*x^6 - 17496*x^7)/(12*(2 + 3*x + x^2)^4) - 1471*Log[1 + x] + 1472*
Log[2 + x]
```

fricas [A] time = 1.23, size = 165, normalized size = 1.59

$$-\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2-17664\left(x^8+12\,x^7+62\,x^6+\right.}{12\,(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 32
1*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)/(x^
8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

giac [A] time = 0.98, size = 62, normalized size = 0.60

$$-\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\,(x+2)^4(x+1)^4}+1472\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x + 2)
) - 1471*log(abs(x + 1))
```

maple [A] time = 0.29, size = 60, normalized size = 0.58

method	result
norman	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{\left(x^2+3x+2\right)^4}-1471\ln(1+x)+1472\ln(2+x)$
risch	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{\left(x^2+3x+2\right)^4}-1471\ln(1+x)+1472\ln(2+x)$
default	$-\frac{128}{(2+x)^4}-\frac{256}{3(2+x)^3}-\frac{384}{(2+x)^2}-\frac{1024}{2+x}+1472\ln(2+x)+\frac{1}{4(1+x)^4}-\frac{14}{3(1+x)^3}+\frac{48}{(1+x)^2}-\frac{434}{1+x}-1471\ln(1+x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-229950*x^3-85880*x-67824*x^5-15350*x^6-1458*x^7-651951/4*x^4-571502/3*x^2
-48820/3)/(x^2+3*x+2)^4-1471*ln(1+x)+1472*ln(2+x)
```

maxima [A] time = 0.55, size = 90, normalized size = 0.87

$$-\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\left(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16\right)}+1472\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4
+ 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)
```

mupad [B] time = 0.21, size = 90, normalized size = 0.87

$$1472\,\ln(x+2)-1471\,\ln(x+1)-\frac{1458\,x^7+15350\,x^6+67824\,x^5+\frac{651951\,x^4}{4}+229950\,x^3+\frac{571502\,x^2}{3}+85880\,x+\cdots}{x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x + x^2 + 2)^5,x)
```

```
[Out] 1472*log(x + 2) - 1471*log(x + 1) - (85880*x + (571502*x^2)/3 + 229950*x^3
+ (651951*x^4)/4 + 67824*x^5 + 15350*x^6 + 1458*x^7 + 48820/3)/(96*x + 248*
x^2 + 360*x^3 + 321*x^4 + 180*x^5 + 62*x^6 + 12*x^7 + x^8 + 16)
```

sympy [A] time = 0.21, size = 90, normalized size = 0.87

$$\frac{-17496x^7-184200x^6-813888x^5-1955853x^4-2759400x^3-2286008x^2-1030560x-195280}{12x^8+144x^7+744x^6+2160x^5+3852x^4+4320x^3+2976x^2+1152x+192}-1471\log(x+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**2+3*x+2)**5,x)
```

```
[Out] (-17496*x**7 - 184200*x**6 - 813888*x**5 - 1955853*x**4 - 2759400*x**3 - 22
86008*x**2 - 1030560*x - 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5
 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) + 14
72*log(x + 2)
```

3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$

Optimal. Leaf size=102

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{62x+73}{3\left(2x^2+5x+3\right)^3}+\frac{(2x+1)(6x+7)}{4\left(2x^2+5x+3\right)^4}+2480\log(x+1)-2480\log(2x+3)$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {738, 638, 614, 616, 31}

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{62x+73}{3\left(2x^2+5x+3\right)^3}+\frac{(2x+1)(6x+7)}{4\left(2x^2+5x+3\right)^4}+2480\log(x+1)-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3 + 5*x + 2
*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5
*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 + 2*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2}{\left(3+5x+2x^2\right)^5}\,dx &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}-\frac{1}{4}\int \frac{-28-72x}{\left(3+5x+2x^2\right)^4}\,dx\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}+\frac{310}{3}\int \frac{1}{\left(3+5x+2x^2\right)^3}\,dx\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}-620\int \frac{1}{\left(3+5x+2x^2\right)^2}\,dx\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+2480\int \frac{}{3}\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+4960\int \frac{}{2}\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+2480\log(\end{aligned}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.97

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{31(4x+5)}{6\left(2x^2+5x+3\right)^3}-\frac{10x+11}{4\left(2x^2+5x+3\right)^4}+2480\log(2(x+1))-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] -1/4*(11 + 10*x)/(3 + 5*x + 2*x^2)^4 + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)^
3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x + 2
*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*x]
```

IntegrateAlgebraic [A] time = 0.07, size = 66, normalized size = 0.65

$$\frac{238080x^7+2083200x^6+7757440x^5+15934000x^4+19495776x^3+14209160x^2+5712464x+977397}{12\left(2x^2+5x+3\right)^4}+2480\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] (977397 + 5712464*x + 14209160*x^2 + 19495776*x^3 + 15934000*x^4 + 7757440*
x^5 + 2083200*x^6 + 238080*x^7)/(12*(3 + 5*x + 2*x^2)^4) + 2480*Log[1 + x]
- 2480*Log[3 + 2*x]
```

fricas [A] time = 0.89, size = 173, normalized size = 1.70

$$\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2-29760\left(16\,x^8+160\,x^7+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7 +
696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x + 1
) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
 2580*x^3 + 1566*x^2 + 540*x + 81)
```

giac [A] time = 1.07, size = 66, normalized size = 0.65

$$\frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 + 5712464\,x + 977397}{12\left(2\,x^2 + 5\,x + 3\right)^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="giac")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2*x
 + 3)) + 2480*log(abs(x + 1))
```

maple [A] time = 0.30, size = 64, normalized size = 0.63

method	result
norman	$\frac{173600x^6+\frac{1428116}{3}x+19840x^7+\frac{1939360}{3}x^5+1624648x^3+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{\left(2x^2+5x+3\right)^4} + 2480\ln(1+x) - 2480\ln(3+$
risch	$\frac{173600x^6+\frac{1428116}{3}x+19840x^7+\frac{1939360}{3}x^5+1624648x^3+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{\left(2x^2+5x+3\right)^4} + 2480\ln(1+x) - 2480\ln(3+$
default	$\frac{16}{(3+2x)^4} + \frac{256}{3(3+2x)^3} + \frac{328}{(3+2x)^2} + \frac{1360}{3+2x} - 2480\ln(3+2x) - \frac{1}{4(1+x)^4} + \frac{14}{3(1+x)^3} - \frac{52}{(1+x)^2} + \frac{560}{1+x} + 2480\ln(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2/(2*x^2+5*x+3)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (173600*x^6+1428116/3*x+19840*x^7+1939360/3*x^5+1624648*x^3+3552290/3*x^2+3
983500/3*x^4+325799/4)/(2*x^2+5*x+3)^4+2480*ln(1+x)-2480*ln(3+2*x)
```

maxima [A] time = 0.62, size = 94, normalized size = 0.92

$$\frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 + 5712464\,x + 977397}{12\left(16\,x^8 + 160\,x^7 + 696\,x^6 + 1720\,x^5 + 2641\,x^4 + 2580\,x^3 + 1566\,x^2 + 540\,x + 81\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5
 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*log(2*x + 3) + 2480*
log(x + 1)
```

mupad [B] time = 0.21, size = 85, normalized size = 0.83

$$\frac{1240\,x^7 + 10850\,x^6 + \frac{121210\,x^5}{3} + \frac{995875\,x^4}{12} + \frac{203081\,x^3}{2} + \frac{1776145\,x^2}{24} + \frac{357029\,x}{12} + \frac{325799}{64}}{x^8 + 10\,x^7 + \frac{87\,x^6}{2} + \frac{215\,x^5}{2} + \frac{2641\,x^4}{16} + \frac{645\,x^3}{4} + \frac{783\,x^2}{8} + \frac{135\,x}{4} + \frac{81}{16}} - 4960\,\mathrm{atanh}\left(4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2/(5*x + 2*x^2 + 3)^5,x)
```

```
[Out] ((357029*x)/12 + (1776145*x^2)/24 + (203081*x^3)/2 + (995875*x^4)/12 + (121
210*x^5)/3 + 10850*x^6 + 1240*x^7 + 325799/64)/((135*x)/4 + (783*x^2)/8 + (
645*x^3)/4 + (2641*x^4)/16 + (215*x^5)/2 + (87*x^6)/2 + 10*x^7 + x^8 + 81/1
6) - 4960*atanh(4*x + 5)
```

sympy [A] time = 0.23, size = 90, normalized size = 0.88

$$\frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{192x^8 + 1920x^7 + 8352x^6 + 20640x^5 + 31692x^4 + 30960x^3 + 18792x^2 + 6480x + 972} + 2480\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

```
[Out] (238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3
+ 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 + 2
0640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*log
(x + 1) - 2480*log(x + 3/2)
```

3.206 $\int \frac{(a-bx^2)^3}{x^7} \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^2)^3}{x^7} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a-bx)^3}{x^4} \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}-\frac{b^3}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 37, normalized size = 0.92

$$b^3(-\log(x))-\frac{a\left(2a^2-9abx^2+18b^2x^4\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^2)^3/x^7,x]
```

```
[Out] -1/12*(a*(2*a^2 - 9*a*b*x^2 + 18*b^2*x^4))/x^6 - b^3*Log[x]
```

fricas [A] time = 1.22, size = 39, normalized size = 0.98

$$-\frac{12\,b^3x^6\log(x)+18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

giac [A] time = 0.97, size = 47, normalized size = 1.18

$$-\frac{1}{2}\,b^3\log\left(x^2\right)+\frac{11\,b^3x^6-18\,ab^2x^4+9\,a^2bx^2-2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)/
x^6
```

maple [A] time = 0.26, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}-b^3\ln(x)$	35
norman	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln(x)$	37
risch	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln(x)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

maxima [A] time = 0.51, size = 39, normalized size = 0.98

$$-\frac{1}{2}\,b^3\log\left(x^2\right)-\frac{18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

mupad [B] time = 0.20, size = 37, normalized size = 0.92

$$-b^3\,\ln(x)-\frac{\frac{a^3}{6}-\frac{3\,a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^3/x^7,x)
```

```
[Out] - b^3*log(x) - (a^3/6 - (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

sympy [A] time = 0.25, size = 37, normalized size = 0.92

$$-b^3 \log(x) - \frac{2a^3 - 9a^2bx^2 + 18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**3/x**7,x)
[Out] -b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

3.207 $\int \frac{x^{13}}{\left(a^4+x^4\right)^5} \, dx$

Optimal. Leaf size=83

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 288, 199, 203}

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^4 + x^4)^5,x]
```

```
[Out] -x^10/(16*(a^4 + x^4)^4) - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(128*(a^4 +
 x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^2/a^2])/(256*a^6)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a^4+x^4\right)^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a^4+x^2\right)^5}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} + \frac{5}{16}\,\text{Subst}\left(\int \frac{x^4}{\left(a^4+x^2\right)^4}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} + \frac{5}{32}\,\text{Subst}\left(\int \frac{x^2}{\left(a^4+x^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5}{128}\,\text{Subst}\left(\int \frac{1}{\left(a^4+x^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5x^2}{256a^4\left(a^4+x^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx,\right.}{256a^4}$$

$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5x^2}{256a^4\left(a^4+x^4\right)} + \frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.75

$$\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right) - \frac{a^2x^2\left(15a^{12}+55a^8x^4+73a^4x^8-15x^{12}\right)}{\left(a^4+x^4\right)^4}}{768a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^4 + x^4)^5,x]
```

```
[Out] (-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4) +
 15*ArcTan[x^2/a^2])/(768*a^6)
```

IntegrateAlgebraic [A] time = 0.03, size = 62, normalized size = 0.75

$$\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6} - \frac{x^2\left(15a^{12}+55a^8x^4+73a^4x^8-15x^{12}\right)}{768a^4\left(a^4+x^4\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13/(a^4 + x^4)^5,x]
```

```
[Out] -1/768*(x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4*(a^4 + x^4)
^4) + (5*ArcTan[x^2/a^2])/(256*a^6)
```

fricas [A] time = 0.77, size = 113, normalized size = 1.36

$$-\frac{15\,a^{14}x^2 + 55\,a^{10}x^6 + 73\,a^6x^{10} - 15\,a^2x^{14} - 15\left(a^{16} + 4\,a^{12}x^4 + 6\,a^8x^8 + 4\,a^4x^{12} + x^{16}\right)\arctan\left(\frac{x^2}{a^2}\right)}{768\left(a^{22} + 4\,a^{18}x^4 + 6\,a^{14}x^8 + 4\,a^{10}x^{12} + a^6x^{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")
```

```
[Out] -1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 15*(a^16 +
4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/a^2))/(a^22 + 4*a^18
*x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)
```

giac [A] time = 0.96, size = 58, normalized size = 0.70

$$\frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{15\,a^{12}x^2 + 55\,a^8x^6 + 73\,a^4x^{10} - 15\,x^{14}}{768\left(a^4 + x^4\right)^4 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="giac")
```

```
[Out] 5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 -
 15*x^14)/((a^4 + x^4)^4*a^4)
```

maple [A] time = 0.29, size = 55, normalized size = 0.66

method	result	size
risch	$\frac{-\frac{5a^8x^2}{256}-\frac{55a^4x^6}{768}-\frac{73x^{10}}{768}+\frac{5x^{14}}{256a^4}}{\left(a^4+x^4\right)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$	55
default	$\frac{\frac{5x^{14}}{128a^4}-\frac{73x^{10}}{384}-\frac{55a^4x^6}{384}-\frac{5a^8x^2}{128}}{2\left(a^4+x^4\right)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^4+x^4)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/256*a^8*x^2-55/768*a^4*x^6-73/768*x^10+5/256/a^4*x^14)/(a^4+x^4)^4+5/25
6*arctan(x^2/a^2)/a^6
```

maxima [A] time = 1.11, size = 83, normalized size = 1.00

$$-\frac{15\,a^{12}x^2 + 55\,a^8x^6 + 73\,a^4x^{10} - 15\,x^{14}}{768\left(a^{20} + 4\,a^{16}x^4 + 6\,a^{12}x^8 + 4\,a^8x^{12} + a^4x^{16}\right)} + \frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")
```

```
[Out] -1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 + 4*a^16*x^
4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(x^2/a^2)/a^6
```

mupad [B] time = 0.15, size = 79, normalized size = 0.95

$$\frac{5\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{\frac{73\,x^{10}}{768} + \frac{55\,a^4\,x^6}{768} + \frac{5\,a^8\,x^2}{256} - \frac{5\,x^{14}}{256\,a^4}}{a^{16} + 4\,a^{12}\,x^4 + 6\,a^8\,x^8 + 4\,a^4\,x^{12} + x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^4 + x^4)^5,x)
```

```
[Out] (5*atan(x^2/a^2))/(256*a^6) - ((73*x^10)/768 + (55*a^4*x^6)/768 + (5*a^8*x^
2)/256 - (5*x^14)/(256*a^4))/(a^16 + x^16 + 4*a^4*x^12 + 6*a^8*x^8 + 4*a^12
*x^4)
```

sympy [C] time = 0.69, size = 102, normalized size = 1.23

$$\frac{-15a^{12}x^2-55a^8x^6-73a^4x^{10}+15x^{14}}{768a^{20}+3072a^{16}x^4+4608a^{12}x^8+3072a^8x^{12}+768a^4x^{16}}+\frac{-\frac{5i\log\left(-ia^2+x^2\right)}{512}+\frac{5i\log\left(ia^2+x^2\right)}{512}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(a**4+x**4)**5,x)
```

```
[Out] (-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a**20 + 307
2*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*a**4*x**16) + (-5*I*
log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x**2)/512)/a**6
```

3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2} \left(1+x^2\right) dx$

Optimal. Leaf size=49

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 1820, 266, 43}

$$\frac{2x^{13/2}}{13}-\frac{2x^6}{3}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}-x^4+\frac{8x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right) dx &= \int \left(2-\sqrt{x}\right)^2 x^{5/2}\left(1+x^2\right) dx\\ &= \int \left(\left(-2+\sqrt{x}\right)^2 x^{5/2}+\left(-2+\sqrt{x}\right)^2 x^{9/2}\right) dx\\ &= \int \left(-2+\sqrt{x}\right)^2 x^{5/2} dx+\int \left(-2+\sqrt{x}\right)^2 x^{9/2} dx\\ &= 2\,\text{Subst}\left(\int (-2+x)^2x^6\,dx, x, \sqrt{x}\right)+2\,\text{Subst}\left(\int (-2+x)^2x^{10}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(4x^6-4x^7+x^8\right) dx, x, \sqrt{x}\right)+2\,\text{Subst}\left(\int \left(4x^{10}-4x^{11}+x^{12}\right) d\right.\\ &= \frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

IntegrateAlgebraic [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

fricas [A] time = 1.63, size = 37, normalized size = 0.76

$$-\frac{2}{3}\,x^6-x^4+\frac{2}{9009}\left(693\,x^6+3276\,x^5+1001\,x^4+5148\,x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*sqrt(x)
```

giac [A] time = 0.99, size = 31, normalized size = 0.63

$$\frac{2}{13}\,x^{\frac{13}{2}}-\frac{2}{3}\,x^6+\frac{8}{11}\,x^{\frac{11}{2}}+\frac{2}{9}\,x^{\frac{9}{2}}-x^4+\frac{8}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

maple [A] time = 0.29, size = 32, normalized size = 0.65

method	result	size
derivativedivides	$\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
default	$\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
trager	$-\frac{(2x^5+2x^4+5x^3+5x^2+5x+5)(-1+x)}{3}+\frac{2x^{\frac{7}{2}}(693x^3+3276x^2+1001x+5148)}{9009}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

maxima [A] time = 0.53, size = 31, normalized size = 0.63

$$\frac{2}{13}x^{\frac{13}{2}}-\frac{2}{3}x^6+\frac{8}{11}x^{\frac{11}{2}}+\frac{2}{9}x^{\frac{9}{2}}-x^4+\frac{8}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

mupad [B] time = 0.19, size = 31, normalized size = 0.63

$$\frac{8\,x^{7/2}}{7}-\frac{2\,x^6}{3}-x^4+\frac{2\,x^{9/2}}{9}+\frac{8\,x^{11/2}}{11}+\frac{2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x - 2*x^(1/2))^2*(x^2 + 1),x)
```

```
[Out] (8*x^(7/2))/7 - (2*x^6)/3 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 + (2*x^(1
3/2))/13
```

sympy [A] time = 2.60, size = 42, normalized size = 0.86

$$\frac{2x^{\frac{13}{2}}}{13}+\frac{8x^{\frac{11}{2}}}{11}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{7}{2}}}{7}-\frac{2x^6}{3}-x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

```
[Out] 2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2*x**6/3 -
x**4
```

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx$

Optimal. Leaf size=55

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Rubi [A] time = 0.24, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 1584, 1820}

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x^(3/5) + x^(3/2))^2*(-x^(2/3)/3 + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx &= \int \left(-3+x^{9/10}\right)^2 x^{6/5} \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx\\
&= \int \left(-\frac{1}{3}+4x^{5/6}\right) \left(-3+x^{9/10}\right)^2 x^{28/15}\, dx\\
&= 30\,\text{Subst}\left(\int x^{85} \left(-\frac{1}{3}+4x^{25}\right) \left(-3+x^{27}\right)^2 dx, x, \sqrt[30]{x}\right)\\
&= 30\,\text{Subst}\left(\int \left(-3x^{85}+36x^{110}+2x^{112}-24x^{137}-\frac{x^{139}}{3}+4x^{164}\right) dx, x\right.\\
&= -\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.00

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

IntegrateAlgebraic [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

fricas [A] time = 1.18, size = 31, normalized size = 0.56

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fri
cas")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

giac [A] time = 1.05, size = 31, normalized size = 0.56

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="gia
c")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

maple [A] time = 0.29, size = 32, normalized size = 0.58

method	result	size
derivativedivides	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32
default	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x,method=_RETURNVERBOSE
)
```

```
[Out] -45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x^
(14/3)+8/11*x^(11/2)
```

maxima [A] time = 0.50, size = 31, normalized size = 0.56

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="max
ima")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.56

$$\frac{8\,x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120\,x^{23/5}}{23}+\frac{360\,x^{37/10}}{37}-\frac{45\,x^{43/15}}{43}+\frac{60\,x^{113/30}}{113}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(3/2) - 3*x^(3/5))^2*(x^(2/3)/3 - 4*x^(3/2)),x)
```

```
[Out] (8*x^(11/2))/11 - x^(14/3)/14 - (120*x^(23/5))/23 + (360*x^(37/10))/37 - (4
5*x^(43/15))/43 + (60*x^(113/30))/113
```

sympy [A] time = 2.75, size = 48, normalized size = 0.87

$$\frac{60x^{\frac{113}{30}}}{113}-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

```
[Out] 60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x**(23/5)/2
3 - x**(14/3)/14 + 8*x**(11/2)/11
```

3.210 $\int \frac{1}{1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=22

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 190, 43}

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x])^(-1),x]
```

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx, x, 1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \sqrt{1+x}\right)\\
&= 2\sqrt{1+x}-2\log\left(1+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x])^(-1),x]
```

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[1 + x])^(-1),x]
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

fricas [A] time = 1.11, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

giac [A] time = 0.84, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

maple [A] time = 0.08, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{1+x}\right)+2\sqrt{1+x}$	19
trager	$2\sqrt{1+x}-\ln\left(2\sqrt{1+x}+2+x\right)$	22
default	$2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln(x)$	31
meijerg	$\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{1+x}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{2\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

maxima [A] time = 0.72, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

mupad [B] time = 0.13, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\ln\left(\sqrt{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 1),x)
```

```
[Out] 2*(x + 1)^(1/2) - 2*log((x + 1)^(1/2) + 1)
```

sympy [A] time = 0.14, size = 19, normalized size = 0.86

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.211 $\int \frac{x}{1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}(x+1)^{3/2}-x$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {371}

$$\frac{2}{3}(x+1)^{3/2}-x$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[1 + x]),x]
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \left(-1+\sqrt{x}\right)\,dx,x,1+x\right)\\ &= -x+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.27

$$2\left(\frac{1}{3}(x+1)^{3/2}-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[1 + x]),x]
[Out] 2*(-1/2*x + (1 + x)^(3/2)/3)
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}-x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + Sqrt[1 + x]),x]
[Out] -x + (2*(1 + x)^(3/2))/3
```

fricas [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - x
```

giac [A] time = 0.92, size = 12, normalized size = 0.80

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

maple [A] time = 0.02, size = 13, normalized size = 0.87

method	result	size
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
trager	$-x+\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1+x}$	16
meijerg	$\frac{-\frac{\sqrt{\pi}\,(12x+8)}{6}+\frac{\sqrt{\pi}\,(8+8x)\sqrt{1+x}}{6}}{2\sqrt{\pi}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+x)^(3/2)-1-x
```

maxima [A] time = 0.52, size = 12, normalized size = 0.80

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

mupad [B] time = 0.03, size = 11, normalized size = 0.73

$$\frac{2\,(x+1)^{3/2}}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(1/2) + 1),x)
[Out] (2*(x + 1)^(3/2))/3 - x
```

sympy [B] time = 0.97, size = 22, normalized size = 1.47

$$\frac{2x\sqrt{x+1}}{3}-x+\frac{2\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)**(1/2)),x)
[Out] 2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} \, dx$

Optimal. Leaf size=25

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {431, 376, 77}

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} \, dx &= \text{Subst}\left(\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx, x, 1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x} \, dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right) \, dx, x, \sqrt{1+x}\right)\\
&= x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.96

$$x+4\left(\sqrt{x+1}+\log\left(1-\sqrt{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
[Out] x + 4*(Sqrt[1 + x] + Log[1 - Sqrt[1 + x]])
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 0.92

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
[Out] x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]
```

fricas [A] time = 0.91, size = 19, normalized size = 0.76

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

giac [A] time = 0.87, size = 21, normalized size = 0.84

$$x+4\sqrt{x+1}+4\log\left(\left|\sqrt{x+1}-1\right|\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x + 1) + 4*log(abs(sqrt(x + 1) - 1)) + 1
```

maple [A] time = 0.27, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
default	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
trager	$-1+x+4\sqrt{1+x}+2\ln\left(2\sqrt{1+x}-2-x\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

maxima [A] time = 0.48, size = 20, normalized size = 0.80

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

mupad [B] time = 0.22, size = 19, normalized size = 0.76

$$x+4\ln\left(\sqrt{x+1}-1\right)+4\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + 1)/((x + 1)^(1/2) - 1),x)
[Out] x + 4*log((x + 1)^(1/2) - 1) + 4*(x + 1)^(1/2)
```

sympy [A] time = 0.16, size = 20, normalized size = 0.80

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}} \, dx$

Optimal. Leaf size=33

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2012, 1593, 266, 43}

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2012

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}} \, dx &= \text{Subst}\left(\int \frac{1}{-\sqrt{x}+x^{2/3}} \, dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(-1+\sqrt[6]{x}\right)\sqrt{x}} \, dx, x, 1+x\right)\\
&= 6\,\text{Subst}\left(\int \frac{x^2}{-1+x} \, dx, x, \sqrt[6]{1+x}\right)\\
&= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right) dx, x, \sqrt[6]{1+x}\right)\\
&= 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)
\end{aligned}$$

Mathematica [A]   time = 0.02, size = 33, normalized size = 1.00

$$3\left(\sqrt[3]{x+1}+2\sqrt[6]{x+1}+2\log\left(1-\sqrt[6]{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
[Out] 3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[1 - (1 + x)^(1/6)])
```

IntegrateAlgebraic [A]   time = 0.01, size = 31, normalized size = 0.94

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(\sqrt[6]{x+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[-1 + (1 + x)^(1/6)]
```

fricas [A]   time = 0.72, size = 25, normalized size = 0.76

$$3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

giac [A]   time = 1.18, size = 26, normalized size = 0.79

$$3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left(\left|(x+1)^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="giac")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log(abs((x + 1)^(1/6) - 1))
```

maple [A]   time = 0.04, size = 26, normalized size = 0.79

method	result
derivativedivides	$3\,(1+x)^{\frac{1}{3}}+6\,(1+x)^{\frac{1}{6}}+6\ln\left((1+x)^{\frac{1}{6}}-1\right)$
default	$6\,(1+x)^{\frac{1}{6}}+3\,(1+x)^{\frac{1}{3}}+\ln(x)+2\ln\left((1+x)^{\frac{1}{6}}-1\right)-\ln\left((1+x)^{\frac{1}{3}}+(1+x)^{\frac{1}{6}}+1\right)-2\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)^(2/3)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 3*(1+x)^(1/3)+6*(1+x)^(1/6)+6*ln((1+x)^(1/6)-1)
```

maxima [A]   time = 0.52, size = 25, normalized size = 0.76

$$3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

mupad [B] time = 0.24, size = 25, normalized size = 0.76

$$6\ln\left((x+1)^{1/6}-1\right)+3\,(x+1)^{1/3}+6\,(x+1)^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^(1/2) - (x + 1)^(2/3)),x)
[Out] 6*log((x + 1)^(1/6) - 1) + 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{2}{3}}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
[Out] Integral(1/((x + 1)**(2/3) - sqrt(x + 1)), x)
```

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx &= 4\,\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt[4]{x}\right)\\
&= -3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{3}{7}\left(\sqrt[4]{x}+1\right)^{4/3}\left(4\sqrt[4]{x}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
[Out] (3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

IntegrateAlgebraic [A] time = 0.03, size = 56, normalized size = 1.93

$$\frac{12}{7}\sqrt[3]{\sqrt[4]{x}+1}\,\sqrt{x}+\frac{3}{7}\sqrt[3]{\sqrt[4]{x}+1}\,\sqrt[4]{x}-\frac{9}{7}\sqrt[3]{\sqrt[4]{x}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] (-9*(1 + x^(1/4))^(1/3))/7 + (3*(1 + x^(1/4))^(1/3)*x^(1/4))/7 + (12*(1 + x
^(1/4))^(1/3)*Sqrt[x])/7
```

fricas [A] time = 1.12, size = 19, normalized size = 0.66

$$\frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

giac [A] time = 0.92, size = 19, normalized size = 0.66

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="giac")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

maple [C] time = 0.30, size = 17, normalized size = 0.59

method	result	size
meijerg	$2\sqrt{x}\ \mathrm{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],-x^{\frac{1}{4}}\right)$	17
derivativedivides	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20
default	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/4))^(1/3)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)*hypergeom([-1/3,2],[3],-x^(1/4))
```

maxima [A] time = 0.49, size = 19, normalized size = 0.66

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

mupad [B] time = 0.54, size = 16, normalized size = 0.55

$$\frac{3\left(x^{1/4}+1\right)^{4/3}\left(4\,x^{1/4}-3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4) + 1)^(1/3)/x^(1/2),x)
```

```
[Out] (3*(x^(1/4) + 1)^(4/3)*(4*x^(1/4) - 3))/7
```

sympy [B] time = 1.35, size = 134, normalized size = 4.62

$$\frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x}+\frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x}{7x^{\frac{5}{4}}+7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

```
[Out] 12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)*(x**(1/4)
 + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4) + 7*x) + 15*x**(3/
2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 9*x*(x**(1/4) + 1)**(1/3)/(7*
x**(5/4) + 7*x) + 9*x/(7*x**(5/4) + 7*x)
```

3.215 $\int \frac{1}{x^3(1+x)^{3/2}} \, dx$

Optimal. Leaf size=52

$$-\frac{1}{2x^2\sqrt{x+1}}+\frac{5}{4x\sqrt{x+1}}+\frac{15}{4\sqrt{x+1}}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {51, 63, 207}

$$-\frac{5\sqrt{x+1}}{2x^2}+\frac{2}{x^2\sqrt{x+1}}+\frac{15\sqrt{x+1}}{4x}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] 2/(x^2*Sqrt[1 + x]) - (5*Sqrt[1 + x])/(2*x^2) + (15*Sqrt[1 + x])/(4*x) - (1
5*ArcTanh[Sqrt[1 + x]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1+x)^{3/2}} \, dx &= \frac{2}{x^2\sqrt{1+x}}+5\int \frac{1}{x^3\sqrt{1+x}} \, dx\\
&= \frac{2}{x^2\sqrt{1+x}}-\frac{5\sqrt{1+x}}{2x^2}-\frac{15}{4}\int \frac{1}{x^2\sqrt{1+x}} \, dx\\
&= \frac{2}{x^2\sqrt{1+x}}-\frac{5\sqrt{1+x}}{2x^2}+\frac{15\sqrt{1+x}}{4x}+\frac{15}{8}\int \frac{1}{x\sqrt{1+x}} \, dx\\
&= \frac{2}{x^2\sqrt{1+x}}-\frac{5\sqrt{1+x}}{2x^2}+\frac{15\sqrt{1+x}}{4x}+\frac{15}{4}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx,x,\sqrt{1+x}\right)\\
&= \frac{2}{x^2\sqrt{1+x}}-\frac{5\sqrt{1+x}}{2x^2}+\frac{15\sqrt{1+x}}{4x}-\frac{15}{4}\tanh^{-1}\left(\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};x+1\right)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)),x]
[Out] (2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + x])/Sqrt[1 + x]
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 0.71

$$\frac{15x^2+5x-2}{4x^2\sqrt{x+1}}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + x)^(3/2)),x]
[Out] (-2 + 5*x + 15*x^2)/(4*x^2*Sqrt[1 + x]) - (15*ArcTanh[Sqrt[1 + x]])/4
```

fricas [A] time = 1.10, size = 63, normalized size = 1.21

$$-\frac{15\left(x^3+x^2\right)\log\left(\sqrt{x+1}+1\right)-15\left(x^3+x^2\right)\log\left(\sqrt{x+1}-1\right)-2\left(15\,x^2+5\,x-2\right)\sqrt{x+1}}{8\left(x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")
[Out] -1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1)
- 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)
```

giac [A] time = 1.01, size = 49, normalized size = 0.94

$$\frac{2}{\sqrt{x+1}}+\frac{7\,(x+1)^{\frac{3}{2}}-9\,\sqrt{x+1}}{4\,x^2}-\frac{15}{8}\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="giac")
[Out] 2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(x
 + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))
```

maple [A] time = 0.30, size = 30, normalized size = 0.58

method	result
risch	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2}-\frac{15\,\mathrm{arctanh}\left(\sqrt{1+x}\right)}{4}$
trager	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2}-\frac{15\ln\left(\frac{2\sqrt{1+x}+2+x}{x}\right)}{8}$
derivativedivides	$\frac{2}{\sqrt{1+x}}+\frac{1}{8\left(1+\sqrt{1+x}\right)^2}+\frac{7}{8\left(1+\sqrt{1+x}\right)}-\frac{15\ln\left(1+\sqrt{1+x}\right)}{8}-\frac{1}{8\left(-1+\sqrt{1+x}\right)^2}+\frac{7}{8\left(-1+\sqrt{1+x}\right)}+\frac{15\ln\left(-1+\sqrt{\cdots}\right)}{8}$
default	$\frac{2}{\sqrt{1+x}}+\frac{1}{8\left(1+\sqrt{1+x}\right)^2}+\frac{7}{8\left(1+\sqrt{1+x}\right)}-\frac{15\ln\left(1+\sqrt{1+x}\right)}{8}-\frac{1}{8\left(-1+\sqrt{1+x}\right)^2}+\frac{7}{8\left(-1+\sqrt{1+x}\right)}+\frac{15\ln\left(-1+\sqrt{\cdots}\right)}{8}$

meijerg	$\frac{\frac{\sqrt{\pi}\left(-47x^{2}-24x+8\right)}{16x^{2}}-\frac{\sqrt{\pi}\left(-60x^{2}-20x+8\right)}{16x^{2}\sqrt{1+x}}-\frac{15\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{4}+\frac{15\left(\frac{47}{30}-2\ln(2)+\ln(x)\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^{2}}+\frac{3\sqrt{\pi}}{2x}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(15*x^2+5*x-2)/(1+x)^(1/2)/x^2-15/4*arctanh((1+x)^(1/2))
```

maxima [A] time = 0.68, size = 55, normalized size = 1.06

$$\frac{15\,(x+1)^{2}-25\,x-17}{4\left((x+1)^{\frac{5}{2}}-2\,(x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)}-\frac{15}{8}\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x +
1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)
```

mupad [B] time = 0.05, size = 43, normalized size = 0.83

$$-\frac{15\,\mathrm{atanh}\left(\sqrt{x+1}\right)}{4}-\frac{\frac{25\,x}{4}-\frac{15\,(x+1)^{2}}{4}+\frac{17}{4}}{\sqrt{x+1}-2\,(x+1)^{3/2}+(x+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(3/2)),x)
```

```
[Out] - (15*atanh((x + 1)^(1/2)))/4 - ((25*x)/4 - (15*(x + 1)^2)/4 + 17/4)/((x +
1)^(1/2) - 2*(x + 1)^(3/2) + (x + 1)^(5/2))
```

sympy [B] time = 3.27, size = 3966, normalized size = 76.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(
x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(
9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt
(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*
(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*
I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(13/2
)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420*I*pi*(x + 1)**
(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*
(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5
/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**(11/2)*acoth(sqrt(
```

```
x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 840*I*pi*(x + 1)**(11/2)/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 2100*(x + 1)**(9/2)*acoth(sqrt(x + 1))/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 1050*I*pi*(x + 1)**(9/2)/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 1680*(x + 1)**(7/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 840*I*pi*(x + 1)**(7/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(1
5/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
 840*(x + 1)**(5/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*
(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
420*I*pi*(x + 1)**(5/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x +
1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
+ 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x + 1)**(3
/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1
)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) +
 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)
**(3/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 30*sqrt(x + 1)*acoth(sqrt(x + 1
))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x +
 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2)
- 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*sqrt(x + 1)/(8*(x + 1)**(17/
2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(
x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2)
 + 8*sqrt(x + 1)) + 30*(x + 1)**8/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) +
 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 230*(
x + 1)**7/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 4
48*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)*
*(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 766*(x + 1)**6/(8*(x + 1)**(1
7/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560
*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/
2) + 8*sqrt(x + 1)) - 1446*(x + 1)**5/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/
2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(
x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1
690*(x + 1)**4/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2
) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
+ 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1250*(x + 1)**3/(8*(x +
1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2)
 + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1
)**(3/2) + 8*sqrt(x + 1)) + 570*(x + 1)**2/(8*(x + 1)**(17/2) - 64*(x + 1)*
*(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) -
448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)
) - 146*(x + 1)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/
2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
 + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 16/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)), Abs(x + 1) > 1), (-15*(x + 1)**(17/2)*atanh(sqrt(x + 1))/(4*
```

```
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(15/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(13/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(11/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 1050*(x + 1)**(9/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(7/2)*atanh(sqrt(x + 1))/(4*(
x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(1
1/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
 + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(5/2)*atanh(sqrt(x + 1))/(4*(x
 + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11
/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
+ 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(3/2)*atanh(sqrt(x + 1))/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) - 15*sqrt(x + 1)*atanh(sqrt(x + 1))/(4*(x + 1)*
*(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) +
280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**
(3/2) + 4*sqrt(x + 1)) + 15*(x + 1)**8/(4*(x + 1)**(17/2) - 32*(x + 1)**(15
/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*
(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) -
115*(x + 1)**7/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2
) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x
+ 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 383*(x + 1)**6/(4*(x + 1
)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
+ 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)
**(3/2) + 4*sqrt(x + 1)) - 723*(x + 1)**5/(4*(x + 1)**(17/2) - 32*(x + 1)**
(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 2
24*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1))
 + 845*(x + 1)**4/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(1
3/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*
(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 625*(x + 1)**3/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) + 285*(x + 1)**2/(4*(x + 1)**(17/2) - 32*(x + 1
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2)
- 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x +
1)) - 73*(x + 1)/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13
/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(
x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 8/(4*(x + 1)**(17/2) -
 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4
*sqrt(x + 1)), True))
```

3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$

Optimal. Leaf size=118

$$-\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}+\frac{3003}{64\sqrt{1-x}}-\frac{429}{64(1-x)^{5/2}x}+\frac{1001}{64(1-x)^{3/2}}+\frac{3003}{320(1-x)^{5/2}}-\frac{3003}{64}\text{ t}$$

Rubi [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {51, 63, 206}

$$-\frac{1001\sqrt{1-x}}{32x^2}-\frac{1001\sqrt{1-x}}{40x^3}-\frac{429\sqrt{1-x}}{20x^4}+\frac{286}{15\sqrt{1-x}\,x^4}+\frac{26}{15(1-x)^{3/2}x^4}+\frac{2}{5(1-x)^{5/2}x^4}-\frac{3003\sqrt{1-x}}{64x}-\frac{3003}{64}\text{ ta}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] 2/(5*(1 - x)^(5/2)*x^4) + 26/(15*(1 - x)^(3/2)*x^4) + 286/(15*Sqrt[1 - x]*x
^4) - (429*Sqrt[1 - x])/(20*x^4) - (1001*Sqrt[1 - x])/(40*x^3) - (1001*Sqrt
[1 - x])/(32*x^2) - (3003*Sqrt[1 - x])/(64*x) - (3003*ArcTanh[Sqrt[1 - x]])
/64
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{2}{5(1-x)^{5/2}x^4} + \frac{13}{5}\int \frac{1}{(1-x)^{5/2}x^5}\,dx$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{143}{15}\int \frac{1}{(1-x)^{3/2}x^5}\,dx$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} + \frac{429}{5}\int \frac{1}{\sqrt{1-x}\,x^5}\,dx$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} + \frac{3003}{40}\int \frac{1}{\sqrt{1-x}\,x^4}\,dx$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} + \frac{1001}{16}\int \frac{1}{\sqrt{1-x}}$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} +$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} -$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} -$$
$$= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}\,x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} -$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.22

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},5;-\frac{3}{2};1-x\right)}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] (2*Hypergeometric2F1[-5/2, 5, -3/2, 1 - x])/(5*(1 - x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 66, normalized size = 0.56

$$\frac{\sqrt{1-x}\left(-45045x^6+105105x^5-69069x^4+6435x^3+1430x^2+520x+240\right)}{960(x-1)^3x^4} - \frac{3003}{64}\tanh^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] (Sqrt[1 - x]*(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 -
45045*x^6))/(960*(-1 + x)^3*x^4) - (3003*ArcTanh[Sqrt[1 - x]])/64
```

fricas [A] time = 0.98, size = 125, normalized size = 1.06

$$-\frac{45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}+1\right)-45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}-1\right)+2\left(45045\right.}{1920\left(x^7-3x^6+3x^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/1920*(45045*(x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) + 1) - 45045*(x
^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) - 1) + 2*(45045*x^6 - 105105*x^5
```

```
 + 69069*x^4 - 6435*x^3 - 1430*x^2 - 520*x - 240)*sqrt(-x + 1))/(x^7 - 3*x^
6 + 3*x^5 - x^4)
```

giac [A] time = 0.92, size = 104, normalized size = 0.88

$$\frac{2\left(225\,(x-1)^2-25\,x+28\right)}{15\,(x-1)^2\sqrt{-x+1}}-\frac{3249\,(x-1)^3\sqrt{-x+1}+10633\,(x-1)^2\sqrt{-x+1}-11767\,(-x+1)^{\frac{3}{2}}+4431\,\sqrt{-x}}{192\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192*(3249*(x
- 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11767*(-x + 1)^(3/2) +
 4431*sqrt(-x + 1))/x^4 - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(abs
(sqrt(-x + 1) - 1))
```

maple [A] time = 0.33, size = 59, normalized size = 0.50

method	result
risch	$\frac{45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240}{960x^4\sqrt{1-x}\,(-1+x)^2}-\frac{3003\,\mathrm{arctanh}\left(\sqrt{1-x}\right)}{64}$
trager	$-\frac{\left(45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240\right)\sqrt{1-x}}{960(-1+x)^3x^4}+\frac{3003\ln\left(\frac{-2+x+2\sqrt{1-x}}{x}\right)}{128}$
meijerg	$\frac{\frac{\sqrt{\pi}\,\left(-329177x^4+110880x^3+30240x^2+8960x+1920\right)}{7680x^4}-\frac{\sqrt{\pi}\,\left(-180180x^6+420420x^5-276276x^4+25740x^3+5720x^2+2080x+960\right)}{3840x^4(1-x)^{\frac{5}{2}}}-\frac{3003\sqrt{\pi}}{}}{\sqrt{\pi}}$
derivativedivides	$-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+\frac{3003\ln\left(\sqrt{1-x}-1\right)}{128}+\frac{2}{5(1-x)^{\frac{5}{2}}}+-\frac{}{3}$
default	$-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+\frac{3003\ln\left(\sqrt{1-x}-1\right)}{128}+\frac{2}{5(1-x)^{\frac{5}{2}}}+-\frac{}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*(45045*x^6-105105*x^5+69069*x^4-6435*x^3-1430*x^2-520*x-240)/x^4/(1-x
)^(1/2)/(-1+x)^2-3003/64*arctanh((1-x)^(1/2))
```

maxima [A] time = 0.52, size = 111, normalized size = 0.94

$$\frac{45045\,(x-1)^6+165165\,(x-1)^5+219219\,(x-1)^4+119691\,(x-1)^3+18304\,(x-1)^2-1664\,x+2048}{960\left((-x+1)^{\frac{13}{2}}-4\,(-x+1)^{\frac{11}{2}}+6\,(-x+1)^{\frac{9}{2}}-4\,(-x+1)^{\frac{7}{2}}+(-x+1)^{\frac{5}{2}}\right)}-\frac{3003}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 119691*(x -
1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2) - 4*(-x + 1)^(11/2
) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x + 1)^(5/2)) - 3003/128*log(sq
rt(-x + 1) + 1) + 3003/128*log(sqrt(-x + 1) - 1)
```

mupad [B] time = 0.21, size = 96, normalized size = 0.81

$$\frac{\frac{286\,(x-1)^2}{15}-\frac{26\,x}{15}+\frac{39897\,(x-1)^3}{320}+\frac{73073\,(x-1)^4}{320}+\frac{11011\,(x-1)^5}{64}+\frac{3003\,(x-1)^6}{64}+\frac{32}{15}}{(1-x)^{5/2}-4\,(1-x)^{7/2}+6\,(1-x)^{9/2}-4\,(1-x)^{11/2}+(1-x)^{13/2}}-\frac{3003\,\mathrm{atanh}\left(\sqrt{1-x}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x)^(7/2)),x)
```

```
[Out] ((286*(x - 1)^2)/15 - (26*x)/15 + (39897*(x - 1)^3)/320 + (73073*(x - 1)^4)
/320 + (11011*(x - 1)^5)/64 + (3003*(x - 1)^6)/64 + 32/15)/((1 - x)^(5/2) -
 4*(1 - x)^(7/2) + 6*(1 - x)^(9/2) - 4*(1 - x)^(11/2) + (1 - x)^(13/2)) - (
3003*atanh((1 - x)^(1/2)))/64
```

sympy [C] time = 20.37, size = 971, normalized size = 8.23

$$\begin{cases} -\frac{45045ix^7\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{-960x^7+2880x^6-2880x^5+960x^4} + \frac{45045ix^6\sqrt{x-1}}{-960x^7+2880x^6-2880x^5+960x^4} + \frac{135135ix^6\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{-960x^7+2880x^6-2880x^5+960x^4} - \frac{105105ix^5\sqrt{x-1}}{-960x^7+2880x^6-2880x^5+960x^4} - \\ -\frac{45045x^7\log(x)}{-1920x^7+5760x^6-5760x^5+1920x^4} + \frac{90090x^7\log\left(\sqrt{1-x}+1\right)}{-1920x^7+5760x^6-5760x^5+1920x^4} - \frac{45045i\pi x^7}{-1920x^7+5760x^6-5760x^5+1920x^4} + \frac{90090x^6\sqrt{1-x}}{-1920x^7+5760x^6-5760x^5+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/x**5,x)
```

```
[Out] Piecewise((-45045*I*x**7*asin(1/sqrt(x))/(-960*x**7 + 2880*x**6 - 2880*x**5
 + 960*x**4) + 45045*I*x**6*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 2880*x**5
+ 960*x**4) + 135135*I*x**6*asin(1/sqrt(x))/(-960*x**7 + 2880*x**6 - 2880*x
**5 + 960*x**4) - 105105*I*x**5*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 2880*x
**5 + 960*x**4) - 135135*I*x**5*asin(1/sqrt(x))/(-960*x**7 + 2880*x**6 - 28
80*x**5 + 960*x**4) + 69069*I*x**4*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 288
0*x**5 + 960*x**4) + 45045*I*x**4*asin(1/sqrt(x))/(-960*x**7 + 2880*x**6 -
2880*x**5 + 960*x**4) - 6435*I*x**3*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 28
80*x**5 + 960*x**4) - 1430*I*x**2*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 2880
*x**5 + 960*x**4) - 520*I*x*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 2880*x**5
+ 960*x**4) - 240*I*sqrt(x - 1)/(-960*x**7 + 2880*x**6 - 2880*x**5 + 960*x*
*4), Abs(x) > 1), (-45045*x**7*log(x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 +
 1920*x**4) + 90090*x**7*log(sqrt(1 - x) + 1)/(-1920*x**7 + 5760*x**6 - 576
0*x**5 + 1920*x**4) - 45045*I*pi*x**7/(-1920*x**7 + 5760*x**6 - 5760*x**5 +
 1920*x**4) + 90090*x**6*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 +
1920*x**4) + 135135*x**6*log(x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*
x**4) - 270270*x**6*log(sqrt(1 - x) + 1)/(-1920*x**7 + 5760*x**6 - 5760*x**
5 + 1920*x**4) + 135135*I*pi*x**6/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 192
0*x**4) - 210210*x**5*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 192
0*x**4) - 135135*x**5*log(x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**
4) + 270270*x**5*log(sqrt(1 - x) + 1)/(-1920*x**7 + 5760*x**6 - 5760*x**5 +
 1920*x**4) - 135135*I*pi*x**5/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x
**4) + 138138*x**4*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x
**4) + 45045*x**4*log(x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4) -
 90090*x**4*log(sqrt(1 - x) + 1)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920
*x**4) + 45045*I*pi*x**4/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4) -
 12870*x**3*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4) -
2860*x**2*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4) - 10
40*x*sqrt(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4) - 480*sqr
t(1 - x)/(-1920*x**7 + 5760*x**6 - 5760*x**5 + 1920*x**4), True))
```

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt[3]{x-1}}{4x^4}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {51, 58, 618, 204, 31}

$$\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{\sqrt[3]{x-1}}{4x^4}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 + x)^(1/3))
/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1 - 2*(-1 + x)^(1/3))
/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x)^(1/3)])/81 - (55*Log[x])/243
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^{2/3}x^5}\,dx &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11}{12}\int \frac{1}{(-1+x)^{2/3}x^4}\,dx\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{22}{27}\int \frac{1}{(-1+x)^{2/3}x^3}\,dx\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55}{81}\int \frac{1}{(-1+x)^{2/3}x^2}\,dx\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}+\frac{110}{243}\int \frac{1}{(-1+x)^{2/3}x}\,dx\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}-\frac{55\log(x)}{243}+\frac{55}{81}\text{Subst}\left(\int \frac{1}{1+x}\,dx\right.\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}+\frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right)-\frac{55\log(x)}{243}\\ &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{55}{81}\log\left(1\right.\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.21

$$3\sqrt[3]{x-1}\,{}_2F_1\left(\frac{1}{3},5;\frac{4}{3};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] 3*(-1 + x)^(1/3)*Hypergeometric2F1[1/3, 5, 4/3, 1 - x]
```

IntegrateAlgebraic [A] time = 0.07, size = 97, normalized size = 0.93

$$\frac{\sqrt[3]{x-1}\left(220x^3+132x^2+99x+81\right)}{324x^4}+\frac{110}{243}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55}{243}\log\left((x-1)^{2/3}-\sqrt[3]{x-1}+1\right)-\frac{110\tan^{-1}\left(\frac{1}{\sqrt{3}}-\right.}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] ((-1 + x)^(1/3)*(81 + 99*x + 132*x^2 + 220*x^3))/(324*x^4) - (110*ArcTan[1/
Sqrt[3] - (2*(-1 + x)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (110*Log[1 + (-1 + x)
^(1/3)])/243 - (55*Log[1 - (-1 + x)^(1/3) + (-1 + x)^(2/3)])/243
```

fricas [A] time = 1.19, size = 86, normalized size = 0.83

$$\frac{440\sqrt{3}\,x^4\arctan\left(\frac{2}{3}\sqrt{3}\,(x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-220\,x^4\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+440\,x^4\log\left((x-1)^{\frac{1}{3}}+1\right)+3\,(2}{972\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/972*(440*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 22
0*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 440*x^4*log((x - 1)^(1/3) +
1) + 3*(220*x^3 + 132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

giac [A] time = 1.14, size = 82, normalized size = 0.79

$$\frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,x^4}-\frac{55}{243}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/x^
4 - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/
3) + 1)
```

maple [C] time = 0.54, size = 85, normalized size = 0.82

method	result
meijerg	$\frac{\left(-\mathrm{signum}(-1+x)\right)^{\frac{2}{3}}\left(\frac{308\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[1,1,\frac{17}{3}\right],[2,6],x\right)}{729}+\frac{110\left(\frac{877}{1320}+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)}{243}-\frac{\Gamma\left(\frac{2}{3}\right)}{4x^4}-\frac{2\Gamma\left(\frac{2}{3}\right)}{9x^3}-\frac{5\Gamma\left(\frac{2}{3}\right)}{18x^2}\right.}{\Gamma\left(\frac{2}{3}\right)\mathrm{signum}(-1+x)^{\frac{2}{3}}}$
risch	$\frac{220x^4-88x^3-33x^2-18x-81}{324x^4(-1+x)^{\frac{2}{3}}}+\frac{110\left(-\mathrm{signum}(-1+x)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\right.}{243\Gamma\left(\frac{2}{3}\right)\mathrm{signum}(-1+x)^{\frac{2}{3}}}$
derivativedivides	$-\frac{-75(-1+x)^{\frac{7}{3}}+190(-1+x)^2-350(-1+x)^{\frac{5}{3}}+\frac{1157(-1+x)^{\frac{4}{3}}}{4}+\frac{149}{4}-138x-116(-1+x)^{\frac{2}{3}}+137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}}-(-1+x)^{\frac{1}{3}}+1\right)^4}-\frac{55\ln\left((-1+x)^{\frac{2}{3}}\right.}{24}$
default	$-\frac{-75(-1+x)^{\frac{7}{3}}+190(-1+x)^2-350(-1+x)^{\frac{5}{3}}+\frac{1157(-1+x)^{\frac{4}{3}}}{4}+\frac{149}{4}-138x-116(-1+x)^{\frac{2}{3}}+137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}}-(-1+x)^{\frac{1}{3}}+1\right)^4}-\frac{55\ln\left((-1+x)^{\frac{2}{3}}\right.}{24}$
trager	$\frac{\left(220x^3+132x^2+99x+81\right)(-1+x)^{\frac{1}{3}}}{324x^4}+\frac{110\ln\left(\frac{144(-1+x)^{\frac{2}{3}}\mathrm{RootOf}\left(2304\_Z^2+48\_Z+1\right)+4608\,\mathrm{RootOf}\left(2304\_Z^2+48\_Z+1\right)^2x+144}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(2/3)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/GAMMA(2/3)/signum(-1+x)^(2/3)*(-signum(-1+x))^(2/3)*(308/729*GAMMA(2/3)*x
*hypergeom([1,1,17/3],[2,6],x)+110/243*(877/1320+1/6*Pi*3^(1/2)-3/2*ln(3)+l
n(x)+I*Pi)*GAMMA(2/3)-1/4*GAMMA(2/3)/x^4-2/9*GAMMA(2/3)/x^3-5/18*GAMMA(2/3)
/x^2-40/81*GAMMA(2/3)/x)
```

maxima [A] time = 1.39, size = 105, normalized size = 1.01

$$\frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\left((x-1)^4+4\,(x-1)^3+6\,(x-1)^2+4\,x-3\right)}-\frac{55}{243}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/((
x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x - 3) - 55/243*log((x - 1)^(2/3)
- (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/3) + 1)
```

mupad [B] time = 0.21, size = 120, normalized size = 1.15

$$\frac{110\ln\left(\frac{12100\,(x-1)^{1/3}}{6561}+\frac{12100}{6561}\right)}{243}+\frac{\frac{133\,(x-1)^{1/3}}{81}+\frac{341\,(x-1)^{4/3}}{108}+\frac{22\,(x-1)^{7/3}}{9}+\frac{55\,(x-1)^{10/3}}{81}}{4\,x+6\,(x-1)^2+4\,(x-1)^3+(x-1)^4-3}-\ln\left(\frac{55}{27}-\frac{110\,(x-1)^{1/3}}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x - 1)^(2/3)),x)
```

```
[Out] (110*log((12100*(x - 1)^(1/3))/6561 + 12100/6561))/243 + ((133*(x - 1)^(1/3
))/81 + (341*(x - 1)^(4/3))/108 + (22*(x - 1)^(7/3))/9 + (55*(x - 1)^(10/3)
)/81)/(4*x + 6*(x - 1)^2 + 4*(x - 1)^3 + (x - 1)^4 - 3) - log((3^(1/2)*55i)
/27 - (110*(x - 1)^(1/3))/27 + 55/27)*((3^(1/2)*55i)/243 + 55/243) + log((1
10*(x - 1)^(1/3))/27 + (3^(1/2)*55i)/27 - 55/27)*((3^(1/2)*55i)/243 - 55/24
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(2/3)/x**5,x)
```

```
[Out] Timed out
```

3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1-x}{1+x}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x)+2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x)-2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{x+1}\left(\sqrt{x+1}(x-1)+2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*Ar
cSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 1.00

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 - x)/(1 + x)],x]
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

fricas [A] time = 0.92, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

giac [A] time = 0.92, size = 29, normalized size = 0.76

$$\frac{1}{2}\pi\mathrm{sgn}(x+1)+\arcsin(x)\mathrm{sgn}(x+1)+\sqrt{-x^2+1}\,\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

maple [A] time = 0.16, size = 39, normalized size = 1.03

method	result
default	$\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(1+x)(-1+x)}}$
risch	$(1+x)\sqrt{-\frac{-1+x}{1+x}}-\frac{\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(1+x)(-1+x)}}{-1+x}$
trager	$(1+x)\sqrt{-\frac{-1+x}{1+x}}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{-1+x}{1+x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{-1+x}{1+x}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(1+x)*(-1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x)
)
```

maxima [A] time = 1.38, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

mupad [B] time = 0.19, size = 43, normalized size = 1.13

$$-2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
```

```
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}(b-x)^2\sqrt{\frac{x-a}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{x-a}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{\frac{x-a}{b-x}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1960, 455, 385, 203}

$$\frac{1}{2}(b-x)^2\sqrt{-\frac{a-x}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{-\frac{a-x}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] ((a - 5*b)*Sqrt[-((a - x)/(b - x))]*(b - x))/4 + (Sqrt[-((a - x)/(b - x))]*
(b - x)^2)/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[-((a - x)/(b - x))]])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\left((2(a-b))\,\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)}{\left(1+x^2\right)^3}\,dx, x, \sqrt{\frac{-a+x}{b-x}}\right)\right)$$
$$= \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}\,(b-x)^2 - \frac{1}{2}(-a+b)\,\text{Subst}\left(\int \frac{-a+b-4bx^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{b-x}}\right)$$
$$= \frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}\,(b-x) + \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}\,(b-x)^2 - \frac{1}{4}((a-b)(a+3b))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.$$
$$= \frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}\,(b-x) + \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}\,(b-x)^2 - \frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Mathematica [A] time = 0.29, size = 115, normalized size = 1.25

$$\frac{\sqrt{\frac{x-a}{b-x}}\left((b-x)(a-3b-2x)\sqrt{\frac{a-x}{a-b}} - \sqrt{a-b}\,(a+3b)\sqrt{b-x}\,\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)\right)}{4\sqrt{\frac{a-x}{a-b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] (Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*Sqrt[(a - x)/(a - b)]*(b - x) - Sq
rt[a - b]*(a + 3*b)*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*Sqrt[
(a - x)/(a - b)])
```

IntegrateAlgebraic [A] time = 0.08, size = 80, normalized size = 0.87

$$\frac{1}{4}\left(-a^2-2ab+3b^2\right)\tan^{-1}\left(\sqrt{\frac{x-a}{b-x}}\right) + \frac{1}{4}\sqrt{\frac{x-a}{b-x}}\left(ab-ax-3b^2+bx+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] (Sqrt[(-a + x)/(b - x)]*(a*b - 3*b^2 - a*x + b*x + 2*x^2))/4 + ((-a^2 - 2*a
*b + 3*b^2)*ArcTan[Sqrt[(-a + x)/(b - x)]])/4
```

fricas [A] time = 1.03, size = 73, normalized size = 0.79

$$-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) + \frac{1}{4}\left(ab-3\,b^2-(a-b)x+2\,x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(a*b - 3*b^
2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

giac [A] time = 1.03, size = 103, normalized size = 1.12

$$\frac{1}{8}\left(a^2\text{sgn}\left(-b+x\right)+2\,ab\text{sgn}\left(-b+x\right)-3\,b^2\text{sgn}\left(-b+x\right)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\text{sgn}\left(-a+b\right)-\frac{1}{4}\sqrt{-ab+ax+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*sgn(-b + x) + 2*a*b*sgn(-b + x) - 3*b^2*sgn(-b + x))*arcsin((a + b
 - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a*sgn(-b +
 x) - 3*b*sgn(-b + x) - 2*x*sgn(-b + x))
```

maple [B] time = 0.10, size = 195, normalized size = 2.12

method	result
risch	$\frac{(a-3b-2x)(b-x)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(a-x)(b-x)}}{4\sqrt{-(-a+x)(-b+x)}}+\frac{\left(\frac{\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-x^2+(a+b)x-ab}}\right)ab}{4}+\frac{\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-x^2+(a+b)x-ab}}\right)a^2}{8}-\frac{3\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-x^2+(a+b)x-ab}}\right)b^2}{8}\right)\sqrt{-\frac{a-}{b-}}}{a-x}$
default	$\frac{\sqrt{-\frac{a-x}{b-x}}\,(b-x)\left(\arctan\left(\frac{a-2x+b}{2\sqrt{-ab+ax+bx-x^2}}\right)a^2+2b\arctan\left(\frac{a-2x+b}{2\sqrt{-ab+ax+bx-x^2}}\right)a-3\arctan\left(\frac{a-2x+b}{2\sqrt{-ab+ax+bx-x^2}}\right)b^2+2\sqrt{-ab+ax+bx-x^2}\,a-6\sqrt{-a}\right.}{8\sqrt{-(a-x)(b-x)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-a+x)/(b-x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a-3*b-2*x)*(b-x)/(-(-a+x)*(-b+x))^(1/2)*(-(a-x)/(b-x))^(1/2)*(-(a-x)*(
b-x))^(1/2)+(1/4*arctan((x-1/2*a-1/2*b)/(-x^2+(a+b)*x-a*b)^(1/2))*a*b+1/8*a
rctan((x-1/2*a-1/2*b)/(-x^2+(a+b)*x-a*b)^(1/2))*a^2-3/8*arctan((x-1/2*a-1/2
*b)/(-x^2+(a+b)*x-a*b)^(1/2))*b^2)*(-(a-x)/(b-x))^(1/2)*(-(a-x)*(b-x))^(1/2
)/(a-x)
```

maxima [A] time = 1.45, size = 130, normalized size = 1.41

$$-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)-\frac{\left(a^2-6\,ab+5\,b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2\,ab-3\,b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*((a^2 - 6*a
*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3*b^2)*sqrt(-(a - x)/
(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b - x) + 1)
```

mupad [B] time = 0.34, size = 140, normalized size = 1.52

$$-\frac{\sqrt{-\frac{a-x}{b-x}}\,\left(\frac{a^2\,1\mathrm{i}}{4}+\frac{a\,b\,1\mathrm{i}}{2}-\frac{b^2\,3\mathrm{i}}{4}\right)\,1\mathrm{i}-{\left(-\frac{a-x}{b-x}\right)}^{3/2}\,\left(\frac{a^2\,1\mathrm{i}}{4}-\frac{a\,b\,3\mathrm{i}}{2}+\frac{b^2\,5\mathrm{i}}{4}\right)\,1\mathrm{i}}{\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1}-\frac{\mathrm{atan}\left(\sqrt{-\frac{a-x}{b-x}}\right)\,(a-b)\,(a+3\,b)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(a - x)/(b - x))^(1/2),x)
```

```
[Out] - ((-(a - x)/(b - x))^(1/2)*((a*b*1i)/2 + (a^2*1i)/4 - (b^2*3i)/4)*1i - (-(
a - x)/(b - x))^(3/2)*((a^2*1i)/4 - (a*b*3i)/2 + (b^2*5i)/4)*1i)/((a - x)^2
/(b - x)^2 - (2*(a - x))/(b - x) + 1) - (atan((-(a - x)/(b - x))^(1/2))*(a
- b)*(a + 3*b))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))**(1/2),x)
[Out] Integral(x*sqrt((-a + x)/(b - x)), x)
```

3.220 $\int \frac{\sqrt{-5+x}\,\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\,\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\,\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1586, 178, 92, 203, 93, 206}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\,\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\,\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqrt
[-5 + x]]/(3*Sqrt[5])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 178

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[
((e + f*x)^(p - 1)*(g + h*x)^q)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c -
 a*d), Int[((e + f*x)^(p - 1)*(g + h*x)^q)/(c + d*x), x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx &= \int \frac{\sqrt{3+x}}{\sqrt{-5+x}\,(-1+x)(5+x)}\,dx\\ &= \frac{1}{3}\int \frac{1}{\sqrt{-5+x}\sqrt{3+x}\,(5+x)}\,dx+\frac{2}{3}\int \frac{1}{\sqrt{-5+x}\,(-1+x)\sqrt{3+x}}\,dx\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{2-10x^2}\,dx,x,\frac{\sqrt{3+x}}{\sqrt{-5+x}}\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{16+x^2}\,dx,x,\sqrt{-5+x}\sqrt{3\right.\\ &= \frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}\end{aligned}$$

Mathematica [A]   time = 0.15, size = 47, normalized size = 0.87

$$\frac{1}{15}\left(5\tan^{-1}\left(\sqrt{\frac{x-5}{x+3}}\right)+\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{\frac{x-5}{x+3}}}{\sqrt{5}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] (5*ArcTan[Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[(-5 + x)/(3 + x)]/
Sqrt[5]])/15
```

IntegrateAlgebraic [B]   time = 0.52, size = 121, normalized size = 2.24

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)-\frac{\tanh^{-1}\left(\frac{-\frac{11x}{\sqrt{5}}+\frac{\sqrt{x-5}\left(3\sqrt{x+3}+2\right)}{\sqrt{5}}+6\sqrt{5}\sqrt{x+3}-7\sqrt{5}}{\sqrt{x-5}\left(2\sqrt{x+3}-6\right)-2\sqrt{x+3}+6}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 - ArcTanh[(-7*Sqrt[5] - (11*x)/Sqrt[
5] + 6*Sqrt[5]*Sqrt[3 + x] + (Sqrt[-5 + x]*(2 + 3*Sqrt[3 + x]))/Sqrt[5])/(6
 - 2*Sqrt[3 + x] + Sqrt[-5 + x]*(-6 + 2*Sqrt[3 + x]))]/(3*Sqrt[5])
```

fricas [A]   time = 0.90, size = 65, normalized size = 1.20

$$\frac{1}{30}\sqrt{5}\log\left(\frac{\sqrt{x+3}\sqrt{x-5}\left(3\sqrt{5}+5\right)+\sqrt{5}\,(3x+5)+9x+15}{x+5}\right)+\frac{1}{3}\arctan\left(\frac{1}{4}\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x +
5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x +
1/4)
```

giac [B] time = 0.96, size = 74, normalized size = 1.37

$$-\frac{1}{30}\sqrt{5}\log\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\sqrt{5}+12}\right)-\frac{1}{3}\arctan\left(\frac{1}{8}\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(x
 + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) - s
qrt(x - 5))^2)
```

maple [A] time = 0.40, size = 64, normalized size = 1.19

method	result	size
default	$\frac{\sqrt{-5+x}\,\sqrt{3+x}\left(\sqrt{5}\,\operatorname{arctanh}\left(\frac{(5+3x)\sqrt{5}}{5\sqrt{x^2-2x-15}}\right)-5\arctan\left(\frac{4}{\sqrt{x^2-2x-15}}\right)\right)}{30\sqrt{x^2-2x-15}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(-5+x)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x
-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+3}\,\sqrt{x-5}}{\left(x^2-25\right)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

mupad [B] time = 0.61, size = 95, normalized size = 1.76

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x+3}\,\sqrt{x-5}-2\sqrt{2}\,\sqrt{x-5}}{x-2\sqrt{2}\,\sqrt{x+3}+3}\right)}{3}-\frac{\sqrt{5}\,\operatorname{atanh}\left(-\frac{\sqrt{5}\,\sqrt{x+3}\,\sqrt{x-5}-2\sqrt{2}\,\sqrt{5}\,\sqrt{x-5}}{5x-10\sqrt{2}\,\sqrt{x+3}+15}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 3)^(1/2)*(x - 5)^(1/2))/((x^2 - 25)*(x - 1)),x)
```

```
[Out] atan(((x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*(x - 5)^(1/2))/(x - 2*2^(1/2)
*(x + 3)^(1/2) + 3))/3 - (5^(1/2)*atanh(-(5^(1/2)*(x + 3)^(1/2)*(x - 5)^(1/
2) - 2*2^(1/2)*5^(1/2)*(x - 5)^(1/2))/(5*x - 10*2^(1/2)*(x + 3)^(1/2) + 15)
))/15
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

```
[Out] Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}dx$

Optimal. Leaf size=304

$$\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}+\frac{x\left(1-x^2\right)^{5/4}}{6\sqrt{1-x}}+\frac{7\left(1-x^2\right)^{5/4}}{24\sqrt{1-x}}+\frac{1}{24}(x+1)^{3/4}(1-x)^{5/4}+\frac{5}{16}\sqrt[4]{x+1}\,(1-x)^{3/4}-\frac{1}{16}(x+1)^{3/4}\sqrt[4]{1-x}$$

Rubi [A] time = 0.82, antiderivative size = 319, normalized size of antiderivative = 1.05, number of steps used = 33, number of rules used = 16, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2103, 795, 675, 50, 63, 240, 211, 1165, 628, 1162, 617, 204, 1633, 793, 331, 297}

$$\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}+\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}+\frac{1}{6}(1-x)^{7/4}(x+1)^{5/4}+\frac{1}{24}(1-x)^{5/4}(x+1)^{3/4}-\frac{1}{16}\sqrt[4]{1-x}\,(x+1)^{3/4}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 +
x])),x]
```

```
[Out] (-5*(1 - x)^(3/4)*(1 + x)^(1/4))/48 + (5*(1 - x)^(7/4)*(1 + x)^(1/4))/24 -
((1 - x)^(1/4)*(1 + x)^(3/4))/16 + ((1 - x)^(5/4)*(1 + x)^(3/4))/24 + ((1 -
 x)^(7/4)*(1 + x)^(5/4))/6 + (Sqrt[1 + x]*(1 - x^2)^(5/4))/6 + (1 - x^2)^(9
/4)/(3*(1 - x)^(3/2)) - (3*ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)
])/(8*Sqrt[2]) + (3*ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*S
qrt[2]) + Log[1 + Sqrt[1 - x]/Sqrt[1 + x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)
^(1/4)]/(8*Sqrt[2]) - Log[1 + Sqrt[1 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 - x)^(1
/4))/(1 + x)^(1/4)]/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
```

```
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+x}\,\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = -\left(\frac{1}{2}\int x\sqrt{1+x}\,\sqrt[4]{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{x(1+x)\sqrt[4]{1-x^2}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}-\frac{1}{12}\int \sqrt{1+x}\,\sqrt[4]{1-x^2}\,dx-\frac{1}{2}\int \frac{x\left(1-x^2\right)^{5/4}}{(1-x)^{3/2}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{1}{12}\int \sqrt[4]{1-x}\,(1+x)^{3/4}\,dx-\frac{1}{2}\int \frac{(1-x^{\cdots}}{\sqrt{1-\cdots}}$$

$$= \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{1}{16}\int \frac{\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\,d\cdots$$

$$= -\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1\cdots}$$

$$= \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}\,(1+x)^{3/4}+\frac{1}{24}(1-\cdots$$

Mathematica [C] time = 0.51, size = 153, normalized size = 0.50

$$\frac{\sqrt[4]{1-x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1-x}{2}\right)}{8\sqrt[4]{2}\,\sqrt[4]{x+1}}+\frac{5\left(1-x^2\right)^{3/4}\;{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1-x}{2}\right)}{24\;2^{3/4}(x+1)^{3/4}}-\frac{1}{48}\sqrt{x+1}\,\sqrt[4]{1-x^2}\left(8x^2-\frac{\sqrt{1-x^2}\left(8x^2+22x+29\right)}{x+1}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqr
t[1 + x])),x]
```

```
[Out] -1/48*(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*(29 +
 22*x + 8*x^2))/(1 + x))) + ((1 - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/
```

```
4, (1 - x)/2])/(8*2^(1/4)*(1 + x)^(1/4)) + (5*(1 - x^2)^(3/4)*Hypergeometri
c2F1[3/4, 3/4, 7/4, (1 - x)/2])/(24*2^(3/4)*(1 + x)^(3/4))
```

IntegrateAlgebraic [A] time = 42.80, size = 294, normalized size = 0.97

$$\frac{1}{48}\sqrt{x+1}\,\sqrt[4]{1-x^2}\,\left(-8x^2-2x+7\right)+\frac{1}{48}\sqrt{1-x}\,\sqrt[4]{1-x^2}\,\left(8x^2+22x+29\right)-\frac{\tan^{-1}\left(\frac{-\frac{\sqrt{1-x^2}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{x+1}\,\sqrt[4]{1-x^2}}\right)}{16\sqrt{2}}-\frac{5\tan^{-1}\left(\cdots\right)}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 -
 x] - Sqrt[1 + x])),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 2*x - 8*x^2)*(1 - x^2)^(1/4))/48 + (Sqrt[1 - x]*(1 - x^2)
^(1/4)*(29 + 22*x + 8*x^2))/48 - ArcTan[(1/Sqrt[2] + x/Sqrt[2] - Sqrt[1 - x
^2]/Sqrt[2])/(Sqrt[1 + x]*(1 - x^2)^(1/4))]/(16*Sqrt[2]) - (5*ArcTan[(-(1/S
qrt[2]) + x/Sqrt[2] + Sqrt[1 - x^2]/Sqrt[2])/(Sqrt[1 - x]*(1 - x^2)^(1/4))]
)/(16*Sqrt[2]) + (5*ArcTanh[(Sqrt[2]*Sqrt[1 - x]*(1 - x^2)^(1/4))/(-1 + x -
 Sqrt[1 - x^2])])/(16*Sqrt[2]) + ArcTanh[(Sqrt[2]*Sqrt[1 + x]*(1 - x^2)^(1/
4))/(1 + x + Sqrt[1 - x^2])]/(16*Sqrt[2])
```

fricas [B] time = 1.29, size = 577, normalized size = 1.90

$$-\frac{1}{48}\left(8\,x^2+2\,x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+\frac{1}{48}\left(8\,x^2+22\,x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}-\frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x+1)\cdots}{\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^2 + 22*x +
 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - 1/16*sqrt(2)*arctan((sqrt(2)*(x + 1)*s
qrt((sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x + 1)
) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - 1)/(x + 1)) - 1/16*sqrt(2)*a
rctan((sqrt(2)*(x + 1)*sqrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sq
rt(-x^2 + 1) - 1)/(x + 1)) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + 1)/
(x + 1)) - 5/16*sqrt(2)*arctan((sqrt(2)*(x - 1)*sqrt((sqrt(2)*(-x^2 + 1)^(1
/4)*sqrt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - sqrt(2)*(-x^2 + 1)^(1
/4)*sqrt(-x + 1) - x + 1)/(x - 1)) - 5/16*sqrt(2)*arctan((sqrt(2)*(x - 1)*s
qrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x -
1)) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + x - 1)/(x - 1)) + 1/64*sqrt(2
)*log(4*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x
+ 1)) - 1/64*sqrt(2)*log(-4*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sqr
t(-x^2 + 1) - 1)/(x + 1)) + 5/64*sqrt(2)*log(4*(sqrt(2)*(-x^2 + 1)^(1/4)*sq
rt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - 5/64*sqrt(2)*log(-4*(sqrt(2
)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}\,x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="giac")
```

```
[Out] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-x^{2}+1\right)^{\frac{1}{4}} \sqrt{1+x}}{\sqrt{1-x}\, \left(\sqrt{1-x}-\sqrt{1+x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

```
[Out] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-x^{2}+1\right)^{\frac{1}{4}} \sqrt{x+1}\, x^{2}}{\sqrt{-x+1} \left(\sqrt{x+1}-\sqrt{-x+1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="maxima")
```

```
[Out] -integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^{2} \left(1-x^{2}\right)^{1/4} \sqrt{x+1}}{\left(\sqrt{x+1}-\sqrt{1-x}\right) \sqrt{1-x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*(
1 - x)^(1/2)),x)
```

```
[Out] -int((x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*(
1 - x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1+
x)**(1/2)),x)
```

```
[Out] Timed out
```

3.222 $\int \frac{\sqrt{1-x}\,x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$

Optimal. Leaf size=292

$$-\frac{1}{12}(1-x)^{2/3}\sqrt[3]{x+1}\,(1-3x)-\frac{1}{4}(1-x)(x+3)+\frac{1}{12}\sqrt[3]{1-x}\,(x+1)^{2/3}(3x+1)+\frac{1}{12}\sqrt[6]{1-x}\,(x+1)^{5/6}(3x+2)-\frac{1}{12}(1-x)^{5/6}\sqrt[6]{}$$

Rubi [A] time = 1.85, antiderivative size = 522, normalized size of antiderivative = 1.79, number of steps used = 46, number of rules used = 21, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6688, 6742, 50, 60, 517, 195, 216, 675, 890, 63, 240, 209, 634, 618, 204, 628, 203, 26, 21, 331, 295}

$$\frac{x^2}{4}+\frac{1}{4}\sqrt{1-x^2}\,x+\frac{x}{2}-\frac{1}{4}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{4}(1-x)^{7/6}(x+1)^{5/6}+\frac{5}{12}\sqrt[6]{1-x}\,(x+1)^{5/6}-\frac{1}{4}(1-x)^{4/3}(x+1)^{2/3}+\frac{1}{3}\sqrt[3]{1-x}\,(x$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)
^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] x/2 + x^2/4 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/12 + ((1 - x)^(2/3)*(1 + x)^(
1/3))/6 - ((1 - x)^(5/3)*(1 + x)^(1/3))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3))/3
 - ((1 - x)^(4/3)*(1 + x)^(2/3))/4 + (5*(1 - x)^(1/6)*(1 + x)^(5/6))/12 - (
(1 - x)^(7/6)*(1 + x)^(5/6))/4 - ((1 - x)^(5/6)*(1 + x)^(7/6))/4 + (x*Sqrt[
1 - x^2])/4 + ArcSin[x]/4 - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3 + (2*
ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3])
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - (2*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/
(Sqrt[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - Log[1 - x]/9 + Log[1 + x]/9 + Log[1
 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (
Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/
(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) - Log[1
 + (1 + x)^(1/3)/(1 - x)^(1/3)]/3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 890

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{1-x}\,x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \int \frac{x\sqrt[3]{1+x}}{-\sqrt[3]{1-x}+\sqrt[6]{1-x^2}}\,dx$$

$$= \int \left(\frac{1}{2}(1-x)^{2/3}\sqrt[3]{1+x} + \frac{1}{2}\sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2} + \frac{1}{2}\sqrt[3]{1+x}\sqrt[3]{1-\right.$$

$$= \frac{1}{2}\int (1-x)^{2/3}\sqrt[3]{1+x}\,dx + \frac{1}{2}\int \sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2}\,dx + \frac{1}{2}\int$$

$$= -\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x} + \frac{1}{6}\int \frac{(1-x)^{2/3}}{(1+x)^{2/3}}\,dx + \frac{1}{2}\int \sqrt[3]{1-x}\,(1+x)^{2/3}\,d$$

$$= \frac{x}{2} + \frac{x^2}{4} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{4/3}(1+$$

$$= \frac{x}{2} + \frac{x^2}{4} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x} + \frac{1}{3}\sqrt[3]{1-x}\,(1+$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{}$$

Mathematica [C] time = 0.78, size = 348, normalized size = 1.19

$$-\frac{5\sqrt{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1-x}{2}\right)}{6\sqrt[6]{2}\sqrt[3]{1-x}\sqrt{x+1}}-\frac{2^{2/3}\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{3\sqrt[3]{x+1}}-\frac{1}{12}\sqrt[3]{x+1}\left(-4\,2^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)+\frac{(3x+}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (
1 - x)^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] ((1 - x)^(5/6)*(1 + x)^(5/6)*ArcSin[x])/(4*(1 - x^2)^(5/6)) - (5*Sqrt[1 - x
^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2])/(6*2^(1/6)*(1 - x)^(1/3)*S
qrt[1 + x]) - (2^(2/3)*(1 - x^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1
- x)/2])/(3*(1 + x)^(1/3)) - ((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(
1 - x)^(1/3)*x*(2 + x))/(1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6)
 - (1 + 3*x)*(1 - x^2)^(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((
10 + 3*x)*(1 - x^2)^(5/6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3,
4/3, (1 + x)/2]))/12 - (7*(1 - x^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6,
 (1 - x)/2])/(30*2^(5/6)*(1 + x)^(5/6))
```

IntegrateAlgebraic [F] time = 117.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\,x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(
1/3)) + (1 - x)^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.28, size = 865, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x +
 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1)*
(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5/6)
 - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(-x +
 1)^(1/3))/(x + 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(
x + 1)^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 5/72*sqrt(3)*log(100*(sqrt(3)*(x +
1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) +
5/72*sqrt(3)*log(-100*(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(
2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 7/72*sqrt(3)*log(196*(sqrt(3)*(x + 1)^(
1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 7/72
*sqrt(3)*log(-196*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)^(1/3)
*(-x + 1)^(2/3) + 1)/(x - 1)) + 1/2*x + 5/18*arctan(-(sqrt(3)*(x + 1) - 2*(
x + 1)*sqrt((sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x +
 1)^(1/3) + 1)/(x + 1)) + 2*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 5/18*a
rctan((sqrt(3)*(x + 1) + 2*(x + 1)*sqrt(-(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1
/6) - x - (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 2*(x + 1)^(5/6)*(-x
+ 1)^(1/6))/(x + 1)) + 7/18*arctan(-(sqrt(3)*(x - 1) - 2*(x - 1)*sqrt((sqrt
(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x
 - 1)) + 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) + 7/18*arctan((sqrt(3)*(x
```

```
- 1) + 2*(x - 1)*sqrt(-(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)
^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) - 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x -
1)) - 5/18*arctan((-x + 1)^(1/6)/(x + 1)^(1/6)) - 7/18*arctan((x + 1)^(1/6)
*(-x + 1)^(5/6)/(x - 1)) - 1/2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2
/9*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) + 1/9*log((x - (x +
1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1)^(2/3) + 1)/(x + 1)) - 1/9*
log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(
x - 1)) + 2/9*log(-(x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F**] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\,(1+x)^{\frac{2}{3}}\,\sqrt{1-x}}{-(1-x)^{\frac{5}{6}}\,(1+x)^{\frac{1}{3}}+(1-x)^{\frac{2}{3}}\,\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

```
[Out] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{2}{3}}x\sqrt{-x+1}}{\sqrt{x+1}\,(-x+1)^{\frac{2}{3}}-(x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1
)^(1/3)*(-x + 1)^(5/6)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{1-x}\,{(x+1)}^{2/3}}{{(1-x)}^{2/3}\,\sqrt{x+1}-{(1-x)}^{5/6}\,{(x+1)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^
(5/6)*(x + 1)^(1/3)),x)
```

```
[Out] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^
(5/6)*(x + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{1-x}\,(x+1)^{\frac{2}{3}}}{-(1-x)^{\frac{5}{6}}\sqrt[3]{x+1}+(1-x)^{\frac{2}{3}}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(2
/3)*(1+x)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(1 - x)*(x + 1)**(2/3)/(-(1 - x)**(5/6)*(x + 1)**(1/3) + (1
- x)**(2/3)*sqrt(x + 1)), x)
```

3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6719, 37}

$$\frac{3(1-x)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx &= \frac{\left((-1+x)^{4/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}}\,dx}{\sqrt[3]{(-1+x)^4(1+x)^2}}\\ &= \frac{3(1-x)(1+x)}{2\sqrt[3]{(1-x)^4(1+x)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.42, size = 46, normalized size = 1.84

$$-\frac{3\left(x^6-2x^5-x^4+4x^3-x^2-2x+1\right)^{2/3}}{2(x-1)^3(x+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (-3*(1 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6)^(2/3))/(2*(-1 + x)^3*(1 + x
))
```

fricas [B] time = 0.94, size = 47, normalized size = 1.88

$$-\frac{3\left(x^{6}-2x^{5}-x^{4}+4x^{3}-x^{2}-2x+1\right)^{\frac{2}{3}}}{2\left(x^{4}-2x^{3}+2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x -
 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)^{2}(x-1)^{4}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

maple [A] time = 0.04, size = 22, normalized size = 0.88

method	result	size
gosper	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^{4}(1+x)^{2}\right)^{\frac{1}{3}}}$	22
risch	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^{4}(1+x)^{2}\right)^{\frac{1}{3}}}$	22
trager	$-\frac{3\left(x^{6}-2x^{5}-x^{4}+4x^{3}-x^{2}-2x+1\right)^{\frac{2}{3}}}{2(1+x)(-1+x)^{3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^4*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)^{2}(x-1)^{4}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

mupad [**B**] time = 0.24, size = 25, normalized size = 1.00

$$-\frac{3\left((x-1)^4 (x+1)^2\right)^{2/3}}{2(x-1)^3 (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^4*(x + 1)^2)^(1/3),x)
[Out] -(3*((x - 1)^4*(x + 1)^2)^(2/3))/(2*(x - 1)^3*(x + 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^4 (x+1)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

Optimal. Leaf size=25

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6719, 37}

$$-\frac{4(1-x)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

```
[Out] (-4*(1 - x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx &= \frac{\left((-1+x)^{3/4}(2+x)^{5/4}\right)\int \frac{1}{(-1+x)^{3/4}(2+x)^{5/4}}\,dx}{\sqrt[4]{(-1+x)^3(2+x)^5}}\\ &= -\frac{4(1-x)(2+x)}{3\sqrt[4]{-(1-x)^3(2+x)^5}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

```
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

IntegrateAlgebraic [A] time = 26.03, size = 25, normalized size = 1.00

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

```
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

fricas [B] time = 0.90, size = 69, normalized size = 2.76

$$\frac{4 \left(x^8 + 7 x^7 + 13 x^6 - 11 x^5 - 50 x^4 - 8 x^3 + 64 x^2 + 16 x - 32\right)^{\frac{3}{4}}}{3 \left(x^6 + 6 x^5 + 9 x^4 - 8 x^3 - 24 x^2 + 16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^(
3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+2)^5 (x-1)^3\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

maple [A] time = 0.06, size = 22, normalized size = 0.88

method	result	size
gosper	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^3(2+x)^5\right)^{\frac{1}{4}}}$	22
risch	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^3(2+x)^5\right)^{\frac{1}{4}}}$	22
trager	$\frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3(-1+x)^2(2+x)^4}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^3*(2+x)^5)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+2)^5 (x-1)^3\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

mupad [B] time = 0.26, size = 25, normalized size = 1.00

$$\frac{4 \left((x-1)^3 (x+2)^5\right)^{3/4}}{3 (x-1)^2 (x+2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^3*(x + 2)^5)^(1/4),x)
```

```
[Out] (4*((x - 1)^3*(x + 2)^5)^(3/4))/(3*(x - 1)^2*(x + 2)^4)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{(x-1)^3 (x+2)^5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
```

```
[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

Optimal. Leaf size=53

$$\frac{9(x-1)^2(x+1)}{16\sqrt[3]{(x-1)^7(x+1)^2}}-\frac{3(x-1)(x+1)}{8\sqrt[3]{(x-1)^7(x+1)^2}}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6719, 45, 37}

$$\frac{9(x+1)(1-x)^2}{16\sqrt[3]{-(1-x)^7(x+1)^2}}+\frac{3(x+1)(1-x)}{8\sqrt[3]{-(1-x)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(8*(-((1 - x)^7*(1 + x)^2))^(1/3)) + (9*(1 - x)^2*(1 +
x))/(16*(-((1 - x)^7*(1 + x)^2))^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx &= \frac{\left((-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{7/3}(1+x)^{2/3}}\,dx}{\sqrt[3]{(-1+x)^7(1+x)^2}}\\
&= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}}-\frac{\left(3(-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}}\,dx}{8\sqrt[3]{(-1+x)^7(1+x)^2}}\\
&= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}}+\frac{9(1-x)^2(1+x)}{16\sqrt[3]{-(1-x)^7(1+x)^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.57

$$\frac{3(x-1)(x+1)(3x-5)}{16\sqrt[3]{(x-1)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

IntegrateAlgebraic [A] time = 2.52, size = 56, normalized size = 1.06

$$\frac{3(3x-5)\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{2/3}}{16(x-1)^6(x+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(-5 + 3*x)*(-1 + 5*x - 8*x^2 + 14*x^4 - 14*x^5 + 8*x^7 - 5*x^8 + x^9)^(2
/3))/(16*(-1 + x)^6*(1 + x))
```

fricas [A] time = 0.91, size = 77, normalized size = 1.45

$$\frac{3\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{\frac{2}{3}}(3x-5)}{16\left(x^7-5x^6+9x^5-5x^4-5x^3+9x^2-5x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x -
 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

maple [A] time = 0.05, size = 27, normalized size = 0.51

method	result	size
gosper	$\frac{3(1+x)(-1+x)(3x-5)}{16\left((-1+x)^7(1+x)^2\right)^{\frac{1}{3}}}$	27
risch	$\frac{3(-1+x)\left(3x^2-2x-5\right)}{16\left((-1+x)^7(1+x)^2\right)^{\frac{1}{3}}}$	29
trager	$\frac{3(3x-5)\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{\frac{2}{3}}}{16(-1+x)^6(1+x)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^7*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*(1+x)*(-1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

mupad [B] time = 0.23, size = 30, normalized size = 0.57

$$\frac{3\,(3\,x-5)\,\left((x-1)^7\,(x+1)^2\right)^{2/3}}{16\,(x-1)^6\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^7*(x + 1)^2)^(1/3),x)
```

```
[Out] (3*(3*x - 5)*((x - 1)^7*(x + 1)^2)^(2/3))/(16*(x - 1)^6*(x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^7\,(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)}{\sqrt[3]{(x-1)^2(x+1)}}+1}{\sqrt{3}}\right)$$

Rubi [B] time = 0.12, antiderivative size = 188, normalized size of antiderivative = 2.81, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2067, 2064, 60}

$$-\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{8}{3}(x-1)\right)}{2\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}}-\frac{\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{3-3x}}+1\right)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] -(((3 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(3 - 3*x)^(1/3))])/(3^(1/6)*(1 - x - x^2 + x^3)^(1/3))) - ((3 - 3*x)^(2
/3)*(1 + x)^(1/3)*Log[(-8*(-1 + x))/3])/(2*3^(2/3)*(1 - x - x^2 + x^3)^(1/3
)) - (3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 + x)^(1/3))
/(3 - 3*x)^(1/3)])/(2*(1 - x - x^2 + x^3)^(1/3))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, -\frac{1}{3}+x\right)$$
$$= \frac{\left(4\,2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right)}{3\sqrt[3]{1-x-x^2+x^3}}$$
$$= -\frac{\sqrt{3}\,(1-x)^{2/3}\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{\sqrt[3]{1-x-x^2+x^3}} - \frac{(1-x)^{2/3}\sqrt[3]{1+x}\,\log(1-x)}{2\sqrt[3]{1-x-x^2+x^3}} - \frac{3(1-x}{}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.73

$$\frac{3(x-1)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{\sqrt[3]{2}\,\sqrt[3]{(x-1)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - x)/2])/(2^(
1/3)*((-1 + x)^2*(1 + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 132, normalized size = 1.97

$$-\log\left(\sqrt[3]{x^3-x^2-x+1}-x+1\right)+\frac{1}{2}\log\left(x^2+\left(x^3-x^2-x+1\right)^{2/3}+(x-1)\sqrt[3]{x^3-x^2-x+1}-2x+1\right)-\sqrt{3}\tan^{-}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x - x^2 + x^3)^(1/3))/(-2 + 2*x + (1 - x - x
^2 + x^3)^(1/3))]) - Log[1 - x + (1 - x - x^2 + x^3)^(1/3)] + Log[1 - 2*x +
 x^2 + (-1 + x)*(1 - x - x^2 + x^3)^(1/3) + (1 - x - x^2 + x^3)^(2/3)]/2
```

fricas [B] time = 0.90, size = 128, normalized size = 1.91

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x-1)+2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3\,(x-1)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2\,x+\left(x^3-x^2-x+\right.}{x^2-2\,x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3))
/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 -
 x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (x^3 - x^2 - x + 1)^(
1/3) - 1)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

maple [C] time = 0.44, size = 370, normalized size = 5.52

method	result
trager	$-\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2-4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2-x+1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^2*(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((4*RootOf(_Z^2-_Z+1)^2*x^2-4*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*
(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)*x-4*RootOf(_Z^2
-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)+2*RootOf(_Z^2-_Z+1)*x+3*
(x^3-x^2-x+1)^(1/3)*x+x^2+2*RootOf(_Z^2-_Z+1)-3*(x^3-x^2-x+1)^(1/3)-1)/(-1+
x))+RootOf(_Z^2-_Z+1)*ln(-(2*RootOf(_Z^2-_Z+1)^2*x^2-2*RootOf(_Z^2-_Z+1)^2*
x+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(2/3)-5*RootOf(_Z^2-_Z+1)*x^2+6*RootOf(
_Z^2-_Z+1)*x-3*(x^3-x^2-x+1)^(2/3)+3*(x^3-x^2-x+1)^(1/3)*x+2*x^2-RootOf(_Z^
2-_Z+1)-3*(x^3-x^2-x+1)^(1/3)-4*x+2)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left((x-1)^2\,(x+1)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^2*(x + 1))^(1/3),x)
```

```
[Out] int(1/((x - 1)^2*(x + 1))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^2\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

Optimal. Leaf size=122

$$-\frac{4(x-2)(x+1)}{3\sqrt{(x-2)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}\,(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{(x-2)(x+1)^3}}+\frac{2\sqrt{x-2}\,(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{(x-2)(x+1)^3}}$$

Rubi [A] time = 0.35, antiderivative size = 133, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1593, 6719, 1614, 21, 105, 54, 215, 93, 204}

$$\frac{4(2-x)(x+1)}{3\sqrt{-(2-x)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}\,(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{-(2-x)(x+1)^3}}+\frac{2\sqrt{x-2}\,(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(x+1)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] (4*(2 - x)*(1 + x))/(3*Sqrt[-((2 - x)*(1 + x)^3)]) + (2*Sqrt[-2 + x]*(1 + x
)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[-((2 - x)*(1 + x)^3)] - (Sqrt[2
]*Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]])/Sq
rt[-((2 - x)*(1 + x)^3)]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx &= \int \frac{1+x^2}{x\sqrt{(-2+x)(1+x)^3}}\,dx \\
&= \frac{\left(\sqrt{-2+x}\,(1+x)^{3/2}\right)\int \frac{1+x^2}{\sqrt{-2+x}\,x(1+x)^{3/2}}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} - \frac{\left(2\sqrt{-2+x}\,(1+x)^{3/2}\right)\int \frac{-\frac{3}{2}-\frac{3x}{2}}{\sqrt{-2+x}\,x\sqrt{1+x}}\,dx}{3\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}\,(1+x)^{3/2}\right)\int \frac{\sqrt{1+x}}{\sqrt{-2+x}\,x}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}\,(1+x)^{3/2}\right)\int \frac{1}{\sqrt{-2+x}\,\sqrt{1+x}}\,dx}{\sqrt{(-2+x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}\,(1+x)^{3/\cdots}\right.}{\sqrt{(-2+\cdots}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(2\sqrt{-2+x}\,(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-1-2x^2}\,dx, x, \frac{\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}} + \frac{\left(2\sqrt{\cdots}\right.}{\cdots} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{2\sqrt{-2+x}\,(1+x)^{3/2}\sinh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(1+x)^3}} - \frac{\sqrt{2}\,\sqrt{-2+x}\,(1+x)^{\cdots}}{\sqrt{-(2-x\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 114, normalized size = 0.93

$$-\frac{(x+1)\left(-4(2-x)^{3/2}-6(x-2)\sqrt{x+1}\,\sin^{-1}\left(\frac{\sqrt{2-x}}{\sqrt{3}}\right)-3\sqrt{2}\sqrt{-(x-2)^2}\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{\frac{x-2}{x+1}}}{\sqrt{2}}\right)\right)}{3\sqrt{2-x}\sqrt{(x-2)(x+1)^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] -1/3*((1 + x)*(-4*(2 - x)^(3/2) - 6*(-2 + x)*Sqrt[1 + x]*ArcSin[Sqrt[2 - x]
/Sqrt[3]] - 3*Sqrt[2]*Sqrt[-(-2 + x)^2]*Sqrt[1 + x]*ArcTan[Sqrt[(-2 + x)/(1
 + x)]/Sqrt[2]]))/(Sqrt[2 - x]*Sqrt[(-2 + x)*(1 + x)^3])
```

IntegrateAlgebraic [A] time = 0.37, size = 97, normalized size = 0.80

$$-\frac{4\sqrt{x^4+x^3-3x^2-5x-2}}{3(x+1)^2}+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{x^4+x^3-3x^2-5x-2}}{\sqrt{2}\,(x+1)^2}\right)+2\tanh^{-1}\left(\frac{\sqrt{x^4+x^3-3x^2-5x-2}}{(x+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] (-4*Sqrt[-2 - 5*x - 3*x^2 + x^3 + x^4])/(3*(1 + x)^2) + Sqrt[2]*ArcTan[Sqrt
[-2 - 5*x - 3*x^2 + x^3 + x^4]/(Sqrt[2]*(1 + x)^2)] + 2*ArcTanh[Sqrt[-2 - 5
*x - 3*x^2 + x^3 + x^4]/(1 + x)^2]
```

fricas [A] time = 0.79, size = 142, normalized size = 1.16

$$\frac{3\sqrt{2}\left(x^2+2x+1\right)\arctan\left(-\frac{\sqrt{2}\left(x^2+x\right)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3\left(x^2+2x+1\right)\log\left(-\frac{2x^2+x-2\sqrt{x^4+x^3-3x^2-5x-\cdots}}{x+1}\cdots\right.}{3\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sqr
t(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(-(
2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*sqr
t(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)
```

giac [A] time = 0.83, size = 177, normalized size = 1.45

$$-\frac{\sqrt{2}\arcsin\left(\frac{4}{3x}+\frac{1}{3}\right)}{2\,\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}+\frac{\log\left(\left|2\sqrt{2}+\frac{2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3}{\frac{4}{x}+1}+3\right|\right)}{\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}-\frac{\log\left(\left|-2\sqrt{2}+\frac{2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3}{\frac{4}{x}+1}+3\right|\right)}{\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}+\frac{\cdots}{3\left(\frac{2\sqrt{2}\sqrt{-\frac{1}{x}-\cdots}}{\frac{4}{x}+\cdots}\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arcsin(4/3/x + 1/3)/sgn(1/x^2 + 1/x^3) + log(abs(2*sqrt(2) + (
2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) + 3))/sgn(1/x^2 + 1/x^3) -
log(abs(-2*sqrt(2) + (2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) + 3))
/sgn(1/x^2 + 1/x^3) + 8/3*sqrt(2)/(((2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/
(4/x + 1) - 1)*sgn(1/x^2 + 1/x^3))
```

maple [A] time = 0.24, size = 86, normalized size = 0.70

method	result
risch	$-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^{3}}}+\frac{\left(\ln\left(-\frac{1}{2}+x+\sqrt{x^{2}-x-2}\right)+\frac{\sqrt{2}\,\arctan\left(\frac{(-4-x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)}{2}\right)(1+x)\sqrt{(1+x)(-2+x)}}{\sqrt{(-2+x)(1+x)^{3}}}$
default	$\frac{\left(-3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)x+6\ln\left(-\frac{1}{2}+x+\sqrt{x^{2}-x-2}\right)x-3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)+6\ln\left(-\frac{1}{2}+x+\sqrt{x^{2}-x-2}\right)-8\sqrt{x^{2}-x-2}\right)\sqrt{(1+x)(-}{6\sqrt{(-2+x)(1+x)^{3}}}$
trager	$-\frac{4\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}}{3(1+x)^{2}}+\ln\left(\frac{2x^{2}+2\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}+x-1}{1+x}\right)+\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)x^{2}+5\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\sqrt{x}}{x(1+x)}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+(ln(-1/2+x+(x^2-x-2)^(1/2))+1/2*2^
(1/2)*arctan(1/4*(-4-x)*2^(1/2)/(x^2-x-2)^(1/2)))/((-2+x)*(1+x)^3)^(1/2)*(1
+x)*((1+x)*(-2+x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^{3}(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^{3}\,(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2),x)
```

```
[Out] int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+1}{x\sqrt{(x-2)\,(x+1)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{(x-1)^2(x+1)}}{x}+\frac{\log(x)}{6}-\frac{2}{3}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)-\frac{1}{2}\log\left(\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\ldots}{\ldots}\right)}{\ldots}$$

Rubi [B] time = 0.33, antiderivative size = 404, normalized size of antiderivative = 2.69, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2081, 2077, 97, 157, 60, 91}

$$-\frac{\sqrt[3]{x^3-x^2-x+1}}{x}+\frac{\sqrt[3]{x^3-x^2-x+1}\log(x)}{2\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}}-\frac{3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{4(x+1)}{3}\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}}-\frac{3\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{\ldots}{\ldots}\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] -((1 - x - x^2 + x^3)^(1/3)/x) - (3*3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTa
n[1/Sqrt[3] - (2*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3
)*(1 + x)^(1/3)) - (3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTan[1/Sqrt[3] + (2
*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3)*(1 + x)^(1/3))
 + ((1 - x - x^2 + x^3)^(1/3)*Log[x])/(2*3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1
/3)) - (3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(4*(1 + x))/3])/(2*(3 - 3*x)^
(2/3)*(1 + x)^(1/3)) - (3*3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[1 + (3 - 3*
x)^(1/3)/(3^(1/3)*(1 + x)^(1/3))])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3)) - (3^(
2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(2/3)^(2/3)*(3 - 3*x)^(1/3) - (2^(2/3)*(
1 + x)^(1/3))/3^(1/3)])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \text{Subst}\left(\int \frac{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)}{4\,2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} + \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{-\frac{64}{27}-\frac{32x}{9}}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\right.}{4\,2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\left(4\sqrt[3]{2}\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx,\right.}{9(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\sqrt{3}\sqrt[3]{1-x-x^2+x^3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{(1-x)^{2/3}\sqrt[3]{1+x}} - \frac{\sqrt[3]{1-x-x^2+}}{\sqrt{3}}$$

Mathematica [C] time = 0.07, size = 112, normalized size = 0.75

$$\frac{\sqrt[3]{(x-1)^2(x+1)}\left(3(x+1)\left(3\,2^{2/3}\sqrt[3]{1-x}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)-2x+2\right)-2\left(\frac{1}{x}+1\right)^{2/3}\sqrt[3]{\frac{x-1}{x}}\,xF_1\left(1;\frac{1}{3},\frac{2}{3};2;\frac{1}{x},-\frac{1}{x}\right.\right.}{6x\left(x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] (((-1 + x)^2*(1 + x))^(1/3)*(-2*(1 + x^(-1))^(2/3)*((-1 + x)/x)^(1/3)*x*App
ellF1[1, 1/3, 2/3, 2, x^(-1), -x^(-1)] + 3*(1 + x)*(2 - 2*x + 3*2^(2/3)*(1
- x)^(1/3)*x*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2])))/(6*x*(-1 + x^2)
)
```

IntegrateAlgebraic [A] time = 8.04, size = 247, normalized size = 1.65

$$\sqrt[3]{x-1}\,(x+1)^{2/3}\sqrt[3]{(x-1)^2(x+1)}\left(-\frac{(x-1)^{2/3}\sqrt[3]{x+1}}{x}-\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)-\frac{1}{3}\log\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)+\frac{1}{6}\log\left((\right.\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] ((-1 + x)^(1/3)*(1 + x)^(2/3)*((-1 + x)^2*(1 + x))^(1/3)*(-(((-1 + x)^(2/3)
*(1 + x)^(1/3))/x) - ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x)^(1/3) - (1
+ x)^(1/3))]/Sqrt[3] + Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x)^(
1/3) + (1 + x)^(1/3))] - Log[(-1 + x)^(1/3) - (1 + x)^(1/3)] - Log[(-1 + x)
^(1/3) + (1 + x)^(1/3)]/3 + Log[(-1 + x)^(2/3) - (-1 + x)^(1/3)*(1 + x)^(1/
3) + (1 + x)^(2/3)]/6 + Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) +
 (1 + x)^(2/3)]/2))/(-1 + x^2)
```

fricas [B] time = 0.88, size = 280, normalized size = 1.87

$$6\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3(x-1)}\right)-2\sqrt{3}\,x\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3(x-1)}\right)+3\,x\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1
)^(1/3))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*(x
^3 - x^2 - x + 1)^(1/3))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(1/3
)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*log((
x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) +
 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1))
 - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 - x^2 - x
 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

maple [C] time = 4.80, size = 1247, normalized size = 8.31

method	result

risch	$-\frac{\left((-1+x)^{2}(1+x)\right)^{\frac{1}{3}}}{x}+\left(-\frac{\ln\left(-\frac{-157880368143+288529720857x+4262769939861x^{5}-2841846626574x^{3}+4262769939861x^{4}-2395436537574x^{2}+3346\cdots}{\cdots}\right)}{\cdots}\right.$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^2*(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((-1+x)^2*(1+x))^(1/3)/x+(-1/3*ln(-(-157880368143+288529720857*x+426276993
9861*x^5-2841846626574*x^3+4262769939861*x^4-2395436537574*x^2-108655360*Ro
otOf(_Z^2-3*_Z+9)^2-12395048712*RootOf(_Z^2-3*_Z+9)+2933694720*RootOf(_Z^2-
3*_Z+9)^2*x^4-1955796480*RootOf(_Z^2-3*_Z+9)^2*x^3-21459433600*RootOf(_Z^2-
3*_Z+9)^2*x^2-19612292480*RootOf(_Z^2-3*_Z+9)^2*x+334666315224*RootOf(_Z^2-
3*_Z+9)*x^5+1030402198152*(x^3+x^2-x-1)^(2/3)*x^3-5266768885533*(x^3+x^2-x-
1)^(1/3)*x^4+343467399384*(x^3+x^2-x-1)^(2/3)*x^2-3511179257022*(x^3+x^2-x-
1)^(1/3)*x^3+65021838093*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-1144891331
28*(x^3+x^2-x-1)^(2/3)*x+2340786171348*(x^3+x^2-x-1)^(1/3)*x^2+52300823301*
RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)+390131028558*(x^3+x^2-x-1)^(1/3)*x+
2933694720*RootOf(_Z^2-3*_Z+9)^2*x^5-195065514279*(x^3+x^2-x-1)^(1/3)-38163
044376*(x^3+x^2-x-1)^(2/3)-1755589628511*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^
(2/3)*x^3+1412122229127*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^4-5851965
42837*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^2+941414819418*RootOf(_Z^2-
3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^3+195065514279*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x
-1)^(2/3)*x-627609879612*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^2-104601
646602*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x-223110876816*RootOf(_Z^2-3
*_Z+9)*x^3-477460395840*RootOf(_Z^2-3*_Z+9)*x^2-266744567736*RootOf(_Z^2-3*
_Z+9)*x+334666315224*RootOf(_Z^2-3*_Z+9)*x^4)/x/(1+x))+1/9*RootOf(_Z^2-3*_Z
+9)*ln(-(33401336760+117256110840*x-901836092520*x^5+601224061680*x^3-90183
6092520*x^4+685078835760*x^2-529078624*RootOf(_Z^2-3*_Z+9)^2+53888059173*Ro
otOf(_Z^2-3*_Z+9)+14285122848*RootOf(_Z^2-3*_Z+9)^2*x^4-9523415232*RootOf(_
Z^2-3*_Z+9)^2*x^3-104493028240*RootOf(_Z^2-3*_Z+9)^2*x^2-95498691632*RootOf
(_Z^2-3*_Z+9)^2*x-1454977597671*RootOf(_Z^2-3*_Z+9)*x^5-4236366687381*(x^3+
x^2-x-1)^(2/3)*x^3+5266768885533*(x^3+x^2-x-1)^(1/3)*x^4-1412122229127*(x^3
+x^2-x-1)^(2/3)*x^2+3511179257022*(x^3+x^2-x-1)^(1/3)*x^3-65021838093*RootO
f(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)+470707409709*(x^3+x^2-x-1)^(2/3)*x-23407
86171348*(x^3+x^2-x-1)^(1/3)*x^2-12721014792*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x
-1)^(1/3)-390131028558*(x^3+x^2-x-1)^(1/3)*x+14285122848*RootOf(_Z^2-3*_Z+9
)^2*x^5+195065514279*(x^3+x^2-x-1)^(1/3)+156902469903*(x^3+x^2-x-1)^(2/3)+1
755589628511*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^3-343467399384*RootO
f(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^4+585196542837*RootOf(_Z^2-3*_Z+9)*(x^
3+x^2-x-1)^(2/3)*x^2-228978266256*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x
^3-195065514279*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x+152652177504*Root
Of(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^2+25442029584*RootOf(_Z^2-3*_Z+9)*(x^
3+x^2-x-1)^(1/3)*x+969985065114*RootOf(_Z^2-3*_Z+9)*x^3+1047579629778*RootO
f(_Z^2-3*_Z+9)*x^2+131482623837*RootOf(_Z^2-3*_Z+9)*x-1454977597671*RootOf(
_Z^2-3*_Z+9)*x^4)/x/(1+x)))*((-1+x)^2*(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/(
-1+x)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)(x-1)^{2}\right)^{\frac{1}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left((x-1)^2\,(x+1)\right)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^2*(x + 1))^(1/3)/x^2,x)
[Out] int(((x - 1)^2*(x + 1))^(1/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x-1)^2\,(x+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
[Out] Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}} dx$

Optimal. Leaf size=43

$$\frac{1-x}{12(x^2-2x-3)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{1-x}{12(x^2-2x-3)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx &= \frac{1-x}{12(-3-2x+x^2)^{3/2}}-\frac{1}{6}\int \frac{1}{(-3-2x+x^2)^{3/2}}\,dx\\ &= \frac{1-x}{12(-3-2x+x^2)^{3/2}}-\frac{1-x}{24\sqrt{-3-2x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.63

$$\frac{(x-1)(x^2-2x-5)}{24(x^2-2x-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] ((-1 + x)*(-5 - 2*x + x^2))/(24*(-3 - 2*x + x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 39, normalized size = 0.91

$$\frac{\sqrt{x^2-2x-3}\left(x^3-3x^2-3x+5\right)}{24(x-3)^2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (Sqrt[-3 - 2*x + x^2]*(5 - 3*x - 3*x^2 + x^3))/(24*(-3 + x)^2*(1 + x)^2)
```

fricas [B] time = 1.07, size = 64, normalized size = 1.49

$$\frac{x^4-4\,x^3-2\,x^2+\left(x^3-3\,x^2-3\,x+5\right)\sqrt{x^2-2\,x-3}+12\,x+9}{24\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) + 1
2*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

giac [A] time = 0.66, size = 23, normalized size = 0.53

$$\frac{((x-3)x-3)x+5}{24\left(x^2-2\,x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

maple [A] time = 0.33, size = 26, normalized size = 0.60

method	result	size
trager	$\frac{x^3-3x^2-3x+5}{24\left(x^2-2x-3\right)^{\frac{3}{2}}}$	26
risch	$\frac{x^3-3x^2-3x+5}{24\left(x^2-2x-3\right)^{\frac{3}{2}}}$	26
gosper	$\frac{(1+x)(-3+x)\left(x^3-3x^2-3x+5\right)}{24\left(x^2-2x-3\right)^{\frac{5}{2}}}$	32
default	$-\frac{2x-2}{24\left(x^2-2x-3\right)^{\frac{3}{2}}}+\frac{2x-2}{48\sqrt{x^2-2x-3}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(3/2)
```

maxima [A] time = 0.53, size = 51, normalized size = 1.19

$$\frac{x}{24\sqrt{x^2-2\,x-3}}-\frac{1}{24\sqrt{x^2-2\,x-3}}-\frac{x}{12\left(x^2-2\,x-3\right)^{\frac{3}{2}}}+\frac{1}{12\left(x^2-2\,x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x -
 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

mupad [B] time = 0.05, size = 27, normalized size = 0.63

$$-\frac{(4\,x-4)\,\left(-8\,x^2+16\,x+40\right)}{768\,{\left(x^2-2\,x-3\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x - 3)^(5/2),x)
```

```
[Out] -((4*x - 4)*(16*x - 8*x^2 + 40))/(768*(x^2 - 2*x - 3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2x-3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-3)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 3)**(-5/2), x)
```

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=42

$$\frac{(3-x)\sqrt{x+1}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2067, 2064, 63, 206}

$$\frac{(3-x)\sqrt{x+1}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)$$
$$= \frac{\left(128(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\sqrt{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt{3}\sqrt{9+3x-5x^2+x^3}}$$
$$= \frac{\left(16(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\frac{128}{3}-2x^2}\,dx, x, \frac{4\sqrt{1+x}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{9+3x-5x^2+x^3}}$$
$$= \frac{(3-x)\sqrt{1+x}\tanh^{-1}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.88

$$-\frac{(x-3)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
[Out] -(((-3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)])
```

IntegrateAlgebraic [A] time = 0.03, size = 26, normalized size = 0.62

$$-\tanh^{-1}\left(\frac{2x-6}{\sqrt{x^3-5x^2+3x+9}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
[Out] -ArcTanh[(-6 + 2*x)/Sqrt[9 + 3*x - 5*x^2 + x^3]]
```

fricas [A] time = 0.74, size = 62, normalized size = 1.48

$$-\frac{1}{2}\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)+\frac{1}{2}\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*log(-(2*x -
 sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

giac [A] time = 0.65, size = 34, normalized size = 0.81

$$-\frac{\log\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sgn}(x-3)}+\frac{\log\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sgn}(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x
- 3)
```

maple [A] time = 0.14, size = 35, normalized size = 0.83

method	result	size
trager	$-\frac{\ln\left(\frac{x^2+4\sqrt{x^3-5x^2+3x+9}+2x-15}{(-3+x)^2}\right)}{2}$	35
default	$\frac{(-3+x)\sqrt{1+x}\,\left(\ln\left(\sqrt{1+x}-2\right)-\ln\left(\sqrt{1+x}+2\right)\right)}{2\sqrt{x^3-5x^2+3x+9}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((x^2+4*(x^3-5*x^2+3*x+9)^(1/2)+2*x-15)/(-3+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3 - 5\,x^2 + 3\,x + 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3 - 5\,x^2 + 3\,x + 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2),x)
```

```
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3 - 5x^2 + 3x + 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

3.231 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}}{512\left(x^3-5x^2+3x+9\right.}$$

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2067, 2064, 51, 63, 206}

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}}{512\left(x^3-5x^2+3x+9\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^2*(1 + x))
/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1 + x))/(256*(9 + 3*x
- 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)^(3/2)*ArcTanh[Sqrt[1 + x]/2])
/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
```

```
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(2097152(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^3\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)}{81\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(20480(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^2\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/}}\right.}{27\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(80(3-x)^3(1+x)^{3/2}\right)\text{Sub}}{3\sqrt{3}\ (9}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.25

$$\frac{(x-3)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{x+1}{4}\right)}{32\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((-3 + x)*Hypergeometric2F1[-1/2, 3, 1/2, (1 + x)/4])/(32*Sqrt[(-3 + x)^2*(
1 + x)])
```

IntegrateAlgebraic [A] time = 3.62, size = 67, normalized size = 0.48

$$\frac{(x-3)\sqrt{x+1}\left(\frac{15x^2-70x+43}{256(x-3)^2\sqrt{x+1}}-\frac{15}{512}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)\right)}{\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((-3 + x)*Sqrt[1 + x]*((43 - 70*x + 15*x^2)/(256*(-3 + x)^2*Sqrt[1 + x]) -
(15*ArcTanh[Sqrt[1 + x]/2])/512))/Sqrt[(-3 + x)^2*(1 + x)]
```

fricas [A] time = 0.56, size = 138, normalized size = 0.99

$$-\frac{15\left(x^4-8x^3+18x^2-27\right)\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-15\left(x^4-8x^3+18x^2-27\right)\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)}{1024\left(x^4-8x^3+18x^2-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x +
 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 - 5
*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 - 70*
x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.11, size = 71, normalized size = 0.51

method	result
risch	$\frac{15x^2-70x+43}{256(-3+x)\sqrt{(1+x)(-3+x)^2}}+\frac{\left(-\frac{15\ln\left(\sqrt{1+x}+2\right)}{1024}+\frac{15\ln\left(\sqrt{1+x}-2\right)}{1024}\right)\sqrt{1+x}\,(-3+x)}{\sqrt{(1+x)(-3+x)^2}}$
trager	$\frac{\left(15x^2-70x+43\right)\sqrt{x^3-5x^2+3x+9}}{256(-3+x)^3(1+x)}+\frac{15\ln\left(\frac{-x^2+4\sqrt{x^3-5x^2+3x+9}-2x+15}{(-3+x)^2}\right)}{1024}$
default	$\frac{(-3+x)^3(1+x)\left(15(1+x)^{\frac{5}{2}}\ln\left(\sqrt{1+x}-2\right)-15(1+x)^{\frac{5}{2}}\ln\left(\sqrt{1+x}+2\right)-120(1+x)^{\frac{3}{2}}\ln\left(\sqrt{1+x}-2\right)+120(1+x)^{\frac{3}{2}}\ln\left(\sqrt{1+x}+2\right)+240\ln\left(\sqrt{1+\right.\right.}{1024\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}\left(\sqrt{1+x}-2\right)^2\left(\sqrt{1+x}+2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*(15*x^2-70*x+43)/(-3+x)/((1+x)*(-3+x)^2)^(1/2)+(-15/1024*ln((1+x)^(1/
2)+2)+15/1024*ln((1+x)^(1/2)-2))/((1+x)*(-3+x)^2)^(1/2)*(1+x)^(1/2)*(-3+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2),x)
```

```
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}\log\left(1-\frac{x-3}{\sqrt[3]{x^3-5x^2+3x+9}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-3)}{\sqrt[3]{x^3-5x^2+3x+9}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Rubi [B] time = 0.12, antiderivative size = 188, normalized size of antiderivative = 2.51, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2067, 2064, 60}

$$-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{32}{3}(x-3)\right)}{2\,3^{2/3}\sqrt[3]{x^3-5x^2+3x+9}}-\frac{\sqrt[3]{3}(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{9-3x}}+1\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{\sqrt[6]{3}\sqrt[3]{x^3-5x^2+3x}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] -(((9 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(9 - 3*x)^(1/3))])/(3^(1/6)*(9 + 3*x - 5*x^2 + x^3)^(1/3))) - ((9 - 3*x
)^(2/3)*(1 + x)^(1/3)*Log[(-32*(-3 + x))/3])/(2*3^(2/3)*(9 + 3*x - 5*x^2 +
x^3)^(1/3)) - (3^(1/3)*(9 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 +
x)^(1/3))/(9 - 3*x)^(1/3)])/(2*(9 + 3*x - 5*x^2 + x^3)^(1/3))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(16\,2^{2/3}(3-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{2/3}\sqrt[3]{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt[3]{9+3x-5x^2+x^3}}$$

$$= -\frac{\sqrt{3}\,(3-x)^{2/3}\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{3-x}}\right)}{\sqrt[3]{9+3x-5x^2+x^3}} - \frac{(3-x)^{2/3}\sqrt[3]{1+x}\,\log(3-x)}{2\sqrt[3]{9+3x-5x^2+x^3}} - \frac{3(3-}{}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.65

$$\frac{3(x-3)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{3-x}{4}\right)}{2^{2/3}\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*(-3 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 - x)/4])/(2^(
2/3)*((-3 + x)^2*(1 + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 132, normalized size = 1.76

$$-\log\left(\sqrt[3]{x^3-5x^2+3x+9}-x+3\right)+\frac{1}{2}\log\left(x^2+\left(x^3-5x^2+3x+9\right)^{2/3}+(x-3)\sqrt[3]{x^3-5x^2+3x+9}-6x+9\right)-$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(9 + 3*x - 5*x^2 + x^3)^(1/3))/(-6 + 2*x + (9 + 3
*x - 5*x^2 + x^3)^(1/3))]) - Log[3 - x + (9 + 3*x - 5*x^2 + x^3)^(1/3)] + L
og[9 - 6*x + x^2 + (-3 + x)*(9 + 3*x - 5*x^2 + x^3)^(1/3) + (9 + 3*x - 5*x^
2 + x^3)^(2/3)]/2
```

fricas [A] time = 1.34, size = 128, normalized size = 1.71

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x-3)+2\,\sqrt{3}\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}}{3\,(x-3)}\right)+\frac{1}{2}\,\log\left(\frac{x^2+\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}(x-3)-6\,x+\left(x^3-\right.}{x^2-6\,x+9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 3) + 2*sqrt(3)*(x^3 - 5*x^2 + 3*x + 9)^(1
/3))/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3) - 6*x
+ (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) - log(-(x - (x^3 - 5*
x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

maple [C] time = 0.47, size = 672, normalized size = 8.96

method	result
trager	$\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\ln\left(-\frac{20\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2 x^2-60\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2 x+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}+27\,\mathrm{RootOf}\left(\_Z^2-3\_\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*RootOf(_Z^2-3*_Z+9)*ln(-(20*RootOf(_Z^2-3*_Z+9)^2*x^2-60*RootOf(_Z^2-3*
_Z+9)^2*x+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)+27*RootOf(_Z^2-3*_
Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x-33*RootOf(_Z^2-3*_Z+9)*x^2-81*RootOf(_Z^2-3*
_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)-6*RootOf(_Z^2-3*_Z+9)*x-216*(x^3-5*x^2+3*x+9)
^(2/3)-216*(x^3-5*x^2+3*x+9)^(1/3)*x-36*x^2+315*RootOf(_Z^2-3*_Z+9)+648*(x^
3-5*x^2+3*x+9)^(1/3)+360*x-756)/(-3+x))-1/3*ln(-(20*RootOf(_Z^2-3*_Z+9)^2*x
^2-60*RootOf(_Z^2-3*_Z+9)^2*x-27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3
)-27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x-87*RootOf(_Z^2-3*_Z+9)*x
^2+81*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)+366*RootOf(_Z^2-3*_Z+9)*x
-135*(x^3-5*x^2+3*x+9)^(2/3)-135*(x^3-5*x^2+3*x+9)^(1/3)*x+45*x^2-315*RootO
f(_Z^2-3*_Z+9)+405*(x^3-5*x^2+3*x+9)^(1/3)-198*x+189)/(-3+x))*RootOf(_Z^2-3
*_Z+9)+ln(-(20*RootOf(_Z^2-3*_Z+9)^2*x^2-60*RootOf(_Z^2-3*_Z+9)^2*x-27*Root
Of(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)-27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3
*x+9)^(1/3)*x-87*RootOf(_Z^2-3*_Z+9)*x^2+81*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+
3*x+9)^(1/3)+366*RootOf(_Z^2-3*_Z+9)*x-135*(x^3-5*x^2+3*x+9)^(2/3)-135*(x^3
-5*x^2+3*x+9)^(1/3)*x+45*x^2-315*RootOf(_Z^2-3*_Z+9)+405*(x^3-5*x^2+3*x+9)^
(1/3)-198*x+189)/(-3+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^3-5\,x^2+3\,x+9\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3),x)
```

```
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^{3}-5x^{2}+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

3.233 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=29

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2067, 2064, 37}

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(512\sqrt[3]{2}\,(3-x)^{4/3}(1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{4/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)}{9\left(9+3x-5x^2+x^3\right)^{2/3}}$$

$$= \frac{3(3-x)(1+x)}{4\left(9+3x-5x^2+x^3\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.79

$$-\frac{3(x-3)(x+1)}{4\left((x-3)^2(x+1)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
[Out] (-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

IntegrateAlgebraic [A] time = 3.22, size = 26, normalized size = 0.90

$$-\frac{3\sqrt[3]{x^3-5x^2+3x+9}}{4(x-3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
[Out] (-3*(9 + 3*x - 5*x^2 + x^3)^(1/3))/(4*(-3 + x))
```

fricas [A] time = 0.88, size = 22, normalized size = 0.76

$$-\frac{3\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}{4(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")
[Out] -3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="giac")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

maple [A] time = 0.03, size = 20, normalized size = 0.69

method	result	size
risch	$-\frac{3(-3+x)(1+x)}{4\left((1+x)(-3+x)^2\right)^{\frac{2}{3}}}$	20
trager	$-\frac{3\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}{4(-3+x)}$	23
gosper	$-\frac{3(1+x)(-3+x)}{4\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(2/3),x,method=_RETURNVERBOSE)
[Out] -3/4/((1+x)*(-3+x)^2)^(2/3)*(-3+x)*(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.83

$$-\frac{3\,(x^3 - 5\,x^2 + 3\,x + 9)^{1/3}}{4\,(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(2/3),x)
[Out] -(3*(3*x - 5*x^2 + x^3 + 9)^(1/3))/(4*(x - 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

3.234 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{4/3}}\,dx$

Optimal. Leaf size=92

$$-\frac{27(x+1)(3-x)^3}{320\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{3(x+1)(3-x)}{20\left(x^3-5x^2+3x+9\right)^{4/3}}$$

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2067, 2064, 45, 37}

$$-\frac{27(x+1)(3-x)^3}{320\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{3(x+1)(3-x)}{20\left(x^3-5x^2+3x+9\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 - x)^2*(1 + x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*(1 + x))/(320*(9 + 3*x - 5*x^2 + x^3)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*Simplify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{4/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(262144\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{8/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)}{81\left(9+3x-5x^2+x^3\right)^{4/3}}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{\left(4096\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{5/3}}\right.}{45\left(9+3x-5x^2+x^3\right)^{4/3}}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{\left(16\ 2^{2/3}(3-x)^{8/3}(1+\right.}{}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}} - \frac{27(3-x)^3(1+x)}{320\left(9+3x-5x^2+x\right.}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.35

$$\frac{3\left(9x^2-42x+29\right)}{320(x-3)\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(29 - 42*x + 9*x^2))/(320*(-3 + x)*((-3 + x)^2*(1 + x))^(1/3))
```

IntegrateAlgebraic [A] time = 4.88, size = 37, normalized size = 0.40

$$\frac{3(x-3)(x+1)\left(9(x+1)^2-60(x+1)+80\right)}{320\left((x-3)^2(x+1)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(-3 + x)*(1 + x)*(80 - 60*(1 + x) + 9*(1 + x)^2))/(320*((-3 + x)^2*(1 +
x))^(4/3))
```

fricas [A] time = 0.67, size = 44, normalized size = 0.48

$$\frac{3\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}\left(9x^2-42x+29\right)}{320\left(x^4-8x^3+18x^2-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*x
^2 - 27)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="giac")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

maple [A] time = 0.04, size = 29, normalized size = 0.32

method	result	size
risch	$\frac{\frac{27}{320}x^2-\frac{63}{160}x+\frac{87}{320}}{(-3+x)\left((1+x)(-3+x)^2\right)^{\frac{1}{3}}}$	29
gosper	$\frac{3(1+x)(-3+x)\left(9x^2-42x+29\right)}{320\left(x^3-5x^2+3x+9\right)^{\frac{4}{3}}}$	34
trager	$\frac{3\left(9x^2-42x+29\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}{320(-3+x)^3(1+x)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/320*(9*x^2-42*x+29)/(-3+x)/((1+x)*(-3+x)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

mupad [B] time = 0.08, size = 37, normalized size = 0.40

$$\frac{3\,\left(9\,x^2-42\,x+29\right)\,{\left(x^3-5\,x^2+3\,x+9\right)}^{2/3}}{320\,\left(x+1\right)\,{\left(x-3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(4/3),x)
[Out] (3*(9*x^2 - 42*x + 29)*(3*x - 5*x^2 + x^3 + 9)^(2/3))/(320*(x + 1)*(x - 3)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}-5x^{2}+3x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 3*x - 2*x^2],x]
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{41}}}\,dx, x, 3-4x\right)}{\sqrt{82}}\\
&= -\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.10, size = 32, normalized size = 1.68

$$\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-2x^2+3x+4}-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[4 + 3*x - 2*x^2],x]
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x)/(-2 + Sqrt[4 + 3*x - 2*x^2])]
```

fricas [B] time = 0.86, size = 33, normalized size = 1.74

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x^2+3x+4}-2\sqrt{2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*(sqrt(2)*sqrt(-2*x^2 + 3*x + 4) - 2*sqrt(2))/x)
```

giac [A] time = 0.66, size = 16, normalized size = 0.84

$$\frac{1}{2}\sqrt{2}\arcsin\left(\frac{1}{41}\sqrt{41}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arcsin(1/41*sqrt(41)*(4*x - 3))
```

maple [A] time = 0.36, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{2}\arcsin\left(\frac{4\sqrt{41}\left(x-\frac{3}{4}\right)}{41}\right)}{2}$	15
trager	$\frac{\mathrm{RootOf}(\_Z^2+2)\ln\left(-4\,\mathrm{RootOf}(\_Z^2+2)x+3\,\mathrm{RootOf}(\_Z^2+2)+4\sqrt{-2x^2+3x+4}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+3*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

maxima [A] time = 1.30, size = 16, normalized size = 0.84

$$-\frac{1}{2}\sqrt{2}\arcsin\left(-\frac{1}{41}\sqrt{41}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

mupad [B] time = 0.20, size = 16, normalized size = 0.84

$$\frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{41}\,(4x-3)}{41}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 2*x^2 + 4)^(1/2),x)
```

```
[Out] (2^(1/2)*asin((41^(1/2)*(4*x - 3))/41))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^2+3x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+3*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**2 + 3*x + 4), x)
```

3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}} \, dx$

Optimal. Leaf size=8

$$-\sin^{-1}(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x - x^2],x]
```

```
[Out] -ArcSin[2 - x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+4x-x^2}} \, dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} \, dx, x, 4-2x\right)\right)\\ &= -\sin^{-1}(2-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x - x^2],x]
```

```
[Out] -ArcSin[2 - x]
```

IntegrateAlgebraic [B] time = 0.09, size = 23, normalized size = 2.88

$$-2\tan^{-1}\left(\frac{\sqrt{-x^2+4x-3}}{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-3 + 4*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[-3 + 4*x - x^2]/(-1 + x)]
```

fricas [B] time = 0.98, size = 29, normalized size = 3.62

$$-\arctan\left(\frac{\sqrt{-x^2+4\,x-3}\,(x-2)}{x^2-4\,x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))
```

giac [A] time = 0.63, size = 4, normalized size = 0.50

$$\arcsin\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="giac")
[Out] arcsin(x - 2)
```

maple [A] time = 0.32, size = 5, normalized size = 0.62

method	result	size
default	$\arcsin\left(-2+x\right)$	5
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+2\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2+4x-3}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+4*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-2+x)
```

maxima [A] time = 1.17, size = 8, normalized size = 1.00

$$-\arcsin\left(-x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x + 2)
```

mupad [B] time = 0.18, size = 4, normalized size = 0.50

$$\mathrm{asin}\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - x^2 - 3)^(1/2),x)
[Out] asin(x - 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+4*x-3)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 + 4*x - 3), x)
```

3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}} \, dx$

Optimal. Leaf size=12

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-5x-3x^2}} \, dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, -5-6x\right)}{\sqrt{3}}\\ &= \frac{\sin^{-1}(5+6x)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

IntegrateAlgebraic [B] time = 0.10, size = 33, normalized size = 2.75

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3x^2-5x-2}}{\sqrt{3}\,(x+1)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2 - 5*x - 3*x^2],x]
[Out] (-2*ArcTan[Sqrt[-2 - 5*x - 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]
```

fricas [B] time = 0.63, size = 40, normalized size = 3.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2-5x-2}(6x+5)}{6\left(3x^2+5x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 + 5
*x + 2))
```

giac [A] time = 0.63, size = 11, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\arcsin(6x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

maple [A] time = 0.29, size = 12, normalized size = 1.00

method	result	size
default	$\frac{\arcsin(5+6x)\sqrt{3}}{3}$	12
trager	$\frac{\mathrm{RootOf}(\_Z^2+3)\ln\left(-6\,\mathrm{RootOf}(\_Z^2+3)x-5\,\mathrm{RootOf}(\_Z^2+3)+6\sqrt{-3x^2-5x-2}\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-5*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(5+6*x)*3^(1/2)
```

maxima [A] time = 1.10, size = 11, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\arcsin(6x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

mupad [B] time = 0.22, size = 11, normalized size = 0.92

$$\frac{\sqrt{3}\,\mathrm{asin}(6x+5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 5*x - 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*asin(6*x + 5))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2-5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 - 5*x - 2), x)
```

3.238 $\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx &= \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

IntegrateAlgebraic [C] time = 0.09, size = 55, normalized size = 1.77

$$-\frac{i\tanh^{-1}\left(\frac{x^2}{2\sqrt{5}}+\frac{i\sqrt{1-x^2}\,x}{2\sqrt{5}}+\frac{2}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] ((-1/2*I)*ArcTanh[2/Sqrt[5] + x^2/(2*Sqrt[5]) + ((I/2)*x*Sqrt[1 - x^2])/Sqr
t[5]])/Sqrt[5]
```

fricas [A] time = 1.25, size = 23, normalized size = 0.74

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{2\sqrt{5}\sqrt{-x^2+1}}{5x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(5)*arctan(2/5*sqrt(5)*sqrt(-x^2 + 1)/x)
```

giac [B] time = 0.63, size = 51, normalized size = 1.65

$$\frac{1}{20}\sqrt{5}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{5}\,x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/x
^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

maple [A] time = 0.29, size = 29, normalized size = 0.94

method	result	size
default	$-\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-x^2+1}\,x}{2x^2-2}\right)}{10}$	29
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+5\right)\ln\left(-\frac{9\operatorname{RootOf}\left(\_Z^2+5\right)x^2-20\sqrt{-x^2+1}\,x-4\operatorname{RootOf}\left(\_Z^2+5\right)}{x^2+4}\right)}{20}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*5^(1/2)*arctan(1/2*5^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^2+4\right)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

mupad [B] time = 0.52, size = 79, normalized size = 2.55

$$\frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(-1+x\,2\mathrm{i})\,1\mathrm{i}}{5}-\sqrt{1-x^2}\,1\mathrm{i}}{x-2\mathrm{i}}\right)1\mathrm{i}}{20}-\frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(1+x\,2\mathrm{i})\,1\mathrm{i}}{5}+\sqrt{1-x^2}\,1\mathrm{i}}{x+2\mathrm{i}}\right)1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 4)),x)
```

```
[Out] (5^(1/2)*log(((5^(1/2)*(x*2i - 1)*1i)/5 - (1 - x^2)^(1/2)*1i)/(x - 2i))*1i)
/20 - (5^(1/2)*log(((5^(1/2)*(x*2i + 1)*1i)/5 + (1 - x^2)^(1/2)*1i)/(x + 2i
))*1i)/20
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,\left(x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}\,x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}\,x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{4-15x^2}\,dx, x, \frac{x}{\sqrt{1+4x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{15}\,x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}\,x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

IntegrateAlgebraic [A] time = 0.08, size = 48, normalized size = 1.55

$$\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{15}}-\frac{\sqrt{4x^2+1}\,x}{2\sqrt{15}}+\frac{4}{\sqrt{15}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ArcTanh[4/Sqrt[15] + x^2/Sqrt[15] - (x*Sqrt[1 + 4*x^2])/(2*Sqrt[15])]/(2*Sq
rt[15])
```

fricas [B] time = 0.94, size = 54, normalized size = 1.74

$$\frac{1}{60}\sqrt{15}\log\left(\frac{961\,x^2+8\sqrt{15}\left(31\,x^2+4\right)+4\sqrt{4\,x^2+1}\left(31\sqrt{15}\,x+120\,x\right)+124}{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(3
1*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))
```

giac [B] time = 0.64, size = 57, normalized size = 1.84

$$-\frac{1}{60}\sqrt{15}\log\left(\frac{\left(2\,x-\sqrt{4\,x^2+1}\right)^2-8\sqrt{15}+31}{\left(2\,x-\sqrt{4\,x^2+1}\right)^2+8\sqrt{15}+31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - sq
rt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

maple [A] time = 0.29, size = 22, normalized size = 0.71

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{15}}{2\sqrt{4x^2+1}}\right)\sqrt{15}}{30}$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-15\right)\ln\left(\frac{31\operatorname{RootOf}\left(\_Z^2-15\right)x^2+60\sqrt{4x^2+1}\,x+4\operatorname{RootOf}\left(\_Z^2-15\right)}{x^2+4}\right)}{60}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,x^2+1}\left(x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

mupad [B] time = 0.48, size = 61, normalized size = 1.97

$$-\frac{\sqrt{15}\left(\ln\left(x-2\mathrm{i}\right)-\ln\left(x+\frac{\sqrt{15}\,\sqrt{x^2+\frac{1}{4}}}{4}-\frac{1}{8}\mathrm{i}\right)\right)}{60}+\frac{\sqrt{15}\left(\ln\left(x+2\mathrm{i}\right)-\ln\left(x-\frac{\sqrt{15}\,\sqrt{x^2+\frac{1}{4}}}{4}+\frac{1}{8}\mathrm{i}\right)\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)*(4*x^2 + 1)^(1/2)),x)
```

```
[Out] (15^(1/2)*(log(x + 2i) - log(x - (15^(1/2)*(x^2 + 1/4)^(1/2))/4 + 1i/8)))/6
0 - (15^(1/2)*(log(x - 2i) - log(x + (15^(1/2)*(x^2 + 1/4)^(1/2))/4 - 1i/8)
))/60
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^2+4\right)\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(3-x^2\right)\sqrt{5-x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(3-x)\sqrt{5-x}}\,dx,x,x^2\right)\\ &= -\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\sqrt{5-x^2}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.04, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((3 - x^2)*Sqrt[5 - x^2]),x]
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

fricas [B] time = 0.92, size = 48, normalized size = 2.00

$$\frac{1}{8}\sqrt{2}\log\left(\frac{x^4-4\sqrt{2}\left(x^2-7\right)\sqrt{-x^2+5}-22x^2+89}{x^4-6x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log((x^4 - 4*sqrt(2)*(x^2 - 7)*sqrt(-x^2 + 5) - 22*x^2 + 89)/(x
^4 - 6*x^2 + 9))
```

giac [B] time = 0.62, size = 42, normalized size = 1.75

$$\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) +
sqrt(-x^2 + 5)))
```

maple [C] time = 0.30, size = 49, normalized size = 2.04

method	result	size
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x^2-7\,\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{-x^2+5}}{x^2-3}\right)}{4}$	49
default	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(4+2\sqrt{3}\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{3}\right)^2+2\sqrt{3}\left(x+\sqrt{3}\right)+2}}\right)}{4}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(4-2\sqrt{3}\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{3}\right)^2-2\sqrt{3}\left(x-\sqrt{3}\right)+2}}\right)}{4}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+3)/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*x^2-7*RootOf(_Z^2-2)+4*(-x^2+5)^(1/
2))/(x^2-3))
```

maxima [B] time = 1.38, size = 112, normalized size = 4.67

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|}+\frac{4}{|2x+2\sqrt{3}|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|}+\frac{4}{|2x-2\sqrt{3}|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*
x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt(2)*log(-sqrt(3) + 2
*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3)) + 4/abs(2*x - 2*sqrt(3))))
```

mupad [B] time = 0.79, size = 78, normalized size = 3.25

$$\frac{\sqrt{2}\left(\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{3}\,x+5)\,1i}{2}+\sqrt{5-x^2}\,1i}{x+\sqrt{3}}\right)+\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{3}\,x-5)\,1i}{2}-\sqrt{5-x^2}\,1i}{x-\sqrt{3}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^2 - 3)*(5 - x^2)^(1/2)),x)
```

```
[Out] (2^(1/2)*(log(((2^(1/2)*(3^(1/2)*x + 5)*1i)/2 + (5 - x^2)^(1/2)*1i)/(x + 3^
(1/2))) + log(((2^(1/2)*(3^(1/2)*x - 5)*1i)/2 - (5 - x^2)^(1/2)*1i)/(x - 3^
(1/2)))))/4
```

sympy [A] time = 5.72, size = 61, normalized size = 2.54

$$-\begin{cases} -\frac{\sqrt{2}\,\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} > \frac{1}{2} \\ -\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} < \frac{1}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) > 1/2),
(-sqrt(2)*atanh(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) < 1/2))
```

3.241 $\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3-x}\,(5-x)}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{3-x^2}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

fricas [A] time = 1.15, size = 32, normalized size = 1.28

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)\sqrt{-x^2+3}}{4\left(x^2-3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)*sqrt(-x^2 + 3)/(x^2 - 3))
```

giac [A] time = 0.59, size = 20, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

maple [C] time = 0.30, size = 48, normalized size = 1.92

method	result	size
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x^2-\mathrm{RootOf}\left(\_Z^2+2\right)-4\sqrt{-x^2+3}}{x^2-5}\right)}{4}$	48
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4+2\sqrt{5}\,\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{5}\right)^2+2\sqrt{5}\,\left(x+\sqrt{5}\right)-2}}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4-2\sqrt{5}\,\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{5}\right)^2-2\sqrt{5}\,\left(x-\sqrt{5}\right)-2}}\right)}{4}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+5)/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^2+2)*ln((RootOf(_Z^2+2)*x^2-RootOf(_Z^2+2)-4*(-x^2+3)^(1/2))/
(x^2-5))
```

maxima [B] time = 1.37, size = 101, normalized size = 4.04

$$-\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x+2\sqrt{5}\right|}+\frac{2\sqrt{3}}{\left|2x+2\sqrt{5}\right|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x-2\sqrt{5}\right|}-\frac{2\sqrt{3}}{\left|2x-2\sqrt{5}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x + 2*sqr
t(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5
)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/abs(2*x - 2*sqrt(5))))
```

mupad [B] time = 0.76, size = 83, normalized size = 3.32

$$-\frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,\left(\sqrt{5}\,x+3\right)}{2}+\sqrt{3-x^2}\ 1\mathrm{i}}{x+\sqrt{5}}\right)\ 1\mathrm{i}}{4}-\frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,\left(\sqrt{5}\,x-3\right)}{2}-\sqrt{3-x^2}\ 1\mathrm{i}}{x-\sqrt{5}}\right)\ 1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((3 - x^2)^(1/2)*(x^2 - 5)),x)
```

```
[Out] - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x + 3))/2 + (3 - x^2)^(1/2)*1i)/(x + 5^(1
/2)))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x - 3))/2 - (3 - x^2)^(1/2)*1
i)/(x - 5^(1/2)))*1i)/4
```

sympy [A] time = 5.54, size = 24, normalized size = 0.96

$$-\frac{\sqrt{2}\ \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2
```

3.242 $\int \frac{1}{\sqrt{2+x^2}\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1175, 377, 206, 203}

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] -ArcTan[x/Sqrt[2 + x^2]]/2 - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2+x^2}\left(-1+x^4\right)}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(1-x^2\right)\sqrt{2+x^2}}\,dx\right)-\frac{1}{2}\int \frac{1}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-3x^2}\,dx,x,\frac{x}{\sqrt{2+x^2}}\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{2+x^2}}\right)\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 96, normalized size = 2.23

$$\frac{1}{12}\left(-3\tan^{-1}\left(\frac{-x+2i}{\sqrt{x^2+2}}\right)+3\tan^{-1}\left(\frac{x+2i}{\sqrt{x^2+2}}\right)+\sqrt{3}\tanh^{-1}\left(\frac{2-x}{\sqrt{3}\sqrt{x^2+2}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{x+2}{\sqrt{3}\sqrt{x^2+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] (-3*ArcTan[(2*I - x)/Sqrt[2 + x^2]] + 3*ArcTan[(2*I + x)/Sqrt[2 + x^2]] + S
qrt[3]*ArcTanh[(2 - x)/(Sqrt[3]*Sqrt[2 + x^2])] - Sqrt[3]*ArcTanh[(2 + x)/(
Sqrt[3]*Sqrt[2 + x^2])])/12
```

IntegrateAlgebraic [A] time = 0.09, size = 65, normalized size = 1.51

$$\frac{1}{2}\tan^{-1}\left(x^2-\sqrt{x^2+2}\,x+1\right)-\frac{\tanh^{-1}\left(-\frac{x^2}{\sqrt{3}}+\frac{\sqrt{x^2+2}\,x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] ArcTan[1 + x^2 - x*Sqrt[2 + x^2]]/2 - ArcTanh[1/Sqrt[3] - x^2/Sqrt[3] + (x*
Sqrt[2 + x^2])/Sqrt[3]]/(2*Sqrt[3])
```

fricas [B] time = 0.99, size = 72, normalized size = 1.67

$$\frac{1}{12}\sqrt{3}\log\left(\frac{4x^2-\sqrt{3}\left(2x^2+1\right)-\sqrt{x^2+2}\left(2\sqrt{3}x-3x\right)+2}{x^2-1}\right)-\frac{1}{2}\arctan\left(-x^2+\sqrt{x^2+2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((4*x^2 - sqrt(3)*(2*x^2 + 1) - sqrt(x^2 + 2)*(2*sqrt(3)*x
- 3*x) + 2)/(x^2 - 1)) - 1/2*arctan(-x^2 + sqrt(x^2 + 2)*x - 1)
```

giac [B] time = 0.65, size = 74, normalized size = 1.72

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right)+\frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x - s
qrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)
```

maple [B] time = 0.28, size = 70, normalized size = 1.63

method	result	size
default	$-\frac{\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4+2x)\sqrt{3}}{6\sqrt{(-1+x)^2+1+2x}}\right)}{12}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4-2x)\sqrt{3}}{6\sqrt{(1+x)^2+1-2x}}\right)}{12}$	70

trager	$\frac{\mathrm{RootOf}(\_Z^2-3)\ln\left(-\frac{-2\,\mathrm{RootOf}(\_Z^2-3)x^2+3\sqrt{x^2+2}\,x-\mathrm{RootOf}(\_Z^2-3)}{(1+x)(-1+x)}\right)}{12}-\frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(\frac{\sqrt{x^2+2}\,x+\mathrm{RootOf}(\_Z^2+1)}{x^2+1}\right)}{4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(x/(x^2+2)^(1/2))-1/12*3^(1/2)*arctanh(1/6*(4+2*x)*3^(1/2)/((-1+
x)^2+1+2*x)^(1/2))+1/12*3^(1/2)*arctanh(1/6*(4-2*x)*3^(1/2)/((1+x)^2+1-2*x)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

mupad [B] time = 0.11, size = 107, normalized size = 2.49

$$\frac{\sqrt{3}\left(\ln(x-1)-\ln\left(x+\sqrt{3}\sqrt{x^2+2}+2\right)\right)}{12}-\frac{\sqrt{3}\left(\ln(x+1)-\ln\left(\sqrt{3}\sqrt{x^2+2}-x+2\right)\right)}{12}+\frac{\ln\left(\sqrt{x^2+2}+2-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 2)^(1/2)*(x^4 - 1)),x)
```

```
[Out] (log((x^2 + 2)^(1/2) - x*1i + 2)*1i)/4 - (log(x*1i + (x^2 + 2)^(1/2) + 2)*1
i)/4 + (log(x - 1i)*1i)/4 - (log(x + 1i)*1i)/4 + (3^(1/2)*(log(x - 1) - log
(x + 3^(1/2)*(x^2 + 2)^(1/2) + 2)))/12 - (3^(1/2)*(log(x + 1) - log(3^(1/2)
*(x^2 + 2)^(1/2) - x + 2)))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)(x+1)(x^2+1)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)
```

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1033, 724, 206, 688, 207}

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqr
t[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx = \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-12+4x^2}\,dx, x, \sqrt{4+2x+x^2}\right) - \text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{10}{\sqrt{4+}}\right.$$
$$= -\frac{\tanh^{-1}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.98

$$\frac{1}{42}\left(-3\sqrt{7}\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)-7\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{(x+1)^2+3}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
[Out] (-3*Sqrt[7]*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])] - 7*Sqrt[3]*Ar
cTanh[Sqrt[3 + (1 + x)^2]/Sqrt[3]])/42
```

IntegrateAlgebraic [A] time = 0.20, size = 80, normalized size = 1.29

$$\frac{\tanh^{-1}\left(-\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}+\frac{x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{7}}-\frac{x}{\sqrt{7}}+\frac{1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
[Out] ArcTanh[1/Sqrt[3] + x/Sqrt[3] - Sqrt[4 + 2*x + x^2]/Sqrt[3]]/Sqrt[3] - ArcT
anh[1/Sqrt[7] - x/Sqrt[7] + Sqrt[4 + 2*x + x^2]/Sqrt[7]]/Sqrt[7]
```

fricas [A] time = 1.09, size = 74, normalized size = 1.19

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) -
 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x +
 1))
```

giac [B] time = 0.73, size = 109, normalized size = 1.76

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4) + 1))

maple [A] time = 0.42, size = 49, normalized size = 0.79

method	result	size
default	$-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$	49
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}-5\,\mathrm{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{14}-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}+\mathrm{RootOf}\left(\_Z^2-3\right)}{1+x}\right)}{6}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))

maxima [A] time = 1.30, size = 54, normalized size = 0.87

$$-\frac{1}{14}\sqrt{7}\,\operatorname{arsinh}\left(\frac{4\sqrt{3}\,x}{3\,|2\,x-2|}+\frac{10\sqrt{3}}{3\,|2\,x-2|}\right)-\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(\frac{2\sqrt{3}}{|2\,x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")

[Out] -1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/abs(2*x - 2)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(x^2-1\right)\sqrt{x^2+2\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)),x)

[Out] int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)

[Out] Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}\left(-8+x^3\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2074, 724, 206, 1025, 982, 204, 1024}

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] -ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 + 3
*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 + 2*x +
x^2]]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{5+2x+x^2}\left(-8+x^3\right)}\,dx &= \int \left(\frac{1}{12(-2+x)\sqrt{5+2x+x^2}}+\frac{-4-x}{12\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(-2+x)\sqrt{5+2x+x^2}}\,dx+\frac{1}{12}\int \frac{-4-x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{2+2x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14+}{\sqrt{5+2x}}\right.\\
&= -\frac{\tanh^{-1}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sqrt{5+2x+x^2}\right)+\operatorname{Subst}\left(\right.\\
&= -\frac{\tan^{-1}\left(\frac{2+2x}{2\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.36, size = 159, normalized size = 1.94

$$\frac{1}{312}\left(-2\sqrt{13}\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)-13\left(\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2\left(\sqrt[3]{-1}-2\right)x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2+2x+5}}\right)+\left(\sqrt{3}-i\right)\tan^{-1}\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 2*(-2 + (-1)^(1/3))*x)/(Sqr
t[2 - (2*I)*Sqrt[3]]*Sqrt[5 + 2*x + x^2])] + (-I + Sqrt[3])*ArcTan[(1 - (5*
I)*Sqrt[3] - 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 + 2*x +
x^2])]) - 2*Sqrt[13]*ArcTanh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])])/312
```

IntegrateAlgebraic [A] time = 0.36, size = 119, normalized size = 1.45

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{(x+1)\sqrt{x^2+2x+5}}{\sqrt{3}}+\frac{2x}{\sqrt{3}}+\frac{4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+5}}{\sqrt{13}}-\frac{x}{\sqrt{13}}+\frac{2}{\sqrt{13}}\right)}{6\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] ArcTan[4/Sqrt[3] + (2*x)/Sqrt[3] + x^2/Sqrt[3] - ((1 + x)*Sqrt[5 + 2*x + x^
2])/Sqrt[3]]/(4*Sqrt[3]) + ArcTanh[Sqrt[5 + 2*x + x^2]]/12 - ArcTanh[2/Sqrt
[13] - x/Sqrt[13] + Sqrt[5 + 2*x + x^2]/Sqrt[13]]/(6*Sqrt[13])
```

fricas [B] time = 1.00, size = 151, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{156}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) +
1/156*sqrt(13)*log((sqrt(13)*(3*x + 7) + sqrt(x^2 + 2*x + 5)*(3*sqrt(13) -
13) - 9*x - 21)/(x - 2)) - 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x
 + 6) + 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)
```

giac [B] time = 0.72, size = 164, normalized size = 2.00

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)+\frac{1}{156}\sqrt{13}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqrt
(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(abs
(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) + 2*s
qrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*s
qrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

maple [A] time = 0.58, size = 69, normalized size = 0.84

method	result
default	$\frac{\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{12}-\frac{\sqrt{13}\operatorname{arctanh}\left(\frac{(14+6x)\sqrt{13}}{26\sqrt{(-2+x)^2+1+6x}}\right)}{156}$
trager	$\operatorname{RootOf}\left(144\_Z^2-12\_Z+1\right)\ln\left(-\frac{-2880\operatorname{RootOf}\left(144\_Z^2-12\_Z+1\right)^2x+126\operatorname{RootOf}\left(144\_Z^2-12\_Z+1\right)\sqrt{x^2+2x+5}+3}{12\operatorname{RootOf}\left(144\_\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)
^(1/2)*(2*x+2))-1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+1+6
*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3-8)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3-8\right)\,\sqrt{x^2+2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-2)\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.245 $\int \frac{x}{\left(4+x+x^2\right)\sqrt{5+4x+4x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1025, 982, 207, 1024, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1 +
 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
```

```
- b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(4+x+x^2\right)\sqrt{5+4x+4x^2}}\,dx = \frac{1}{8}\int \frac{4+8x}{\left(4+x+x^2\right)\sqrt{5+4x+4x^2}}\,dx - \frac{1}{2}\int \frac{1}{\left(4+x+x^2\right)\sqrt{5+4x+4x^2}}\,dx$$
$$= 4\,\mathrm{Subst}\left(\int \frac{1}{-240+11x^2}\,dx, x, \frac{4+8x}{\sqrt{5+4x+4x^2}}\right) - \mathrm{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x,$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

Mathematica [C] time = 0.11, size = 114, normalized size = 1.81

$$\frac{\left(\sqrt{15}-i\right)\tan^{-1}\left(\frac{-2i\sqrt{15}x-i\sqrt{15}+4}{\sqrt{11}\sqrt{4x^2+4x+5}}\right)+\left(\sqrt{15}+i\right)\tan^{-1}\left(\frac{2i\sqrt{15}x+i\sqrt{15}+4}{\sqrt{11}\sqrt{4x^2+4x+5}}\right)}{2\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ((-I + Sqrt[15])*ArcTan[(4 - I*Sqrt[15] - (2*I)*Sqrt[15]*x)/(Sqrt[11]*Sqrt[
5 + 4*x + 4*x^2])] + (I + Sqrt[15])*ArcTan[(4 + I*Sqrt[15] + (2*I)*Sqrt[15]
*x)/(Sqrt[11]*Sqrt[5 + 4*x + 4*x^2])])/(2*Sqrt[165])
```

IntegrateAlgebraic [C] time = 0.24, size = 102, normalized size = 1.62

$$\frac{1}{2}\mathrm{RootSum}\left[\#1^4-4\#1^3+58\#1^2-108\#1+69\&, \frac{\#1^2\log\left(-\#1+\sqrt{4x^2+4x+5}-2x\right)-5\log\left(-\#1+\sqrt{4x^2+4x}\right.}{\#1^3-3\#1^2+29\#1-27}\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] RootSum[69 - 108*#1 + 58*#1^2 - 4*#1^3 + #1^4 & , (-5*Log[-2*x + Sqrt[5 + 4
*x + 4*x^2] - #1] + Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1]*#1^2)/(-27 + 29*
#1 - 3*#1^2 + #1^3) & ]/2
```

fricas [B] time = 1.17, size = 307, normalized size = 4.87

$$\frac{2}{165}\sqrt{165}\sqrt{15}\arctan\left(\frac{1}{60}\sqrt{2}\sqrt{4x^2-\sqrt{4x^2+4x+5}(2x+1)+4x-\sqrt{165}+16}\left(\sqrt{165}\sqrt{15}+15\sqrt{15}\right)+\frac{1}{60}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/165*sqrt(165)*sqrt(15)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x
+ 5)*(2*x + 1) + 4*x - sqrt(165) + 16)*(sqrt(165)*sqrt(15) + 15*sqrt(15)) +
 1/60*sqrt(165)*sqrt(15)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*
sqrt(15) + 15*sqrt(15)) + 1/4*sqrt(15)*(2*x + 1)) + 2/165*sqrt(165)*sqrt(15
)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x +
sqrt(165) + 16)*(sqrt(165)*sqrt(15) - 15*sqrt(15)) + 1/60*sqrt(165)*sqrt(15
)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*sqrt(15) - 15*sqrt(15))
 - 1/4*sqrt(15)*(2*x + 1)) - 1/330*sqrt(165)*log(460800*x^2 - 115200*sqrt(4
```

```
*x^2 + 4*x + 5)*(2*x + 1) + 460800*x + 115200*sqrt(165) + 1843200) + 1/330*
sqrt(165)*log(460800*x^2 - 115200*sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 460800*
x - 115200*sqrt(165) + 1843200)
```

giac [B] time = 0.71, size = 165, normalized size = 2.62

$$\frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2x-\sqrt{4x^2+4x+5}+1}{\sqrt{15}+\sqrt{11}}\right)-\frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2x-\sqrt{4x^2+4x+5}+1}{\sqrt{15}-\sqrt{11}}\right)-\frac{1}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(15
) + sqrt(11))) - 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x +
 5) + 1)/(sqrt(15) - sqrt(11))) - 1/330*sqrt(165)*log(90000*(2*x - sqrt(4*x
^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) + sqrt(11))^2) + 1/330*sqrt(165)*log
(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) - sqrt(11))^2)
```

maple [A] time = 0.75, size = 53, normalized size = 0.84

method	result
default	$\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}\,\sqrt{11}}{11}\right)\sqrt{11}}{11}-\frac{\sqrt{165}\,\operatorname{arctanh}\left(\frac{\sqrt{165}\,(8x+4)}{60\sqrt{4x^2+4x+5}}\right)}{165}$
trager	$\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)\ln\left(-\frac{3524400\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)^5 x+111270\operatorname{RootOf}\left(27225\_Z\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*arc
tanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{4x^2+4x+5}\,(x^2+x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{4x^2+4x+5}\,\left(x^2+x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)),x)
```

```
[Out] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2+x+4\right)\sqrt{4x^2+4x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=56

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1036, 1030, 207, 203}

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(1
 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{3+x}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx = -\left(\frac{1}{2}\int \frac{-4-4x}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx\right) + \frac{1}{2}\int \frac{2-2x}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{1}{-8+x^2}\,dx, x, \frac{-2-2x}{\sqrt{1+x+x^2}}\right) + 16\,\text{Subst}\left(\int \frac{1}{32+x^2}\,dx, x, \frac{-4+4x}{\sqrt{1+x+x}}\right.$$
$$= -2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

Mathematica [C] time = 0.03, size = 80, normalized size = 1.43

$$\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left((2+i)\tan^{-1}\left(\frac{\sqrt[4]{-1}\,((2+i)x+(1+2i))}{2\sqrt{x^2+x+1}}\right)+(1+2i)\tanh^{-1}\left(\frac{(-1)^{3/4}((1+2i)x+(2+i))}{2\sqrt{x^2+x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] (1/2 + I/2)*(-1)^(3/4)*((2 + I)*ArcTan[((-1)^(1/4)*((1 + 2*I) + (2 + I)*x))
/(2*Sqrt[1 + x + x^2])] + (1 + 2*I)*ArcTanh[((-1)^(3/4)*((2 + I) + (1 + 2*I
)*x))/(2*Sqrt[1 + x + x^2])])
```

IntegrateAlgebraic [C] time = 0.22, size = 103, normalized size = 1.84

$$\frac{1}{2}\text{RootSum}\left[\#1^4+2\#1^2-4\#1+2\&, \frac{\#1^2\log\left(-\#1+\sqrt{x^2+x+1}-x\right)-6\#1\log\left(-\#1+\sqrt{x^2+x+1}-x\right)+2\,\text{lo}}{\#1^3+\#1-1}\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (2*Log[-x + Sqrt[1 + x + x^2] - #1] -
6*Log[-x + Sqrt[1 + x + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x + x^2] - #1]*#1
^2)/(-1 + #1 + #1^3) & ]/2
```

fricas [B] time = 0.92, size = 303, normalized size = 5.41

$$\frac{4}{5}\sqrt{10}\sqrt{5}\arctan\left(\frac{1}{25}\sqrt{5}\sqrt{\sqrt{10}\sqrt{5}(x-1)+10x^2-\sqrt{x^2+x+1}\left(\sqrt{10}\sqrt{5}+10x\right)+5x+15}\left(\sqrt{10}\sqrt{5}+10\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(10)*sqrt(5)*arctan(1/25*sqrt(5)*sqrt(sqrt(10)*sqrt(5)*(x - 1) + 10
*x^2 - sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) + 10*x) + 5*x + 15)*(sqrt(10)*sq
rt(5) + 10) + 1/5*sqrt(10)*sqrt(5)*(x + 1) - 1/5*sqrt(x^2 + x + 1)*(sqrt(10
)*sqrt(5) + 10) + 2*x + 1) + 4/5*sqrt(10)*sqrt(5)*arctan(1/5*sqrt(10)*sqrt(
5)*(x + 1) + 1/50*sqrt(-20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2
 + x + 1)*(sqrt(10)*sqrt(5) - 10*x) + 100*x + 300)*(sqrt(10)*sqrt(5) - 10)
- 1/5*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 10) - 2*x - 1) - 1/10*sqrt(10)*
sqrt(5)*log(20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 - 20*sqrt(x^2 + x + 1)*(s
qrt(10)*sqrt(5) + 10*x) + 100*x + 300) + 1/10*sqrt(10)*sqrt(5)*log(-20*sqrt
(10)*sqrt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 1
0*x) + 100*x + 300)
```

giac [B] time = 0.68, size = 152, normalized size = 2.71

$$-\frac{1}{2}\sqrt{2}\left(\pi+4\arctan\left(-\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{2}+2\right)-\sqrt{2}-1\right)\right)+\frac{1}{2}\sqrt{2}\left(\pi+4\arctan\left(\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(pi + 4*arctan(-(x - sqrt(x^2 + x + 1))*(sqrt(2) + 2) - sqrt(2
) - 1)) + 1/2*sqrt(2)*(pi + 4*arctan((x - sqrt(x^2 + x + 1))*(sqrt(2) - 2)
+ sqrt(2) - 1)) - 1/2*sqrt(2)*log((x + sqrt(2) - sqrt(x^2 + x + 1) - 1)^2 +
 (x - sqrt(x^2 + x + 1) + 1)^2) + 1/2*sqrt(2)*log((x - sqrt(2) - sqrt(x^2 +
 x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2)
```

maple [B] time = 0.72, size = 128, normalized size = 2.29

method	result
default	$\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}}{2}\right)-2\arctan\left(\frac{\sqrt{2}\,(-1+x)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,(-1-x)}\right)\right)}{\sqrt{\frac{\frac{(-1+x)^2}{(-1-x)^2}+3}{\left(1+\frac{-1+x}{-1-x}\right)^2}}\,\left(1+\frac{-1+x}{-1-x}\right)}$
trager	$-\frac{2\ln\left(\frac{-12\,\mathrm{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^5 x-172\,\mathrm{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^3 x+320\,\mathrm{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^2\sqrt{x^2+x+1}+40\,\mathrm{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^3+}{2x\,\mathrm{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^2+3x-4}\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)^2+3)^(1/2
)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)*(-1+x)/(-1-x)))/(((
-1+x)^2/(-1-x)^2+3)/(1+(-1+x)/(-1-x))^2)^(1/2)/(1+(-1+x)/(-1-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\sqrt{x^2+x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+3}{\left(x^2+1\right)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)),x)
```

```
[Out] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(x^2+1\right)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1035, 1029, 207, 203}

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]

[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - Ar
cTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\left(4+4x+3x^2\right)}dx = -\left(\frac{1}{42}\int \frac{-70-70x}{\sqrt{-1+6x+x^2}\left(4+4x+3x^2\right)}dx\right)+\frac{1}{42}\int \frac{-28+14}{\sqrt{-1+6x+x^2}\left(4+\right.}$$
$$= -\left(\frac{896}{3}\operatorname{Subst}\left(\int \frac{1}{-200704+28x^2}dx, x, \frac{-224-224x}{\sqrt{-1+6x+x^2}}\right)\right)-\frac{2800}{3}\operatorname{Subs}$$
$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}$$

Mathematica [C] time = 0.45, size = 174, normalized size = 2.49

$$-\frac{\sqrt{7-4i\sqrt{2}}\left(8\sqrt{2}+13i\right)\tan^{-1}\left(\frac{\left(-7-2i\sqrt{2}\right)x-6i\sqrt{2}+9}{\sqrt{7\left(7-4i\sqrt{2}\right)}\sqrt{x^2+6x-1}}\right)+\sqrt{7+4i\sqrt{2}}\left(8\sqrt{2}-13i\right)\tan^{-1}\left(\frac{\left(-7+2i\sqrt{2}\right)x+6i\sqrt{2}+9}{\sqrt{7\left(7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}}\right)}{108\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] -1/108*(Sqrt[7 - (4*I)*Sqrt[2]]*(13*I + 8*Sqrt[2])*ArcTan[(9 - (6*I)*Sqrt[2
] + (-7 - (2*I)*Sqrt[2])*x)/(Sqrt[7*(7 - (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^
2])] + Sqrt[7 + (4*I)*Sqrt[2]]*(-13*I + 8*Sqrt[2])*ArcTan[(9 + (6*I)*Sqrt[2
] + (-7 + (2*I)*Sqrt[2])*x)/(Sqrt[7*(7 + (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^
2])])/Sqrt[14]
```

IntegrateAlgebraic [C] time = 0.30, size = 123, normalized size = 1.76

$$\operatorname{RootSum}\left[3\#1^4-8\#1^3+46\#1^2-104\#1+171\&, \frac{\#1^2\log\left(-\#1+\sqrt{x^2+6x-1}-x\right)-\#1\log\left(-\#1+\sqrt{x^2+6x-\right.}{3\#1^3-6\#1^2+23\#1-}\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] RootSum[171 - 104*#1 + 46*#1^2 - 8*#1^3 + 3*#1^4 & , (4*Log[-x + Sqrt[-1 +
6*x + x^2] - #1] - Log[-x + Sqrt[-1 + 6*x + x^2] - #1]*#1 + Log[-x + Sqrt[-
1 + 6*x + x^2] - #1]*#1^2)/(-26 + 23*#1 - 6*#1^2 + 3*#1^3) & ]
```

fricas [B] time = 1.19, size = 311, normalized size = 4.44

$$\frac{1}{84}\sqrt{14}\sqrt{2}\log\left(13068\sqrt{14}\sqrt{2}(x-2)+78408x^2-13068\sqrt{x^2+6x-1}\left(\sqrt{14}\sqrt{2}+6x+4\right)+287496x+2874\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/84*sqrt(14)*sqrt(2)*log(13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 - 1306
8*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) + 6*x + 4) + 287496*x + 287496) - 1
/84*sqrt(14)*sqrt(2)*log(-13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 + 1306
8*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) - 6*x - 4) + 287496*x + 287496) - 5
/42*sqrt(14)*arctan(1/24*sqrt(3)*sqrt(sqrt(14)*sqrt(2)*(x - 2) + 6*x^2 - sq
rt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) + 6*x + 4) + 22*x + 22)*(sqrt(14) + sqr
t(2)) + 1/8*sqrt(2)*(x + 3) + 1/8*sqrt(14)*(x + 1) - 1/8*sqrt(x^2 + 6*x - 1
```

```
)*(sqrt(14) + sqrt(2))) - 5/42*sqrt(14)*arctan(-1/8*sqrt(2)*(x + 3) + 1/8*s
qrt(14)*(x + 1) + 1/1584*sqrt(-13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 +
 13068*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) - 6*x - 4) + 287496*x + 287496
)*(sqrt(14) - sqrt(2)) - 1/8*sqrt(x^2 + 6*x - 1)*(sqrt(14) - sqrt(2)))
```

giac [B] time = 0.66, size = 257, normalized size = 3.67

$$-\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan(2)+\arctan\left(\frac{1}{8}\left(x-\sqrt{x^2+6x-1}\right)\left(\sqrt{14}+\sqrt{2}\right)+\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right)+\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] -5/84*sqrt(7)*sqrt(2)*(arctan(2) + arctan(1/8*(x - sqrt(x^2 + 6*x - 1))*(sq
rt(14) + sqrt(2)) + 1/8*sqrt(14) + 3/8*sqrt(2))) + 5/84*sqrt(7)*sqrt(2)*(ar
ctan(1/2) + arctan(-1/8*(x - sqrt(x^2 + 6*x - 1))*(sqrt(14) - sqrt(2)) - 1/
8*sqrt(14) + 3/8*sqrt(2))) + 1/42*sqrt(7)*log(4*(4*sqrt(7)*sqrt(2) + 3*x +
sqrt(7) - 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) + 2)^2 + 16*(sqrt(7)*sqrt(2) -
3*x - sqrt(7) - sqrt(2) + 3*sqrt(x^2 + 6*x - 1) - 2)^2) - 1/42*sqrt(7)*log(
4*(4*sqrt(7)*sqrt(2) + 3*x - sqrt(7) + 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) +
2)^2 + 16*(sqrt(7)*sqrt(2) - 3*x + sqrt(7) + sqrt(2) + 3*sqrt(x^2 + 6*x - 1
) - 2)^2)
```

maple [B] time = 0.95, size = 158, normalized size = 2.26

method	result
default	$-\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\left(4\sqrt{7}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,\sqrt{7}}{21}\right)-5\sqrt{14}\,\arctan\left(\frac{\sqrt{14}\,\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,(-2+x)}{4\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)(-1-x)}\right)\right)}{84\sqrt{-\frac{3\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)}{\left(1+\frac{-2+x}{-1-x}\right)^2}}\left(1+\frac{-2+x}{-1-x}\right)}$
trager	$\operatorname{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)\ln\left(-\frac{1568802816x\operatorname{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^5+6019776\operatorname{RootOf}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(4*7^(1/2)*arctanh(1/21*(-6*(-2+x)^2/
(-1-x)^2+15)^(1/2)*7^(1/2))-5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(-2+x)^2/(-1
-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x)))/(-3*(2*(-2+x)^2/(-1
-x)^2-5)/(1+(-2+x)/(-1-x))^2)^(1/2)/(1+(-2+x)/(-1-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(3x^2+4x+4)\sqrt{x^2+6x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\,\left(3x^2+4x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)),x)
```

```
[Out] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\left(3x^2+4x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}\,(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}\,(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1035, 1029, 206, 204}

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}\,(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}\,(-x(A+B)+A+B)}{2\sqrt{10x^2-22x+13}\,(A+B)}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35])
 - ((A + B)*ArcTanh[(Sqrt[35]*(A + B - (A + B)*x))/(2*(A + B)*Sqrt[13 - 22*
x + 10*x^2])])/(2*Sqrt[35])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{B+Ax}{\left(17-18x+5x^2\right)\sqrt{13-22x+10x^2}}\,dx = \frac{1}{70}\int \frac{140(A+B)-70(A+B)x}{\left(17-18x+5x^2\right)\sqrt{13-22x+10x^2}}\,dx - \frac{1}{70}\int \frac{70(2A+}{\left(17-18x+5\right.}$$
$$= \left(560(A+B)^2\right)\text{Subst}\left(\int \frac{1}{313600(A+B)^2-140x^2}\,dx, x, \frac{-140(A+}{\sqrt{13-}}\right.$$
$$= -\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}} - \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(A+B-(A+B)}{2(A+B)\sqrt{13-22x+10}}\right.}{2\sqrt{35}}$$

Mathematica [C] time = 0.12, size = 94, normalized size = 1.18

$$\frac{((4-i)A+(2-i)B)\tan^{-1}\left(\frac{(2-18i)-(1-18i)x}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)+((1-4i)A+(1-2i)B)\tanh^{-1}\left(\frac{(18-i)x-(18-2i)}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)}{4\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] (((4 - I)*A + (2 - I)*B)*ArcTan[((2 - 18*I) - (1 - 18*I)*x)/(Sqrt[35]*Sqrt[
13 - 22*x + 10*x^2])] + ((1 - 4*I)*A + (1 - 2*I)*B)*ArcTanh[((-18 + 2*I) +
(18 - I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])])/(4*Sqrt[35])
```

IntegrateAlgebraic [C] time = 1.13, size = 143, normalized size = 1.79

$$\frac{1}{70}\left((1+4i)\sqrt{35}A+(1+2i)\sqrt{35}B\right)\tanh^{-1}\left(\frac{(2-i)\sqrt{10x^2-22x+13}+(-2+i)\sqrt{10}x+(4-i)\sqrt{10}}{\sqrt{35}}\right)-\frac{1}{70}i\left((4+\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2])
,x]
```

```
[Out] (((1 + 4*I)*Sqrt[35]*A + (1 + 2*I)*Sqrt[35]*B)*ArcTanh[((4 - I)*Sqrt[10] -
(2 - I)*Sqrt[10]*x + (2 - I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]])/70 - (I/7
0)*((4 + I)*Sqrt[35]*A + (2 + I)*Sqrt[35]*B)*ArcTanh[((4 + I)*Sqrt[10] - (2
 + I)*Sqrt[10]*x + (2 + I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [B] time = 0.99, size = 629, normalized size = 7.86

$$\frac{2\sqrt{35}\left(2A^2+3AB+B^2\right)\sqrt{A^2+2AB+B^2}\left(\arctan(3)+\arctan\left(-\frac{5\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)\left(300\sqrt{14}-1129\right)-7658\sqrt{35}+}{2329\sqrt{35}-4358\sqrt{10}}\right.\right.}{35\left(15A^2+14AB+3B^2-\sqrt{289A^4+612A^3B+494A^2B^2+180AB^3+25B^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac
")
```

```
[Out] 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(3) + ar
ctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(300*sqrt(14) - 1129) - 76
58*sqrt(35) + 14361*sqrt(10))/(2329*sqrt(35) - 4358*sqrt(10))))/(15*A^2 + 1
4*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2 + 180*A*B^3 + 25*B^4
)) - 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(1/
7) + arctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(62556*sqrt(14) + 2
45977) - 1617962*sqrt(35) - 3089577*sqrt(10))/(496201*sqrt(35) + 929846*sqr
t(10))))/(15*A^2 + 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2
+ 180*A*B^3 + 25*B^4)) + 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(25*(546
*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 2807*sqrt(10)*x - 234*s
qrt(35)*sqrt(14) - 1014*sqrt(14)*sqrt(10) - 1203*sqrt(35) - 5213*sqrt(10) -
 2807*sqrt(10*x^2 - 22*x + 13))^2 + 25*(78*sqrt(14)*(sqrt(10)*x - sqrt(10*x
^2 - 22*x + 13)) + 401*sqrt(10)*x + 48*sqrt(35)*sqrt(14) + 208*sqrt(14)*sqr
t(10) + 141*sqrt(35) + 611*sqrt(10) - 401*sqrt(10*x^2 - 22*x + 13))^2) - 1/
140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(625*(18*sqrt(14)*(sqrt(10)*x - sqr
t(10*x^2 - 22*x + 13)) - 75*sqrt(10)*x + 8*sqrt(35)*sqrt(14) - 24*sqrt(14)*
sqrt(10) - 37*sqrt(35) + 111*sqrt(10) + 75*sqrt(10*x^2 - 22*x + 13))^2 + 62
5*(6*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 25*sqrt(10)*x + 6*s
qrt(35)*sqrt(14) - 18*sqrt(14)*sqrt(10) - 25*sqrt(35) + 75*sqrt(10) + 25*sq
rt(10*x^2 - 22*x + 13))^2)
```

maple [B] time = 0.46, size = 192, normalized size = 2.40

method	result
default	$\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\left(\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)A-4\arctan\left(\frac{\sqrt{35}\,(-2+x)}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,(1-x)}\right)A+\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)B-2\arctan \cdots\right.}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(1+\frac{-2+x}{1-x}\right)^2}}\left(1+\frac{-2+x}{1-x}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9)
^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x
))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/2)/
((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((-2+x)^2/(1-x)^2+9)/(1+(-2+x)
/(1-x))^2)^(1/2)/(1+(-2+x)/(1-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}\,\left(5\,x^2-18\,x+17\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B+A\,x}{\left(5\,x^2-18\,x+17\right)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

```
[Out] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax + B}{\left(5x^2 - 18x + 17\right)\sqrt{10x^2 - 22x + 13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1029, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = 8\,\text{Subst}\left(\int \frac{1}{64-140x^2}\,dx, x, \frac{2-2x}{\sqrt{13-22x+10x^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

Mathematica [C] time = 0.04, size = 76, normalized size = 2.00

$$\frac{i\left(\tan^{-1}\left(\frac{(2-18i)-(1-18i)x}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)+i\tanh^{-1}\left(\frac{(18-i)x-(18-2i)}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)\right)}{4\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ((I/4)*(ArcTan[((2 - 18*I) - (1 - 18*I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x^
2])] + I*ArcTanh[((-18 + 2*I) + (18 - I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x
^2])]))/Sqrt[35]
```

IntegrateAlgebraic [B] time = 0.50, size = 91, normalized size = 2.39

$$-\frac{\tanh^{-1}\left(\frac{-50x^2+\left(5\sqrt{10}\,x-9\sqrt{10}\right)\sqrt{10x^2-22x+13}+145x-135}{-2\sqrt{35}\,\sqrt{10x^2-22x+13}+10\sqrt{14}\,x-20\sqrt{14}}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),
x]
```

```
[Out] -1/2*ArcTanh[(-135 + 145*x - 50*x^2 + (-9*Sqrt[10] + 5*Sqrt[10]*x)*Sqrt[13
- 22*x + 10*x^2])/(-20*Sqrt[14] + 10*Sqrt[14]*x - 2*Sqrt[35]*Sqrt[13 - 22*x
 + 10*x^2])]/Sqrt[35]
```

fricas [B] time = 0.68, size = 81, normalized size = 2.13

$$\frac{1}{280}\sqrt{35}\log\left(\frac{11225\,x^4-47220\,x^3-8\sqrt{35}\left(75\,x^3-233\,x^2+245\,x-87\right)\sqrt{10\,x^2-22\,x+13}+75534\,x^2-5437}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/280*sqrt(35)*log((11225*x^4 - 47220*x^3 - 8*sqrt(35)*(75*x^3 - 233*x^2 +
245*x - 87)*sqrt(10*x^2 - 22*x + 13) + 75534*x^2 - 54372*x + 14849)/(25*x^4
 - 180*x^3 + 494*x^2 - 612*x + 289))
```

giac [B] time = 1.84, size = 231, normalized size = 6.08

$$\frac{1}{140}\sqrt{35}\log\left(\left|21875000000\sqrt{14}\left(\sqrt{10}\,x-\sqrt{10\,x^2-22\,x+13}\right)^2+82031250000\left(\sqrt{10}\,x-\sqrt{10\,x^2-22\,x+13}\right)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/140*sqrt(35)*log(abs(21875000000*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*
x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 - 918750
00000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 172812500000*sqrt(
10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 240625000000*sqrt(14) + 91328
1250000)) - 1/140*sqrt(35)*log(abs(-21875000000*sqrt(14)*(sqrt(10)*x - sqrt
(10*x^2 - 22*x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13
))^2 + 91875000000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 17281
2500000*sqrt(10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 240625000000*sqr
t(14) + 913281250000))
```

maple [C] time = 0.36, size = 82, normalized size = 2.16

method	result	size
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{75\,\mathrm{RootOf}\left(\_Z^2-35\right)x^2-158\,\mathrm{RootOf}\left(\_Z^2-35\right)x+140\sqrt{10x^2-22x+13}\,x+87\,\mathrm{RootOf}\left(\_Z^2-35\right)-140\sqrt{10x^2-22x+13}}{5x^2-18x+17}\right)}{140}$	82

default	$-\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\,\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(1+\frac{-2+x}{1-x}\right)^2}}\left(1+\frac{-2+x}{1-x}\right)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/140*RootOf(_Z^2-35)*ln(-(75*RootOf(_Z^2-35)*x^2-158*RootOf(_Z^2-35)*x+14
0*(10*x^2-22*x+13)^(1/2)*x+87*RootOf(_Z^2-35)-140*(10*x^2-22*x+13)^(1/2))/(
5*x^2-18*x+17))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt{10\,x^2-22\,x+13}\left(5\,x^2-18\,x+17\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x-2}{\left(5\,x^2-18\,x+17\right)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

```
[Out] int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\left(5x^2-18x+17\right)\sqrt{10x^2-22x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

3.250 $\int x^4\sqrt{5-x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{25}{16}\sqrt{5-x^2}\,x+\frac{1}{6}\sqrt{5-x^2}\,x^5-\frac{5}{24}\sqrt{5-x^2}\,x^3+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{6}\sqrt{5-x^2}\,x^5-\frac{5}{24}\sqrt{5-x^2}\,x^3-\frac{25}{16}\sqrt{5-x^2}\,x+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5 - x^2])/6
 + (125*ArcSin[x/Sqrt[5]])/16
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{5-x^2}\,dx &= \frac{1}{6}x^5\sqrt{5-x^2}+\frac{5}{6}\int\frac{x^4}{\sqrt{5-x^2}}\,dx\\
&= -\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{25}{8}\int\frac{x^2}{\sqrt{5-x^2}}\,dx\\
&= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\int\frac{1}{\sqrt{5-x^2}}\,dx\\
&= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.62

$$\frac{1}{48}\left(x\sqrt{5-x^2}\,\left(8x^4-10x^2-75\right)+375\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4) + 375*ArcSin[x/Sqrt[5]])/48
```

IntegrateAlgebraic [C] time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{48}\sqrt{5-x^2}\left(8x^5-10x^3-75x\right)+\frac{125}{16}i\log\left(\sqrt{5-x^2}-ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (Sqrt[5 - x^2]*(-75*x - 10*x^3 + 8*x^5))/48 + ((125*I)/16)*Log[(-I)*x + Sqr
t[5 - x^2]]
```

fricas [A] time = 0.53, size = 42, normalized size = 0.65

$$\frac{1}{48}\left(8x^5-10x^3-75x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(-x^2 + 5)/
x)
```

giac [A] time = 0.60, size = 36, normalized size = 0.55

$$\frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}\,x+\frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)*
x)
```

maple [A] time = 0.31, size = 40, normalized size = 0.62

method	result	size
risch	$-\frac{x\left(8x^4-10x^2-75\right)\left(x^2-5\right)}{48\sqrt{-x^2+5}}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	40
default	$-\frac{x^3\left(-x^2+5\right)^{\frac{3}{2}}}{6}-\frac{5x\left(-x^2+5\right)^{\frac{3}{2}}}{8}+\frac{25x\sqrt{-x^2+5}}{16}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	49
meijerg	$\frac{125i\left(\frac{i\sqrt{\pi}\,x\sqrt{5}\left(-\frac{8}{5}x^4+2x^2+15\right)\sqrt{-\frac{x^2}{5}+1}}{300}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{4}\right)}{4\sqrt{\pi}}$	52
trager	$\frac{x\left(8x^4-10x^2-75\right)\sqrt{-x^2+5}}{48}+\frac{125\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+5}+x\right)}{16}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*x*(8*x^4-10*x^2-75)*(x^2-5)/(-x^2+5)^(1/2)+125/16*arcsin(1/5*x*5^(1/2
))
```

maxima [A] time = 1.22, size = 48, normalized size = 0.74

$$-\frac{1}{6}\left(-x^{2}+5\right)^{\frac{3}{2}}x^{3}-\frac{5}{8}\left(-x^{2}+5\right)^{\frac{3}{2}}x+\frac{25}{16}\sqrt{-x^{2}+5}\,x+\frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-x^2 + 5)*x
 + 125/16*arcsin(1/5*sqrt(5)*x)
```

mupad [B] time = 0.03, size = 35, normalized size = 0.54

$$\frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{16}-\sqrt{5-x^{2}}\,\left(-\frac{x^{5}}{6}+\frac{5\,x^{3}}{24}+\frac{25\,x}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5 - x^2)^(1/2),x)
```

```
[Out] (125*asin((5^(1/2)*x)/5))/16 - (5 - x^2)^(1/2)*((25*x)/16 + (5*x^3)/24 - x^
5/6)
```

sympy [A] time = 4.51, size = 155, normalized size = 2.38

$$\begin{cases} \frac{ix^{7}}{6\sqrt{x^{2}-5}}-\frac{25ix^{5}}{24\sqrt{x^{2}-5}}-\frac{25ix^{3}}{48\sqrt{x^{2}-5}}+\frac{125ix}{16\sqrt{x^{2}-5}}-\frac{125i\,\mathrm{acosh}\left(\frac{\sqrt{5}\,x}{5}\right)}{16} & \text{for}\ \frac{|x^{2}|}{5}>1 \\ -\frac{x^{7}}{6\sqrt{5-x^{2}}}+\frac{25x^{5}}{24\sqrt{5-x^{2}}}+\frac{25x^{3}}{48\sqrt{5-x^{2}}}-\frac{125x}{16\sqrt{5-x^{2}}}+\frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*I
*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqrt(
5)*x/5)/16, Abs(x**2)/5 > 1), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt(
5 - x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 125*
asin(sqrt(5)*x/5)/16, True))
```

3.251 $\int \frac{1}{x^6 \sqrt{2+x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{x^2+2}}{15x}-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 264}

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -Sqrt[2 + x^2]/(10*x^5) + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/(15*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\sqrt{2+x^2}}\,dx &= -\frac{\sqrt{2+x^2}}{10x^5}-\frac{2}{5}\int \frac{1}{x^4\sqrt{2+x^2}}\,dx\\
&= -\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}+\frac{2}{15}\int \frac{1}{x^2\sqrt{2+x^2}}\,dx\\
&= -\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}-\frac{\sqrt{2+x^2}}{15x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$-\frac{\sqrt{x^2+2}\left(2x^4-2x^2+3\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -1/30*(Sqrt[2 + x^2]*(3 - 2*x^2 + 2*x^4))/x^5
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 0.57

$$\frac{\sqrt{x^2+2}\left(-2x^4+2x^2-3\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*(-3 + 2*x^2 - 2*x^4))/(30*x^5)
```

fricas [A] time = 0.64, size = 31, normalized size = 0.63

$$-\frac{2\,x^5+\left(2\,x^4-2\,x^2+3\right)\sqrt{x^2+2}}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5
```

giac [A] time = 0.63, size = 51, normalized size = 1.04

$$\frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4-5\left(x-\sqrt{x^2+2}\right)^2+2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2-2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x^
2 + 2))^2 - 2)^5
```

maple [A] time = 0.32, size = 25, normalized size = 0.51

method	result	size
gosper	$-\frac{\sqrt{x^2+2}\,\left(2x^4-2x^2+3\right)}{30x^5}$	25
trager	$-\frac{\sqrt{x^2+2}\,\left(2x^4-2x^2+3\right)}{30x^5}$	25
meijerg	$-\frac{\sqrt{2}\,\left(\frac{2}{3}x^4-\frac{2}{3}x^2+1\right)\sqrt{1+\frac{x^2}{2}}}{10x^5}$	30
risch	$-\frac{2x^6+2x^4-x^2+6}{30x^5\,\sqrt{x^2+2}}$	30
default	$-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{15x}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

maxima [A] time = 1.09, size = 37, normalized size = 0.76

$$-\frac{\sqrt{x^2+2}}{15\,x}+\frac{\sqrt{x^2+2}}{15\,x^3}-\frac{\sqrt{x^2+2}}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5
```

mupad [B] time = 0.03, size = 25, normalized size = 0.51

$$-\sqrt{x^2+2}\,\left(\frac{1}{15\,x}-\frac{1}{15\,x^3}+\frac{1}{10\,x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^2 + 2)^(1/2)),x)
```

```
[Out] -(x^2 + 2)^(1/2)*(1/(15*x) - 1/(15*x^3) + 1/(10*x^5))
```

sympy [A] time = 2.49, size = 41, normalized size = 0.84

$$-\frac{\sqrt{1+\frac{2}{x^2}}}{15}+\frac{\sqrt{1+\frac{2}{x^2}}}{15x^2}-\frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**2+2)**(1/2),x)
```

```
[Out] -sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x*
*4)
```

3.252 $\int \frac{1}{\left(3+2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=49

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\left(2x^2+3\right)^{3/2}}+\frac{x}{15\left(2x^2+3\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\left(2x^2+3\right)^{3/2}}+\frac{x}{15\left(2x^2+3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/(405*Sqrt[
3 + 2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(3+2x^2\right)^{7/2}} \, dx &= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4}{15}\int \frac{1}{\left(3+2x^2\right)^{5/2}} \, dx \\
&= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8}{135}\int \frac{1}{\left(3+2x^2\right)^{3/2}} \, dx \\
&= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8x}{405\sqrt{3+2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 2*x^2)^(-7/2),x]
[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

fricas [A] time = 0.58, size = 44, normalized size = 0.90

$$\frac{\left(32\,x^5+120\,x^3+135\,x\right)\sqrt{2\,x^2+3}}{405\left(8\,x^6+36\,x^4+54\,x^2+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")
[Out] 1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4 + 54*x^2 +
 27)
```

giac [A] time = 0.64, size = 26, normalized size = 0.53

$$\frac{\left(8\left(4\,x^2+15\right)x^2+135\right)x}{405\left(2\,x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="giac")
[Out] 1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

maple [A] time = 0.28, size = 25, normalized size = 0.51

method	result	size
gosper	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
trager	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
risch	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
meijerg	$\frac{\sqrt{3}\,x\left(\frac{32}{9}x^4+\frac{40}{3}x^2+15\right)}{1215\left(1+\frac{2x^2}{3}\right)^{\frac{5}{2}}}$	28
default	$\frac{x}{15(2x^2+3)^{\frac{5}{2}}}+\frac{4x}{135(2x^2+3)^{\frac{3}{2}}}+\frac{8x}{405\sqrt{2x^2+3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

maxima [A] time = 0.54, size = 37, normalized size = 0.76

$$\frac{8\,x}{405\,\sqrt{2\,x^2+3}}+\frac{4\,x}{135\,\left(2\,x^2+3\right)^{\frac{3}{2}}}+\frac{x}{15\,\left(2\,x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x^2 + 3)^(5
/2)
```

mupad [B] time = 0.05, size = 187, normalized size = 3.82

$$\frac{2\,\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{405\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)}+\frac{2\,\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{405\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)}+\frac{\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{1440\,\left(-x^3+\frac{3\mathrm{i}\,\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}-\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)}+\frac{\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{1440\,\left(-x^3-\frac{3\mathrm{i}\,\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}+\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)}+\frac{\surd}{259}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 3)^(7/2),x)
```

```
[Out] (2*2^(1/2)*(x^2 + 3/2)^(1/2))/(405*(x - (6^(1/2)*1i)/2)) + (2*2^(1/2)*(x^2
+ 3/2)^(1/2))/(405*(x + (6^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 3/2)^(1/2))/(144
0*((9*x)/2 - (6^(1/2)*3i)/4 + (6^(1/2)*x^2*3i)/2 - x^3)) + (2^(1/2)*(x^2 +
3/2)^(1/2))/(1440*((9*x)/2 + (6^(1/2)*3i)/4 - (6^(1/2)*x^2*3i)/2 - x^3)) +
(2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i + x^2 - 3/2))
+ (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i - x^2 + 3/2)
)
```

sympy [B] time = 9.96, size = 139, normalized size = 2.84

$$\frac{32x^5}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}+\frac{120x^3}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)**(7/2),x)
```

```
[Out] 32*x**5/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x**2 + 3) + 3645*sqr
t(2*x**2 + 3)) + 120*x**3/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x*
*2 + 3) + 3645*sqrt(2*x**2 + 3)) + 135*x/(1620*x**4*sqrt(2*x**2 + 3) + 4860
*x**2*sqrt(2*x**2 + 3) + 3645*sqrt(2*x**2 + 3))
```

3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=12

$$\log \left(a+\sqrt{x^2+1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2155, 31}

$$\log \left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

```
[Out] Log[a + Sqrt[1 + x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+a\sqrt{1+x}}\,dx,x,x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt{1+x^2}\right)\\ &= \log \left(a+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$\log \left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

```
[Out] Log[a + Sqrt[1 + x^2]]
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$\log \left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

```
[Out] Log[a + Sqrt[1 + x^2]]
```

fricas [B] time = 0.59, size = 62, normalized size = 5.17

$$\frac{1}{2}\log\left(-a^2+x^2+1\right)-\frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}\,(a+x)+1\right)+\frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}\,(a-x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) +
1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

giac [A] time = 0.64, size = 11, normalized size = 0.92

$$\log\left(\left|a+\sqrt{x^2+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a + sqrt(x^2 + 1)))
```

maple [B] time = 0.07, size = 328, normalized size = 27.33

method	result
default	$-\frac{\sqrt{\left(x-\sqrt{(a-1)(1+a)}\right)^2+2\sqrt{(a-1)(1+a)}\left(x-\sqrt{(a-1)(1+a)}\right)+a^2}}{2a}+\frac{a\ln\left(\frac{2a^2+2\sqrt{(a-1)(1+a)}\left(x-\sqrt{(a-1)(1+a)}\right)+2\sqrt{a^2}\sqrt{\left(x-\sqrt{(a-1)(1+a)}\right)^2+2\sqrt{}}}{x-\sqrt{(a-1)(1+a)}}\right.}{2\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x^2+a*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*((x-((a-1)*(1+a))^(1/2))^2+2*((a-1)*(1+a))^(1/2)*(x-((a-1)*(1+a))^(1
/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2+2*((a-1)*(1+a))^(1/2)*(x-((a-1)
*(1+a))^(1/2))+2*(a^2)^(1/2)*((x-((a-1)*(1+a))^(1/2))^2+2*((a-1)*(1+a))^(1/
2)*(x-((a-1)*(1+a))^(1/2))+a^2)^(1/2))/(x-((a-1)*(1+a))^(1/2)))+1/a*(x^2+1)
^(1/2)-1/2/a*((x+((a-1)*(1+a))^(1/2))^2-2*((a-1)*(1+a))^(1/2)*(x+((a-1)*(1+
a))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((a-1)*(1+a))^(1/2)*(x+
((a-1)*(1+a))^(1/2))+2*(a^2)^(1/2)*((x+((a-1)*(1+a))^(1/2))^2-2*((a-1)*(1+a
))^(1/2)*(x+((a-1)*(1+a))^(1/2))+a^2)^(1/2))/(x+((a-1)*(1+a))^(1/2)))+1/2*l
n(-a^2+x^2+1)
```

maxima [A] time = 0.46, size = 10, normalized size = 0.83

$$\log\left(a+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a + sqrt(x^2 + 1))
```

mupad [B] time = 0.27, size = 154, normalized size = 12.83

$$\frac{\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)}{2}+\frac{\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)}{2}-\frac{a\left(\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}-x\,\sqrt{a-1}\,\sqrt{a}\right.\right.}{2\sqrt{(a-1)\,(a+1)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*(x^2 + 1)^(1/2) + x^2 + 1),x)
```

```
[Out] log(x + (a - 1)^(1/2)*(a + 1)^(1/2))/2 + log(x - (a - 1)^(1/2)*(a + 1)^(1/2
))/2 - (a*(log(x + (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^2)
^(1/2) - x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(1/2
)) - (a*(log(x - (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^2)^(
1/2) + x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(1/2))
```

sympy [**B**] time = 3.00, size = 53, normalized size = 4.42

$$-\frac{a\left(-\frac{\log\left(2a+2\sqrt{x^2+1}\right)}{a}+\frac{\log\left(-2\sqrt{x^2+1}\right)}{a}\right)}{2}+\frac{\log\left(a\sqrt{x^2+1}+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

```
[Out] -a*(-log(2*a + 2*sqrt(x**2 + 1))/a + log(-2*sqrt(x**2 + 1))/a)/2 + log(a*sq
rt(x**2 + 1) + x**2 + 1)/2
```

3.254 $\int \frac{1-x+x^2}{\left(1+x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 215}

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+x^2}{\left(1+x^2\right)^{3/2}} \, dx &= \frac{1}{\sqrt{1+x^2}}+\int \frac{1}{\sqrt{1+x^2}} \, dx\\ &= \frac{1}{\sqrt{1+x^2}}+\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

IntegrateAlgebraic [B] time = 0.17, size = 26, normalized size = 2.17

$$\frac{1}{\sqrt{x^2+1}}-\log\left(\sqrt{x^2+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x + x^2)/(1 + x^2)^(3/2),x]
[Out] 1/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

fricas [B] time = 0.66, size = 37, normalized size = 3.08

$$-\frac{(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] -((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)
```

giac [B] time = 0.67, size = 22, normalized size = 1.83

$$\frac{1}{\sqrt{x^2+1}}-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.35, size = 11, normalized size = 0.92

method	result	size
default	$\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$	11
risch	$\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$	11
trager	$\frac{1}{\sqrt{x^2+1}}-\ln\left(x-\sqrt{x^2+1}\right)$	23
meijerg	$\frac{x}{\sqrt{x^2+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^2+1}}}{\sqrt{\pi}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)+1/(x^2+1)^(1/2)
```

maxima [A] time = 1.43, size = 10, normalized size = 0.83

$$\frac{1}{\sqrt{x^2+1}}+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] 1/sqrt(x^2 + 1) + arcsinh(x)
```

mupad [B] time = 0.19, size = 24, normalized size = 2.00

$$\frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)+\sqrt{x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 1)/(x^2 + 1)^(3/2),x)
```

```
[Out] (asinh(x) + x^2*asinh(x) + (x^2 + 1)^(1/2))/(x^2 + 1)
```

sympy [**B**] time = 12.85, size = 29, normalized size = 2.42

$$\frac{x^2 \operatorname{asinh}(x)}{x^2+1}+\frac{\operatorname{asinh}(x)}{x^2+1}+\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {402, 215, 377, 206}

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx &= \int \frac{1}{\sqrt{1+x^2}}\,dx-\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)}\,dx\\ &= \sinh^{-1}(x)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right)\\ &= \sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

IntegrateAlgebraic [B] time = 0.09, size = 57, normalized size = 2.11

$$-\log\left(\sqrt{x^2+1}-x\right)-\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{2}}-\frac{\sqrt{x^2+1}\,x}{\sqrt{2}}+\sqrt{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] -(ArcTanh[Sqrt[2] + x^2/Sqrt[2] - (x*Sqrt[1 + x^2])/Sqrt[2]]/Sqrt[2]) - Log
[-x + Sqrt[1 + x^2]]
```

fricas [B] time = 0.57, size = 67, normalized size = 2.48

$$\frac{1}{4}\sqrt{2}\log\left(\frac{9x^2-2\sqrt{2}\left(3x^2+2\right)-2\sqrt{x^2+1}\left(3\sqrt{2}x-4x\right)+6}{x^2+2}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))
```

giac [B] time = 0.63, size = 64, normalized size = 2.37

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1)
)^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.28, size = 23, normalized size = 0.85

method	result	size
default	$\operatorname{arcsinh}(x)-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{2}$	23
trager	$-\ln\left(x-\sqrt{x^2+1}\right)+\frac{\operatorname{RootOf}(\_Z^2-2)\ln\left(-\frac{3\operatorname{RootOf}(\_Z^2-2)x^2-4x\sqrt{x^2+1}+2\operatorname{RootOf}(\_Z^2-2)}{x^2+2}\right)}{4}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{x^2+1}\,x}{x^2+2}+\int \frac{\sqrt{x^2+1}\,x^4}{x^6+5\,x^4+8\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x^
2 + 4), x)
```

mupad [B] time = 0.17, size = 77, normalized size = 2.85

$$\operatorname{asinh}(x)+\frac{\sqrt{2}\,\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4}-\frac{\sqrt{2}\,\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2}\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x^2 + 2),x)
```

```
[Out] asinh(x) + (2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/
2)*1i + 1)))/4 - (2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2
^(1/2)*x*1i + 1)))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

3.256 $\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\left(x^2+2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {382, 377, 206}

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/(
4*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)^2}\,dx &= -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)}+\frac{3}{4}\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)}\,dx\\ &= -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right)\\ &= -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)}+\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.21, size = 74, normalized size = 1.54

$$\frac{\sqrt{x^2+1}\left(3\sqrt{2}\sqrt{\frac{x^2}{x^2+1}}\left(x^2+2\right)\tanh^{-1}\left(\sqrt{\frac{x^2}{2x^2+2}}\right)-2x^2\right)}{8x\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] (Sqrt[1 + x^2]*(-2*x^2 + 3*Sqrt[2]*Sqrt[x^2/(1 + x^2)]*(2 + x^2)*ArcTanh[Sq
rt[x^2/(2 + 2*x^2)]]))/(8*x*(2 + x^2))
```

IntegrateAlgebraic [A] time = 0.08, size = 64, normalized size = 1.33

$$\frac{3\tanh^{-1}\left(\frac{x^2}{\sqrt{2}}-\frac{\sqrt{x^2+1}\,x}{\sqrt{2}}+\sqrt{2}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -1/4*(x*Sqrt[1 + x^2])/(2 + x^2) + (3*ArcTanh[Sqrt[2] + x^2/Sqrt[2] - (x*Sq
rt[1 + x^2])/Sqrt[2]])/(4*Sqrt[2])
```

fricas [B] time = 0.53, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}\left(x^2+2\right)\log\left(\frac{9x^2+2\sqrt{2}\left(3x^2+2\right)+2\sqrt{x^2+1}\left(3\sqrt{2}x+4x\right)+6}{x^2+2}\right)-4x^2-4\sqrt{x^2+1}\,x-8}{16\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 2)*log((9*x^2 + 2*sqrt(2)*(3*x^2 + 2) + 2*sqrt(x^2 +
 1)*(3*sqrt(2)*x + 4*x) + 6)/(x^2 + 2)) - 4*x^2 - 4*sqrt(x^2 + 1)*x - 8)/(x
^2 + 2)
```

giac [B] time = 0.64, size = 101, normalized size = 2.10

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 +
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^2
+ 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

maple [A] time = 0.28, size = 38, normalized size = 0.79

method	result	size

risch	$\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$	38
default	$\frac{x}{4\sqrt{x^2+1}\left(\frac{x^2}{x^2+1}-2\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}$	46
trager	$-\frac{x\sqrt{x^2+1}}{4(x^2+2)}-\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^2-4x\sqrt{x^2+1}+2\,\mathrm{RootOf}\left(\_Z^2-2\right)}{x^2+2}\right)}{16}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+2)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

mupad [B] time = 0.09, size = 117, normalized size = 2.44

$$-\frac{3\sqrt{2}\,\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16}+\frac{3\sqrt{2}\,\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^2),x)
```

```
[Out] (3*2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i + 1
)))/16 - (3*2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/
2)*1i + 1)))/16 - (x^2 + 1)^(1/2)/(8*(x - 2^(1/2)*1i)) - (x^2 + 1)^(1/2)/(8
*(x + 2^(1/2)*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

3.257 $\int \frac{x^2}{\left(-6+x^2\right)\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=41

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,x}{\sqrt{x^2-2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {483, 217, 206, 377, 207}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(-6+x^2\right)\sqrt{-2+x^2}}\,dx &= 6\int \frac{1}{\left(-6+x^2\right)\sqrt{-2+x^2}}\,dx + \int \frac{1}{\sqrt{-2+x^2}}\,dx \\
&= 6\,\text{Subst}\left(\int \frac{1}{-6+4x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right) \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,x}{\sqrt{-2+x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$
\log\left(\sqrt{x^2-2}+x\right) - \sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,x}{\sqrt{x^2-2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] -(Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]) + Log[x + Sqrt[-2 + x^2]
]
```

IntegrateAlgebraic [A] time = 0.12, size = 66, normalized size = 1.61

$$
-\log\left(\sqrt{x^2-2}-x\right) - \sqrt{\frac{3}{2}}\tanh^{-1}\left(-\frac{x^2}{2\sqrt{6}} + \frac{\sqrt{x^2-2}\,x}{2\sqrt{6}} + \sqrt{\frac{3}{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] -(Sqrt[3/2]*ArcTanh[Sqrt[3/2] - x^2/(2*Sqrt[6]) + (x*Sqrt[-2 + x^2])/(2*Sqr
t[6])]) - Log[-x + Sqrt[-2 + x^2]]
```

fricas [B] time = 0.67, size = 77, normalized size = 1.88

$$
\frac{1}{4}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}\left(5x^2-6\right) - 25x^2 + 2\left(5\sqrt{3}\sqrt{2}x - 12x\right)\sqrt{x^2-2} + 30}{x^2-6}\right) - \log\left(-x+\sqrt{x^2-2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(5*x^2 - 6) - 25*x^2 + 2*(5*sqr
t(3)*sqrt(2)*x - 12*x)*sqrt(x^2 - 2) + 30)/(x^2 - 6)) - log(-x + sqrt(x^2 -
 2))
```

giac [B] time = 0.66, size = 72, normalized size = 1.76

$$
-\frac{1}{4}\sqrt{6}\log\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2 - 8\sqrt{6} - 20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2 + 8\sqrt{6} - 20\right|}\right) - \frac{1}{2}\log\left(\left(x-\sqrt{x^2-2}\right)^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(6)*log(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs(2*(x - s
qrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*log((x - sqrt(x^2 - 2))^2)
```

maple [C] time = 0.31, size = 64, normalized size = 1.56

method	result	size
trager	$-\ln\left(x-\sqrt{x^2-2}\right)-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-6\right)x^2+12\sqrt{x^2-2}\,x-6\,\mathrm{RootOf}\left(\_Z^2-6\right)}{x^2-6}\right)}{4}$	64
default	$\ln\left(x+\sqrt{x^2-2}\right)-\frac{\sqrt{6}\,\mathrm{arctanh}\left(\frac{8+2\sqrt{6}\,(x-\sqrt{6})}{4\sqrt{(x-\sqrt{6})^2+2\sqrt{6}\,(x-\sqrt{6})+4}}\right)}{4}+\frac{\sqrt{6}\,\mathrm{arctanh}\left(\frac{8-2\sqrt{6}\,(x+\sqrt{6})}{4\sqrt{(x+\sqrt{6})^2-2\sqrt{6}\,(x+\sqrt{6})+4}}\right)}{4}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2-6)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x-(x^2-2)^(1/2))-1/4*RootOf(_Z^2-6)*ln(-(5*RootOf(_Z^2-6)*x^2+12*(x^2-2
)^(1/2)*x-6*RootOf(_Z^2-6))/(x^2-6))
```

maxima [B] time = 1.46, size = 107, normalized size = 2.61

$$\frac{1}{12}\sqrt{6}\left(2\sqrt{6}\log\left(x+\sqrt{x^2-2}\right)-3\log\left(\sqrt{6}+\frac{4\sqrt{x^2-2}}{\left|2x-2\sqrt{6}\right|}+\frac{8}{\left|2x-2\sqrt{6}\right|}\right)+3\log\left(-\sqrt{6}+\frac{4\sqrt{x^2-2}}{\left|2x+2\sqrt{6}\right|}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^2
 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4*s
qrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^2-2}\,\left(x^2-6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)),x)
```

```
[Out] int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-6\right)\sqrt{x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1-x^2}\,x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {527, 12, 377, 203}

$$\frac{\sqrt{1-x^2}\,x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x^2}{\sqrt{1-x^2}\left(1+x^2\right)^2}\,dx &= \frac{x\sqrt{1-x^2}}{1+x^2}-\frac{1}{4}\int -\frac{16}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx\\ &= \frac{x\sqrt{1-x^2}}{1+x^2}+4\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx\\ &= \frac{x\sqrt{1-x^2}}{1+x^2}+4\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{x\sqrt{1-x^2}}{1+x^2}+2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 1.81

$$\frac{\sqrt{1-x^2}\,x}{x^2+1}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}+\frac{3x\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\right)}{\sqrt{2}\sqrt{-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2] + (
3*x*ArcTanh[Sqrt[2]*Sqrt[x^2/(-1 + x^2)]])/(Sqrt[2]*Sqrt[-x^2])
```

IntegrateAlgebraic [A] time = 0.12, size = 54, normalized size = 1.15

$$\frac{x\sqrt{1-x^2}}{x^2+1}-2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{1-x^2}}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) - 2*Sqrt[2]*ArcTan[(Sqrt[2]*x*Sqrt[1 - x^2])/(-
1 + x^2)]
```

fricas [A] time = 0.70, size = 50, normalized size = 1.06

$$-\frac{2\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)-\sqrt{-x^2+1}\,x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) - sqrt(-x^2 + 1)
*x)/(x^2 + 1)
```

giac [B] time = 0.65, size = 123, normalized size = 2.62

$$\sqrt{2}\left(\pi\text{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}\,x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{2\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1
)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)
```

maple [A] time = 0.31, size = 53, normalized size = 1.13

method	result	size
risch	$-\frac{x\left(x^{2}-1\right)}{\left(x^{2}+1\right)\sqrt{-x^{2}+1}}-2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{-x^{2}+1}\ x}{x^{2}-1}\right)$	53
trager	$\frac{x\sqrt{-x^{2}+1}}{x^{2}+1}+\mathrm{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^{2}+2\right)x^{2}+4\sqrt{-x^{2}+1}\ x+\mathrm{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)$	66
default	$-2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{-x^{2}+1}\ x}{x^{2}-1}\right)-\frac{\sqrt{-x^{2}+1}\ x}{2\left(x^{2}-1\right)\left(\frac{\left(-x^{2}+1\right)x^{2}}{\left(x^{2}-1\right)^{2}}+\frac{1}{2}\right)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(x^2-1)/(x^2+1)/(-x^2+1)^(1/2)-2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(
x^2-1)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+5}{\left(x^{2}+1\right)^{2}\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

mupad [B] time = 0.13, size = 115, normalized size = 2.45

$$\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\ (-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^{2}}\ 1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i}-\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\ (1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^{2}}\ 1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i}+\frac{\sqrt{1-x^{2}}}{2\ (x-\mathrm{i})}+\frac{\sqrt{1-x^{2}}}{2\ (x+1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 5)/((1 - x^2)^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] 2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i -
 2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i))*1i
+ (1 - x^2)^(1/2)/(2*(x - 1i)) + (1 - x^2)^(1/2)/(2*(x + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+5}{\sqrt{-(x-1)\,(x+1)}\ \left(x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 5)/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=88

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.24, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 1591, 190, 43, 6740, 203, 402, 216, 377}

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 + Sq
rt[1 - x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
```

```
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \int \left(\frac{4x}{5+\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\right)dx$$
$$= 4\int \frac{x}{5+\sqrt{1-x^2}}\,dx-\int \frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{5+\sqrt{x}}\,dx,x,1-x^2\right)\right)-\int \left(1-\frac{5}{5+\sqrt{1-x^2}}\right)dx$$
$$= -x-4\,\text{Subst}\left(\int \frac{x}{5+x}\,dx,x,\sqrt{1-x^2}\right)+5\int \frac{1}{5+\sqrt{1-x^2}}\,dx$$
$$= -x-4\,\text{Subst}\left(\int \left(1-\frac{5}{5+x}\right)dx,x,\sqrt{1-x^2}\right)+5\int \left(\frac{5}{24+x^2}-\frac{\sqrt{1-x^2}}{24+x^2}\right)dx$$
$$= -x-4\sqrt{1-x^2}+20\log\left(5+\sqrt{1-x^2}\right)-5\int \frac{\sqrt{1-x^2}}{24+x^2}\,dx+25\int \frac{1}{24+x^2}\,dx$$
$$= -x-4\sqrt{1-x^2}+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)+5\int \frac{1}{\sqrt{1-x^2}}\,dx-125\int \frac{}{\sqrt{1}}$$
$$= -x-4\sqrt{1-x^2}+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)-125\,\text{Subst}\left(\int \frac{}{24}\right.$$
$$= -x-4\sqrt{1-x^2}+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-\right.$$

Mathematica [A] time = 0.20, size = 137, normalized size = 1.56

$$-4\sqrt{1-x^2}+10\log\left(x^2+24\right)-10\log\left(\left(x^2+24\right)^2\right)+10\log\left(\left(x^2+24\right)\left(-x^2+10\sqrt{1-x^2}+26\right)\right)+\frac{25\tan^{-1}\left(\frac{4x^2+40}{10\sqrt{}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 + (25*ArcTan[(96 + 4*x^2 + 409*x*Sqrt[1 - x^2])/(10*Sqrt[6]*(-1 + 17*x^2))
])/(2*Sqrt[6]) + 10*Log[24 + x^2] - 10*Log[(24 + x^2)^2] + 10*Log[(24 + x^2
)*(26 - x^2 + 10*Sqrt[1 - x^2])]
```

IntegrateAlgebraic [A] time = 0.32, size = 108, normalized size = 1.23

$$-4\sqrt{1-x^2}-20\log\left(\sqrt{1-x^2}-1\right)+20\log\left(-x^2+4\sqrt{1-x^2}-4\right)+10\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)-\frac{25\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{1-x^2}-1}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 10*ArcTan[x/(-1 + Sqrt[1 - x^2])] - (25*ArcTan[(Sqrt
[3/2]*x)/(-1 + Sqrt[1 - x^2])])/Sqrt[6] - 20*Log[-1 + Sqrt[1 - x^2]] + 20*L
og[-4 - x^2 + 4*Sqrt[1 - x^2]]
```

fricas [B] time = 0.59, size = 160, normalized size = 1.82

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2x}\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3x}\right)-x-4\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*sq
rt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2 +
1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)/x)
 + 10*log(x^2 + 24) - 10*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 10*log((x
^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)
```

giac [B] time = 0.69, size = 135, normalized size = 1.53

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3x}\right)-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2x}\right)-x-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(s
qrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1) -
 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(3*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3)
```

maple [A] time = 0.92, size = 82, normalized size = 0.93

method	result
default	$\frac{25\arctan\left(\frac{x\sqrt{6}}{12}\right)\sqrt{6}}{12}+10\ln\left(x^2+24\right)-x+5\arcsin(x)+\frac{25\sqrt{6}\arctan\left(\frac{5\sqrt{6}\sqrt{-x^2+1}x}{12(x^2-1)}\right)}{12}-4\sqrt{-x^2+1}+20\text{ a}$
trager	$-x-4\sqrt{-x^2+1}+5\operatorname{RootOf}\left(\_Z^2+8\_Z+17\right)\ln\left(\frac{-25\operatorname{RootOf}\left(\_Z^2+8\_Z+17\right)^2\operatorname{RootOf}\left(24\_Z^2-192\_Z+409\right)x-}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(1
/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arcta
nh(1/5*(-x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x - 4\sqrt{-x^2+1} + 5\int \frac{1}{\sqrt{x+1}\sqrt{-x+1}+5}\,dx + 20\log\left(\sqrt{-x^2+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) +
20*log(sqrt(-x^2 + 1) + 5)
```

mupad [B] time = 0.38, size = 159, normalized size = 1.81

$$5\,\mathrm{asin}(x) - x - 4\sqrt{1-x^2} - \frac{\sqrt{24}\,\ln\left(\frac{\frac{2\sqrt{6}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x-\sqrt{6}\,2\mathrm{i}}\right)\left(125+\sqrt{24}\,100\mathrm{i}\right)1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(\frac{-\frac{\sqrt{24}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x+\sqrt{24}\,1\mathrm{i}}\right)\left(-125+\right.}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - (1 - x^2)^(1/2))/((1 - x^2)^(1/2) + 5),x)
```

```
[Out] 5*asin(x) - x - 4*(1 - x^2)^(1/2) - (24^(1/2)*log(((2*6^(1/2)*x)/5 + (1 - x
^2)^(1/2)*1i + 1i/5)/(x - 6^(1/2)*2i))*(24^(1/2)*100i + 125)*1i)/240 - (24^
(1/2)*log(((1 - x^2)^(1/2)*1i - (24^(1/2)*x)/5 + 1i/5)/(x + 24^(1/2)*1i))*(
24^(1/2)*100i - 125)*1i)/240 - (24^(1/2)*log(x - 6^(1/2)*2i)*(24^(1/2)*20i
+ 25)*1i)/48 - (24^(1/2)*log(x + 24^(1/2)*1i)*(24^(1/2)*20i - 25)*1i)/48
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)
```

3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx$

Optimal. Leaf size=136

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}\,x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\ldots$$

Rubi [A] time = 1.50, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6742, 195, 215, 634, 618, 204, 628, 1020, 12, 1081, 1037, 1031, 206, 261}

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}\,x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x
]
```

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[x
])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[(1
 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2])])
/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1031

```
Int[((g_) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b
 - 2*a*h)*(b^2 - 4*a*c) - b*d*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g
*c)*x, x]/Sqrt[d + f*x^2]], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[b*h^2*d - 2*g*h*(c*d - a*f) - g^2*b*f, 0]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + b^2*d*f, 2]}, Dist
[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f - q) + (h*(c*d - a*f + q) + g*b*f)*
x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[
h*b*d - g*(c*d - a*f + q) + (h*(c*d - a*f - q) + g*b*f)*x, x]/((a + b*x + c
*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b
^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1081

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \int\left(-\frac{x^2}{1-x^3+\sqrt{1+x^2}+x^2\sqrt{1+x^2}}-\frac{2x^2}{\sqrt{1+x^2}\left(-1+x^3-\left(1+x^2\right)^{3/2}\right.}\right.$$

$$=-\left(2\int\frac{x^2}{\sqrt{1+x^2}\left(-1+x^3-\left(1+x^2\right)^{3/2}\right)}\,dx\right)-\int\frac{x^2}{1-x^3+\sqrt{1+x^2}+}$$

$$=-\left(2\int\left(-\frac{1}{3}+\frac{2}{9\sqrt{1+x^2}}-\frac{x}{3\sqrt{1+x^2}}+\frac{2x}{3\left(3+2x+3x^2\right)}+\frac{}{9\sqrt{1+x^2}}\right.\right.$$

$$=\frac{8x}{9}-\frac{x^2}{6}-\frac{1}{9}\int\frac{-3-5x}{3+2x+3x^2}\,dx-\frac{2}{9}\int\frac{3+5x}{\sqrt{1+x^2}\left(3+2x+3x^2\right)}\,dx-$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{4}{9}\sinh^{-1}(x)+\frac{1}{18}\int\frac{4}{\sqrt{1+x^2}\left(3\right.}$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{11}{18}\sinh^{-1}(x)-\frac{7}{54}\log\left(3+2x+\right.$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{11}{18}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1}{2}\right.$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1}{2}\right.$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1}{2}\right.$$

$$=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1}{2}\right.$$

Mathematica [C] time = 1.01, size = 261, normalized size = 1.92

$$\frac{1}{162}\left(-27x^2-27\sqrt{x^2+1}\,x+144\sqrt{x^2+1}-21\log\left(3x^2+2x+3\right)+\sqrt{1-2i\sqrt{2}}\left(11\sqrt{2}-i\right)\tanh^{-1}\left(\frac{-2i\sqrt{2}}{\sqrt{2+4i}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/
2))),x]
```

```
[Out] (144*x - 27*x^2 + 144*Sqrt[1 + x^2] - 27*x*Sqrt[1 + x^2] - 123*ArcSinh[x] +
 24*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])] + Sqrt[1 - (2*I)*Sqrt[2]]*(-I + 1
1*Sqrt[2])*ArcTanh[(3 - x - (2*I)*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[
1 + x^2])] + I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(3 - x + (2*I)*Sqrt[2]*x)/(S
qrt[2 - (4*I)*Sqrt[2]]*Sqrt[1 + x^2])] + 11*Sqrt[2 + (4*I)*Sqrt[2]]*ArcTanh
[(3 - x + (2*I)*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[1 + x^2])] - 21*Lo
g[3 + 2*x + 3*x^2])/162
```

IntegrateAlgebraic [A] time = 0.32, size = 117, normalized size = 0.86

$$\frac{1}{18}\sqrt{x^2+1}\,(16-3x)+\frac{1}{18}\left(16x-3x^2\right)+\frac{55}{54}\log\left(\sqrt{x^2+1}-x\right)-\frac{7}{27}\log\left(-x^2+(x+1)\sqrt{x^2+1}-x-2\right)+\frac{8}{27}\sqrt{2}\tan^{-}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 +
 x^2)^(3/2))),x]
```

```
[Out] (16*x - 3*x^2)/18 + ((16 - 3*x)*Sqrt[1 + x^2])/18 + (8*Sqrt[2]*ArcTan[1/Sqr
t[2] + x/Sqrt[2] - Sqrt[1 + x^2]/Sqrt[2]])/27 + (55*Log[-x + Sqrt[1 + x^2]]
)/54 - (7*Log[-2 - x - x^2 + (1 + x)*Sqrt[1 + x^2]])/27
```

fricas [A] time = 0.68, size = 170, normalized size = 1.25

$$-\frac{1}{6}x^2-\frac{1}{18}\sqrt{x^2+1}\,(3x-16)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\,(3x+1)\right)+\frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\,(3x-1)+\frac{3}{2}\sqrt{2}\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*
(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqrt(
x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(x^2
 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/54*l
og(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) + 41/54
*log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.67, size = 176, normalized size = 1.29

$$-\frac{1}{6}x^2-\frac{1}{18}\sqrt{x^2+1}\,(3x-16)+\frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(3x-3\sqrt{x^2+1}-1\right)\right)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\,(3x+1)\right)-\frac{4}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)
*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1))
- 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1) + 1)) + 8/9*x + 7/54*
log(3*(x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) + 1) - 7/54*log((x - sq
rt(x^2 + 1))^2 + 2*x - 2*sqrt(x^2 + 1) + 3) - 7/54*log(3*x^2 + 2*x + 3) + 4
1/54*log(-x + sqrt(x^2 + 1))
```

maple [B] time = 0.05, size = 654, normalized size = 4.81

$$-\frac{x^2}{6}+\frac{8x}{9}-\frac{7\ln\left(3x^2+2x+3\right)}{54}+\frac{4\sqrt{2}\arctan\left(\frac{(6x+2)\sqrt{2}}{8}\right)}{27}-\frac{41\,\mathrm{arcsinh}(x)}{54}-\frac{\sqrt{2}\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{2(\cdots}{(\cdots}}}{2\left(\frac{(1+\cdots}{(1-\cdots}\right.}\right.\right.}{12\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}\cdots}{\left(1+\frac{1+x}{1-x}\cdots\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2)
)-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arcta
n(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+
5*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x)
)^2)^(1/2)/(1+(1+x)/(1-x))+3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2
)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/
(1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/
(1-x))^2)^(1/2)/(1+(1+x)/(1-x))-1/6*x*(x^2+1)^(1/2)+8/9*(x^2+1)^(1/2)+1/216
*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x
)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+43*arctanh((2*(1+x)^2
/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/(1+(1+x)/
(1-x))-1/36*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*arctan(1/2*2^(
1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+arctanh((
2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/
(1+(1+x)/(1-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)+\int -\frac{3\,x^{10}-4\,x^9+5\,}{2\left(2\,x^{13}+7\,x^{11}-4\,x^{10}+11\,x^9-11\,x^8+13\,x^7-13\,x^6+11\,x^5-11\,x^4+4\,x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8 -
 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*x^
8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10 -
2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt(x^2
 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)
```

mupad [B] time = 0.62, size = 216, normalized size = 1.59

$$\frac{8\,x}{9}-\frac{41\,\mathrm{asinh}(x)}{54}-\left(\frac{x}{6}-\frac{8}{9}\right)\sqrt{x^2+1}-\frac{x^2}{6}+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(-\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)1\mathrm{i}}{8}+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(\frac{1}{2}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*((x^2 + 1)^(1/2) - 2))/((x^2 + 1)^(1/2)*((x^2 + 1)^(3/2) - x^3 +
1)),x)
```

```
[Out] (8*x)/9 - (41*asinh(x))/54 - (x/6 - 8/9)*(x^2 + 1)^(1/2) - x^2/6 + (2^(1/2)
*log(x - (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 - 16/27)*1i)/8 + (2^(1/2)*
log(x + (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 + 16/27)*1i)/8 + (2^(1/2)*(
(2^(1/2)*44i)/81 + 4/81)*(log(x + (2^(1/2)*2i)/3 + 1/3) - log(((2^(1/2)*1i)
/3 + 2/3)*(x^2 + 1)^(1/2) - x/3 - (2^(1/2)*x*2i)/3 + 1))*1i)/(8*(((2^(1/2)*
2i)/3 + 1/3)^2 + 1)^(1/2)) + (2^(1/2)*((2^(1/2)*44i)/81 - 4/81)*(log(x - (2
^(1/2)*2i)/3 + 1/3) - log((2^(1/2)*x*2i)/3 - ((2^(1/2)*1i)/3 - 2/3)*(x^2 +
1)^(1/2) - x/3 + 1))*1i)/(8*(((2^(1/2)*2i)/3 - 1/3)^2 + 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2)
,x)
```

```
[Out] Timed out
```

3.261 $\int x\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=64

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {640, 612, 620, 203}

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] -(r*(r - x)*Sqrt[2*r*x - x^2])/2 - (2*r*x - x^2)^(3/2)/3 + r^3*ArcTan[x/Sqr
t[2*r*x - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{2rx-x^2}\,dx &= -\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r\int\sqrt{2rx-x^2}\,dx\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+\frac{1}{2}r^3\int\frac{1}{\sqrt{2rx-x^2}}\,dx\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{2rx-x^2}}\right)\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.12, size = 72, normalized size = 1.12

$$\frac{1}{6}\sqrt{-x(x-2r)}\left(\frac{6r^{5/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}}-3r^2-rx+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 + (6*r^(5/2)*ArcSin[Sqrt[x]/(S
qrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r])))/6
```

IntegrateAlgebraic [C] time = 0.18, size = 113, normalized size = 1.77

$$-\frac{1}{2}ir^3\tanh^{-1}\left(\frac{x}{r}+\frac{i\sqrt{2rx-x^2}}{r}\right)+\frac{1}{6}\sqrt{2rx-x^2}\left(-3r^2-rx+2x^2\right)+\frac{1}{4}ir^3\log\left(r^2-2ix\sqrt{2rx-x^2}+2rx-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[2*r*x - x^2]*(-3*r^2 - r*x + 2*x^2))/6 - (I/2)*r^3*ArcTanh[x/r + (I*S
qrt[2*r*x - x^2])/r] + (I/4)*r^3*Log[r^2 + 2*r*x - 2*x^2 - (2*I)*x*Sqrt[2*r
*x - x^2]]
```

fricas [A] time = 0.66, size = 51, normalized size = 0.80

$$-r^3\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{6}\left(3\,r^2+rx-2\,x^2\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -r^3*arctan(sqrt(2*r*x - x^2)/x) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt(2*r*x - x
^2)
```

giac [A] time = 0.65, size = 45, normalized size = 0.70

$$-\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}(r)-\frac{1}{6}\left(3\,r^2+(r-2\,x)x\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x -
x^2)
```

maple [A] time = 0.31, size = 60, normalized size = 0.94

method	result	size
risch	$-\frac{(3r^2+rx-2x^2)x(2r-x)}{6\sqrt{-x(-2r+x)}}+\frac{r^3\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}$	60
default	$-\frac{(2rx-x^2)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*r^2+r*x-2*x^2)*x*(2*r-x)/(-x*(-2*r+x))^(1/2)+1/2*r^3*arctan((x-r)/(
2*r*x-x^2)^(1/2))
```

maxima [A] time = 1.17, size = 63, normalized size = 0.98

$$-\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)-\frac{1}{2}\,\sqrt{2\,rx-x^2}\,r^2+\frac{1}{2}\,\sqrt{2\,rx-x^2}\,rx-\frac{1}{3}\left(2\,rx-x^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*r^3*arcsin((r - x)/r) - 1/2*sqrt(2*r*x - x^2)*r^2 + 1/2*sqrt(2*r*x - x
^2)*r*x - 1/3*(2*r*x - x^2)^(3/2)
```

mupad [B] time = 0.10, size = 56, normalized size = 0.88

$$-\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}-\frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*r*x - x^2)^(1/2),x)
```

```
[Out] - ((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 - (r^3*log(x - r - (x*(
2*r - x))^(1/2)*1i)*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-x\left(-2r+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-x*(-2*r + x)), x)
```

3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=89

$$\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {670, 640, 612, 620, 203}

$$-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}+\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/12 - (x*(2
*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{1}{4}x\left(2rx-x^2\right)^{3/2} + \frac{1}{4}(5r)\int x\sqrt{2rx-x^2}\,dx$$
$$= -\frac{5}{12}r\left(2rx-x^2\right)^{3/2} - \frac{1}{4}x\left(2rx-x^2\right)^{3/2} + \frac{1}{4}\left(5r^2\right)\int \sqrt{2rx-x^2}\,dx$$
$$= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r\left(2rx-x^2\right)^{3/2} - \frac{1}{4}x\left(2rx-x^2\right)^{3/2} + \frac{1}{8}\left(5r^4\right)\int \frac{1}{\sqrt{2rx-x^2}}\,dx$$
$$= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r\left(2rx-x^2\right)^{3/2} - \frac{1}{4}x\left(2rx-x^2\right)^{3/2} + \frac{1}{4}\left(5r^4\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.$$
$$= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r\left(2rx-x^2\right)^{3/2} - \frac{1}{4}x\left(2rx-x^2\right)^{3/2} + \frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.90

$$\frac{1}{24}\sqrt{-x(x-2r)}\left(\frac{30r^{7/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}} - 15r^3 - 5r^2x - 2rx^2 + 6x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 + (30*r^(7/2)*A
rcSin[Sqrt[x]/(Sqrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r])))/24
```

IntegrateAlgebraic [C] time = 0.20, size = 121, normalized size = 1.36

$$-\frac{5}{8}ir^4\tanh^{-1}\left(\frac{x}{r}+\frac{i\sqrt{2rx-x^2}}{r}\right)+\frac{5}{16}ir^4\log\left(r^2-2ix\sqrt{2rx-x^2}+2rx-2x^2\right)+\frac{1}{24}\sqrt{2rx-x^2}\left(-15r^3-5r^2x-2rx^2\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[2*r*x - x^2]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3))/24 - ((5*I)/8)*r^
4*ArcTanh[x/r + (I*Sqrt[2*r*x - x^2])/r] + ((5*I)/16)*r^4*Log[r^2 + 2*r*x -
 2*x^2 - (2*I)*x*Sqrt[2*r*x - x^2]]
```

fricas [A] time = 0.69, size = 60, normalized size = 0.67

$$-\frac{5}{4}r^4\arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right) - \frac{1}{24}\left(15r^3+5r^2x+2rx^2-6x^3\right)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/4*r^4*arctan(sqrt(2*r*x - x^2)/x) - 1/24*(15*r^3 + 5*r^2*x + 2*r*x^2 - 6
*x^3)*sqrt(2*r*x - x^2)
```

giac [A] time = 0.66, size = 54, normalized size = 0.61

$$-\frac{5}{8}r^4\arcsin\left(\frac{r-x}{r}\right)\text{sgn}(r) - \frac{1}{24}\left(15r^3+\left(5r^2+2(r-3x)x\right)x\right)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

[Out] -5/8*r^4*arcsin((r - x)/r)*sgn(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r - 3*x)*x)*x)*sqrt(2*r*x - x^2)

maple [A] time = 0.29, size = 69, normalized size = 0.78

method	result	size
risch	$-\frac{\left(15r^{3}+5r^{2}x+2r\,x^{2}-6x^{3}\right)x(2r-x)}{24\sqrt{-x(-2r+x)}}+\frac{5r^{4}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{8}$	69
default	$-\frac{x\left(2rx-x^{2}\right)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{\left(2rx-x^{2}\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^{2}}}{4}+\frac{r^{2}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}\right)\right)}{4}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/24*(15*r^3+5*r^2*x+2*r*x^2-6*x^3)*x*(2*r-x)/(-x*(-2*r+x))^(1/2)+5/8*r^4*arctan((x-r)/(2*r*x-x^2)^(1/2))

maxima [A] time = 1.18, size = 81, normalized size = 0.91

$$-\frac{5}{8}\,r^{4}\arcsin\left(\frac{r-x}{r}\right)-\frac{5}{8}\,\sqrt{2\,rx-x^{2}}\,r^{3}+\frac{5}{8}\,\sqrt{2\,rx-x^{2}}\,r^{2}x-\frac{5}{12}\,\left(2\,rx-x^{2}\right)^{\frac{3}{2}}r-\frac{1}{4}\,\left(2\,rx-x^{2}\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")

[Out] -5/8*r^4*arcsin((r - x)/r) - 5/8*sqrt(2*r*x - x^2)*r^3 + 5/8*sqrt(2*r*x - x^2)*r^2*x - 5/12*(2*r*x - x^2)^(3/2)*r - 1/4*(2*r*x - x^2)^(3/2)*x

mupad [B] time = 0.32, size = 75, normalized size = 0.84

$$-\frac{x\,{\left(2\,r\,x-x^2\right)}^{3/2}}{4}-\frac{5\,r\,\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*r*x - x^2)^(1/2),x)

[Out] - (x*(2*r*x - x^2)^(3/2))/4 - (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{-x\left(-2r+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*r*x-x**2)**(1/2),x)

[Out] Integral(x**2*sqrt(-x*(-2*r + x)), x)

3.263 $\int x^3\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=113

$$\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {670, 640, 612, 620, 203}

$$-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2))/12 - (7*
r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))/5 + (7*r^5*ArcTan[x
/Sqrt[2*r*x - x^2]])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{5}(7r)\int x^2\sqrt{2rx-x^2}\,dx$$
$$= -\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^2\right)\int x\sqrt{2rx-x^2}\,dx$$
$$= -\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^3\right)\int\sqrt{2rx-x^2}\,dx$$
$$= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{8}$$
$$= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}$$
$$= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.78

$$\frac{1}{120}\sqrt{-x(x-2r)}\left(\frac{210r^{9/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}}-105r^4-35r^3x-14r^2x^2-6rx^3+24x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^4
 + (210*r^(9/2)*ArcSin[Sqrt[x]/(Sqrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r]))
)/120
```

IntegrateAlgebraic [C] time = 0.23, size = 129, normalized size = 1.14

$$-\frac{7}{8}ir^5\tanh^{-1}\left(\frac{x}{r}+\frac{i\sqrt{2rx-x^2}}{r}\right)+\frac{7}{16}ir^5\log\left(r^2-2ix\sqrt{2rx-x^2}+2rx-2x^2\right)+\frac{1}{120}\sqrt{2rx-x^2}\left(-105r^4-35r^3x\right.$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[2*r*x - x^2]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^4))/1
20 - ((7*I)/8)*r^5*ArcTanh[x/r + (I*Sqrt[2*r*x - x^2])/r] + ((7*I)/16)*r^5*
Log[r^2 + 2*r*x - 2*x^2 - (2*I)*x*Sqrt[2*r*x - x^2]]
```

fricas [A] time = 0.60, size = 68, normalized size = 0.60

$$-\frac{7}{4}r^5\arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right)-\frac{1}{120}\left(105r^4+35r^3x+14r^2x^2+6rx^3-24x^4\right)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x + 14*r^2*x
^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

giac [A] time = 0.63, size = 63, normalized size = 0.56

$$-\frac{7}{8}r^5\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}(r)-\frac{1}{120}\left(105r^4+\left(35r^3+2\left(7r^2+3(r-4x)x\right)x\right)x\right)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 + 3
*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

maple [A] time = 0.29, size = 77, normalized size = 0.68

method	result	size
risch	$-\frac{\left(105r^4+35r^3x+14r^2x^2+6r\,x^3-24x^4\right)x(2r-x)}{120\sqrt{-x(-2r+x)}}+\frac{7r^5\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	77
default	$-\frac{x^2\left(2rx-x^2\right)^{\frac{3}{2}}}{5}+\frac{7r\left(-\frac{x\left(2rx-x^2\right)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{\left(2rx-x^2\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)\right)}{4}\right)}{5}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(105*r^4+35*r^3*x+14*r^2*x^2+6*r*x^3-24*x^4)*x*(2*r-x)/(-x*(-2*r+x))
^(1/2)+7/8*r^5*arctan((x-r)/(2*r*x-x^2)^(1/2))
```

maxima [A] time = 1.10, size = 101, normalized size = 0.89

$$-\frac{7}{8}r^5\arcsin\left(\frac{r-x}{r}\right)-\frac{7}{8}\sqrt{2\,rx-x^2}\,r^4+\frac{7}{8}\sqrt{2\,rx-x^2}\,r^3x-\frac{7}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r^2-\frac{7}{20}\left(2\,rx-x^2\right)^{\frac{3}{2}}rx-\frac{1}{5}\left(2\,rx-x^2\right)^{\frac{3}{2}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r) - 7/8*sqrt(2*r*x - x^2)*r^4 + 7/8*sqrt(2*r*x - x
^2)*r^3*x - 7/12*(2*r*x - x^2)^(3/2)*r^2 - 7/20*(2*r*x - x^2)^(3/2)*r*x - 1
/5*(2*r*x - x^2)^(3/2)*x^2
```

mupad [B] time = 0.14, size = 96, normalized size = 0.85

$$-\frac{7\,r\left(\frac{x\left(2\,r\,x-x^2\right)^{3/2}}{4}+\frac{5\,r\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,(2\,r-x)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}\right)}{4}\right)}{5}-\frac{x^2\,\left(2\,r\,x-x^2\right)^{3/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*r*x - x^2)^(1/2),x)
```

```
[Out] - (7*r*((x*(2*r*x - x^2)^(3/2))/4 + (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*
r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4))/5 -
 (x^2*(2*r*x - x^2)^(3/2))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-x\,(-2r+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-x*(-2*r + x)), x)
```

3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {984, 688, 204, 724, 206}

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -ArcTan[Sqrt[2*x + x^2]]/2 - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])]/(
2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{2}\int \frac{1}{(-1-x)\sqrt{2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{2x+x^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-4-4x^2}\,dx, x, \sqrt{2x+x^2}\right) - \text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{2+4x}{\sqrt{2x+x^2}}\right)$$
$$= -\frac{1}{2}\tan^{-1}\left(\sqrt{2x+x^2}\right) - \frac{\tanh^{-1}\left(\frac{2+4x}{2\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.31

$$-\frac{\sqrt{x}\sqrt{x+2}\left(3\tan^{-1}\left(\sqrt{\frac{x}{x+2}}\right)+\sqrt{3}\tanh^{-1}\left(\sqrt{3}\sqrt{\frac{x}{x+2}}\right)\right)}{3\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -1/3*(Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[Sqrt[x/(2 + x)]] + Sqrt[3]*ArcTanh[Sqrt
[3]*Sqrt[x/(2 + x)]]))/Sqrt[x*(2 + x)]
```

IntegrateAlgebraic [A] time = 0.19, size = 57, normalized size = 1.16

$$\tan^{-1}\left(-\sqrt{x^2+2x}+x+1\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x}}{\sqrt{3}}-\frac{x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[1 + x - Sqrt[2*x + x^2]] - ArcTanh[1/Sqrt[3] - x/Sqrt[3] + Sqrt[2*x
+ x^2]/Sqrt[3]]/Sqrt[3]
```

fricas [A] time = 0.58, size = 62, normalized size = 1.27

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}(2x+1)+\sqrt{x^2+2x}\left(2\sqrt{3}-3\right)-4x-2}{x-1}\right) - \arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(sqrt(3)*(2*x + 1) + sqrt(x^2 + 2*x)*(2*sqrt(3) - 3) - 4*x
 - 2)/(x - 1)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

giac [A] time = 0.71, size = 71, normalized size = 1.45

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x}+2\right|}{\left|-2x+2\sqrt{3}+2\sqrt{x^2+2x}+2\right|}\right) - \arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2*
sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

maple [A] time = 0.30, size = 42, normalized size = 0.86

method	result	size
default	$-\frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{(2+4x)\sqrt{3}}{6\sqrt{(-1+x)^2-1+4x}}\right)}{6}+\frac{\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)}{2}$	42
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2-3\right)x+3\sqrt{x^2+2x}-\mathrm{RootOf}\left(\_Z^2-3\right)}{-1+x}\right)}{6}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{x^2+2x}}{1+x}\right)}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*3^(1/2)*arctanh(1/6*(2+4*x)*3^(1/2)/((-1+x)^2-1+4*x)^(1/2))+1/2*arctan
(1/((1+x)^2-1)^(1/2))
```

maxima [A] time = 1.33, size = 54, normalized size = 1.10

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^2+2x}}{|2x-2|}+\frac{6}{|2x-2|}+2\right)+\frac{1}{2}\arcsin\left(\frac{2}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2*x - 2) +
2) + 1/2*arcsin(2/abs(2*x + 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+2x}\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x^2 - 1)),x)
```

```
[Out] int(1/((2*x + x^2)^(1/2)*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}\,(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {834, 806, 724, 204}

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 + x)) + ArcT
an[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}+\frac{1}{6}\int \frac{-7+5x}{(1+x)^2\sqrt{2x-x^2}}\,dx$$
$$= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{1}{2}\int \frac{1}{(1+x)\sqrt{2x-x^2}}\,dx$$
$$= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{-2+4x}{\sqrt{2x-x^2}}\right)$$
$$= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{\tan^{-1}\left(\frac{-2+4x}{2\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.09, size = 72, normalized size = 0.91

$$\frac{x\left(4x^2+x-18\right)-2\sqrt{3}\sqrt{x-2}\sqrt{x}(x+1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{x-2}{x}}}{\sqrt{3}}\right)}{6\sqrt{-((x-2)x)}(x+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
[Out] (x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[Sqr
t[(-2 + x)/x]/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

IntegrateAlgebraic [A] time = 0.31, size = 57, normalized size = 0.72

$$\frac{\sqrt{2x-x^2}(-4x-9)}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{2x-x^2}}{\sqrt{3}x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
[Out] ((-9 - 4*x)*Sqrt[2*x - x^2])/(6*(1 + x)^2) + ArcTan[Sqrt[2*x - x^2]/(Sqrt[3
]*x)]/Sqrt[3]
```

fricas [A] time = 0.84, size = 64, normalized size = 0.81

$$\frac{2\sqrt{3}\left(x^2+2x+1\right)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right)-\sqrt{-x^2+2x}(4x+9)}{6\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x) - sqr
t(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)
```

giac [B] time = 0.67, size = 147, normalized size = 1.86

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2+2x}-1\right)}{x-1}-1\right)\right)+\frac{\frac{34\left(\sqrt{-x^2+2x}-1\right)}{x-1}-\frac{39\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}+\frac{18\left(\sqrt{-x^2+2x}-1\right)^3}{(x-1)^3}-26}{24\left(\frac{\sqrt{-x^2+2x}-1}{x-1}-\frac{\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1/
24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x - 1)
^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)/(x
 - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2
```

maple [A] time = 0.29, size = 56, normalized size = 0.71

method	result	size
risch	$\frac{x(-2+x)(9+4x)}{6(1+x)^2\sqrt{-x(-2+x)}}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$	56
trager	$-\frac{(9+4x)\sqrt{-x^2+2x}}{6(1+x)^2}+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-x^2+2x}-\mathrm{RootOf}\left(\_Z^2+3\right)}{1+x}\right)}{6}$	71
default	$-\frac{5\sqrt{-(1+x)^2+1+4x}}{6(1+x)^2}-\frac{2\sqrt{-(1+x)^2+1+4x}}{3(1+x)}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(-2+x)*(9+4*x)/(1+x)^2/(-x*(-2+x))^(1/2)-1/6*3^(1/2)*arctan(1/6*(-2+4
*x)*3^(1/2)/(-(1+x)^2+1+4*x)^(1/2))
```

maxima [A] time = 1.33, size = 66, normalized size = 0.84

$$-\frac{1}{6}\sqrt{3}\arcsin\left(\frac{2x}{|x+1|}-\frac{1}{|x+1|}\right)-\frac{5\sqrt{-x^2+2x}}{6\left(x^2+2x+1\right)}-\frac{2\sqrt{-x^2+2x}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^2 + 2*x)/(
x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x-2}{\sqrt{2x-x^2}\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3),x)
```

```
[Out] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x-2}{\sqrt{-x(x-2)}\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

3.266 $\int \frac{1}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=12

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {619, 215}

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x+x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x\right)}{\sqrt{3}}\\ &= \sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

IntegrateAlgebraic [A] time = 0.06, size = 20, normalized size = 1.67

$$-\log\left(2\sqrt{x^2+x+1}-2x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + x + x^2],x]
[Out] -Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]
```

fricas [A] time = 0.71, size = 18, normalized size = 1.50

$$-\log\left(-2\,x + 2\,\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.65, size = 18, normalized size = 1.50

$$-\log\left(-2\,x + 2\,\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.35, size = 10, normalized size = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)$	10
trager	$-\ln\left(2\sqrt{x^2+x+1}-1-2x\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(2/3*(1/2+x)*3^(1/2))
```

maxima [A] time = 1.40, size = 11, normalized size = 0.92

$$\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}\,(2\,x + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 0.24, size = 12, normalized size = 1.00

$$\ln\left(x + \sqrt{x^2 + x + 1} + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + 1)^(1/2),x)
[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + x + 1), x)
```

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{x^2+x+1}\,x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {742, 779, 619, 215}

$$\frac{1}{3}\sqrt{x^2+x+1}\,x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (7*ArcSinh[
(1 + 2*x)/Sqrt[3]])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx &= \frac{1}{3}x^2\sqrt{1+x+x^2} + \frac{1}{3}\int \frac{\left(-2-\frac{5x}{2}\right)x}{\sqrt{1+x+x^2}}\,dx \\ &= \frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7}{16}\int \frac{1}{\sqrt{1+x+x^2}}\,dx \\ &= \frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x\right)}{16\sqrt{3}} \\ &= \frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7}{16}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.77

$$\frac{1}{48}\left(2\sqrt{x^2+x+1}\,\left(8x^2-10x-1\right)+21\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2) + 21*ArcSinh[(1 + 2*x)/Sqrt[3]])/4
8
```

IntegrateAlgebraic [A] time = 0.08, size = 47, normalized size = 0.89

$$\frac{1}{24}\sqrt{x^2+x+1}\,\left(8x^2-10x-1\right)-\frac{7}{16}\log\left(2\sqrt{x^2+x+1}\,-2x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 - (7*Log[-1 - 2*x + 2*Sqrt[1 + x
 + x^2]])/16
```

fricas [A] time = 0.73, size = 39, normalized size = 0.74

$$\frac{1}{24}\left(8\,x^2-10\,x-1\right)\sqrt{x^2+x+1}\,-\frac{7}{16}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(8*x^2 - 10*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
+ 1) - 1)
```

giac [A] time = 0.63, size = 39, normalized size = 0.74

$$\frac{1}{24}\,(2\,(4\,x-5)x-1)\sqrt{x^2+x+1}\,-\frac{7}{16}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
 + 1) - 1)
```

maple [A] time = 0.35, size = 33, normalized size = 0.62

method	result	size
risch	$\frac{\left(8x^{2}-10x-1\right)\sqrt{x^{2}+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{16}$	33
trager	$\left(\frac{1}{3}x^{2}-\frac{5}{12}x-\frac{1}{24}\right)\sqrt{x^{2}+x+1}-\frac{7\ln\left(2\sqrt{x^{2}+x+1}-1-2x\right)}{16}$	39
default	$\frac{x^{2}\sqrt{x^{2}+x+1}}{3}-\frac{5x\sqrt{x^{2}+x+1}}{12}-\frac{\sqrt{x^{2}+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{16}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*x^2-10*x-1)*(x^2+x+1)^(1/2)+7/16*arcsinh(2/3*(1/2+x)*3^(1/2))
```

maxima [A] time = 1.26, size = 48, normalized size = 0.91

$$\frac{1}{3}\sqrt{x^{2}+x+1}\,x^{2}-\frac{5}{12}\sqrt{x^{2}+x+1}\,x-\frac{1}{24}\sqrt{x^{2}+x+1}+\frac{7}{16}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(x^2 + x +
1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3}}{\sqrt{x^{2}+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x + x^2 + 1)^(1/2),x)
```

```
[Out] int(x^3/(x + x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{x^{2}+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2 + x + 1), x)
```

3.268 $\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {613}

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

IntegrateAlgebraic [A] time = 0.15, size = 19, normalized size = 1.00

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

fricas [B] time = 0.63, size = 34, normalized size = 1.79

$$\frac{2\left(2\,x^2+\sqrt{x^2+x+1}\,(2\,x+1)+2\,x+2\right)}{3\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)
```

giac [A] time = 0.70, size = 15, normalized size = 0.79

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

maple [A] time = 0.34, size = 16, normalized size = 0.84

method	result	size
gosper	$\frac{\frac{4x}{3}+\frac{2}{3}}{\sqrt{x^2+x+1}}$	16
default	$\frac{\frac{4x}{3}+\frac{2}{3}}{\sqrt{x^2+x+1}}$	16
trager	$\frac{\frac{4x}{3}+\frac{2}{3}}{\sqrt{x^2+x+1}}$	16
risch	$\frac{\frac{4x}{3}+\frac{2}{3}}{\sqrt{x^2+x+1}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

maxima [A] time = 0.59, size = 22, normalized size = 1.16

$$\frac{4x}{3\sqrt{x^2+x+1}}+\frac{2}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{4\left(x+\frac{1}{2}\right)}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + 1)^(3/2),x)
```

```
[Out] (4*(x + 1/2))/(3*(x + x^2 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(-3/2), x)
```

3.269 $\int \frac{x}{\left(1+x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=17

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {636}

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(1+x+x^2\right)^{3/2}}\,dx=-\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] time = 0.04, size = 17, normalized size = 1.00

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

IntegrateAlgebraic [A] time = 0.13, size = 17, normalized size = 1.00

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

fricas [B] time = 0.67, size = 28, normalized size = 1.65

$$-\frac{2\left(x^2+\sqrt{x^2+x+1}\,(x+2)+x+1\right)}{3\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)
```

giac [A] time = 0.65, size = 13, normalized size = 0.76

$$-\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="giac")
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1)
```

maple [A] time = 0.35, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
trager	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
risch	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
default	$-\frac{1}{\sqrt{x^2+x+1}}-\frac{1+2x}{3\sqrt{x^2+x+1}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)
```

maxima [A] time = 0.43, size = 22, normalized size = 1.29

$$-\frac{2\,x}{3\,\sqrt{x^2+x+1}}-\frac{4}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.88

$$-\frac{2\,x+4}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^2 + 1)^(3/2),x)
[Out] -(2*x + 4)/(3*(x + x^2 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**(3/2),x)
[Out] Integral(x/(x**2 + x + 1)**(3/2), x)
```

3.270 $\int \frac{x^3}{\left(1+x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {738, 779, 619, 215}

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x + x^2])/3 -
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(1+x+x^2\right)^{3/2}}\,dx = -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{x(4+2x)}{\sqrt{1+x+x^2}}\,dx$$
$$= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{1}{2}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)$$
$$= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.86

$$\frac{6x^2-9\sqrt{x^2+x+1}\,\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+14x+10}{6\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x + x^2)^(3/2),x]
[Out] (10 + 14*x + 6*x^2 - 9*Sqrt[1 + x + x^2]*ArcSinh[(1 + 2*x)/Sqrt[3]])/(6*Sqr
t[1 + x + x^2])
```

IntegrateAlgebraic [A] time = 0.18, size = 47, normalized size = 0.84

$$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}+\frac{3}{2}\log\left(2\sqrt{x^2+x+1}-2x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 + x + x^2)^(3/2),x]
[Out] (5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) + (3*Log[-1 - 2*x + 2*Sqrt[1 + x +
x^2]])/2
```

fricas [A] time = 0.58, size = 64, normalized size = 1.14

$$\frac{19\,x^2+18\left(x^2+x+1\right)\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)+4\left(3\,x^2+7\,x+5\right)\sqrt{x^2+x+1}+19\,x+19}{12\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] 1/12*(19*x^2 + 18*(x^2 + x + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 4*(3*
x^2 + 7*x + 5)*sqrt(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)
```

giac [A] time = 0.64, size = 38, normalized size = 0.68

$$\frac{(3\,x+7)x+5}{3\,\sqrt{x^2+x+1}}+\frac{3}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x + 1
) - 1)
```

maple [A] time = 0.35, size = 33, normalized size = 0.59

method	result	size
risch	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} - \frac{3\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{2}$	33
trager	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} + \frac{3\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{2}$	40
default	$\frac{x^2}{\sqrt{x^2+x+1}} + \frac{3x}{2\sqrt{x^2+x+1}} + \frac{5}{4\sqrt{x^2+x+1}} + \frac{\frac{5}{12}+\frac{5x}{6}}{\sqrt{x^2+x+1}} - \frac{3\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*x^2+7*x+5)/(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*(1/2+x)*3^(1/2))
```

maxima [A] time = 1.11, size = 47, normalized size = 0.84

$$\frac{x^2}{\sqrt{x^2+x+1}} + \frac{7\,x}{3\,\sqrt{x^2+x+1}} + \frac{5}{3\,\sqrt{x^2+x+1}} - \frac{3}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 + x + 1) - 3
/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(x^2+x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x + x^2 + 1)^(3/2),x)
```

```
[Out] int(x^3/(x + x^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {742, 640, 612, 619, 215}

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (x*(1 + x +
 x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule Rubi steps

$$\begin{aligned}\int x^2\sqrt{1+x+x^2}\,dx &= \frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{4}\int\left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx\\ &= -\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{16}\int\sqrt{1+x+x^2}\,dx\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\int\frac{1}{\sqrt{1+x+x^2}}\,d\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{128}\sqrt{3}\,\text{Subst}\left(\int\frac{}{\sqrt{1}}\right.\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.71

$$\frac{1}{384}\left(2\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3) + 9*ArcSinh[(1 + 2*x)/Sq
rt[3]])/384
```

IntegrateAlgebraic [A] time = 0.09, size = 52, normalized size = 0.80

$$\frac{1}{192}\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)-\frac{3}{128}\log\left(2\sqrt{x^2+x+1}-2x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/192 - (3*Log[-1 - 2*x + 2
*Sqrt[1 + x + x^2]])/128
```

fricas [A] time = 0.71, size = 44, normalized size = 0.68

$$\frac{1}{192}\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2*s
qrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.65, size = 44, normalized size = 0.68

$$\frac{1}{192}\left(2\left(4\left(6\,x+1\right)x+7\right)x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2
*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.35, size = 38, normalized size = 0.58

method	result	size
risch	$\frac{\left(48x^{3}+8x^{2}+14x-37\right)\sqrt{x^{2}+x+1}}{192}+\frac{3\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{128}$	38
trager	$\left(\frac{1}{4}x^{3}+\frac{1}{24}x^{2}+\frac{7}{96}x-\frac{37}{192}\right)\sqrt{x^{2}+x+1}-\frac{3\ln\left(2\sqrt{x^{2}+x+1}-1-2x\right)}{128}$	44
default	$\frac{x\left(x^{2}+x+1\right)^{\frac{3}{2}}}{4}-\frac{5\left(x^{2}+x+1\right)^{\frac{3}{2}}}{24}+\frac{(1+2x)\sqrt{x^{2}+x+1}}{64}+\frac{3\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{128}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*x^3+8*x^2+14*x-37)*(x^2+x+1)^(1/2)+3/128*arcsinh(2/3*(1/2+x)*3^(1
/2))
```

maxima [A] time = 1.16, size = 56, normalized size = 0.86

$$\frac{1}{4}\left(x^{2}+x+1\right)^{\frac{3}{2}}x-\frac{5}{24}\left(x^{2}+x+1\right)^{\frac{3}{2}}+\frac{1}{32}\sqrt{x^{2}+x+1}\,x+\frac{1}{64}\sqrt{x^{2}+x+1}+\frac{3}{128}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(x^2 + x +
1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 0.13, size = 61, normalized size = 0.94

$$\frac{3\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128}-\frac{\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{4}-\frac{5\,\left(8\,x^2+2\,x+5\right)\sqrt{x^2+x+1}}{192}+\frac{x\,\left(x^2+x+1\right)^{3/2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + x^2 + 1)^(1/2),x)
```

```
[Out] (3*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/
2))/4 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/192 + (x*(x + x^2 + 1)^(3
/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x^{2}+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2 + x + 1), x)
```

3.272 $\int \left(1+x+x^2\right)^{3/2} \, dx$

Optimal. Leaf size=55

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {612, 619, 215}

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(3/2),x]
```

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (2
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(1+x+x^2\right)^{3/2} \, dx &= \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{9}{16}\int \sqrt{1+x+x^2} \, dx \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\int \frac{1}{\sqrt{1+x+x^2}} \, dx \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{128}\left(9\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} \, dx,\right. \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.84

$$\frac{1}{128}\left(2\sqrt{x^2+x+1}\left(16x^3+24x^2+42x+17\right)+27\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3) + 27*ArcSinh[(1 + 2*x)/S
qrt[3]])/128
```

IntegrateAlgebraic [A] time = 0.15, size = 52, normalized size = 0.95

$$\frac{1}{64}\sqrt{x^2+x+1}\left(16x^3+24x^2+42x+17\right)-\frac{27}{128}\log\left(2\sqrt{x^2+x+1}-2x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x + x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3))/64 - (27*Log[-1 - 2*x + 2
*Sqrt[1 + x + x^2]])/128
```

fricas [A] time = 0.77, size = 44, normalized size = 0.80

$$\frac{1}{64}\left(16\,x^3+24\,x^2+42\,x+17\right)\sqrt{x^2+x+1}-\frac{27}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(16*x^3 + 24*x^2 + 42*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2*
sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.63, size = 44, normalized size = 0.80

$$\frac{1}{64}\left(2\left(4\left(2\,x+3\right)x+21\right)x+17\right)\sqrt{x^2+x+1}-\frac{27}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x +
2*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.35, size = 38, normalized size = 0.69

method	result	size
risch	$\frac{\left(16x^3+24x^2+42x+17\right)\sqrt{x^2+x+1}}{64}+\frac{27\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{128}$	38
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{8}+\frac{9(1+2x)\sqrt{x^2+x+1}}{64}+\frac{27\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{128}$	43
trager	$\left(\frac{1}{4}x^3+\frac{3}{8}x^2+\frac{21}{32}x+\frac{17}{64}\right)\sqrt{x^2+x+1}+\frac{27\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(16*x^3+24*x^2+42*x+17)*(x^2+x+1)^(1/2)+27/128*arcsinh(2/3*(1/2+x)*3^(
1/2))
```

maxima [A] time = 1.29, size = 56, normalized size = 1.02

$$\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9}{32}\sqrt{x^2+x+1}\,x+\frac{9}{64}\sqrt{x^2+x+1}+\frac{27}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x^2 + x + 1
)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 0.21, size = 43, normalized size = 0.78

$$\frac{27\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128}+\frac{\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{3/2}}{4}+\frac{9\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)^(3/2),x)
```

```
[Out] (27*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 + ((x + 1/2)*(x + x^2 + 1)^(3/2
))/4 + (9*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/16
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)**(3/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(3/2), x)
```

3.273 $\int \left(1+x+x^2\right)^{5/2} dx$

Optimal. Leaf size=74

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {612, 619, 215}

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(5/2),x]
```

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)^(3/2))/64
 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1 + 2*x)/Sqrt[3]])/10
24
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(1+x+x^2\right)^{5/2} dx &= \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{5}{8}\int \left(1+x+x^2\right)^{3/2} dx \\
&= \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{45}{128}\int \sqrt{1+x+x^2}\, dx \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+ \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+ \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.76

$$\frac{2\sqrt{x^2+x+1}\left(256x^5+640x^4+1264x^3+1256x^2+1142x+383\right)+405\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^
5) + 405*ArcSinh[(1 + 2*x)/Sqrt[3]])/3072
```

IntegrateAlgebraic [A] time = 0.28, size = 62, normalized size = 0.84

$$\frac{\sqrt{x^2+x+1}\left(256x^5+640x^4+1264x^3+1256x^2+1142x+383\right)}{1536}-\frac{135\log\left(2\sqrt{x^2+x+1}-2x-1\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x + x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^5)
)/1536 - (135*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/1024
```

fricas [A] time = 0.60, size = 54, normalized size = 0.73

$$\frac{1}{1536}\left(256\,x^5+640\,x^4+1264\,x^3+1256\,x^2+1142\,x+383\right)\sqrt{x^2+x+1}-\frac{135}{1024}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(256*x^5 + 640*x^4 + 1264*x^3 + 1256*x^2 + 1142*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.62, size = 54, normalized size = 0.73

$$\frac{1}{1536}\,(2\,(4\,(2\,(8\,(2\,x+5)x+79)x+157)x+571)x+383)\sqrt{x^2+x+1}-\frac{135}{1024}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.35, size = 48, normalized size = 0.65

method	result	size
risch	$\frac{\left(256x^5+640x^4+1264x^3+1256x^2+1142x+383\right)\sqrt{x^2+x+1}}{1536}+\frac{135\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{1024}$	48
trager	$\left(\frac{1}{6}x^5+\frac{5}{12}x^4+\frac{79}{96}x^3+\frac{157}{192}x^2+\frac{571}{768}x+\frac{383}{1536}\right)\sqrt{x^2+x+1}-\frac{135\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{1024}$	54
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{5}{2}}}{12}+\frac{5(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{64}+\frac{45(1+2x)\sqrt{x^2+x+1}}{512}+\frac{135\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{1024}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536*(256*x^5+640*x^4+1264*x^3+1256*x^2+1142*x+383)*(x^2+x+1)^(1/2)+135/1
024*arcsinh(2/3*(1/2+x)*3^(1/2))
```

maxima [A] time = 1.43, size = 77, normalized size = 1.04

$$\frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x+\frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}}+\frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{45}{256}\sqrt{x^2+x+1}\,x+\frac{45}{512}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2 + x + 1)^(
3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x + 1)*x + 45/512*sqr
t(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 0.07, size = 56, normalized size = 0.76

$$\frac{135\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{1024}+\frac{5\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{3/2}}{32}+\frac{\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{5/2}}{6}+\frac{45\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)^(5/2),x)
```

```
[Out] (135*log(x + (x + x^2 + 1)^(1/2) + 1/2))/1024 + (5*(x + 1/2)*(x + x^2 + 1)^
(3/2))/32 + ((x + 1/2)*(x + x^2 + 1)^(5/2))/6 + (45*(x/2 + 1/4)*(x + x^2 +
1)^(1/2))/128
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+x+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)**(5/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(5/2), x)
```

3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {730, 724, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x + x^2]),x]

[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx &= -\frac{\sqrt{1+x+x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{x}+\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= -\frac{\sqrt{1+x+x^2}}{x}+\frac{1}{2}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

IntegrateAlgebraic [A] time = 0.08, size = 33, normalized size = 0.87

$$-\frac{\sqrt{x^2+x+1}}{x}-\tanh^{-1}\left(x-\sqrt{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -(Sqrt[1 + x + x^2]/x) - ArcTanh[x - Sqrt[1 + x + x^2]]
```

fricas [A] time = 0.68, size = 52, normalized size = 1.37

$$\frac{x\log\left(-x+\sqrt{x^2+x+1}+1\right)-x\log\left(-x+\sqrt{x^2+x+1}-1\right)-2\,x-2\,\sqrt{x^2+x+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*log(-x + sqrt(x^2 + x + 1) + 1) - x*log(-x + sqrt(x^2 + x + 1) - 1)
 - 2*x - 2*sqrt(x^2 + x + 1))/x
```

giac [B] time = 0.65, size = 67, normalized size = 1.76

$$\frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1}+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2*log(abs(-
x + sqrt(x^2 + x + 1) + 1)) - 1/2*log(abs(-x + sqrt(x^2 + x + 1) - 1))
```

maple [A] time = 0.35, size = 31, normalized size = 0.82

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$	31
risch	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$	31
trager	$-\frac{\sqrt{x^2+x+1}}{x}+\frac{\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

maxima [A] time = 1.03, size = 37, normalized size = 0.97

$$-\frac{\sqrt{x^2+x+1}}{x}+\frac{1}{2}\operatorname{arsinh}\left(\frac{\sqrt{3}\,x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x
))
```

mupad [B] time = 0.03, size = 31, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\frac{x}{2}+1}{\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + x^2 + 1)^(1/2)),x)
```

```
[Out] atanh((x/2 + 1)/(x + x^2 + 1)^(1/2))/2 - (x + x^2 + 1)^(1/2)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

3.275 $\int \frac{1}{x^3 \sqrt{1+x+x^2}} dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x^2+x+1}}{4x} - \frac{\sqrt{x^2+x+1}}{2x^2} + \frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {744, 806, 724, 206}

$$\frac{3\sqrt{x^2+x+1}}{4x} - \frac{\sqrt{x^2+x+1}}{2x^2} + \frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx &= -\frac{\sqrt{1+x+x^2}}{2x^2}-\frac{1}{2}\int \frac{\frac{3}{2}+x}{x^2\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}-\frac{1}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.75

$$\frac{1}{8}\left(\frac{2\sqrt{x^2+x+1}\,(3x-2)}{x^2}+\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((2*(-2 + 3*x)*Sqrt[1 + x + x^2])/x^2 + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2
])])/8
```

IntegrateAlgebraic [A] time = 0.14, size = 42, normalized size = 0.74

$$\frac{(3x-2)\sqrt{x^2+x+1}}{4x^2}-\frac{1}{4}\tanh^{-1}\left(x-\sqrt{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

fricas [A] time = 0.71, size = 63, normalized size = 1.11

$$\frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6\,x^2+2\,\sqrt{x^2+x+1}\,(3\,x-2)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1) -
 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

giac [A] time = 0.67, size = 84, normalized size = 1.47

$$\frac{\left(x-\sqrt{x^2+x+1}\right)^3+9\,x-9\,\sqrt{x^2+x+1}+8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2}+\frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt(
x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*log
(abs(-x + sqrt(x^2 + x + 1) - 1))
```

maple [A] time = 0.36, size = 42, normalized size = 0.74

method	result	size
trager	$\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2}-\frac{\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{8}$	42
risch	$\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}}+\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	42
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{3\sqrt{x^2+x+1}}{4x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-2+3*x)/x^2*(x^2+x+1)^(1/2)-1/8*ln((-2-x+2*(x^2+x+1)^(1/2))/x)
```

maxima [A] time = 1.24, size = 50, normalized size = 0.88

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|}+\frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt(
3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^3*(x + x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {740, 806, 724, 206}

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x) + (3*ArcT
anh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x+x^2\right)^{3/2}}\,dx = \frac{2(1-x)}{3x\sqrt{1+x+x^2}} + \frac{2}{3}\int \frac{\frac{5}{2}-x}{x^2\sqrt{1+x+x^2}}\,dx$$
$$= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} - \frac{3}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx$$
$$= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + 3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+x}{\sqrt{1+x+x^2}}\right)$$
$$= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + \frac{3}{2}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.81

$$\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right) - \frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] -1/3*(3 + 7*x + 5*x^2)/(x*Sqrt[1 + x + x^2]) + (3*ArcTanh[(2 + x)/(2*Sqrt[1
 + x + x^2])])/2
```

IntegrateAlgebraic [A] time = 0.20, size = 45, normalized size = 0.73

$$\frac{-5x^2-7x-3}{3x\sqrt{x^2+x+1}} - 3\tanh^{-1}\left(x-\sqrt{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-3 - 7*x - 5*x^2)/(3*x*Sqrt[1 + x + x^2]) - 3*ArcTanh[x - Sqrt[1 + x + x^2
]]
```

fricas [B] time = 0.59, size = 94, normalized size = 1.52

$$-\frac{10\,x^3+10\,x^2-9\left(x^3+x^2+x\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\left(x^3+x^2+x\right)\log\left(-x+\sqrt{x^2+x+1}-1\right)+2}{6\left(x^3+x^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) + 1) +
 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(5*x^2 + 7*x + 3)*sq
rt(x^2 + x + 1) + 10*x)/(x^3 + x^2 + x)
```

giac [A] time = 0.62, size = 80, normalized size = 1.29

$$-\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}}+\frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1}+\frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2
 + x + 1))^2 - 1) + 3/2*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 3/2*log(abs(
-x + sqrt(x^2 + x + 1) - 1))
```

maple [A] time = 0.36, size = 41, normalized size = 0.66

method	result	size
risch	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	41
trager	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}-\frac{3\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{2}$	47
default	$-\frac{1}{x\sqrt{x^2+x+1}}-\frac{3}{2\sqrt{x^2+x+1}}-\frac{5(1+2x)}{6\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(5*x^2+7*x+3)/x/(x^2+x+1)^(1/2)+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

maxima [A] time = 1.42, size = 58, normalized size = 0.94

$$-\frac{5\,x}{3\,\sqrt{x^2+x+1}}-\frac{7}{3\,\sqrt{x^2+x+1}}-\frac{1}{\sqrt{x^2+x+1}\,x}+\frac{3}{2}\,\mathrm{arsinh}\left(\frac{\sqrt{3}\,x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x)
+ 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(x^2+x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^2*(x + x^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(x^{2} + x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

3.277 $\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {740, 834, 806, 724, 206}

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*x^2) + (37
*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}}\,dx &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{\frac{7}{2}-2x}{x^3\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}-\frac{1}{3}\int \frac{\frac{37}{4}+\frac{7x}{2}}{x^2\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}+\frac{3}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+}{\sqrt{1+}}\right.\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}-\frac{3}{8}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 0.82

$$\frac{74x^3+46x^2-9\sqrt{x^2+x+1}\,x^2\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+30x-12}{24x^2\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-12 + 30*x + 46*x^2 + 74*x^3 - 9*x^2*Sqrt[1 + x + x^2]*ArcTanh[(2 + x)/(2*
Sqrt[1 + x + x^2])])/(24*x^2*Sqrt[1 + x + x^2])
```

IntegrateAlgebraic [A] time = 0.22, size = 52, normalized size = 0.66

$$\frac{3}{4}\tanh^{-1}\left(x-\sqrt{x^2+x+1}\right)+\frac{37x^3+23x^2+15x-6}{12x^2\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-6 + 15*x + 23*x^2 + 37*x^3)/(12*x^2*Sqrt[1 + x + x^2]) + (3*ArcTanh[x - S
qrt[1 + x + x^2]])/4
```

fricas [A] time = 0.67, size = 107, normalized size = 1.35

$$\frac{74\,x^4+74\,x^3+74\,x^2-9\left(x^4+x^3+x^2\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\left(x^4+x^3+x^2\right)\log\left(-x+\sqrt{x^2+x+1}-\right.}{24\left(x^4+x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(74*x^4 + 74*x^3 + 74*x^2 - 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x
+ 1) + 1) + 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(37*x^3
 + 23*x^2 + 15*x - 6)*sqrt(x^2 + x + 1))/(x^4 + x^3 + x^2)
```

giac [A] time = 0.63, size = 117, normalized size = 1.48

$$\frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}-\frac{3\left(x-\sqrt{x^2+x+1}\right)^3+8\left(x-\sqrt{x^2+x+1}\right)^2-13\,x+13\,\sqrt{x^2+x+1}-16}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2}-\frac{3}{8}\log\left(\left|-x+\sqrt{x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^3 + 8*(x -
 sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 16)/((x - sqrt(x^2 +
x + 1))^2 - 1)^2 - 3/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 3/8*log(abs(-
x + sqrt(x^2 + x + 1) - 1))
```

maple [A] time = 0.35, size = 46, normalized size = 0.58

method	result	size
risch	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\;x^2}-\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	46
trager	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\;x^2}+\frac{3\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{8}$	52
default	$-\frac{1}{2x^2\sqrt{x^2+x+1}}+\frac{5}{4x\sqrt{x^2+x+1}}+\frac{3}{8\sqrt{x^2+x+1}}+\frac{\frac{37}{24}+\frac{37x}{12}}{\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(37*x^3+23*x^2+15*x-6)/(x^2+x+1)^(1/2)/x^2-3/8*arctanh(1/2*(2+x)/(x^2+
x+1)^(1/2))
```

maxima [A] time = 1.09, size = 71, normalized size = 0.90

$$\frac{37\,x}{12\,\sqrt{x^2+x+1}}+\frac{23}{12\,\sqrt{x^2+x+1}}+\frac{5}{4\,\sqrt{x^2+x+1}\,x}-\frac{1}{2\,\sqrt{x^2+x+1}\,x^2}-\frac{3}{8}\,\mathrm{arsinh}\left(\frac{\sqrt{3}\,x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x^2 + x + 1
)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3
*sqrt(3)/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(x^2+x+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^3*(x + x^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(x^2 + x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {724, 206}

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1-x}{\sqrt{1+x+x^2}}\right)\right)\\ &= -\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

IntegrateAlgebraic [A] time = 0.14, size = 18, normalized size = 0.82

$$2\tanh^{-1}\left(-\sqrt{x^2+x+1}+x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[1 + x + x^2]),x]
[Out] 2*ArcTanh[1 + x - Sqrt[1 + x + x^2]]
```

fricas [A] time = 0.61, size = 30, normalized size = 1.36

$$-\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

giac [A] time = 0.66, size = 32, normalized size = 1.45

$$-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))
```

maple [A] time = 0.36, size = 22, normalized size = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)$	22
trager	$-\ln\left(\frac{2\sqrt{x^2+x+1}+1-x}{1+x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

maxima [A] time = 1.24, size = 25, normalized size = 1.14

$$\operatorname{arsinh}\left(\frac{\sqrt{3}\,x}{3\,|x+1|}-\frac{\sqrt{3}}{3\,|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(x+1)\,\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + x^2 + 1)^(1/2)),x)
[Out] int(1/((x + 1)*(x + x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
[Out] Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

3.279 $\int \frac{1}{\sqrt{4+2x+x^2}\left(-x+x^3\right)}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.28, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1593, 6725, 724, 206, 1033, 688, 207}

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqr
t[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+2x+x^2}\left(-x+x^3\right)}\,dx = \int \frac{1}{x\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx$$
$$= \int \left(-\frac{1}{x\sqrt{4+2x+x^2}} + \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\right)dx$$
$$= -\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx + \int \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx$$
$$= \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx + 2\,\mathrm{Subst}\Bigg(\int$$
$$= \frac{1}{2}\tanh^{-1}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) + 2\,\mathrm{Subst}\left(\int \frac{1}{-12+4x^2}\,dx, x, \sqrt{4+2x+x^2}\right) -$$
$$= \frac{1}{2}\tanh^{-1}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.97

$$\frac{1}{42}\left(21\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right) - 3\sqrt{7}\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right) - 7\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{(x+1)^2+3}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] (21*ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])] - 3*Sqrt[7]*ArcTanh[(5 + 2*x)/
(Sqrt[7]*Sqrt[4 + 2*x + x^2])] - 7*Sqrt[3]*ArcTanh[Sqrt[3 + (1 + x)^2]/Sqrt
[3]])/42
```

IntegrateAlgebraic [A] time = 0.23, size = 105, normalized size = 1.22

$$-\tanh^{-1}\left(\frac{x}{2} - \frac{1}{2}\sqrt{x^2+2x+4}\right) + \frac{\tanh^{-1}\left(-\frac{\sqrt{x^2+2x+4}}{\sqrt{3}} + \frac{x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{7}} - \frac{x}{\sqrt{7}} + \frac{1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] -ArcTanh[x/2 - Sqrt[4 + 2*x + x^2]/2] + ArcTanh[1/Sqrt[3] + x/Sqrt[3] - Sqr
t[4 + 2*x + x^2]/Sqrt[3]]/Sqrt[3] - ArcTanh[1/Sqrt[7] - x/Sqrt[7] + Sqrt[4
+ 2*x + x^2]/Sqrt[7]]/Sqrt[7]
```

fricas [A] time = 0.63, size = 110, normalized size = 1.28

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)+\frac{1}{2}\log\left(-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) -
 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x +
 1)) + 1/2*log(-x + sqrt(x^2 + 2*x + 4) + 2) - 1/2*log(-x + sqrt(x^2 + 2*x
+ 4) - 2)
```

giac [B] time = 0.73, size = 147, normalized size = 1.71

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)+\frac{1}{2}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x
 + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x
- 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4)
 + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x + sqrt
(x^2 + 2*x + 4) - 2))
```

maple [A] time = 0.42, size = 69, normalized size = 0.80

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{8+2x}{4\sqrt{x^2+2x+4}}\right)}{2}-\frac{\sqrt{7}\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}-5\,\mathrm{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{14}-\frac{\ln\left(\frac{-4-x+2\sqrt{x^2+2x+4}}{x}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-3\right)+}{1+x}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(10+4*
x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x\right)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x-x^3\right)\sqrt{x^2+2\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x - x^3)*(2*x + x^2 + 4)^(1/2)),x)
```

```
[Out] -int(1/((x - x^3)*(2*x + x^2 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x-1)\,(x+1)\,\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 843, 619, 215, 724, 206}

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5 + 2*
x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx &= \frac{\sqrt{4+2x+x^2}}{1-x}+\frac{1}{2}\int \frac{2+2x}{(-1+x)\sqrt{4+2x+x^2}}\,dx\\ &= \frac{\sqrt{4+2x+x^2}}{1-x}+2\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx+\int \frac{1}{\sqrt{4+2x+x^2}}\,dx\\ &= \frac{\sqrt{4+2x+x^2}}{1-x}-4\,\mathrm{Subst}\left(\int \frac{1}{28-x^2}\,dx,x,\frac{10+4x}{\sqrt{4+2x+x^2}}\right)+\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2\right.}{2\sqrt{3}}\\ &= \frac{\sqrt{4+2x+x^2}}{1-x}+\sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right)-\frac{2\tanh^{-1}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.98

$$-\frac{\sqrt{x^2+2x+4}}{x-1}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] -(Sqrt[4 + 2*x + x^2]/(-1 + x)) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5
+ 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

IntegrateAlgebraic [A] time = 0.23, size = 81, normalized size = 1.31

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\log\left(\sqrt{x^2+2x+4}-x-1\right)-\frac{4\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{7}}-\frac{x}{\sqrt{7}}+\frac{1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) - (4*ArcTanh[1/Sqrt[7] - x/Sqrt[7] + Sqrt[4 + 2
*x + x^2]/Sqrt[7]])/Sqrt[7] - Log[-1 - x + Sqrt[4 + 2*x + x^2]]
```

fricas [A] time = 0.57, size = 92, normalized size = 1.48

$$\frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)-7(x-1)\log\left(-x+\sqrt{x^2+2x+4}-1\right)-7x-7\sqrt{x^2+2}}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt
(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) - 1)
 - 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

giac [A] time = 0.66, size = 33, normalized size = 0.53

$$-\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}\,\mathrm{sgn}\left(\frac{1}{x-1}\right)+\mathrm{sgn}\left(\frac{1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="giac")
```

```
[Out] -sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*sgn(1/(x - 1)) + sgn(1/(x - 1))
```

maple [A] time = 0.38, size = 56, normalized size = 0.90

method	result
risch	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$
trager	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\ln\left(1+x+\sqrt{x^2+2x+4}\right)+\frac{2\,\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}-5\,\mathrm{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{7}$
default	$-\frac{\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}}{7(-1+x)}+\frac{2\sqrt{(-1+x)^2+3+4x}}{7}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}+\frac{(2x+2)\sqrt{(-1+x)^2+3}}{14}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+4)^(1/2)/(-1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-1+x)*(x^2+2*x+4)^(1/2)+arcsinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh(
1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))
```

maxima [A] time = 1.30, size = 61, normalized size = 0.98

$$-\frac{2}{7}\sqrt{7}\,\operatorname{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right)-\frac{\sqrt{x^2+2x+4}}{x-1}+\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) - s
qrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2,x)
```

```
[Out] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

```
[Out] Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

3.281 $\int \frac{3+2x}{\left(3+2x+x^2\right)^2 \sqrt{4+2x+x^2}} \, dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{x^2+2x+4}\,(3-x)}{4\left(x^2+2x+3\right)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1016, 1025, 982, 204, 1024, 206}

$$-\frac{\sqrt{x^2+2x+4}\,(3-x)}{4\left(x^2+2x+3\right)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -((3 - x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - ArcTan[(1 + x)/(Sqrt[2
]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 + 2*x + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
```

```
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{3+2x}{\left(3+2x+x^2\right)^2\sqrt{4+2x+x^2}}\,dx = -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)} + \frac{1}{8}\int \frac{-10-8x}{\left(3+2x+x^2\right)\sqrt{4+2x+x^2}}\,dx$$

$$= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)} - \frac{1}{4}\int \frac{1}{\left(3+2x+x^2\right)\sqrt{4+2x+x^2}}\,dx - \frac{1}{2}\int \frac{}{(3}$$

$$= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)} + 2\,\mathrm{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{4+2x+x^2}\right) + \mathrm{Su}$$

$$= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)} - \frac{\tan^{-1}\left(\frac{2+2x}{2\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \tanh^{-1}\left(\sqrt{4+2x+x^2}\right.$$

Mathematica [A] time = 0.40, size = 146, normalized size = 1.92

$$\frac{1}{32}\left(8\left(\frac{\sqrt{x^2+2x+4}\,(x-3)}{x^2+2x+3} - 2\log\left(\left(x^2+2x+3\right)^2\right) + 2\log\left(\left(x^2+2x+3\right)\left(x^2+2\sqrt{x^2+2x+4}+2x+5\right)\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (-4*Sqrt[2]*ArcTan[(Sqrt[2]*(4 + 10*x + 5*x^2))/(12 + 4*x^2 + 11*Sqrt[4 + 2
*x + x^2] + x*(8 + 11*Sqrt[4 + 2*x + x^2]))] + 8*(((-3 + x)*Sqrt[4 + 2*x +
x^2])/(3 + 2*x + x^2) - 2*Log[(3 + 2*x + x^2)^2] + 2*Log[(3 + 2*x + x^2)*(5
 + 2*x + x^2 + 2*Sqrt[4 + 2*x + x^2])]))/32
```

IntegrateAlgebraic [C] time = 1.68, size = 244, normalized size = 3.21

$$\frac{\sqrt{x^2+2x+4}\,(x-3)}{4\left(x^2+2x+3\right)} - i\tan^{-1}\left(-\sqrt{\frac{1}{9}-\frac{2i\sqrt{2}}{9}}\sqrt{x^2+2x+4} + \sqrt{\frac{1}{9}-\frac{2i\sqrt{2}}{9}}\,x + \sqrt{\frac{1}{9}-\frac{2i\sqrt{2}}{9}}\right) + i\tan^{-1}\left(-\sqrt{\frac{1}{9}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] ((-3 + x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - I*ArcTan[Sqrt[1/9 - ((
2*I)/9)*Sqrt[2]] + Sqrt[1/9 - ((2*I)/9)*Sqrt[2]]*x - Sqrt[1/9 - ((2*I)/9)*S
qrt[2]]*Sqrt[4 + 2*x + x^2]] + I*ArcTan[Sqrt[1/9 + ((2*I)/9)*Sqrt[2]] + Sqr
t[1/9 + ((2*I)/9)*Sqrt[2]]*x - Sqrt[1/9 + ((2*I)/9)*Sqrt[2]]*Sqrt[4 + 2*x +
 x^2]] + ArcTan[3/Sqrt[2] + Sqrt[2]*x + x^2/Sqrt[2] - ((1 + x)*Sqrt[4 + 2*x
 + x^2])/Sqrt[2]]/(4*Sqrt[2])
```

fricas [B] time = 0.57, size = 174, normalized size = 2.29

$$\sqrt{2}\left(x^2+2x+3\right)\arctan\left(-\tfrac{1}{2}\sqrt{2}(x+2)+\tfrac{1}{2}\sqrt{2}\sqrt{x^2+2x+4}\right)-\sqrt{2}\left(x^2+2x+3\right)\arctan\left(-\tfrac{1}{2}\sqrt{2}x+\tfrac{1}{2}\sqrt{2}\sqrt{x^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqrt
(x^2 + 2*x + 4)) - sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt
(2)*sqrt(x^2 + 2*x + 4)) + 2*x^2 - 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2
*x + 4)*(x + 2) + 3*x + 5) + 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x + 4
)*x + x + 3) + 2*sqrt(x^2 + 2*x + 4)*(x - 3) + 4*x + 6)/(x^2 + 2*x + 3)
```

giac [B] time = 0.66, size = 235, normalized size = 3.09

$$\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}+2\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}\right)\right)+\frac{4\left(x-\ldots\right.}{2\left(\left(x-\sqrt{x^2+2x+\ldots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(2
)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 + 2
*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x + 4)
 + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8*(x
- sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log((x -
 sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log((x - s
qrt(x^2 + 2*x + 4))^2 + 2)
```

maple [A] time = 0.72, size = 64, normalized size = 0.84

method	result
risch	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}+\operatorname{arctanh}\left(\sqrt{x^2+2x+4}\right)-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}(2x+2)}{4\sqrt{x^2+2x+4}}\right)}{8}$
default	$-\frac{1}{2\left(\sqrt{x^2+2x+4}-1\right)}-\frac{\ln\left(\sqrt{x^2+2x+4}-1\right)}{2}-\frac{1}{2\left(\sqrt{x^2+2x+4}+1\right)}+\frac{\ln\left(\sqrt{x^2+2x+4}+1\right)}{2}+\frac{\frac{3}{4}+\frac{3x}{4}}{\sqrt{x^2+2x+4}\left(\frac{(1+x)^2}{x^2+2x+4}+2\right)}-\frac{\arctan\left(\frac{(1+\ldots}{2\sqrt{x^2\ldots}}\right)}{8\ldots}$
trager	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}-3\ln\left(\frac{48384\operatorname{RootOf}(384\_Z^2-128\_Z+11)^2x+960\sqrt{x^2+2x+4}\operatorname{RootOf}(384\_Z^2-128\_Z+11)-15312\operatorname{RootOf}(38\ldots}{48\operatorname{RootOf}(384\_Z^2-128\_Z+\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3+x)/(x^2+2*x+3)*(x^2+2*x+4)^(1/2)+arctanh((x^2+2*x+4)^(1/2))-1/8*2^(
1/2)*arctan(1/4*2^(1/2)/(x^2+2*x+4)^(1/2)*(2*x+2))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{\sqrt{x^2+2x+4}\left(x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+3}{\left(x^2+2x+3\right)^2\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)),x)
```

```
[Out] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{\left(x^2+2x+3\right)^2\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\left(-3+x+2x^2\right)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1586, 1638, 650}

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \int \frac{x^2(3+2x)}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx$$
$$= \int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx$$
$$= \sqrt{-3+2x+x^2} + \int \frac{1}{(-1+x)\sqrt{-3+2x+x^2}}\,dx$$
$$= \sqrt{-3+2x+x^2} + \frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.72

$$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
[Out] (-9 + 3*x + 2*x^2)/(2*Sqrt[-3 + 2*x + x^2])
```

IntegrateAlgebraic [A] time = 0.13, size = 26, normalized size = 0.72

$$\frac{(2x-3)\sqrt{x^2+2x-3}}{2(x-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),
x]
[Out] ((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

fricas [A] time = 0.68, size = 22, normalized size = 0.61

$$\frac{\sqrt{x^2+2\,x-3}\,(2\,x-3)}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas"
)
[Out] 1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

giac [A] time = 0.65, size = 30, normalized size = 0.83

$$\sqrt{x^2+2\,x-3} + \frac{2}{x-\sqrt{x^2+2\,x-3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

maple [A] time = 0.34, size = 21, normalized size = 0.58

method	result	size
gosper	$\frac{(-3+2x)(3+x)}{2\sqrt{x^2+2x-3}}$	21
trager	$\frac{(-3+2x)\sqrt{x^2+2x-3}}{2x-2}$	23
risch	$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$	23
default	$\sqrt{x^2+2x-3}-\frac{\sqrt{(-1+x)^2-4+4x}}{2(-1+x)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-3+2*x)*(3+x)/(x^2+2*x-3)^(1/2)
```

maxima [A] time = 1.42, size = 28, normalized size = 0.78

$$\sqrt{x^2+2x-3}-\frac{\sqrt{x^2+2x-3}}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

mupad [B] time = 0.27, size = 19, normalized size = 0.53

$$\frac{\left(x-\frac{3}{2}\right)\sqrt{x^2+2x-3}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2*x^3)/((x + 2*x^2 - 3)*(2*x + x^2 - 3)^(1/2)),x)
```

```
[Out] ((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+3)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{2}\sqrt{x^2+x+2}\,x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

Rubi [A] time = 0.21, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6728, 640, 619, 215, 742, 1025, 982, 204, 1024, 206}

$$\frac{1}{2}\sqrt{x^2+x+2}\,x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqrt
[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[Sq
rt[2 + x + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}dx = \int \left(-\frac{x}{\sqrt{2+x+x^2}} + \frac{x^2}{\sqrt{2+x+x^2}} + \frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}}\right)dx$$

$$= -\int \frac{x}{\sqrt{2+x+x^2}}dx + \int \frac{x^2}{\sqrt{2+x+x^2}}dx + \int \frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}}d$$

$$= -\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} + \frac{1}{2}\int \frac{1}{\sqrt{2+x+x^2}}dx + \frac{1}{2}\int \frac{-2-\frac{3x}{2}}{\sqrt{2+x+x}}$$

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{5}{8}\int \frac{1}{\sqrt{2+x+x^2}}dx + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+}}\right.}{2}$$

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} + \frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{7}}\right) + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x}}\right.}{\sqrt{3}}$$

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{1}{8}\sinh^{-1}\left(\frac{1+2x}{\sqrt{7}}\right) + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x}}\right.}{\sqrt{3}}$$

Mathematica [C] time = 0.16, size = 134, normalized size = 1.54

$$\frac{1}{24}\left(6\sqrt{x^2+x+2}(2x-7) - 4i\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{-2i\sqrt{3}x - i\sqrt{3}+7}{4\sqrt{x^2+x+2}}\right) + 4i\left(\sqrt{3}+3i\right)\tanh^{-1}\left(\frac{2i\sqrt{3}x+i\sqrt{3}}{4\sqrt{x^2+x+}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (6*(-7 + 2*x)*Sqrt[2 + x + x^2] - 3*ArcSinh[(1 + 2*x)/Sqrt[7]] - (4*I)*(-3*
I + Sqrt[3])*ArcTanh[(7 - I*Sqrt[3] - (2*I)*Sqrt[3]*x)/(4*Sqrt[2 + x + x^2]
)] + (4*I)*(3*I + Sqrt[3])*ArcTanh[(7 + I*Sqrt[3] + (2*I)*Sqrt[3]*x)/(4*Sqr
t[2 + x + x^2])])/24
```

IntegrateAlgebraic [C] time = 1.59, size = 254, normalized size = 2.92

$$\frac{1}{4}\sqrt{x^2+x+2}(2x-7) + \frac{1}{8}\log\left(2\sqrt{x^2+x+2}-2x-1\right) + i\tan^{-1}\left(-2\sqrt{-\frac{1}{49}-\frac{4i\sqrt{3}}{49}}\sqrt{x^2+x+2} + 2\sqrt{-\frac{1}{49}-\frac{4}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] ((-7 + 2*x)*Sqrt[2 + x + x^2])/4 + I*ArcTan[Sqrt[-1/49 - ((4*I)/49)*Sqrt[3]
] + 2*Sqrt[-1/49 - ((4*I)/49)*Sqrt[3]]*x - 2*Sqrt[-1/49 - ((4*I)/49)*Sqrt[3
]]*Sqrt[2 + x + x^2]] - I*ArcTan[Sqrt[-1/49 + ((4*I)/49)*Sqrt[3]] + 2*Sqrt[
-1/49 + ((4*I)/49)*Sqrt[3]]*x - 2*Sqrt[-1/49 + ((4*I)/49)*Sqrt[3]]*Sqrt[2 +
 x + x^2]] - ArcTan[2/Sqrt[3] + (2*x)/Sqrt[3] + (2*x^2)/Sqrt[3] - ((1 + 2*x
)*Sqrt[2 + x + x^2])/Sqrt[3]]/Sqrt[3] + Log[-1 - 2*x + 2*Sqrt[2 + x + x^2]]
/8
```

fricas [B] time = 0.72, size = 147, normalized size = 1.69

$$\frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3) + \frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3)
 + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x -
1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*
x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) + 1/8*
log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

giac [B] time = 0.68, size = 148, normalized size = 1.70

$$\frac{1}{4}\sqrt{x^2+x+2}\,(2\,x-7)-\frac{1}{3}\sqrt{3}\,\arctan\left(-\frac{1}{3}\sqrt{3}\left(2\,x-2\,\sqrt{x^2+x+2}+3\right)\right)+\frac{1}{3}\sqrt{3}\,\arctan\left(-\frac{1}{3}\sqrt{3}\left(2\,x-2\,\sqrt{x^2+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*
sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x^2
 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 + x
+ 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) + 1)
+ 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

maple [A] time = 2.71, size = 63, normalized size = 0.72

method	result
risch	$\frac{(2x-7)\sqrt{x^2+x+2}}{4}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(\frac{1}{2}+x\right)}{7}\right)}{8}-\operatorname{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$
default	$\frac{x\sqrt{x^2+x+2}}{2}-\frac{7\sqrt{x^2+x+2}}{4}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(\frac{1}{2}+x\right)}{7}\right)}{8}-\operatorname{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$
trager	$\left(\frac{x}{2}-\frac{7}{4}\right)\sqrt{x^2+x+2}+\frac{\ln\left(\frac{1194390076660265776589788621316411229554703558 64x-1547697292817558076095613967756493895336593544\,\mathrm{Ro}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x-7)*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)*(1/2+x))-arctanh((x^2+x
+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x^2+x+2}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+1}{\left(x^2+x+1\right)\,\sqrt{x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)),x)
```

```
[Out] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 + 1}{\left(x^2 + x + 1\right)\sqrt{x^2 + x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

3.284 $\int \frac{1}{\left(4+2x+x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x + x^2)^(3/2)
) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(4+2x+x^2\right)^{7/2}}\,dx &= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4}{15}\int \frac{1}{\left(4+2x+x^2\right)^{5/2}}\,dx\\
&= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8}{135}\int \frac{1}{\left(4+2x+x^2\right)^{3/2}}\,dx\\
&= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8(1+x)}{405\sqrt{4+2x+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.67

$$\frac{(x+1)\left(8x^4+32x^3+108x^2+152x+203\right)}{405\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5/
2))
```

IntegrateAlgebraic [A] time = 0.29, size = 39, normalized size = 0.67

$$\frac{(x+1)\left(8x^4+32x^3+108x^2+152x+203\right)}{405\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5/
2))
```

fricas [B] time = 0.77, size = 98, normalized size = 1.69

$$\frac{8\,x^6+48\,x^5+192\,x^4+448\,x^3+768\,x^2+\left(8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x+203\right)\sqrt{x^2+2\,x+4}+768}{405\left(x^6+6\,x^5+24\,x^4+56\,x^3+96\,x^2+96\,x+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 140
*x^3 + 260*x^2 + 355*x + 203)*sqrt(x^2 + 2*x + 4) + 768*x + 512)/(x^6 + 6*x
^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)
```

giac [A] time = 0.63, size = 33, normalized size = 0.57

$$\frac{(4\left(\left(2\left(x+5\right)x+35\right)x+65\right)x+355)x+203}{405\left(x^2+2\,x+4\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="giac")
```

```
[Out] 1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5/
2)
```

maple [A] time = 0.37, size = 38, normalized size = 0.66

method	result	size
gosper	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405\left(x^2+2x+4\right)^{\frac{5}{2}}}$	38
trager	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405\left(x^2+2x+4\right)^{\frac{5}{2}}}$	38
risch	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405\left(x^2+2x+4\right)^{\frac{5}{2}}}$	38
default	$\frac{2x+2}{30\left(x^2+2x+4\right)^{\frac{5}{2}}}+\frac{\frac{4}{135}+\frac{4x}{135}}{\left(x^2+2x+4\right)^{\frac{3}{2}}}+\frac{\frac{8}{405}+\frac{8x}{405}}{\sqrt{x^2+2x+4}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)
```

maxima [A] time = 0.63, size = 76, normalized size = 1.31

$$\frac{8\,x}{405\,\sqrt{x^2+2\,x+4}}+\frac{8}{405\,\sqrt{x^2+2\,x+4}}+\frac{4\,x}{135\,\left(x^2+2\,x+4\right)^{\frac{3}{2}}}+\frac{4}{135\,\left(x^2+2\,x+4\right)^{\frac{3}{2}}}+\frac{x}{15\,\left(x^2+2\,x+4\right)^{\frac{5}{2}}}+\frac{}{15\,\left(x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x/(x^2 + 2*
x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2) +
 1/15/(x^2 + 2*x + 4)^(5/2)
```

mupad [B] time = 0.23, size = 69, normalized size = 1.19

$$\frac{51\,x+8\,x\,\left(x^2+2\,x+4\right)^2+8\,\left(x^2+2\,x+4\right)^2+12\,x^2+12\,x\,\left(x^2+2\,x+4\right)+75}{\left(x^2+2\,x+4\right)^{3/2}\,\left(405\,x^2+810\,x+1620\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + x^2 + 4)^(7/2),x)
```

```
[Out] (51*x + 8*x*(2*x + x^2 + 4)^2 + 8*(2*x + x^2 + 4)^2 + 12*x^2 + 12*x*(2*x +
x^2 + 4) + 75)/((2*x + x^2 + 4)^(3/2)*(810*x + 405*x^2 + 1620))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x+4\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)**(7/2),x)
```

```
[Out] Integral((x**2 + 2*x + 4)**(-7/2), x)
```

3.285 $\int \frac{1}{\left(1+8x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] -(4 + 3*x)/(39*(1 + 8*x + 3*x^2)^(3/2)) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x +
 3*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+8x+3x^2\right)^{5/2}}\,dx &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}-\frac{2}{13}\int \frac{1}{\left(1+8x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}+\frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.70

$$\frac{(3x+4)\left(18x^2+48x-7\right)}{507\left(3x^2+8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] ((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 57, normalized size = 1.21

$$\frac{3\sqrt{3x^2+8x+1}\left(54x^3+216x^2+171x-28\right)}{169\left(-3x+\sqrt{13}-4\right)^2\left(3x+\sqrt{13}+4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] (3*Sqrt[1 + 8*x + 3*x^2]*(-28 + 171*x + 216*x^2 + 54*x^3))/(169*(-4 + Sqrt[
13] - 3*x)^2*(4 + Sqrt[13] + 3*x)^2)
```

fricas [A] time = 0.65, size = 73, normalized size = 1.55

$$-\frac{252\,x^4+1344\,x^3+1960\,x^2-\left(54\,x^3+216\,x^2+171\,x-28\right)\sqrt{3\,x^2+8\,x+1}+448\,x+28}{507\left(9\,x^4+48\,x^3+70\,x^2+16\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sqr
t(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)
```

giac [A] time = 0.65, size = 27, normalized size = 0.57

$$\frac{9\,(6\,(x+4)x+19)x-28}{507\left(3\,x^2+8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

maple [A] time = 0.31, size = 30, normalized size = 0.64

method	result	size
gosper	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
trager	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
risch	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
default	$-\frac{6x+8}{78(3x^2+8x+1)^{\frac{3}{2}}}+\frac{6x+8}{169\sqrt{3x^2+8x+1}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

maxima [A] time = 0.44, size = 59, normalized size = 1.26

$$\frac{6\,x}{169\,\sqrt{3\,x^2+8\,x+1}}+\frac{8}{169\,\sqrt{3\,x^2+8\,x+1}}-\frac{x}{13\left(3\,x^2+8\,x+1\right)^{\frac{3}{2}}}-\frac{4}{39\left(3\,x^2+8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^2
 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

mupad [B] time = 0.05, size = 29, normalized size = 0.62

$$\frac{(12\,x+16)\,\left(72\,x^2+192\,x-28\right)}{8112\,{\left(3\,x^2+8\,x+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x + 3*x^2 + 1)^(5/2),x)
```

```
[Out] ((12*x + 16)*(192*x + 72*x^2 - 28))/(8112*(8*x + 3*x^2 + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+8x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+8*x+1)**(5/2),x)
```

```
[Out] Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -(2 - 3*x)/(57*(5 + 4*x - 3*x^2)^(3/2)) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x -
 3*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5+4x-3x^2\right)^{5/2}}\,dx &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}+\frac{2}{19}\int \frac{1}{\left(5+4x-3x^2\right)^{3/2}}\,dx\\ &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}-\frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.70

$$-\frac{(3x-2)\left(18x^2-24x-49\right)}{1083\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -1/1083*((-2 + 3*x)*(-49 - 24*x + 18*x^2))/(5 + 4*x - 3*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.31, size = 57, normalized size = 1.21

$$-\frac{3\sqrt{-3x^2+4x+5}\left(54x^3-108x^2-99x+98\right)}{361\left(-3x+\sqrt{19}+2\right)^2\left(3x+\sqrt{19}-2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] (-3*Sqrt[5 + 4*x - 3*x^2]*(98 - 99*x - 108*x^2 + 54*x^3))/(361*(2 + Sqrt[19
] - 3*x)^2*(-2 + Sqrt[19] + 3*x)^2)
```

fricas [A] time = 0.53, size = 51, normalized size = 1.09

$$-\frac{\left(54\,x^3-108\,x^2-99\,x+98\right)\sqrt{-3\,x^2+4\,x+5}}{1083\left(9\,x^4-24\,x^3-14\,x^2+40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*x^4 - 24*x
^3 - 14*x^2 + 40*x + 25)
```

giac [A] time = 0.71, size = 39, normalized size = 0.83

$$-\frac{(9\,(6\,(x-2)x-11)x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\left(3\,x^2-4\,x-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x^2 - 4*x -
 5)^2
```

maple [A] time = 0.39, size = 30, normalized size = 0.64

method	result	size
gosper	$-\frac{54x^3-108x^2-99x+98}{1083\left(-3x^2+4x+5\right)^{\frac{3}{2}}}$	30
default	$-\frac{-6x+4}{114\left(-3x^2+4x+5\right)^{\frac{3}{2}}}-\frac{-6x+4}{361\sqrt{-3x^2+4x+5}}$	40
trager	$-\frac{\left(54x^3-108x^2-99x+98\right)\sqrt{-3x^2+4x+5}}{1083\left(3x^2-4x-5\right)^2}$	42
risch	$\frac{54x^3-108x^2-99x+98}{1083\left(3x^2-4x-5\right)\sqrt{-3x^2+4x+5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1083*(54*x^3-108*x^2-99*x+98)/(-3*x^2+4*x+5)^(3/2)
```

maxima [A] time = 0.68, size = 59, normalized size = 1.26

$$\frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}}-\frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}}+\frac{x}{19\left(-3\,x^2+4\,x+5\right)^{\frac{3}{2}}}-\frac{2}{57\left(-3\,x^2+4\,x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1/19*x/(-3*
x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

mupad [B] time = 0.20, size = 29, normalized size = 0.62

$$\frac{(12\,x-8)\,\left(-72\,x^2+96\,x+196\right)}{17328\,{\left(-3\,x^2+4\,x+5\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 5)^(5/2),x)
```

```
[Out] ((12*x - 8)*(96*x - 72*x^2 + 196))/(17328*(4*x - 3*x^2 + 5)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+4x+5\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

```
[Out] Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 684, 619, 215}

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \int \left(-\frac{1}{(1+x)^2}+\frac{\sqrt{2+2x+x^2}}{(1+x)^2}\right)dx$$
$$= \frac{1}{1+x}+\int \frac{\sqrt{2+2x+x^2}}{(1+x)^2}\,dx$$
$$= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\int \frac{1}{\sqrt{2+2x+x^2}}\,dx$$
$$= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4}}}\,dx,x,2+2x\right)$$
$$= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\sinh^{-1}(1+x)$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.03

$$\frac{-\sqrt{x^2+2x+2}+(x+1)\sinh^{-1}(x+1)+1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 - Sqrt[2 + 2*x + x^2] + (1 + x)*ArcSinh[1 + x])/(1 + x)
```

IntegrateAlgebraic [A] time = 0.29, size = 46, normalized size = 1.59

$$\frac{\sqrt{x^2+2x+2}}{-x-1}-\log\left(\sqrt{x^2+2x+2}-x-1\right)+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) + Sqrt[2 + 2*x + x^2]/(-1 - x) - Log[-1 - x + Sqrt[2 + 2*x + x
^2]]
```

fricas [A] time = 0.68, size = 39, normalized size = 1.34

$$-\frac{(x+1)\log\left(-x+\sqrt{x^2+2x+2}-1\right)+x+\sqrt{x^2+2x+2}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(-x + sqrt(x^2 + 2*x + 2) - 1) + x + sqrt(x^2 + 2*x + 2))/(x +
 1)
```

giac [B] time = 0.66, size = 60, normalized size = 2.07

$$\frac{2}{\left(x-\sqrt{x^2+2x+2}\right)^2+2x-2\sqrt{x^2+2x+2}}+\frac{1}{x+1}-\log\left(-x+\sqrt{x^2+2x+2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1) -
 log(-x + sqrt(x^2 + 2*x + 2) - 1)
```

maple [A] time = 0.14, size = 40, normalized size = 1.38

method	result	size
default	$-\frac{\left((1+x)^2+1\right)^{\frac{3}{2}}}{1+x}+(1+x)\sqrt{(1+x)^2+1}+\operatorname{arcsinh}(1+x)+\frac{1}{1+x}$	40
trager	$-\frac{x}{1+x}-\frac{\sqrt{x^2+2x+2}}{1+x}-\ln\left(\sqrt{x^2+2x+2}-1-x\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^2+2*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\frac{1}{x+1}+\int \frac{\sqrt{x^2+2x+2}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2 + 2)^(1/2) + 1),x)
```

```
[Out] 1/(x + 1) + int((2*x + x^2 + 2)^(1/2)/(x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx$

Optimal. Leaf size=45

$$\sqrt{x^2+x+1}+2\log\left(\sqrt{x^2+x+1}+x\right)-x-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2116, 893}

$$\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^2]] - (3*L
og[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx &= 2\,\text{Subst}\left(\int \frac{1+x+x^2}{x(1+2x)^2} \, dx, x, x+\sqrt{1+x+x^2}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{x}-\frac{3}{2(1+2x)^2}-\frac{3}{2(1+2x)}\right) dx, x, x+\sqrt{1+x+x^2}\right)\\ &= \frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)}+2\log\left(x+\sqrt{1+x+x^2}\right)-\frac{3}{2}\log\left(1+2\left(x+\sqrt{1+x+x\right.\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.31

$$\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^2]] - (3*L
og[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

IntegrateAlgebraic [A] time = 0.12, size = 54, normalized size = 1.20

$$\sqrt{x^2+x+1}+2\log\left(\sqrt{x^2+x+1}-x-2\right)-\frac{1}{2}\log\left(2\sqrt{x^2+x+1}-2x-1\right)-x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] -x + Sqrt[1 + x + x^2] + 2*Log[-2 - x + Sqrt[1 + x + x^2]] - Log[-1 - 2*x +
 2*Sqrt[1 + x + x^2]]/2
```

fricas [A] time = 0.78, size = 63, normalized size = 1.40

$$-x+\sqrt{x^2+x+1}+\log(x+1)-\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x + sqrt(x^2 + x + 1) + log(x + 1) - log(-x + sqrt(x^2 + x + 1)) + log(-x
+ sqrt(x^2 + x + 1) - 2) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.65, size = 66, normalized size = 1.47

$$-x+\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)+\log\left(|x+1|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs(
x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1)
 - 2))
```

maple [A] time = 0.12, size = 52, normalized size = 1.16

method	result	size
default	$\sqrt{(1+x)^2-x}-\frac{\operatorname{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{2}-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)-x+\ln(1+x)$	52
trager	$\sqrt{x^2+x+1}-x-\frac{\ln\left(\frac{2x^2\sqrt{x^2+x+1}+2x^3+8\sqrt{x^2+x+1}\,x+9x^2+14\sqrt{x^2+x+1}+12x+13}{(1+x)^4}\right)}{2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*(1/2+x)*3^(1/2))-arctanh(1/2*(1-x)/((1+x)
^2-x)^(1/2))-x+ln(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\ln(x+1) - x + \int \frac{\sqrt{x^2+x+1}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + x^2 + 1)^(1/2)),x)
```

```
[Out] log(x + 1) - x + int((x + x^2 + 1)^(1/2)/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}} \, dx$

Optimal. Leaf size=79

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6742, 742, 640, 612, 619, 215}

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -x^3/9 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/2))
/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx &= \int \left(-\frac{x^2}{3}-\frac{2x^3}{3}+\frac{2}{3}x^2\sqrt{1+x+x^2}\right)dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{2}{3}\int x^2\sqrt{1+x+x^2}\,dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{6}\int\left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{24}\int\sqrt{1+x+x^2}\,dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+
\end{aligned}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 0.90

$$\frac{1}{576}\left(-96x^4-64x^3+96\left(x^2+x+1\right)^{3/2}x-80\left(x^2+x+1\right)^{3/2}+6(2x+1)\sqrt{x^2+x+1}+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] (-64*x^3 - 96*x^4 + 6*(1 + 2*x)*Sqrt[1 + x + x^2] - 80*(1 + x + x^2)^(3/2)
+ 96*x*(1 + x + x^2)^(3/2) + 9*ArcSinh[(1 + 2*x)/Sqrt[3]])/576
```

IntegrateAlgebraic [A] time = 0.19, size = 67, normalized size = 0.85

$$-\frac{1}{64}\log\left(2\sqrt{x^2+x+1}-2x-1\right)+\frac{1}{18}\left(-3x^4-2x^3\right)+\frac{1}{288}\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/288 + (-2*x^3 - 3*x^4)/18
 - Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/64
```

fricas [A] time = 0.66, size = 54, normalized size = 0.68

$$-\frac{1}{6}x^4-\frac{1}{9}x^3+\frac{1}{288}\left(48x^3+8x^2+14x-37\right)\sqrt{x^2+x+1}-\frac{1}{64}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) -
 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.62, size = 54, normalized size = 0.68

$$-\frac{1}{6}x^4-\frac{1}{9}x^3+\frac{1}{288}(2(4(6x+1)x+7)x-37)\sqrt{x^2+x+1}-\frac{1}{64}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1)
 - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.07, size = 55, normalized size = 0.70

method	result	size
trager	$-\frac{(2+3x)x^3}{18}+\frac{\left(\frac{1}{2}x^3+\frac{1}{12}x^2+\frac{7}{48}x-\frac{37}{96}\right)\sqrt{x^2+x+1}}{3}-\frac{\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{64}$	55
default	$-\frac{x^3}{9}-\frac{x^4}{6}+\frac{x\left(x^2+x+1\right)^{\frac{3}{2}}}{6}-\frac{5\left(x^2+x+1\right)^{\frac{3}{2}}}{36}+\frac{(1+2x)\sqrt{x^2+x+1}}{96}+\frac{\operatorname{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{64}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(2+3*x)*x^3+1/3*(1/2*x^3+1/12*x^2+7/48*x-37/96)*(x^2+x+1)^(1/2)-1/64*
ln(2*(x^2+x+1)^(1/2)-1-2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

mupad [B] time = 0.07, size = 71, normalized size = 0.90

$$\frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{64}-\frac{\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{6}-\frac{x^3}{9}-\frac{x^4}{6}-\frac{5\left(8x^2+2x+5\right)\sqrt{x^2+x+1}}{288}+\frac{x\left(x^2+x+1\right)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x + 2*(x + x^2 + 1)^(1/2) + 1),x)
```

```
[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)/64 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/2))/6
 - x^3/9 - x^4/6 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/288 + (x*(x +
x^2 + 1)^(3/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=80

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.34, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 734, 843, 619, 215, 724, 206, 6740}

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \int \left(-\frac{3x}{-1+\sqrt{1+x+x^2}}+\frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\right)dx$$

$$= -\left(3\int \frac{x}{-1+\sqrt{1+x+x^2}}\,dx\right)+\int \frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$$

$$= -\left(3\int \left(\frac{1}{1+x}+\frac{\sqrt{1+x+x^2}}{1+x}\right)dx\right)+\int \left(1+\frac{1}{-1+\sqrt{1+x+x^2}}\right)dx$$

$$= x-3\log(1+x)-3\int \frac{\sqrt{1+x+x^2}}{1+x}\,dx+\int \frac{1}{-1+\sqrt{1+x+x^2}}\,dx$$

$$= x-3\sqrt{1+x+x^2}-3\log(1+x)+\frac{3}{2}\int \frac{-1+x}{(1+x)\sqrt{1+x+x^2}}\,dx+\int \left(\frac{1}{-1-x}+\frac{1}{x}\right.$$

$$= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)+\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx-3\int \frac{}{(1+x)\sqrt{}}$$

$$= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)-\frac{1}{2}\int \frac{-2-x}{x\sqrt{1+x+x^2}}\,dx+\frac{1}{2}\int \frac{}{(1+x)}$$

$$= x-3\sqrt{1+x+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\,\mathrm{l}$$

$$= x-3\sqrt{1+x+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\,\mathrm{l}$$

$$= x-3\sqrt{1+x+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)-\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

Mathematica [A] time = 0.12, size = 80, normalized size = 1.00

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```

IntegrateAlgebraic [A] time = 0.26, size = 72, normalized size = 0.90

$$-3\sqrt{x^2+x+1}-8\log\left(\sqrt{x^2+x+1}-x-2\right)+2\log\left(\sqrt{x^2+x+1}-x-1\right)+\frac{1}{2}\log\left(2\sqrt{x^2+x+1}-2x-1\right)+x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] - 8*Log[-2 - x + Sqrt[1 + x + x^2]] + 2*Log[-1 - x
+ Sqrt[1 + x + x^2]] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

fricas [A] time = 0.71, size = 99, normalized size = 1.24

$$x-3\sqrt{x^2+x+1}-4\log\left(x+1\right)+\log\left(x\right)-\log\left(-x+\sqrt{x^2+x+1}+1\right)+4\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x + 1
) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1) -
4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.67, size = 105, normalized size = 1.31

$$x-3\sqrt{x^2+x+1}-\frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)-4\log\left(|x+1|\right)+\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)+4\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(a
bs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(abs
(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log(ab
s(-x + sqrt(x^2 + x + 1) - 2))
```

maple [A] time = 0.38, size = 80, normalized size = 1.00

method	result
default	$-4\ln\left(1+x\right)+\ln\left(x\right)+x-4\sqrt{\left(1+x\right)^2-x}+\frac{5\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{2}+4\,\mathrm{arctanh}\left(\frac{1-x}{2\sqrt{\left(1+x\right)^2-x}}\right)+\sqrt{x^2+x}$
trager	$-1+x-3\sqrt{x^2+x+1}+\frac{\ln\left(\frac{-8-96x+5493060x^7+1790544x^5-1526x^3+3865870x^6+445596x^4-507x^2+4224608x^9+8448x^{14}+5593140x^8+458x^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*ln(1+x)+ln(x)+x-4*((1+x)^2-x)^(1/2)+5/2*arcsinh(2/3*(1/2+x)*3^(1/2))+4*a
rctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))+(x^2+x+1)^(1/2)-arctanh(1/2*(2+x)/(x^2+
x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}x^2 + \frac{1}{2}x + \int -\frac{3x^3 + 2x^2 - x}{2\left(x^2 + x - 2\sqrt{x^2 + x + 1} + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] 3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2
+ x + 1) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$x - 4\ln(x+1) + \ln(x) - \int \frac{(3x-1)\sqrt{x^2 + x + 1}}{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - (x + x^2 + 1)^(1/2))/((x + x^2 + 1)^(1/2) - 1),x)
```

```
[Out] x - 4*log(x + 1) + log(x) - int(((3*x - 1)*(x + x^2 + 1)^(1/2))/(x*(x + 1))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{\sqrt{x^2 + x + 1} - 1}\,dx - \int \left(-\frac{\sqrt{x^2 + x + 1}}{\sqrt{x^2 + x + 1} - 1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

```
[Out] -Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/(
sqrt(x**2 + x + 1) - 1), x)
```

3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}\sqrt{x^2+2x+4}\,(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2$$

Rubi [A] time = 0.53, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {6742, 734, 843, 619, 215, 724, 206, 612}

$$\frac{1}{2}\sqrt{x^2+2x+4}\,(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x + x
^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 + (
43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*S
qrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^
2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \left(-\frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}-\frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\right) d$$

$$= -\int \frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\,dx - \int \frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+}$$

$$= -\int \left(-\frac{\sqrt{1+x+x^2}}{3+x}-\frac{\sqrt{4+2x+x^2}}{3+x}\right) dx - \int \left(-\sqrt{1+x+x^2}+\frac{3\sqrt{1}}{}\right.$$

$$= -\left(3\int \frac{\sqrt{1+x+x^2}}{3+x}\,dx\right) - 3\int \frac{\sqrt{4+2x+x^2}}{3+x}\,dx + \int \sqrt{1+x+x^2}\,dx$$

$$= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{2}(1+x)$$

$$= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{2}(1+x)$$

$$= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{2}(1+x)$$

$$= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{2}(1+x)$$

Mathematica [A] time = 0.34, size = 151, normalized size = 0.96

$$\frac{1}{8}\left(2\left(2\sqrt{x^2+x+1}\,x+2\sqrt{x^2+2x+4}\,x-7\sqrt{x^2+x+1}-6\sqrt{x^2+2x+4}-8\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (44*ArcSinh[(1 + x)/Sqrt[3]] + 43*ArcSinh[(1 + 2*x)/Sqrt[3]] + 2*(-7*Sqrt[1
 + x + x^2] + 2*x*Sqrt[1 + x + x^2] - 6*Sqrt[4 + 2*x + x^2] + 2*x*Sqrt[4 +
```

```
2*x + x^2] - 8*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*Sqrt[1 + x + x^2])] + 8
*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]))/8
```

IntegrateAlgebraic [A] time = 2.99, size = 165, normalized size = 1.04

$$\frac{1}{2}\sqrt{x^2+2x+4}\,(x-3)+\frac{1}{4}(2x-7)\sqrt{x^2+x+1}-\frac{43}{8}\log\left(2\sqrt{x^2+x+1}-2x-1\right)-\frac{11}{2}\log\left(\sqrt{x^2+2x+4}-x-1\right)-4$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] ((-7 + 2*x)*Sqrt[1 + x + x^2])/4 + ((-3 + x)*Sqrt[4 + 2*x + x^2])/2 - 4*Sqr
t[7]*ArcTanh[3/Sqrt[7] + x/Sqrt[7] - Sqrt[1 + x + x^2]/Sqrt[7]] - 4*Sqrt[7]
*ArcTanh[3/Sqrt[7] + x/Sqrt[7] - Sqrt[4 + 2*x + x^2]/Sqrt[7]] - (43*Log[-1
- 2*x + 2*Sqrt[1 + x + x^2]])/8 - (11*Log[-1 - x + Sqrt[4 + 2*x + x^2]])/2
```

fricas [A] time = 0.73, size = 155, normalized size = 0.98

$$\frac{1}{4}\sqrt{x^2+x+1}\,(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}\,(x-3)+2\sqrt{7}\log\left(\frac{2\sqrt{7}\,(5x+1)+2\sqrt{x^2+x+1}\,\left(5\sqrt{7}-14\right)-25x-}{x+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt(
7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x -
 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2*sq
rt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4) - 1) - 4
3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

maple [A] time = 0.03, size = 140, normalized size = 0.89

method	result
default	$-2\sqrt{(3+x)^2-5x-8}+\frac{43\,\mathrm{arcsinh}\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{8}+2\sqrt{7}\,\,\mathrm{arctanh}\left(\frac{(-1-5x)\sqrt{7}}{14\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2-4x-5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*(1/2+x)*3^(1/2))+2*7^(1/2)*arctan
h(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11/2
*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x)^2
-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x+1}{\sqrt{x^2+x+1}-\sqrt{x^2+2\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)),x)
[Out] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

3.292 $\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {51, 63, 203}

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*x^3),x]
```

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1 + x]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx &= \frac{\sqrt{-1+x}}{2x^2}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{8}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.54

$$2\sqrt{x-1}\, {}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*x^3),x]
[Out] 2*Sqrt[-1 + x]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x]
```

IntegrateAlgebraic [A] time = 0.02, size = 32, normalized size = 0.78

$$\frac{\sqrt{x-1}\,(3x+2)}{4x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + x]*x^3),x]
[Out] (Sqrt[-1 + x]*(2 + 3*x))/(4*x^2) + (3*ArcTan[Sqrt[-1 + x]])/4
```

fricas [A] time = 0.50, size = 28, normalized size = 0.68

$$\frac{3\,x^2\arctan\left(\sqrt{x-1}\right)+(3\,x+2)\sqrt{x-1}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

giac [A] time = 0.61, size = 29, normalized size = 0.71

$$\frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,x^2}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="giac")
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))
```

maple [A] time = 0.32, size = 30, normalized size = 0.73

method	result
derivativedivides	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}$
default	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}$
risch	$\frac{3x^2-x-2}{4x^2\sqrt{-1+x}}+\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}$
trager	$\frac{(2+3x)\sqrt{-1+x}}{4x^2}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-1+x}+x-2}{x}\right)}{8}$
meijerg	$\frac{\sqrt{-\mathrm{signum}(-1+x)}\left(\frac{\sqrt{\pi}\,\left(-7x^2+8x+8\right)}{16x^2}-\frac{\sqrt{\pi}\,(12x+8)\sqrt{1-x}}{16x^2}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-x}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^2}-\frac{\sqrt{\pi}}{2x}\right)}{\sqrt{\pi}\,\sqrt{\mathrm{signum}(-1+x)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

maxima [A] time = 1.37, size = 38, normalized size = 0.93

$$\frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\left((x-1)^2+2\,x-1\right)}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4*arctan(sq
rt(x - 1))
```

mupad [B] time = 0.04, size = 29, normalized size = 0.71

$$\frac{3\,\mathrm{atan}\left(\sqrt{x-1}\right)}{4}+\frac{3\,\sqrt{x-1}}{4\,x}+\frac{\sqrt{x-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x - 1)^(1/2)),x)
```

```
[Out] (3*atan((x - 1)^(1/2)))/4 + (3*(x - 1)^(1/2))/(4*x) + (x - 1)^(1/2)/(2*x^2)
```

sympy [A] time = 3.08, size = 131, normalized size = 3.20

$$\begin{cases} \frac{3i\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4}-\frac{3i}{4\sqrt{x}\,\sqrt{-1+\frac{1}{x}}}+\frac{i}{4x^{\frac{3}{2}}\,\sqrt{-1+\frac{1}{x}}}+\frac{i}{2x^{\frac{5}{2}}\,\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|}>1 \\ -\frac{3\,\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)}{4}+\frac{3}{4\sqrt{x}\,\sqrt{1-\frac{1}{x}}}-\frac{1}{4x^{\frac{3}{2}}\,\sqrt{1-\frac{1}{x}}}-\frac{1}{2x^{\frac{5}{2}}\,\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x
**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3
*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1
/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))
```

3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3/x)^(4/3)*x^2),x]
[Out] -(1 - 3/x)^(-1/3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{\sqrt[3]{\frac{x-3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
[Out] -((-3 + x)/x)^(-1/3)
```

IntegrateAlgebraic [A] time = 0.01, size = 19, normalized size = 1.46

$$-\frac{\left(\frac{x-3}{x}\right)^{2/3} x}{x-3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - 3/x)^(4/3)*x^2),x]
[Out] -((((-3 + x)/x)^(2/3)*x)/(-3 + x))
```

fricas [A] time = 0.52, size = 17, normalized size = 1.31

$$-\frac{x\left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")
[Out] -x*((x - 3)/x)^(2/3)/(x - 3)
```

giac [A] time = 0.65, size = 11, normalized size = 0.85

$$-\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="giac")
[Out] -1/((x - 3)/x)^(1/3)
```

maple [A] time = 0.03, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
default	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
risch	$-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$	12
gosper	$-\frac{-3+x}{x\left(\frac{-3+x}{x}\right)^{\frac{4}{3}}}$	18
trager	$-\frac{x\left(-\frac{3-x}{x}\right)^{\frac{2}{3}}}{-3+x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3/x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/(1-3/x)^(1/3)
```

maxima [A] time = 0.52, size = 11, normalized size = 0.85

$$-\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")
[Out] -1/(-3/x + 1)^(1/3)
```

mupad [B] time = 0.46, size = 11, normalized size = 0.85

$$-\frac{1}{\left(1-\frac{3}{x}\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - 3/x)^(4/3)),x)
[Out] -1/(1 - 3/x)^(1/3)
```

sympy [A] time = 0.98, size = 10, normalized size = 0.77

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)**(4/3)/x**2,x)
[Out] -1/(1 - 3/x)**(1/3)
```

3.294 $\int \frac{(-1+3x)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 58, 618, 204, 31}

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*
x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+3x)^{4/3}}{x^2}\,dx &= -\frac{(-1+3x)^{4/3}}{x}+4\int \frac{\sqrt[3]{-1+3x}}{x}\,dx\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}-4\int \frac{1}{x(-1+3x)^{2/3}}\,dx\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+2\log(x)-6\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+3x}\right)-6\,\text{Subst}\Big(\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+2\log(x)-6\log\left(1+\sqrt[3]{-1+3x}\right)+12\,\text{Subst}\Big(\int \frac{1}{-3-x^2}\,d\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right)+2\log(x)-6\log\Big(1+\sqrt[3]{-1}\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.37

$$\frac{9}{7}(3x-1)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};1-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x)^(4/3)/x^2,x]
[Out] (9*(-1 + 3*x)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 - 3*x])/7
```

IntegrateAlgebraic [A] time = 0.06, size = 88, normalized size = 1.24

$$\frac{\sqrt[3]{3x-1}\,(9x+1)}{x}-4\log\left(\sqrt[3]{3x-1}+1\right)+2\log\left((3x-1)^{2/3}-\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x)^(4/3)/x^2,x]
[Out] ((-1 + 3*x)^(1/3)*(1 + 9*x))/x + 4*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-1 + 3*x)
^(1/3))/Sqrt[3]] - 4*Log[1 + (-1 + 3*x)^(1/3)] + 2*Log[1 - (-1 + 3*x)^(1/3)
 + (-1 + 3*x)^(2/3)]
```

fricas [A] time = 0.70, size = 80, normalized size = 1.13

$$-\frac{4\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\,(3x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\,x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4\,x\log\left((3x-1)^{\frac{1}{3}}+1\right)-(9}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")
[Out] -(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log((
3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9*x
 + 1)*(3*x - 1)^(1/3))/x
```

giac [A] time = 0.65, size = 76, normalized size = 1.07

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)+9(3x-1)^{\frac{1}{3}}+\frac{(3x-1)^{\frac{1}{3}}}{x}+2\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\log\Big(($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

maple [C] time = 0.60, size = 67, normalized size = 0.94

method	result
meijerg	$-\frac{4\mathrm{signum}\left(-\frac{1}{3}+x\right)^{\frac{4}{3}}\left(-\frac{3\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,3],3x\right)}{2}+3\left(2+\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{3\Gamma\left(\frac{2}{3}\right)}{4x}\right)}{3\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(-\frac{1}{3}+x\right)\right)^{\frac{4}{3}}}$
derivativedivides	$9\left(-1+3x\right)^{\frac{1}{3}}-\frac{1}{1+(-1+3x)^{\frac{1}{3}}}-4\ln\left(1+\left(-1+3x\right)^{\frac{1}{3}}\right)+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left(\left(-1+3x\right)^{\frac{2}{3}}\right.$
default	$9\left(-1+3x\right)^{\frac{1}{3}}-\frac{1}{1+(-1+3x)^{\frac{1}{3}}}-4\ln\left(1+\left(-1+3x\right)^{\frac{1}{3}}\right)+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left(\left(-1+3x\right)^{\frac{2}{3}}\right.$
risch	$\frac{(-1+3x)^{\frac{1}{3}}}{x}+\frac{\left(-\frac{4(-1+3x)^{\frac{2}{3}}\left(-\mathrm{signum}\left(-\frac{1}{3}+x\right)\right)^{\frac{2}{3}}\left(2\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],3x\right)+\left(\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{\left((-1+3x)^2\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(-\frac{1}{3}+x\right)^{\frac{2}{3}}}+\frac{9(-1+3x)^{\frac{2}{3}}(-\cdots}{\cdots}\right.}{(-1+3x)^{\frac{2}{3}}}$
trager	$\frac{(1+9x)(-1+3x)^{\frac{1}{3}}}{x}-4\ln\left(\frac{(-1+3x)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x-\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2+\mathrm{RootOf}(\_\cdots}{x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+3*x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/GAMMA(2/3)*signum(-1/3+x)^(4/3)/(-signum(-1/3+x))^(4/3)*(-3/2*GAMMA(2/
3)*x*hypergeom([2/3,1,1],[2,3],3*x)+3*(2+1/6*Pi*3^(1/2)-1/2*ln(3)+ln(x)+I*P
i)*GAMMA(2/3)+3/4*GAMMA(2/3)/x)
```

maxima [A] time = 1.09, size = 76, normalized size = 1.07

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(3x-1\right)^{\frac{1}{3}}-1\right)\right)+9\left(3x-1\right)^{\frac{1}{3}}+\frac{\left(3x-1\right)^{\frac{1}{3}}}{x}+2\log\left(\left(3x-1\right)^{\frac{2}{3}}-\left(3x-1\right)^{\frac{1}{3}}+1\right)-4\log\left((3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

mupad [B] time = 0.21, size = 90, normalized size = 1.27

$$9\left(3x-1\right)^{1/3}-4\ln\left(144\left(3x-1\right)^{1/3}+144\right)+\frac{\left(3x-1\right)^{1/3}}{x}+\ln\left(18-36\left(3x-1\right)^{1/3}+\sqrt{3}\,18i\right)\left(2+\sqrt{3}\,2i\right)-\ln\left(36\,(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 1)^(4/3)/x^2,x)
```

```
[Out] 9*(3*x - 1)^(1/3) - 4*log(144*(3*x - 1)^(1/3) + 144) + (3*x - 1)^(1/3)/x +
log(3^(1/2)*18i - 36*(3*x - 1)^(1/3) + 18)*(3^(1/2)*2i + 2) - log(3^(1/2)*1
8i + 36*(3*x - 1)^(1/3) - 18)*(3^(1/2)*2i - 2)
```

sympy [C] time = 2.27, size = 541, normalized size = 7.62

$$\frac{189\sqrt[3]{3}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84\left(x-\frac{1}{3}\right)\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)**(4/3)/x**2,x)
```

```
[Out] 189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/
3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3)*
exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/
3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_pol
ar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/
3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)
*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) +
 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I
*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*p
i) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*p
i/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*g
amma(10/3))
```

3.295 $\int (4-3x)^{4/3}x^2\,dx$

Optimal. Leaf size=40

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*x)^(4/3)*x^2,x]
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (4-3x)^{4/3}x^2\,dx &= \int \left(\frac{16}{9}(4-3x)^{4/3}-\frac{8}{9}(4-3x)^{7/3}+\frac{1}{9}(4-3x)^{10/3}\right)dx\\ &= -\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.58

$$-\frac{1}{455}(4-3x)^{7/3}\left(35x^2+28x+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*x)^(4/3)*x^2,x]
[Out] -1/455*((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))
```

IntegrateAlgebraic [A] time = 0.01, size = 33, normalized size = 0.82

$$\frac{1}{455}\sqrt[3]{4-3x}\left(-315x^4+588x^3-32x^2-64x-256\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 - 3*x)^(4/3)*x^2,x]
[Out] ((4 - 3*x)^(1/3)*(-256 - 64*x - 32*x^2 + 588*x^3 - 315*x^4))/455
```

fricas [A] time = 0.62, size = 29, normalized size = 0.72

$$-\frac{1}{455}\left(315\,x^4-588\,x^3+32\,x^2+64\,x+256\right)(-3\,x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")
```

```
[Out] -1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

giac [A] time = 0.63, size = 49, normalized size = 1.22

$$-\frac{1}{117}\,(3\,x-4)^4(-3\,x+4)^{\frac{1}{3}}-\frac{4}{45}\,(3\,x-4)^3(-3\,x+4)^{\frac{1}{3}}-\frac{16}{63}\,(3\,x-4)^2(-3\,x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="giac")
```

```
[Out] -1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)^(1/3) - 1
6/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

maple [C] time = 0.30, size = 18, normalized size = 0.45

method	result	size
meijerg	$\frac{4\,2^{\frac{2}{3}}x^3\,\mathrm{hypergeom}\left(\left[-\frac{4}{3},3\right],[4],\frac{3x}{4}\right)}{3}$	18
gosper	$-\frac{\left(35x^2+28x+16\right)(4-3x)^{\frac{7}{3}}}{455}$	20
derivativedivides	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
default	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
trager	$\left(-\frac{9}{13}x^4+\frac{84}{65}x^3-\frac{32}{455}x^2-\frac{64}{455}x-\frac{256}{455}\right)(4-3x)^{\frac{1}{3}}$	29
risch	$\frac{\left(315x^4-588x^3+32x^2+64x+256\right)(-4+3x)}{455(4-3x)^{\frac{2}{3}}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-3*x)^(4/3)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*2^(2/3)*x^3*hypergeom([-4/3,3],[4],3/4*x)
```

maxima [A] time = 0.61, size = 28, normalized size = 0.70

$$-\frac{1}{117}\,(-3\,x+4)^{\frac{13}{3}}+\frac{4}{45}\,(-3\,x+4)^{\frac{10}{3}}-\frac{16}{63}\,(-3\,x+4)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")
```

```
[Out] -1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x + 4)^(7/3)
```

mupad [B] time = 0.20, size = 23, normalized size = 0.58

$$-\frac{(4-3\,x)^{7/3}\,\left(1092\,x+35\,(3\,x-4)^2-416\right)}{4095}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4 - 3*x)^(4/3),x)
```

```
[Out] -((4 - 3*x)^(7/3)*(1092*x + 35*(3*x - 4)^2 - 416))/4095
```

sympy [B] time = 1.75, size = 180, normalized size = 4.50

$$\begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} & \text{for } \frac{3|x|}{4} > 1 \\ -\frac{9x^4\sqrt[3]{4-3x}}{13} + \frac{84x^3\sqrt[3]{4-3x}}{65} - \frac{32x^2\sqrt[3]{4-3x}}{455} - \frac{64x\sqrt[3]{4-3x}}{455} - \frac{256\sqrt[3]{4-3x}}{455} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)**(4/3)*x**2,x)
```

```
[Out] Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1/
3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x -
4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, 3*Abs(x)/
4 > 1), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x**
2*(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3)/4
55, True))
```

3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=48

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {266, 50, 63, 298, 203, 206}

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(1-2x)^{3/4}}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-2x}\,x}\,dx, x, \sqrt[3]{x}\right)\\ &= 4\left(1-2\sqrt[3]{x}\right)^{3/4} - 6\,\text{Subst}\left(\int \frac{x^2}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right)\\ &= 4\left(1-2\sqrt[3]{x}\right)^{3/4} - 6\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right) + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right)\\ &= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

IntegrateAlgebraic [A] time = 7.82, size = 48, normalized size = 1.00

$$4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

fricas [A] time = 0.65, size = 52, normalized size = 1.08

$$4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

giac [A] time = 8.18, size = 53, normalized size = 1.10

$$4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left|\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log(abs((-2*x^(1/3) + 1)^(1/4) - 1))
```

maple [A] time = 0.31, size = 53, normalized size = 1.10

method	result
derivativedivides	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)$
default	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)$
meijerg	$-\frac{9\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(\frac{2\pi\sqrt{2}\,x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1,1\right],[2,2],2x^{\frac{1}{3}}\right)}{\Gamma\left(\frac{3}{4}\right)}-\frac{4\left(\frac{4}{3}-2\ln(2)-\frac{\pi}{2}+\frac{\ln(x)}{3}+i\pi\right)\pi\sqrt{2}}{3\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x^(1/3))^(3/4)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1-2*x^(1/3))^(3/4)+3*ln((1-2*x^(1/3))^(1/4)-1)-3*ln((1-2*x^(1/3))^(1/4)+
1)+6*arctan((1-2*x^(1/3))^(1/4))
```

maxima [A] time = 1.07, size = 52, normalized size = 1.08

$$4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

mupad [B] time = 0.87, size = 36, normalized size = 0.75

$$6\,\mathrm{atan}\left({\left(1-2\,x^{1/3}\right)}^{1/4}\right)-6\,\mathrm{atanh}\left({\left(1-2\,x^{1/3}\right)}^{1/4}\right)+4\,{\left(1-2\,x^{1/3}\right)}^{3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 2*x^(1/3))^(3/4)/x,x)
```

```
[Out] 6*atan((1 - 2*x^(1/3))^(1/4)) - 6*atanh((1 - 2*x^(1/3))^(1/4)) + 4*(1 - 2*x
^(1/3))^(3/4)
```

sympy [C]   time = 2.09, size = 51, normalized size = 1.06

$$-\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}\,e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x**(1/3))**(3/4)/x,x)
```

```
[Out] -3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), 
1/(2*x**(1/3))))/gamma(1/4)
```

3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (3 - 2*Sqrt
[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(3-2x)^{3/4}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{27}{8(3-2x)^{3/4}}-\frac{27}{8}\sqrt[4]{3-2x}+\frac{9}{8}(3-2x)^{5/4}-\frac{1}{8}(3-2x)^{9/4}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.52

$$-\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}\left(5x^{3/2}+10x+24\sqrt{x}+144\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65
```

IntegrateAlgebraic [A] time = 0.30, size = 80, normalized size = 1.16

$$-\frac{4}{13}\sqrt[4]{3-2\sqrt{x}}\,x^{3/2}-\frac{8}{13}\sqrt[4]{3-2\sqrt{x}}\,x-\frac{96}{65}\sqrt[4]{3-2\sqrt{x}}\,\sqrt{x}-\frac{576}{65}\sqrt[4]{3-2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-576*(3 - 2*Sqrt[x])^(1/4))/65 - (96*(3 - 2*Sqrt[x])^(1/4)*Sqrt[x])/65 - (
8*(3 - 2*Sqrt[x])^(1/4)*x)/13 - (4*(3 - 2*Sqrt[x])^(1/4)*x^(3/2))/13
```

fricas [A] time = 0.58, size = 25, normalized size = 0.36

$$-\frac{4}{65}\left((5x+24)\sqrt{x}+10x+144\right)\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
```

```
[Out] -4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

giac [A] time = 0.60, size = 63, normalized size = 0.91

$$-\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}-\frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="giac")
```

```
[Out] -1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2*
sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(
1/4)
```

maple [C] time = 0.29, size = 20, normalized size = 0.29

method	result	size
meijerg	$\frac{3^{\frac{1}{4}}x^{2}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},4\right],[5],\frac{2\sqrt{x}}{3}\right)}{6}$	20
derivativedivides	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46
default	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3-2*x^(1/2))^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/4)*x^2*hypergeom([3/4,4],[5],2/3*x^(1/2))
```

maxima [A] time = 0.47, size = 45, normalized size = 0.65

$$\frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}}-\frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

```
[Out] 1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt(
x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

mupad [B] time = 0.30, size = 45, normalized size = 0.65

$$\frac{27\left(3-2\sqrt{x}\right)^{5/4}}{10}-\frac{27\left(3-2\sqrt{x}\right)^{1/4}}{2}-\frac{\left(3-2\sqrt{x}\right)^{9/4}}{2}+\frac{\left(3-2\sqrt{x}\right)^{13/4}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3 - 2*x^(1/2))^(3/4),x)
```

```
[Out] (27*(3 - 2*x^(1/2))^(5/4))/10 - (27*(3 - 2*x^(1/2))^(1/4))/2 - (3 - 2*x^(1/
2))^(9/4)/2 + (3 - 2*x^(1/2))^(13/4)/26
```

sympy [B] time = 2.25, size = 3305, normalized size = 47.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x**(1/2))**(3/4),x)
```

```
[Out] Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-
37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(
17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
+ 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4)*ex
p(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 18954
0*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900
*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2*sqrt(x
) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x
**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4
)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sqrt(3)*x**(
21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2*sqrt(x) - 3)
**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/
2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**1
0 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(2*sqrt(x) - 3)**(1/
4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37
440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17
/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
47385*3**(1/4)*x**8) - 8960*3**(1/4)*x**12*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*
pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 18432*3**(1/4)*x**11*(2*sqrt(x) - 3)**(1/4
)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**
(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4
160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3
**(1/4)*x**8) + 965520*3**(1/4)*x**10*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(1/4)*x**(21/2
) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x
**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8)
+ 2548584*3**(1/4)*x**9*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1
/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 416
0*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**
(1/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(
1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3
**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*3**(1/
4)*x**8*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) -
```

```
280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11
 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 41
9904*sqrt(3)*x**8/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8), 2*Abs(sqrt(x))/3 > 1), (-1280*3
**(1/4)*x**(25/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 28080
0*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14
0400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 26304*3
**(1/4)*x**(23/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 28080
0*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14
0400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 200016*
3**(1/4)*x**(21/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 2808
00*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 1
40400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776
*sqrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2123820*3**(1/4)*x**(19/2)*(3
 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2)
 - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10
+ 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
 + 47385*3**(1/4)*x**8) + 1609632*3**(1/4)*x**(17/2)*(3 - 2*sqrt(x))**(1/4)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37440*3**(1/4)*x**(
21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/
4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x*
*8) + 8960*3**(1/4)*x**12*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2)
 - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x*
*11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) +
 18432*3**(1/4)*x**11*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 2
80800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11
+ 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 368
64*sqrt(3)*x**11/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 965520*3**(1/4)*x**10*(3 - 2*sq
rt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1895
40*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31590
0*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(
1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 41
60*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3*
*(1/4)*x**8) - 2548584*3**(1/4)*x**9*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4
)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*
3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1
/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/
4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**
(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 419904*3**(1/4)
*x**8*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x
**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4
)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*sqrt(3)*x**8
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8), True))
```

3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*A
rcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) + (5*ArcTan[1 + Sqrt
[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[
x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2
*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{(-1+2x)^{5/4}}{x^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} + \frac{5}{2}\text{Subst}\left(\int \frac{\sqrt[4]{-1+2x}}{x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1}{x(-1+2x)^{3/4}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right) + \frac{5}{4}\text{Subs}$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right) +$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\,\text{l}}{}$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.18

$$\frac{32}{9}\left(2\sqrt{x}-1\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},3;\frac{13}{4};1-2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] (32*(-1 + 2*Sqrt[x])^(9/4)*Hypergeometric2F1[9/4, 3, 13/4, 1 - 2*Sqrt[x]])/
9
```

IntegrateAlgebraic [A] time = 0.34, size = 132, normalized size = 0.68

$$-\frac{9\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}} + \frac{\sqrt[4]{2\sqrt{x}-1}}{x} - \frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2\sqrt{x}-1}}{\sqrt{2\sqrt{x}-1}-1}\right)}{2\sqrt{2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2\sqrt{x}-1}}{\sqrt{2\sqrt{x}-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] (-1 + 2*Sqrt[x])^(1/4)/x - (9*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*ArcT
an[(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))/(-1 + Sqrt[-1 + 2*Sqrt[x]])])/(2*Sqrt[2
]) + (5*ArcTanh[(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))/(1 + Sqrt[-1 + 2*Sqrt[x]])
])/(2*Sqrt[2])
```

fricas [A] time = 0.61, size = 202, normalized size = 1.05

$$-\frac{20\sqrt{2}\,x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1}-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}-1\right)+20\sqrt{2}\,x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt
(2*sqrt(x) - 1) + 1) - sqrt(2)*(2*sqrt(x) - 1)^(1/4) - 1) + 20*sqrt(2)*x*ar
ctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) -
 1) + 4) - sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 1) - 5*sqrt(2)*x*log(4*sqrt(2)*(
2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) - 1) + 4) + 5*sqrt(2)*x*log(-4*sqrt
(2)*(2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) - 1) + 4) + 4*(9*sqrt(x) - 2)*
(2*sqrt(x) - 1)^(1/4))/x
```

giac [A] time = 0.60, size = 142, normalized size = 0.74

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 1/4*(9*(
2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/x
```

maple [C] time = 0.32, size = 85, normalized size = 0.44

method	result
meijerg	$\frac{5\mathrm{signum}\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}\left(\frac{\Gamma\left(\frac{3}{4}\right)\sqrt{x}\ \mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,4],2\sqrt{x}\right)}{4}+\frac{\left(-2\ln(2)+\frac{\pi}{2}-\frac{3}{2}+\frac{\ln(x)}{2}+i\pi\right)\Gamma\left(\frac{3}{4}\right)}{2}-\frac{2\Gamma\left(\frac{3}{4}\right)}{5x}+\frac{2\Gamma\left(\frac{3}{4}\right)}{\sqrt{x}}\right)}{2\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(-1+2\sqrt{x}\right)\right)^{\frac{5}{4}}}$
derivativedivides	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\arctan\left(-1+\right.\right.}{8}$
default	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\arctan\left(-1+\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x^(1/2))^(5/4)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2/GAMMA(3/4)*signum(-1+2*x^(1/2))^(5/4)/(-signum(-1+2*x^(1/2)))^(5/4)*(1/
4*GAMMA(3/4)*x^(1/2)*hypergeom([1,1,7/4],[2,4],2*x^(1/2))+1/2*(-2*ln(2)+1/2
*Pi-3/2+1/2*ln(x)+I*Pi)*GAMMA(3/4)-2/5*GAMMA(3/4)/x+2*GAMMA(3/4)/x^(1/2))
```

maxima [A] time = 1.20, size = 157, normalized size = 0.81

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(2*sq
rt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqrt(x)
- 1)
```

mupad [B] time = 1.28, size = 77, normalized size = 0.40

$$-\frac{5\left(2\sqrt{x}-1\right)^{1/4}}{4x}-\frac{9\left(2\sqrt{x}-1\right)^{5/4}}{4x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{4}+\frac{5}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^(1/2) - 1)^(5/4)/x^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 - 1i/2))*(5/4 + 5i/4) - (9*
(2*x^(1/2) - 1)^(5/4))/(4*x) - (5*(2*x^(1/2) - 1)^(1/4))/(4*x) + 2^(1/2)*at
an(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 + 1i/2))*(5/4 - 5i/4)
```

sympy [C] time = 5.63, size = 44, normalized size = 0.23

$$-\frac{4\sqrt[4]{2}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

```
[Out] -4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*pi)/(2*sqrt
(x)))/(x**(3/8)*gamma(7/4))
```

3.299 $\int x^6 \sqrt[3]{1+x^7}\,dx$

Optimal. Leaf size=13

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(1 + x^7)^(1/3),x]
[Out] (3*(1 + x^7)^(4/3))/28
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(1 + x^7)^(1/3),x]
[Out] (3*(1 + x^7)^(4/3))/28
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(1 + x^7)^(1/3),x]
[Out] (3*(1 + x^7)^(4/3))/28
```

fricas [A] time = 0.65, size = 9, normalized size = 0.69

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

giac [A] time = 0.63, size = 9, normalized size = 0.69

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

maple [A] time = 0.28, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\frac{3\left(x^7+1\right)^{\frac{4}{3}}}{28}$	10
default	$\frac{3\left(x^7+1\right)^{\frac{4}{3}}}{28}$	10
risch	$\frac{3\left(x^7+1\right)^{\frac{4}{3}}}{28}$	10
trager	$\left(\frac{3}{28}+\frac{3x^7}{28}\right)\left(x^7+1\right)^{\frac{1}{3}}$	16
meijerg	$\frac{x^7 \operatorname{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],-x^7\right)}{7}$	17
gosper	$\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)\left(x^7+1\right)^{\frac{1}{3}}}{28}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^7+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/28*(x^7+1)^(4/3)
```

maxima [A] time = 0.48, size = 9, normalized size = 0.69

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

mupad [B] time = 0.29, size = 9, normalized size = 0.69

$$\frac{3\left(x^7+1\right)^{4/3}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^7 + 1)^(1/3),x)
```

```
[Out] (3*(x^7 + 1)^(4/3))/28
```

sympy [B] time = 0.54, size = 26, normalized size = 2.00

$$\frac{3x^7\sqrt[3]{x^7+1}}{28}+\frac{3\sqrt[3]{x^7+1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**7+1)**(1/3),x)
```

```
[Out] 3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

3.300 $\int \frac{x^6}{(1+x^7)^{5/3}} dx$

Optimal. Leaf size=13

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^7)^(5/3),x]
[Out] -3/(14*(1 + x^7)^(2/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^6}{\left(1+x^7\right)^{5/3}} dx = -\frac{3}{14\left(1+x^7\right)^{2/3}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^7)^(5/3),x]
[Out] -3/(14*(1 + x^7)^(2/3))
```

IntegrateAlgebraic [A] time = 0.03, size = 13, normalized size = 1.00

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(1 + x^7)^(5/3),x]
[Out] -3/(14*(1 + x^7)^(2/3))
```

fricas [A] time = 0.58, size = 9, normalized size = 0.69

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")
[Out] -3/14/(x^7 + 1)^(2/3)
```

giac [A] time = 0.62, size = 9, normalized size = 0.69

$$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="giac")
[Out] -3/14/(x^7 + 1)^(2/3)
```

maple [A] time = 0.27, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$	10
default	$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$	10
trager	$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$	10
risch	$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$	10
meijerg	$\frac{x^{7}\,\mathrm{hypergeom}\left(\left[1,\frac{5}{3}\right],[2],-x^{7}\right)}{7}$	17
gosper	$-\frac{3(1+x)\left(x^{6}-x^{5}+x^{4}-x^{3}+x^{2}-x+1\right)}{14\left(x^{7}+1\right)^{\frac{5}{3}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^7+1)^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/14/(x^7+1)^(2/3)
```

maxima [A] time = 0.60, size = 9, normalized size = 0.69

$$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
[Out] -3/14/(x^7 + 1)^(2/3)
```

mupad [B] time = 0.32, size = 9, normalized size = 0.69

$$-\frac{3}{14\left(x^{7}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^7 + 1)^(5/3),x)
```

```
[Out] -3/(14*(x^7 + 1)^(2/3))
```

sympy [A] time = 0.54, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^{7}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**7+1)**(5/3),x)
```

```
[Out] -3/(14*(x**7 + 1)**(2/3))
```

3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$

Optimal. Leaf size=59

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 58, 618, 204, 31}

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] -ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) - Log[x]/18 +
 Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{7}\,\mathrm{Subst}\left(\int \frac{1}{x(-27+2x)^{2/3}}\,dx, x, x^7\right)$$
$$= -\frac{\log(x)}{18} + \frac{1}{42}\,\mathrm{Subst}\left(\int \frac{1}{3+x}\,dx, x, \sqrt[3]{-27+2x^7}\right) + \frac{1}{14}\,\mathrm{Subst}\left(\int \frac{1}{9-3x+x^2}\,dx, x$$
$$= -\frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right) - \frac{1}{7}\,\mathrm{Subst}\left(\int \frac{1}{-27-x^2}\,dx, x, -3+2\sqrt[3]{-27+$$
$$= -\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)$$

Mathematica [A] time = 0.03, size = 84, normalized size = 1.42

$$\frac{1}{63}\log\left(\sqrt[3]{2x^7-27}+3\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{2/3} - 3\sqrt[3]{2x^7-27}+9\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{2x^7-27}-3}{3\sqrt{3}}\right)}{21\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] ArcTan[(-3 + 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) + Log[3 + (-2
7 + 2*x^7)^(1/3)]/63 - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)]
/126
```

IntegrateAlgebraic [A] time = 0.08, size = 86, normalized size = 1.46

$$\frac{1}{63}\log\left(\sqrt[3]{2x^7-27}+3\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{2/3} - 3\sqrt[3]{2x^7-27}+9\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] -1/21*ArcTan[1/Sqrt[3] - (2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/Sqrt[3] + Log
[3 + (-27 + 2*x^7)^(1/3)]/63 - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7
)^(2/3)]/126
```

fricas [A] time = 0.57, size = 66, normalized size = 1.12

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{2}{9}\sqrt{3}\left(2x^7-27\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}} - 3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left(2x^7-27\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/63*sqrt(3)*arctan(2/9*sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)) - 1/126*l
og((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(
1/3) + 3)
```

giac [A] time = 0.62, size = 65, normalized size = 1.10

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}} - 3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left|\left(2x^7-27\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log(abs((2*x^7 - 27)^(1/
3) + 3))
```

maple [C] time = 9.79, size = 74, normalized size = 1.25

method	result
meijerg	$\frac{\left(-\mathrm{signum}\left(-1+\frac{2x^7}{27}\right)\right)^{\frac{2}{3}}\left(\frac{4\Gamma\left(\frac{2}{3}\right)x^7\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],\frac{2x^7}{27}\right)}{81}+\left(\frac{\pi\sqrt{3}}{6}-\frac{9\ln(3)}{2}+7\ln(x)+\ln(2)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{63\,\mathrm{signum}\left(-1+\frac{2x^7}{27}\right)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$
trager	$-\frac{\ln\left(-\frac{3421730148318\,\mathrm{RootOf}\left(81\_Z^2+9\_Z+1\right)^2x^7-279878182758\,\mathrm{RootOf}\left(81\_Z^2+9\_Z+1\right)x^7-7423011100x^7+1660604640093\left(2x^7-27\right)^{\frac{2}{3}}\mathrm{RootOf}\left(81\_Z^2+9\_\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*x^7-27)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/63/signum(-1+2/27*x^7)^(2/3)*(-signum(-1+2/27*x^7))^(2/3)*(4/81*GAMMA(2/3
)*x^7*hypergeom([1,1,5/3],[2,2],2/27*x^7)+(1/6*Pi*3^(1/2)-9/2*ln(3)+7*ln(x)
+ln(2)+I*Pi)*GAMMA(2/3))/GAMMA(2/3)
```

maxima [A] time = 1.30, size = 64, normalized size = 1.08

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right)-\frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2x^7-27\right)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3) +
 3)
```

mupad [B] time = 0.46, size = 76, normalized size = 1.29

$$\frac{\ln\left(\frac{\left(2x^7-27\right)^{1/3}}{49}+\frac{3}{49}\right)}{63}-\ln\left(\frac{27}{14}-\frac{9\left(2x^7-27\right)^{1/3}}{7}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(\frac{1}{126}+\frac{\sqrt{3}\,1\mathrm{i}}{126}\right)+\ln\left(\frac{9\left(2x^7-27\right)^{1/3}}{7}-\frac{27}{14}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x^7 - 27)^(2/3)),x)
```

```
[Out] log((2*x^7 - 27)^(1/3)/49 + 3/49)/63 - log((3^(1/2)*27i)/14 - (9*(2*x^7 - 2
7)^(1/3))/7 + 27/14)*((3^(1/2)*1i)/126 + 1/126) + log((3^(1/2)*27i)/14 + (9
*(2*x^7 - 27)^(1/3))/7 - 27/14)*((3^(1/2)*1i)/126 - 1/126)
```

sympy [C] time = 1.02, size = 42, normalized size = 0.71

$$-\frac{\sqrt[3]{2}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x**7-27)**(2/3),x)
```

```
[Out] -2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*pi)/(2*x**7
))/(14*x**(14/3)*gamma(5/3))
```

3.302 $\int \frac{(1+x^7)^{2/3}}{x^8} \, dx$

Optimal. Leaf size=70

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3]])/(7*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^7\right)^{2/3}}{x^8} dx &= \frac{1}{7}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2} dx, x, x^7\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7} + \frac{2}{21}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}} dx, x, x^7\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7} - \frac{\log(x)}{3} - \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1+x^7}\right) + \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2} dx, x\right.\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7} - \frac{\log(x)}{3} + \frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right) - \frac{2}{7}\operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2\sqrt[3]{1+x^7}\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7} + \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.37

$$\frac{3}{35}\left(x^7+1\right)^{5/3} {}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^7+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] (3*(1 + x^7)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^7])/35
```

IntegrateAlgebraic [A] time = 0.09, size = 90, normalized size = 1.29

$$-\frac{\left(x^7+1\right)^{2/3}}{7x^7}+\frac{2}{21}\log\left(\sqrt[3]{x^7+1}-1\right)-\frac{1}{21}\log\left(\left(x^7+1\right)^{2/3}+\sqrt[3]{x^7+1}+1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{7\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -1/7*(1 + x^7)^(2/3)/x^7 + (2*ArcTan[1/Sqrt[3] + (2*(1 + x^7)^(1/3))/Sqrt[3
]])/(7*Sqrt[3]) + (2*Log[-1 + (1 + x^7)^(1/3)])/21 - Log[1 + (1 + x^7)^(1/3
) + (1 + x^7)^(2/3)]/21
```

fricas [A] time = 0.65, size = 79, normalized size = 1.13

$$\frac{2\sqrt{3}\,x^7\arctan\left(\frac{2}{3}\sqrt{3}\left(x^7+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^7\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+2\,x^7\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)-3\,(}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(3)*x^7*arctan(2/3*sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) - x^7
*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2*x^7*log((x^7 + 1)^(1/3) - 1
) - 3*(x^7 + 1)^(2/3))/x^7
```

giac [A] time = 0.64, size = 67, normalized size = 0.96

$$\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}-\frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\log\left(\left|\left(x^7+1\right)^{\frac{1}{3}}-1\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="giac")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log(abs((x
^7 + 1)^(1/3) - 1))
```

maple [C] time = 9.89, size = 76, normalized size = 1.09

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{\pi\sqrt{3}\,x^7\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,3],-x^7\right)}{9\Gamma\left(\frac{2}{3}\right)}-\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}-1+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)x^7}\right)}{21\pi}$
risch	$-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^7\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^7\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$
trager	$-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}+\frac{2\ln\left(-\frac{3914010\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^7-2502441\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^7-71266x^7+6095754\left(x^7+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)-391401}{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7+1)^(2/3)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/Pi*3^(1/2)*GAMMA(2/3)*(1/9*Pi*3^(1/2)/GAMMA(2/3)*x^7*hypergeom([1,1,4
/3],[2,3],-x^7)-2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)-1+7*ln(x))*Pi*3^(1/2)/GAMMA(
2/3)+Pi*3^(1/2)/GAMMA(2/3)/x^7)
```

maxima [A] time = 1.35, size = 66, normalized size = 0.94

$$\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}-\frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log((x^7 +
 1)^(1/3) - 1)
```

mupad [B] time = 0.37, size = 92, normalized size = 1.31

$$\frac{2\ln\left(\frac{4\left(x^7+1\right)^{1/3}}{49}-\frac{4}{49}\right)}{21}+\ln\left(\frac{4\left(x^7+1\right)^{1/3}}{49}-9\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)-\ln\left(\frac{4\left(x^7+1\right)^{1/3}}{49}-9\left(\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7 + 1)^(2/3)/x^8,x)
```

```
[Out] (2*log((4*(x^7 + 1)^(1/3))/49 - 4/49))/21 + log((4*(x^7 + 1)^(1/3))/49 - 9*
((3^(1/2)*1i)/21 - 1/21)^2)*((3^(1/2)*1i)/21 - 1/21) - log((4*(x^7 + 1)^(1/
3))/49 - 9*((3^(1/2)*1i)/21 + 1/21)^2)*((3^(1/2)*1i)/21 + 1/21) - (x^7 + 1)
^(2/3)/(7*x^7)
```

sympy [C] time = 1.42, size = 34, normalized size = 0.49

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^{7}}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**7+1)**(2/3)/x**8,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x**(7/3)*ga
mma(4/3))
```

3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2} \, dx$

Optimal. Leaf size=68

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx &= -\frac{\sqrt[4]{3+4x^4}}{x}+4\int \frac{x^2}{\left(3+4x^4\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{3+4x^4}}{x}+4\,\mathrm{Subst}\left(\int \frac{x^2}{1-4x^4}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right)\\
&= -\frac{\sqrt[4]{3+4x^4}}{x}+\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right)-\mathrm{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right)\\
&= -\frac{\sqrt[4]{3+4x^4}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica **[C]** time = 0.01, size = 27, normalized size = 0.40

$$-\frac{\sqrt[4]{3}\ {}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{4x^4}{3}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (-4*x^4)/3])/x)
```

IntegrateAlgebraic **[A]** time = 0.16, size = 68, normalized size = 1.00

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

fricas **[B]** time = 5.60, size = 146, normalized size = 2.15

$$-\frac{2\sqrt{2}\,x\arctan\left(\frac{4}{3}\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+\frac{2}{3}\sqrt{2}\left(4x^4+3\right)^{\frac{3}{4}}x\right)-\sqrt{2}\,x\log\left(-256\,x^8-192\,x^4-4\sqrt{2}\left(16\,x^5+3\,x\right)(4}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(2)*x*arctan(4/3*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 2/3*sqrt(2)*(4
*x^4 + 3)^(3/4)*x) - sqrt(2)*x*log(-256*x^8 - 192*x^4 - 4*sqrt(2)*(16*x^5 +
 3*x)*(4*x^4 + 3)^(3/4) - 8*sqrt(2)*(16*x^7 + 9*x^3)*(4*x^4 + 3)^(1/4) - 16
*(8*x^6 + 3*x^2)*sqrt(4*x^4 + 3) - 9) + 8*(4*x^4 + 3)^(1/4))/x
```

giac **[A]** time = 0.67, size = 83, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

maple [C] time = 3.74, size = 20, normalized size = 0.29

method	result
meijerg	$-\frac{3^{\frac{1}{4}} \text{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{4x^4}{3}\right)}{x}$
risch	$-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{4\,3^{\frac{1}{4}} x^3 \text{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{4x^4}{3}\right)}{9}$
trager	$-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}-\frac{\text{RootOf}\left(\_Z^2-2\right) \ln\left(4\,\text{RootOf}\left(\_Z^2-2\right)\sqrt{4x^4+3}\;x^2+8\,\text{RootOf}\left(\_Z^2-2\right)x^4-4\left(4x^4+3\right)^{\frac{3}{4}}x-8x^3\left(4x^4+3\right)^{\frac{1}{4}}+3\,\text{RootOf}\left(\_Z^2-\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+3)^(1/4)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3^(1/4)/x*hypergeom([-1/4,-1/4],[3/4],-4/3*x^4)
```

maxima [A] time = 1.40, size = 83, normalized size = 1.22

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{4}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

mupad [B] time = 0.42, size = 18, normalized size = 0.26

$$-\frac{3^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{4\,x^4}{3}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4 + 3)^(1/4)/x^2,x)
```

```
[Out] -(3^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(4*x^4)/3))/x
```

sympy [C] time = 1.03, size = 41, normalized size = 0.60

$$\frac{\sqrt[4]{3}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+3)**(1/4)/x**2,x)
```

```
[Out] 3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar(I*pi)/3)/
(4*x*gamma(3/4))
```

3.304 $\int x^2 \left(3+4x^4\right)^{5/4} dx$

Optimal. Leaf size=93

$$-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}\,x^3$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 331, 298, 203, 206}

$$\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}\,x^3-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(128*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int x^2\left(3+4x^4\right)^{5/4}dx=\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{15}{8}\int x^2\sqrt[4]{3+4x^4}\,dx$$
$$=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\int\frac{x^2}{\left(3+4x^4\right)^{3/4}}dx$$
$$=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\text{Subst}\left(\int\frac{x^2}{1-4x^4}dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)$$
$$=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{128}\text{Subst}\left(\int\frac{1}{1-2x^2}dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)-\frac{45}{128}\text{S}$$
$$=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.28

$$\sqrt[4]{3}\,x^3\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] 3^(1/4)*x^3*Hypergeometric2F1[-5/4, 3/4, 7/4, (-4*x^4)/3]
```

IntegrateAlgebraic [A] time = 0.18, size = 83, normalized size = 0.89

$$-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{1}{32}\sqrt[4]{4x^4+3}\left(16x^7+27x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] ((3 + 4*x^4)^(1/4)*(27*x^3 + 16*x^7))/32 - (45*ArcTan[(Sqrt[2]*x)/(3 + 4*x^
4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128
*Sqrt[2])
```

fricas [A] time = 0.70, size = 105, normalized size = 1.13

$$\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{45}{512}\sqrt{2}\log\left(8x^4+4\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}\,x^2+2\sqrt{2}\left(4x^4+3\right)^{\frac{3}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45/512*sqrt(2)*log
(8*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt(2
)*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)
```

giac [A] time = 0.65, size = 110, normalized size = 1.18

$$\frac{1}{32}x^8\left(\frac{9\left(4x^4+3\right)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}-\frac{20\left(4x^4+3\right)^{\frac{1}{4}}}{x}\right)+\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^4+\right.}{x}}{\sqrt{2}+\frac{\left(4x^4+\right.}{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="giac")
```

```
[Out] 1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45/
256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(-(
sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

maple [C] time = 3.41, size = 19, normalized size = 0.20

method	result
meijerg	$3^{\frac{1}{4}} x^{3} \mathrm{hypergeom}\left(\left[-\frac{5}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4x^{4}}{3}\right)$
risch	$\frac{x^{3}\left(16x^{4}+27\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{5\,3^{\frac{1}{4}} x^{3} \mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4x^{4}}{3}\right)}{32}$
trager	$\frac{x^{3}\left(16x^{4}+27\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{45 \mathrm{RootOf}\left(\_Z^{2}+2\right) \ln\left(4 \mathrm{RootOf}\left(\_Z^{2}+2\right) \sqrt{4x^{4}+3}\, x^{2}-8 \mathrm{RootOf}\left(\_Z^{2}+2\right) x^{4}+4\left(4x^{4}+3\right)^{\frac{3}{4}} x-8x^{3}\left(4x^{4}+3\right)^{\frac{1}{4}}-3\right.}{512}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^4+3)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 3^(1/4)*x^3*hypergeom([-5/4,3/4],[7/4],-4/3*x^4)
```

maxima [A] time = 1.24, size = 130, normalized size = 1.40

$$\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^{4}+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}}\right)+\frac{9\left(\frac{20\left(4x^{4}+3\right)^{\frac{1}{4}}}{x}-\frac{9\left(4x^{4}+3\right)^{\frac{5}{4}}}{x^{5}}\right)}{32\left(\frac{8\left(4x^{4}+3\right)}{x^{4}}-\frac{\left(4x^{4}+3\right)^{2}}{x^{8}}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="maxima")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log
(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) + 9/32*(
20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x
^4 + 3)^2/x^8 - 16)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(4x^{4}+3\right)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^4 + 3)^(5/4),x)
```

```
[Out] int(x^2*(4*x^4 + 3)^(5/4), x)
```

sympy [C] time = 1.57, size = 41, normalized size = 0.44

$$\frac{3\sqrt[4]{3}\,x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{4x^{4}e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**4+3)**(5/4),x)
```

```
[Out] 3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*pi
)/3)/(4*gamma(7/4))
```

3.305 $\int x^6 \sqrt[4]{3+4x^4}\, dx$

Optimal. Leaf size=93

$$\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}+\frac{1}{8}\sqrt[4]{4x^4+3}\,x^7+\frac{3}{128}\sqrt[4]{4x^4+3}\,x^3$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{1}{8}\sqrt[4]{4x^4+3}\,x^7+\frac{3}{128}\sqrt[4]{4x^4+3}\,x^3+\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(512*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int x^6\sqrt[4]{3+4x^4}\,dx &= \frac{1}{8}x^7\sqrt[4]{3+4x^4}+\frac{3}{8}\int\frac{x^6}{\left(3+4x^4\right)^{3/4}}\,dx\\ &= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{128}\int\frac{x^2}{\left(3+4x^4\right)^{3/4}}\,dx\\ &= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{128}\mathrm{Subst}\left(\int\frac{x^2}{1-4x^4}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\ &= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{512}\mathrm{Subst}\left(\int\frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)+\frac{27}{512}\mathrm{Subs}\\ &= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}+\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.46

$$\frac{1}{32}x^3\left(\left(4x^4+3\right)^{5/4}-3\sqrt[4]{3}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (x^3*((3 + 4*x^4)^(5/4) - 3*3^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-4*x
^4)/3]))/32
```

IntegrateAlgebraic [A] time = 0.18, size = 83, normalized size = 0.89

$$\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}+\frac{1}{128}\sqrt[4]{4x^4+3}\left(16x^7+3x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] ((3 + 4*x^4)^(1/4)*(3*x^3 + 16*x^7))/128 + (27*ArcTan[(Sqrt[2]*x)/(3 + 4*x^
4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512
*Sqrt[2])
```

fricas [A] time = 0.51, size = 105, normalized size = 1.13

$$-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{27}{2048}\sqrt{2}\log\left(8x^4-4\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}\,x^2-2\sqrt{2}\left(4x^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(8*x^4 - 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 - 2*sqr
t(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4)
```

giac [A] time = 0.67, size = 109, normalized size = 1.17

$$\frac{1}{128}\,x^8\left(\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}+\frac{12\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}\right)-\frac{27}{1024}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)+\frac{27}{2048}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)/x) - 27/1
024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*log(-
(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

maple [C] time = 3.32, size = 20, normalized size = 0.22

method	result
meijerg	$\frac{3^{\frac{1}{4}} x^7 \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{7}{4}\right],\left[\frac{11}{4}\right],-\frac{4x^4}{3}\right)}{7}$
risch	$\frac{x^3\left(16x^4+3\right)\left(4x^4+3\right)^{\frac{1}{4}}}{128}-\frac{3\,3^{\frac{1}{4}} x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{4x^4}{3}\right)}{128}$
trager	$\frac{x^3\left(16x^4+3\right)\left(4x^4+3\right)^{\frac{1}{4}}}{128}+\frac{27\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)\sqrt{4x^4+3}\;x^2-8\,\mathrm{RootOf}\left(\_Z^2-2\right)x^4+4\left(4x^4+3\right)^{\frac{3}{4}}x+8x^3\left(4x^4+3\right)^{\frac{1}{4}}-3\right.}{2048}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(4*x^4+3)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*3^(1/4)*x^7*hypergeom([-1/4,7/4],[11/4],-4/3*x^4)
```

maxima [A] time = 1.20, size = 129, normalized size = 1.39

$$-\frac{27}{1024}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)+\frac{27}{2048}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{9\left(\frac{12\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}+\frac{\left(4\,x^4+3\right)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8\left(4\,x^4+3\right)}{x^4}-\frac{\left(4\,x^4+3\right)^2}{x^8}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - 9/1
28*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4
*x^4 + 3)^2/x^8 - 16)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(4\,x^4+3\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(4*x^4 + 3)^(1/4),x)
```

```
[Out] int(x^6*(4*x^4 + 3)^(1/4), x)
```

sympy [C] time = 1.32, size = 39, normalized size = 0.42

$$\frac{\sqrt[4]{3}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(4*x**4+3)**(1/4),x)
```

```
[Out] 3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_polar(I*pi)
/3)/(4*gamma(11/4))
```

3.306 $\int \sqrt[3]{x(1-x^2)}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}\sqrt[3]{x\left(1-x^2\right)}\,x-\frac{1}{4}\log\left(\sqrt[3]{x\left(1-x^2\right)}+x\right)+\frac{\tan^{-1}\left(\frac{2x-\sqrt[3]{x\left(1-x^2\right)}}{\sqrt{3}\sqrt[3]{x\left(1-x^2\right)}}\right)}{2\sqrt{3}}+\frac{\log(x)}{12}$$

Rubi [B] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 2.15, number of steps used = 12, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {1979, 2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}\sqrt[3]{x-x^3}\,x+\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{12\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{6\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\tan^{-1}\left(1 -\ldots\right.}{2\sqrt{3}\left(x-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3))/2 - (x^(2/3)*(1 - x^2)^(2/3)*ArcTan[(1 - (2*x^(2/3))/(1
 - x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x - x^3)^(2/3)) + (x^(2/3)*(1 - x^2)^(
2/3)*Log[1 + x^(4/3)/(1 - x^2)^(2/3) - x^(2/3)/(1 - x^2)^(1/3)])/(12*(x - x
^3)^(2/3)) - (x^(2/3)*(1 - x^2)^(2/3)*Log[1 + x^(2/3)/(1 - x^2)^(1/3)])/(6*
(x - x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \int \sqrt[3]{x-x^3}\,dx$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{1}{3}\int \frac{x}{\left(x-x^3\right)^{2/3}}\,dx$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1-x^2\right)^{2/3}}\,dx}{3\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1-x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{2\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+}{1-x\cdot}\right.}{6\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x^{2/}}{\sqrt[3]{1-}}\right.}{12\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12\left(x-x^3\right)^{2/3}} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} -$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}} + \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12\left(x-x^3\right)^{2/3}} -$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.43

$$\frac{3x\sqrt[3]{x-x^3}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (3*x*(x - x^3)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, x^2])/(4*(1 - x^2)^(
1/3))
```

IntegrateAlgebraic [A] time = 0.15, size = 105, normalized size = 1.13

$$\frac{1}{2}\sqrt[3]{x-x^3}\,x-\frac{1}{6}\log\left(\sqrt[3]{x-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x-x^3}-x}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(-\sqrt[3]{x-x^3}\,x+\left(x-x^3\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3))/2 + ArcTan[(Sqrt[3]*x)/(-x + 2*(x - x^3)^(1/3))]/(2*Sqr
t[3]) - Log[x + (x - x^3)^(1/3)]/6 + Log[x^2 - x*(x - x^3)^(1/3) + (x - x^3
)^(2/3)]/12
```

fricas [A] time = 0.74, size = 99, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}\left(-x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(16754327161\,x^2-2707204793\right)+10524305234\sqrt{3}\left(-x^3\right.}{81835897185\,x^2-1102302937}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan((44032959556*sqrt(3)*(-x^3 + x)^(1/3)*x - sqrt(3)*(1675
4327161*x^2 - 2707204793) + 10524305234*sqrt(3)*(-x^3 + x)^(2/3))/(81835897
185*x^2 - 1102302937)) + 1/2*(-x^3 + x)^(1/3)*x - 1/12*log(3*(-x^3 + x)^(1/
3)*x + 3*(-x^3 + x)^(2/3) + 1)
```

giac [A] time = 0.68, size = 69, normalized size = 0.74

$$\frac{1}{2}x^2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{12}\log\left(\left(\frac{1}{x^2}-1\right)^{\frac{2}{3}}-\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{6}\log\left(\left|\left(\frac{1}{x^2}-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*x^2*(1/x^2 - 1)^(1/3) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 - 1)^(
1/3) - 1)) + 1/12*log((1/x^2 - 1)^(2/3) - (1/x^2 - 1)^(1/3) + 1) - 1/6*log(
abs((1/x^2 - 1)^(1/3) + 1))
```

maple [C] time = 3.23, size = 15, normalized size = 0.16

method	result
meijerg	$\frac{3x^{\frac{4}{3}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^2\right)}{4}$
trager	$\frac{x\left(-x^3+x\right)^{\frac{1}{3}}}{2}-\frac{\ln\left(4959\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^2-6768\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+x\right)^{\frac{2}{3}}-22833\left(-x^3+x\right)^{\frac{1}{3}}\mathrm{RootOf}\left(9\_Z^2-3\_Z+\right.\right.}{}$
risch	$\frac{x\left(-x\left(x^2-1\right)\right)^{\frac{1}{3}}}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+6\_Z+36\right)\ln\left(-\frac{47\,\mathrm{RootOf}\left(\_Z^2+6\_Z+36\right)^2x^4+3207\,\mathrm{RootOf}\left(\_Z^2+6\_Z+36\right)x^4+2925\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+6\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(-x^2+1))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

mupad [B] time = 0.37, size = 29, normalized size = 0.31

$$\frac{3\,x\,\left(x-x^3\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ x^2\right)}{4\,\left(1-x^2\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x^2 - 1))^(1/3),x)
```

```
[Out] (3*x*(x - x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1 - x^2)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x**2+1))**(1/3),x)
```

```
[Out] Integral((x*(1 - x**2))**(1/3), x)
```

3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$

Optimal. Leaf size=126

$$\frac{3}{40}\sqrt{\left(\sqrt[3]{x}+1\right)x}\,x^{2/3}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{\left(\sqrt[3]{x}+1\right)x}}\right)+\frac{3}{5}\sqrt{\left(\sqrt[3]{x}+1\right)x}\,x-\frac{7}{80}\sqrt{\left(\sqrt[3]{x}+1\right)x}\,\sqrt[3]{x}+\frac{7}{64}\sqrt{\left(\sqrt[3]{x}+1\right)x}\,-$$

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1979, 2004, 2024, 2010, 2029, 206}

$$\frac{3}{5}\sqrt{x^{4/3}+x}\,x+\frac{3}{40}\sqrt{x^{4/3}+x}\,x^{2/3}-\frac{7}{80}\sqrt{x^{4/3}+x}\,\sqrt[3]{x}+\frac{7}{64}\sqrt{x^{4/3}+x}-\frac{21\sqrt{x^{4/3}+x}}{128\sqrt[3]{x}}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (7*Sqrt[x + x^(4/3)])/64 - (21*Sqrt[x + x^(4/3)])/(128*x^(1/3)) - (7*x^(1/3
)*Sqrt[x + x^(4/3)])/80 + (3*x^(2/3)*Sqrt[x + x^(4/3)])/40 + (3*x*Sqrt[x +
x^(4/3)])/5 + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx &= \int \sqrt{x+x^{4/3}}\,dx \\
&= \frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{1}{10}\int \frac{x}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}-\frac{7}{80}\int \frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\,dx \\
&= -\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{7}{96}\int \frac{\sqrt[3]{x}}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}-\frac{7}{128}\int \frac{1}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{7}{256} \\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{21}{128} \\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{21}{128}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.66

$$\frac{\sqrt{x^{4/3}+x}\left(\sqrt{\sqrt[3]{x}+1}\,\sqrt[6]{x}\left(384x^{4/3}-56x^{2/3}+48x+70\sqrt[3]{x}-105\right)+105\sinh^{-1}\left(\sqrt[6]{x}\right)\right)}{640\sqrt{\sqrt[3]{x}+1}\,\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (Sqrt[x + x^(4/3)]*(Sqrt[1 + x^(1/3)]*x^(1/6)*(-105 + 70*x^(1/3) - 56*x^(2/
3) + 48*x + 384*x^(4/3)) + 105*ArcSinh[x^(1/6)]))/(640*Sqrt[1 + x^(1/3)]*Sq
rt[x])
```

IntegrateAlgebraic [A] time = 0.50, size = 69, normalized size = 0.55

$$\frac{\sqrt{x^{4/3}+x}\left(384x^{4/3}-56x^{2/3}+48x+70\sqrt[3]{x}-105\right)}{640\sqrt[3]{x}}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 384*x^(4/3)))/(
640*x^(1/3)) + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

fricas [A] time = 74.92, size = 87, normalized size = 0.69

$$\frac{35\,x\log\left(\frac{32\,x^{2}+48\,x^{\frac{5}{3}}+2\left(16\,x^{\frac{4}{3}}+16\,x+3\,x^{\frac{2}{3}}\right)\sqrt{x^{\frac{4}{3}}+x}+18\,x^{\frac{4}{3}}+x}{x}\right)+2\left(384\,x^{2}+3\,(16\,x-35)x^{\frac{2}{3}}-56\,x^{\frac{4}{3}}+70\,x\right)\sqrt{x^{\frac{4}{3}}+x}}{1280\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(35*x*log((32*x^2 + 48*x^(5/3) + 2*(16*x^(4/3) + 16*x + 3*x^(2/3))*s
qrt(x^(4/3) + x) + 18*x^(4/3) + x)/x) + 2*(384*x^2 + 3*(16*x - 35)*x^(2/3)
- 56*x^(4/3) + 70*x)*sqrt(x^(4/3) + x))/x
```

giac [A] time = 0.93, size = 66, normalized size = 0.52

$$\frac{1}{1280}\left(2\left(2\left(4\left(6\,x^{\frac{1}{3}}\left(8\,x^{\frac{1}{3}}+1\right)-7\right)x^{\frac{1}{3}}+35\right)x^{\frac{1}{3}}-105\right)\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-105\,\log\left(\left|2\,\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-2\,x^{\frac{1}{3}}-1\right|\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(2*(4*(6*x^(1/3)*(8*x^(1/3) + 1) - 7)*x^(1/3) + 35)*x^(1/3) - 105
)*sqrt(x^(2/3) + x^(1/3)) - 105*log(abs(2*sqrt(x^(2/3) + x^(1/3)) - 2*x^(1/
3) - 1)))*sgn(x)
```

maple [A] time = 0.29, size = 51, normalized size = 0.40

method	result
meijerg	$-\frac{3\left(\frac{\sqrt{\pi}\,x^{\frac{1}{6}}\left(-1152x^{\frac{4}{3}}-144x+168x^{\frac{2}{3}}-210x^{\frac{1}{3}}+315\right)\sqrt{1+x^{\frac{1}{3}}}}{2880}-\frac{7\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{6}}\right)}{64}\right)}{2\sqrt{\pi}}$
derivativedivides	$\frac{\sqrt{\left(1+x^{\frac{1}{3}}\right)x}\,\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}+105\ln\left(\frac{1}{2}+x^{\frac{1}{3}}+\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\right)-\right.}{1280x^{\frac{1}{3}}\sqrt{\left(1+x^{\frac{1}{3}}\right)x^{\frac{1}{3}}}}$
default	$\frac{\sqrt{\left(1+x^{\frac{1}{3}}\right)x}\,\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}+105\ln\left(\frac{1}{2}+x^{\frac{1}{3}}+\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\right)-\right.}{1280x^{\frac{1}{3}}\sqrt{\left(1+x^{\frac{1}{3}}\right)x^{\frac{1}{3}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x^(1/3))*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/Pi^(1/2)*(1/2880*Pi^(1/2)*x^(1/6)*(-1152*x^(4/3)-144*x+168*x^(2/3)-210
*x^(1/3)+315)*(1+x^(1/3))^(1/2)-7/64*Pi^(1/2)*arcsinh(x^(1/6)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x\left(x^{\frac{1}{3}}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*(x^(1/3) + 1)), x)
```

mupad [B] time = 0.29, size = 27, normalized size = 0.21

$$\frac{2\,x\,\sqrt{x+x^{4/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{9}{2};\;\frac{11}{2};\;-x^{1/3}\right)}{3\,\sqrt{x^{1/3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^(1/3) + 1))^(1/2),x)
```

```
[Out] (2*x*(x + x^(4/3))^(1/2)*hypergeom([-1/2, 9/2], 11/2, -x^(1/3)))/(3*(x^(1/3
) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x\left(\sqrt[3]{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x**(1/3))*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x**(1/3) + 1)), x)
```

3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1469, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] -ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(-1+x^4\right)\sqrt{1+2x^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+2x^2}}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \frac{1+2x^4}{\sqrt{1+2x^8}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.85

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{6x^8+3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(1 + 2*x^4)/Sqrt[3 + 6*x^8]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.42, size = 48, normalized size = 1.41

$$\frac{\tanh^{-1}\left(-\frac{\sqrt{2x^8+1}}{\sqrt{3}}-\sqrt{\frac{2}{3}}x^4+\sqrt{\frac{2}{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] ArcTanh[Sqrt[2/3] - Sqrt[2/3]*x^4 - Sqrt[1 + 2*x^8]/Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.61, size = 49, normalized size = 1.44

$$\frac{1}{12}\sqrt{3}\log\left(\frac{2x^4-\sqrt{3}\left(2x^4+1\right)-\sqrt{2x^8+1}\left(\sqrt{3}-3\right)+1}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((2*x^4 - sqrt(3)*(2*x^4 + 1) - sqrt(2*x^8 + 1)*(sqrt(3) -
3) + 1)/(x^4 - 1))
```

giac [B] time = 0.66, size = 70, normalized size = 2.06

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2x^8+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(2
*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1)))
```

maple [C] time = 0.30, size = 58, normalized size = 1.71

method	result	size
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)x^4+\mathrm{RootOf}\left(\_Z^2-3\right)+3\sqrt{2x^8+1}}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{12}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^2-3)*ln(-(2*RootOf(_Z^2-3)*x^4+RootOf(_Z^2-3)+3*(2*x^8+1)^(
1/2))/(-1+x)/(1+x)/(x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{2x^8+1}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)
```

mupad [B] time = 0.52, size = 35, normalized size = 1.03

$$-\frac{\sqrt{3}\left(\ln\left(x^4+\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{x^8+\frac{1}{2}}}{2}+\frac{1}{2}\right)-\ln\left(x^4-1\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x^4 - 1)*(2*x^8 + 1)^(1/2)),x)
```

```
[Out] -(3^(1/2)*(log(x^4 + (2^(1/2)*3^(1/2)*(x^8 + 1/2)^(1/2))/2 + 1/2) - log(x^4
 - 1)))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{(x-1)(x+1)\left(x^2+1\right)\sqrt{2x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)
```

```
[Out] Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)
```

3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

Optimal. Leaf size=58

$$-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)+\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}$$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 640, 612, 619, 215}

$$\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] -((1 + 2*x^5)*Sqrt[1 + x^5 + x^10])/40 + (1 + x^5 + x^10)^(3/2)/15 - (3*Arc
Sinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{5}\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx,x,x^5\right)$$
$$= \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{1}{10}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx,x,x^5\right)$$
$$= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{3}{80}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{1}{80}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,\right.$$
$$= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{3}{80}\sinh^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.81

$$\frac{1}{240}\left(2\sqrt{x^{10}+x^5+1}\left(8x^{10}+2x^5+5\right)-9\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] (2*Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10) - 9*ArcSinh[(1 + 2*x^5)/Sqrt[3
]])/240
```

IntegrateAlgebraic [A] time = 0.13, size = 55, normalized size = 0.95

$$\frac{1}{120}\sqrt{x^{10}+x^5+1}\left(8x^{10}+2x^5+5\right)+\frac{3}{80}\log\left(-2x^5+2\sqrt{x^{10}+x^5+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] (Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10))/120 + (3*Log[-1 - 2*x^5 + 2*Sqr
t[1 + x^5 + x^10]])/80
```

fricas [A] time = 0.72, size = 47, normalized size = 0.81

$$\frac{1}{120}\left(8x^{10}+2x^5+5\right)\sqrt{x^{10}+x^5+1}+\frac{3}{80}\log\left(-2x^5+2\sqrt{x^{10}+x^5+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(8*x^10 + 2*x^5 + 5)*sqrt(x^10 + x^5 + 1) + 3/80*log(-2*x^5 + 2*sqrt(
x^10 + x^5 + 1) - 1)
```

giac [A] time = 0.64, size = 49, normalized size = 0.84

$$\frac{1}{120}\sqrt{x^{10}+x^5+1}\left(2\left(4x^5+1\right)x^5+5\right)+\frac{3}{80}\log\left(-2x^5+2\sqrt{x^{10}+x^5+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*sq
rt(x^10 + x^5 + 1) - 1)
```

maple [A] time = 0.36, size = 47, normalized size = 0.81

method	result	size
trager	$\left(\frac{1}{15}x^{10}+\frac{1}{60}x^{5}+\frac{1}{24}\right)\sqrt{x^{10}+x^{5}+1}-\frac{3\ln\left(2x^{5}+2\sqrt{x^{10}+x^{5}+1}+1\right)}{80}$	47
risch	$\frac{\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}}{120}+\frac{3\ln\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)}{80}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^10+x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/15*x^10+1/60*x^5+1/24)*(x^10+x^5+1)^(1/2)-3/80*ln(2*x^5+2*(x^10+x^5+1)^(
1/2)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{10}+x^{5}+1}\,x^{9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

mupad [B] time = 0.29, size = 43, normalized size = 0.74

$$\frac{\sqrt{x^{10}+x^{5}+1}\,\left(8\,x^{10}+2\,x^{5}+5\right)}{120}-\frac{3\,\ln\left(\sqrt{x^{10}+x^{5}+1}+x^{5}+\frac{1}{2}\right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^5 + x^10 + 1)^(1/2),x)
```

```
[Out] ((x^5 + x^10 + 1)^(1/2)*(2*x^5 + 8*x^10 + 5))/120 - (3*log((x^5 + x^10 + 1)
^(1/2) + x^5 + 1/2))/80
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{9}\sqrt{\left(x^{2}+x+1\right)\left(x^{8}-x^{7}+x^{5}-x^{4}+x^{3}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

```
[Out] Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x + 1
)), x)
```

3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

Optimal. Leaf size=71

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 744, 806, 724, 206}

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] -Sqrt[4 + 2*x^2 + x^4]/(16*x^4) + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x^2) + ArcT
anh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}-\frac{1}{16}\operatorname{Subst}\left(\int \frac{3+x}{x^2\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}-\frac{1}{64}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{32}\operatorname{Subst}\left(\int \frac{1}{16-x^2}\,dx,x,\frac{2\left(4+x^2\right)}{\sqrt{4+2x^2+x^4}}\right)\\ &= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{128}\tanh^{-1}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.77

$$\frac{1}{128}\left(\frac{2\sqrt{x^4+2x^2+4}\left(3x^2-4\right)}{x^4}+\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
[Out] ((2*(-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 + ArcTanh[(4 + x^2)/(2*Sqrt[4 +
 2*x^2 + x^4])])/128
```

IntegrateAlgebraic [A] time = 0.21, size = 60, normalized size = 0.85

$$\frac{\left(3x^2-4\right)\sqrt{x^4+2x^2+4}}{64x^4}-\frac{1}{64}\tanh^{-1}\left(\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4+2x^2+4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
[Out] ((-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/(64*x^4) - ArcTanh[x^2/2 - Sqrt[4 + 2*
x^2 + x^4]/2]/64
```

fricas [A] time = 0.76, size = 81, normalized size = 1.14

$$\frac{x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)-x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}-2\right)+6\,x^4+2\,\sqrt{x^4+2\,x^2+4}\left(3\,x^2-4\right)}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/128*(x^4*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2) - x^4*log(-x^2 + sqrt(x^4
+ 2*x^2 + 4) - 2) + 6*x^4 + 2*sqrt(x^4 + 2*x^2 + 4)*(3*x^2 - 4))/x^4
```

giac [A] time = 0.67, size = 112, normalized size = 1.58

$$\frac{\left(x^2-\sqrt{x^4+2\,x^2+4}\right)^3+36\,x^2-36\,\sqrt{x^4+2\,x^2+4}+64}{32\left(\left(x^2-\sqrt{x^4+2\,x^2+4}\right)^2-4\right)^2}-\frac{1}{128}\log\left(x^2-\sqrt{x^4+2\,x^2+4}+2\right)+\frac{1}{128}\log\left(-x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*((x^2 - sqrt(x^4 + 2*x^2 + 4))^3 + 36*x^2 - 36*sqrt(x^4 + 2*x^2 + 4) +
 64)/((x^2 - sqrt(x^4 + 2*x^2 + 4))^2 - 4)^2 - 1/128*log(x^2 - sqrt(x^4 + 2
*x^2 + 4) + 2) + 1/128*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2)
```

maple [A] time = 0.22, size = 52, normalized size = 0.73

method	result	size
trager	$\frac{(3x^2-4)\sqrt{x^4+2x^2+4}}{64x^4}+\frac{\ln\left(\frac{x^2+2\sqrt{x^4+2x^2+4}+4}{x^2}\right)}{128}$	52
default	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
risch	$\frac{3x^6+2x^4+4x^2-16}{64x^4\sqrt{x^4+2x^2+4}}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
elliptic	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4+2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(3*x^2-4)/x^4*(x^4+2*x^2+4)^(1/2)+1/128*ln((x^2+2*(x^4+2*x^2+4)^(1/2)+
4)/x^2)
```

maxima [A] time = 1.21, size = 52, normalized size = 0.73

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}+\frac{4\sqrt{3}}{3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 + 1/128*arc
sinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1251, 838, 206}

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 838

```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Dist[(4*f*(a - d))/(b*d - a*e), Subst[Int[1/(4*(a
 - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[e
*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt{1+3x+x^2}}\,dx, x, x^2\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2\left(1+x^2\right)}{\sqrt{1+3x^2+x^4}}\right)\\
&= \tanh^{-1}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)
\end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.71

$$\frac{1}{2}\left(\tanh^{-1}\left(\frac{2x^2+3}{2\sqrt{x^4+3x^2+1}}\right)+\tanh^{-1}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] (ArcTanh[(3 + 2*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])] + ArcTanh[(2 + 3*x^2)/(2*Sq
rt[1 + 3*x^2 + x^4])])/2
```

IntegrateAlgebraic [B] time = 0.23, size = 52, normalized size = 2.48

$$-\frac{1}{2}\log\left(-2x^2+2\sqrt{x^4+3x^2+1}-3\right)-\tanh^{-1}\left(x^2-\sqrt{x^4+3x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] -ArcTanh[x^2 - Sqrt[1 + 3*x^2 + x^4]] - Log[-3 - 2*x^2 + 2*Sqrt[1 + 3*x^2 +
 x^4]]/2
```

fricas [B] time = 0.61, size = 59, normalized size = 2.81

$$-\frac{1}{2}\log\left(4x^4+11x^2-\sqrt{x^4+3x^2+1}\left(4x^2+5\right)+5\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5) + 1/2*log(
-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

giac [B] time = 0.63, size = 69, normalized size = 3.29

$$-\frac{1}{2}\log\left(2x^2-2\sqrt{x^4+3x^2+1}+3\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(2*x^2 - 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(-x^2 + sqrt(x^4 + 3
*x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 3*x^2 + 1) - 1)
```

maple [A] time = 0.20, size = 27, normalized size = 1.29

method	result	size
trager	$-\ln\left(\frac{-x^2+\sqrt{x^4+3x^2+1}-1}{x}\right)$	27
default	$\frac{\ln\left(\frac{3}{2}+x^2+\sqrt{x^4+3x^2+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)}{2}$	46
elliptic	$\frac{\ln\left(\frac{3}{2}+x^2+\sqrt{x^4+3x^2+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((-x^2+(x^4+3*x^2+1)^(1/2)-1)/x)
```

maxima [B] time = 0.48, size = 52, normalized size = 2.48

$$\frac{1}{2}\log\left(2x^2+2\sqrt{x^4+3x^2+1}+3\right)+\frac{1}{2}\log\left(\frac{2\sqrt{x^4+3x^2+1}}{x^2}+\frac{2}{x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4 + 3*x^2 +
 1)/x^2 + 2/x^2 + 3)
```

mupad [B] time = 0.81, size = 49, normalized size = 2.33

$$\frac{\ln\left(\frac{1}{x^2}\right)}{2}+\frac{\ln\left(\sqrt{x^4+3\,x^2+1}+x^2+\frac{3}{2}\right)}{2}+\frac{\ln\left(\frac{2\,\sqrt{x^4+3\,x^2+1}}{3}+x^2+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x*(3*x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] log(1/x^2)/2 + log((3*x^2 + x^4 + 1)^(1/2) + x^2 + 3/2)/2 + log((2*(3*x^2 +
 x^4 + 1)^(1/2))/3 + x^2 + 2/3)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

3.312 $\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx$

Optimal. Leaf size=17

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(-3x^2+x^4\right)^{8/5}$$

Mathematica [C] time = 0.06, size = 75, normalized size = 4.41

$$\frac{5\left(x^2\left(x^2-3\right)\right)^{3/5}\left(16x^4\,{}_2F_1\left(-\frac{3}{5},\frac{13}{5};\frac{18}{5};\frac{x^2}{3}\right)-39x^2\,{}_2F_1\left(-\frac{3}{5},\frac{8}{5};\frac{13}{5};\frac{x^2}{3}\right)\right)}{208\left(1-\frac{x^2}{3}\right)^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(x^2*(-3 + x^2))^(3/5)*(-39*x^2*Hypergeometric2F1[-3/5, 8/5, 13/5, x^2/3
] + 16*x^4*Hypergeometric2F1[-3/5, 13/5, 18/5, x^2/3]))/(208*(1 - x^2/3)^(3
/5))
```

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

fricas [A] time = 0.60, size = 13, normalized size = 0.76

$$\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

giac [A] time = 0.63, size = 13, normalized size = 0.76

$$\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="giac")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

maple [A] time = 0.31, size = 14, normalized size = 0.82

method	result	size
default	$\frac{5\left(x^4-3x^2\right)^{\frac{8}{5}}}{16}$	14
gosper	$\frac{5\left(x^4-3x^2\right)^{\frac{3}{5}}x^2\left(x^2-3\right)}{16}$	22
trager	$\frac{5\left(x^4-3x^2\right)^{\frac{3}{5}}x^2\left(x^2-3\right)}{16}$	22
risch	$\frac{5x^2\left(x^2\left(x^2-3\right)\right)^{\frac{3}{5}}\left(x^2-3\right)}{16}$	22
meijerg	$\frac{5\,3^{\frac{3}{5}}\operatorname{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{26}{5}}\operatorname{hypergeom}\left(\left[-\frac{3}{5},\frac{13}{5}\right],\left[\frac{18}{5}\right],\frac{x^2}{3}\right)}{13\left(-\operatorname{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}-\frac{15\,3^{\frac{3}{5}}\operatorname{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{16}{5}}\operatorname{hypergeom}\left(\left[-\frac{3}{5},\frac{8}{5}\right],\left[\frac{13}{5}\right],\frac{x^2}{3}\right)}{16\left(-\operatorname{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*(x^4-3*x^2)^(8/5)
```

maxima [A] time = 0.60, size = 13, normalized size = 0.76

$$\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

mupad [B] time = 0.28, size = 21, normalized size = 1.24

$$\frac{5\,x^2\,\left(x^2-3\right)\,{\left(x^4-3\,x^2\right)}^{3/5}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^3)*(x^4 - 3*x^2)^(3/5),x)
```

```
[Out] (5*x^2*(x^2 - 3)*(x^4 - 3*x^2)^(3/5))/16
```

sympy [B] time = 1.31, size = 36, normalized size = 2.12

$$\frac{5x^4\left(x^4-3x^2\right)^{\frac{3}{5}}}{16}-\frac{15x^2\left(x^4-3x^2\right)^{\frac{3}{5}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

```
[Out] 5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16
```

3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$

Optimal. Leaf size=46

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

Rubi [A] time = 0.19, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6742, 266, 43, 261}

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)^
(9/4))/243
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx = \int \left(-\frac{2x^5}{\left(-1+3x^3\right)^{3/4}}+\frac{3x^8}{\left(-1+3x^3\right)^{3/4}}-\frac{x^2}{\sqrt[12]{-1+3x^3}}\right)dx$$
$$= -\left(2\int \frac{x^5}{\left(-1+3x^3\right)^{3/4}}\,dx\right)+3\int \frac{x^8}{\left(-1+3x^3\right)^{3/4}}\,dx-\int \frac{x^2}{\sqrt[12]{-1+3x^3}}\,dx$$
$$= -\frac{4}{33}\left(-1+3x^3\right)^{11/12}-\frac{2}{3}\text{Subst}\left(\int \frac{x}{(-1+3x)^{3/4}}\,dx,x,x^3\right)+\text{Subst}\left(\int$$
$$= -\frac{4}{33}\left(-1+3x^3\right)^{11/12}-\frac{2}{3}\text{Subst}\left(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{1}{3}\sqrt[4]{-1+3x}\right)dx,\right.$$
$$= -\frac{4}{27}\sqrt[4]{-1+3x^3}-\frac{4}{33}\left(-1+3x^3\right)^{11/12}+\frac{4}{243}\left(-1+3x^3\right)^{9/4}$$

Mathematica [A] time = 0.08, size = 40, normalized size = 0.87

$$-\frac{4\sqrt[4]{3x^3-1}\left(-99x^6+66x^3+81\left(3x^3-1\right)^{2/3}+88\right)}{2673}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
[Out] (-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/2673
```

IntegrateAlgebraic [A] time = 0.04, size = 43, normalized size = 0.93

$$\frac{4}{243}\sqrt[4]{3x^3-1}\left(9x^6-6x^3-8\right)-\frac{4}{33}\left(3x^3-1\right)^{11/12}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(
3/4),x]
[Out] (-4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)^(1/4)*(-8 - 6*x^3 + 9*x^6))/
243
```

fricas [A] time = 0.62, size = 35, normalized size = 0.76

$$\frac{4}{243}\left(9\,x^6-6\,x^3-8\right)\left(3\,x^3-1\right)^{\frac{1}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
fricas")
[Out] 4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(11/12)
```

giac [A] time = 0.62, size = 34, normalized size = 0.74

$$\frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
giac")
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

maple [C] time = 0.35, size = 116, normalized size = 2.52

method	result
meijerg	$-\frac{\left(-\mathrm{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}} x^6 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},2\right],[3],3x^3\right)}{3\mathrm{signum}\left(3x^3-1\right)^{\frac{3}{4}}}+\frac{\left(-\mathrm{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}} x^9 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},3\right],[4],3x^3\right)}{3\mathrm{signum}\left(3x^3-1\right)^{\frac{3}{4}}}-\frac{\left(-\mathrm{signum}\left(3x^3-1\right)\right)^{\frac{1}{12}} \cdots}{3\mathrm{sig}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom([3/4,2],[
3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^9*hypergeom(
[3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)*(-signum(3*x^3-1))^(1/12)*x^3
*hypergeom([1/12,1],[2],3*x^3)
```

maxima [A] time = 0.57, size = 34, normalized size = 0.74

$$\frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
maxima")
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

mupad [B] time = 0.37, size = 34, normalized size = 0.74

$$-\left(3\,x^3-1\right)^{1/4}\,\left(\frac{8\,x^3}{81}-\frac{4\,x^6}{27}+\frac{4\,\left(3\,x^3-1\right)^{2/3}}{33}+\frac{32}{243}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*x^3 - 1)^(2/3) + 2*x^5 - 3*x^8)/(3*x^3 - 1)^(3/4),x)
```

```
[Out] -(3*x^3 - 1)^(1/4)*((8*x^3)/81 - (4*x^6)/27 + (4*(3*x^3 - 1)^(2/3))/33 + 32
/243)
```

sympy [C] time = 3.94, size = 221, normalized size = 4.80

$$-\frac{4\left(3x^3-1\right)^{\frac{11}{12}}}{33}-2\left(\begin{cases}\frac{4x^3\sqrt[4]{3x^3-1}}{45}+\frac{16\sqrt[4]{3x^3-1}}{135} & \text{for}\ 3\left|x^3\right|>1\\ -\frac{4x^3\sqrt[4]{1-3x^3}\,e^{-\frac{3i\pi}{4}}}{45}-\frac{16\sqrt[4]{1-3x^3}\,e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise}\end{cases}\right)+3\left(\begin{cases}\frac{4x^6\sqrt[4]{3x^3-1}}{81}+\frac{32x^3\sqrt[4]{3x^3-1}}{1215}+\frac{128\sqrt[4]{3x^3-1}}{3645} & \cdots\\ \frac{4x^6\sqrt[4]{1-3x^3}\,e^{\frac{i\pi}{4}}}{81}+\frac{32x^3\sqrt[4]{1-3x^3}\,e^{\frac{i\pi}{4}}}{1215}+\frac{128\sqrt[4]{1-\cdots}}{364\cdots} & \cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

```
[Out] -4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 +
16*(3*x**3 - 1)**(1/4)/135, 3*Abs(x**3) > 1), (-4*x**3*(1 - 3*x**3)**(1/4)*
exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) + 3*P
iecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1215
+ 128*(3*x**3 - 1)**(1/4)/3645, 3*Abs(x**3) > 1), (4*x**6*(1 - 3*x**3)**(1/
4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 128*(1 -
 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))
```

3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=78

$$-\frac{\log(x^3-1)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}x^2}{(x^3+2)^{2/3}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))]/3^(5/6)) + Log[1 - (3
^(1/3)*x)/(2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3^(2/3)*x^2)/(2 + x^3)^(2
/3) + (3^(1/3)*x)/(2 + x^3)^(1/3)]/(6*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{-2-\sqrt[3]{3}\,x}{1+\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{2+ \cdots}}\right)$$
$$= \frac{\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{3}+ \cdots}{1+\sqrt[3]{3}\,x \cdots}\right.}{\cdots}$$
$$= \frac{\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}x^2}{\left(2+x^3\right)^{2/3}}+\frac{\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{6\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{\sqrt[3]{3}}$$
$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}x^2}{\left(2+x^3\right)^{2/3}}+\frac{\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}\right)}{6\sqrt[3]{3}}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+2}}\right)-\log\left(\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+2}}+\frac{3^{2/3}x^2}{\left(x^3+2\right)^{2/3}}+1\right)\right)-6\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[1
 - (3^(1/3)*x)/(2 + x^3)^(1/3)] - Log[1 + (3^(2/3)*x^2)/(2 + x^3)^(2/3) + (3
^(1/3)*x)/(2 + x^3)^(1/3)]))/(6*3^(5/6))
```

IntegrateAlgebraic [A] time = 0.24, size = 113, normalized size = 1.45

$$\frac{\log\left(3^{2/3}\sqrt[3]{x^3+2}-3x\right)}{3\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+2}+\sqrt[3]{3}\,x}\right)}{3^{5/6}}-\frac{\log\left(3^{2/3}\sqrt[3]{x^3+2}\,x+\sqrt[3]{3}\left(x^3+2\right)^{2/3}+3x^2\right)}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(2 + x^3)^(1/3))]/3^(5/6)) + Log[-3*x +
 3^(2/3)*(2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[3*x^2 + 3^(2/3)*x*(2 + x^3)^(1/
3) + 3^(1/3)*(2 + x^3)^(2/3)]/(6*3^(1/3))
```

fricas [B] time = 3.67, size = 232, normalized size = 2.97

$$\frac{1}{27} \cdot 3^{\frac{2}{3}} \log\left(\frac{9 \cdot 3^{\frac{1}{3}}\left(x^3+2\right)^{\frac{1}{3}} x^2 - 2 \cdot 3^{\frac{2}{3}}\left(x^3-1\right) - 9\left(x^3+2\right)^{\frac{2}{3}} x}{x^3-1}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(\frac{3 \cdot 3^{\frac{2}{3}}\left(7 x^4+2 x\right)\left(x^3+2\right)^{\frac{2}{3}} + 3^{\frac{1}{3}}(3}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*(
x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*(x
^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 + 2)
^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*
x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*x^3 +
 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^6 + 24
*x^3 - 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^3-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

maple [C] time = 2.74, size = 902, normalized size = 11.56

method	result
trager	$\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-9\right)^2+9\_Z\,\mathrm{RootOf}\left(\_Z^3-9\right)+81\_Z^2\right)\ln\left(-\frac{27\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-9\right)^2+9\_Z\,\mathrm{R}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln(-(27*RootOf(RootOf(
_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+12*RootOf(Roo
tOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+15*(x^3+2)^
(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2
*x+45*(x^3+2)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*Ro
otOf(_Z^3-9)*x^2+7*(x^3+2)^(1/3)*RootOf(_Z^3-9)^2*x^2+36*RootOf(RootOf(_Z^3
-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+16*RootOf(_Z^3-9)*x^3+21*x*(x^3+2)^(
2/3)+18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+8*RootOf(_Z^3-
9))/(-1+x)/(x^2+x+1))-1/9*ln((-27*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z
^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+15*(x^3+2)^(2/3)*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*x+45*(x^3+2)^(1/3)*RootOf(R
ootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2-2*(x^3+2)^(
1/3)*RootOf(_Z^3-9)^2*x^2+9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*
_Z^2)*x^3-3*RootOf(_Z^3-9)*x^3-6*x*(x^3+2)^(2/3)+18*RootOf(RootOf(_Z^3-9)^2
+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-6*RootOf(_Z^3-9))/(-1+x)/(x^2+x+1))*RootOf(_Z
^3-9)-ln((-27*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf
```

```
(_Z^3-9)^2*x^3+9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootO
f(_Z^3-9)^3*x^3+15*(x^3+2)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*RootOf(_Z^3-9)^2*x+45*(x^3+2)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z
*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2-2*(x^3+2)^(1/3)*RootOf(_Z^3-9)^
2*x^2+9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-3*RootOf(_
Z^3-9)*x^3-6*x*(x^3+2)^(2/3)+18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)
+81*_Z^2)-6*RootOf(_Z^3-9))/(-1+x)/(x^2+x+1))*RootOf(RootOf(_Z^3-9)^2+9*_Z*
RootOf(_Z^3-9)+81*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3-1\right)\,\left(x^3+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)),x)
[Out] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\sqrt[3]{x^3+2}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
[Out] Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {377, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(
2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 +
x^4)^(1/4)]/(4*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x}}\right.$$

$$= -\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{}}{\sqrt[4]{2}}\right.}{2\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x}}\right.}{4\sqrt{}}$$

Mathematica [A] time = 0.06, size = 120, normalized size = 0.85

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)-\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)+\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*x)/(2 +
 x^4)^(1/4)] - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(2 + x^4)^(1/4)] + L
og[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 + x^4)^(1/4)])/(4*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.21, size = 85, normalized size = 0.60

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+2}}{\sqrt{x^4+2}-x^2}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+2}}{\sqrt{x^4+2}+x^2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x*(2 + x^4)^(1/4))/(-x^2 + Sqrt[2 + x^4])]/(2*Sqrt[2]) + Ar
cTanh[(Sqrt[2]*x*(2 + x^4)^(1/4))/(x^2 + Sqrt[2 + x^4])]/(2*Sqrt[2])
```

fricas [B] time = 6.77, size = 388, normalized size = 2.75

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+2\right)^{\frac{5}{4}}-\left(2x^5-\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+2\right)^{\frac{5}{4}}+4x\right)\sqrt{\frac{x^4+\sqrt{2}\left(x^4+2\right)^{\frac{1}{4}}x}{}}}{2\left(x^5+2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/
4) - (2*x^5 - sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*
sqrt((x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^
4 + 2)^(3/4)*x + 1)/(x^4 + 1)))/(x^5 + 2*x)) + 1/4*sqrt(2)*arctan(1/2*(sqrt
(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + (2*x^5 + sqrt(2)*(x^4 +
 2)^(3/4)*x^2 + sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*sqrt((x^4 - sqrt(2)*(x^4 + 2
)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1
)))/(x^5 + 2*x)) + 1/16*sqrt(2)*log(4*(x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 +
2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - 1/16*sqrt
(2)*log(4*(x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2
)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 1.34, size = 150, normalized size = 1.06

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{\sqrt{x^4+2}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2-\left(x^4+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3+\left(x^4+2\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^4+1\right)}{x^4+1}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{(x^4+2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*RootOf(_Z^4+1)*ln(((x^4+2)^(1/2)*RootOf(_Z^4+1)^3*x^2-(x^4+2)^(1/4)*Ro
otOf(_Z^4+1)^2*x^3+(x^4+2)^(3/4)*x-RootOf(_Z^4+1))/(x^4+1))+1/4*RootOf(_Z^4
+1)^3*ln(((x^4+2)^(1/4)*RootOf(_Z^4+1)^2*x^3-(x^4+2)^(1/2)*RootOf(_Z^4+1)*x
^2+(x^4+2)^(3/4)*x+RootOf(_Z^4+1)^3)/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^4+1\right)\left(x^4+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)),x)
```

```
[Out] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)\sqrt[4]{x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

```
[Out] Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 239}

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5}{3}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx\\ &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.44

$$\frac{1}{18}\left(6\left(x^3+2\right)^{2/3}x+10\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right)-10\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-5\log\left(\frac{x}{\sqrt[3]{x^3+2}}+\frac{x^2}{\left(x^3+2\right)^{2/3}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3
]] + 10*Log[1 - x/(2 + x^3)^(1/3)] - 5*Log[1 + x^2/(2 + x^3)^(2/3) + x/(2 +
 x^3)^(1/3)])/18
```

IntegrateAlgebraic [A] time = 0.15, size = 94, normalized size = 1.49

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{9}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+2}+x}\right)}{3\sqrt{3}}-\frac{5}{18}\log\left(\sqrt[3]{x^3+2}\,x+\left(x^3+2\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3))])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/9 - (5*Log[x^2 + x*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)])/18
```

fricas [A] time = 0.86, size = 86, normalized size = 1.37

$$\frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x+\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+2\right)^{\frac{1}{3}}}{3x}\right)+\frac{5}{9}\log\left(-\frac{x-\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{18}\log\left(\frac{x^2+\left(x^3+2\right)^{\frac{1}{3}}x+\left(x^3+\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
+ 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3 +
 2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3-1}{\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

maple [C] time = 1.63, size = 29, normalized size = 0.46

method	result
risch	$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}-\frac{5\,2^{\frac{2}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^3}{2}\right)}{6}$
meijerg	$-\frac{2^{\frac{2}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^3}{2}\right)}{2}+\frac{2^{\frac{2}{3}}x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{4}{3}\right],\left[\frac{7}{3}\right],-\frac{x^3}{2}\right)}{8}$
trager	$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}+\frac{5\ln\left(-4\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^3+6\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(x^3+2\right)^{\frac{1}{3}}x^2-8\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x^3+3x\left(x^3+2\right)^{\frac{2}{3}}-4x^3\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

[Out] 1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)

maxima [A] time = 1.21, size = 94, normalized size = 1.49

$$\frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\left(x^3+2\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+2}{x^3}-1\right)}-\frac{5}{18}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{\left(x^3+2\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")

[Out] 5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^(2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(2/3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3-1}{\left(x^3+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x^3 + 2)^(1/3),x)

[Out] int((x^3 - 1)/(x^3 + 2)^(1/3), x)

sympy [C] time = 2.00, size = 71, normalized size = 1.13

$$\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)}-\frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(x**3+2)**(1/3),x)

[Out] 2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)/(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi)/2)/(6*gamma(4/3))

3.317 $\int \frac{\left(1+x^4\right)^{3/4}}{\left(2+x^4\right)^2} \, dx$

Optimal. Leaf size=74

$$\frac{\left(x^4+1\right)^{3/4} x}{8\left(x^4+2\right)}+\frac{3 \tan^{-1}\left(\frac{x}{\sqrt[4]{2} \sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}+\frac{3 \tanh^{-1}\left(\frac{x}{\sqrt[4]{2} \sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {378, 377, 212, 206, 203}

$$\frac{\left(x^4+1\right)^{3/4} x}{8\left(x^4+2\right)}+\frac{3 \tan^{-1}\left(\frac{x}{\sqrt[4]{2} \sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}+\frac{3 \tanh^{-1}\left(\frac{x}{\sqrt[4]{2} \sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+x^4\right)^{3/4}}{\left(2+x^4\right)^2}\,dx = \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)} + \frac{3}{8}\int \frac{1}{\sqrt[4]{1+x^4}\left(2+x^4\right)}\,dx$$

$$= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)$$

$$= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}}$$

$$= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)} + \frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\,2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\,2^{3/4}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.55

$$\frac{x\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{x^4}{x^4+2}\right)}{2\,2^{3/4}\sqrt[4]{x^4+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*Hypergeometric2F1[-3/4, 1/4, 5/4, -(x^4/(2 + x^4))])/(2*2^(3/4)*(2 + x^4
)^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 74, normalized size = 1.00

$$\frac{\left(x^4+1\right)^{3/4}x}{8\left(x^4+2\right)} + \frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\,2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

fricas [B] time = 11.89, size = 242, normalized size = 3.27

$$-\frac{12\cdot 8^{\frac{3}{4}}\left(x^4+2\right)\arctan\left(-\frac{8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{1}{4}}\left(3x^4+2\right)\right)}{2\left(x^4+2\right)}\right)-3\cdot 8^{\frac{3}{4}}\left(x^4+2\right)\log\left(\frac{8\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3 \ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/512*(12*8^(3/4)*(x^4 + 2)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8
^(1/4)*(x^4 + 1)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*
x^4 + 2)))/(x^4 + 2)) - 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*
x^3 + 8*8^(1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4
)*x)/(x^4 + 2)) + 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 -
```

```
8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(
x^4 + 2)) - 64*(x^4 + 1)^(3/4)*x)/(x^4 + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

maple [C] time = 2.83, size = 228, normalized size = 3.08

method	result
risch	$\frac{\left(x^4+1\right)^{\frac{3}{4}} x}{8x^4+16}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(\frac{2\sqrt{x^4+1}\ \mathrm{RootOf}\left(\_Z^4-2\right)^2\ \mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)x^2+2\left(x^4+1\right)^{\frac{1}{4}}\ \mathrm{RootOf}\left(\_Z^4-2\right)^2 x^3-}{x^4+2}\right.}{64}$
trager	$\frac{\left(x^4+1\right)^{\frac{3}{4}} x}{8x^4+16}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{2\sqrt{x^4+1}\ \mathrm{RootOf}\left(\_Z^4-2\right)^2\ \mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)x^2-2\left(x^4+1\right)^{\frac{1}{4}}\ \mathrm{RootOf}\left(\_Z^4-2\right)^2 x^3}{x^4+2}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(3/4)/(x^4+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(x^4+1)^(3/4)/(x^4+2)+3/64*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*(x^4+1
)^(1/2)*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+2*(x^4+1)^(1/4)*
RootOf(_Z^4-2)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^4-4*(x^4+1)^(3/4)*x-
2*RootOf(_Z^2+RootOf(_Z^4-2)^2))/(x^4+2))+3/64*RootOf(_Z^4-2)*ln(-(2*(x^4+1
)^(1/2)*RootOf(_Z^4-2)^3*x^2+2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3+3*RootOf(
_Z^4-2)*x^4+4*(x^4+1)^(3/4)*x+2*RootOf(_Z^4-2))/(x^4+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)^{3/4}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(3/4)/(x^4 + 2)^2,x)
```

```
[Out] int((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

```
[Out] Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$

Optimal. Leaf size=48

$$\frac{97x}{891\sqrt[5]{x^5+3}}+\frac{5x}{297\left(x^5+3\right)^{6/5}}-\frac{5\left(x^5-2\right)x}{33\left(x^5+3\right)^{11/5}}$$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {378, 191}

$$\frac{x\left(2-x^5\right)^2}{33\left(x^5+3\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(x^5+3\right)^{6/5}}+\frac{100x}{891\sqrt[5]{x^5+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (10*x*(2 - x^5))/(297*(3 + x^5)^(6/
5)) + (100*x)/(891*(3 + x^5)^(1/5))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-2+x^5\right)^2}{\left(3+x^5\right)^{16/5}}\,dx &= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}-\frac{20}{33}\int \frac{-2+x^5}{\left(3+x^5\right)^{11/5}}\,dx\\ &= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(3+x^5\right)^{6/5}}+\frac{100}{297}\int \frac{1}{\left(3+x^5\right)^{6/5}}\,dx\\ &= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(3+x^5\right)^{6/5}}+\frac{100x}{891\sqrt[5]{3+x^5}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.54

$$\frac{x\left(97x^{10}+462x^5+1188\right)}{891\left(x^5+3\right)^{11/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
[Out] (x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

IntegrateAlgebraic [A] time = 1.12, size = 27, normalized size = 0.56

$$\frac{97x^{11} + 462x^6 + 1188x}{891\left(x^5+3\right)^{11/5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
[Out] (1188*x + 462*x^6 + 97*x^11)/(891*(3 + x^5)^(11/5))
```

fricas [A] time = 0.66, size = 40, normalized size = 0.83

$$\frac{\left(97\,x^{11} + 462\,x^{6} + 1188\,x\right)\left(x^{5} + 3\right)^{\frac{4}{5}}}{891\,\left(x^{15} + 9\,x^{10} + 27\,x^{5} + 27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")
[Out] 1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10 + 27*x^5
+ 27)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5} - 2\right)^{2}}{\left(x^{5} + 3\right)^{\frac{16}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="giac")
[Out] integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)
```

maple [A] time = 0.32, size = 23, normalized size = 0.48

method	result	size
gosper	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
trager	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
risch	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
meijerg	$\frac{4\,3^{\frac{4}{5}}x\left(\frac{25}{9}x^{10}+\frac{55}{3}x^{5}+33\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}+\frac{3^{\frac{4}{5}}x^{11}}{891\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}-\frac{2\,3^{\frac{4}{5}}x^{6}\left(11+\frac{5x^{5}}{3}\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-2)^2/(x^5+3)^(16/5),x,method=_RETURNVERBOSE)
[Out] 1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

maxima [B] time = 0.57, size = 73, normalized size = 1.52

$$-\frac{4\,x^{11}\left(\frac{11\,\left(x^5+3\right)}{x^5}-\frac{33\,\left(x^5+3\right)^2}{x^{10}}-3\right)}{891\,\left(x^5+3\right)^{\frac{11}{5}}}-\frac{2\,x^{11}\left(\frac{11\,\left(x^5+3\right)}{x^5}-6\right)}{297\,\left(x^5+3\right)^{\frac{11}{5}}}+\frac{x^{11}}{33\,\left(x^5+3\right)^{\frac{11}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")
```

```
[Out] -4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3)^(11/5) -
 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1/33*x^11/(x^5 + 3)^(
11/5)
```

mupad [B] time = 0.26, size = 23, normalized size = 0.48

$$\frac{97\,x^{11}+462\,x^6+1188\,x}{891\,\left(x^5+3\right)^{11/5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 2)^2/(x^5 + 3)^(16/5),x)
```

```
[Out] (1188*x + 462*x^6 + 97*x^11)/(891*(x^5 + 3)^(11/5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

```
[Out] Timed out
```

3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {432, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6))
+ Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3
^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)
^(1/3)]/(6*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 432

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(3x+3x^2+x^3\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx &= \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{-2}{1+\sqrt[3]{3}}\right.\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \frac{1+}{\sqrt[3]{2+(1}}\right.\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}}+\frac{\text{Subst}\left(\right.}{}\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}\right.}{6\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 120, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)-\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(
1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)])
)/(6*3^(5/6))
```

IntegrateAlgebraic [B] time = 0.38, size = 189, normalized size = 2.10

$$\frac{\log\left(-\sqrt[3]{x^3+3x^2+3x+3}+\sqrt[3]{3}x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+\left(x^3+3x^2+3x+3\right)^{2/3}+\left(\sqrt[3]{3}x+\sqrt[3]{3}\right)\sqrt[3]{x^3+3x^2+3}\right.}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (
3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + Log[3^(1/3) + 3^(1/3)*x - (3 + 3*x
 + 3*x^2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^
2 + (3^(1/3) + 3^(1/3)*x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2
+ x^3)^(2/3)]/(6*3^(1/3))
```

fricas [B] time = 15.93, size = 458, normalized size = 5.09

$$-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7x^4+28x^3+42x^2+30x+9\right)\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31x^6+186x^5+465x^4+666\right.}{x^6+6x^5+15x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^3+3x^2+3x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

maple [C] time = 13.60, size = 2501, normalized size = 27.79

method	result	size
trager	Expression too large to display	2501

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln(-(5486785533*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x+1732189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*R
ootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x-4041776571*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3+15322002984*RootOf(_Z^3-9)^
2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2
/3)*x+6809512275*(x^3+3*x^2+3*x+3)^(2/3)-10200674203*RootOf(_Z^3-9)*x^3-306
02022609*RootOf(_Z^3-9)*x^2-32311070361*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf
(_Z^3-9)+81*_Z^2)*x^3-12802499577*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-53038926819*RootOf(RootOf(_Z^3-9)^2+9*_Z*Roo
tOf(_Z^3-9)+81*_Z^2)+1732189959*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+5486785533*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z
^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*x^2-96933211083*RootOf(RootOf(_Z^3-9
)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-96933211083*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)*x-16744502937*RootOf(_Z^3-9)-30602022609*RootOf(
_Z^3-9)*x+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)+6809512275*(x
^3+3*x^2+3*x+3)^(2/3)*x+1828928511*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+577396653*RootOf(RootOf(_Z^3-9)^2+9*_Z*
RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+15322002984*RootOf(_Z^3-9)^2*R
ootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)
+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2+5674993806*RootOf(
_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x-20428536825*RootOf(_Z^3-9)*RootOf(RootO
f(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)-2042853682
5*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+
3*x^2+3*x+3)^(1/3)*x^2-40857073650*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9
*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)*x)/x/(x^2+3*x+3))-1/9*l
n(-(10102924098*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*Root
Of(_Z^3-9)^2*x+1732189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z
^2)*RootOf(_Z^3-9)^3*x-4041776571*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2)*RootOf(_Z^3-9)^3-15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)*x-8512490709*(x
^3+3*x^2+3*x+3)^(2/3)+10778070856*RootOf(_Z^3-9)*x^3+32334212568*RootOf(_Z^
3-9)*x^2+62862638832*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x
^3-23573489562*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootO
f(_Z^3-9)^2+74088110052*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2
)+1732189959*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*x^2+10102924098*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO
f(_Z^3-9)+81*_Z^2)^2*x^2+188587916496*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2)*x^2+188587916496*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*x+12702726366*RootOf(_Z^3-9)+32334212568*RootOf(_Z^3-9)*x-226983
7425*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)-8512490709*(x^3+3*x^2+3*x+3)^
(2/3)*x+3367641366*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*R
ootOf(_Z^3-9)^2*x^3+577396653*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*RootOf(_Z^3-9)^3*x^3-15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*Root
Of(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3
*x^2+3*x+3)^(1/3)*x+25537472127*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z
*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)+25537472127*RootOf(RootOf(
_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/
3)*x^2+51074944254*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)*x)/x/(x^2+3*x+3))*RootOf(_Z^3-9)-ln(-(1
0102924098*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z
^3-9)^2*x+1732189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*R
```

```
ootOf(_Z^3-9)^3*x-4041776571*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81
*_Z^2)*RootOf(_Z^3-9)^3-15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)*x-8512490709*(x^3+3*
x^2+3*x+3)^(2/3)+10778070856*RootOf(_Z^3-9)*x^3+32334212568*RootOf(_Z^3-9)*
x^2+62862638832*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-23
573489562*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^
3-9)^2+74088110052*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+173
2189959*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^
2)*x^2+10102924098*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^
3-9)+81*_Z^2)^2*x^2+188587916496*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*x^2+188587916496*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*
_Z^2)*x+12702726366*RootOf(_Z^3-9)+32334212568*RootOf(_Z^3-9)*x-2269837425*
RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)-8512490709*(x^3+3*x^2+3*x+3)^(2/3)
*x+3367641366*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf
(_Z^3-9)^2*x^3+577396653*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^
2)*RootOf(_Z^3-9)^3*x^3-15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z
^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+
3*x+3)^(1/3)*x+25537472127*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*Root
Of(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)+25537472127*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^
2+51074944254*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81
*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)*x)/x/(x^2+3*x+3))*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^3+3\,x^2+3\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+3\,x^2+3\,x\right)\,{\left(x^3+3\,x^2+3\,x+3\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

3.320 $\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 40, normalized size = 1.74

$$\sqrt[4]{-1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right),-1\right)-2\Pi\left(-i;\,i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[-I, I*Arc
Sinh[(-1)^(1/4)*x], -1])
```

IntegrateAlgebraic [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

fricas [A] time = 1.11, size = 18, normalized size = 0.78

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

maple [A] time = 0.32, size = 22, normalized size = 0.96

method	result	size
elliptic	$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x+\sqrt{x^4+1}}{x^2+1}\right)}{2}$	34
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/2/x*2^(1/2)*(x^4+1)^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] -int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x*
*2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

3.321 $\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.09, size = 36, normalized size = 1.57

$$\sqrt[4]{-1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right),-1\right)-2\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[I, ArcSin
[(-1)^(3/4)*x], -1])
```

IntegrateAlgebraic [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

fricas [B] time = 0.96, size = 42, normalized size = 1.83

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}\,x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+1}{\sqrt{x^4+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [A] time = 0.30, size = 22, normalized size = 0.96

method	result	size
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{x^4+1}}{(1+x)(-1+x)}\right)}{2}$	38
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2/x*2^(1/2)*(x^4+1)^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+1}{\sqrt{x^4+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2+1}{(x^2-1)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2-1}{\sqrt{x^4+1}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{x\sqrt{1+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^4\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^4}\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.31

$$\frac{1}{2}\left(\sinh^{-1}\left(x^2\right)-\tanh^{-1}\left(\sqrt{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] (ArcSinh[x^2] - ArcTanh[Sqrt[1 + x^4]])/2
```

IntegrateAlgebraic [B] time = 0.19, size = 35, normalized size = 2.19

$$\frac{1}{2}\log\left(\sqrt{x^4+1}+x^2\right)-\tanh^{-1}\left(\sqrt{x^4+1}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] -ArcTanh[x^2 + Sqrt[1 + x^4]] + Log[x^2 + Sqrt[1 + x^4]]/2
```

fricas [B] time = 0.75, size = 49, normalized size = 3.06

$$-\frac{1}{2}\log\left(2x^4-x^2-\sqrt{x^4+1}\left(2x^2-1\right)+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) - 1)
```

giac [B] time = 0.64, size = 51, normalized size = 3.19

$$\frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/2*log(x^2 - sqrt(x^4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2*
log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.30, size = 18, normalized size = 1.12

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}+\frac{\operatorname{arcsinh}\left(x^2\right)}{2}$	18
trager	$\ln\left(\frac{x^2+\sqrt{x^4+1}-1}{x}\right)$	18
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}+\frac{\operatorname{arcsinh}\left(x^2\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}\left(x^2\right)}{2}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)+(-2\ln(2)+4\ln(x))\sqrt{\pi}}{4\sqrt{\pi}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))+1/2*arcsinh(x^2)
```

maxima [B] time = 1.17, size = 57, normalized size = 3.56

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

mupad [B] time = 0.15, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x*(x^4 + 1)^(1/2)),x)
```

```
[Out] asinh(x^2)/2 - atanh((x^4 + 1)^(1/2))/2
```

sympy [A] time = 9.51, size = 14, normalized size = 0.88

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x/(x**4+1)**(1/2),x)
```

```
[Out] -asinh(x**(-2))/2 + asinh(x**2)/2
```

3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+1}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x\sqrt{1+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(x^2\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(x^2\right) + \frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.19

$$\frac{1}{2}\left(\tanh^{-1}\left(\sqrt{x^4+1}\right) + \sinh^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] (ArcSinh[x^2] + ArcTanh[Sqrt[1 + x^4]])/2
```

IntegrateAlgebraic [B] time = 0.19, size = 41, normalized size = 2.56

$$\frac{1}{2}\log\left(\sqrt{x^4+1} + x^2 + 1\right) - \tanh^{-1}\left(-2\sqrt{x^4+1} - 2x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] -ArcTanh[1 - 2*x^2 - 2*Sqrt[1 + x^4]] + Log[1 + x^2 + Sqrt[1 + x^4]]/2
```

fricas [B] time = 0.98, size = 47, normalized size = 2.94

$$-\frac{1}{2}\log\left(2x^4 + x^2 - \sqrt{x^4+1}\left(2x^2+1\right) + 1\right) + \frac{1}{2}\log\left(-x^2 + \sqrt{x^4+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) + 1)
```

giac [B] time = 0.60, size = 51, normalized size = 3.19

$$-\frac{1}{2}\log\left(x^2 - \sqrt{x^4+1} + 1\right) + \frac{1}{2}\log\left(-x^2 + \sqrt{x^4+1} + 1\right) - \frac{1}{2}\log\left(-x^2 + \sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/2*log(x^2 - sqrt(x^4 + 1) + 1) + 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2
*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.29, size = 18, normalized size = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
elliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
trager	$-\ln\left(\frac{-x^2+\sqrt{x^4+1}-1}{x}\right)$	22
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)+(-2\ln(2)+4\ln(x))\sqrt{\pi}}{4\sqrt{\pi}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [B] time = 1.28, size = 57, normalized size = 3.56

$$\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)-\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
+ 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

mupad [B] time = 0.23, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x*(x^4 + 1)^(1/2)),x)
[Out] asinh(x^2)/2 + atanh((x^4 + 1)^(1/2))/2
```

sympy [A] time = 5.57, size = 14, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**4+1)**(1/2),x)
[Out] asinh(x**(-2))/2 + asinh(x**2)/2
```

3.324 $\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1698, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 1.32, size = 522, normalized size = 20.08

$$(-1)^{2/3}\left(-\sqrt[3]{-1}\left(\sqrt[3]{-1}-1\right)^2\left(1+\sqrt[3]{-1}\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-2i\sqrt{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*(-((-1)^(1/3)*(-1 + (-1)^(1/3))^2*(1 + (-1)^(1/3))*Sqrt[1 + (-1
)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-
```

```
1)^(2/3)]) - (2*I)*Sqrt[3]*((-1)^(1/3) - x)^2*Sqrt[((-1)^(2/3) - x)/((1 + (
-1)^(1/3))*((-1)^(1/3) - x))]*Sqrt[((-1)^(2/3) + x)/(-1 + x - (-1)^(1/3)*x)
]*Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]*((1 + (-
1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((
-1)^(1/3) - x))]], -3] - 2*(-1)^(1/3)*EllipticPi[-1, ArcSin[Sqrt[(1 + x - (
-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]], -3]) + (2*I)*Sqrt[3]*((
-1)^(1/3) - x)^2*Sqrt[((-1)^(2/3) - x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]
*Sqrt[((-1)^(2/3) + x)/(-1 + x - (-1)^(1/3)*x)]*Sqrt[(1 + x - (-1)^(1/3)*x)
/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[S
qrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]], -3] - 2*(
-1)^(1/3)*EllipticPi[3, ArcSin[Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3)
)*((-1)^(1/3) - x))]], -3])))/((1 - (-1)^(2/3))*Sqrt[1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.21, size = 26, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

fricas [B] time = 0.82, size = 45, normalized size = 1.73

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{x^4+2\,\sqrt{3}\,\sqrt{x^4+x^2+1}\,x+4\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 -
2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+1}{\sqrt{x^4+x^2+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

maple [A] time = 0.56, size = 31, normalized size = 1.19

method	result
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+x^2+1}\,\sqrt{2}\,\sqrt{6}}{6x}\right)\sqrt{6}\,\sqrt{2}}{6}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)x+\sqrt{x^4+x^2+1}}{(1+x)(-1+x)}\right)}{3}$

default	$-\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticPi}\left(\right.}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctanh(1/6*(x^4+x^2+1)^(1/2)*2^(1/2)/x*6^(1/2))*6^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+1}{\sqrt{x^4+x^2+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2+1}{(x^2-1)\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) - I
ntegral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)
```

3.325 $\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=15

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1698, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.13, size = 94, normalized size = 6.27

$$-\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-2\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(\right.\right.}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticF[
I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 2*EllipticPi[(-1)^(1/3), I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.17, size = 15, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

fricas [A] time = 0.72, size = 13, normalized size = 0.87

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] arctan(x/sqrt(x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\sqrt{x^4+x^2+1}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

maple [A] time = 0.49, size = 18, normalized size = 1.20

method	result
elliptic	$-\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)$
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x-\sqrt{x^4+x^2+1}}{x^2+1}\right)$
default	$-\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}\cdots}\right.}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctan(1/x*(x^4+x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{\sqrt{x^4+x^2+1}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$-\int \frac{x^2 - 1}{(x^2 + 1)\sqrt{x^4 + x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] -int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4 + x^2 + 1} + \sqrt{x^4 + x^2 + 1}}\, dx - \int \left(-\frac{1}{x^2\sqrt{x^4 + x^2 + 1} + \sqrt{x^4 + x^2 + 1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) - I
ntegral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)
```

3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1590}

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\frac{\sqrt{1+x^2+x^4}}{x}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

IntegrateAlgebraic [A] time = 0.80, size = 16, normalized size = 1.00

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

fricas [A] time = 0.85, size = 14, normalized size = 0.88

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^4 + x^2 + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\sqrt{x^4+x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

maple [A] time = 0.08, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
trager	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
risch	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
elliptic	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
gosper	$\frac{(x^2+x+1)(x^2-x+1)}{\sqrt{x^4+x^2+1}\,x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/x*(x^4+x^2+1)^(1/2)
```

maxima [A] time = 0.87, size = 22, normalized size = 1.38

$$\frac{\sqrt{x^2+x+1}\,\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

mupad [B] time = 0.06, size = 14, normalized size = 0.88

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x^2 + x^4 + 1)^(1/2)),x)
```

[Out] (x^2 + x^4 + 1)^(1/2)/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{x^2\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x + 1
))), x)
```

3.327 $\int \frac{1-x^2}{\left(1+2ax+x^2\right)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{2x\left(a^2-b+1\right)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Rubi [A] time = 0.20, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2084}

$$\frac{\tan^{-1}\left(\frac{2x\left(a^2-b+1\right)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])
,x]
```

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
+ 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rule 2084

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c),
 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[a^2*(2*a - c), 2]), x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+2ax+x^2\right)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \frac{\tan^{-1}\left(\frac{a+2\left(1+a^2-b\right)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Mathematica [C] time = 6.47, size = 17955, normalized size = 242.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 +
 x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.67, size = 67, normalized size = 0.91

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b-1}\,x}{-\sqrt{2ax^3+2ax+2bx^2+x^4+1}+2ax+x^2+1}\right)}{\sqrt{b-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 +
2*a*x^3 + x^4]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + b]*x)/(1 + 2*a*x + x^2 - Sqrt[1 + 2*a
*x + 2*b*x^2 + 2*a*x^3 + x^4])])/Sqrt[-1 + b])
```

fricas [A] time = 1.66, size = 252, normalized size = 3.41

$$\left[\frac{\sqrt{2}\log\left(\frac{4\,a^{3}x^{3}+\left(a^{2}+2\,b-2\right)x^{4}+4\,a^{3}x+2\left(2\,a^{4}+5\,a^{2}-2\left(2\,a^{2}+3\right)b+4\,b^{2}+2\right)x^{2}+a^{2}-\frac{2\,\sqrt{2}\,\sqrt{2\,ax^{3}+x^{4}+2\,bx^{2}+2\,ax+1}\left((ab-a)x^{2}+ab-2\left(a^{2}-\left(a^{2}+2\right)b+b^{2}+1\right)x-a\right)}{\sqrt{b-1}}+2}{4\,ax^{3}+x^{4}+2\left(2\,a^{2}+1\right)x^{2}+4\,ax+1}\right)}{4\sqrt{b-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5*
a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x^4
 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^2 +
 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 + 4*a*
x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(2*a*x
^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(a^2 -
b + 1)*x + a))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^{2}-1}{\sqrt{2\,ax^{3}+x^{4}+2\,bx^{2}+2\,ax+1}\left(2\,ax+x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

maple [C] time = 0.39, size = 247419, normalized size = 3343.50

method	result	size
default	Expression too large to display	247419
elliptic	Expression too large to display	258804

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}-1}{\sqrt{2\,ax^{3}+x^{4}+2\,bx^{2}+2\,ax+1}\left(2\,ax+x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-1}{\left(x^2+2\,a\,x+1\right)\,\sqrt{x^4+2\,a\,x^3+2\,b\,x^2+2\,a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/
2)),x)
```

```
[Out] -int((x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/
2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2),
x)
```

```
[Out] -Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*s
qrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x
**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2
+ x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*
x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)
```

3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [A] time = 1.12, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

IntegrateAlgebraic [C] time = 0.26, size = 96, normalized size = 4.36

$$i\tanh^{-1}\left(\sqrt{2}\,x^4+\frac{\sqrt{x^4+1}\left(-2x^2+i\sqrt{2}\,x\sqrt{\sqrt{x^4+1}-x^2}\right)}{\sqrt{2}}-i\sqrt{\sqrt{x^4+1}-x^2}\,x^3+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[
1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

fricas [B] time = 1.99, size = 62, normalized size = 2.82

$$-\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+\left(7\,x^5-x\right)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

3.329 $\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx$

Optimal. Leaf size=24

$$\tan^{-1}\left(\frac{x}{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right)\end{aligned}$$

Mathematica [A] time = 0.09, size = 26, normalized size = 1.08

$$\cot^{-1}\left(\frac{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] Could not integrate
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+\left(x^{2\,n}+1\right)^{\left(\frac{1}{n}\right)}}\left(x^{2\,n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

```
[Out] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+\left(x^{2\,n}+1\right)^{\left(\frac{1}{n}\right)}}\left(x^{2\,n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(x^{2\,n}+1\right)\,\sqrt{{\left(x^{2\,n}+1\right)}^{1/n}-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)),x)
[Out] int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{2} + \left(x^{2n} + 1\right)^{\frac{1}{n}}} \left(x^{2n} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)
```

3.330 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.92, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.60, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

maple [A] time = 0.03, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

maxima [A] time = 0.57, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B] time = 0.18, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.331 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x) - \frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.84, size = 10, normalized size = 0.91

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

giac [A] time = 0.61, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.30, size = 11, normalized size = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.50, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$\sin(x) - \frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] sin(x) - sin(x)^3/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

3.332 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]^4,x]
```

[Out] Could not integrate

fricas [A] time = 0.79, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\cos(x)^3-5\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

giac [A] time = 0.59, size = 16, normalized size = 0.67

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

maple [A] time = 0.31, size = 17, normalized size = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

maxima [A] time = 0.50, size = 16, normalized size = 0.67

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.67

$$\frac{3x}{8}-\frac{\sin(2x)}{4}+\frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

sympy [A] time = 0.06, size = 24, normalized size = 1.00

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

3.333 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^6(x)\,dx &= \frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{6}\int \cos^4(x)\,dx\\ &= \frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{8}\int \cos^2(x)\,dx\\ &= \frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^6,x]
[Out] Could not integrate
```

fricas [A] time = 0.67, size = 25, normalized size = 0.74

$$\frac{1}{48}\left(8\,\cos(x)^5+10\,\cos(x)^3+15\,\cos(x)\right)\sin(x)+\frac{5}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

giac [A] time = 0.58, size = 22, normalized size = 0.65

$$\frac{5}{16}\,x+\frac{1}{192}\,\sin\left(6\,x\right)+\frac{3}{64}\,\sin\left(4\,x\right)+\frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="giac")
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

maple [A] time = 0.32, size = 23, normalized size = 0.68

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+\frac{75x\left(\tan^8\left(\frac{x}{2}\right)\right)}{16}+15x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

maxima [A] time = 0.44, size = 24, normalized size = 0.71

$$-\frac{1}{48}\,\sin\left(2\,x\right)^3+\frac{5}{16}\,x+\frac{3}{64}\,\sin\left(4\,x\right)+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{5\,x}{16}+\frac{15\,\sin\left(2\,x\right)}{64}+\frac{3\,\sin\left(4\,x\right)}{64}+\frac{\sin\left(6\,x\right)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] (5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

sympy [**A**] time = 0.07, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

3.334 $\int \sin^8(x)\,dx$

Optimal. Leaf size=44

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 - (7*Cos[x]*
Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \sin^8(x)\,dx &= -\frac{1}{8}\cos(x)\sin^7(x)+\frac{7}{8}\int \sin^6(x)\,dx\\
&= -\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{48}\int \sin^4(x)\,dx\\
&= -\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{64}\int \sin^2(x)\,dx\\
&= -\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35\int 1\,dx}{128}\\
&= \frac{35x}{128}-\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 0.86

$$\frac{35x}{128}-\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)-\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/10
24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^8(x)\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]^8,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.97, size = 31, normalized size = 0.70

$$\frac{1}{384} \left(48 \cos(x)^7 - 200 \cos(x)^5 + 326 \cos(x)^3 - 279 \cos(x)\right) \sin(x) + \frac{35}{128}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35/
128*x
```

giac [A] time = 0.61, size = 28, normalized size = 0.64

$$\frac{35}{128}\, x + \frac{1}{1024}\, \sin(8\, x) - \frac{1}{96}\, \sin(6\, x) + \frac{7}{128}\, \sin(4\, x) - \frac{7}{32}\, \sin(2\, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x)
```

maple [A] time = 0.36, size = 29, normalized size = 0.66

method	result
risch	$\frac{35x}{128} + \frac{\sin(8x)}{1024} - \frac{\sin(6x)}{96} + \frac{7\sin(4x)}{128} - \frac{7\sin(2x)}{32}$
default	$-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8} + \frac{35x}{128}$
norman	$\frac{\frac{35x}{128}-\frac{35\tan\left(\frac{x}{2}\right)}{64}+\frac{1225x\left(\tan^8\left(\frac{x}{2}\right)\right)}{64}+\frac{245x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{16}+\frac{245x\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{32}+\frac{245x\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}-\frac{805\left(\tan^3\left(\frac{x}{2}\right)\right)}{192}+\frac{35x\left(\tan^2\left(\frac{x}{2}\right)\right)}{16}+\frac{245x\left(\tan^4\left(\frac{x}{2}\right)\right)}{32}+\frac{35x\left(\tan^{14}\right.}{16}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 35/128*x+1/1024*sin(8*x)-1/96*sin(6*x)+7/128*sin(4*x)-7/32*sin(2*x)
```

maxima [A] time = 0.58, size = 30, normalized size = 0.68

$$\frac{1}{24}\, \sin(2\, x)^3 + \frac{35}{128}\, x + \frac{1}{1024}\, \sin(8\, x) + \frac{7}{128}\, \sin(4\, x) - \frac{1}{4}\, \sin(2\, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*x
)
```

mupad [B]   time = 0.03, size = 28, normalized size = 0.64

$$\frac{35\,x}{128}-\frac{7\,\sin\left(2\,x\right)}{32}+\frac{7\,\sin\left(4\,x\right)}{128}-\frac{\sin\left(6\,x\right)}{96}+\frac{\sin\left(8\,x\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^8,x)
```

```
[Out] (35*x)/128 - (7*sin(2*x))/32 + (7*sin(4*x))/128 - sin(6*x)/96 + sin(8*x)/10
24
```

sympy [A]   time = 0.07, size = 48, normalized size = 1.09

$$\frac{35x}{128}-\frac{\sin^{7}\left(x\right)\cos\left(x\right)}{8}-\frac{7\sin^{5}\left(x\right)\cos\left(x\right)}{48}-\frac{35\sin^{3}\left(x\right)\cos\left(x\right)}{192}-\frac{35\sin\left(x\right)\cos\left(x\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**8,x)
```

```
[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x)
/192 - 35*sin(x)*cos(x)/128
```

3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=20

$$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\sin(x)\cos(x)$$

Rubi [B] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 3.20, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{3}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[Pi/4 + x/2]*Sin[Pi/4 + x/2])/4 + (Cos[Pi/4 + x/2]^3*Sin[Pi
/4 + x/2])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)dx &= \frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3}{4}\int \sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx\\ &= \frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.05

$$\frac{1}{16}(6x+8\cos(x)-2\sin(x)\cos(x)+3\pi)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.62, size = 37, normalized size = 1.85

$$\frac{1}{4}\left(2\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{3}+3\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")
[Out] 1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi + 1/2*x) +
 3/8*x
```

giac [A] time = 0.59, size = 14, normalized size = 0.70

$$\frac{3}{8}x+\frac{1}{2}\cos(x)-\frac{1}{16}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

maple [A] time = 0.34, size = 15, normalized size = 0.75

method	result
risch	$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$
derivativedivides	$\frac{\left(\cos^{3}\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
default	$\frac{\left(\cos^{3}\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^{3}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3\left(\tan^{5}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}-\frac{5\left(\tan^{7}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3x\left(\tan^{2}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{4}+\frac{3x\left(\tan^{6}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3x\left(\tan^{8}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{8}}{\left(1+\tan^{2}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/4*Pi+1/2*x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/2*cos(x)-1/16*sin(2*x)
```

maxima [A] time = 0.49, size = 23, normalized size = 1.15

$$\frac{3}{16}\pi+\frac{3}{8}x+\frac{1}{16}\sin(\pi+2x)+\frac{1}{2}\sin\left(\frac{1}{2}\pi+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")
[Out] 3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

mupad [B] time = 0.27, size = 20, normalized size = 1.00

$$\frac{3x}{8}+\frac{\sin(\Pi+2x)}{16}+\frac{\sin\left(\frac{\Pi}{2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(Pi/4 + x/2)^4,x)
```

```
[Out] (3*x)/8 + sin(Pi + 2*x)/16 + sin(Pi/2 + x)/2
```

sympy [B] time = 0.65, size = 99, normalized size = 4.95

$$\frac{3x\sin^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)**4,x)
```

```
[Out] 3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)**2/4 + 3*
x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 + pi/4)/4 + 5*sin(x/2
 + pi/4)*cos(x/2 + pi/4)**3/4
```

3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2633}

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -Cos[Pi/12 - 3*x]/3 + Cos[Pi/12 - 3*x]^3/9
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(\frac{\pi}{12}-3x\right)\right)\right)\\ &= -\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -1/4*Cos[Pi/12 - 3*x] + Cos[3*(Pi/12 - 3*x)]/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.78, size = 22, normalized size = 0.71

$$-\frac{1}{9}\left(\cos\left(\frac{5}{12}\pi+3x\right)^2+2\right)\sin\left(\frac{5}{12}\pi+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")
[Out] -1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

giac [A] time = 0.74, size = 23, normalized size = 0.74

$$\frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3-\frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="giac")
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

maple [A] time = 0.54, size = 22, normalized size = 0.71

method	result	size
risch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
derivativedivides	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
default	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
norman	$\frac{-\frac{4\left(\tan^3\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{3}-\frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(5/12*Pi+3*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/36*sin(1/4*Pi+9*x)-1/4*sin(5/12*Pi+3*x)
```

maxima [A] time = 0.57, size = 23, normalized size = 0.74

$$\frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3-\frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

mupad [B] time = 0.25, size = 22, normalized size = 0.71

$$\frac{\sin\left(\frac{5\Pi}{12}+3x\right)\left(\sin\left(\frac{5\Pi}{12}+3x\right)^2-3\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((5*Pi)/12 + 3*x)^3,x)
[Out] (sin((5*Pi)/12 + 3*x)*(sin((5*Pi)/12 + 3*x)^2 - 3))/9
```

sympy [A] time = 0.34, size = 39, normalized size = 1.26

$$-\frac{2\sin^3\left(3x+\frac{5\pi}{12}\right)}{9}-\frac{\sin\left(3x+\frac{5\pi}{12}\right)\cos^2\left(3x+\frac{5\pi}{12}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)**3,x)
```

```
[Out] -2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3
```

3.337 $\int \csc^6(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767}

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6,x]
```

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(x)\,dx &= -\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,\cot(x)\right)\\ &= -\cot(x)-\frac{2\cot^3(x)}{3}-\frac{\cot^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.29

$$-\frac{8\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{4}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6,x]
```

```
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^6,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.48, size = 37, normalized size = 1.76

$$-\frac{8\cos(x)^5-20\cos(x)^3+15\cos(x)}{15\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*s
in(x))
```

giac [A] time = 0.59, size = 20, normalized size = 0.95

$$-\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

maple [A] time = 0.31, size = 18, normalized size = 0.86

method	result	size
default	$\left(-\frac{8}{15}-\frac{\left(\csc^4(x)\right)}{5}-\frac{4\left(\csc^2(x)\right)}{15}\right)\cot(x)$	18
risch	$-\frac{16i\left(10\,e^{4ix}-5\,e^{2ix}+1\right)}{15\left(e^{2ix}-1\right)^5}$	29
norman	$\frac{-\frac{1}{160}-\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{96}-\frac{5\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^8\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^5}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] (-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

maxima [A] time = 0.51, size = 20, normalized size = 0.95

$$-\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

mupad [B] time = 0.20, size = 27, normalized size = 1.29

$$-\frac{8 \cos(x)\sin(x)^4 + 4 \cos(x)\sin(x)^2 + 3 \cos(x)}{15\sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^6,x)
```

```
[Out] -(3*cos(x) + 4*cos(x)*sin(x)^2 + 8*cos(x)*sin(x)^4)/(15*sin(x)^5)
```

sympy [A] time = 0.06, size = 32, normalized size = 1.52

$$-\frac{8\cos(x)}{15\sin(x)} - \frac{4\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**6,x)
```

```
[Out] -8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

3.338 $\int \csc^7(x)\,dx$

Optimal. Leaf size=36

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^7,x]
```

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]^3)/24 - (
Cot[x]*Csc[x]^5)/6
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^7(x)\,dx &= -\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{6}\int \csc^5(x)\,dx\\
&= -\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{8}\int \csc^3(x)\,dx\\
&= -\frac{5}{16}\cot(x)\csc(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{16}\int \csc(x)\,dx\\
&= -\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{5}{16}\cot(x)\csc(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)
\end{aligned}$$

Mathematica [B] time = 0.01, size = 95, normalized size = 2.64

$$-\frac{1}{384}\csc^6\left(\frac{x}{2}\right)-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{5}{64}\csc^2\left(\frac{x}{2}\right)+\frac{1}{384}\sec^6\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{5}{64}\sec^2\left(\frac{x}{2}\right)+\frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{5}{16}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^7,x]
```

```
[Out] (-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[x/2]])/16
+ (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4/64 + Sec[x/2]^6/384
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^7(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^7,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.00, size = 93, normalized size = 2.58

$$\frac{30 \cos(x)^5 - 80 \cos(x)^3 - 15 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 15 \left(\cos(x)^6 - 3 \cos(\right.}{96 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 -
1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(
-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

giac [B] time = 0.62, size = 112, normalized size = 3.11

$$-\frac{\left(\frac{9 (\cos(x)-1)}{\cos(x)+1} - \frac{45 (\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{110 (\cos(x)-1)^3}{(\cos(x)+1)^3} - 1\right)(\cos(x)+1)^3}{384 (\cos(x)-1)^3} - \frac{15 (\cos(x)-1)}{128 (\cos(x)+1)} + \frac{3 (\cos(x)-1)^2}{128 (\cos(x)+1)^2} - \frac{(\cos(x) -}{384 (\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="giac")
```

```
[Out] -1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) + 1)^2 + 11
0*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(cos(x) - 1)^3 - 15/128
*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/384*(c
os(x) - 1)^3/(cos(x) + 1)^3 + 5/32*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.36, size = 32, normalized size = 0.89

method	result	size
default	$\left(-\frac{\left(\csc^5(x)\right)}{6} - \frac{5\left(\csc^3(x)\right)}{24} - \frac{5\csc(x)}{16}\right)\cot(x) + \frac{5\ln(\csc(x)-\cot(x))}{16}$	32
norman	$\frac{-\frac{1}{384} - \frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{128} - \frac{15\left(\tan^4\left(\frac{x}{2}\right)\right)}{128} + \frac{15\left(\tan^8\left(\frac{x}{2}\right)\right)}{128} + \frac{3\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{128} + \frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6} + \frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16}$	58
risch	$\frac{15\,e^{11ix} - 85\,e^{9ix} + 198\,e^{7ix} + 198\,e^{5ix} - 85\,e^{3ix} + 15\,e^{ix}}{24\left(e^{2ix}-1\right)^6} - \frac{5\ln\left(e^{ix}+1\right)}{16} + \frac{5\ln\left(e^{ix}-1\right)}{16}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^7,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-cot(x))
```

maxima [A] time = 0.51, size = 54, normalized size = 1.50

$$\frac{15 \cos(x)^5 - 40 \cos(x)^3 + 33 \cos(x)}{48 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1\right)} - \frac{5}{32} \log(\cos(x) + 1) + \frac{5}{32} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos
(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1)
```

mupad [B] time = 0.25, size = 44, normalized size = 1.22

$$\frac{\frac{5\cos(x)^5}{16}-\frac{5\cos(x)^3}{6}+\frac{11\,\cos(x)}{16}}{\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1}-\frac{5\,\mathrm{atanh}\left(\cos(x)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^7,x)
```

```
[Out] ((11*cos(x))/16 - (5*cos(x)^3)/6 + (5*cos(x)^5)/16)/(3*cos(x)^2 - 3*cos(x)^
4 + cos(x)^6 - 1) - (5*atanh(cos(x)))/16
```

sympy [A] time = 0.17, size = 60, normalized size = 1.67

$$-\frac{-15\cos^{5}(x)+40\cos^{3}(x)-33\cos(x)}{48\cos^{6}(x)-144\cos^{4}(x)+144\cos^{2}(x)-48}+\frac{5\log(\cos(x)-1)}{32}-\frac{5\log(\cos(x)+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**7,x)
```

```
[Out] -(-15*cos(x)**5 + 40*cos(x)**3 - 33*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 +
 144*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x) + 1)/32
```

3.339 $\int \sec^{12}(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767}

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^12,x]
```

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + T
an[x]^11/11
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{12}(x)\,dx &= -\operatorname{Subst}\left(\int \left(1+5x^2+10x^4+10x^6+5x^8+x^{10}\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{5\tan^3(x)}{3}+2\tan^5(x)+\frac{10\tan^7(x)}{7}+\frac{5\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.39

$$\frac{256\tan(x)}{693}+\frac{1}{11}\tan(x)\sec^{10}(x)+\frac{10}{99}\tan(x)\sec^8(x)+\frac{80}{693}\tan(x)\sec^6(x)+\frac{32}{231}\tan(x)\sec^4(x)+\frac{128}{693}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^12,x]
```

```
[Out] (256*Tan[x])/693 + (128*Sec[x]^2*Tan[x])/693 + (32*Sec[x]^4*Tan[x])/231 + (
80*Sec[x]^6*Tan[x])/693 + (10*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{12}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^12,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.81, size = 40, normalized size = 0.98

$$\frac{\left(256\cos(x)^{10}+128\cos(x)^8+96\cos(x)^6+80\cos(x)^4+70\cos(x)^2+63\right)\sin(x)}{693\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="fricas")
```

```
[Out] 1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x)
^2 + 63)*sin(x)/cos(x)^11
```

giac [A] time = 0.61, size = 33, normalized size = 0.80

$$\frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^{9} + \frac{10}{7}\tan(x)^{7} + 2\tan(x)^{5} + \frac{5}{3}\tan(x)^{3} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

maple [A] time = 0.33, size = 37, normalized size = 0.90

method	result	size
default	$-\left(-\frac{256}{693}-\frac{\left(\sec^{10}(x)\right)}{11}-\frac{10\left(\sec^{8}(x)\right)}{99}-\frac{80\left(\sec^{6}(x)\right)}{693}-\frac{32\left(\sec^{4}(x)\right)}{231}-\frac{128\left(\sec^{2}(x)\right)}{693}\right)\tan(x)$	37
risch	$\frac{512i\left(462\,e^{10ix}+330\,e^{8ix}+165\,e^{6ix}+55\,e^{4ix}+11\,e^{2ix}+1\right)}{693\left(1+e^{2ix}\right)^{11}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*sec(x)^4-12
8/693*sec(x)^2)*tan(x)
```

maxima [A] time = 0.49, size = 33, normalized size = 0.80

$$\frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^{9} + \frac{10}{7}\tan(x)^{7} + 2\tan(x)^{5} + \frac{5}{3}\tan(x)^{3} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

mupad [B] time = 0.21, size = 51, normalized size = 1.24

$$\frac{256\sin(x)\cos(x)^{10} + 128\sin(x)\cos(x)^{8} + 96\sin(x)\cos(x)^{6} + 80\sin(x)\cos(x)^{4} + 70\sin(x)\cos(x)^{2} + 63\sin(x)}{693\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^12,x)
```

```
[Out] (63*sin(x) + 70*cos(x)^2*sin(x) + 80*cos(x)^4*sin(x) + 96*cos(x)^6*sin(x) +
 128*cos(x)^8*sin(x) + 256*cos(x)^10*sin(x))/(693*cos(x)^11)
```

sympy [A] time = 0.07, size = 66, normalized size = 1.61

$$\frac{256\sin(x)}{693\cos(x)} + \frac{128\sin(x)}{693\cos^{3}(x)} + \frac{32\sin(x)}{231\cos^{5}(x)} + \frac{80\sin(x)}{693\cos^{7}(x)} + \frac{10\sin(x)}{99\cos^{9}(x)} + \frac{\sin(x)}{11\cos^{11}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**12,x)
```

```
[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(x
)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11*c
os(x)**11)
```

3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$

Optimal. Leaf size=40

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3768, 3770}

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx &= \frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)+\frac{1}{2}\int \csc\left(\frac{\pi}{4}-3x\right)\,dx\\ &= \frac{1}{6}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.74, size = 70, normalized size = 1.75

$$\frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3x\right)}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")
[Out] 1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2*
log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2
```

giac [A] time = 0.64, size = 53, normalized size = 1.32

$$-\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="giac")
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(-sin(1/4*pi + 3*x) + 1)
```

maple [A] time = 0.65, size = 40, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
default	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
norman	$\frac{\frac{\left(\tan^3\left(\frac{\pi}{8}+\frac{3x}{2}\right)\right)}{3}+\frac{\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}}{\left(\tan^2\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)^2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)}{6}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)}{6}$	66
risch	$-\frac{i\left((-1)^{\frac{3}{4}}e^{9ix}-(-1)^{\frac{1}{4}}e^{3ix}\right)}{3\left(ie^{6ix}+1\right)^2}+\frac{\ln\left(e^{\frac{i(\pi+12x)}{4}}+i\right)}{6}-\frac{\ln\left(e^{\frac{i(\pi+12x)}{4}}-i\right)}{6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(1/4*Pi+3*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x))
```

maxima [A] time = 0.51, size = 51, normalized size = 1.28

$$-\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

mupad [B] time = 0.62, size = 35, normalized size = 0.88

$$\frac{\ln\left(\tan\left(\frac{\Pi}{8}+\frac{3x}{2}+\frac{\pi}{4}\right)\right)}{6}+\frac{\tan\left(\frac{\Pi}{4}+3x\right)}{6\cos\left(\frac{\Pi}{4}+3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(Pi/4 + 3*x)^3,x)
```

```
[Out] log(tan(Pi/8 + (3*x)/2 + pi/4))/6 + tan(Pi/4 + 3*x)/(6*cos(Pi/4 + 3*x))
```

sympy [B] time = 1.31, size = 388, normalized size = 9.70

$$-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right.}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)**3,x)
```

```
[Out] -log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/
8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3*x/
2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log
(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*t
an(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**
2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 +
pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*tan(3
*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*
tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6)
```

3.341 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \tan^6(x)\,dx &= \frac{\tan^5(x)}{5}-\int \tan^4(x)\,dx\\
&= -\frac{1}{3}\tan^3(x)+\frac{\tan^5(x)}{5}+\int \tan^2(x)\,dx\\
&= \tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}-\int 1\,dx\\
&= -x+\tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.36

$$-x+\frac{23\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]^6,x]
[Out] Could not integrate
```

fricas [A] time = 0.79, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^{5} - \frac{1}{3}\tan(x)^{3} - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

giac [A] time = 0.59, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^{5} - \frac{1}{3}\tan(x)^{3} - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

maple [A] time = 0.04, size = 19, normalized size = 0.86

method	result	size
norman	$-x+\tan(x)-\frac{\left(\tan^{3}(x)\right)}{3}+\frac{\left(\tan^{5}(x)\right)}{5}$	19
derivativedivides	$\frac{\left(\tan^{5}(x)\right)}{5}-\frac{\left(\tan^{3}(x)\right)}{3}+\tan(x)-\arctan\left(\tan(x)\right)$	21
default	$\frac{\left(\tan^{5}(x)\right)}{5}-\frac{\left(\tan^{3}(x)\right)}{3}+\tan(x)-\arctan\left(\tan(x)\right)$	21
risch	$-x+\frac{2i\left(45\,e^{8ix}+90\,e^{6ix}+140\,e^{4ix}+70\,e^{2ix}+23\right)}{15\left(1+e^{2ix}\right)^{5}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

maxima [A] time = 1.34, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^{5} - \frac{1}{3}\tan(x)^{3} - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$\frac{\tan(x)^{5}}{5} - \frac{\tan(x)^{3}}{3} + \tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
```

```
[Out] tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

sympy [**A**] time = 0.08, size = 31, normalized size = 1.41

$$-x+\frac{\sin^5(x)}{5\cos^5(x)}-\frac{\sin^3(x)}{3\cos^3(x)}+\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**6,x)
```

```
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

3.342 $\int \cot^5(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}+\log(\sin(x))$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^5,x]
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^5(x)\,dx &= -\frac{1}{4}\cot^4(x)-\int \cot^3(x)\,dx\\ &= \frac{\cot^2(x)}{2}-\frac{\cot^4(x)}{4}+\int \cot(x)\,dx\\ &= \frac{\cot^2(x)}{2}-\frac{\cot^4(x)}{4}+\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.80

$$-\frac{1}{4}\csc^4(x)+\csc^2(x)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^5,x]
[Out] Csc[x]^2 - Csc[x]^4/4 + Log[Sin[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.94, size = 40, normalized size = 2.00

$$\frac{2\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4+3\tan(x)^4+2\tan(x)^2-1}{4\tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="fricas")
[Out] 1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1)
/tan(x)^4
```

giac [B] time = 0.83, size = 37, normalized size = 1.85

$$-\frac{3\tan(x)^4-2\tan(x)^2+1}{4\tan(x)^4}-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="giac")
[Out] -1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*l
og(tan(x)^2)
```

maple [A] time = 0.06, size = 26, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{\ln\left(1+\tan^2(x)\right)}{2}-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}$	26
default	$-\frac{\ln\left(1+\tan^2(x)\right)}{2}-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}$	26
norman	$\frac{-\frac{1}{4}+\frac{\left(\tan^2(x)\right)}{2}}{\tan(x)^4}-\frac{\ln\left(1+\tan^2(x)\right)}{2}+\ln(\tan(x))$	27
risch	$-ix-\frac{4\left(e^{6ix}-e^{4ix}+e^{2ix}\right)}{\left(e^{2ix}-1\right)^4}+\ln\left(e^{2ix}-1\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^5,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+tan(x)^2)-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2
```

maxima [A] time = 0.61, size = 22, normalized size = 1.10

$$\frac{4\sin(x)^2-1}{4\sin(x)^4}+\frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="maxima")
[Out] 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

mupad [B] time = 0.27, size = 26, normalized size = 1.30

$$\ln(\tan(x))-\frac{\ln\left(\tan(x)^2+1\right)}{2}+\frac{\frac{\tan(x)^2}{2}-\frac{1}{4}}{\tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^5,x)
```

```
[Out] log(tan(x)) - log(tan(x)^2 + 1)/2 + (tan(x)^2/2 - 1/4)/tan(x)^4
```

sympy [**A**] time = 0.11, size = 19, normalized size = 0.95

$$\frac{4\sin^2(x)-1}{4\sin^4(x)}+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)**5,x)
```

```
[Out] (4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$

Optimal. Leaf size=32

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3473, 8}

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)dx &= -\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)-\int \tan^2\left(\frac{\pi}{4}-\frac{x}{3}\right)dx\\ &= 3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)+\int 1\,dx\\ &= x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 40, normalized size = 1.25

$$-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2\left(\frac{x}{3}+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] -(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.70, size = 70, normalized size = 2.19

$$\frac{4\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)^{2}+\left(x\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-x\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)+2\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-2}{\left(\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-1\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")
```

```
[Out] (4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x)
+ 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*x)
)
```

giac [B] time = 1.12, size = 53, normalized size = 1.66

$$\frac{3}{4}\pi+\frac{1}{8}\tan\left(\frac{1}{8}\pi+\frac{1}{6}x\right)^{3}+x+\frac{15\tan\left(\frac{1}{8}\pi+\frac{1}{6}x\right)^{2}-1}{8\tan\left(\frac{1}{8}\pi+\frac{1}{6}x\right)^{3}}-\frac{15}{8}\tan\left(\frac{1}{8}\pi+\frac{1}{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="giac")
```

```
[Out] 3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1)
/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

maple [A] time = 0.04, size = 38, normalized size = 1.19

method	result	size
derivativedivides	$-\left(\cot^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\mathrm{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
default	$-\left(\cot^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\mathrm{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
norman	$\frac{-1+x\left(\tan^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\left(\tan^{2}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}}$	38
risch	$x+\frac{4i\left(-3\,\mathrm{e}^{\frac{4ix}{3}}-3i\mathrm{e}^{\frac{2ix}{3}}+2\right)}{\left(\mathrm{e}^{\frac{i(3\pi+4x)}{6}}-1\right)^{3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(1/4*Pi+1/3*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -cot(1/4*Pi+1/3*x)^3+3*cot(1/4*Pi+1/3*x)-3/2*Pi+3*arccot(cot(1/4*Pi+1/3*x))
```

maxima [A] time = 1.32, size = 30, normalized size = 0.94

$$\frac{3}{4}\pi+x+\frac{3\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{2}-1}{\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")
```

```
[Out] 3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

mupad [B] time = 0.09, size = 24, normalized size = 0.75

$$-\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)^3+3\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(Pi/4 + x/3)^4,x)
```

```
[Out] x + 3*cot(Pi/4 + x/3) - cot(Pi/4 + x/3)^3
```

sympy [A] time = 0.21, size = 20, normalized size = 0.62

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)**4,x)
```

```
[Out] x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

3.344 $\int \cos^6(x)\sin^4(x)\,dx$

Optimal. Leaf size=56

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[x]^5*Sin[x
])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(x)\sin^4(x)\,dx &= -\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{10}\int \cos^6(x)\sin^2(x)\,dx\\
&= -\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{80}\int \cos^6(x)\,dx\\
&= \frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{1}{32}\int \cos^4(x)\,dx\\
&= \frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{128}\int\\
&= \frac{3}{256}\cos(x)\sin(x)+\frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos\\
&= \frac{3x}{256}+\frac{3}{256}\cos(x)\sin(x)+\frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.82

$$\frac{3x}{256}+\frac{1}{512}\sin(2x)-\frac{1}{256}\sin(4x)-\frac{\sin(6x)}{1024}+\frac{\sin(8x)}{2048}+\frac{\sin(10x)}{5120}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 + S
in[10*x]/5120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^6(x)\sin^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.97, size = 37, normalized size = 0.66

$$\frac{1}{1280}\left(128\cos(x)^9 - 176\cos(x)^7 + 8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{3}{256}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))
*sin(x) + 3/256*x
```

giac [A] time = 0.77, size = 34, normalized size = 0.61

$$\frac{3}{256}x + \frac{1}{5120}\sin(10x) + \frac{1}{2048}\sin(8x) - \frac{1}{1024}\sin(6x) - \frac{1}{256}\sin(4x) + \frac{1}{512}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin(
4*x) + 1/512*sin(2*x)
```

maple [A] time = 0.32, size = 35, normalized size = 0.62

method	result	size
risch	$\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$	35
default	$-\frac{\left(\cos^7(x)\right)\left(\sin^3(x)\right)}{10}-\frac{3\left(\cos^7(x)\right)\sin(x)}{80}+\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{3x}{256}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/256*x+1/5120*sin(10*x)+1/2048*sin(8*x)-1/1024*sin(6*x)-1/256*sin(4*x)+1/5
12*sin(2*x)
```

maxima [A] time = 0.57, size = 24, normalized size = 0.43

$$\frac{1}{320}\sin(2x)^5 + \frac{3}{256}x + \frac{1}{2048}\sin(8x) - \frac{1}{256}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

mupad [B] time = 0.04, size = 38, normalized size = 0.68

$$\left(\frac{\cos(x)^5}{10}+\frac{\cos(x)^3}{16}+\frac{\cos(x)}{32}\right)\sin(x)^5+\frac{3\,x}{256}-\frac{\sin(2\,x)}{128}+\frac{\sin(4\,x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^4,x)
```

```
[Out] (3*x)/256 - sin(2*x)/128 + sin(4*x)/1024 + sin(x)^5*(cos(x)/32 + cos(x)^3/1
6 + cos(x)^5/10)
```

sympy [A] time = 0.07, size = 56, normalized size = 1.00

$$\frac{3x}{256}+\frac{\sin(x)\cos^9(x)}{10}-\frac{11\sin(x)\cos^7(x)}{80}+\frac{\sin(x)\cos^5(x)}{160}+\frac{\sin(x)\cos^3(x)}{128}+\frac{3\sin(x)\cos(x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**4,x)
```

```
[Out] 3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*cos(x)**5/1
60 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

3.345 $\int \cos^6(x)\sin^7(x)\,dx$

Optimal. Leaf size=33

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 270}

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] -Cos[x]^7/7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^6(x)\sin^7(x)\,dx &= -\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)dx,x,\cos(x)\right)\\ &= -\frac{1}{7}\cos^7(x)+\frac{\cos^9(x)}{3}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^{13}(x)}{13}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.67

$$-\frac{5\cos(x)}{1024}-\frac{5\cos(3x)}{4096}+\frac{3\cos(5x)}{4096}+\frac{3\cos(7x)}{14336}-\frac{\cos(9x)}{6144}-\frac{\cos(11x)}{45056}+\frac{\cos(13x)}{53248}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] (-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/143
36 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^6(x)\sin^7(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^6*Sin[x]^7,x]
[Out] Could not integrate
```

fricas [A] time = 0.92, size = 25, normalized size = 0.76

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^{9} - \frac{1}{7}\cos(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

giac [A] time = 0.83, size = 25, normalized size = 0.76

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^{9} - \frac{1}{7}\cos(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

maple [A] time = 0.05, size = 38, normalized size = 1.15

method	result	size
default	$-\frac{(\cos^7(x))(\sin^6(x))}{13} - \frac{6(\sin^4(x))(\cos^7(x))}{143} - \frac{8(\sin^2(x))(\cos^7(x))}{429} - \frac{16(\cos^7(x))}{3003}$	38
risch	$-\frac{5\cos(x)}{1024} + \frac{\cos(13x)}{53248} - \frac{\cos(11x)}{45056} - \frac{\cos(9x)}{6144} + \frac{3\cos(7x)}{14336} + \frac{3\cos(5x)}{4096} - \frac{5\cos(3x)}{4096}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^7,x,method=_RETURNVERBOSE)
[Out] -1/13*cos(x)^7*sin(x)^6-6/143*sin(x)^4*cos(x)^7-8/429*sin(x)^2*cos(x)^7-16/
3003*cos(x)^7
```

maxima [A] time = 0.55, size = 25, normalized size = 0.76

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^{9} - \frac{1}{7}\cos(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

mupad [B] time = 0.20, size = 25, normalized size = 0.76

$$\frac{\cos(x)^{13}}{13} - \frac{3\cos(x)^{11}}{11} + \frac{\cos(x)^{9}}{3} - \frac{\cos(x)^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^7,x)
[Out] cos(x)^9/3 - cos(x)^7/7 - (3*cos(x)^11)/11 + cos(x)^13/13
```

sympy [A] time = 0.07, size = 27, normalized size = 0.82

$$\frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^{9}(x)}{3} - \frac{\cos^{7}(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**7,x)
```

```
[Out] cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

3.346 $\int \sin^{10}(x)\tan(x)\,dx$

Optimal. Leaf size=46

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2590, 266, 43}

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^{10}(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{x}\,dx,x,\cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^5}{x}\,dx,x,\cos^2(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{1}{x}+10x-10x^2+5x^3-x^4\right)dx,x,\cos^2(x)\right)\right)\\ &= \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{10}(x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]^10*Tan[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 38, normalized size = 0.83

$$\frac{1}{10}\cos(x)^{10} - \frac{5}{8}\cos(x)^{8} + \frac{5}{3}\cos(x)^{6} - \frac{5}{2}\cos(x)^{4} + \frac{5}{2}\cos(x)^{2} - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="fricas")
```

```
[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
- log(-cos(x))
```

giac [A] time = 0.82, size = 42, normalized size = 0.91

$$-\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(-\sin(x)^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="giac")
```

```
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^2
 - 1/2*log(-sin(x)^2 + 1)
```

maple [A] time = 0.07, size = 37, normalized size = 0.80

method	result	size
default	$-\frac{\left(\sin^{10}(x)\right)}{10} - \frac{\left(\sin^{8}(x)\right)}{8} - \frac{\left(\sin^{6}(x)\right)}{6} - \frac{\left(\sin^{4}(x)\right)}{4} - \frac{\left(\sin^{2}(x)\right)}{2} - \ln(\cos(x))$	37
risch	$ix + \frac{281\,e^{2ix}}{1024} + \frac{281\,e^{-2ix}}{1024} - \ln\left(1 + e^{2ix}\right) + \frac{\cos(10x)}{5120} - \frac{3\cos(8x)}{1024} + \frac{67\cos(6x)}{3072} - \frac{29\cos(4x)}{256}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^11/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)^2-ln(cos(
x))
```

maxima [A] time = 0.53, size = 40, normalized size = 0.87

$$-\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(\sin(x)^{2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="maxima")
```

```
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^2
 - 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.78

$$-\frac{\sin(x)^{10}}{10}-\frac{\sin(x)^{8}}{8}-\frac{\sin(x)^{6}}{6}-\frac{\sin(x)^{4}}{4}-\frac{\sin(x)^{2}}{2}-\ln\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^11/cos(x),x)
```

```
[Out] - log(cos(x)) - sin(x)^2/2 - sin(x)^4/4 - sin(x)^6/6 - sin(x)^8/8 - sin(x)^
10/10
```

sympy [A] time = 0.10, size = 44, normalized size = 0.96

$$-\log\left(\cos\left(x\right)\right)+\frac{\cos^{10}\left(x\right)}{10}-\frac{5\cos^{8}\left(x\right)}{8}+\frac{5\cos^{6}\left(x\right)}{3}-\frac{5\cos^{4}\left(x\right)}{2}+\frac{5\cos^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**11/cos(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*cos(x)**4/
2 + 5*cos(x)**2/2
```

3.347 $\int \csc^6(x)\sec^6(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2620, 270}

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Tan
[x]^5/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(x)\sec^6(x)\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^6}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(10+\frac{1}{x^6}+\frac{5}{x^4}+\frac{10}{x^2}+5x^2+x^4\right)dx,x,\tan(x)\right)\\ &= -10\cot(x)-\frac{5\cot^3(x)}{3}-\frac{\cot^5(x)}{5}+10\tan(x)+\frac{5\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.29

$$\frac{128\tan(x)}{15}-\frac{128\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{19}{15}\cot(x)\csc^2(x)+\frac{1}{5}\tan(x)\sec^4(x)+\frac{19}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] (-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Tan
[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^6(x)\sec^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.63, size = 55, normalized size = 1.34

$$-\frac{256 \cos(x)^{10} - 640 \cos(x)^8 + 480 \cos(x)^6 - 80 \cos(x)^4 - 10 \cos(x)^2 - 3}{15 \left(\cos(x)^9 - 2 \cos(x)^7 + \cos(x)^5\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4 - 10*cos(x
)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

giac [A] time = 0.83, size = 26, normalized size = 0.63

$$-\frac{32 \left(15 \tan(2x)^4 + 10 \tan(2x)^2 + 3\right)}{15 \tan(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="giac")
```

```
[Out] -32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

maple [C] time = 0.33, size = 38, normalized size = 0.93

method	result	size
risch	$-\frac{512 i \left(10 e^{8 i x}-5 e^{4 i x}+1\right)}{15 \left(e^{2 i x}-1\right)^{5} \left(1+e^{2 i x}\right)^{5}}$	38
default	$\frac{1}{5 \sin(x)^5 \cos(x)^5}-\frac{2}{5 \sin(x)^5 \cos(x)^3}+\frac{16}{15 \sin(x)^3 \cos(x)^3}-\frac{32}{15 \sin(x)^3 \cos(x)}+\frac{128}{15 \sin(x) \cos(x)}-\frac{256 \cot(x)}{15}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^6/sin(x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -512/15*I*(10*exp(8*I*x)-5*exp(4*I*x)+1)/(exp(2*I*x)-1)^5/(1+exp(2*I*x))^5
```

maxima [A] time = 0.49, size = 37, normalized size = 0.90

$$\frac{1}{5} \tan(x)^5 + \frac{5}{3} \tan(x)^3 - \frac{150 \tan(x)^4 + 25 \tan(x)^2 + 3}{15 \tan(x)^5} + 10 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 + 3)/tan(x)^
5 + 10*tan(x)
```

mupad [B] time = 0.12, size = 27, normalized size = 0.66

$$-\frac{32 \left(\frac{\cos(2x)}{3} - \frac{\cos(6x)}{6} + \frac{\cos(10x)}{30}\right)}{\sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6*sin(x)^6),x)
```

```
[Out] -(32*(cos(2*x)/3 - cos(6*x)/6 + cos(10*x)/30))/sin(2*x)^5
```

sympy [**A**] time = 0.08, size = 44, normalized size = 1.07

$$-\frac{256\cos(2x)}{15\sin(2x)}-\frac{128\cos(2x)}{15\sin^3(2x)}-\frac{32\cos(2x)}{5\sin^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**6/sin(x)**6,x)
```

```
[Out] -256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*cos(2*x)/(
5*sin(2*x)**5)
```

3.348 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x)+\frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(x)\sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^2*Sin[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 0.70, size = 19, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos(x)^3-\cos(x)\right)\sin(x)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

giac [A] time = 1.00, size = 10, normalized size = 0.42

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

maple [A] time = 0.05, size = 11, normalized size = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{\left(\cos^3(x)\right)\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/32*sin(4*x)
```

maxima [A] time = 0.57, size = 10, normalized size = 0.42

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos(x)\,{\sin(x)}^3}{4}-\frac{\cos(x)\,\sin(x)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

sympy [**A**] time = 0.07, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

3.349 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\sin^4(x)\,dx &= -\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{8}\int \cos^4(x)\sin^2(x)\,dx\\ &= -\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{1}{16}\int \cos^4(x)\,dx\\ &= \frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{64}\int \cos^2(x)\,dx\\ &= \frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3\int 1}{128}\\ &= \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.48

$$\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^4(x)\sin^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^4*Sin[x]^4,x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 31, normalized size = 0.67

$$\frac{1}{128}\left(16\cos(x)^7 - 24\cos(x)^5 + 2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

giac [A] time = 0.98, size = 16, normalized size = 0.35

$$\frac{3}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

maple [A] time = 0.34, size = 17, normalized size = 0.37

method	result
risch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
default	$-\frac{\left(\cos^5(x)\right)\left(\sin^3(x)\right)}{8}-\frac{\left(\cos^5(x)\right)\sin(x)}{16}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{64}+\frac{3x}{128}$
norman	$\frac{\frac{3x}{128}-\frac{3\tan\left(\frac{x}{2}\right)}{64}+\frac{105x\left(\tan^8\left(\frac{x}{2}\right)\right)}{64}+\frac{21x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{16}+\frac{21x\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{32}+\frac{21x\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}-\frac{23\left(\tan^3\left(\frac{x}{2}\right)\right)}{64}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{16}+\frac{21x\left(\tan^4\left(\frac{x}{2}\right)\right)}{32}+\frac{3x\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{16}+\frac{3x(}{}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 3/128*x+1/1024*sin(8*x)-1/128*sin(4*x)
```

maxima [A] time = 0.49, size = 16, normalized size = 0.35

$$\frac{3}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.70

$$\left(\frac{\cos(x)^3}{8}+\frac{\cos(x)}{16}\right)\sin(x)^5+\frac{3\,x}{128}-\frac{\sin\left(2\,x\right)}{64}+\frac{\sin\left(4\,x\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x)
[Out] (3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

sympy [A] time = 0.07, size = 31, normalized size = 0.67

$$\frac{3x}{128}-\frac{\sin^3\left(2x\right)\cos\left(2x\right)}{128}-\frac{3\sin\left(2x\right)\cos\left(2x\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**4,x)
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

3.350 $\int \cos^6(x)\sin^6(x)\,dx$

Optimal. Leaf size=68

$$\frac{5x}{1024}-\frac{1}{12}\sin^5(x)\cos^7(x)-\frac{1}{24}\sin^3(x)\cos^7(x)-\frac{1}{64}\sin(x)\cos^7(x)+\frac{1}{384}\sin(x)\cos^5(x)+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)c}{102}$$

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{5x}{1024}-\frac{1}{12}\sin^5(x)\cos^7(x)-\frac{1}{24}\sin^3(x)\cos^7(x)-\frac{1}{64}\sin(x)\cos^7(x)+\frac{1}{384}\sin(x)\cos^5(x)+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)c}{102}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 + (Cos[x]^5*
Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3)/24 - (Cos[x]^7*Sin
[x]^5)/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(x)\sin^6(x)\,dx &= -\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5}{12}\int \cos^6(x)\sin^4(x)\,dx\\
&= -\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{1}{8}\int \cos^6(x)\sin^2(x)\,dx\\
&= -\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{1}{64}\int \cos^6(x)\,dx\\
&= \frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5}{384}\\
&= \frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7\\
&= \frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\\
&= \frac{5x}{1024}+\frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.44

$$\frac{5x}{1024}-\frac{15\sin(4x)}{8192}+\frac{3\sin(8x)}{8192}-\frac{\sin(12x)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^6,x]
[Out] (5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^6(x)\sin^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^6*Sin[x]^6,x]
[Out] Could not integrate
```

fricas [A] time = 0.79, size = 43, normalized size = 0.63

$$-\frac{1}{3072}\left(256\cos(x)^{11}-640\cos(x)^{9}+432\cos(x)^{7}-8\cos(x)^{5}-10\cos(x)^{3}-15\cos(x)\right)\sin(x)+\frac{5}{1024}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
[Out] -1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5 - 10*cos(
x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

giac [A] time = 0.89, size = 22, normalized size = 0.32

$$\frac{5}{1024}x-\frac{1}{24576}\sin(12x)+\frac{3}{8192}\sin(8x)-\frac{15}{8192}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
[Out] 5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

maple [A] time = 0.31, size = 23, normalized size = 0.34

method	result	size
risch	$\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$	23
default	$-\frac{\left(\cos^7(x)\right)\left(\sin^5(x)\right)}{12}-\frac{\left(\cos^7(x)\right)\left(\sin^3(x)\right)}{24}-\frac{\left(\cos^7(x)\right)\sin(x)}{64}+\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{384}+\frac{5x}{1024}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/1024*x-1/24576*sin(12*x)+3/8192*sin(8*x)-15/8192*sin(4*x)
```

maxima [A] time = 0.47, size = 24, normalized size = 0.35

$$\frac{1}{6144}\sin(4x)^3+\frac{5}{1024}x+\frac{3}{8192}\sin(8x)-\frac{1}{512}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

mupad [B] time = 0.03, size = 44, normalized size = 0.65

$$\left(\frac{\cos(x)^5}{12}+\frac{\cos(x)^3}{24}+\frac{\cos(x)}{64}\right)\sin(x)^7+\frac{5\,x}{1024}-\frac{15\,\sin\left(2\,x\right)}{4096}+\frac{3\,\sin\left(4\,x\right)}{4096}-\frac{\sin\left(6\,x\right)}{12288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^6,x)
```

```
[Out] (5*x)/1024 - (15*sin(2*x))/4096 + (3*sin(4*x))/4096 - sin(6*x)/12288 + sin(
x)^7*(cos(x)/64 + cos(x)^3/24 + cos(x)^5/12)
```

sympy [A] time = 0.07, size = 46, normalized size = 0.68

$$\frac{5x}{1024}-\frac{\sin^5\left(2x\right)\cos\left(2x\right)}{768}-\frac{5\sin^3\left(2x\right)\cos\left(2x\right)}{3072}-\frac{5\sin\left(2x\right)\cos\left(2x\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**6,x)
```

```
[Out] 5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072 - 5*sin(2
*x)*cos(2*x)/2048
```

3.351 $\int \cos^8(x)\sin^8(x)\,dx$

Optimal. Leaf size=90

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin}{}$$

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49152 + (7*C
os[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x]^9*Sin[x])/256 - (5
*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^8(x)\sin^8(x)\,dx &= -\frac{1}{16}\cos^9(x)\sin^7(x) + \frac{7}{16}\int \cos^8(x)\sin^6(x)\,dx \\ &= -\frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \frac{5}{32}\int \cos^8(x)\sin^4(x)\,dx \\ &= -\frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \frac{5}{128}\int \cos^8(x)\sin^2(x) \\ &= -\frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \\ &= \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos \\ &= \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(\\ &= \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x \\ &= \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\,s \\ &= \frac{35x}{32768} + \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.42

$$\frac{35x}{32768} - \frac{7\sin(4x)}{16384} + \frac{7\sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 +
Sin[16*x]/524288
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^8(x)\sin^8(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.78, size = 55, normalized size = 0.61

$$\frac{1}{98304}\left(6144\cos(x)^{15} - 21504\cos(x)^{13} + 25856\cos(x)^{11} - 10880\cos(x)^9 + 48\cos(x)^7 + 56\cos(x)^5 + 70\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)^
9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/32768
*x
```

giac [A] time = 0.63, size = 28, normalized size = 0.31

$$\frac{35}{32768}x + \frac{1}{524288}\sin(16x) - \frac{1}{49152}\sin(12x) + \frac{7}{65536}\sin(8x) - \frac{7}{16384}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7/
16384*sin(4*x)
```

maple [A] time = 0.32, size = 29, normalized size = 0.32

method	result
risch	$\frac{35x}{32768}+\frac{\sin(16x)}{524288}-\frac{\sin(12x)}{49152}+\frac{7\sin(8x)}{65536}-\frac{7\sin(4x)}{16384}$
default	$-\frac{\left(\cos^9(x)\right)\left(\sin^7(x)\right)}{16}-\frac{\left(\cos^9(x)\right)\left(\sin^5(x)\right)}{32}-\frac{5\left(\cos^9(x)\right)\left(\sin^3(x)\right)}{384}-\frac{\left(\cos^9(x)\right)\sin(x)}{256}+\frac{\left(\cos^7(x)+\frac{7\left(\cos^5(x)\right)}{6}+\frac{35\left(\cos^3(x)\right)}{24}+\frac{35\cos}{16}\right.}{2048}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8*sin(x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 35/32768*x+1/524288*sin(16*x)-1/49152*sin(12*x)+7/65536*sin(8*x)-7/16384*si
n(4*x)
```

maxima [A] time = 0.65, size = 30, normalized size = 0.33

$$\frac{1}{12288}\sin(4x)^3+\frac{35}{32768}x+\frac{1}{524288}\sin(16x)+\frac{7}{65536}\sin(8x)-\frac{1}{2048}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) - 1
/2048*sin(4*x)
```

mupad [B] time = 0.04, size = 56, normalized size = 0.62

$$\left(\frac{\cos(x)^7}{16}+\frac{\cos(x)^5}{32}+\frac{5\cos(x)^3}{384}+\frac{\cos(x)}{256}\right)\sin(x)^9+\frac{35x}{32768}-\frac{7\sin(2x)}{8192}+\frac{7\sin(4x)}{32768}-\frac{\sin(6x)}{24576}+\frac{\sin(8x)}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8*sin(x)^8,x)
```

```
[Out] (35*x)/32768 - (7*sin(2*x))/8192 + (7*sin(4*x))/32768 - sin(6*x)/24576 + si
n(8*x)/262144 + sin(x)^9*(cos(x)/256 + (5*cos(x)^3)/384 + cos(x)^5/32 + cos
(x)^7/16)
```

sympy [A] time = 0.08, size = 61, normalized size = 0.68

$$\frac{35x}{32768}-\frac{\sin^7(2x)\cos(2x)}{4096}-\frac{7\sin^5(2x)\cos(2x)}{24576}-\frac{35\sin^3(2x)\cos(2x)}{98304}-\frac{35\sin(2x)\cos(2x)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**8*sin(x)**8,x)
```

```
[Out] 35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35*
sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\sin^2(x)\right)}{2m+1}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2577}

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1),\frac{1}{2}(2m+3),\sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.85

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}-m,m+\frac{1}{2};m+\frac{3}{2};\sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m,
 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos(x)^{2m}\sin(x)^{2m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
[Out] integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\cos^{2m}(x)\right)\left(\sin^{2m}(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)
[Out] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

mupad [B] time = 0.75, size = 52, normalized size = 0.76

$$-\frac{\cos(x)^{2m+1}\sin(x)^{2m+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2}-m, m+\frac{1}{2};\ m+\frac{3}{2};\ \cos(x)^2\right)}{(2m+1)\left(\sin(x)^2\right)^{m+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)
[Out] -(cos(x)^(2*m + 1)*sin(x)^(2*m + 1)*hypergeom([1/2 - m, m + 1/2], m + 3/2,
cos(x)^2))/((2*m + 1)*(sin(x)^2)^(m + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
[Out] Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2620, 14}

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] -Cot[Pi/4 + 2*x]^2/4 + Log[Tan[Pi/4 + 2*x]]/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\frac{1+x^2}{x^3}\,dx,x,\tan\left(\frac{\pi}{4}+2x\right)\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan\left(\frac{\pi}{4}+2x\right)\right)\\&=-\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 1.41

$$\frac{1}{4}\left(-\csc^2\left(2x+\frac{\pi}{4}\right)+2\log\left(\sin\left(2x+\frac{\pi}{4}\right)\right)-2\log\left(\cos\left(\frac{1}{4}(8x+\pi)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] (-Csc[Pi/4 + 2*x]^2 - 2*Log[Cos[(Pi + 8*x)/4]] + 2*Log[Sin[Pi/4 + 2*x]])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.86, size = 71, normalized size = 2.22

$$-\frac{\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(\cos\left(\frac{1}{4}\pi+2x\right)^2\right)-\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi+2x\right)^2+\frac{1}{4}\right)-1}{4\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2*
x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 - 1
)
```

giac [B] time = 0.75, size = 132, normalized size = 4.12

$$-\frac{\left(\frac{4\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right)\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}+\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}+\frac{1}{4}\log\left(-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}\right)-\frac{1}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="giac")
```

```
[Out] -1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi +
 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*
pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1))
- 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

maple [A] time = 0.09, size = 25, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	25
default	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	25
risch	$\frac{i e^{4ix}}{\left(i e^{4ix}-1\right)^2}+\frac{\ln\left(i e^{4ix}-1\right)}{2}-\frac{\ln\left(i e^{4ix}+1\right)}{2}$	48
norman	$\frac{-\frac{1}{16}-\frac{\left(\tan^4\left(\frac{\pi}{8}+x\right)\right)}{16}}{\tan\left(\frac{\pi}{8}+x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

maxima [A] time = 0.47, size = 41, normalized size = 1.28

$$-\frac{1}{4\sin\left(\frac{1}{4}\pi+2x\right)^2}-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2-1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(1
/4*pi + 2*x)^2)
```

mupad [B] time = 0.28, size = 24, normalized size = 0.75

$$\frac{\ln\left(\tan\left(\frac{\Pi}{4}+2\,x\right)\right)}{2}-\frac{1}{4\sin\left(\frac{\Pi}{4}+2\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(Pi/4 + 2*x)*sin(Pi/4 + 2*x)^3),x)
```

```
[Out] log(tan(Pi/4 + 2*x))/2 - 1/(4*sin(Pi/4 + 2*x)^2)
```

sympy [B] time = 1.66, size = 54, normalized size = 1.69

$$-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}+\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2}-\frac{\tan^2\left(x+\frac{\pi}{8}\right)}{16}-\frac{1}{16\tan^2\left(x+\frac{\pi}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

```
[Out] -log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/2
 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

3.354 $\int \sec^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan^2(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^2(x)\tan^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.89, size = 14, normalized size = 1.75

$$-\frac{\left(\cos(x)^2-1\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

giac [A] time = 0.62, size = 6, normalized size = 0.75

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
[Out] 1/3*tan(x)^3
```

maple [A] time = 0.06, size = 11, normalized size = 1.38

method	result	size
default	$\frac{\sin^3(x)}{3\cos(x)^3}$	11
risch	$-\frac{2i\left(3\,e^{4ix}+1\right)}{3\left(1+e^{2ix}\right)^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*sin(x)^3/cos(x)^3
```

maxima [A] time = 0.49, size = 6, normalized size = 0.75

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
[Out] 1/3*tan(x)^3
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x)^2,x)
[Out] tan(x)^3/3
```

sympy [B] time = 0.07, size = 17, normalized size = 2.12

$$-\frac{\sin(x)}{3\cos(x)}+\frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**2,x)
[Out] -sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

3.355 $\int \cot^3(x) \csc(x) \, dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2606}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \cot^3(x) \csc(x) \, dx &= -\text{Subst}\left(\int \left(-1 + x^2\right) \, dx, x, \csc(x)\right) \\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^3(x) \csc(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]^3*Csc[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.84, size = 22, normalized size = 2.00

$$\frac{3 \cos(x)^2 - 2}{3 \left(\cos(x)^2 - 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="fricas")
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.60, size = 14, normalized size = 1.27

$$\frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="giac")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

maple [B] time = 0.31, size = 32, normalized size = 2.91

method	result	size
default	$-\frac{\cos^4(x)}{3\sin(x)^3} + \frac{\cos^4(x)}{3\sin(x)} + \frac{(2+\cos^2(x))\sin(x)}{3}$	32
risch	$\frac{2i(3\,e^{5ix}-2\,e^{3ix}+3\,e^{ix})}{3(e^{2ix}-1)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x),x,method=_RETURNVERBOSE)
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.56, size = 14, normalized size = 1.27

$$\frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="maxima")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

mupad [B] time = 0.32, size = 11, normalized size = 1.00

$$\frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/sin(x),x)
[Out] (sin(x)^2 - 1/3)/sin(x)^3
```

sympy [A] time = 0.10, size = 15, normalized size = 1.36

$$-\frac{1 - 3\sin^2(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x),x)
[Out] -(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

3.356 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^3(x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^3*Tan[x],x]
[Out] Could not integrate
```

fricas [A] time = 0.62, size = 6, normalized size = 0.75

$$\frac{1}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
[Out] 1/3/cos(x)^3
```

giac [A] time = 0.63, size = 6, normalized size = 0.75

$$\frac{1}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

maple [A] time = 0.06, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\left(\sec^3(x)\right)}{3}$	7
default	$\frac{\left(\sec^3(x)\right)}{3}$	7
risch	$\frac{8\,e^{3ix}}{3\left(1+e^{2ix}\right)^3}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/3*sec(x)^3
```

maxima [A] time = 0.47, size = 6, normalized size = 0.75

$$\frac{1}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

mupad [B] time = 0.32, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

3.357 $\int \cot^2(x)\csc^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\csc^3(x)\,dx &= -\frac{1}{4}\cot(x)\csc^3(x)-\frac{1}{4}\int \csc^3(x)\,dx\\ &= \frac{1}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)-\frac{1}{8}\int \csc(x)\,dx\\ &= \frac{1}{8}\tanh^{-1}(\cos(x))+\frac{1}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] time = 0.02, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)+\frac{1}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)-\frac{1}{32}\sec^2\left(\frac{x}{2}\right)-\frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/2
]^2/32 + Sec[x/2]^4/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^2(x)\csc^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.73, size = 68, normalized size = 2.62

$$-\frac{2\cos(x)^3-\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+}{16\left(\cos(x)^4-2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + (co
s(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x))/(cos(x)^4 - 2*c
os(x)^2 + 1)
```

giac [B] time = 0.62, size = 47, normalized size = 1.81

$$-\frac{\frac{1}{\cos(x)}+\cos(x)}{8\left(\left(\frac{1}{\cos(x)}+\cos(x)\right)^2-4\right)}+\frac{1}{32}\log\left(\left|\frac{1}{\cos(x)}+\cos(x)+2\right|\right)-\frac{1}{32}\log\left(\left|\frac{1}{\cos(x)}+\cos(x)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*log(abs(1/cos(x
) + cos(x) + 2)) - 1/32*log(abs(1/cos(x) + cos(x) - 2))
```

maple [A] time = 0.08, size = 36, normalized size = 1.38

method	result	size
default	$-\frac{\cos^3(x)}{4\sin(x)^4}-\frac{\cos^3(x)}{8\sin(x)^2}-\frac{\cos(x)}{8}-\frac{\ln(\csc(x)-\cot(x))}{8}$	36
risch	$-\frac{e^{7ix}+7e^{5ix}+7e^{3ix}+e^{ix}}{4\left(e^{2ix}-1\right)^4}+\frac{\ln\left(e^{ix}+1\right)}{8}-\frac{\ln\left(e^{ix}-1\right)}{8}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*csc(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(x)^3/sin(x)^4-1/8/sin(x)^2*cos(x)^3-1/8*cos(x)-1/8*ln(csc(x)-cot(x
))
```

maxima [A] time = 0.53, size = 38, normalized size = 1.46

$$-\frac{\cos(x)^3+\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{1}{16}\log\left(\cos(x)+1\right)-\frac{1}{16}\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(cos(x) + 1)
 - 1/16*log(cos(x) - 1)
```

mupad [B] time = 0.32, size = 24, normalized size = 0.92

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64}-\frac{1}{64\tan\left(\frac{x}{2}\right)^4}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/sin(x)^3,x)
```

```
[Out] tan(x/2)^4/64 - 1/(64*tan(x/2)^4) - log(tan(x/2))/8
```

sympy [A] time = 0.15, size = 41, normalized size = 1.58

$$\frac{-\cos^3(x)-\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{\log(\cos(x)-1)}{16}+\frac{\log(\cos(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*csc(x)**3,x)
```

```
[Out] (-cos(x)**3 - cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(x) - 1)/16
 + log(cos(x) + 1)/16
```

3.358 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\csc^4(x)\,dx &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\ &= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\ &= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^3(x)\csc^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]^3*Csc[x]^4,x]
[Out] Could not integrate
```

fricas [B] time = 0.69, size = 30, normalized size = 1.76

$$\frac{3\cos(x)^2-1}{12\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

giac [A] time = 0.61, size = 14, normalized size = 0.82

$$\frac{3\sin(x)^2-2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

maple [A] time = 0.07, size = 22, normalized size = 1.29

method	result	size
default	$-\frac{\cos^4(x)}{6\sin(x)^6}-\frac{\cos^4(x)}{12\sin(x)^4}$	22
norman	$\frac{-\frac{1}{384}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6}$	34
risch	$\frac{4\,e^{8ix}+\frac{8\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}-1\right)^6}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

maxima [A] time = 0.50, size = 14, normalized size = 0.82

$$\frac{3\sin(x)^2-2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

mupad [B] time = 0.07, size = 13, normalized size = 0.76

$$\frac{\frac{\sin(x)^2}{4}-\frac{1}{6}}{\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] (sin(x)^2/4 - 1/6)/sin(x)^6
```

sympy [A] time = 0.10, size = 15, normalized size = 0.88

$$-\frac{2-3\sin^2(x)}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**7,x)
```

```
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

3.359 $\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx$

Optimal. Leaf size=31

$$\frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2622, 270}

$$\frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^(13/2)*Sin[x]^5,x]
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx &= \text{Subst}\left(\int \sqrt{x} \left(-1+x^2\right)^2 \, dx, x, \sec(x)\right) \\
&= \text{Subst}\left(\int \left(\sqrt{x} - 2x^{5/2} + x^{9/2}\right) dx, x, \sec(x)\right) \\
&= \frac{2}{3} \sec^{\frac{3}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{11} \sec^{\frac{11}{2}}(x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.77

$$\frac{1}{924}(44 \cos(2x) + 77 \cos(4x) + 135) \sec^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
[Out] ((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^(13/2)*Sin[x]^5,x]
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 20, normalized size = 0.65

$$\frac{2\left(77\cos(x)^4-66\cos(x)^2+21\right)}{231\cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

giac [A] time = 0.65, size = 20, normalized size = 0.65

$$\frac{2\left(77\cos(x)^4-66\cos(x)^2+21\right)}{231\cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

maple [B] time = 0.20, size = 49, normalized size = 1.58

method	result	size
default	$\frac{\frac{32\left(\sin^8\left(\frac{x}{2}\right)\right)}{3}-\frac{64\left(\sin^6\left(\frac{x}{2}\right)\right)}{3}+\frac{96\left(\sin^4\left(\frac{x}{2}\right)\right)}{7}-\frac{64\left(\sin^2\left(\frac{x}{2}\right)\right)}{21}+\frac{64}{231}}{\left(-2\left(\sin^2\left(\frac{x}{2}\right)\right)+1\right)^{\frac{11}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^(3/2)*tan(x)^5,x,method=_RETURNVERBOSE)
[Out] 32/231/(-2*sin(1/2*x)^2+1)^(11/2)*(77*sin(1/2*x)^8-154*sin(1/2*x)^6+99*sin(
1/2*x)^4-22*sin(1/2*x)^2+2)
```

maxima [A] time = 0.54, size = 19, normalized size = 0.61

$$\frac{2}{3\cos(x)^{\frac{3}{2}}}-\frac{4}{7\cos(x)^{\frac{7}{2}}}+\frac{2}{11\cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
[Out] 2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

mupad [B] time = 1.12, size = 22, normalized size = 0.71

$$\frac{2\left(\frac{1}{\cos(x)}\right)^{11/2}\left(77\cos(x)^4-66\cos(x)^2+21\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5*(1/cos(x))^(3/2),x)
[Out] (2*(1/cos(x))^(11/2)*(77*cos(x)^4 - 66*cos(x)^2 + 21))/231
```

sympy [A] time = 178.37, size = 39, normalized size = 1.26

$$\frac{2\tan^{4}(x)\sec^{\frac{3}{2}}(x)}{11}-\frac{16\tan^{2}(x)\sec^{\frac{3}{2}}(x)}{77}+\frac{64\sec^{\frac{3}{2}}(x)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**(3/2)*tan(x)**5,x)
```

```
[Out] 2*tan(x)**4*sec(x)**(3/2)/11 - 16*tan(x)**2*sec(x)**(3/2)/77 + 64*sec(x)**(
3/2)/231
```

3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2607, 14}

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx &= \text{Subst}\left(\int x^{3/2}\left(1+x^2\right)\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^{3/2}+x^{7/2}\right)\,dx,x,\tan(x)\right)\\ &= \frac{2}{5}\tan^{\frac{5}{2}}(x)+\frac{2}{9}\tan^{\frac{9}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.05

$$\frac{2}{45}(2\cos(2x)+7)\tan^{\frac{5}{2}}(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^4*Tan[x]^(3/2),x]
[Out] Could not integrate
```

fricas [B]   time = 0.86, size = 27, normalized size = 1.29

$$-\frac{2\left(4\,\cos(x)^{4}+\cos(x)^{2}-5\right)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\,\cos(x)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
[Out] -2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

giac [A]   time = 0.60, size = 13, normalized size = 0.62

$$\frac{2}{9}\,\tan(x)^{\frac{9}{2}}+\frac{2}{5}\,\tan(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

maple [A]   time = 0.50, size = 26, normalized size = 1.24

method	result	size
default	$\frac{2\left(4\left(\cos^{2}(x)\right)+5\right)\sin(x)\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{3}{2}}}{45\cos(x)^{3}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45*(4*cos(x)^2+5)*sin(x)*(sin(x)/cos(x))^(3/2)/cos(x)^3
```

maxima [A]   time = 0.60, size = 13, normalized size = 0.62

$$\frac{2}{9}\,\tan(x)^{\frac{9}{2}}+\frac{2}{5}\,\tan(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

mupad [B]   time = 1.15, size = 32, normalized size = 1.52

$$-\frac{4\,\sqrt{\sin\left(2\,x\right)}\,\left(2\cos\left(2\,x\right)^2+5\,\cos\left(2\,x\right)-7\right)}{45\,{\left(\cos\left(2\,x\right)+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(3/2)/cos(x)^4,x)
[Out] -(4*sin(2*x)^(1/2)*(5*cos(2*x) + 2*cos(2*x)^2 - 7))/(45*(cos(2*x) + 1)^(5/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(x)\sec^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**(3/2),x)
```

```
[Out] Integral(tan(x)**(3/2)*sec(x)**4, x)
```

3.361 $\int \cot^4(x) \csc^3(x) \, dx$

Optimal. Leaf size=38

$$-\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{16} \cot(x) \csc(x)$$

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$-\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{16} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Csc[x]^3,x]

[Out] -ArcTanh[Cos[x]]/16 - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 - (Cot[x]^3*
Csc[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(x) \csc^3(x) \, dx &= -\frac{1}{6} \cot^3(x) \csc^3(x) - \frac{1}{2} \int \cot^2(x) \csc^3(x) \, dx \\
&= \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \int \csc^3(x) \, dx \\
&= -\frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{16} \int \csc(x) \, dx \\
&= -\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x)
\end{aligned}$$

Mathematica [B] time = 0.02, size = 95, normalized size = 2.50

$$-\frac{1}{384} \csc^6\left(\frac{x}{2}\right)+\frac{1}{64} \csc^4\left(\frac{x}{2}\right)-\frac{1}{64} \csc^2\left(\frac{x}{2}\right)+\frac{1}{384} \sec^6\left(\frac{x}{2}\right)-\frac{1}{64} \sec^4\left(\frac{x}{2}\right)+\frac{1}{64} \sec^2\left(\frac{x}{2}\right)+\frac{1}{16} \log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{16} \log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -1/64*Csc[x/2]^2 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log[
Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^4(x)\csc^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.68, size = 93, normalized size = 2.45

$$\frac{6\cos(x)^5+16\cos(x)^3-3\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^6-3\cos(x)^4\right.}{96\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(-1/
2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

giac [A] time = 0.61, size = 44, normalized size = 1.16

$$\frac{3\cos(x)^5+8\cos(x)^3-3\cos(x)}{48\left(\cos(x)^2-1\right)^3}-\frac{1}{32}\log\left(\cos(x)+1\right)+\frac{1}{32}\log\left(-\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(x
) + 1) + 1/32*log(-cos(x) + 1)
```

maple [A] time = 0.09, size = 52, normalized size = 1.37

method	result	size
default	$-\frac{\cos^5(x)}{6\sin(x)^6}-\frac{\cos^5(x)}{24\sin(x)^4}+\frac{\cos^5(x)}{48\sin(x)^2}+\frac{\left(\cos^3(x)\right)}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$	52
risch	$\frac{3\,e^{11ix}+47\,e^{9ix}+78\,e^{7ix}+78\,e^{5ix}+47\,e^{3ix}+3\,e^{ix}}{24\left(e^{2ix}-1\right)^6}-\frac{\ln\left(e^{ix}+1\right)}{16}+\frac{\ln\left(e^{ix}-1\right)}{16}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*csc(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x)^5+1/48*c
os(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

maxima [A] time = 0.44, size = 54, normalized size = 1.42

$$\frac{3\cos(x)^5+8\cos(x)^3-3\cos(x)}{48\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}-\frac{1}{32}\log\left(\cos(x)+1\right)+\frac{1}{32}\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

mupad [B] time = 0.28, size = 57, normalized size = 1.50

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16}+\frac{\frac{\tan\left(\frac{x}{2}\right)^4}{128}+\frac{\tan\left(\frac{x}{2}\right)^2}{128}-\frac{1}{384}}{\tan\left(\frac{x}{2}\right)^6}-\frac{\tan\left(\frac{x}{2}\right)^2}{128}-\frac{\tan\left(\frac{x}{2}\right)^4}{128}+\frac{\tan\left(\frac{x}{2}\right)^6}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/sin(x)^3,x)
```

```
[Out] log(tan(x/2))/16 + (tan(x/2)^2/128 + tan(x/2)^4/128 - 1/384)/tan(x/2)^6 - t
an(x/2)^2/128 - tan(x/2)^4/128 + tan(x/2)^6/384
```

sympy [A] time = 0.16, size = 56, normalized size = 1.47

$$-\frac{-3\cos^5(x)-8\cos^3(x)+3\cos(x)}{48\cos^6(x)-144\cos^4(x)+144\cos^2(x)-48}+\frac{\log(\cos(x)-1)}{32}-\frac{\log(\cos(x)+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*csc(x)**3,x)
```

```
[Out] -(-3*cos(x)**5 - 8*cos(x)**3 + 3*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 14
4*cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/32
```

3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=76

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2611, 3768, 3770}

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4 + (Sec[Pi
/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx &= \frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{4}\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx\\ &= -\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{8}\int \csc\left(\right.\\ &= -\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.97

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^4\left(\frac{1}{4}(2x+\pi)\right)-\frac{1}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^2\left(\frac{1}{4}(2x+\pi)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 + (
Sec[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.73, size = 82, normalized size = 1.08

$$-\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2\right.}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/4*pi + 1/2
*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1/2*x)^2 - 2)*sin(1/4
*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4
```

giac [A] time = 0.95, size = 95, normalized size = 1.25

$$\frac{\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)^2-4\right)}-\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+2\right|\right)+\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x) +
 sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4
*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*
x) - 2))
```

maple [A] time = 0.56, size = 76, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4}+\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$	76
default	$\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4}+\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$	76
risch	$\frac{i\left(-(-1)^{\frac{3}{4}}e^{\frac{7ix}{2}}+7(-1)^{\frac{1}{4}}e^{\frac{5ix}{2}}+7(-1)^{\frac{3}{4}}e^{\frac{3ix}{2}}-(-1)^{\frac{1}{4}}e^{\frac{ix}{2}}\right)}{2\left(ie^{ix}+1\right)^4}+\frac{\ln\left(e^{\frac{i(\pi+2x)}{4}}-i\right)}{4}-\frac{\ln\left(e^{\frac{i(\pi+2x)}{4}}+i\right)}{4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)^3/cos(1/4
*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/2*x)+tan(1/4*Pi+1/2*
x))
```

maxima [A] time = 0.67, size = 74, normalized size = 0.97

$$\frac{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^3+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4-2\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2+1\right)}-\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1/2*x)^4 -
2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2*x) + 1) + 1/8*log(s
in(1/4*pi + 1/2*x) - 1)
```

mupad [B] time = 6.38, size = 75, normalized size = 0.99

$$\frac{2\left(\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^7}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^5}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^3}{4}+\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)}{4}\right)}{\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^2-1\right)^4}-\frac{\operatorname{atanh}\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(Pi/4 + x/2)^2/cos(Pi/4 + x/2)^3,x)
```

```
[Out] (2*(tan(Pi/8 + x/4)/4 + (7*tan(Pi/8 + x/4)^3)/4 + (7*tan(Pi/8 + x/4)^5)/4 +
 tan(Pi/8 + x/4)^7/4))/(tan(Pi/8 + x/4)^2 - 1)^4 - atanh(tan(Pi/8 + x/4))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

```
[Out] Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)
```

3.363 $\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$

Optimal. Leaf size=88

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\sin(x))+4ax+4a\cot(x)+(4-a)a\log(\cos(x))+\frac{x}{2}+\cos^4(x)+2\cos$$

Rubi [A] time = 0.55, antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1805, 1802, 635, 203, 260}

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\tan(x))+\frac{1}{2}(8a+1)x+4a\cot(x)+4(a+1)\log(\cos(x))+\cos^4(x)(1-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] ((1 + 8*a)*x)/2 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + 4*(1 + a)*Log[Cos[x]] + (
4 + a^2)*Log[Tan[x]] + Cos[x]^4*(1 - Tan[x]) + a^2*Tan[x] + (a^2*Tan[x]^3)/
3 + (Cos[x]^2*(4 + Tan[x]))/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)\left(a-2x+2ax^2+ax^4\right)^2}{x^3\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)$$

$$= \cos^4(x)(1-\tan(x)) - \frac{1}{4}\text{Subst}\left(\int \frac{-4a^2+16ax-4\left(4+3a^2\right)x^2 -$$

$$= \cos^4(x)(1-\tan(x)) + \frac{1}{2}\cos^2(x)(4+\tan(x)) + \frac{1}{8}\text{Subst}\left(\int \frac{8a^2 -$$

$$= \cos^4(x)(1-\tan(x)) + \frac{1}{2}\cos^2(x)(4+\tan(x)) + \frac{1}{8}\text{Subst}\left(\int \left(8a^2 +\right.\right.$$

$$= 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + \left(4+a^2\right)\log(\tan(x)) + \cos^4(x)(1-\tan($$

$$= 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + \left(4+a^2\right)\log(\tan(x)) + \cos^4(x)(1-\tan($$

$$= \frac{1}{2}(1+8a)x + 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + 4(1+a)\log(\cos(x)) + (4$$

Mathematica [A] time = 2.02, size = 127, normalized size = 1.44

$$-\frac{2\sin(x)\cos^3(x)\left(\sin(2x)-a\sec^2(x)\right)^2\left(-8a^2(\cos(2x)+2)\sec^2(x)-3\cot(x)\left(-4a^2\csc^2(x)+8a^2\log(\sin(x))\right.\right.}{3(-4a+2\text{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] (-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(2
 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] - 4
*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] + 8*a
^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.95, size = 178, normalized size = 2.02

$$\frac{24\cos(x)^9 + 24\cos(x)^7 + 3(4(8a+1)x-27)\cos(x)^5 + 3\left(4a^2-4(8a+1)x+11\right)\cos(x)^3 - 12\left(\left(a^2-4a\right)\text{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5 + 3*(4*a^
2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(x)^5 - (a^2 - 4*a)*c
os(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)^5 - (a^2 + 4)*cos(x)^3)*log(-
```

```
1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*cos(x
)^4 + 2*a^2*cos(x)^2 + 2*a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

giac [A] time = 0.64, size = 149, normalized size = 1.69

$$\frac{1}{3}a^2\tan(x)^3+a^2\tan(x)+\frac{1}{2}(8a+1)x-2(a+1)\log\left(\tan(x)^2+1\right)+\left(a^2+4\right)\log\left(|\tan(x)|\right)-\frac{a^2\tan(x)^6-4a\tan(x)^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*log(tan(x)^2 +
1) + (a^2 + 4)*log(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a*tan(x)^6 + 3*a^2*
tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)^5 + 3*a^2*tan(x)^2 - 16*a*t
an(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 + tan(x)^3 + a^2 - 8*a*tan(x) - 6*tan(x
)^2)/(tan(x)^3 + tan(x))^2
```

maple [B] time = 0.56, size = 210, normalized size = 2.39

method	result
default	$\frac{2(\cos^8(x))}{\sin(x)^2}+2\left(\cos^6(x)\right)+\cos^4(x)+2\left(\cos^2(x)\right)+4\ln\left(\sin(x)\right)-\frac{4(\cos^7(x))}{\sin(x)}-4\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\right.$
risch	$\frac{x}{2}-4ixa+4ax+\frac{ie^{4ix}}{16}+\frac{e^{4ix}}{16}-\frac{ie^{-4ix}}{16}+\frac{3e^{2ix}}{4}+\frac{3e^{-2ix}}{4}+\frac{e^{-4ix}}{16}-4ix+\frac{2a(12ie^{8ix}+3ae^{8ix}+6iae^{6ix}+24ie^{6ix}+9ae^{6ix}-}{3(e^{2ix}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/sin(x)^2*cos(x)^8+2*cos(x)^6+cos(x)^4+2*cos(x)^2+4*ln(sin(x))-4/sin(x)*co
s(x)^7-4*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+1/2*x-2/sin(x)^2*cos(x)
^6+8/sin(x)*cos(x)^5+8*(cos(x)^3+3/2*cos(x))*sin(x)-4*a*(-1/2*cot(x)^2-ln(s
in(x)))-4*a*(-cot(x)-x)-4*cot(x)-4*a/sin(x)^2+a^2*(-1/2/sin(x)^2+ln(tan(x))
)-a^2*(1/sin(x)/cos(x)-2*cot(x))-4*a*(-1/2/sin(x)^2+ln(tan(x)))+a^2*(1/3/si
n(x)/cos(x)^3+4/3/sin(x)/cos(x)-8/3*cot(x))
```

maxima [A] time = 1.35, size = 115, normalized size = 1.31

$$\frac{1}{3}\left(\tan(x)^3+3\tan(x)\right)a^2-\frac{1}{2}a^2\left(\frac{1}{\sin(x)^2}+\log\left(\sin(x)^2-1\right)-\log\left(\sin(x)^2\right)\right)+4a\left(x+\frac{1}{\tan(x)}\right)+2a\log\left(-\sin(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) - l
og(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8*c
os(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

mupad [B] time = 0.38, size = 133, normalized size = 1.51

$$a^2\tan(x)-\frac{\tan(x)^4\left(\frac{a^2}{2}-2\right)-4a\tan(x)+\frac{a^2}{2}-\tan(x)^5\left(4a+\frac{1}{2}\right)-\tan(x)^3\left(8a-\frac{1}{2}\right)+\tan(x)^2\left(a^2-3\right)}{\tan(x)^6+2\tan(x)^4+\tan(x)^2}-\ln(\tan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^3 + 1)*(sin(2*x) - a/cos(x)^2)^2,x)
```

```
[Out] a^2*tan(x) - (tan(x)^4*(a^2/2 - 2) - 4*a*tan(x) + a^2/2 - tan(x)^5*(4*a + 1
/2) - tan(x)^3*(8*a - 1/2) + tan(x)^2*(a^2 - 3))/(tan(x)^2 + 2*tan(x)^4 + t
an(x)^6) - log(tan(x) - 1i)*(a*(2 + 2i) + (2 + 1i/4)) - log(tan(x) + 1i)*(a
*(2 - 2i) + (2 - 1i/4)) + (a^2*tan(x)^3)/3 + log(tan(x))*(a^2 + 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

```
[Out] Timed out
```

3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx$

Optimal. Leaf size=70

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\,c$$

Rubi [A] time = 0.15, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4401, 2637, 2668, 2669, 2635, 8, 641}

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\,c$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] - (99*Cos[x
]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4)/8 - (3*
Sin[x]^5)/80
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \int \left(4-3\cos(x)+2(-4+3\cos(x))\sin(x)-\frac{3}{2}(-4+3\cos(x))\sin^2(x)+\frac{1}{2}\right.$$
$$= 4x-\frac{1}{16}\int(-4+3\cos(x))\sin^4(x)\,dx+\frac{1}{2}\int(-4+3\cos(x))\sin^3(x)\,dx-$$
$$= 4x-3\sin(x)-\frac{3\sin^3(x)}{2}-\frac{3\sin^5(x)}{80}-\frac{1}{54}\text{Subst}\left(\int(-4+x)\left(9-x^2\right)dx\right.$$
$$= 4x+8\cos(x)-3\cos^2(x)-3\sin(x)-3\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos$$
$$= 7x+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-\frac{3\sin}{}$$
$$= \frac{227x}{32}+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-$$

Mathematica [A] time = 0.09, size = 74, normalized size = 1.06

$$\frac{227x}{32}-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x)+\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x)+\frac{3}{64}\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*Cos[4*x])/6
4 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])/256 + Sin[4*x]/128
- (3*Sin[5*x])/1280
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 54, normalized size = 0.77

$$\frac{3}{8}\cos(x)^4-\frac{2}{3}\cos(x)^3-\frac{15}{4}\cos(x)^2-\frac{1}{160}\left(6\cos(x)^4-10\cos(x)^3-252\cos(x)^2+505\cos(x)+726\right)\sin(x)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")
```

```
[Out] 3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x)
^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)
```

giac [A] time = 0.60, size = 54, normalized size = 0.77

$$\frac{227}{32}x+\frac{3}{64}\cos(4x)-\frac{1}{6}\cos(3x)-\frac{27}{16}\cos(2x)+\frac{19}{2}\cos(x)-\frac{3}{1280}\sin(5x)+\frac{1}{128}\sin(4x)+\frac{99}{256}\sin(3x)-\frac{25}{16}\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="giac")
```

```
[Out] 227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3/
1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/128
*sin(x)
```

maple [A] time = 0.38, size = 55, normalized size = 0.79

method	result
risch	$\frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{531\sin(x)}{128}-\frac{3\sin(5x)}{1280}+\frac{3\cos(4x)}{64}+\frac{\sin(4x)}{128}-\frac{\cos(3x)}{6}+\frac{99\sin(3x)}{256}-\frac{27\cos(2x)}{16}-\frac{25\sin(2x)}{16}$
default	$\frac{227x}{32}+8\cos(x)-3\cos(x)\sin(x)+\frac{2\left(2+\sin^2(x)\right)\cos(x)}{3}-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{16}-3\sin(x)-3\left(\cos^2(x)\right)-\frac{3(}{}$
norman	$\frac{-29\left(\tan^8\left(\frac{x}{2}\right)\right)-\frac{57\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{5}+\frac{97\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{128\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+\frac{227x}{32}-\frac{391\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}-\frac{306\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}-\frac{185\left(\tan^7\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^9\left(\frac{x}{2}\right)\right)}{16}+\frac{1135x\left(\tan^2\left(\frac{x}{2}\right)\right)}{32}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-3*cos(x))*(1-1/2*sin(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 227/32*x+19/2*cos(x)-531/128*sin(x)-3/1280*sin(5*x)+3/64*cos(4*x)+1/128*sin
(4*x)-1/6*cos(3*x)+99/256*sin(3*x)-27/16*cos(2*x)-25/16*sin(2*x)
```

maxima [A] time = 0.45, size = 54, normalized size = 0.77

$$-\frac{3}{80}\sin(x)^5+\frac{3}{8}\sin(x)^4-\frac{2}{3}\cos(x)^3-\frac{3}{2}\sin(x)^3-3\cos(x)^2+\frac{227}{32}x+10\cos(x)+\frac{1}{128}\sin(4x)-\frac{25}{16}\sin(2x)-3\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")
```

```
[Out] -3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*cos(x)^2 +
227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x) - 3*sin(x)
```

mupad [B] time = 0.38, size = 94, normalized size = 1.34

$$-\frac{6\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^9}{5}+6\cos\left(\frac{x}{2}\right)^8+\frac{17\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^7}{5}-\frac{52\cos\left(\frac{x}{2}\right)^6}{3}+\frac{93\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^5}{10}+2\cos\left(\frac{x}{2}\right)^4-\frac{191\sin\left(\frac{x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*cos(x) - 4)*(sin(x)/2 - 1)^4,x)
```

```
[Out] (227*x)/32 - (191*cos(x/2)^3*sin(x/2))/8 + (93*cos(x/2)^5*sin(x/2))/10 + (1
7*cos(x/2)^7*sin(x/2))/5 - (6*cos(x/2)^9*sin(x/2))/5 + 28*cos(x/2)^2 + 2*co
s(x/2)^4 - (52*cos(x/2)^6)/3 + 6*cos(x/2)^8 + (3*cos(x/2)*sin(x/2))/16
```

sympy [A] time = 1.60, size = 148, normalized size = 2.11

$$\frac{3x\sin^4(x)}{32}+\frac{3x\sin^2(x)\cos^2(x)}{16}+3x\sin^2(x)+\frac{3x\cos^4(x)}{32}+3x\cos^2(x)+4x-\frac{3\sin^5(x)}{80}+\frac{3\sin^4(x)}{8}-\frac{5\sin^3(x)\cos(}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

```
[Out] 3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)*
*4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)**4/8 - 5*sin(x)**3*
cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x) - 3*sin(x)*cos(x)**3/32 - 3*
sin(x)*cos(x) - 3*sin(x) + 4*cos(x)**3/3 - 3*cos(x)**2 + 8*cos(x)
```

3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$

Optimal. Leaf size=33

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3528, 3525, 3475}

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin[
x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx &= (3-2\cot(x))^3+\int \left(-\frac{9}{2}-10\cot(x)\right)(3-2\cot(x))^2\,dx\\
&= 5(3-2\cot(x))^2+(3-2\cot(x))^3+\int \left(-\frac{67}{2}-21\cot(x)\right)(3-2\cot(x))\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\int \cot(x)\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.88

$$-\frac{285x}{2}-148\cot(x)+56\csc^2(x)+4\log(\sin(x))-8\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 - 148*Cot[x] + 56*Csc[x]^2 - 8*Cot[x]*Csc[x]^2 + 4*Log[Sin[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.86, size = 71, normalized size = 2.15

$$\frac{4\left(\cos(2x)-1\right)\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)\sin(2x)-296\cos(2x)^2-(285x\cos(2x)-285x+224)\sin(2x)+32\cos(2x)+328}{2\left(\cos(2x)-1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos(2*x)^2 -
(285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) + 328)/((cos(2*x) - 1
)*sin(2*x))
```

giac [B] time = 0.67, size = 75, normalized size = 2.27

$$\tan\left(\frac{1}{2}x\right)^3+14\tan\left(\frac{1}{2}x\right)^2-\frac{285}{2}x-\frac{22\tan\left(\frac{1}{2}x\right)^3+225\tan\left(\frac{1}{2}x\right)^2-42\tan\left(\frac{1}{2}x\right)+3}{3\tan\left(\frac{1}{2}x\right)^3}-4\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+4\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)+75\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="giac")
```

```
[Out] tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 + 225*tan(1
/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*log(tan(1/2*x)^2 + 1) + 4*log
(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

maple [A] time = 0.06, size = 35, normalized size = 1.06

method	result	size
derivativedivides	$-8\left(\cot^3(x)\right)+56\left(\cot^2(x)\right)-156\cot(x)-2\ln\left(\cot^2(x)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$	35
default	$-8\left(\cot^3(x)\right)+56\left(\cot^2(x)\right)-156\cot(x)-2\ln\left(\cot^2(x)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$	35
norman	$\frac{-8-156\left(\tan^2(x)\right)-\frac{285x\left(\tan^3(x)\right)}{2}+56\tan(x)}{\tan(x)^3}+4\ln(\tan(x))-2\ln\left(1+\tan^2(x)\right)$	40
risch	$-\frac{285x}{2}-4ix+\frac{\left(-\frac{224}{1873}-\frac{264i}{1873}\right)\left(1873\,e^{4ix}-1260i e^{2ix}-3358\,e^{2ix}+1221+1036i\right)}{\left(e^{2ix}-1\right)^3}+4\ln\left(e^{2ix}-1\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/2-3*cot(x))*(3-2*cot(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -8*cot(x)^3+56*cot(x)^2-156*cot(x)-2*ln(cot(x)^2+1)+285/4*Pi-285/2*arccot(c
ot(x))
```

maxima [A] time = 1.27, size = 36, normalized size = 1.09

$$-\frac{285}{2}\,x-\frac{4\left(39\,\tan(x)^{2}-14\,\tan(x)+2\right)}{\tan(x)^{3}}-2\,\log\left(\tan(x)^{2}+1\right)+4\,\log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")
```

```
[Out] -285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x)^2 + 1) +
 4*log(tan(x))
```

mupad [B] time = 0.52, size = 75, normalized size = 2.27

$$x\left(-\frac{285}{2}-4\mathrm{i}\right)+4\,\ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)+\frac{64\mathrm{i}}{3\,\mathrm{e}^{x\,2\mathrm{i}}-3\,\mathrm{e}^{x\,4\mathrm{i}}+\mathrm{e}^{x\,6\mathrm{i}}-1}+\frac{-224+96\mathrm{i}}{1+\mathrm{e}^{x\,4\mathrm{i}}-2\,\mathrm{e}^{x\,2\mathrm{i}}}+\frac{-224-264\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cot(x) - 3)^3*(3*cot(x) - 1/2),x)
```

```
[Out] 4*log(exp(x*2i) - 1) - x*(285/2 + 4i) + 64i/(3*exp(x*2i) - 3*exp(x*4i) + ex
p(x*6i) - 1) - (224 - 96i)/(exp(x*4i) - 2*exp(x*2i) + 1) - (224 + 264i)/(ex
p(x*2i) - 1)
```

sympy [A] time = 0.58, size = 39, normalized size = 1.18

$$-\frac{285x}{2}-2\log\left(\tan^{2}(x)+1\right)+4\log\left(\tan(x)\right)-\frac{156}{\tan(x)}+\frac{56}{\tan^{2}(x)}-\frac{8}{\tan^{3}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

```
[Out] -285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56/tan(x)**2
 - 8/tan(x)**3
```

3.366 $\int \cos(5x)\sec^5(x)\,dx$

Optimal. Leaf size=16

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1153, 203}

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \cos(5x)\sec^5(x)\,dx &= \text{Subst}\left(\int \frac{1-10x^2+5x^4}{1+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(-15+5x^2+\frac{16}{1+x^2}\right)dx,x,\tan(x)\right)\\
&= -15\tan(x)+\frac{5\tan^3(x)}{3}+16\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= 16x-15\tan(x)+\frac{5\tan^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.25

$$16x-\frac{50\tan(x)}{3}+\frac{5}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(5x)\sec^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[5*x]*Sec[x]^5,x]
[Out] Could not integrate
```

fricas [A] time = 0.68, size = 26, normalized size = 1.62

$$\frac{48\,x\cos(x)^3-5\left(10\,\cos(x)^2-1\right)\sin(x)}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
[Out] 1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

giac [A] time = 0.62, size = 14, normalized size = 0.88

$$\frac{5}{3}\,\tan(x)^3+16\,x-15\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

maple [A] time = 0.33, size = 21, normalized size = 1.31

method	result	size
default	$16x-5\left(-\frac{2}{3}-\frac{\left(\sec^2(x)\right)}{3}\right)\tan(x)-20\tan(x)$	21
risch	$16x-\frac{20i\left(6\,e^{4ix}+9\,e^{2ix}+5\right)}{3\left(1+e^{2ix}\right)^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/cos(x)^5,x,method=_RETURNVERBOSE)
[Out] 16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

maxima [A] time = 1.28, size = 14, normalized size = 0.88

$$\frac{5}{3}\,\tan(x)^3+16\,x-15\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

mupad [B] time = 0.31, size = 26, normalized size = 1.62

$$\frac{48\,x\cos(x)^3-50\,\sin(x)\cos(x)^2+5\,\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/cos(x)^5,x)
[Out] (5*sin(x) + 48*x*cos(x)^3 - 50*cos(x)^2*sin(x))/(3*cos(x)^3)
```

sympy [A] time = 17.46, size = 24, normalized size = 1.50

$$16x - \frac{20\sin(x)}{\cos(x)} + \frac{5\tan^3(x)}{3} + 5\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)**5,x)
```

```
[Out] 16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)
```

3.367 $\int \cos(4x)\sec(x)\,dx$

Optimal. Leaf size=12

$$\tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4364, 1153, 206}

$$\tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(4x)\sec(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{1-x^2}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(-8x^2+\frac{1}{1-x^2}\right)dx,x,\sin(x)\right)\\ &= -\frac{8}{3}\sin^3(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(x)\right)\\ &= \tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\tanh^{-1}(\sin(x))-\frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[x],x]
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(4x)\sec(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[4*x]*Sec[x],x]
[Out] Could not integrate
```

fricas [B] time = 0.80, size = 27, normalized size = 2.25

$$\frac{8}{3}\left(\cos(x)^2 - 1\right)\sin(x) + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(-\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="fricas")
[Out] 8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

giac [B] time = 0.60, size = 23, normalized size = 1.92

$$-\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(-\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="giac")
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

maple [B] time = 0.33, size = 22, normalized size = 1.83

method	result	size
default	$\ln\left(\sec(x) + \tan(x)\right) + \frac{8\left(2+\cos^2(x)\right)\sin(x)}{3} - 8\sin(x)$	22
risch	$i\mathrm{e}^{ix} - i\mathrm{e}^{-ix} + \ln\left(\mathrm{e}^{ix} + i\right) - \ln\left(\mathrm{e}^{ix} - i\right) + \frac{2\sin(3x)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x),x,method=_RETURNVERBOSE)
[Out] ln(sec(x)+tan(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

maxima [B] time = 0.45, size = 21, normalized size = 1.75

$$-\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(\sin(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="maxima")
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

mupad [B] time = 0.27, size = 10, normalized size = 0.83

$$\operatorname{atanh}\left(\sin(x)\right) - \frac{8\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x),x)
```

```
[Out] atanh(sin(x)) - (8*sin(x)^3)/3
```

sympy [A] time = 1.70, size = 24, normalized size = 2.00

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}-\frac{8\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

3.368 $\int \cos(x)\cos(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[4*x],x]
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(4x)\,dx=\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[4*x],x]
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cos(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*Cos[4*x],x]
[Out] Could not integrate
```

fricas [A] time = 0.59, size = 18, normalized size = 1.06

$$\frac{1}{15}\left(24\cos(x)^4-8\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

giac [A] time = 0.61, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin\left(5x\right) + \frac{1}{6}\sin\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

maple [A] time = 0.12, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{2\left(\tan^2(2x)\right)\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin\left(5x\right) + \frac{1}{6}\sin\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

mupad [B] time = 0.19, size = 13, normalized size = 0.76

$$\frac{\sin\left(3x\right)}{6} + \frac{\sin\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*cos(x),x)
[Out] sin(3*x)/6 + sin(5*x)/10
```

sympy [A] time = 0.55, size = 20, normalized size = 1.18

$$-\frac{\sin\left(x\right)\cos\left(4x\right)}{15} + \frac{4\sin\left(4x\right)\cos\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x)
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

3.369 $\int \cos(4x)\sec^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{35}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)-\frac{29}{8}\tan(x)\sec(x)$$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4364, 1157, 385, 206}

$$\frac{35}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)-\frac{29}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \cos(4x)\sec^5(x)\,dx = \text{Subst}\left(\int \frac{1-8x^2+8x^4}{\left(1-x^2\right)^3}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\text{Subst}\left(\int \frac{-3+32x^2}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{35}{8}\tanh^{-1}(\sin(x)) - \frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)$$

Mathematica [A] time = 0.05, size = 26, normalized size = 1.00

$$\frac{1}{8}\left(35\tanh^{-1}(\sin(x)) - 27\tan(x)\sec^3(x) + 29\tan^3(x)\sec(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[x]^5,x]
[Out] (35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(4x)\sec^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[4*x]*Sec[x]^5,x]
[Out] Could not integrate
```

fricas [B] time = 0.91, size = 43, normalized size = 1.65

$$\frac{35\cos(x)^4\log(\sin(x)+1) - 35\cos(x)^4\log(-\sin(x)+1) - 2\left(29\cos(x)^2-2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
[Out] 1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1) - 2*(29*co
s(x)^2 - 2)*sin(x))/cos(x)^4
```

giac [A] time = 0.59, size = 38, normalized size = 1.46

$$\frac{29\sin(x)^3 - 27\sin(x)}{8\left(\sin(x)^2-1\right)^2} + \frac{35}{16}\log(\sin(x)+1) - \frac{35}{16}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
[Out] 1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*log(sin(x) + 1) - 35
/16*log(-sin(x) + 1)
```

maple [A] time = 0.36, size = 31, normalized size = 1.19

method	result	size
default	$-\left(-\frac{\left(\sec^3(x)\right)}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{35\ln(\sec(x)+\tan(x))}{8}-4\sec(x)\tan(x)$	31
risch	$\frac{i\left(29\,e^{7ix}+21\,e^{5ix}-21\,e^{3ix}-29\,e^{ix}\right)}{4\left(1+e^{2ix}\right)^4}+\frac{35\ln\left(e^{ix}+i\right)}{8}-\frac{35\ln\left(e^{ix}-i\right)}{8}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(sec(x)+tan(x))-4*sec(x)*tan(x)
```

maxima [B] time = 0.45, size = 54, normalized size = 2.08

$$\frac{5\,\sin(x)^3-3\,\sin(x)}{8\left(\sin(x)^4-2\,\sin(x)^2+1\right)}+\frac{3\,\sin(x)}{\sin(x)^2-1}+\frac{35}{16}\log\left(\sin(x)+1\right)-\frac{35}{16}\log\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x)/(sin(x)^
2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

mupad [B] time = 0.23, size = 33, normalized size = 1.27

$$\frac{35\,\mathrm{atanh}\left(\sin(x)\right)}{8}-\frac{\frac{27\,\sin(x)}{8}-\frac{29\,\sin(x)^3}{8}}{\sin(x)^4-2\,\sin(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x)^5,x)
```

```
[Out] (35*atanh(sin(x)))/8 - ((27*sin(x))/8 - (29*sin(x)^3)/8)/(sin(x)^4 - 2*sin(
x)^2 + 1)
```

sympy [B] time = 17.15, size = 75, normalized size = 2.88

$$-\frac{35\log\left(\sin\left(x\right)-1\right)}{16}+\frac{35\log\left(\sin\left(x\right)+1\right)}{16}-\frac{3\sin^3\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5\sin\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{8\sin\left(x\right)}{2\sin^2\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)**5,x)
```

```
[Out] -35*log(sin(x) - 1)/16 + 35*log(sin(x) + 1)/16 - 3*sin(x)**3/(8*sin(x)**4 -
 16*sin(x)**2 + 8) + 5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 8*sin(x)/(
2*sin(x)**2 - 2)
```

3.370 $\int \cos^4(x)\cos(4x)\,dx$

Optimal. Leaf size=38

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2637}

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Cos[4*x],x]
```

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\cos(4x)\,dx &= \int \left(\frac{1}{16}+\frac{1}{4}\cos(2x)+\frac{3}{8}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{16}\cos(8x)\right)dx\\ &= \frac{x}{16}+\frac{1}{16}\int \cos(8x)\,dx+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(6x)\,dx+\frac{3}{8}\int \cos(4x)\,dx\\ &= \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Cos[4*x],x]
```

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^4(x)\cos(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^4*Cos[4*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.85, size = 31, normalized size = 0.82

$$\frac{1}{48}\left(48\cos(x)^{7}-8\cos(x)^{5}+2\cos(x)^{3}+3\cos(x)\right)\sin(x)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

giac [A] time = 0.63, size = 28, normalized size = 0.74

$$\frac{1}{16}x+\frac{1}{128}\sin(8x)+\frac{1}{24}\sin(6x)+\frac{3}{32}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)
```

maple [A] time = 0.15, size = 29, normalized size = 0.76

method	result	size
default	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29
risch	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cos(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

maxima [A] time = 0.46, size = 30, normalized size = 0.79

$$-\frac{1}{6}\sin(2x)^{3}+\frac{1}{16}x+\frac{1}{128}\sin(8x)+\frac{3}{32}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.30, size = 36, normalized size = 0.95

$$\frac{x}{16}+\frac{\frac{\tan(x)^{7}}{16}+\frac{11\tan(x)^{5}}{48}+\frac{5\tan(x)^{3}}{48}+\frac{15\tan(x)}{16}}{\left(\tan(x)^{2}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*cos(x)^4,x)
```

```
[Out] x/16 + ((15*tan(x))/16 + (5*tan(x)^3)/48 + (11*tan(x)^5)/48 + tan(x)^7/16)/
(tan(x)^2 + 1)^4
```

sympy [B] time = 24.53, size = 139, normalized size = 3.66

$$\frac{x\sin^4(x)\cos(4x)}{16} - \frac{x\sin^3(x)\sin(4x)\cos(x)}{4} - \frac{3x\sin^2(x)\cos^2(x)\cos(4x)}{8} + \frac{x\sin(x)\sin(4x)\cos^3(x)}{4} + \frac{x\cos^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cos(4*x),x)
```

```
[Out] x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*cos
(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/16
 - sin(x)**4*sin(4*x)/24 - 5*sin(x)**3*cos(x)*cos(4*x)/48 - 11*sin(x)*cos(x
)**3*cos(4*x)/48 + 7*sin(4*x)*cos(x)**4/24
```

3.371 $\int \cos(5x) \csc^5(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4366, 1247, 698}

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Csc[x]^5,x]
```

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \cos(5x)\csc^5(x)\,dx &= -\text{Subst}\left(\int \frac{x\left(5-20x^2+16x^4\right)}{\left(1-x^2\right)^3}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{5-20x+16x^2}{(1-x)^3}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^3}-\frac{12}{(-1+x)^2}-\frac{16}{-1+x}\right)dx,x,\cos^2(x)\right)\right)\\
&= 6\csc^2(x)-\frac{\csc^4(x)}{4}+16\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Csc[x]^5,x]
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(5x)\csc^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[5*x]*Csc[x]^5,x]
[Out] Could not integrate
```

fricas [B] time = 0.97, size = 43, normalized size = 2.15

$$-\frac{24\,\cos(x)^2 - 64\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\,\sin(x)\right) - 23}{4\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
[Out] -1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)) - 23)/(c
os(x)^4 - 2*cos(x)^2 + 1)
```

giac [A] time = 0.63, size = 21, normalized size = 1.05

$$\frac{24\,\sin(x)^2 - 1}{4\,\sin(x)^4} + 16\,\log\left(|\sin(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
[Out] 1/4*(24*sin(x)^2 - 1)/sin(x)^4 + 16*log(abs(sin(x)))
```

maple [A] time = 0.14, size = 35, normalized size = 1.75

method	result	size
default	$-\frac{5}{4\sin(x)^4} + \frac{5\left(\cos^4(x)\right)}{\sin(x)^4} - 4\left(\cot^4(x)\right) + 8\left(\cot^2(x)\right) + 16\ln\left(\sin(x)\right)$	35
risch	$-16ix - \frac{4\left(6\,e^{6ix} - 11\,e^{4ix} + 6\,e^{2ix}\right)}{\left(e^{2ix} - 1\right)^4} + 16\ln\left(e^{2ix} - 1\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/sin(x)^5,x,method=_RETURNVERBOSE)
[Out] -5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin(x))
```

maxima [A] time = 0.43, size = 33, normalized size = 1.65

$$\frac{5}{\sin(x)^2} + \frac{4\,\sin(x)^2 - 1}{4\,\sin(x)^4} + \frac{11}{2}\,\log\left(\sin(x)^2\right) + 5\,\log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) + 5*log(sin
(x))
```

mupad [B] time = 0.07, size = 21, normalized size = 1.05

$$8\,\ln\left(\sin(x)^2\right)+\frac{6\sin(x)^2-\frac{1}{4}}{\sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/sin(x)^5,x)
```

```
[Out] 8*log(sin(x)^2) + (6*sin(x)^2 - 1/4)/sin(x)^4
```

sympy [A] time = 24.84, size = 22, normalized size = 1.10

$$8\log\left(\sin^2(x)\right)+\frac{6}{\sin^2(x)}-\frac{1}{4\sin^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)**5,x)
```

```
[Out] 8*log(sin(x)**2) + 6/sin(x)**2 - 1/(4*sin(x)**4)
```

3.372 $\int \csc^4(x) \sin(4x) \, dx$

Optimal. Leaf size=12

$$-2\csc^2(x) - 8\log(\sin(x))$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$-2\csc^2(x) - 8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4*Sin[4*x],x]
```

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(x)\sin(4x)\,dx &= \text{Subst}\left(\int \frac{4-8x^2}{x^3}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{4}{x^3}-\frac{8}{x}\right)dx, x, \sin(x)\right)\\
&= -2\csc^2(x) - 8\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-2\csc^2(x) - 8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4*Sin[4*x],x]
```

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^4(x)\sin(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^4*Sin[4*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.67, size = 25, normalized size = 2.08

$$-\frac{2\left(4\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\sin(x)\right)-1\right)}{\cos(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
[Out] -2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

giac [A] time = 0.60, size = 13, normalized size = 1.08

$$-\frac{2}{\sin(x)^2} - 8 \log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
[Out] -2/sin(x)^2 - 8*log(abs(sin(x)))
```

maple [A] time = 0.13, size = 19, normalized size = 1.58

method	result	size
default	$\frac{2}{\sin(x)^2} - 4\left(\cot^2(x)\right) - 8\ln(\sin(x))$	19
risch	$8ix + \frac{8\,e^{2ix}}{\left(e^{2ix}-1\right)^2} - 8\ln\left(e^{2ix}-1\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

maxima [A] time = 0.43, size = 19, normalized size = 1.58

$$-\frac{2}{\sin(x)^2} - 2 \log\left(\sin(x)^2\right) - 4 \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
[Out] -2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

mupad [B] time = 0.30, size = 35, normalized size = 2.92

$$8 \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 8 \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{2\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x)^4,x)
[Out] 8*log(tan(x/2)^2 + 1) - 8*log(tan(x/2)) - 1/(2*tan(x/2)^2) - tan(x/2)^2/2
```

sympy [A] time = 5.89, size = 14, normalized size = 1.17

$$-8\log(\sin(x)) - \frac{2}{\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)**4,x)
[Out] -8*log(sin(x)) - 2/sin(x)**2
```

3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

Optimal. Leaf size=64

$$-\frac{x}{2\sqrt{3}}+\frac{1}{2}\log(\sin(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}-\frac{1}{4}\log(\sin(x)\cos(x)+1)$$

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {705, 29, 634, 618, 204, 628}

$$-\frac{x}{2\sqrt{3}}-\frac{1}{4}\log\left(\tan^2(x)+\tan(x)+1\right)+\frac{1}{2}\log(\tan(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] -x/(2*Sqrt[3]) + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*Sin[x])]/(
2*Sqrt[3]) + Log[Tan[x]]/2 - Log[1 + Tan[x] + Tan[x]^2]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{2+\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(2+2x+2x^2\right)}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,\tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-2-2x}{2+2x+2x^2}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\log(\tan(x))-\frac{1}{4}\text{Subst}\left(\int \frac{2+4x}{2+2x+2x^2}\,dx,x,\tan(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{2+2x+2x^2}\,dx,x,\text{ta}\right.\\
&= \frac{1}{2}\log(\tan(x))-\frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right)+\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,2+4\tan(x)\right)\\
&= -\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}}+\frac{1}{2}\log(\tan(x))-\frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.61

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)+6\log(\sin(x))-3\log(\sin(2x)+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin[
2*x]])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 64, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-\frac{1}{8}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right)+\frac{1}{4}\log\left(-\frac{1}{4}\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) - 1/8*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/4*log(-1/4*co
s(x)^2 + 1/4)
```

giac [A] time = 0.64, size = 75, normalized size = 1.17

$$-\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right)-\frac{1}{4}\log\left(\tan(x)^2+\tan(x)+1\right)+\frac{1}{2}\log(|\text{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) - 1/4*log(tan(x)^2
 + tan(x) + 1) + 1/2*log(abs(tan(x)))
```

maple [A] time = 0.26, size = 35, normalized size = 0.55

method	result	size
default	$-\frac{\ln\left(1+\tan(x)+\tan^2(x)\right)}{4}-\frac{\sqrt{3}\ \arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(\tan(x))}{2}$	35
risch	$-\frac{\ln\left(e^{2ix}-i\sqrt{3}+2i\right)}{4}+\frac{i\ln\left(e^{2ix}-i\sqrt{3}+2i\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}+i\sqrt{3}+2i\right)}{4}-\frac{i\ln\left(e^{2ix}+i\sqrt{3}+2i\right)\sqrt{3}}{12}+\frac{\ln\left(e^{2ix}-1\right)}{2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)/(2+sin(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(1+tan(x)+tan(x)^2)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2))+1/2
*ln(tan(x))
```

maxima [B] time = 1.07, size = 208, normalized size = 3.25

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-2\left(4\sin\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+16\cos\left(2x\right)^2+8\cos\left(2x\right)\sin\left(4x\right)+\sin\left(4x\right)^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*c
os(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x) +
 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*x)^2
 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2 + si
n(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x
)^2 - 4*sqrt(3) + 7)))
```

mupad [B] time = 0.34, size = 47, normalized size = 0.73

$$\frac{\ln\left(\tan(x)\right)}{2}+\ln\left(\tan(x)+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(\tan(x)+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)*(sin(2*x) + 2)),x)
```

```
[Out] log(tan(x))/2 + log(tan(x) - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/4)
- log(tan(x) + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(x\right)}{\left(\sin\left(2x\right)+2\right)\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\csc^2(x)-4\right)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4356, 266, 36, 31, 29}

$$\log(\sin(x))-\frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[x]*Sec[3*x],x]
```

```
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(x)\sec(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1-4x^2\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-4x)x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+2\,\text{Subst}\left(\int \frac{1}{1-4x}\,dx, x, \sin^2(x)\right)\\ &= \log(\sin(x))-\frac{1}{2}\log\left(1-4\sin^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.55

$$\log(\sin(x)) - \frac{1}{2}\log\left(1 - 4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(x)\sec(3x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*Cot[x]*Sec[3*x],x]
[Out] Could not integrate
```

fricas [A] time = 0.81, size = 17, normalized size = 1.55

$$-\frac{1}{2}\log\left(4\cos(x)^2 - 3\right) + \log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")
[Out] -1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

giac [B] time = 0.64, size = 24, normalized size = 2.18

$$\frac{1}{2}\log\left(-\cos(x)^2 + 1\right) - \frac{1}{2}\log\left(\left|4\cos(x)^2 - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="giac")
[Out] 1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))
```

maple [B] time = 0.15, size = 27, normalized size = 2.45

method	result	size
default	$\frac{\ln(1+\cos(x))}{2} - \frac{\ln\left(4\left(\cos^2(x)\right)-3\right)}{2} + \frac{\ln(-1+\cos(x))}{2}$	27
risch	$\ln\left(e^{2ix} - 1\right) - \frac{\ln\left(e^{4ix}-e^{2ix}+1\right)}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x)/sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1+cos(x))-1/2*ln(4*cos(x)^2-3)+1/2*ln(-1+cos(x))
```

maxima [B] time = 0.45, size = 92, normalized size = 8.36

$$-\frac{1}{4}\log\left(-2\left(\cos\left(2x\right) - 1\right)\cos\left(4x\right) + \cos\left(4x\right)^2 + \cos\left(2x\right)^2 + \sin\left(4x\right)^2 - 2\sin\left(4x\right)\sin\left(2x\right) + \sin\left(2x\right)^2 - 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
- 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/2*log(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 0.62, size = 25, normalized size = 2.27

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^4-14\tan\left(\frac{x}{2}\right)^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(cos(3*x)*sin(x)),x)
```

```
[Out] log(tan(x/2)) - log(tan(x/2)^4 - 14*tan(x/2)^2 + 1)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2(x)}{\sin(x)\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/cos(3*x)/sin(x),x)
```

```
[Out] Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal. Leaf size=7

$$-\tan^{-1}(\cos(2x))$$

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {12, 1107, 617, 204}

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

```
[Out] -ArcTan[Cos[2*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-2x+2x^2}\,dx, x, \sin^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-2\sin^2(x)\right)\\
&= -\tan^{-1}\left(1-2\sin^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 7, normalized size = 1.00

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
[Out] -ArcTan[Cos[2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
[Out] Could not integrate
```

fricas [A] time = 1.08, size = 11, normalized size = 1.57

$$-\arctan\left(2\cos(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
[Out] -arctan(2*cos(x)^2 - 1)
```

giac [A] time = 0.61, size = 9, normalized size = 1.29

$$\arctan\left(2\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="giac")
[Out] arctan(2*sin(x)^2 - 1)
```

maple [A] time = 0.12, size = 12, normalized size = 1.71

method	result	size
derivativedivides	$-\arctan\left(2\left(\cos^2(x)\right)-1\right)$	12
default	$-\arctan\left(2\left(\cos^2(x)\right)-1\right)$	12
risch	$-\frac{i\ln\left(e^{4ix}+2ie^{2ix}+1\right)}{2}+\frac{i\ln\left(e^{4ix}-2ie^{2ix}+1\right)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(x)^4+sin(x)^4),x,method=_RETURNVERBOSE)
[Out] -arctan(2*cos(x)^2-1)
```

maxima [A] time = 0.96, size = 9, normalized size = 1.29

$$\arctan\left(2\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
[Out] arctan(2*sin(x)^2 - 1)
```

mupad [B] time = 0.27, size = 5, normalized size = 0.71

$$\mathrm{atan}\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(x)^4 + sin(x)^4),x)
```

```
[Out] atan(tan(x)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
```

```
[Out] Timed out
```

3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{\sin(x)+\sqrt{3}\cos(x)+2\left(2+\sqrt{3}\right)}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3124, 617, 204}

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(3-4\sqrt{3}\right)\sin(x)+\left(4-\sqrt{3}\right)\cos(x)}{\left(4-\sqrt{3}\right)\sin(x)-\left(3-4\sqrt{3}\right)\cos(x)+2\left(5+2\sqrt{3}\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[((4 - Sqrt[3])*Cos[x] + (3 - 4*Sqrt[3])*Sin[x])/(2*(
5 + 2*Sqrt[3]) - (3 - 4*Sqrt[3])*Cos[x] + (4 - Sqrt[3])*Sin[x])]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4+\sqrt{3}+2x+\left(4-\sqrt{3}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,1+\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)\right)\\
&= \frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(4-\sqrt{3}\right)\cos(x)+\left(3-4\sqrt{3}\right)\sin(x)}{2\left(5+2\sqrt{3}\right)-\left(3-4\sqrt{3}\right)\cos(x)+\left(4-\sqrt{3}\right)\sin(x)}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.62

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt{3}-4\right)\tan\left(\frac{x}{2}\right)-1}{2\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
[Out] -(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4+\sqrt{3}\,\cos(x)+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
[Out] Could not integrate
```

fricas [A] time = 1.25, size = 38, normalized size = 0.72

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2\left(\left(4\sqrt{3}\cos(x)+3\right)\sin(x)+\sqrt{3}\cos(x)+3\right)}{3\left(4\cos(x)^2-3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x) + 3)*sin(x) + sqrt(3)*cos(x) + 3)
/(4*cos(x)^2 - 3))
```

giac [A] time = 0.64, size = 78, normalized size = 1.47

$$\frac{\left(x+2\arctan\left(\frac{\sqrt{3}\cos(x)-8\sqrt{3}\sin(x)+\sqrt{3}+4\cos(x)+7\sin(x)+4}{8\sqrt{3}\cos(x)+\sqrt{3}\sin(x)+8\sqrt{3}-7\cos(x)+4\sin(x)+19}\right)\right)\left(\sqrt{3}+4\right)}{2\left(4\sqrt{3}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="giac")
[Out] 1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x) +
 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x) +
4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)
```

maple [A] time = 0.21, size = 43, normalized size = 0.81

method	result	size
default	$-\frac{52\arctan\left(\frac{26\tan\left(\frac{x}{2}\right)+2\sqrt{3}+8}{16\sqrt{3}+12}\right)}{\left(\sqrt{3}-4\right)\left(16\sqrt{3}+12\right)}$	43
risch	$-\frac{i\sqrt{3}\,\ln\left(-\frac{i\sqrt{3}}{2}-\frac{3}{2}+i+\sqrt{3}+e^{ix}\right)}{6}+\frac{i\sqrt{3}\,\ln\left(e^{ix}+\frac{3}{2}+i+\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}{6}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+sin(x)+cos(x)*3^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(1
/2)+12))
```

maxima [A] time = 0.96, size = 27, normalized size = 0.51

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{\left(\sqrt{3}-4\right)\sin(x)}{\cos(x)+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1) - 1))
```

mupad [B] time = 0.21, size = 23, normalized size = 0.43

$$-\frac{\sqrt{12}\,\mathrm{atan}\left(\frac{\sqrt{12}\,\left(\tan\left(\frac{x}{2}\right)\left(\sqrt{3}-4\right)-1\right)}{12}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 3^(1/2)*cos(x) + 4),x)
```

```
[Out] -(12^(1/2)*atan((12^(1/2)*(tan(x/2)*(3^(1/2) - 4) - 1))/12))/6
```

sympy [B] time = 9.89, size = 107, normalized size = 2.02

$$-\frac{710490629196485166087273623623712231666542242569 69\sqrt{3}\left(\mathrm{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)\right.}{-21314718875894554982618208708711366949996267277 0907+1230605868069891879698900427181521 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

```
[Out] -7104906291964851660872736236237122316665422425 6969*sqrt(3)*(atan(-tan(x/2)
/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-21314
71887589455498261820870871136694999626727 70907 + 12306058680698918796989004
27181521271409146030597 00*sqrt(3)) + 12306058680698918796989004271815212714
09146030597 00*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*fl
oor((x/2 - pi/2)/pi))/(-213147188758945549826182087087113669499962672770907
 + 1230605868069891879698900427181521271409146030597 00*sqrt(3))
```

3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{3\sin(x)+3\cos(x)+8}\right)}{\sqrt{23}}$$

Rubi [B] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 2.85, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3124, 618, 206}

$$\frac{\log\left(\sqrt{23}\sin(x)-4\sin(x)-4\sqrt{23}\cos(x)+19\cos(x)+4\left(5-\sqrt{23}\right)\right)}{2\sqrt{23}}-\frac{\log\left(-\sqrt{23}\sin(x)-4\sin(x)+4\sqrt{23}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] -Log[4*(5 + Sqrt[23]) + 19*Cos[x] + 4*Sqrt[23]*Cos[x] - 4*Sin[x] - Sqrt[23]
*Sin[x]]/(2*Sqrt[23]) + Log[4*(5 - Sqrt[23]) + 19*Cos[x] - 4*Sqrt[23]*Cos[x
] - 4*Sin[x] + Sqrt[23]*Sin[x]]/(2*Sqrt[23])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{7+8x-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{92-x^2}\,dx, x, 8-2\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{\log\left(4\left(5+\sqrt{23}\right)+19\cos(x)+4\sqrt{23}\cos(x)-4\sin(x)-\sqrt{23}\sin(x)\right)}{2\sqrt{23}}+\frac{\text{lo}}{}\end{aligned}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.67

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-4}{\sqrt{23}}\right)}{\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.04, size = 66, normalized size = 2.00

$$\frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8\left(\sqrt{23}-3\right)\cos(x)-2\left(4\sqrt{23}-7\cos(x)+12\right)\sin(x)-3\sqrt{23}-48}{8\,(4\cos(x)+3)\sin(x)+24\cos(x)+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*log(-(6*sqrt(23)*cos(x)^2 + 8*(sqrt(23) - 3)*cos(x) - 2*(4*sq
rt(23) - 7*cos(x) + 12)*sin(x) - 3*sqrt(23) - 48)/(8*(4*cos(x) + 3)*sin(x)
+ 24*cos(x) + 25))
```

giac [A] time = 0.68, size = 37, normalized size = 1.12

$$-\frac{1}{23}\sqrt{23}\log\left(\frac{\left|-2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}{\left|2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="giac")
```

```
[Out] -1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*t
an(1/2*x) - 8))
```

maple [A] time = 0.12, size = 20, normalized size = 0.61

method	result	size
default	$\frac{2\sqrt{23}\,\mathrm{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-8\right)\sqrt{23}}{46}\right)}{23}$	20
risch	$\frac{\sqrt{23}\,\ln\left(e^{ix}+\frac{3}{8}+\frac{3i}{8}-\frac{\sqrt{23}}{8}+\frac{i\sqrt{23}}{8}\right)}{23}-\frac{\sqrt{23}\,\ln\left(e^{ix}+\frac{3}{8}+\frac{3i}{8}+\frac{\sqrt{23}}{8}-\frac{i\sqrt{23}}{8}\right)}{23}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+4*cos(x)+4*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*23^(1/2)*arctanh(1/46*(2*tan(1/2*x)-8)*23^(1/2))
```

maxima [A] time = 0.97, size = 39, normalized size = 1.18

$$-\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23) + sin(x)
/(cos(x) + 1) - 4))
```

mupad [B] time = 0.08, size = 17, normalized size = 0.52

$$\frac{2\sqrt{23}\,\operatorname{atanh}\left(\frac{\sqrt{23}\,\left(\tan\left(\frac{x}{2}\right)-4\right)}{23}\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(x) + 4*sin(x) + 3),x)
```

```
[Out] (2*23^(1/2)*atanh((23^(1/2)*(tan(x/2) - 4))/23))/23
```

sympy [A] time = 0.59, size = 39, normalized size = 1.18

$$\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23}-\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2) - sqrt(23)
 - 4)/23
```

3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {203}

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+9x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 0.33

$$\frac{1}{3}\tan^{-1}(3\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[3*Tan[x]]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.24, size = 21, normalized size = 0.78

$$-\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-9}{6\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

giac [A] time = 0.63, size = 20, normalized size = 0.74

$$\frac{1}{3}\,x-\frac{1}{3}\,\arctan\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

maple [A] time = 0.10, size = 8, normalized size = 0.30

method	result	size
default	$\frac{\arctan(3\tan(x))}{3}$	8
risch	$-\frac{i\ln\left(e^{2ix}-\frac{1}{2}\right)}{6}+\frac{i\ln\left(e^{2ix}-2\right)}{6}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-3*cos(x)^2+5*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(3*tan(x))
```

maxima [A] time = 0.96, size = 7, normalized size = 0.26

$$\frac{1}{3}\,\arctan\left(3\,\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(3*tan(x))
```

mupad [B] time = 0.28, size = 16, normalized size = 0.59

$$\frac{x}{3}-\frac{\mathrm{atan}\left(\tan(x)\right)}{3}+\frac{\mathrm{atan}\left(3\tan(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*sin(x)^2 - 3*cos(x)^2 + 4),x)
```

```
[Out] x/3 - atan(tan(x))/3 + atan(3*tan(x))/3
```

sympy [B] time = 13.65, size = 219, normalized size = 8.11

$$\frac{4478554083\sqrt{17-12\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}+\frac{3166815962\sqrt{2}\sqrt{17-12\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\right.}{2305195203+1630019160\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

```
[Out] 4478554083*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi
*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 3166815962*sqr
t(2)*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi*floor
((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 131836323*sqrt(12*sq
rt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/p
i))/(2305195203 + 1630019160*sqrt(2)) + 93222358*sqrt(2)*sqrt(12*sqrt(2) +
17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi))/(230
5195203 + 1630019160*sqrt(2))
```

3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

Optimal. Leaf size=28

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {801, 635, 203, 260}

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{x}{(2+x)^2\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{2}{5(2+x)^2}-\frac{3}{25(2+x)}+\frac{4+3x}{25\left(1+x^2\right)}\right)dx,x,\tan(x)\right)\\
&= -\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}+\frac{1}{25}\text{Subst}\left(\int \frac{4+3x}{1+x^2}\,dx,x,\tan(x)\right)\\
&= -\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}+\frac{3}{25}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,\tan(x)\right)+\frac{4}{25}\\
&= \frac{4x}{25}-\frac{3}{25}\log(\cos(x))-\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 1.46

$$\frac{4x-3\log(\sin(x)+2\cos(x))+\cot(x)(8x-6\log(\sin(x)+2\cos(x)))-5}{50\cot(x)+25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin[
x]])/(25 + 50*Cot[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.00, size = 46, normalized size = 1.64

$$-\frac{3\left(\tan(x)+2\right)\log\left(\frac{\tan(x)^2+4\,\tan(x)+4}{\tan(x)^2+1}\right)-8\left(x-1\right)\tan(x)-16\,x-4}{50\left(\tan(x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1)) - 8*(x
- 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

giac [A] time = 0.63, size = 29, normalized size = 1.04

$$\frac{4}{25}\,x+\frac{2}{5\left(\tan(x)+2\right)}+\frac{3}{50}\,\log\left(\tan(x)^2+1\right)-\frac{3}{25}\,\log\left(\left|\tan(x)+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) +
2))
```

maple [A] time = 0.21, size = 31, normalized size = 1.11

method	result	size
default	$\frac{3\ln\left(1+\tan^2(x)\right)}{50}+\frac{4\arctan(\tan(x))}{25}+\frac{2}{5(2+\tan(x))}-\frac{3\ln(2+\tan(x))}{25}$	31
norman	$\frac{\frac{8x}{25}+\frac{4x\tan(x)}{25}+\frac{2}{5}}{2+\tan(x)}-\frac{3\ln(2+\tan(x))}{25}+\frac{3\ln\left(1+\tan^2(x)\right)}{50}$	35
risch	$\frac{4x}{25}+\frac{3ix}{25}+\frac{16}{25\left(5\,e^{2ix}+3+4i\right)}-\frac{12i}{25\left(5\,e^{2ix}+3+4i\right)}-\frac{3\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{25}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+4*cot(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/50*ln(1+tan(x)^2)+4/25*arctan(tan(x))+2/5/(2+tan(x))-3/25*ln(2+tan(x))
```

maxima [A] time = 0.96, size = 28, normalized size = 1.00

$$\frac{4}{25}\,x+\frac{2}{5\,(\tan(x)+2)}+\frac{3}{50}\,\log\left(\tan(x)^2+1\right)-\frac{3}{25}\,\log\left(\tan(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan(x) + 2)
```

mupad [B] time = 0.32, size = 38, normalized size = 1.36

$$\frac{2}{5\,\left(\tan(x)+2\right)}-\frac{3\,\ln\left(\tan(x)+2\right)}{25}+\ln\left(\tan(x)-\mathrm{i}\right)\,\left(\frac{3}{50}-\frac{2}{25}\mathrm{i}\right)+\ln\left(\tan(x)+1\mathrm{i}\right)\,\left(\frac{3}{50}+\frac{2}{25}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cot(x) + tan(x) + 4),x)
```

```
[Out] log(tan(x) - 1i)*(3/50 - 2i/25) - (3*log(tan(x) + 2))/25 + log(tan(x) + 1i)
*(3/50 + 2i/25) + 2/(5*(tan(x) + 2))
```

sympy [B] time = 0.49, size = 102, normalized size = 3.64

$$\frac{8x\tan\left(x\right)}{50\tan\left(x\right)+100}+\frac{16x}{50\tan\left(x\right)+100}-\frac{6\log\left(\tan\left(x\right)+2\right)\tan\left(x\right)}{50\tan\left(x\right)+100}-\frac{12\log\left(\tan\left(x\right)+2\right)}{50\tan\left(x\right)+100}+\frac{3\log\left(\tan^2\left(x\right)+1\right)\tan\left(\right.}{50\tan\left(x\right)+100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan(x) + 2)*t
an(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x) + 100) + 3*log(tan(
x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(tan(x)**2 + 1)/(50*tan(x) + 100
) + 20/(50*tan(x) + 100)
```

3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {614, 618, 204}

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] + 2*Cos[x]*S
in[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x] + 2*Tan[x]^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(2+x+2x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}+\frac{4}{15}\text{Subst}\left(\int \frac{1}{2+x+2x^2}\,dx,x,\tan(x)\right)\\
&= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}-\frac{8}{15}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,1+4\tan(x)\right)\\
&= \frac{8x}{15\sqrt{15}}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}}+\frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 58, normalized size = 0.87

$$\frac{(\sin(2x)+4)\sec^2(x)\left(15(\cos(2x)-15)+8\sqrt{15}\,(\sin(2x)+4)\tan^{-1}\left(\frac{4\tan(x)+1}{\sqrt{15}}\right)\right)}{900(\sin(x)+2\sec(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*Ta
n[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.88, size = 61, normalized size = 0.91

$$\frac{4\left(\sqrt{15}\,\cos(x)\sin(x)+2\,\sqrt{15}\right)\arctan\left(\frac{8\,\sqrt{15}\,\cos(x)\sin(x)+\sqrt{15}}{15\left(2\,\cos(x)^2-1\right)}\right)+15\,\cos(x)^2-120}{225\,(\cos(x)\sin(x)+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/225*(4*(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))*arctan(1/15*(8*sqrt(15)*cos(
x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) + 15*cos(x)^2 - 120)/(cos(x)*sin(x)
 + 2)
```

giac [A] time = 0.64, size = 78, normalized size = 1.16

$$\frac{8}{225}\,\sqrt{15}\left(x+\arctan\left(-\frac{\sqrt{15}\,\sin(2\,x)-\cos(2\,x)-4\,\sin(2\,x)-1}{\sqrt{15}\,\cos(2\,x)+\sqrt{15}\,-4\,\cos(2\,x)+\sin(2\,x)+4}\right)\right)+\frac{4\,\tan(x)+1}{15\left(2\,\tan(x)^2+\tan(x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="giac")
```

```
[Out] 8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1)
/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*tan
(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

maple [A] time = 0.22, size = 39, normalized size = 0.58

method	result	size
default	$\frac{1+4\tan(x)}{30+15\tan(x)+30\left(\tan^2(x)\right)}+\frac{8\sqrt{15}\,\arctan\left(\frac{(1+4\tan(x))\sqrt{15}}{15}\right)}{225}$	39
risch	$\frac{\left(\frac{8}{3615}-\frac{2i}{241}\right)\left(241\,e^{2ix}-15+4i\right)}{e^{4ix}+8ie^{2ix}-1}+\frac{4i\sqrt{15}\,\ln\left(e^{2ix}+i\sqrt{15}+4i\right)}{225}-\frac{4i\sqrt{15}\,\ln\left(e^{2ix}-i\sqrt{15}+4i\right)}{225}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sec(x)+sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/15*(1+4*tan
(x))*15^(1/2))
```

maxima [A] time = 0.96, size = 38, normalized size = 0.57

$$\frac{8}{225}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\left(4\tan(x)+1\right)\right)+\frac{4\tan(x)+1}{15\left(2\tan(x)^2+\tan(x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan(x) + 1)/(
2*tan(x)^2 + tan(x) + 2)
```

mupad [B] time = 0.43, size = 120, normalized size = 1.79

$$\frac{4\sqrt{15}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^3}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^2}{15}+\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{15}}{15}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{15}}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{15}\right)\right)}{225}-\frac{\frac{7\tan\left(\frac{x}{2}\right)^3}{30}}{\tan\left(\frac{x}{2}\right)^4-\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 2/cos(x))^2,x)
```

```
[Out] (4*15^(1/2)*(2*atan((2*15^(1/2)*tan(x/2))/5 + 15^(1/2)/15 - (2*15^(1/2)*tan
(x/2)^2)/15 + (2*15^(1/2)*tan(x/2)^3)/15) - 2*atan(15^(1/2)/15 - (2*15^(1/2
)*tan(x/2))/15)))/225 - ((2*tan(x/2)^2)/15 - (7*tan(x/2))/30 + (7*tan(x/2)^
3)/30)/(tan(x/2) + 2*tan(x/2)^2 - tan(x/2)^3 + tan(x/2)^4 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sin(x)+2\sec(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))**2,x)
```

```
[Out] Integral((sin(x) + 2*sec(x))**(-2), x)
```

3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$

Optimal. Leaf size=55

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 203}

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/(6*Sqrt[6]
) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3+2x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{6\left(3+2\tan^2(x)\right)}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{6\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}}+\frac{\tan(x)}{6\left(3+2\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.98

$$\frac{(\cos(2x)+5)\sec^4(x)\left(6\sin(2x)+\sqrt{6}(\cos(2x)+5)\tan^{-1}\left(\sqrt{\frac{2}{3}}\tan(x)\right)\right)}{144\left(2\sec^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] ((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x]) +
 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cos(x) + 2\sec(x))^2}\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.24, size = 58, normalized size = 1.05

$$-\frac{\left(\sqrt{6}\cos(x)^2 + 2\sqrt{6}\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2 - 2\sqrt{6}}{12\cos(x)\sin(x)}\right) - 12\cos(x)\sin(x)}{72\left(\cos(x)^2 + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")
```

```
[Out] -1/72*((sqrt(6)*cos(x)^2 + 2*sqrt(6))*arctan(1/12*(5*sqrt(6)*cos(x)^2 - 2*s
qrt(6))/(cos(x)*sin(x))) - 12*cos(x)*sin(x))/(cos(x)^2 + 2)
```

giac [A] time = 0.63, size = 61, normalized size = 1.11

$$\frac{1}{36}\sqrt{6}\left(x + \arctan\left(-\frac{\sqrt{6}\sin(2x) - 2\sin(2x)}{\sqrt{6}\cos(2x) + \sqrt{6} - 2\cos(2x) + 2}\right)\right) + \frac{\tan(x)}{6\left(2\tan(x)^2 + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x)
 + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

maple [A] time = 0.14, size = 29, normalized size = 0.53

method	result	size
default	$\frac{\tan(x)}{18+12\left(\tan^2(x)\right)} + \frac{\sqrt{6}\arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)}{36}$	29
risch	$\frac{i\left(5\,e^{2ix}+1\right)}{3\,e^{4ix}+30\,e^{2ix}+3} + \frac{i\sqrt{6}\ln\left(e^{2ix}+2\sqrt{6}+5\right)}{72} - \frac{i\sqrt{6}\ln\left(e^{2ix}-2\sqrt{6}+5\right)}{72}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+2*sec(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

maxima [A] time = 0.97, size = 28, normalized size = 0.51

$$\frac{1}{36}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\tan(x)\right) + \frac{\tan(x)}{6\left(2\tan(x)^2 + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

mupad [B] time = 0.39, size = 77, normalized size = 1.40

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\,\sqrt{6}\,\tan\left(\frac{x}{2}\right)}{12}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}\,\tan\left(\frac{x}{2}\right)}{4}\right)\right)}{72}+\frac{\frac{\tan\left(\frac{x}{2}\right)}{9}-\frac{\tan\left(\frac{x}{2}\right)^3}{9}}{\tan\left(\frac{x}{2}\right)^4+\frac{2\tan\left(\frac{x}{2}\right)^2}{3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 2/cos(x))^2,x)
```

```
[Out] (6^(1/2)*(2*atan((5*6^(1/2)*tan(x/2))/12 + (6^(1/2)*tan(x/2)^3)/4) + 2*atan
((6^(1/2)*tan(x/2))/4)))/72 + (tan(x/2)/9 - tan(x/2)^3/9)/((2*tan(x/2)^2)/3
 + tan(x/2)^4 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos\left(x\right)+2\sec\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))**2,x)
```

```
[Out] Integral((cos(x) + 2*sec(x))**(-2), x)
```

3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

Optimal. Leaf size=42

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

Rubi [A] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3628, 3529, 3531, 3530}

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[x])^2) - 2
9/(50*(1 + 3*Tan[x]))
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{7}{10(1+3\tan(x))^2}+\frac{1}{10}\int \frac{8-34\tan(x)}{(1+3\tan(x))^2}\,dx$$
$$= -\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}+\frac{1}{100}\int \frac{-94-58\tan(x)}{1+3\tan(x)}\,dx$$
$$= -\frac{67x}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28}{125}\int \frac{3-\tan(x)}{1+3\tan(x)}\,dx$$
$$= -\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x))-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}$$

Mathematica [A] time = 0.22, size = 70, normalized size = 1.67

$$-\frac{670x+560\log(3\sin(x)+\cos(x))-4\cos(2x)(134x+112\log(3\sin(x)+\cos(x))-405)+6\sin(2x)(67x+5}{500(3\sin(x)+\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] -1/500*(-1305 + 670*x + 560*Log[Cos[x] + 3*Sin[x]] - 4*Cos[2*x]*(-405 + 134
*x + 112*Log[Cos[x] + 3*Sin[x]]) + 6*(-90 + 67*x + 56*Log[Cos[x] + 3*Sin[x]
])*Sin[2*x])/(Cos[x] + 3*Sin[x])^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.09, size = 77, normalized size = 1.83

$$-\frac{9\,(134\,x-1)\tan(x)^2+56\left(9\,\tan(x)^2+6\,\tan(x)+1\right)\log\left(\frac{9\,\tan(x)^2+6\,\tan(x)+1}{\tan(x)^2+1}\right)+12\,(67\,x+72)\tan(x)+134\,x}{500\left(9\,\tan(x)^2+6\,\tan(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")
```

```
[Out] -1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan(
x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 639)
/(9*tan(x)^2 + 6*tan(x) + 1)
```

giac [A] time = 0.64, size = 39, normalized size = 0.93

$$-\frac{67}{250}\,x-\frac{87\,\tan(x)+64}{50\,(3\,\tan(x)+1)^2}+\frac{14}{125}\,\log\left(\tan(x)^2+1\right)-\frac{28}{125}\,\log\left(|3\,\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="giac")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*log(tan(x)^2 +
1) - 28/125*log(abs(3*tan(x) + 1))
```

maple [A] time = 0.10, size = 45, normalized size = 1.07

method	result	size
derivativedivides	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$	45
default	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$	45
risch	$-\frac{67x}{250}+\frac{28ix}{125}+\frac{\left(-\frac{36}{24125}-\frac{621i}{48250}\right)\left(965\,e^{2ix}-324+768i\right)}{\left(5\,e^{2ix}-4+3i\right)^2}-\frac{28\ln\left(e^{2ix}-\frac{4}{5}+\frac{3i}{5}\right)}{125}$	49
norman	$\frac{\frac{297\tan(x)}{50}+\frac{288\left(\tan^2(x)\right)}{25}-\frac{67x}{250}-\frac{201x\tan(x)}{125}-\frac{603x\left(\tan^2(x)\right)}{250}}{(1+3\tan(x))^2}-\frac{28\ln(1+3\tan(x))}{125}+\frac{14\ln\left(1+\tan^2(x)\right)}{125}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 14/125*ln(1+tan(x)^2)-67/250*arctan(tan(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*
tan(x))-28/125*ln(1+3*tan(x))
```

maxima [A] time = 1.00, size = 44, normalized size = 1.05

$$-\frac{67}{250}\,x-\frac{87\,\tan(x)+64}{50\left(9\,\tan(x)^2+6\,\tan(x)+1\right)}+\frac{14}{125}\,\log\left(\tan(x)^2+1\right)-\frac{28}{125}\,\log\left(3\,\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log(
tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

mupad [B] time = 0.28, size = 48, normalized size = 1.14

$$-\frac{28\,\ln\left(\tan(x)+\frac{1}{3}\right)}{125}-\frac{\frac{29\tan(x)}{150}+\frac{32}{225}}{\tan(x)^2+\frac{2\tan(x)}{3}+\frac{1}{9}}+\ln\left(\tan(x)-\mathrm{i}\right)\left(\frac{14}{125}+\frac{67}{500}\mathrm{i}\right)+\ln\left(\tan(x)+1\mathrm{i}\right)\left(\frac{14}{125}-\frac{67}{500}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(x) + 6*tan(x)^2 - 5)/(3*tan(x) + 1)^3,x)
```

```
[Out] log(tan(x) - 1i)*(14/125 + 67i/500) - (28*log(tan(x) + 1/3))/125 + log(tan(
x) + 1i)*(14/125 - 67i/500) - ((29*tan(x))/150 + 32/225)/((2*tan(x))/3 + ta
n(x)^2 + 1/9)
```

sympy [B] time = 0.56, size = 252, normalized size = 6.00

$$-\frac{603x\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{402x\tan(x)}{2250\tan^2(x)+1500\tan(x)+250}-\frac{67x}{2250\tan^2(x)+1500\tan(x)+250}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

```
[Out] -603*x*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 402*x*tan(x)/(2250*
tan(x)**2 + 1500*tan(x) + 250) - 67*x/(2250*tan(x)**2 + 1500*tan(x) + 250)
- 504*log(tan(x) + 1/3)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 33
6*log(tan(x) + 1/3)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) - 56*log(ta
```

```
n(x) + 1/3)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 252*log(tan(x)**2 + 1)*t
an(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) + 168*log(tan(x)**2 + 1)*tan(
x)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 28*log(tan(x)**2 + 1)/(2250*tan(x
)**2 + 1500*tan(x) + 250) - 435*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250)
 - 320/(2250*tan(x)**2 + 1500*tan(x) + 250)
```

3.383 $\int \cos^2(x) \sec(3x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {206}

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sec[3*x],x]
```

```
[Out] ArcTanh[2*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x) \sec(3x) \, dx &= \text{Subst}\left(\int \frac{1}{1-4x^2} \, dx, x, \sin(x)\right)\\ &= \frac{1}{2} \tanh^{-1}(2 \sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sec[3*x],x]
```

```
[Out] ArcTanh[2*Sin[x]]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(x) \sec(3x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^2*Sec[3*x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.03, size = 19, normalized size = 2.11

$$\frac{1}{4} \log(2 \sin(x) + 1) - \frac{1}{4} \log(-2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

giac [B] time = 0.65, size = 21, normalized size = 2.33

$$\frac{1}{4}\log\left(|2\sin(x)+1|\right)-\frac{1}{4}\log\left(|2\sin(x)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
[Out] 1/4*log(abs(2*sin(x) + 1)) - 1/4*log(abs(2*sin(x) - 1))
```

maple [B] time = 0.15, size = 20, normalized size = 2.22

method	result	size
default	$-\frac{\ln(2\sin(x)-1)}{4}+\frac{\ln(1+2\sin(x))}{4}$	20
risch	$\frac{\ln\left(ie^{ix}+e^{2ix}-1\right)}{4}-\frac{\ln\left(-ie^{ix}+e^{2ix}-1\right)}{4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(2*sin(x)-1)+1/4*ln(1+2*sin(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)^2}{\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
[Out] integrate(cos(x)^2/cos(3*x), x)
```

mupad [B] time = 0.36, size = 7, normalized size = 0.78

$$\frac{\operatorname{atanh}(2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x),x)
[Out] atanh(2*sin(x))/2
```

sympy [B] time = 5.46, size = 76, normalized size = 8.44

$$-\frac{\log(\sin(3x)-1)}{12}+\frac{\log(\sin(3x)+1)}{12}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/cos(3*x),x)
[Out] -log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/6 + log(ta
n(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 + log(tan(x/2)**2 + 4*
tan(x/2) + 1)/12
```

3.384 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(2x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.38, size = 174, normalized size = 11.60

$$\frac{4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)+\log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right)+2i\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2
]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*S
qrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(2x)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[2*x]*Sin[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.22, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

giac [B] time = 0.68, size = 49, normalized size = 3.27

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

maple [A] time = 0.12, size = 13, normalized size = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.98, size = 129, normalized size = 8.60

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

mupad [B] time = 0.13, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x)
[Out] Integral(sin(x)/cos(2*x), x)
```

3.385 $\int \sec(2x)\sin^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {298, 203, 206}

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -x/2 + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(2x)\sin^2(x)\,dx &= \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{x}{2}+\frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.65

$$-\frac{x}{2}-\frac{1}{4}\log(\cos(x)-\sin(x))+\frac{1}{4}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -1/2*x - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(2x)\sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[2*x]*Sin[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 0.81, size = 26, normalized size = 1.53

$$-\frac{1}{2}\,x+\frac{1}{8}\,\log\left(2\,\cos(x)\sin(x)+1\right)-\frac{1}{8}\,\log\left(-2\,\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
[Out] -1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x) + 1)
```

giac [A] time = 0.65, size = 20, normalized size = 1.18

$$-\frac{1}{2}\,x+\frac{1}{4}\,\log\left(|\tan(x)+1|\right)-\frac{1}{4}\,\log\left(|\tan(x)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
[Out] -1/2*x + 1/4*log(abs(tan(x) + 1)) - 1/4*log(abs(tan(x) - 1))
```

maple [A] time = 0.14, size = 21, normalized size = 1.24

method	result	size
default	$-\frac{\ln(\tan(x)-1)}{4}-\frac{\arctan(\tan(x))}{2}+\frac{\ln(1+\tan(x))}{4}$	21
risch	$-\frac{x}{2}-\frac{\ln\left(e^{2ix}-i\right)}{4}+\frac{\ln\left(e^{2ix}+i\right)}{4}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/cos(2*x),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(tan(x)-1)-1/2*arctan(tan(x))+1/4*ln(1+tan(x))
```

maxima [B] time = 1.02, size = 128, normalized size = 7.53

$$-\frac{1}{2}\,x-\frac{1}{8}\,\log\left(2\,\cos(x)^2+2\,\sin(x)^2+2\,\sqrt{2}\,\cos(x)+2\,\sqrt{2}\,\sin(x)+2\right)+\frac{1}{8}\,\log\left(2\,\cos(x)^2+2\,\sin(x)^2+2\,\sqrt{2}\,\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
[Out] -1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*s
in(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)
*sin(x) + 2) - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(
2)*sin(x) + 2)
```

mupad [B] time = 0.24, size = 9, normalized size = 0.53

$$\frac{\operatorname{atanh}\left(\tan(x)\right)}{2}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/cos(2*x),x)
```

```
[Out] atanh(tan(x))/2 - x/2
```

sympy [A] time = 1.13, size = 22, normalized size = 1.29

$$-\frac{x}{2}-\frac{\log\left(\sin\left(2x\right)-1\right)}{8}+\frac{\log\left(\sin\left(2x\right)+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/cos(2*x),x)
```

```
[Out] -x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

3.386 $\int \sec(3x)\sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4366, 446, 72}

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[3*x]*Sin[x]^3,x]
```

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \sec(3x)\sin^3(x)\,dx &= -\text{Subst}\left(\int \frac{-1+x^2}{x\left(3-4x^2\right)}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(3-4x)x}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3x}+\frac{1}{3(-3+4x)}\right)dx,x,\cos^2(x)\right)\right)\\
&= \frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(1-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[3*x]*Sin[x]^3,x]
[Out] Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(3x)\sin^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[3*x]*Sin[x]^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.23, size = 19, normalized size = 0.90

$$-\frac{1}{24}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
[Out] -1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

giac [A] time = 0.61, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\sin(x)^2+1\right)-\frac{1}{24}\log\left(\left|4\sin(x)^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
[Out] 1/6*log(-sin(x)^2 + 1) - 1/24*log(abs(4*sin(x)^2 - 1))
```

maple [A] time = 0.14, size = 18, normalized size = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln\left(4\left(\cos^2(x)\right)-3\right)}{24}$	18
risch	$-\frac{ix}{4}+\frac{\ln\left(1+e^{2ix}\right)}{3}-\frac{\ln\left(e^{4ix}-e^{2ix}+1\right)}{24}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/cos(3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

maxima [B] time = 0.98, size = 81, normalized size = 3.86

$$-\frac{1}{48}\log\left(-2\left(\cos(2x)-1\right)\cos(4x)+\cos(4x)^2+\cos(2x)^2+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
[Out] -1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.12, size = 15, normalized size = 0.71

$$\frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2 - \frac{3}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/cos(3*x),x)
```

```
[Out] log(cos(x))/3 - log(cos(x)^2 - 3/4)/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(x)}{\cos(3x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/cos(3*x),x)
```

```
[Out] Integral(sin(x)**3/cos(3*x), x)
```

3.387 $\int \cos(x)\csc(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 31, 29}

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3-4x^2\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x],x]
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\csc(3x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*Csc[3*x],x]
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 19, normalized size = 0.90

$$-\frac{1}{6}\log\left(4\cos(x)^2-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="fricas")
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

giac [A] time = 0.61, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\cos(x)^2+1\right)-\frac{1}{6}\log\left(\left|4\cos(x)^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="giac")
[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

maple [C] time = 0.14, size = 27, normalized size = 1.29

method	result	size
risch	$\frac{\ln\left(e^{2ix}-1\right)}{3}-\frac{\ln\left(e^{4ix}+e^{2ix}+1\right)}{6}$	27
default	$\frac{\ln(1+\cos(x))}{6}+\frac{\ln(-1+\cos(x))}{6}-\frac{\ln(2\cos(x)-1)}{6}-\frac{\ln(2\cos(x)+1)}{6}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

maxima [B] time = 1.00, size = 129, normalized size = 6.14

$$-\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 0.26, size = 17, normalized size = 0.81

$$\frac{\ln\left(\sin(x)\right)}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x)
```

```
[Out] log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

sympy [A] time = 1.33, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^2(x) - 3\right)}{6} + \frac{\log\left(\sin(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

3.388 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx,x,\sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sin(x)\right)-2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x],x]
```

```
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(4x)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[4*x]*Sin[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.02, size = 50, normalized size = 1.92

$$\frac{1}{8}\,\sqrt{2}\,\log\left(-\frac{2\,\cos(x)^2-2\,\sqrt{2}\,\sin(x)-3}{2\,\cos(x)^2-1}\right)-\frac{1}{8}\,\log\left(\sin(x)+1\right)+\frac{1}{8}\,\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

giac [B] time = 0.64, size = 48, normalized size = 1.85

$$-\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin(x)\right|}{\left|2\,\sqrt{2}+4\,\sin(x)\right|}\right)-\frac{1}{8}\,\log\left(\sin(x)+1\right)+\frac{1}{8}\,\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

maple [A] time = 0.17, size = 28, normalized size = 1.08

method	result	size
default	$-\frac{\ln(1+\sin(x))}{8}+\frac{\ln(-1+\sin(x))}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}$	28
risch	$\frac{\ln\left(e^{ix}-i\right)}{4}-\frac{\ln\left(e^{ix}+i\right)}{4}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{8}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(1+sin(x))+1/8*ln(-1+sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 1.04, size = 171, normalized size = 6.58

$$\frac{1}{16}\,\sqrt{2}\,\log\left(2\,\cos(x)^2+2\,\sin(x)^2+2\,\sqrt{2}\,\cos(x)+2\,\sqrt{2}\,\sin(x)+2\right)-\frac{1}{16}\,\sqrt{2}\,\log\left(2\,\cos(x)^2+2\,\sin(x)^2+2\,\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 0.47, size = 27, normalized size = 1.04

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4}-\frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(4*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

sympy [B] time = 8.14, size = 294, normalized size = 11.31

$$\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808}+\frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808}-\frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808}-\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808}+\frac{27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x)
```

```
[Out] 27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt(2
) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) + 2
7720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*
log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1 +
 sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2)
- 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/(110880*
sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(
2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808)
```

3.389 $\int \csc(4x)\sin^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1130, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \csc(4x)\sin^3(x)\,dx&=\text{Subst}\left(\int \frac{x^2}{4-12x^2+8x^4}\,dx,x,\sin(x)\right)\\&=2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\sin(x)\right)\\&=-\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.34, size = 218, normalized size = 8.38

$$2\log\left(2\sin(x)+\sqrt{2}\right)+4\sqrt{2}\log\left(\cos\left(\tfrac{x}{2}\right)-\sin\left(\tfrac{x}{2}\right)\right)-4\sqrt{2}\log\left(\sin\left(\tfrac{x}{2}\right)+\cos\left(\tfrac{x}{2}\right)\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\right.$$
$$\overline{}\;16$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
 - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqr
```

```
t[2])*Cos[x/2] - Sin[x/2])] + 4*Sqrt[2]*Log[Cos[x/2] - Sin[x/2]] - 4*Sqrt[2
]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Co
s[x] - Sqrt[2]*Sin[x]] - Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(16*Sqrt
[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(4x)\sin^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.01, size = 50, normalized size = 1.92

$$\frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

giac [B] time = 0.66, size = 48, normalized size = 1.85

$$-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

maple [A] time = 0.19, size = 28, normalized size = 1.08

method	result	size
default	$\frac{\ln(-1+\sin(x))}{8}-\frac{\ln(1+\sin(x))}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}$	28
risch	$-\frac{\ln\left(e^{ix}+i\right)}{4}+\frac{\ln\left(e^{ix}-i\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{16}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{16}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(-1+sin(x))-1/8*ln(1+sin(x))+1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 1.04, size = 171, normalized size = 6.58

$$\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 0.29, size = 27, normalized size = 1.04

$$\frac{\sqrt{2}\ \mathrm{atanh}\left(\sqrt{2}\ \sin(x)\right)}{8}-\frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/sin(4*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/8 - atanh(sin(x/2)/cos(x/2))/2
```

sympy [B] time = 20.13, size = 294, normalized size = 11.31

$$\frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}}+\frac{2894292447518688\sqrt{2}\ \log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}}-\frac{4093}{16372590}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/sin(4*x),x)
```

```
[Out] 4093147632754948*log(tan(x/2) - 1)/(16372590531019792 + 11577169790074752*s
qrt(2)) + 2894292447518688*sqrt(2)*log(tan(x/2) - 1)/(16372590531019792 + 1
1577169790074752*sqrt(2)) - 4093147632754948*log(tan(x/2) + 1)/(16372590531
019792 + 11577169790074752*sqrt(2)) - 2894292447518688*sqrt(2)*log(tan(x/2)
 + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) + 1447146223759344*lo
g(tan(x/2) - 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt(2)) +
 1023286908188737*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(16372590531019792 +
11577169790074752*sqrt(2)) + 1447146223759344*log(tan(x/2) + 1 + sqrt(2))/(
16372590531019792 + 11577169790074752*sqrt(2)) + 1023286908188737*sqrt(2)*l
og(tan(x/2) + 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt(2))
- 1447146223759344*log(tan(x/2) - sqrt(2) - 1)/(16372590531019792 + 1157716
9790074752*sqrt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/
(16372590531019792 + 11577169790074752*sqrt(2)) - 1447146223759344*log(tan(
x/2) - sqrt(2) + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) - 10232
86908188737*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(16372590531019792 + 115771
69790074752*sqrt(2))
```

3.390 $\int \sqrt{1+\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2646}

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[2*x]],x]
```

```
[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.56

$$\frac{\sqrt{\sin(2x)+1}\,(\sin(x)-\cos(x))}{\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[2*x]],x]
```

```
[Out] ((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1+\sin(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sin[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.93, size = 34, normalized size = 2.12

$$-\frac{(\cos(2x)-\sin(2x)+1)\sqrt{\sin(2x)+1}}{\cos(2x)+\sin(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
[Out] -(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```

giac [A] time = 0.61, size = 17, normalized size = 1.06

$$\sqrt{2}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+x\right)\right)\sin\left(-\frac{1}{4}\,\pi+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*sgn(cos(-1/4*pi + x))*sin(-1/4*pi + x)
```

maple [A] time = 0.16, size = 22, normalized size = 1.38

method	result	size
default	$\frac{(\sin(2x)-1)\sqrt{1+\sin(2x)}}{\cos(2x)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (sin(2*x)-1)*(1+sin(2*x))^(1/2)/cos(2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(2\,x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

mupad [B] time = 0.23, size = 21, normalized size = 1.31

$$\frac{\left(\sin\left(2\,x\right)-1\right)\,\sqrt{\sin\left(2\,x\right)+1}}{\cos\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(2*x) + 1)^(1/2),x)
[Out] ((sin(2*x) - 1)*(sin(2*x) + 1)^(1/2))/cos(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(2x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))**(1/2),x)
[Out] Integral(sqrt(sin(2*x) + 1), x)
```

3.391 $\int \sqrt{1-\sin(2x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2646}

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sin[2*x]],x]
```

```
[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.59

$$\frac{\sqrt{1-\sin(2x)}\,(\sin(x)+\cos(x))}{\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sin[2*x]],x]
```

```
[Out] ((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\sin(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sin[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.79, size = 35, normalized size = 2.06

$$\frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")
[Out] (cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)
```

giac [A] time = 0.65, size = 29, normalized size = 1.71

$$-\sqrt{2}\left(\cos\left(-\frac{1}{4}\,\pi + x\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + x\right)\right) - \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="giac")
[Out] -sqrt(2)*(cos(-1/4*pi + x)*sgn(sin(-1/4*pi + x)) - sgn(sin(-1/4*pi + x)))
```

maple [A] time = 0.15, size = 31, normalized size = 1.82

method	result	size
default	$-\frac{(\sin(2x)-1)(1+\sin(2x))}{\cos(2x)\sqrt{1-\sin(2x)}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(sin(2*x)-1)*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin\left(2\,x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-sin(2*x) + 1), x)
```

mupad [B] time = 0.22, size = 23, normalized size = 1.35

$$\frac{\sqrt{1-\sin\left(2\,x\right)}\,\left(\sin\left(2\,x\right)+1\right)}{\cos\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin(2*x))^(1/2),x)
[Out] ((1 - sin(2*x))^(1/2)*(sin(2*x) + 1))/cos(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \sin\left(2x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))**(1/2),x)
[Out] Integral(sqrt(1 - sin(2*x)), x)
```

3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\sin(2x)}{\sqrt{1+\cos(2x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.74

$$-\frac{\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\cos(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] -((Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]))/Sqrt[1 + C
os[2*x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.15, size = 55, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\cos(2x)^2 - 2\sqrt{2}\sqrt{\cos(2x)+1}\sin(2x) - 2\cos(2x) - 3}{\cos(2x)^2 + 2\cos(2x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(2*x)^2 - 2*sqrt(2)*sqrt(cos(2*x) + 1)*sin(2*x) - 2*co
s(2*x) - 3)/(cos(2*x)^2 + 2*cos(2*x) + 1))
```

giac [A] time = 0.63, size = 41, normalized size = 1.52

$$\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)}{8\,\mathrm{sgn}(\cos(x))} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)}{8\,\mathrm{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/8*sqrt(2)*log(a
bs(1/sin(x) + sin(x) - 2))/sgn(cos(x))
```

maple [C] time = 0.09, size = 9, normalized size = 0.33

method	result	size
default	$\frac{\sqrt{2}\,\mathrm{am}^{-1}(x\vert 1)}{2}$	9
risch	$-\frac{\sqrt{2}\,\ln\left(e^{ix}-i\right)\cos(x)}{\sqrt{\left(1+e^{2ix}\right)^2 e^{-2ix}}}+\frac{\sqrt{2}\,\ln\left(e^{ix}+i\right)\cos(x)}{\sqrt{\left(1+e^{2ix}\right)^2 e^{-2ix}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*InverseJacobiAM(x,1)
```

maxima [A] time = 1.14, size = 41, normalized size = 1.52

$$\frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right) - \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.48

$$\frac{\sqrt{2}\,\mathrm{asinh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*asinh(sin(x)/cos(x)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(2x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(2*x) + 1), x)
```

3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(2x)}{\sqrt{1-\cos(2x)}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.10

$$-\frac{\sin(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] -(((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.84, size = 58, normalized size = 1.93

$$\frac{1}{4}\,\sqrt{2}\,\log\left(-\frac{(\cos(2\,x)+3)\sin(2\,x)-2\left(\sqrt{2}\,\cos(2\,x)+\sqrt{2}\right)\sqrt{-\cos(2\,x)+1}}{(\cos(2\,x)-1)\sin(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-((cos(2*x) + 3)*sin(2*x) - 2*(sqrt(2)*cos(2*x) + sqrt(2))*
sqrt(-cos(2*x) + 1))/((cos(2*x) - 1)*sin(2*x)))
```

giac [A] time = 0.64, size = 16, normalized size = 0.53

$$\frac{\sqrt{2}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,\mathrm{sgn}\left(\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(tan(1/2*x)))/sgn(sin(x))
```

maple [A] time = 0.14, size = 17, normalized size = 0.57

method	result	size
default	$-\frac{\sin(x)\operatorname{arctanh}(\cos(x))\sqrt{2}}{\sqrt{2-2\cos(2x)}}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{ix}-1\right)\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^{2}e^{-2ix}}}-\frac{\sqrt{2}\,\ln\left(e^{ix}+1\right)\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^{2}e^{-2ix}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

maxima [B] time = 1.17, size = 101, normalized size = 3.37

$$-\frac{1}{4}\,\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,\arctan\left(\sin(2\,x),\cos(2\,x)\right)\right)^{2}+\sin\left(\frac{1}{2}\,\arctan\left(\sin(2\,x),\cos(2\,x)\right)\right)^{2}+2\,\cos\left(\frac{1}{2}\,\arctan\left(\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(s
in(2*x), cos(2*x)))^2 + 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1) + 1/4*s
qrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(sin(2*x
), cos(2*x)))^2 - 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)
```

mupad [B] time = 0.24, size = 28, normalized size = 0.93

$$-\frac{\sqrt{2}\,\sin(2\,x)\,\mathrm{atanh}\left(\sqrt{\cos(x)^{2}}\right)}{2\,\sqrt{1-\cos(2\,x)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(2*x))^(1/2),x)
```

```
[Out] -(2^(1/2)*sin(2*x)*atanh((cos(x)^2)^(1/2)))/(2*(1 - cos(2*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1 - \cos(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cos(2*x)), x)
```

3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2650, 2649, 206}

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/(6*Sqrt[2]) - Sin[3*x]/(6*(
1 - Cos[3*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx &= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1-\cos(3x)}}\,dx\\
&= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\frac{\sin(3x)}{\sqrt{1-\cos(3x)}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 61, normalized size = 1.15

$$-\frac{\sin^3\left(\frac{3x}{2}\right)\left(\csc^2\left(\frac{3x}{4}\right)-\sec^2\left(\frac{3x}{4}\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)\right)}{12(1-\cos(3x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -1/12*((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(3
*x)/4]^2)*Sin[(3*x)/2]^3)/(1 - Cos[3*x])^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.92, size = 107, normalized size = 2.02

$$\frac{\left(\sqrt{2}\cos\left(3\,x\right)-\sqrt{2}\right)\log\left(-\frac{(\cos(3\,x)+3)\sin(3\,x)-2\left(\sqrt{2}\cos(3\,x)+\sqrt{2}\right)\sqrt{-\cos(3\,x)+1}}{(\cos(3\,x)-1)\sin(3\,x)}\right)\sin\left(3\,x\right)+4\left(\cos\left(3\,x\right)+1\right)\sqrt{-\cos\left(3\right.}}{24\left(\cos\left(3\,x\right)-1\right)\sin\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((cos(3*x) + 3)*sin(3*x) - 2*(sqrt(
2)*cos(3*x) + sqrt(2))*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x)))*sin(
3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x))
```

giac [A] time = 0.81, size = 59, normalized size = 1.11

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}}{\tan\left(\frac{3}{2}\,x\right)^{2}}+\log\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}+1\right)-\log\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}-1\right)\right)}{24\,\mathrm{sgn}\left(\tan\left(\frac{3}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*sqrt(tan(3/2*x)^2 + 1)/tan(3/2*x)^2 + log(sqrt(tan(3/2*x)^
2 + 1) + 1) - log(sqrt(tan(3/2*x)^2 + 1) - 1))/sgn(tan(3/2*x))
```

maple [A] time = 0.14, size = 52, normalized size = 0.98

method	result	size
default	$-\frac{\left(\frac{\cos\left(\frac{3x}{2}\right)}{2}+\frac{\left(\ln\left(1+\cos\left(\frac{3x}{2}\right)\right)-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)\right)\left(\sin^{2}\left(\frac{3x}{2}\right)\right)}{4}\right)\sqrt{2}}{3\sin\left(\frac{3x}{2}\right)\sqrt{2-2\cos(3x)}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(3*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(1/2*cos(3/2*x)+1/4*(ln(1+cos(3/2*x))-ln(cos(3/2*x)-1))*sin(3/2*x)^2)/
sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)
```

maxima [B] time = 1.22, size = 433, normalized size = 8.17

$$4\left(\sin\left(6\,x\right)-2\,\sin\left(3\,x\right)\right)\cos\left(\frac{3}{2}\,\pi+\frac{3}{2}\,\arctan\left(\sin\left(3\,x\right),\cos\left(3\,x\right)\right)\right)-4\left(\sin\left(6\,x\right)-2\,\sin\left(3\,x\right)\right)\cos\left(\frac{1}{2}\,\pi+\frac{1}{2}\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(sin(6*x) - 2*sin(3*x))*cos(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x)
)) - 4*(sin(6*x) - 2*sin(3*x))*cos(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))
) + (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^2 +
 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos(3*x) - 1)*log(cos(1/2*arctan2(s
in(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos(1/2*
arctan2(sin(3*x), cos(3*x))) + 1) - (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)
^2 - 4*cos(3*x)^2 - sin(6*x)^2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos
(3*x) - 1)*log(cos(1/2*arctan2(sin(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin
(3*x), cos(3*x)))^2 - 2*cos(1/2*arctan2(sin(3*x), cos(3*x))) + 1) - 4*(cos(
6*x) - 2*cos(3*x) + 1)*sin(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x))) + 4*(c
os(6*x) - 2*cos(3*x) + 1)*sin(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))))/(s
qrt(2)*cos(6*x)^2 + 4*sqrt(2)*cos(3*x)^2 + sqrt(2)*sin(6*x)^2 - 4*sqrt(2)*s
in(6*x)*sin(3*x) + 4*sqrt(2)*sin(3*x)^2 - 2*(2*sqrt(2)*cos(3*x) - sqrt(2))*
cos(6*x) - 4*sqrt(2)*cos(3*x) + sqrt(2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(1-\cos\left(3\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(3*x))^(3/2),x)
```

```
[Out] int(1/(1 - cos(3*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1 - \cos{\left(3x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))**(3/2),x)
```

```
[Out] Integral((1 - cos(3*x))**(-3/2), x)
```

3.395 $\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal. Leaf size=73

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sqrt[1 - Sin
[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx &= \frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{8}{5}\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2} dx\\ &= \frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{32}{15}\int \sqrt{1-\sin\left(\frac{2x}{3}\right)}\\ &= \frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}+\frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.17, size = 76, normalized size = 1.04

$$\frac{\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\left(150\sin\left(\frac{x}{3}\right)-25\sin(x)-3\sin\left(\frac{5x}{3}\right)+150\cos\left(\frac{x}{3}\right)+25\cos(x)-3\cos\left(\frac{5x}{3}\right)\right)}{20\left(\cos\left(\frac{x}{3}\right)-\sin\left(\frac{x}{3}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] ((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150*
Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.79, size = 71, normalized size = 0.97

$$-\frac{\left(3\,\cos\left(\frac{2}{3}\,x\right)^{3}-11\,\cos\left(\frac{2}{3}\,x\right)^{2}+\left(3\,\cos\left(\frac{2}{3}\,x\right)^{2}+14\,\cos\left(\frac{2}{3}\,x\right)-32\right)\sin\left(\frac{2}{3}\,x\right)-46\,\cos\left(\frac{2}{3}\,x\right)-32\right)\sqrt{-\sin\left(\frac{2}{3}\,x\right)}}{5\left(\cos\left(\frac{2}{3}\,x\right)-\sin\left(\frac{2}{3}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos(2/3*x) -
32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) + 1)/(cos(2/3*x) - si
n(2/3*x) + 1)
```

giac [B] time = 0.75, size = 110, normalized size = 1.51

$$-\frac{1}{20}\,\sqrt{2}\left(5\,\cos\left(\frac{1}{4}\,\pi+x\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)+90\,\cos\left(\frac{1}{4}\,\pi-\frac{1}{3}\,x\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)-3\,\cos\left(\frac{1}{4}\,\pi-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2)*(5*cos(1/4*pi + x)*sgn(sin(-1/4*pi + 1/3*x)) + 90*cos(1/4*pi
- 1/3*x)*sgn(sin(-1/4*pi + 1/3*x)) - 3*cos(1/4*pi - 5/3*x)*sgn(sin(-1/4*pi
+ 1/3*x)) + 60*sgn(sin(-1/4*pi + 1/3*x))*sin(1/4*pi + 1/3*x) + 20*sgn(sin(-
1/4*pi + 1/3*x))*sin(1/4*pi - x) - 128*sgn(sin(-1/4*pi + 1/3*x)))
```

maple [A] time = 0.15, size = 47, normalized size = 0.64

method	result	size
default	$-\frac{\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(\sin\left(\frac{2x}{3}\right)+1\right)\left(3\left(\sin^{2}\left(\frac{2x}{3}\right)\right)-14\sin\left(\frac{2x}{3}\right)+43\right)}{5\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2/3*x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-1+sin(2/3*x))*(sin(2/3*x)+1)*(3*sin(2/3*x)^2-14*sin(2/3*x)+43)/cos(2
/3*x)/(1-sin(2/3*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sin\left(\frac{2}{3}x\right)+1\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-sin(2/3*x) + 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin((2*x)/3))^(5/2),x)
```

```
[Out] int((1 - sin((2*x)/3))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))**(5/2),x)
```

```
[Out] Integral((1 - sin(2*x/3))**(5/2), x)
```

3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

Rubi [A] time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4356, 14}

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Sin[x]]/2 +
(1 + 2*Sin[x])^(3/2)/12
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2+2\sqrt[4]{1+2x}}{(1+2x)^{3/2}}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x-2x^4+x^8}{x^3}\,dx, x, \sqrt[4]{1+2\sin(x)}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{3}{x^3}+\frac{8}{x^2}-2x+x^5\right)dx, x, \sqrt[4]{1+2\sin(x)}\right)\\ &= \frac{3}{4\sqrt{1+2\sin(x)}}-\frac{4}{\sqrt[4]{1+2\sin(x)}}-\frac{1}{2}\sqrt{1+2\sin(x)}+\frac{1}{12}(1+2\sin(\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.65

$$-\frac{4\sin(x)+24\sqrt[4]{2\sin(x)+1}+\cos(2x)-3}{6\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2
),x]
```

```
[Out] -1/6*(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/Sqrt[1 + 2*Sin[x]
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin
[x])^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 40, normalized size = 0.73

$$-\frac{\left(\cos(x)^2+2\,\sin(x)-2\right)\sqrt{2\,\sin(x)+1}+12\,(2\,\sin(x)+1)^{\frac{3}{4}}}{3\,(2\,\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x) + 1)^(3/4
))/(2*sin(x) + 1)
```

giac [A] time = 0.65, size = 43, normalized size = 0.78

$$\frac{1}{12}\,(2\,\sin(x)+1)^{\frac{3}{2}}-\frac{16\,(2\,\sin(x)+1)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin(x)+1}}-\frac{1}{2}\,\sqrt{2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x)
 + 1) - 1/2*sqrt(2*sin(x) + 1)
```

maple [A] time = 0.48, size = 31, normalized size = 0.56

$$\frac{\sin^2(x)-2\sin(x)-12\,(1+2\sin(x))^{\frac{1}{4}}+1}{3\sqrt{1+2\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)
```

```
[Out] 1/3/(1+2*sin(x))^(1/2)*(sin(x)^2-2*sin(x)-12*(1+2*sin(x))^(1/4)+1)
```

maxima [A] time = 0.43, size = 43, normalized size = 0.78

$$\frac{1}{12}\,(2\,\sin(x)+1)^{\frac{3}{2}}-\frac{16\,(2\,\sin(x)+1)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin(x)+1}}-\frac{1}{2}\,\sqrt{2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x)
 + 1) - 1/2*sqrt(2*sin(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int -\frac{\cos(x)\left(2\,(2\,\sin(x)+1)^{1/4}-\cos(x)^2\right)}{(2\,\sin(x)+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2),x)
[Out] -int(-(cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
[Out] Timed out
```

3.397 $\int \sqrt{\tan(x)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x
]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt{\tan(x)}\, dx = \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\, dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\, dx, x, \sqrt{\tan(x)}\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\, dx, x, \sqrt{\tan(x)}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\, dx, x, \sqrt{\tan(x)}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\, dx, x, \sqrt{\tan(x)}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\, dx, x, \sqrt{\tan(x)}\right) + \frac{\text{Sub}}{}$$
$$= \frac{\log\left(1-\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\, dx, x, 1-\right.}{\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log(}{}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.24

$$\frac{2}{3}\tan^{\frac{3}{2}}(x)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[x]],x]
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x)}\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[Tan[x]],x]
[Out] Could not integrate
```

fricas [B] time = 0.92, size = 180, normalized size = 1.84

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}-1\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{-\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt((sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) + cos(x) +
 sin(x))/cos(x)) - sqrt(2)*sqrt(sin(x)/cos(x)) - 1) - sqrt(2)*arctan(sqrt(2
)*sqrt(-(sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x)) - sq
rt(2)*sqrt(sin(x)/cos(x)) + 1) - 1/4*sqrt(2)*log(4*(sqrt(2)*sqrt(sin(x)/cos
(x))*cos(x) + cos(x) + sin(x))/cos(x)) + 1/4*sqrt(2)*log(-4*(sqrt(2)*sqrt(s
in(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x))
```

giac [A] time = 0.60, size = 80, normalized size = 0.82

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

maple [A] time = 0.12, size = 49, normalized size = 0.50

method	result	size
lookup	$\frac{\left(\sqrt{\tan(x)}\right)\cos(x)\sqrt{2}\ \arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\ \ln\left(\cos(x)+\sqrt{2}\ \left(\sqrt{\tan(x)}\right)\cos(x)+\sin(x)\right)}{2}$	49
default	$\frac{\left(\sqrt{\tan(x)}\right)\cos(x)\sqrt{2}\ \arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\ \ln\left(\cos(x)+\sqrt{2}\ \left(\sqrt{\tan(x)}\right)\cos(x)+\sin(x)\right)}{2}$	49
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}{1+\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)\right)}{4}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x))
-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.82

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
```

```
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

mupad [B] time = 0.13, size = 65, normalized size = 0.66

$$\frac{\sqrt{2}\,\left(\ln\left(\sqrt{2}\,\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\,\sqrt{\tan(x)}+1\right)\right)}{4}+\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan(x)}-1\right)+\mathrm{atan}\left(\sqrt{2}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*tan
(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1/2
)*tan(x)^(1/2) + 1)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)), x)
```

3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{3}{20}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^2(5x)+1\right)$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3476, 329, 275, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[5*x]^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/
3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\left(1+x^2\right)}\,dx, x, \tan(5x)\right)\\
&= \frac{3}{5}\,\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x, \sqrt[3]{\tan(5x)}\right)\\
&= \frac{3}{10}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)+\frac{1}{10}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right)-\frac{1}{20}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)+\frac{3}{20}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,\right.\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right)-\frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)-\frac{3}{10}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1\right.\\
&= -\frac{1}{10}\sqrt{3}\,\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right)-\frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 69, normalized size = 1.21

$$
\frac{1}{10}\sqrt{3}\,\tan^{-1}\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right)+\frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[5*x]^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/
3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[5*x]^(-1/3),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 54, normalized size = 0.95

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan\left(5x\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{20}\log\left(\tan\left(5x\right)^{\frac{4}{3}}-\tan\left(5x\right)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan\left(5x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(3)*arctan(2/3*sqrt(3)*tan(5*x)^(2/3) - 1/3*sqrt(3)) - 1/20*log(ta
n(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\tan\left(5x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(5*x)^(-1/3), x)
```

maple [A] time = 0.10, size = 53, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{\ln\left(1-\left(\tan^{\frac{2}{3}}(5x)\right)+\tan^{\frac{4}{3}}(5x)\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}+\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}$	53
default	$-\frac{\ln\left(1-\left(\tan^{\frac{2}{3}}(5x)\right)+\tan^{\frac{4}{3}}(5x)\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}+\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(5*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*ln(1-tan(5*x)^(2/3)+tan(5*x)^(4/3))+1/10*3^(1/2)*arctan(1/3*(2*tan(5*
x)^(2/3)-1)*3^(1/2))+1/10*ln(1+tan(5*x)^(2/3))
```

maxima [A] time = 0.95, size = 52, normalized size = 0.91

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(5x\right)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\log\left(\tan\left(5x\right)^{\frac{4}{3}}-\tan\left(5x\right)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan\left(5x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)
^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

mupad [B] time = 0.69, size = 67, normalized size = 1.18

$$\frac{\ln\left(81\tan\left(5x\right)^{2/3}+81\right)}{10}-\ln\left(81-162\tan\left(5x\right)^{2/3}+\sqrt{3}\,81\mathrm{i}\right)\left(\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right)+\ln\left(162\tan\left(5x\right)^{2/3}-81+\sqrt{3}\,81\mathrm{i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(5*x)^(1/3),x)
```

```
[Out] log(81*tan(5*x)^(2/3) + 81)/10 - log(3^(1/2)*81i - 162*tan(5*x)^(2/3) + 81)
*((3^(1/2)*1i)/20 + 1/20) + log(3^(1/2)*81i + 162*tan(5*x)^(2/3) - 81)*((3^
(1/2)*1i)/20 - 1/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)**(1/3),x)
```

```
[Out] Integral(tan(5*x)**(-1/3), x)
```

3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3483, 3536, 3535, 203, 207}

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
+ (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3535

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(-2*d^2)/f, Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3536

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
```

```
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx &= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx\\ &= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{250}\int \frac{27+9\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx-\frac{1}{250}\int \frac{-13+39\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx\\ &= -\frac{3}{25\sqrt{4+3\tan(2x)}}-\frac{81}{250}\text{Subst}\left(\int \frac{1}{162+x^2}\,dx,x,\frac{9-27\tan(2x)}{\sqrt{4+3\tan(2x)}}\right)+\frac{1521}{250}\text{Subst}\Bigg(\\ &= -\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}}+\frac{13\tanh^{-1}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{4+3\tan(2x)}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 73, normalized size = 0.84

$$-\frac{(3+4i)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\left(\frac{4}{25}-\frac{3i}{25}\right)(3\tan(2x)+4)\right)+(3-4i)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\left(\frac{4}{25}+\frac{3i}{25}\right)(3\tan(2x)+4)\right)}{50\sqrt{3\tan(2x)+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] -1/50*((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*T
an[2*x])] + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(4
+ 3*Tan[2*x])])/Sqrt[4 + 3*Tan[2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.92, size = 541, normalized size = 6.22

$$-\frac{36\left(7\sqrt{10}\sqrt{5}\cos(2x)^2+24\sqrt{10}\sqrt{5}\cos(2x)\sin(2x)+9\sqrt{10}\sqrt{5}\right)\arctan\left(\frac{1}{25}\sqrt{15}\sqrt{10}\sqrt{5}\sqrt{\frac{\sqrt{10}\sqrt{5}\sqrt{\frac{4\cos(2x)+}{\cos(}}}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5000*(36*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(10)*sqrt(5)*cos(2*x)*s
in(2*x) + 9*sqrt(10)*sqrt(5))*arctan(1/25*sqrt(15)*sqrt(10)*sqrt(5)*sqrt((s
qrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(
2*x) + 5*sin(2*x))/cos(2*x)) - 1/5*sqrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*si
n(2*x))/cos(2*x)) - 3) + 36*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(10)*sq
rt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*arctan(1/25*sqrt(15)*sqrt(10)
*sqrt(5)*sqrt(-(sqrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*c
```

```
os(2*x) - 15*cos(2*x) - 5*sin(2*x))/cos(2*x)) - 1/5*sqrt(10)*sqrt(5)*sqrt((
4*cos(2*x) + 3*sin(2*x))/cos(2*x)) + 3) - 13*(7*sqrt(10)*sqrt(5)*cos(2*x)^2
 + 24*sqrt(10)*sqrt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*log(9375*(sq
rt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(2
*x) + 5*sin(2*x))/cos(2*x)) + 13*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(1
0)*sqrt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*log(-9375*(sqrt(10)*sqrt
(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) - 15*cos(2*x) - 5*sin
(2*x))/cos(2*x)) + 600*(4*cos(2*x)^2 + 3*cos(2*x)*sin(2*x))*sqrt((4*cos(2*x
) + 3*sin(2*x))/cos(2*x)))/(7*cos(2*x)^2 + 24*cos(2*x)*sin(2*x) + 9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*tan(2*x) + 4)^(-3/2), x)
```

maple [A] time = 0.23, size = 130, normalized size = 1.49

method	result
derivativedivides	$-\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}+\frac{13\sqrt{2}\,\ln(9+3\tan(2x)\cdots}{10\cdots}$
default	$-\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}+\frac{13\sqrt{2}\,\ln(9+3\tan(2x)\cdots}{10\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+3*tan(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1
/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))+13/1000*2^(1/2)*
ln(9+3*tan(2*x)+3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2
*(4+3*tan(2*x))^(1/2)+3*2^(1/2))*2^(1/2))-3/25/(4+3*tan(2*x))^(1/2)
```

maxima [B] time = 2.14, size = 3213, normalized size = 36.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(
8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 2000*
(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*
x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*cos(8
*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4
))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 - 80*(
48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(25*(cos(4*
x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x) + 48*sin(4*
x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8
*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*arctan2(-3*cos
(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) +
```

```
4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3,
 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*25^(1/4)*(25*cos
(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) +
25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan2(-8/3*cos(4*x)^2 +
 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*
x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/
9)) + cos(4*x) - 4/3*sin(4*x), 1/3*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4
+ 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(
4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos
(4*x) + 25)^(1/4)*cos(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*s
in(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) +
 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) - 4/3*cos(4*x) - sin
(4*x) - 4/3) + 18*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) +
 3*sin(8*x) + 4))^2)*arctan2(2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*
cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 +
 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) +
13)^(1/4)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3
, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))
 + 4/3*sin(4*x) + 1, 2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)
^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(
4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4
)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*c
os(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 4/3*c
os(4*x) + 4/3) + 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2
*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
 + 3*sin(8*x) + 4))^2)*log(-2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 6
4*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x
)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*
x) + 25)^(1/4)*(4*cos(4*x) + 3*sin(4*x) + 4)*cos(1/2*arctan2(-8/3*cos(4*x)^
2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos
(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) +
25/9)) + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos
(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48
*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*cos(1/2*arctan2
(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*co
s(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 3
2/9*cos(4*x) + 25/9))^2 + 2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*
cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^
3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x)
 + 25)^(1/4)*(3*cos(4*x) - 4*sin(4*x))*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/
9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^
2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9))
 + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^
2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(
4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*sin(1/2*arctan2(-8/3*
cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x)
, 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*co
s(4*x) + 25/9))^2 + 25/9*cos(4*x)^2 + 25/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3
*sin(4*x) + 16/9) - 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(
1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4
*x) + 3*sin(8*x) + 4))^2)*log(16/9*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 1
6*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3
 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x)
+ 13)^(1/4)*(cos(4*x) + 1)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8
```

```
/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*
sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 +
(8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^
2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)*cos(1/2*a
rctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 -
 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))^2 + 4/9*4^(1/4)*(4
*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) +
17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4)*(4*sin(4*x) + 3)*sin(1/2*arctan2(
32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*s
in(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 +
 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^
2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*
x) + 34*cos(4*x) + 13)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*c
os(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(
4*x) + 16/9))^2 + 16/9*cos(4*x)^2 + 16/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3*s
in(4*x) + 25/9))*(2*(32*cos(4*x) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^
2 + 64*cos(4*x)^2 + 16*(3*cos(4*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)
^2 + 64*sin(4*x)^2 + 64*cos(4*x) + 48*sin(4*x) + 25)^(1/4))/((2*(32*cos(4*x
) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^2 + 64*cos(4*x)^2 + 16*(3*cos(4
*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)^2 + 64*sin(4*x)^2 + 64*cos(4*x
) + 48*sin(4*x) + 25)^(1/4)*(cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sin(1/2*arctan2
(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin
(8*x) + 4))^2))
```

mupad [B] time = 0.44, size = 63, normalized size = 0.72

$$-\frac{3}{25\sqrt{3\tan(2x)+4}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2x)+4}\,\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)\left(\frac{9}{500}+\frac{13}{500}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*tan(2*x) + 4)^(3/2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 - 3i/10))*(9/500 + 13i/50
0) + 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 + 3i/10))*(9/500 - 1
3i/500) - 3/(25*(3*tan(2*x) + 4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))**(3/2),x)
```

```
[Out] Integral((3*tan(2*x) + 4)**(-3/2), x)
```

3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

Rubi [A] time = 0.15, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4342, 43}

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

```
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx &= \text{Subst}\left(\int\left(\frac{3x}{(4-3x)^{3/2}}+\frac{1}{-4+3x}\right)dx,x,\tan(x)\right)\\ &= \frac{1}{3}\log(4-3\tan(x))+3\,\text{Subst}\left(\int\frac{x}{(4-3x)^{3/2}}\,dx,x,\tan(x)\right)\\ &= \frac{1}{3}\log(4-3\tan(x))+3\,\text{Subst}\left(\int\left(\frac{4}{3(4-3x)^{3/2}}-\frac{1}{3\sqrt{4-3x}}\right)dx,x\right.\\ &= \frac{1}{3}\log(4-3\tan(x))+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2}{3}\sqrt{4-3\tan(x)}\end{aligned}$$

Mathematica [A] time = 1.26, size = 38, normalized size = 0.95

$$\frac{-6\tan(x)+\sqrt{4-3\tan(x)}\,\log(4-3\tan(x))+16}{3\sqrt{4-3\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),
x]
```

```
[Out] (16 + Log[4 - 3*Tan[x]]*Sqrt[4 - 3*Tan[x]] - 6*Tan[x])/(3*Sqrt[4 - 3*Tan[x]
])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x
])^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.98, size = 82, normalized size = 2.05

$$\frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)-(4\cos(x)-3\sin(x))\log\left(\cos(x)^2\right)+4\sqrt{\frac{4\cos(x)}{\cos}}}{6(4\cos(x)-3\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9/4) - (4*c
os(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*sin(x))/cos(x))*(8*c
os(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

giac [A] time = 0.63, size = 31, normalized size = 0.78

$$\frac{2}{3}\sqrt{-3\tan(x)+4}+\frac{8}{3\sqrt{-3\tan(x)+4}}+\frac{1}{3}\log\left(|-3\tan(x)+4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x) +
 4))
```

maple [B] time = 0.77, size = 219, normalized size = 5.48

method	result
default	$\frac{(-1+\cos(x))^2(1+\cos(x))^2\left(16\sqrt{\frac{4\cos(x)-3\sin(x)}{\cos(x)}}\cos(x)+4\cos(x)\ln\left(-\frac{-2+2\cos(x)+\sin(x)}{\sin(x)}\right)+4\cos(x)\ln\left(-\frac{-2\sin(x)-1+\cos(x)}{\sin(x)}\right)-4\cos(x)\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*(-1+cos(x))^2*(1+cos(x))^2*(16*((4*cos(x)-3*sin(x))/cos(x))^(1/2)*cos(x
)+4*cos(x)*ln(-(-2+2*cos(x)+sin(x))/sin(x))+4*cos(x)*ln(-(-2*sin(x)-1+cos(x
))/sin(x))-4*cos(x)*ln(-(-1+cos(x)-sin(x))/sin(x))-4*cos(x)*ln(-(-1+cos(x)+
sin(x))/sin(x))-6*sin(x)*((4*cos(x)-3*sin(x))/cos(x))^(1/2)-3*sin(x)*ln(-(-
2+2*cos(x)+sin(x))/sin(x))-3*sin(x)*ln(-(-2*sin(x)-1+cos(x))/sin(x))+3*sin(
x)*ln(-(-1+cos(x)-sin(x))/sin(x))+3*sin(x)*ln(-(-1+cos(x)+sin(x))/sin(x)))/
(4*cos(x)-3*sin(x))/sin(x)^4
```

maxima [A] time = 0.43, size = 30, normalized size = 0.75

$$\frac{2}{3}\sqrt{-3\tan(x)+4}+\frac{8}{3\sqrt{-3\tan(x)+4}}+\frac{1}{3}\log\left(-3\tan(x)+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*tan(x) + 4)
```

mupad [B] time = 1.42, size = 105, normalized size = 2.62

$$\frac{\ln\left(e^{x\,2i}\left(-\frac{16}{3}-4i\right)-\frac{16}{3}+4i\right)}{3}-\frac{\ln\left(e^{x\,2i}\left(\frac{16}{3}-4i\right)+\frac{16}{3}-4i\right)}{3}+\frac{2\,e^{x\,1i}\cos(x)\left(\frac{32\,e^{x\,1i}\cos(x)}{3}-4\,e^{x\,1i}\sin(x)\right)\sqrt{4-\frac{3}{c}}}{8\,e^{x\,2i}+8\cos(2\,x)\,e^{x\,2i}-6\sin(2\,x)\,e^{x\,2i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*tan(x) - (4 - 3*tan(x))^(1/2))/(cos(x)^2*(4 - 3*tan(x))^(3/2)),x)
```

```
[Out] log(- exp(x*2i)*(16/3 + 4i) - (16/3 - 4i))/3 - log(exp(x*2i)*(16/3 - 4i) +
(16/3 - 4i))/3 + (2*exp(x*1i)*cos(x)*((32*exp(x*1i)*cos(x))/3 - 4*exp(x*1i)
*sin(x))*(4 - (3*sin(x))/cos(x))^(1/2))/(8*exp(x*2i) + 8*cos(2*x)*exp(x*2i)
 - 6*sin(2*x)*exp(x*2i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{4-3\tan(x)}}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\,dx-\int\left(-\frac{3\tan(x)}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
```

```
[Out] -Integral(sqrt(4 - 3*tan(x))/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*sq
rt(4 - 3*tan(x))*cos(x)**2), x) - Integral(-3*tan(x)/(-3*sqrt(4 - 3*tan(x))
*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x)
```

3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{x}{2}+\frac{\tan^{-1}\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{2}\log(\cos(x))+\frac{\tanh^{-1}\left(\frac{\tan(x)+1}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.37, antiderivative size = 133, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3670, 6725, 1831, 297, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{x}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)-\frac{\log\left(\tan(x)-\sqrt{2}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -x/2 + ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[T
an[x]]]/Sqrt[2] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] - Log[1 - Sqrt[2]*S
qrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/
(2*Sqrt[2]) + (1 - Sqrt[Tan[x]])^(-1)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \text{Subst}\left(\int \frac{x}{\left(-1+\sqrt{x}\right)^2(1+x^2)}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^3}{(-1+x)^2(1+x^4)}\,dx, x, \sqrt{\tan(x)}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}+\frac{1}{2(-1+x)}-\frac{x(1+x)^2}{2(1+x^4)}\right)dx, x, \sqrt{\tan(x)}\right)$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \frac{x(1+x)^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \left(\frac{2x^2}{1+x^4}+\frac{x(1+x^2)}{1+x^4}\right)dx, x, \sqrt{\tan(x}$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)-\text{Subst}\Big(\int$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right)+\text{Subst}\Big(\int$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\frac{1}{2}\text{Subst}\Big(\int$$
$$= -\frac{x}{2}+\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}+\frac{\log(}{}$$
$$= -\frac{x}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x))+\log(1$$

Mathematica [C] time = 0.28, size = 62, normalized size = 0.74

$$-\frac{2}{3}\tan^{\frac{3}{2}}(x)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(x)\right)-\frac{1}{2}\tan^{-1}(\tan(x))+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -1/2*ArcTan[Tan[x]] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Tan
[x]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] Could not integrate
```

fricas [C] time = 3.25, size = 603, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*sqrt(-I) - I + 1)*(tan(x) - 1)*log(-1/2*(2*sqrt(-I) - I + 1)^2*(
4*(-1)^(1/4) + 2*I + 1) - (2*(-1)^(1/4) + I + 1)^3 - ((2*(-1)^(1/4) + I + 1
)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) + 4*(2*(-1)^(1/4) + I +
1)^2 + 6*sqrt(tan(x)) - 16*(-1)^(1/4) - 8*I - 9) + 2*(2*(-1)^(1/4) + I + 1)
*(tan(x) - 1)*log((2*(-1)^(1/4) + I + 1)^3 - 7/2*(2*(-1)^(1/4) + I + 1)^2 +
 6*sqrt(tan(x)) + 14*(-1)^(1/4) + 7*I + 14) - ((2*sqrt(-I) - I + 1)*(tan(x)
 - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) - 4*sqrt(-3/16*(2*sqrt(-I) - I
+ 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(
1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)*log(
1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/4) +
I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^(1/4)
 + I + 1)^2 + sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I +
1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I
 - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4) - I +
 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - ((2*sqrt(-I) - I + 1)*(t
an(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) + 4*sqrt(-3/16*(2*sqrt(-I)
 - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(
-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)
*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/
4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^
(1/4) + I + 1)^2 - sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) +
 I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) +
1/2*I - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4)
- I + 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - 8*(tan(x) - 1)*log(
sqrt(tan(x)) - 1) + 8*sqrt(tan(x)) + 8)/(tan(x) - 1)
```

giac [A] time = 0.67, size = 111, normalized size = 1.32

$$-\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)-\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) - 1/2*(sq
rt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + 1/4*sqrt(2)*lo
g(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)
) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) - 1/4*log(tan(x)^2 + 1) + log(abs(sq
rt(tan(x)) - 1))
```

maple [A] time = 0.08, size = 94, normalized size = 1.12

method	result
derivativedivides	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}{1+\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)\right)}{4}-\frac{\ln}{}$
default	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}{1+\sqrt{2}\left(\sqrt{\tan(x)}\right)+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)\right)}{4}-\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-1+tan(x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(tan(x))-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(x)^(1/2)+tan(x))/(1+2^(1
/2)*tan(x)^(1/2)+tan(x)))+2*arctan(1+2^(1/2)*tan(x)^(1/2))+2*arctan(-1+2^(1
/2)*tan(x)^(1/2)))-1/4*ln(1+tan(x)^2)-1/(-1+tan(x)^(1/2))+ln(-1+tan(x)^(1/2
))
```

maxima [A] time = 0.97, size = 117, normalized size = 1.39

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{8}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) -
1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) -
 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/8*sqr
t(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(tan(x)
) - 1) + log(sqrt(tan(x)) - 1)
```

mupad [B] time = 1.27, size = 228, normalized size = 2.71

$$\ln\left(612\sqrt{\tan(x)}-612\right)-\frac{1}{\sqrt{\tan(x)}-1}+\left(\sum_{k=1}^{4}\ln\left(4\sqrt{\tan(x)}+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2\sqrt{\tan(x)}\,80+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(tan(x)^(1/2) - 1)^2,x)
```

```
[Out] log(612*tan(x)^(1/2) - 612) - 1/(tan(x)^(1/2) - 1) + symsum(log(4*tan(x)^(1
/2) + 80*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2*tan(x)^(1/2) + 448*ro
ot(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3*tan(x)^(1/2) + 128*root(z^4 + z^
3 + z^2/2 - z/8 + 1/64, z, k)^4*tan(x)^(1/2) + 32*root(z^4 + z^3 + z^2/2 -
z/8 + 1/64, z, k)^2 - 384*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3 - 25
6*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4 - 48*root(z^4 + z^3 + z^2/2
- z/8 + 1/64, z, k)*tan(x)^(1/2) - 4)*root(z^4 + z^3 + z^2/2 - z/8 + 1/64,
z, k), k, 1, 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{(x)}}{\left(\sqrt{\tan{(x)}}-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

```
[Out] Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4306}

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[Sin[2*x]],x]
[Out] -ArcSin[Cos[x] - Sin[x]]/2 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{1}{2}\left(-\sin^{-1}(\cos(x)-\sin(x))-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/Sqrt[Sin[2*x]],x]
[Out] Could not integrate
```

fricas [B] time = 0.92, size = 137, normalized size = 4.42

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-s}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) + 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

maple [C] time = 0.18, size = 266, normalized size = 8.58

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)\right)-\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(tan(1/2*x)+1
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((tan(1/2*x)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2
*x)^2+2*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*El
lipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*
x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))
+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(
1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(2*x)^(1/2),x)
```

```
[Out] int(sin(x)/sin(2*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4305}

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]/Sqrt[Sin[2*x]],x]
[Out] Could not integrate
```

fricas [B] time = 1.00, size = 137, normalized size = 4.42

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

maple [C] time = 0.18, size = 98, normalized size = 3.16

method	result	size
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] int(cos(x)/sin(2*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\,\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4302, 4305}

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\,\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/4 - (Cos
[x]*Sqrt[Sin[2*x]])/2
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sqrt{\sin(2x)}\,dx &= -\frac{1}{2}\cos(x)\sqrt{\sin(2x)}+\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx\\ &= -\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)-\frac{1}{2}\cos(x)\sqrt{\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.91

$$\frac{1}{4}\left(-\sin^{-1}(\cos(x)-\sin(x))-2\sqrt{\sin(2x)}\,\cos(x)+\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] - 2*Cos[x
]*Sqrt[Sin[2*x]])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sqrt{\sin(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]*Sqrt[Sin[2*x]],x]
[Out] Could not integrate
```

fricas [B] time = 0.94, size = 151, normalized size = 3.36

$$-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)+\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{8}\arctan\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*
sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1)
) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1
)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) +
 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(2x)}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

maple [C] time = 0.17, size = 171, normalized size = 3.80

method	result
default	$\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\,(\tan^2(\frac{x}{2})-1)\left(\sqrt{\tan(\frac{x}{2})+1}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,\mathrm{EllipticF}\left(\sqrt{\tan(\frac{x}{2})+1},\frac{\sqrt{2}}{2}\right)(\tan^2(\frac{x}{2}))+\sqrt{\tan(\frac{x}{2})+1}\,\sqrt{-2\,\mathrm{ta}}\right.}{\sqrt{\tan(\frac{x}{2})(\tan^2(\frac{x}{2})-1)}\,\sqrt{\tan^3(\frac{x}{2})-\tan(\frac{x}{2})}\,(1+\tan^2(\frac{x}{2}))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((tan(1/2*x)+1)^(1/2)
*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2)
,1/2*2^(1/2))*tan(1/2*x)^2+(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-t
an(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))+2*tan(1/2*x)^3
-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)/(1+tan(1/2*x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(2x)}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sin(2x)}\,\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(1/2)*sin(x),x)
```

```
[Out] int(sin(2*x)^(1/2)*sin(x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4401, 4301, 4306, 4302, 4305}

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] -Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2 + (Cos[x]*Sqrt[Sin[2*x]])/2 + (Sin
[x]*Sqrt[Sin[2*x]])/2
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \int \left(\cos(x)\sqrt{\sin(2x)}-\sin(x)\sqrt{\sin(2x)}\right)dx$$
$$= \int \cos(x)\sqrt{\sin(2x)}\,dx - \int \sin(x)\sqrt{\sin(2x)}\,dx$$
$$= \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x)} - \frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx + \frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}$$
$$= -\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.91

$$\frac{1}{2}\left(\sin(x)\sqrt{\sin(2x)}+\sqrt{\sin(2x)}\cos(x)-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
[Out] (-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*Sq
rt[Sin[2*x]])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
[Out] Could not integrate
```

fricas [B] time = 0.66, size = 76, normalized size = 1.62

$$\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x)) + \frac{1}{8}\log\left(-32\cos(x)^4+4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/8*log(-32*cos(x)^4 +
4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin
(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

maple [C] time = 0.31, size = 442, normalized size = 9.40

method	result

default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(-3\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(\tan\left(\frac{x}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(-3*(tan(1/2*x)+1)^(1
/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1
/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x
)^2+4*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*(tan
(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2)
,1/2*2^(1/2))*tan(1/2*x)^2+4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4-3
*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF
((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1
))^(1/2)+4*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)
*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*EllipticE((tan(1/2*x)+1)^
(1/2),1/2*2^(1/2))-2*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1
/2*x)^3+4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+2*(tan(1/2*x)*(tan(1
/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1
/2)/(1+tan(1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x
)-1)*(tan(1/2*x)+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sin(2x)}\,\left(\cos(x)-\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(1/2)*(cos(x) - sin(x)),x)
```

```
[Out] int(sin(2*x)^(1/2)*(cos(x) - sin(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx$

Optimal. Leaf size=61

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} - \frac{1}{16}\sin^{-1}(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4294, 4308, 4305}

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} - \frac{1}{16}\sin^{-1}(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 + Si
n[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4294

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Si
n[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{1}{4}\int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx \\ &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\int \csc(x)\sqrt{\sin(2x)}\,dx \\ &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{8}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx \\ &= -\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.82

$$\frac{1}{80}\left(2\sqrt{\sin(2x)}\sec(x)\left(\sec^2(x)-6\right)+5\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*S
ec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.01, size = 181, normalized size = 2.97

$$\frac{10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\cos(x)^3 - 10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)\cos(x)^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*s
in(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*cos(x)^3 - 10*arctan(-(2*sqrt(2)*s
qrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x)))*cos(x)^3 - 5*cos(x
)^3*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*
cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) - 48*cos(
x)^3 - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

maple [C] time = 0.24, size = 510, normalized size = 8.36

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(5\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\left(\tan^{14}\left(\frac{x}{2}\right)\right)+35\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\mathrm{t}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2688*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(tan(1/2*x)
+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^14+35*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)
+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*t
an(1/2*x)^12+10*tan(1/2*x)^15+105*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2
*x)^10+66*tan(1/2*x)^13+175*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^8-
1014*tan(1/2*x)^11+175*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+2002*
tan(1/2*x)^9+105*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-2002*tan(1/
2*x)^7+35*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+1014*tan(1/2*x)^5+
5*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elliptic
F((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))-66*tan(1/2*x)^3-10*tan(1/2*x))/(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(1/2*x)^2)^7/(tan(1/2*x)^3-tan(1/2*x))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(x)^7}{\sin(2x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] int(sin(x)^7/sin(2*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**7/sin(2*x)**(7/2),x)

[Out] Timed out
```

3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=61

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4293, 4307, 4306}

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 - Co
s[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4293

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4307

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx &= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{4}\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx\\
&= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos(x)}{4\sqrt{\sin(2x)}}+\frac{1}{16}\int \sec(x)\sqrt{\sin(2x)}\,dx\\
&= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos(x)}{4\sqrt{\sin(2x)}}+\frac{1}{8}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx\\
&= -\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos(x)}{4\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 0.92

$$\sqrt{\sin(2x)}\left(\frac{3\csc(x)}{20}-\frac{\csc^3(x)}{40}\right)+\frac{1}{16}\left(-\sin^{-1}(\cos(x)-\sin(x))-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((3
*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.02, size = 205, normalized size = 3.36

$$10\left(\cos(x)^2-1\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\sin(x)-10\left(\cos(x)^2-1\right)\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)}}{\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin
(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*sin(x) - 10*(cos(x)
^2 - 1)*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x)))*sin(x) + 5*(cos(x)^2 - 1)*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^
3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 +
 16*cos(x)*sin(x) + 1)*sin(x) + 8*sqrt(2)*(6*cos(x)^2 - 5)*sqrt(cos(x)*sin(
x)) + 48*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

maple [C] time = 0.26, size = 1108, normalized size = 18.16

method	result
default	$\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(192\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\sqrt{\tan\left(\frac{x}{2}\right)+1}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\operatorname{EllipticE}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)}\left(\tan\left(\frac{x}{2}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/
2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^6-96*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ell
ipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/
2*x)+1))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)
*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^10-384*(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-
1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^4+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipti
cF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)
+1))^(1/2)*tan(1/2*x)^4+96*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^8+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan
(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^8+48*(tan(1/2*x)^3-
tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*
x)^8+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1
/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/
2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-96*(tan(1
/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*
(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)
*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-192*(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+14*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*t
an(1/2*x)^6-144*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*
(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^6+96*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+14*tan(1/2*x)^4*(tan(1/2*x)*(ta
n(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+144*(
tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(
1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/
2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)^3-tan(1/2*x))^(1/
2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-(tan(1/2*x
)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))
/(tan(1/2*x)^2-1)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)-1
)*(tan(1/2*x)+1))^(1/2)/(tan(1/2*x)-1)/(tan(1/2*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(x)^7}{\sin(2x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] int(cos(x)^7/sin(2*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

3.408 $\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4292}

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] -1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.00, size = 39, normalized size = 2.44

$$\frac{4\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2 + \left(\cos(x)^2 - 1\right)\sin(x)\right)}{5\left(\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")

[Out] 4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")

[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)

maple [C] time = 0.27, size = 508, normalized size = 31.75

method	result
default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\left(96\sqrt{\tan(\frac{x}{2})(\tan^2(\frac{x}{2})-1)}\,\sqrt{\tan(\frac{x}{2})+1}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,\operatorname{EllipticE}\left(\sqrt{\tan(\frac{x}{2})+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan(\frac{x}{2})(\tan(\frac{x}{2})-\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)^(3/2)/sin(x)^5,x,method=_RETURNVERBOSE)

[Out] 1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^6+28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^4+40*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)-28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2+(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")

[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)

mupad [B] time = 0.56, size = 18, normalized size = 1.12

$$\frac{4\sqrt{\sin(2x)}\,\left(\sin(x)^2-1\right)}{5\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(3/2)/sin(x)^5,x)
```

```
[Out] (4*sin(2*x)^(1/2)*(sin(x)^2 - 1))/(5*sin(x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

```
[Out] Timed out
```

3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\sqrt{\sin(2x)}\,\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\,\sec(x)$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4299, 4291}

$$\frac{1}{5}\sqrt{\sin(2x)}\,\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/Sqrt[Sin[2*x]],x]
[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4299

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Cos[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx &= \frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}+\frac{4}{5}\int \frac{\sec(x)}{\sqrt{\sin(2x)}}\,dx\\ &= \frac{4}{5}\sec(x)\sqrt{\sin(2x)}+\frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.65

$$\frac{1}{5}\sqrt{\sin(2x)}\,\sec(x)\left(\sec^2(x)+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
[Out] (Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 32, normalized size = 1.03

$$\frac{4\cos(x)^3+\sqrt{2}\left(4\cos(x)^2+1\right)\sqrt{\cos(x)\sin(x)}}{5\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(x)^3\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

maple [C] time = 0.21, size = 286, normalized size = 9.23

method	result
default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(5\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\left(\tan^6\left(\frac{x}{2}\right)\right)+15\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\,\mathrm{ta}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^3/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(tan(1/2*x)+1
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+15*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(
1/2*x)^4-14*tan(1/2*x)^7+15*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+
2*tan(1/2*x)^5+5*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))-2*tan(1/2*x)^3+14*tan(1/
2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(1/2*x)^2)^3/(tan(1/2*x)^3-
tan(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(x)^3\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

mupad [B] time = 0.39, size = 20, normalized size = 0.65

$$\frac{\sqrt{\sin\left(2\,x\right)}\,\left(2\,\cos\left(2\,x\right)+3\right)}{5\,{\cos\left(x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x)^(1/2)*cos(x)^3),x)
```

```
[Out] (sin(2*x)^(1/2)*(2*cos(2*x) + 3))/(5*cos(x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**3/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} dx$

Optimal. Leaf size=29

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4308, 4303, 4292}

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} dx &= 2\int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)} dx \\
&= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4}{3}\int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)} dx \\
&= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.83

$$\sqrt{\sin(2x)}\left(\frac{\sec(x)}{2} - \frac{1}{6}\cot(x)\csc(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-1/6*(Cot[x]*Csc[x]) + Sec[x]/2)*Sqrt[Sin[2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.61, size = 43, normalized size = 1.48

$$\frac{4\,\cos(x)^3+\sqrt{2}\left(4\,\cos(x)^2-3\right)\sqrt{\cos(x)\sin(x)}-4\,\cos(x)}{6\left(\cos(x)^3-\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x)) - 4*cos(x))/
(cos(x)^3 - cos(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(2\,x\right)^{\frac{3}{2}}\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

maple [C] time = 0.16, size = 121, normalized size = 4.17

method	result	s
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)-\left(\tan^4\left(\frac{x}{2}\right)\right)+1\right)}{12\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)/sin(2*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*(
tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)-tan(1/2*x)^4+1)/(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(2\,x\right)^{\frac{3}{2}}\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

mupad [B] time = 0.43, size = 29, normalized size = 1.00

$$-\frac{\sqrt{\sin\left(2\,x\right)}\,\left(2\,\cos\left(2\,x\right)-1\right)}{6\,\left(\cos(x)-\cos(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x)^(3/2)*sin(x)),x)
```

```
[Out] -(sin(2*x)^(1/2)*(2*cos(2*x) - 1))/(6*(cos(x) - cos(x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

```
[Out] Timed out
```

3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=68

$$-\frac{9\cos(x)}{16\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}-\frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{33}{32}\tanh^{-1}\left(\frac{1}{2}\sqrt{\sin(2x)}\,\sec(x)\right)$$

Rubi [A] time = 0.86, antiderivative size = 95, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4390, 1619, 63, 207}

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{33\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2))
,x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) - (5*Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (9*C
os[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Si
n[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx = \frac{\sin^5(x)\int \frac{\csc^2(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$
$$= \frac{\sin^5(x)\,\text{Subst}\left(\int \frac{-1+3x+x^2+3x^3}{(2-x)x^{7/2}}\,dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$
$$= \frac{\sin^5(x)\,\text{Subst}\left(\int \left(-\frac{1}{2x^{7/2}}+\frac{5}{4x^{5/2}}+\frac{9}{8x^{3/2}}-\frac{33}{8(-2+x)\sqrt{x}}\right)dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$
$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}-\frac{\left(33\sin^5(x)\right)\text{Subst}\left(\int \right.}{8\sin^{\frac{5}{2}}(2x)}$$
$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}-\frac{\left(33\sin^5(x)\right)\text{Subst}\left(\int \right.}{4\sin^{\frac{5}{2}}(2x)}$$
$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{33\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(}$$

Mathematica [C] time = 6.38, size = 150, normalized size = 2.21

$$\frac{\sqrt{\sin(2x)}\cos(x)(\cos(2x)-3\tan(x))\left(\frac{1}{15}\csc(x)\left(-50\cot(x)+12\csc^2(x)-147\right)-33\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\right.}{16(-6\sin(x)+\text{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^
(5/2)),x]
```

```
[Out] (Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 - 33*S
qrt[Cos[x]/(-2 + 2*Cos[x])]*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] - Elli
pticPi[-2/(-1 + Sqrt[5]), ArcSin[1/Sqrt[Tan[x/2]]], -1] - EllipticPi[(-1 +
Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]])*(Cos[2*x]
 - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] - 6*Sin[x]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*
Sin[2*x]^(5/2)),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.05, size = 136, normalized size = 2.00

$$-\frac{495\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right)\sin(x)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="fricas")
```

```
[Out] -1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
+ 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 495*(cos(x)^
2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(
x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) - 135)
*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(2x) - 3\tan(x))\cos(x)^3}{\left(\sin(x)^2 - \sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5
/2)), x)
```

maple [C] time = 0.61, size = 761, normalized size = 11.19

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(932\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(932*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1
/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-3024*(tan(1/2*x)*(tan(1/2*x)^2-
1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan
(1/2*x)+1))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(
1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^6+3*2^(1/2)*(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2
)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*sum((34*_alpha^3+13*_alpha^2+34*_alpha-21
)*(_alpha^3+2*_alpha-3)*(tan(1/2*x)+1)^(1/2)*(-tan(1/2*x)+1)^(1/2)*(-tan(1/
2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((tan(1/2*x)+1)^(
1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z
^2-_Z+1))*tan(1/2*x)^2+200*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*
(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^5-24*tan(1/2*x)^4*(tan(1/2*
x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
-1920*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^
2-1))^(1/2)-552*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*
(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^4-24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(
tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2+552*(tan(1/2*x
)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(
1/2*x)^2-200*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)*(t
an(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+24*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)
```

```
+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)*(tan(1/2*x)-1)*
(tan(1/2*x)+1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\cos(x)^3\,\left(\cos\left(2\,x\right)-3\,\tan(x)\right)}{\sin\left(2\,x\right)^{5/2}\,\left(\sin\left(2\,x\right)-\sin(x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)
),x)
```

```
[Out] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2
),x)
```

```
[Out] Timed out
```

3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1999, 1954, 1250, 30}

$$\frac{2\tan^2(x)\sec^2(x)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[x]^4*Tan[x]],x]
```

```
[Out] (2*Sec[x]^2*Tan[x]^2)/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 1954

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rule 1999

```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_), x_Symbol] :> Int[(f*x)^m*ExpandToSu
m[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &
& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrin
omialDegree[u, x], 0] &&  !(BinomialMatchQ[z, x] && GeneralizedTrinomialMat
chQ[u, x])
```

Rubi steps

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{x(1+x^2)}{\sqrt{x(1+x^2)^2}}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{x(1+x^2)}{\sqrt{x+2x^3+x^5}}\,dx, x, \tan(x)\right)$$
$$= \frac{\left(\sqrt{\tan(x)}\sqrt{1+2\tan^2(x)+\tan^4(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}(1+x^2)}{\sqrt{1+2x^2+x^4}}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$
$$= \frac{\left(\sec^2(x)\sqrt{\tan(x)}\right)\text{Subst}\left(\int \sqrt{x}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$
$$= \frac{2\sec^2(x)\tan^2(x)}{3\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[Sec[x]^4*Tan[x]],x]
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 15, normalized size = 0.79

$$\frac{2}{3}\sqrt{\frac{\sin(x)}{\cos(x)^5}}\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sin(x)}{\cos(x)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)/cos(x)^5), x)
```

maple [A] time = 0.36, size = 16, normalized size = 0.84

method	result	size
default	$\frac{2\cos(x)\sin(x)\sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)/cos(x)^5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*cos(x)*sin(x)*(sin(x)/cos(x)^5)^(1/2)
```

maxima [A] time = 1.05, size = 6, normalized size = 0.32

$$\frac{2}{3}\tan(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)^(3/2)
```

mupad [B] time = 0.51, size = 15, normalized size = 0.79

$$\frac{\sin(2x)\sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)/cos(x)^5)^(1/2),x)
```

```
[Out] (sin(2*x)*(sin(x)/cos(x)^5)^(1/2))/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)**5)**(1/2),x)
```

```
[Out] Timed out
```

3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}+\frac{3\tan^{-1}\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3\log\left(\sin(x)+\cos(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin\right.}{4\sqrt{2}}$$

Rubi [B] time = 0.24, antiderivative size = 204, normalized size of antiderivative = 2.22, number of steps used = 13, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {6719, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sec^2(x}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^4*Tan[x]])/2 - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec
[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*ArcTan[1 + Sqrt[
2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) +
 (3*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/
(8*Sqrt[2]*Tan[x]^(5/2)) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]
^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x^5}{(1+x^2)^2}}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^{5/2}}{(1+x^2)^2}\,dx, x, \tan(x)\right)}{\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)}{4\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{2\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{4\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan}\right.}{8\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{3\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} + \frac{3\tan}{}$$

Mathematica [A] time = 0.08, size = 66, normalized size = 0.72

$$-\frac{1}{8}\sqrt{\sin(2x)}\csc^3(x)\sqrt{\sin^4(x)\tan(x)}\left(2\sin(x)\sqrt{\sin(2x)} + 3\sin^{-1}(\cos(x)-\sin(x)) + 3\log\left(\sin(x)+\sqrt{\sin(2x)} + \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -1/8*(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[Si
n[2*x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 114.30, size = 1006, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(16*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*cos(x)*sin(x) - 6*
(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(1/2*(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x
)^3 - cos(x))*sin(x) + sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(
x))*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + s
qrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/
(cos(x)^2 - 1)) - sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) +
 2)/(cos(x)^4 - 2*cos(x)^2 + (cos(x)^3 - cos(x))*sin(x) + 1)) - 6*(sqrt(2)*
cos(x)^2 - sqrt(2))*arctan(-1/2*(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x)^3 - co
s(x))*sin(x) - sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*sqrt
((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*cos(x)^2 + sqrt(2)*c
os(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^
2 - 1)) + sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + 2)/(cos
(x)^4 - 2*cos(x)^2 + (cos(x)^3 - cos(x))*sin(x) + 1)) - 6*(sqrt(2)*cos(x)^2
 - sqrt(2))*arctan(((sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)
^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - (2*cos(x)^4 - 3*cos(x)^2 - (sqrt(2)*c
os(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/co
s(x)) + 1)*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x
)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)
) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 2*(cos(x)^3 - cos(x))*sin(x) - 1)) - 6*
(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(((sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin
(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + (2*cos(x)^4 - 3*cos(
x)^2 + (sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)
^2 + 1)*sin(x)/cos(x)) + 1)*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) -
 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 +
 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 2*(cos(x)^3 - cos(x))*
sin(x) - 1)) + 3*(sqrt(2)*cos(x)^2 - sqrt(2))*log((cos(x)^2 + 4*(cos(x)^3 -
 cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)
^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)) - 3*(sqrt(2)*cos(x
)^2 - sqrt(2))*log((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*co
s(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos
(x)) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

maple [C] time = 0.51, size = 318, normalized size = 3.46

method	result
default	$-\frac{\sqrt{32}\,(-1+\cos(x))\left(-3i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{\frac{-1+\cos(x)+\sin(x)}{\sin(x)}}\,\sqrt{-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}}+3i\,\mathrm{Elliptic}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^5/cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*32^(1/2)*(-1+cos(x))*(-3*I*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/s
in(x))^(1/2)*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)+3*I*EllipticPi((-(-1+cos(x)
-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(x))/sin(x))^(1/2)*((
-1+cos(x)+sin(x))/sin(x))^(1/2)*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)-3*Ellipt
```

```
icPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(x))
/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*(-(-1+cos(x)-sin(x))/sin(x
))^(1/2)-3*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*((-1+cos(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*(-(-1+cos
(x)-sin(x))/sin(x))^(1/2)+2*cos(x)^2*2^(1/2)-2*cos(x)*2^(1/2))*(1+cos(x))^2
*(sin(x)^5/cos(x))^(1/2)/sin(x)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^5/cos(x))^(1/2),x)
```

```
[Out] int((sin(x)^5/cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sin^{5}{\left(x \right)}}{\cos{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**5/cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**5/cos(x)), x)
```

3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

Rubi [A] time = 0.14, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6719, 14}

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[x]^3*(Sec[
x]^12*Tan[x]^2)^(1/3))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt[3]{x^2\left(1+x^2\right)^6}}{1+x^2}\,dx,x,\tan(x)\right)\\
&=\frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int x^{2/3}\left(1+x^2\right)\,dx,x,\tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\
&=\frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int \left(x^{2/3}+x^{8/3}\right)\,dx,x,\tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\
&=\frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}+\frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 1.34

$$\frac{3\sin(x)\cos(x)\left(8\left(-\tan^2(x)\right)^{5/6}+3\cos(2x)\left(\left(-\tan^2(x)\right)^{5/6}-1\right)-3\right)\sqrt[3]{\tan^2(x)\sec^{12}(x)}}{55\left(-\tan^2(x)\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*C
os[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 29, normalized size = 0.62

$$\frac{3}{55}\left(6\,\cos(x)^3+5\,\cos(x)\right)\left(-\frac{\cos(x)^2-1}{\cos(x)^{14}}\right)^{\frac{1}{3}}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\left(\frac{\sin^2(x)}{\cos(x)^{14}}\right)^{\frac{1}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

```
[Out] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

maxima [A] time = 1.00, size = 13, normalized size = 0.28

$$\frac{3}{11}\,\tan(x)^{\frac{11}{3}}+\frac{3}{5}\,\tan(x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

mupad [B] time = 3.94, size = 32, normalized size = 0.68

$$\frac{6\,\sin\left(2\,x\right)\,\left(1-\cos\left(2\,x\right)\right)^{1/3}\,\left(3\,\cos\left(2\,x\right)+8\right)}{55\,\left(\cos\left(2\,x\right)+1\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

```
[Out] (6*sin(2*x)*(1 - cos(2*x))^(1/3)*(3*cos(2*x) + 8))/(55*(cos(2*x) + 1)^(7/3)
)
```

sympy [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

```
[Out] Timed out
```

3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Rubi [A] time = 0.20, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6719, 270}

$$-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}+\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^3*Sin[x]^3
)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{\frac{x^{13}}{(1+x^2)^{12}}}\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\ &= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \frac{(1+x^2)^2}{x^{13/4}}\,dx,x,\tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\\ &= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \left(\frac{1}{x^{13/4}}+\frac{2}{x^{5/4}}+x^{3/4}\right)dx,x,\tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\\ &= -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}-\frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}+\frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 0.50

$$-\frac{4\sin(x)\cos(x)(8\cos(2x)-16\cos(4x)+15)}{63\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/(63*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.88, size = 101, normalized size = 1.44

$$\frac{4\left(128\cos(x)^4-144\cos(x)^2+9\right)\left(\left(\cos(x)^{23}-6\cos(x)^{21}+15\cos(x)^{19}-20\cos(x)^{17}+15\cos(x)^{15}-6\cos(\right.\right.}{63\left(\cos(x)^{22}-6\cos(x)^{20}+15\cos(x)^{18}-20\cos(x)^{16}+15\cos(x)^{14}-6\cos(x)^{12}+\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(x
)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/4)
/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 - 6*
cos(x)^12 + cos(x)^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(\cos^{11}(x)\right)\left(\sin^{13}(x)\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

```
[Out] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

maxima [A] time = 1.01, size = 77, normalized size = 1.10

$$\frac{4}{23}\tan(x)^{\frac{23}{4}}+\frac{8}{15}\tan(x)^{\frac{15}{4}}+\frac{4}{7}\tan(x)^{\frac{7}{4}}-\frac{4\left(35\tan(x)^7+161\tan(x)^5+345\tan(x)^3-805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}}+\frac{4\left(21\tan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan(
x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21*t
an(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)
```

mupad [B] time = 3.55, size = 110, normalized size = 1.57

$$-\frac{2^{3/4}\left(-32\cos\left(2x\right)^2+8\cos\left(2x\right)+31\right)\left(924\sin\left(2x\right)-132\sin\left(4x\right)-660\sin\left(6x\right)+165\sin\left(8x\right)+330\sin\left(\right.\right.}{2016\left(\cos\left(2x\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

```
[Out] -(2^(3/4)*(8*cos(2*x) - 32*cos(2*x)^2 + 31)*(924*sin(2*x) - 132*sin(4*x) -
660*sin(6*x) + 165*sin(8*x) + 330*sin(10*x) - 110*sin(12*x) - 110*sin(14*x)
 + 44*sin(16*x) + 22*sin(18*x) - 10*sin(20*x) - 2*sin(22*x) + sin(24*x))^(3
/4))/(2016*(cos(2*x) - 1)^6*(cos(2*x) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

```
[Out] Timed out
```

3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{\sin(2x)}\,\cos(x)\sin^{-1}(\cos(x)-\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sin(2x)}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sqrt{\sin(2x)}\,\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\sqrt{2}\log\Big(\sin(x)$$

Rubi [B] time = 1.54, antiderivative size = 234, normalized size of antiderivative = 2.17, number of steps used = 27, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6719, 6725, 215, 329, 211, 1165, 628, 1162, 617, 204, 321}

$$-2\sec^2(x)\sqrt{\sin(x)\cos^3(x)}-\frac{\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}}+\frac{\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right.}{\sqrt{\tan}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] -2*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]] - Sqrt[2]*ArcSinh[Tan[x]]*Cot[x]*(Sec[x]^
2)^(3/2)*Sqrt[Cos[x]*Sin[x]]*Sqrt[Cos[x]^3*Sin[x]] - (Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[
x]] - (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]
])/(Sqrt[2]*Sqrt[Tan[x]]) + (Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^
2*Sqrt[Cos[x]^3*Sin[x]])/(Sqrt[2]*Sqrt[Tan[x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}\left(1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}\right)}{x}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{2}\sqrt{\frac{x}{1+x^2}}}{\sqrt{x}}+\frac{1}{\sqrt{x}(1+x^2)}-\frac{x^{3/2}}{1+x^2}\right)dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}} - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\,\text{s}\right.}{}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \left(\sqrt{2}\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(}\right.$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}$$

Mathematica [C] time = 0.28, size = 105, normalized size = 0.97

$$\frac{-4\sin(x)\cos^3(x)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\cos^2(x)\right)-3\sqrt[4]{\sin^2(x)}\cos(x)\left(2\sin(x)+\sqrt{\sin(2x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\text{lo}\right.\right.}{3\sqrt[4]{\sin^2(x)}\sqrt{\sin(x)\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] (-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] - 3*Cos[x]*(
Sin[x]^2)^(1/4)*(2*Sin[x] + (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin
[x/2]])*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[x]^3*Sin[x]]*(Sin[x]^2)^(1/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 20.64, size = 611, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(-1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 -
sqrt(2)*sqrt(cos(x)^3*sin(x))*sqrt((4*cos(x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(
x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)) - sqrt(2)*sqrt(cos(x)^3*si
n(x)))/(cos(x)^4 + cos(x)^3*sin(x) - cos(x)^2))*cos(x)^2 + 2*sqrt(2)*arctan
(1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 + sqrt(2)*sqrt(cos(x)^3*s
in(x))*sqrt((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) +
sin(x)) + cos(x))/cos(x)) + sqrt(2)*sqrt(cos(x)^3*sin(x)))/(cos(x)^4 + cos(
x)^3*sin(x) - cos(x)^2))*cos(x)^2 - 2*sqrt(2)*arctan(-1/2*(sqrt(2)*sqrt(cos
(x)^3*sin(x))*(cos(x) - sin(x)) + (2*cos(x)^2*sin(x) - sqrt(2)*sqrt(cos(x)^
3*sin(x))*(cos(x) + sin(x)))*sqrt((4*cos(x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(x
)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)))/(cos(x)^2*sin(x)))*cos(x)^
2 - 2*sqrt(2)*arctan(-1/2*(sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) - sin(x))
- (2*cos(x)^2*sin(x) + sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)))*sqr
t((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) +
cos(x))/cos(x)))/(cos(x)^2*sin(x)))*cos(x)^2 - sqrt(2)*cos(x)^2*log((4*cos(
x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/c
os(x)) + sqrt(2)*cos(x)^2*log((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*
sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)) - sqrt(2)*cos(x)^2*log((cos(x)^
6 - 8*cos(x)^4 + 4*sqrt(cos(x)^3*sin(x))*(cos(x)^2 - 2)*sqrt(cos(x)*sin(x))
 + 8*cos(x)^2)/cos(x)^6) + 8*sqrt(cos(x)^3*sin(x)))/cos(x)^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\sin\left(2\,x\right)} - \cos\left(2\,x\right)}{\sqrt{\cos\left(x\right)^{3}\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

maple [C] time = 0.76, size = 247, normalized size = 2.29

method	result
default	$-\frac{2\cos(x)\sin(x)}{\sqrt{\left(\cos^{3}(x)\right)\sin(x)}}+\frac{2\sqrt{2}\,\cos(x)\sqrt{\cos(x)\sin(x)}\,\operatorname{arctanh}\left(\frac{-1+\cos(x)}{\sin(x)}\right)}{\sqrt{\left(\cos^{3}(x)\right)\sin(x)}}-\frac{\sqrt{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2*cos(x)*sin(x)/(cos(x)^3*sin(x))^(1/2)+2*2^(1/2)*cos(x)*(cos(x)*sin(x))^(
1/2)*arctanh((-1+cos(x))/sin(x))/(cos(x)^3*sin(x))^(1/2)-2^(1/2)*(I*Ellipti
```

```
cPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(
(-(-1+cos(x)-sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF((-(-1
+cos(x)-sin(x))/sin(x))^(1/2),1/2*2^(1/2))+EllipticPi((-(-1+cos(x)-sin(x))/
sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(x)-sin(x))/sin(x)
)^(1/2),1/2+1/2*I,1/2*2^(1/2)))*cos(x)*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*(
(-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*sin(x)^2/(-1+co
s(x))/(cos(x)^3*sin(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*sqrt(2)*integrate(2*((((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) +
 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(
x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4
*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos
(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((cos(1/2*arctan2(sin
(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x
) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arc
tan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
)*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
+ 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
1/2*sqrt(2)*integrate(-2*(((cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x)
+ (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin
(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) -
 sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), co
s(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) - (((cos(4*x) + 1)*cos
(1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(
x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -
cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)
))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
 + 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
 1/2*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/2*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2\,x)-\sqrt{\sin(2\,x)}}{\sqrt{\cos(x)^3\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

```
[Out] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

Optimal. Leaf size=364

$$-\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)+\frac{4}{\sqrt{\tan(x)}}-\sqrt[4]{2}\coth^{-1}\left(\frac{\tan(x)+\sqrt{2}}{2^{3/4}\sqrt{\tan(x)}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\sin(x)\cos^3(x)}}\right)+\sqrt[4]{2}\tan$$

Rubi [A] time = 4.73, antiderivative size = 665, normalized size of antiderivative = 1.83, number of steps used = 66, number of rules used = 21, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.512, Rules used = {6725, 6742, 6719, 325, 329, 297, 1162, 617, 204, 1165, 628, 466, 482, 6733, 15, 29, 266, 36, 31, 260, 444}

$$-\sqrt[4]{2}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)+\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)+\frac{4}{\sqrt{\tan(x)}}+\frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}-\frac{\log(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Tan
[x]]),x]
```

```
[Out] -(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*Sq
rt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[Sq
rt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4*Csc[x]*Sec[x]*Sqrt[Cos[x
]^3*Sin[x]] - (Csc[x]^2*Log[Sec[x]^2]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[C
os[x]*Sin[x]^3])/2 + Csc[x]^2*Log[Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[
x]]*Sqrt[Cos[x]*Sin[x]^3] + (Csc[x]^2*Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x
]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Log[Tan[x]]*Sec[x]^2*Sqrt[Cos[x]*Si
n[x]^3])/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x]*Sin[x]^
3])/(4*Tan[x]^(3/2)) + 4/Sqrt[Tan[x]] + (2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Ta
n[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2^(1/4)*ArcTan[1 + 2
^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2*Sqrt
[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[T
an[x]] + (2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3
*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Se
c[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[
Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Log[Sqrt[
2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4
)*Sqrt[Tan[x]]) + (Log[Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sq
rt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x^3}{(1+x^2)^2}} - \frac{4x}{1+x^2}}{(1+x^2)\left(\sqrt{x} - \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{4x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)} - \frac{\sqrt{\frac{x^3}{(1+x^2)^2}}}{(1+x^2)\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right.}\right.\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{}{(1}\right.$$

$$= 4\,\text{Subst}\left(\int \left(-\frac{1}{2x^{3/2}} - \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2x^2} + \frac{\sqrt{x}}{2(2+x^2)} + \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2(2+x^2)}\right) dx, x, \tan(\right.$$

$$= \frac{4}{\sqrt{\tan(x)}} - 2\,\text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{x^2}\,dx, x, \tan(x)\right) + 2\,\text{Subst}\left(\int \frac{\sqrt{x}}{2+x^2}\,d\right.$$

$$= \frac{4}{\sqrt{\tan(x)}} + 4\,\text{Subst}\left(\int \frac{x^2}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right) - \frac{\left(2\sec^2(x)\sqrt{\cos(x)\sin\right.}{}$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}} - 2\,\text{Subst}\left(\int \frac{\sqrt{2} - x^2}{2+x^4}\,dx,\right.$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}} + \frac{\text{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx,\right.}{2^{3/4}}$$

$$= \frac{\log\left(\sqrt{2} - 2^{3/4}\sqrt{\tan(x)} + \tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2} + 2^{3/4}\sqrt{\tan(x)} + \tan(x)\right)}{2^{3/4}}$$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{\right.}}{}$$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{\right.}}{}$$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 11.37, size = 385, normalized size = 1.06

$$\frac{4\sqrt{2}\cos^2(x)^{3/4}\tan^{\frac{3}{2}}(x)\,_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2\sin^2(x)}{\cos(2x)+3}\right)}{3(\cos(2x)+3)^{3/4}}+\frac{4}{\sqrt{\tan(x)}}+4\csc(x)\sec(x)\sqrt{\sin(x)\cos^3(x)}-\frac{(1+i)\sec^4\left(\frac{x}{2}\right)\sqrt{\sin(x}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sq
rt[Tan[x]]),x]
```

```
[Out] 4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] - (Cos[x]*Csc[x/2]*(4*Log[Sec[x/2]^2]
 - 2*Log[Tan[x/2]] - Log[1 + Tan[x/2]^4])*Sec[x/2]*Sqrt[Cos[x]*Sin[x]^3])/(
8*Sqrt[Cos[x]^3*Sin[x]]) - ((1 + I)*((4 + 4*I)*EllipticPi[-I, ArcSin[Sqrt[T
an[x/2]]], -1] - (4 + 4*I)*EllipticPi[I, ArcSin[Sqrt[Tan[x/2]]], -1] + (-1)
^(1/4)*(-EllipticPi[-(-1)^(1/4), ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(
-1)^(1/4), ArcSin[Sqrt[Tan[x/2]]], -1] - EllipticPi[-(-1)^(3/4), ArcSin[Sqr
t[Tan[x/2]]], -1] + EllipticPi[(-1)^(3/4), ArcSin[Sqrt[Tan[x/2]]], -1]))*Se
c[x/2]^4*Sqrt[Cos[x]^3*Sin[x]])/(Sqrt[Cos[x]*Sec[x/2]^2]*Sqrt[Tan[x/2]]*(-1
 + Tan[x/2]^2)) + 4/Sqrt[Tan[x]] + (Csc[x]^2*(2*Log[Tan[x]] - Log[2 + Tan[x
]^2])*Sqrt[Cos[x]*Sin[x]^3]*Sqrt[Tan[x]])/4 + (4*Sqrt[2]*(Cos[x]^2)^(3/4)*H
ypergeometric2F1[3/4, 3/4, 7/4, (2*Sin[x]^2)/(3 + Cos[2*x])]*Tan[x]^(3/2))/
(3*(3 + Cos[2*x])^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin
[x]] + Sqrt[Tan[x]]),x]
```

```
[Out] Could not integrate
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   not i
nvertible
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\cos(x)\sin(x)^3}-2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)}-\sqrt{\tan(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - sq
rt(tan(x))), x)
```

maple [C] time = 4.60, size = 22964, normalized size = 63.09

method	result	size
default	Expression too large to display	22964

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^
(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*integrate(-1/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sqr
t(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*
sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)
*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2
)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x)
 - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(
sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*c
os(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) +
(sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x)
+ 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt
(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqr
t(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x),
 cos(x) + 1)) + (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) -
 sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos
(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*si
n(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2
)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(
2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqr
t(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*
sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*s
in(1/2*arctan2(sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1
)) - ((((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*s
in(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x)
 + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2
*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x)
 + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))
*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(
2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3
*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*si
n(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arc
tan2(sin(x), cos(x) + 1)) - (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt
(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*
x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2
*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x)
```

```
- sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x
) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(
2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) +
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*s
in(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos
(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x),
cos(2*x) + 1)))/((((2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2
*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + s
in(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(si
n(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*
cos(2*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x
) + sin(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arcta
n2(sin(x), -cos(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2
*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*cos(x
) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2*x)^2
 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 +
(2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*
cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2
*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
^2)*sin(1/2*arctan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 2*integrate(1
/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sqrt(2)*cos(3*x) -
 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x
) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqr
t(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4
*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x)
 + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)
*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*
x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) -
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) +
 (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*
x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*
sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) +
 sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)
*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*a
rctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*si
n(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) +
 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x
) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(
sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1)) - ((((sqrt(2
)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*co
s(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) +
sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*arctan2(sin
(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sq
rt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2
*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)
*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2
)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), co
s(x) + 1)) - (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sq
rt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*c
```

```
os(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2
)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*
x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x
))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*c
os(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*
sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x)
+ sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(
1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x), cos(2*x) + 1)))
/((((2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2
*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*s
in(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) +
 1))^2 + (2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*
cos(2*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2
- 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos
(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2*cos(2*x) - cos
(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)^2 +
2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2*x)*s
in(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x)
- cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)
^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2
*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*sin(1/2*arc
tan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 1/2*log(cos(x)^2 + sin(x)^2
+ 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{2\,\sin\left(2\,x\right)-\sqrt{\cos(x)\,\sin(x)^3}}{\sqrt{\tan(x)}-\sqrt{\cos(x)^3\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x
))^(1/2)),x)
```

```
[Out] -int((2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x
))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1/
2)+tan(x)**(1/2)),x)
```

```
[Out] Timed out
```

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=125

$$-\frac{9\sin^4(x)}{10\left(\sin(x)\cos^5(x)\right)^{2/3}}-\frac{9}{4}\sec^8(x)\left(\sin(x)\cos^5(x)\right)^{4/3}+\frac{3}{14}\tan^4(x)\sqrt[3]{\sin(x)\cos^5(x)}\,\sqrt[3]{\tan(x)\sec^6(x)}+\frac{3}{4}\tan^2(x)\sqrt[3]{s$$

Rubi [A] time = 1.02, antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6719, 6733, 6742, 14}

$$-\frac{9\sin^2(x)\cos^2(x)}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}-\frac{9\sin^4(x)}{10\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^3(x)\cos^3(x)\sqrt[3]{\tan(x)\sec^6(x)}}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^5(x)\cos(x)\sqrt[3]{\tan(x)\sec^6(}}{14\left(\sin(x)\cos^5(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-9*Cos[x]^2*Sin[x]^2)/(4*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sin[x]^4)/(10*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]^5*Sin[x]*(Sec[x]^6*Tan[x])^(1/3))/(2*(Cos[x
]^5*Sin[x])^(2/3)) + (3*Cos[x]^3*Sin[x]^3*(Sec[x]^6*Tan[x])^(1/3))/(4*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]*Sin[x]^5*(Sec[x]^6*Tan[x])^(1/3))/(14*(Cos[
x]^5*Sin[x])^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{-3x+\sqrt[3]{x\left(1+x^2\right)^3}}{\left(\frac{x}{\left(1+x^2\right)^3}\right)^{2/3}\left(1+x^2\right)}\,dx,x,\tan(x)\right)$$

$$= \frac{\left(\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-3x+\sqrt[3]{x\left(1+x^2\right)^3}\right)}{x^{2/3}}\,dx,x,\tan(x)\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(1+x^6\right)\left(-3x^3+\sqrt[3]{x^3\left(1+x^6\right)^3}\right)dx,x,\sqrt[3]{\cdots}\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(-3x^3+\sqrt[3]{x^3\left(1+x^6\right)^3}-x^6\left(3x^3-\sqrt[3]{(x+\cdots}\right.\right.\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \sqrt[3]{x^3\left(1+x^6\right)^3}\,dx\cdots\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(3x^9-x^6\sqrt[3]{x^3(1\cdots}\right.\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6\cdots}}{2\left(\cos^5(x)\sin(x\cdots\right.}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6\cdots}}{2\left(\cos^5(x)\sin(x\cdots\right.}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6\cdots}}{2\left(\cos^5(x)\sin(x\cdots\right.}$$

Mathematica [A] time = 0.35, size = 58, normalized size = 0.46

$$-\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\tan(x)\sec^6(x)}\right)}{2240\left(\sin(x)\cos^5(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Cos
[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^
(2/3),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.31, size = 56, normalized size = 0.45

$$-\frac{3\left(\cos(x)^5\sin(x)\right)^{\frac{1}{3}}\left(21\left(3\cos(x)^2+2\right)\sin(x)-5\left(9\cos(x)^5+3\cos(x)^3+2\cos(x)\right)\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}\right)}{140\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="fricas")
```

```
[Out] -3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9*cos(x)^5
+ 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x)^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}-3\tan(x)}{\left(\cos(x)^5\sin(x)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="giac")
```

```
[Out] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}-3\tan(x)}{\left(\left(\cos^5(x)\right)\sin(x)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

```
[Out] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

maxima [A] time = 1.17, size = 60, normalized size = 0.48

$$-\frac{3}{20}\tan(x)^{\frac{20}{3}}-\frac{3}{7}\tan(x)^{\frac{14}{3}}-\frac{9}{10}\tan(x)^{\frac{10}{3}}-\frac{3}{8}\tan(x)^{\frac{8}{3}}-\frac{9}{4}\tan(x)^{\frac{4}{3}}+\frac{3\left(14\tan(x)^7+60\tan(x)^5+105\tan(x)^3+140\right.}{280\tan(x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="maxima")
```

```
[Out] -3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/8*tan(x)^(
8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x)^5 + 105*tan(x)^3 +
 140*tan(x))/tan(x)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\tan(x) - \left(\frac{\sin(x)}{\cos(x)^7}\right)^{1/3}}{\left(\cos(x)^5 \sin(x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3),x)
[Out] int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x)
[Out] Timed out
```

3.419 $\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx$

Optimal. Leaf size=73

$$-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}-\frac{5}{24}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}-\frac{5}{16}\cos(x)\sqrt{2\cos^2(x)+1}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 195, 215}

$$-\frac{1}{6}\cos(x)(\cos(2x)+2)^{5/2}-\frac{5}{24}\cos(x)(\cos(2x)+2)^{3/2}-\frac{5}{16}\cos(x)\sqrt{\cos(2x)+2}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[2 + Cos[2*x]])/1
6 - (5*Cos[x]*(2 + Cos[2*x])^(3/2))/24 - (Cos[x]*(2 + Cos[2*x])^(5/2))/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx &= -\text{Subst}\left(\int \left(1+2x^2\right)^{5/2}\,dx,x,\cos(x)\right)\\
&= -\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{6}\text{Subst}\left(\int \left(1+2x^2\right)^{3/2}\,dx,x,\cos(x)\right)\\
&= -\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{8}\text{Subst}\left(\int \sqrt{1+2x^2}\,d\right.\\
&= -\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}-\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}\\
&= -\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}-\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}-\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-
\end{aligned}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 0.84

$$\frac{1}{96}\left(-2\sqrt{\cos(2x)+2}\,(92\cos(x)+23\cos(3x)+2\cos(5x))-15\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)+2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
[Out] (-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]*
Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.95, size = 108, normalized size = 1.48

$$-\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1}+\frac{5}{256}\sqrt{2}\log\left(2048\cos(x)^8+2048\cos(x)^6+64($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")
[Out] -1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1) + 5/256*
sqrt(2)*log(2048*cos(x)^8 + 2048*cos(x)^6 + 640*cos(x)^4 + 64*cos(x)^2 - 8*
(128*sqrt(2)*cos(x)^7 + 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 + sqrt(2)
*cos(x))*sqrt(2*cos(x)^2 + 1) + 1)
```

giac [A] time = 0.67, size = 55, normalized size = 0.75

$$-\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x)+\frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="giac")
[Out] -1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/3
2*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

maple [A] time = 0.08, size = 56, normalized size = 0.77

method	result
derivativedivides	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$
default	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*cos(x)^2)^(5/2)*sin(x),x,method=_RETURNVERBOSE)
[Out] -5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arcs
inh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

maxima [A] time = 0.96, size = 55, normalized size = 0.75

$$-\frac{1}{6}\left(2\cos(x)^2+1\right)^{\frac{5}{2}}\cos(x)-\frac{5}{24}\left(2\cos(x)^2+1\right)^{\frac{3}{2}}\cos(x)-\frac{5}{32}\sqrt{2}\,\mathrm{arsinh}\left(\sqrt{2}\cos(x)\right)-\frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*cos(x) - 5
/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x)^2 + 1)*cos(x)
```

mupad [B] time = 0.13, size = 43, normalized size = 0.59

$$-\frac{5\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,\cos(x)\right)}{32}-\frac{\sqrt{2}\,\sqrt{\cos(x)^2+\frac{1}{2}}\,\left(\frac{4\cos(x)^5}{3}+\frac{13\cos(x)^3}{6}+\frac{11\,\cos(x)}{8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(2*cos(x)^2 + 1)^(5/2),x)
```

```
[Out] - (5*2^(1/2)*asinh(2^(1/2)*cos(x)))/32 - (2^(1/2)*(cos(x)^2 + 1/2)^(1/2)*((
11*cos(x))/8 + (13*cos(x)^3)/6 + (4*cos(x)^5)/3))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

```
[Out] Timed out
```

3.420 $\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4356, 195, 216}

$$\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[x]^2])/16
+ (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx &= \text{Subst}\left(\int \left(5-4x^2\right)^{5/2}\,dx,x,\sin(x)\right)\\
&= \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{6}\text{Subst}\left(\int \left(5-4x^2\right)^{3/2}\,dx,x,\sin(x)\right)\\
&= \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{125}{8}\text{Subst}\left(\right.\\
&= \frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{1}{6}\sin(x)\left(5\right.\\
&= \frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{25}{24}\sin(x)\left(5-4\right.
\end{aligned}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.70

$$\frac{1}{96}\left(1875\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)+2(515\sin(x)+90\sin(3x)+8\sin(5x))\sqrt{2\cos(2x)+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
[Out] (1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90*
Sin[3*x] + 8*Sin[5*x]))/96
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
[Out] Could not integrate
```

fricas [A] time = 1.49, size = 88, normalized size = 1.28

$$\frac{1}{48}\left(128\cos(x)^4+264\cos(x)^2+433\right)\sqrt{4\cos(x)^2+1}\sin(x)+\frac{625}{64}\arctan\left(\frac{4\left(8\cos(x)^2-3\right)\sqrt{4\cos(x)^2+1}\sin}{64\cos(x)^4-23\cos(x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin(x) + 625/
64*arctan((4*(8*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(x) - 25*cos(x)*sin(x
))/(64*cos(x)^4 - 23*cos(x)^2 - 16)) + 625/64*arctan(sin(x)/cos(x))
```

giac [A] time = 0.68, size = 41, normalized size = 0.59

$$\frac{1}{48}\left(8\left(16\sin(x)^2-65\right)\sin(x)^2+825\right)\sqrt{-4\sin(x)^2+5}\sin(x)+\frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*sin(x) + 6
25/32*arcsin(2/5*sqrt(5)*sin(x))
```

maple [A] time = 0.23, size = 103, normalized size = 1.49

method	result
default	$\frac{\sqrt{\left(4\left(\cos^2(x)\right)+1\right)\left(\sin^2(x)\right)}\left(512\sqrt{-4\left(\sin^4(x)\right)+5\left(\sin^2(x)\right)}\left(\sin^4(x)\right)-2080\sqrt{-4\left(\sin^4(x)\right)+5\left(\sin^2(x)\right)}\left(\sin^2(x)\right)+3300\sqrt{-4\left(\sin^4(x)\right)+5(}\right.}{192\sin(x)\sqrt{4\left(\cos^2(x)\right)+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/192*((4*cos(x)^2+1)*sin(x)^2)^(1/2)*(512*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*s
in(x)^4-2080*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*sin(x)^2+3300*(-4*sin(x)^4+5*si
n(x)^2)^(1/2)+1875*arcsin(-1+8/5*sin(x)^2))/sin(x)/(4*cos(x)^2+1)^(1/2)
```

maxima [A] time = 0.98, size = 53, normalized size = 0.77

$$\frac{1}{6}\left(-4\sin(x)^2+5\right)^{\frac{5}{2}}\sin(x)+\frac{25}{24}\left(-4\sin(x)^2+5\right)^{\frac{3}{2}}\sin(x)+\frac{125}{16}\sqrt{-4\sin(x)^2+5}\sin(x)+\frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2)*sin(x) +
 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(x)\left(5\cos(x)^2+\sin(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2),x)
```

```
[Out] int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.421 $\int \cos(x) \left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=58

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4356, 195, 217, 203}

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[-1 - 4*Sin
[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}dx = \text{Subst}\left(\int\left(-1-4x^2\right)^{3/2}dx,x,\sin(x)\right)$$
$$= \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}-\frac{3}{4}\text{Subst}\left(\int\sqrt{-1-4x^2}\,dx,x,\sin(x)\right)$$
$$= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)}+\frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}+\frac{3}{8}\text{Subst}\Big($$
$$= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)}+\frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}+\frac{3}{8}\text{Subst}\Big($$
$$= \frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right)-\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)}+\frac{1}{4}\sin(x)\Big($$

Mathematica [A] time = 0.07, size = 61, normalized size = 1.05

$$\frac{\sqrt{2\cos(2x)-3}\left(2(2\sin(3x)-11\sin(x))\sqrt{3-2\cos(2x)}-3\sinh^{-1}(2\sin(x))\right)}{16\sqrt{4\sin^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11*
Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [C] time = 1.31, size = 122, normalized size = 2.10

$$\frac{1}{128}\left(12i\,e^{(4ix)}\log\left(-\frac{1}{2}\sqrt{e^{(4ix)}-3\,e^{(2ix)}+1}\left(4\,e^{(2ix)}-5\right)+2\,e^{(4ix)}-\frac{11}{2}\,e^{(2ix)}+\frac{5}{2}\right)-12i\,e^{(4ix)}\log\left(\sqrt{e^{(4ix)}-3\,e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(12*I*e^(4*I*x)*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1)*(4*e^(2*I*
x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) - 12*I*e^(4*I*x)*log(sqrt(e^(
4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) - 1) + (-16*I*e^(6*I*x) + 88*I*e^(4*I
*x) - 88*I*e^(2*I*x) + 16*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 145*I*e^(4
*I*x))*e^(-4*I*x)
```

giac [C] time = 0.64, size = 41, normalized size = 0.71

$$-\frac{1}{8}i\left(8\sin(x)^2+5\right)\sqrt{4\sin(x)^2+1}\sin(x)+\frac{3}{16}i\log\left(\sqrt{4\sin(x)^2+1}-2\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

[Out] -1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) + 3/16*I*log(sqrt(4*sin(x)^2 + 1) - 2*sin(x))

maple [A] time = 0.22, size = 82, normalized size = 1.41

method	result	size
default	$\frac{\sqrt{\left(4\left(\cos^{2}(x)\right)-5\right)\left(\sin^{2}(x)\right)}\,\left(-32\sqrt{-4\left(\sin^{4}(x)\right)-\left(\sin^{2}(x)\right)}\,\left(\sin^{2}(x)\right)-20\sqrt{-4\left(\sin^{4}(x)\right)-\left(\sin^{2}(x)\right)}+3\arcsin\left(8\left(\sin^{2}(x)\right)+1\right)\right)}{32\sin(x)\sqrt{4\left(\cos^{2}(x)\right)-5}}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/32*((4*cos(x)^2-5)*sin(x)^2)^(1/2)*(-32*(-4*sin(x)^4-sin(x)^2)^(1/2)*sin(x)^2-20*(-4*sin(x)^4-sin(x)^2)^(1/2)+3*arcsin(8*sin(x)^2+1))/sin(x)/(4*cos(x)^2-5)^(1/2)

maxima [C] time = 1.21, size = 36, normalized size = 0.62

$$\frac{1}{4}\left(-4\,\sin(x)^{2}-1\right)^{\frac{3}{2}}\sin(x)-\frac{3}{8}\sqrt{-4\,\sin(x)^{2}-1}\,\sin(x)-\frac{3}{16}i\,\operatorname{arsinh}\left(2\,\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")

[Out] 1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*sin(x) - 3/16*I*arcsinh(2*sin(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cos(x)\left(-\cos(x)^{2}-5\sin(x)^{2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2),x)

[Out] int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)

[Out] Timed out

3.422 $\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4357, 192, 191}

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2)^(3/2)) +
 Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{7/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}}+\frac{2}{5}\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{5/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}}-\frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}}-\frac{2}{15}\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{3/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}}-\frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}}+\frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 37, normalized size = 0.67

$$\frac{\cos(x)(56\cos(2x)+49\cos(4x)+67)}{15\sqrt{2}\,(7\cos(2x)+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/2
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.98, size = 51, normalized size = 0.93

$$\frac{\left(98\cos(x)^5-70\cos(x)^3+15\cos(x)\right)\sqrt{7\cos(x)^2-2}}{30\left(343\cos(x)^6-294\cos(x)^4+84\cos(x)^2-8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)/(343*cos(
x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

giac [A] time = 0.67, size = 30, normalized size = 0.55

$$\frac{\left(14\left(7\cos(x)^2-5\right)\cos(x)^2+15\right)\cos(x)}{30\left(7\cos(x)^2-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

maple [A] time = 0.13, size = 44, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\cos(x)}{10\left(-2+7\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}-\frac{\cos(x)}{15\left(-2+7\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7\left(\cos^2(x)\right)}}$	44
default	$\frac{\cos(x)}{10\left(-2+7\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}-\frac{\cos(x)}{15\left(-2+7\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7\left(\cos^2(x)\right)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*co
s(x)/(-2+7*cos(x)^2)^(1/2)
```

maxima [A] time = 0.42, size = 43, normalized size = 0.78

$$\frac{\cos(x)}{15\sqrt{7\cos(x)^2-2}}-\frac{\cos(x)}{15\left(7\cos(x)^2-2\right)^{\frac{3}{2}}}+\frac{\cos(x)}{10\left(7\cos(x)^2-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(3/2) + 1/1
0*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

mupad [B] time = 0.66, size = 28, normalized size = 0.51

$$\frac{\cos(x)\,\left(98\cos(x)^4-70\cos(x)^2+15\right)}{30\left(7\cos(x)^2-2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x)
```

```
[Out] (cos(x)*(98*cos(x)^4 - 70*cos(x)^2 + 15))/(30*(7*cos(x)^2 - 2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.423 $\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4356, 385, 216}

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx&=\text{Subst}\left(\int \frac{1-2x^2}{\left(2-5x^2\right)^{3/2}}\,dx,x,\sin(x)\right)\\&=\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}+\frac{2}{5}\text{Subst}\left(\int \frac{1}{\sqrt{2-5x^2}}\,dx,x,\sin(x)\right)\\&=\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 39, normalized size = 1.00

$$\frac{1}{50}\left(4\sqrt{5}\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)+\frac{5\sin(x)}{\sqrt{2-5\sin^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.81, size = 100, normalized size = 2.56

$$-\frac{\left(5\sqrt{5}\cos(x)^2-3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos(x)^4-80\sqrt{5}\cos(x)^2+31\sqrt{5}\right)\sqrt{5\cos(x)^2-3}}{10\left(25\cos(x)^4-35\cos(x)^2+12\right)\sin(x)}\right)-5\sqrt{5\cos(x)^2-3}\sin(x)}{50\left(5\cos(x)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 -
80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35*c
os(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)
```

giac [A] time = 0.70, size = 38, normalized size = 0.97

$$\frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin(x)\right)-\frac{\sqrt{-5\sin(x)^2+2}\sin(x)}{10\left(5\sin(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(x
)/(5*sin(x)^2 - 2)
```

maple [B] time = 0.28, size = 58, normalized size = 1.49

method	result	size
default	$\frac{20\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)\sqrt{5}\left(\cos^2(x)\right)+5\sin(x)\sqrt{5\left(\cos^2(x)\right)-3}-12\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)\sqrt{5}}{250\left(\cos^2(x)\right)-150}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/50/(5*cos(x)^2-3)*(20*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2)*cos(x)^2+5*sin(
x)*(5*cos(x)^2-3)^(1/2)-12*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2))
```

maxima [B] time = 1.32, size = 716, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) +
5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*
cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2*x) - 5)*cos(4*x) + 25*cos(4*x
)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 -
20*cos(2*x) + 25)^(1/4)*(sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)^(1/4
)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 - 10*co
s(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*sin(1/2*a
rctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2*x)^2
- 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt(6)*cos(2*x) + 1/2*(25/36
)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 -
 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*cos
(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2
*x)^2 - 5/12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arctan2(1/12*sqrt
(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos
(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 240
1)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/
24*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) + sin(2*x)), 1/12*sqrt(6)*cos(2*x)
+ 1/2*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos(2*x)^2
 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 2401)^(1/4
)*cos(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/24*sin(
2*x)^2 - 5/12*cos(2*x) + 49/24)) - 5/12*sqrt(6))))/(-10*(2*cos(2*x) - 5)*co
s(4*x) + 25*cos(4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 - 20*cos(2*x) + 25)^(1/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(2\,x)\,\cos(x)}{\left(2-5\sin(x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2),x)
```

```
[Out] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 216}

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] -ArcSin[(2*Cos[x])/3]/2 - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (295*Co
s[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1-12x^2+16x^4}{\left(9-4x^2\right)^{5/2}}\,dx, x, \cos(x)\right)$$
$$= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{1}{27}\text{Subst}\left(\int \frac{52+108x^2}{\left(9-4x^2\right)^{3/2}}\,dx, x, \cos(x)\right)$$
$$= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\text{Subst}\left(\int \frac{1}{\sqrt{9-4x^2}}\,dx, x, \cos(\right.$$
$$= -\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}$$

Mathematica [C] time = 0.29, size = 63, normalized size = 1.31

$$\frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(\sqrt{7-2\cos(2x)}+2i\cos(x)\right)}{486(7-2\cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] (2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos[
x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.52, size = 131, normalized size = 2.73

$$\frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{81\cos(x)\sin(x)-4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{-4\cos(x)^2+9}}{64\cos(x)^4-225\cos(x)^2+81}\right)-243\left(16\cos(x)^4-72\right.}{972\left(16\cos(x)^4-72\cos(x)^2+81\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(81*cos(x)*sin(x) - 4*(
8*cos(x)^3 - 9*cos(x))*sqrt(-4*cos(x)^2 + 9))/(64*cos(x)^4 - 225*cos(x)^2 +
 81)) - 243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(sin(x)/cos(x)) - 80*(59
*cos(x)^3 - 108*cos(x))*sqrt(-4*cos(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)^2 +
 81)
```

giac [A] time = 0.68, size = 40, normalized size = 0.83

$$-\frac{20\left(59\cos(x)^2-108\right)\sqrt{-4\cos(x)^2+9}\cos(x)}{243\left(4\cos(x)^2-9\right)^2}-\frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^2
 - 1/2*arcsin(2/3*cos(x))
```

maple [A] time = 0.23, size = 53, normalized size = 1.10

method	result	size
derivativedivides	$-\frac{4\left(\cos^{3}\left(x\right)\right)}{3\left(9-4\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}+\frac{214\cos\left(x\right)}{243\sqrt{9-4\left(\cos^{2}\left(x\right)\right)}}-\frac{\arcsin\left(\frac{2\cos\left(x\right)}{3}\right)}{2}+\frac{26\cos\left(x\right)}{27\left(9-4\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}$	53
default	$-\frac{4\left(\cos^{3}\left(x\right)\right)}{3\left(9-4\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}+\frac{214\cos\left(x\right)}{243\sqrt{9-4\left(\cos^{2}\left(x\right)\right)}}-\frac{\arcsin\left(\frac{2\cos\left(x\right)}{3}\right)}{2}+\frac{26\cos\left(x\right)}{27\left(9-4\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*cos(x)^3/(9-4*cos(x)^2)^(3/2)+214/243*cos(x)/(9-4*cos(x)^2)^(1/2)-1/2*
arcsin(2/3*cos(x))+26/27*cos(x)/(9-4*cos(x)^2)^(3/2)
```

maxima [A] time = 1.00, size = 69, normalized size = 1.44

$$-2\left(\frac{2\cos\left(x\right)^{2}}{\left(-4\cos\left(x\right)^{2}+9\right)^{\frac{3}{2}}}-\frac{3}{\left(-4\cos\left(x\right)^{2}+9\right)^{\frac{3}{2}}}\right)\cos\left(x\right)+\frac{52\cos\left(x\right)}{243\sqrt{-4\cos\left(x\right)^{2}+9}}+\frac{26\cos\left(x\right)}{27\left(-4\cos\left(x\right)^{2}+9\right)^{\frac{3}{2}}}-\frac{1}{2}\arcsin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2))*cos(x)
+ 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-4*cos(x)^2 + 9)^(3/2
) - 1/2*arcsin(2/3*cos(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(5\,x\right)}{{\left(5\,{\cos\left(x\right)}^2+9\,{\sin\left(x\right)}^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x)
```

```
[Out] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4356, 1247, 698}

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/(4*(-5 + 4*Sin[x]^2)^(3/2)) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 + 4*
Sin[x]^2]/8
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x\left(3-10x^2+8x^4\right)}{\left(-5+4x^2\right)^{5/2}}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{3-10x+8x^2}{(-5+4x)^{5/2}}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3}{(-5+4x)^{5/2}}+\frac{5}{2(-5+4x)^{3/2}}+\frac{1}{2\sqrt{-5+4x}}\right)dx, x, \sin^2(x)\right)\\
&= -\frac{1}{4\left(-5+4\sin^2(x)\right)^{3/2}}-\frac{5}{8\sqrt{-5+4\sin^2(x)}}+\frac{1}{8}\sqrt{-5+4\sin^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 28, normalized size = 0.57

$$\frac{11\cos(2x)+\cos(4x)+12}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] (12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.46, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fric
as")
```

```
[Out] 0
```

giac [A] time = 0.64, size = 33, normalized size = 0.67

$$\frac{1}{8}\sqrt{4\sin(x)^2-5}-\frac{20\sin(x)^2-23}{8\left(4\sin(x)^2-5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="giac
")
```

```
[Out] 1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)
```

maple [A] time = 0.14, size = 46, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2\left(\cos^4(x)\right)}{\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}+\frac{7\left(\cos^2(x)\right)}{2\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}+\frac{1}{2\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}$	46
default	$\frac{2\left(\cos^4(x)\right)}{\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}+\frac{7\left(\cos^2(x)\right)}{2\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}+\frac{1}{2\left(-4\left(\cos^2(x)\right)-1\right)^{\frac{3}{2}}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*cos(x)^4/(-4*cos(x)^2-1)^(3/2)+7/2*cos(x)^2/(-4*cos(x)^2-1)^(3/2)+1/2/(-4
*cos(x)^2-1)^(3/2)
```

maxima [B] time = 0.53, size = 192, normalized size = 3.92

$$-\frac{\left(\cos\left(11\,x\right)+14\,\cos\left(9\,x\right)+58\,\cos\left(7\,x\right)+94\,\cos\left(5\,x\right)+58\,\cos\left(3\,x\right)+15\,\cos(x)\right)\cos\left(\frac{5}{2}\,\arctan\left(\sin\left(4\,x\right)+3\,s\right.\right.}{8\left(2\left(3\,\cos\left(2\,x\right)+1\right)co\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] -1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*cos(3*x) +
15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x) + 94*sin(5*x) + 58*sin(3*x) +
13*sin(x))*sin(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)))/(2*(3*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*x)^2
+ 6*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

mupad [B] time = 0.63, size = 28, normalized size = 0.57

$$\frac{2\cos\left(2\,x\right)^2+11\,\cos\left(2\,x\right)+11}{4\left(-2\,\cos\left(2\,x\right)-3\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x)*sin(3*x)*cos(x))/(4*sin(x)^2 - 5)^(5/2),x)
```

```
[Out] (11*cos(2*x) + 2*cos(2*x)^2 + 11)/(4*(- 2*cos(2*x) - 3)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.426 $\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$

Optimal. Leaf size=111

$$2\sqrt{\sin^2(x)-5}-\frac{2\tan^{-1}\left(\frac{\sqrt{\sin^2(x)-5}}{\sqrt{5}}\right)}{\sqrt{5}}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+\frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{\sin^2(x)-5}}\right.$$

Rubi [A] time = 0.57, antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {4401, 4356, 451, 217, 206, 4366, 6725, 203, 261, 1010, 377, 444, 63}

$$2\sqrt{-\cos^2(x)-4}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-\cos^2(}}{\sqrt{5}}\right.}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5 + Sin[
x]^2],x]
```

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4 - Co
s[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5]])/Sqrt[5] - 2*ArcT
anh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 - Cos[x]^2] + (2*Csc[x]*Sqrt[-5
 + Sin[x]^2])/5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \int\left(\frac{2\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}+\frac{\left(-2\cos^3(x)+\cos(2x)\right)\csc\right.$$

$$= 2\int \frac{\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}\,dx+\int \frac{\left(-2\cos^3(x)+\cos(2x\right.}{\sqrt{-5+\sin^2(x}}$$

$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{x^2\sqrt{-5+x^2}}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int\right.$$

$$= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}\;-2\,\text{Subst}\left(\int \frac{1}{\sqrt{-5+x^2}}\right.$$

$$= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}\;+2\,\text{Subst}\left(\int \frac{1}{\sqrt{-4-x^2}}\right.$$

$$= -2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-4-\cos^2(x)}\;+$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2}}\right.$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}}-$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}}-$$

Mathematica [C] time = 2.32, size = 338, normalized size = 3.05

$$\frac{(16-32i)\sqrt{5}\sqrt{\frac{(1+2i)(\cos(x)-2i)}{\cos(x)+1}}\sqrt{\frac{(1-2i)(\cos(x)+2i)}{\cos(x)+1}}\cos^2\left(\frac{x}{2}\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+2i)\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right),-\frac{7}{25}+\frac{24i}{25}\right)-(32-64i)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5
+ Sin[x]^2],x]
```

```
[Out] ((16 - 32*I)*Sqrt[5]*Cos[x/2]^2*Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/(1 + Cos[x
])]*Sqrt[((1 - 2*I)*(2*I + Cos[x]))/(1 + Cos[x])]*EllipticF[ArcSin[((1 + 2*
I)*Tan[x/2])/Sqrt[5]], -7/25 + (24*I)/25] - (32 - 64*I)*Sqrt[5]*Cos[x/2]^2*
Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/(1 + Cos[x])]*Sqrt[((1 - 2*I)*(2*I + Cos[x
]))/(1 + Cos[x])]*EllipticPi[3/5 + (4*I)/5, ArcSin[((1 + 2*I)*Tan[x/2])/Sqr
t[5]], -7/25 + (24*I)/25] - 5*(85 + Sqrt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[
-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]
/Sqrt[10]]*Sqrt[-9 - Cos[2*x]] + 18*Csc[x] + 2*Cos[2*x]*Csc[x] + (10*I)*Sqr
t[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[-9 - Cos[2*x]]] + 5*Cs
c[x]*Sin[3*x]))/(25*Sqrt[2]*Sqrt[-9 - Cos[2*x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))
/Sqrt[-5 + Sin[x]^2],x]
```

```
[Out] Could not integrate
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2 (\sin(x) - 1) \cos(x)^3 - \cos(2x) \sin(x)}{\sqrt{\sin(x)^2 - 5} \sin(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)*
sin(x)^2), x)
```

maple [A] time = 0.44, size = 131, normalized size = 1.18

method	result
default	$-2\ln\left(\sin(x)+\sqrt{-5+\sin^2(x)}\right)+2\sqrt{-5+\sin^2(x)}+\frac{2\sqrt{5}\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin^2(x)}}\right)}{5}+\frac{2\sqrt{-5+\sin^2(x)}}{5\sin(x)}-\frac{\sqrt{(-5+\sin^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2),
x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(sin(x)+(-5+sin(x)^2)^(1/2))+2*(-5+sin(x)^2)^(1/2)+2/5*5^(1/2)*arctan(
5^(1/2)/(-5+sin(x)^2)^(1/2))+2/5*(-5+sin(x)^2)^(1/2)/sin(x)-1/10*((-5+sin(x
)^2)*cos(x)^2)^(1/2)*(-5^(1/2)*arctan(1/5*(3*sin(x)^2-5)*5^(1/2)/(-cos(x)^4
-4*cos(x)^2)^(1/2))-10*arcsin(1+1/2*cos(x)^2))/cos(x)/(-5+sin(x)^2)^(1/2)
```

maxima [C] time = 1.02, size = 115, normalized size = 1.04

$$\frac{2}{5}\sqrt{5}\arcsin\left(\frac{\sqrt{5}}{|\sin(x)|}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)}-\frac{2}{\cos(x)+1}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)}-\frac{2}{\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x)
/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x)/(cos(x
) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)/si
n(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2x)\,\sin(x) - 2\cos(x)^3\,(\sin(x) - 1)}{\sin(x)^2\,\sqrt{\sin(x)^2 - 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1
/2)),x)
```

```
[Out] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)**
2)**(1/2),x)
```

```
[Out] Timed out
```

3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

Optimal. Leaf size=112

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{4\cos^2(x)-1}-\frac{1}{2}\sin(x)\sqrt{8\cos^2(x)-1}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x}{\sqrt{4\cos^2(x}}\right.$$

Rubi [A] time = 0.44, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 402, 216, 377, 204, 195}

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x}{\sqrt{7-8\sin}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]])
/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3 -
4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*Sin[
x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{-1+4x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx, x, \sin(x)\right)$$

$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}+\frac{4x^2}{\sqrt{7-8x^2}-\sqrt{3-4.}}\right.\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx, x, \sin(x)\right)-\text{Subst}\left(\right.$$

$$= 4\,\text{Subst}\left(\int \left(-\frac{1}{4}\sqrt{7-8x^2}-\frac{1}{4}\sqrt{3-4x^2}-\frac{\sqrt{7-8x^2}}{4\left(-1+x^2\right)}-\right.\right.$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{7-8x^2}}{-1+x^2}\,dx, x, \sin(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{3-}}{-1+}\right.$$

$$= -\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}-\frac{1}{4}\text{Su}$$

$$= -\frac{11\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+2\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+\frac{3}{4}$$

$$= -\frac{11\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+2\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+\frac{3}{4}$$

Mathematica [A] time = 0.34, size = 156, normalized size = 1.39

$$\frac{1}{8}\left(5\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+6\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-4\sin(x)\sqrt{2\cos(2x)+1}-4\sin(x)\sqrt{4\cos(2x)+3}+3\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x
]
```

```
[Out] (5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*Ar
cTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[1
+ 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTan[(
7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4*Sqr
t[3 + 4*Cos[2*x]]*Sin[x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin
[x]^2]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.51, size = 195, normalized size = 1.74

$$-\frac{5}{32}\sqrt{2}\arctan\left(\frac{\left(512\sqrt{2}\cos(x)^4-576\sqrt{2}\cos(x)^2+113\sqrt{2}\right)\sqrt{8\cos(x)^2-1}}{16\left(128\cos(x)^4-88\cos(x)^2+9\right)\sin(x)}\right)-\frac{1}{2}\sqrt{8\cos(x)^2-1}\sin(x)-\frac{1}{2}\sqrt{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="fricas")
```

```
[Out] -5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 11
3*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x)))
- 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) + 3/8*a
rctan((4*(8*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x))/(6
4*cos(x)^4 - 71*cos(x)^2 + 16)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arctan(1/
2*(9*cos(x)^2 - 2)/(sqrt(8*cos(x)^2 - 1)*sin(x))) + 3/4*arctan(sqrt(4*cos(x
)^2 - 1)/sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos(3x)}{\sqrt{8\cos(x)^2-1}-\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="giac")
```

```
[Out] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)), x
)
```

maple [F] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\left(\cos^2(x)\right)}+\sqrt{3\left(\cos^2(x)\right)-\left(\sin^2(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

```
[Out] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)),x)
```

```
[Out] -int(-cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{\sqrt{-\sin^2(x) + 3\cos^2(x)} - \sqrt{8\cos^2(x) - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2)
),x)
```

```
[Out] Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1)),
 x)
```

3.428 $\int \left(2-3 \sin^2(x)\right)^{3/5} \sin(4x)\, dx$

Optimal. Leaf size=33

$$\frac{5}{36} \left(2-3 \sin^2(x)\right)^{8/5}-\frac{20}{117} \left(2-3 \sin^2(x)\right)^{13/5}$$

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {12, 444, 43}

$$\frac{5}{36} \left(2-3 \sin^2(x)\right)^{8/5}-\frac{20}{117} \left(2-3 \sin^2(x)\right)^{13/5}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(2-3 \sin^2(x)\right)^{3/5} \sin(4x)\, dx &= \text{Subst}\left(\int 4x \left(2-3x^2\right)^{3/5} \left(1-2x^2\right)\, dx, x, \sin(x)\right)\\
&= 4\,\text{Subst}\left(\int x \left(2-3x^2\right)^{3/5} \left(1-2x^2\right)\, dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int (2-3x)^{3/5}(1-2x)\, dx, x, \sin^2(x)\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{3}(2-3x)^{3/5}+\frac{2}{3}(2-3x)^{8/5}\right) dx, x, \sin^2(x)\right)\\
&= \frac{5}{36} \left(2-3 \sin^2(x)\right)^{8/5}-\frac{20}{117} \left(2-3 \sin^2(x)\right)^{13/5}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.88

$$-\frac{5(3 \cos(2x)+1)^{8/5}(24 \cos(2x)-5)}{936\ 2^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] (-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.50, size = 26, normalized size = 0.79

$$-\frac{5}{468}\left(144\cos(x)^4-135\cos(x)^2+29\right)\left(3\cos(x)^2-1\right)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

giac [A] time = 0.66, size = 35, normalized size = 1.06

$$-\frac{20}{117}\left(3\sin(x)^2-2\right)^2\left(-3\sin(x)^2+2\right)^{\frac{3}{5}}+\frac{5}{36}\left(-3\sin(x)^2+2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="giac")
```

```
[Out] -20/117*(3*sin(x)^2 - 2)^2*(-3*sin(x)^2 + 2)^(3/5) + 5/36*(-3*sin(x)^2 + 2)
^(8/5)
```

maple [A] time = 0.25, size = 38, normalized size = 1.15

method	result	size
default	$\frac{5\left(3\left(\cos^2(x)\right)-1\right)^{\frac{8}{5}}}{12}-\frac{20\left(\frac{1}{2}+\frac{3\cos(2x)}{2}\right)^{\frac{13}{5}}}{117}-\frac{5\left(\frac{1}{2}+\frac{3\cos(2x)}{2}\right)^{\frac{8}{5}}}{18}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*sin(x)^2)^(3/5)*sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*(3*cos(x)^2-1)^(8/5)-20/117*(1/2+3/2*cos(2*x))^(13/5)-5/18*(1/2+3/2*co
s(2*x))^(8/5)
```

maxima [A] time = 0.44, size = 25, normalized size = 0.76

$$-\frac{20}{117}\left(-3\sin(x)^2+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\sin(x)^2+2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sin(4\,x)\,\left(2-3\sin(x)^2\right)^{3/5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5),x)
```

```
[Out] int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

```
[Out] Timed out
```

3.429 $\int \cos(x)\sqrt{\cos(2x)}\, dx$

Optimal. Leaf size=33

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4356, 195, 216}

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\sqrt{\cos(2x)}\, dx &= \text{Subst}\left(\int \sqrt{1-2x^2}\, dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sqrt{\cos(2x)}\sin(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}}\, dx, x, \sin(x)\right)\\
&= \frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sqrt{\cos(2x)}\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{1}{4}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)+2\sin(x)\sqrt{\cos(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.32, size = 77, normalized size = 2.33

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)+\frac{1}{2}\sqrt{2\cos(x)^2-1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*sq
rt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x))) + 1/2*
sqrt(2*cos(x)^2 - 1)*sin(x)
```

giac [A] time = 0.69, size = 27, normalized size = 0.82

$$\frac{1}{4}\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)+\frac{1}{2}\sqrt{-2\sin(x)^2+1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)
```

maple [B] time = 0.16, size = 62, normalized size = 1.88

method	result	size
default	$-\frac{\sqrt{\left(2\left(\cos^2(x)\right)-1\right)\left(\sin^2(x)\right)}\left(-\sqrt{2}\arcsin\left(4\left(\sin^2(x)\right)-1\right)-4\sqrt{-2\left(\sin^4(x)\right)+\sin^2(x)}\right)}{8\sin(x)\sqrt{2\left(\cos^2(x)\right)-1}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(-2^(1/2)*arcsin(4*sin(x)^2-1)-4*(-2*s
in(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

maxima [B] time = 1.15, size = 488, normalized size = 14.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) - 1)*sin(1/2*arctan2(s
in(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
```

```
 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - cos(2*x)*sin
(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*
x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)^2 + si
n(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*
sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), co
s(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) - 1) - ar
ctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(
4*x), cos(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)), (cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\cos(2\,x)}\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(1/2)*cos(x),x)
```

```
[Out] int(cos(2*x)^(1/2)*cos(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)**(1/2),x)
```

```
[Out] Integral(cos(x)*sqrt(cos(2*x)), x)
```

3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$

Optimal. Leaf size=55

$$-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4357, 195, 217, 206}

$$-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Cos[x]*Sqrt[
Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx &= -\text{Subst}\left(\int \left(-1+2x^2\right)^{3/2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{4}\text{Subst}\left(\int \sqrt{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{-1+2x^2}}\,dx, x, \cos(x)\right)\\ &= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\\ &= -\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)\end{aligned}$$

Mathematica [A] time = 0.09, size = 49, normalized size = 0.89

$$-\frac{1}{8}\sqrt{\cos(2x)}\,(\cos(3x)-2\cos(x))-\frac{3\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] -1/8*(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x])) - (3*Log[Sqrt[2]*Cos[x] + Sqrt
[Cos[2*x]]])/(8*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.50, size = 103, normalized size = 1.87

$$-\frac{1}{8}\left(4\cos(x)^3-5\cos(x)\right)\sqrt{2\cos(x)^2-1}+\frac{3}{128}\sqrt{2}\log\Big(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4-64\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)*log(2048*
cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*cos(
x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(2*c
os(x)^2 - 1) + 1)
```

giac [A] time = 0.68, size = 48, normalized size = 0.87

$$-\frac{1}{8}\left(4\cos(x)^2-5\right)\sqrt{2\cos(x)^2-1}\cos(x)+\frac{3}{16}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*log(abs(-s
qrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

maple [A] time = 0.13, size = 55, normalized size = 1.00

method	result	size
default	$-\frac{\left(\cos^{3}(x)\right)\sqrt{2\left(\cos^{2}(x)\right)-1}}{2}+\frac{5\cos(x)\sqrt{2\left(\cos^{2}(x)\right)-1}}{8}-\frac{3\ln\left(\cos(x)\sqrt{2}+\sqrt{2\left(\cos^{2}(x)\right)-1}\right)\sqrt{2}}{16}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x)^3*(2*cos(x)^2-1)^(1/2)+5/8*cos(x)*(2*cos(x)^2-1)^(1/2)-3/16*ln(
cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)
```

maxima [B] time = 1.20, size = 790, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(4
*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(si
n(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(1/2*
arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x)
 + 1))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arct
an2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - 3
*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x
), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4
*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + 3*log(((cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x), c
os(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 2*(
cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1) - 3*log(((co
s(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))
^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x),
 cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) - 2
*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x)
, cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1))
```

mupad [B] time = 0.42, size = 29, normalized size = 0.53

$$-\frac{\cos\left(2\,x\right)^{3/2}\,\cos(x)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \cos\left(2\,x\right)+1\right)}{{\left(-\cos\left(2\,x\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)*sin(x),x)
```

```
[Out] -(cos(2*x)^(3/2)*cos(x)*hypergeom([-3/2, 1/2], 3/2, cos(2*x) + 1))/(-cos(2*
x))^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**(3/2)*sin(x),x)
```

```
[Out] Timed out
```

3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4331}

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Cos[2*x]^(5/2),x]
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Rule 4331

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> -Simp[((m + 2)*(e*Cos[a + b*x])^(m + 1)*Cos[(m + 1)*(a + b*x)])/(d*e*(
m + 1)), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, Abs[m + 2]]
```

Rubi steps

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Cos[2*x]^(5/2),x]
[Out] -1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sin[x]/Cos[2*x]^(5/2),x]
[Out] Could not integrate
```

fricas [B] time = 1.33, size = 39, normalized size = 2.44

$$-\frac{\left(4\cos(x)^3 - 3\cos(x)\right)\sqrt{2\cos(x)^2 - 1}}{3\left(4\cos(x)^4 - 4\cos(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4*cos(x)^2
+ 1)
```

giac [B] time = 0.97, size = 46, normalized size = 2.88

$$\frac{\left(\left(\tan\left(\frac{1}{2}\,x\right)^2 - 15\right)\tan\left(\frac{1}{2}\,x\right)^2 + 15\right)\tan\left(\frac{1}{2}\,x\right)^2 - 1}{3\left(\tan\left(\frac{1}{2}\,x\right)^4 - 6\,\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((tan(1/2*x)^2 - 15)*tan(1/2*x)^2 + 15)*tan(1/2*x)^2 - 1)/(tan(1/2*x)^
4 - 6*tan(1/2*x)^2 + 1)^(3/2)
```

maple [B] time = 0.18, size = 39, normalized size = 2.44

method	result	size
default	$\frac{\sqrt{-2\left(\sin^2(x)\right)+1}\ \cos(x)\left(4\left(\sin^2(x)\right)-1\right)}{12\left(\sin^4(x)\right)-12\left(\sin^2(x)\right)+3}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(4*sin(x)^4-4*sin(x)^2+1)*(-2*sin(x)^2+1)^(1/2)*cos(x)*(4*sin(x)^2-1)
```

maxima [B] time = 1.02, size = 90, normalized size = 5.62

$$-\frac{\sqrt{2}\,\sin\left(\frac{3}{2}\,\arctan\left(\sin\left(4\,x\right),\cos\left(4\,x\right)+1\right)\right)\sin\left(\frac{3}{2}\,\arctan\left(\sin\left(4\,x\right),\cos\left(4\,x\right)\right)\right)+\left(\sqrt{2}\,\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(4\right.\right.\right.\right.}{3\left(\cos\left(4\,x\right)^2+\sin\left(4\,x\right)^2+2\,\cos\left(4\,x\right)+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*arctan2(sin(
4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + sqrt(2))
*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos
(4*x) + 1)^(3/4)
```

mupad [B] time = 0.35, size = 12, normalized size = 0.75

$$-\frac{\cos\left(3\,x\right)}{3\,{\cos\left(2\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x)^(5/2),x)
```

```
[Out] -cos(3*x)/(3*cos(2*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(x\right)}{\cos^{\frac{5}{2}}\left(2x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)**(5/2),x)
```

```
[Out] Integral(sin(x)/cos(2*x)**(5/2), x)
```

3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

Optimal. Leaf size=49

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {4364, 413, 523, 216, 377, 203}

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]])/2 - (S
qrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
```

```
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-2x^2\right)^{3/2}}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{2}\sqrt{\cos(2x)}\,\sec(x)\tan(x) - \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x^2}{\sqrt{1-2x^2}\left(1-x^2\right)}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{2}\sqrt{\cos(2x)}\,\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}\left(1-x^2\right)}\,dx, x, \sin(x)\right) + 4\,\text{Subst}\left(\int\right.\\
&= 2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \frac{1}{2}\sqrt{\cos(2x)}\,\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sin(x)}{\sqrt{\cos(2}}\right.\\
&= 2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\,\sec(x)\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 49, normalized size = 1.00

$$\frac{1}{2}\left(4\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - 5\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \sqrt{\cos(2x)}\,\tan(x)\sec(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] (4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt[
Cos[2*x]]*Sec[x]*Tan[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.97, size = 118, normalized size = 2.41

$$-\frac{2\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)\cos(x)^2 - 5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{2\cos(x)^2-1}\sin(x)}\right)\cos(x)^2 + 2\sqrt{2}}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*
sqrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x)))*cos(
x)^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(x)^
2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

maple [B] time = 0.21, size = 100, normalized size = 2.04

method	result
default	$-\frac{\sqrt{(2(\cos^2(x))-1)(\sin^2(x))}\left(4\sqrt{2}\ \arcsin(4(\cos^2(x))-3)(\cos^2(x))-5\arctan\left(\frac{3(\cos^2(x))-2}{2\sqrt{-2(\sin^4(x))+\sin^2(x)}}\right)(\cos^2(x))+2\sqrt{-2(\sin^4(x))+\sin^2(x)}\right)}{4\cos(x)^2\sin(x)\sqrt{2(\cos^2(x))-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)/cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x)
^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(-2
*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos(2x)}^{3/2}}{{\cos(x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)/cos(x)^3,x)
```

```
[Out] int(cos(2*x)^(3/2)/cos(x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

```
[Out] Timed out
```

3.433 $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=87

$$-\frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)}-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{63\cos(x)}{20\sqrt{\cos(2x)}}+\frac{3\sin^2(x)\cos(x)}{10\cos^{\frac{5}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.21, antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {4377, 12, 452, 288, 217, 206, 4366, 378, 191}

$$-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}+\frac{3\sin^4(x)\cos(x)}{5\cos^{\frac{5}{2}}(2x)}-\frac{4\sin^2(x)\cos(x)}{5\cos^{\frac{3}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (2*Cos[x]^3)/(3*Cos[2
*x]^(3/2)) + (13*Cos[x])/(5*Sqrt[Cos[2*x]]) - (4*Cos[x]*Sin[x]^2)/(5*Cos[2*
x]^(3/2)) + (3*Cos[x]*Sin[x]^4)/(5*Cos[2*x]^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = 3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}\,dx - \int \frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}\,dx$$

$$= -\left(3\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(-1+2x^2\right)^{7/2}}\,dx, x, \cos(x)\right)\right) + \text{Subst}\left(\int \frac{4x^2\left(1-\right.}{\left(-1+2\right.}\right.$$

$$= \frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)} + \frac{12}{5}\,\text{Subst}\left(\int \frac{1-x^2}{\left(-1+2x^2\right)^{5/2}}\,dx, x, \cos(x)\right) + 4\,\text{S}$$

$$= -\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} - \frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)} + \frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)} - \frac{8}{5}\,\text{Subst}\Bigg(\int$$

$$= -\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{13\cos(x)}{5\sqrt{\cos(2x)}} - \frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)} + \frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}$$

$$= -\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{13\cos(x)}{5\sqrt{\cos(2x)}} - \frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)} + \frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}} - \frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{13\cos(x)}{5\sqrt{\cos(2x)}} - \frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}$$

Mathematica [A] time = 0.20, size = 62, normalized size = 0.71

$$\frac{250\cos(x)+45\cos(3x)+169\cos(5x)-120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] (250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[S
qrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2)
,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.37, size = 163, normalized size = 1.87

$$15\left(8\sqrt{2}\cos(x)^6-12\sqrt{2}\cos(x)^4+6\sqrt{2}\cos(x)^2-\sqrt{2}\right)\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4-64\ c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="fricas")
```

```
[Out] 1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 -
sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8
*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2
)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 + 60*
cos(x))*sqrt(2*cos(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 - 1)
```

giac [A] time = 0.73, size = 55, normalized size = 0.63

$$\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)+\frac{\left(\left(169\cos(x)^2-200\right)\cos(x)^2+60\right)\cos(x)}{15\left(2\cos(x)^2-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*c
os(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)
```

maple [B] time = 0.41, size = 180, normalized size = 2.07

method	result
default	$-\frac{120\ln\left(\cos(x)\sqrt{2}+\sqrt{-2\left(\sin^2(x)\right)+1}\right)\sqrt{2}\left(\sin^6(x)\right)+338\sqrt{-2\left(\sin^2(x)\right)+1}\cos(x)\left(\sin^4(x)\right)-180\ln\left(\cos(x)\sqrt{2}+\sqrt{-2\left(\sin^2(x)\right)+1}\right)\sqrt{2}\left(\right.}{30\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*ln(cos(x)*2^(1/2)+(-2*sin(
x)^2+1)^(1/2))*2^(1/2)*sin(x)^6+338*(-2*sin(x)^2+1)^(1/2)*cos(x)*sin(x)^4-1
80*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2^(1/2)*sin(x)^4-276*(-2*sin(x)
^2+1)^(1/2)*sin(x)^2*cos(x)+90*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2^(
1/2)*sin(x)^2+58*(-2*sin(x)^2+1)^(1/2)*cos(x)-15*ln(cos(x)*2^(1/2)+(-2*sin(
x)^2+1)^(1/2))*2^(1/2))
```

maxima [B] time = 1.61, size = 1359, normalized size = 15.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="maxima")
```

```
[Out] 1/48*(4*(4*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x), cos(4*x))) + 4*(sqrt(
2)*cos(4*x) + sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 3*sqrt(2)*cos
(8*x) + 7*sqrt(2)*cos(4*x) + 4*sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x)
+ 1)) + 12*sqrt(2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*a
rctan2(sin(4*x), cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)
^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - 4*(4*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - 4*(sqrt(2)*
cos(4*x) + sqrt(2))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 3*sqrt(2)*sin(8*
x) - 7*sqrt(2)*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1)) - 3*(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2 + sqrt(2)*
sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)
)^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*
sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x
) + sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2
(sin(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2
+ 2*cos(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos
(1/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*
x)))) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x)
+ sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(s
in(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/
2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1) - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) +
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)
))) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(5/4) + 1/20*(((15*co
s(8*x) + 70*cos(4*x) + 43)*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 5*(3*sin(
8*x) + 14*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 12)*cos(5/2*arct
an2(sin(4*x), cos(4*x) + 1)) + 15*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) - (5*(3*sin(8*x) + 14*sin(4*x))*
cos(5/2*arctan2(sin(4*x), cos(4*x))) - (15*cos(8*x) + 70*cos(4*x) + 43)*sin
(5/2*arctan2(sin(4*x), cos(4*x))))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1))
 + 40*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*arctan2(sin(4*
x), cos(4*x) + 1)))/((sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*c
os(4*x) + sqrt(2))*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^2 \left(3\sin(x)^3 - \sin(4x)\cos(x)\right)}{\cos(2x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2),x)
[Out] int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
[Out] Timed out
```

3.434 $\int \left(4-5\sec^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=68

$$-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}+8\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{7}{2}\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 416, 523, 217, 203, 377}

$$8\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{7}{2}\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(Sqrt[5]*Tan
[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(4-5\sec^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(-1-5x^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{-3-35x^2}{\sqrt{-1-5x^2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\left(1+x^2\right)}\,dx, x, \tan(x)\right) - \frac{35}{2}\text{Sub}\\
&= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{35}{2}\text{Subs}\\
&= 8\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\,\tan^{-1}\left(\frac{\sqrt{5}\,\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(
\end{aligned}$$

Mathematica [C] time = 0.20, size = 115, normalized size = 1.69

$$-\frac{\left(4\cos^2(x)-5\right)\sec(x)\sqrt{4-5\sec^2(x)}\left(5\sin(x)\sqrt{2\cos(2x)-3}+16i\cos^2(x)\log\left(\sqrt{2\cos(2x)-3}+2i\sin(x)\right)+\right.}{2(2\cos(2x)-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan[(Sqrt
[5]*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]^2*Log[Sqrt[-3 +
 2*Cos[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[2*x]]*Sin[x]))/(-3 + 2*Cos
[2*x])^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.80, size = 130, normalized size = 1.91

$$\frac{7\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)}{5\sin(x)}\right)\cos(x)+8\,\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right)\cos(x)-8\,\text{arc}}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/si
n(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/co
```

```
s(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x) -
 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x))
/cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-5\sec(x)^2 + 4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

maple [C] time = 0.70, size = 754, normalized size = 11.09

method	result
default	$-\frac{i\left(-70i\left(\cos^2(x)\right)\sin(x)\sqrt{2}\ \sqrt{5}\ \sqrt{-\frac{2\left(2\cos(x)\sqrt{5}+4\cos(x)-2\sqrt{5}-5\right)}{1+\cos(x)}}\ \sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\ (-1+\cos(x))}{\sin(x)}, -\frac{}{9}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/(5^(1/2)+2)/(-9-4*5^(1/2))^(1/2)*(-70*I*cos(x)^2*sin(x)*2^(1/2)*5^(1
/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2*cos(x
)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticPi((-9-4*5^(1/2))
^(1/2)*(-1+cos(x))/sin(x),-1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/
2))^(1/2))+64*I*cos(x)^2*sin(x)*2^(1/2)*5^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos
(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)
/(1+cos(x)))^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(9+
4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))+3*I*cos(x)^2*sin(x)*2
^(1/2)*5^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2
)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticF(I*(-
1+cos(x))*(5^(1/2)+2)/sin(x),9-4*5^(1/2))-140*I*cos(x)^2*sin(x)*2^(1/2)*(-2
*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/
2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*
(-1+cos(x))/sin(x),-1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/
2))+128*I*cos(x)^2*sin(x)*2^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-
5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(
1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(9+4*5^(1/2)),(-9
+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))+6*I*cos(x)^2*sin(x)*2^(1/2)*(-2*(2*
cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/2)-4
*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticF(I*(-1+cos(x))*(5^(1/2)+2)/
sin(x),9-4*5^(1/2))+80*cos(x)^3*5^(1/2)+180*cos(x)^3-80*cos(x)^2*5^(1/2)-18
0*cos(x)^2-100*cos(x)*5^(1/2)-225*cos(x)+100*5^(1/2)+225)*cos(x)*sin(x)*((4
*cos(x)^2-5)/cos(x)^2)^(3/2)/(-1+cos(x))/(4*cos(x)^2-5)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-5\sec(x)^2 + 4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(4-\frac{5}{\cos(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - 5/cos(x)^2)^(3/2),x)
```

```
[Out] int((4 - 5/cos(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4-5\sec^2\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((4 - 5*sec(x)**2)**(3/2), x)
```

3.435 $\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4128, 382, 377, 203}

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan[
x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(-1-5x^2\right)^{3/2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)$$
$$= \frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}$$

Mathematica [A] time = 0.12, size = 79, normalized size = 1.98

$$-\frac{(2\cos(2x)-3)^{3/2}\sec^3(x)\left(10\sin(x)\sqrt{3-2\cos(2x)} + (2\cos(2x)-3)\sinh^{-1}(2\sin(x))\right)}{8\sqrt{-\left(4\sin^2(x)+1\right)^2}\left(4-5\sec^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x])
 + 10*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/((4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*Si
n[x]^2)^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.93, size = 115, normalized size = 2.88

$$-\frac{20\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x) - \left(4\cos(x)^2-5\right)\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right) + (4\cos(x}{16\left(4\cos(x)^2-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)*
arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) +
cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arctan
(sin(x)/cos(x)))/(4*cos(x)^2 - 5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-5\sec(x)^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

maple [C] time = 0.69, size = 473, normalized size = 11.82

method	result
default	$-\frac{i\left(4\left(\cos^2(x)\right)-5\right)\left(2i\sin(x)\sqrt{2}\,\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}+4\cos(x)-2\sqrt{5}-5\right)}{1+\cos(x)}}\,\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\sqrt{5}\ \mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/(-9-4*5^(1/2))^(1/2)/(5^(1/2)+2)*(4*cos(x)^2-5)*(2*I*sin(x)*2^(1/2)*
(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^
(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*5^(1/2)*EllipticPi((-9-4*5^(1
/2))^(1/2)*(-1+cos(x))/sin(x),1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^
(1/2))^(1/2))-I*sin(x)*2^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/
(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2
)*5^(1/2)*EllipticF(I*(-1+cos(x))*(5^(1/2)+2)/sin(x),9-4*5^(1/2))+4*I*sin(x
)*2^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*((2
*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*EllipticPi((-9-4*5^
(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*
5^(1/2))^(1/2))-2*I*sin(x)*2^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)
-5)/(1+cos(x)))^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^
(1/2)*EllipticF(I*(-1+cos(x))*(5^(1/2)+2)/sin(x),9-4*5^(1/2))+20*cos(x)*5^(
1/2)+45*cos(x)-20*5^(1/2)-45)*sin(x)/(-1+cos(x))/cos(x)^3/((4*cos(x)^2-5)/c
os(x)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-5\,\sec(x)^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(4-\frac{5}{\cos(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4 - 5/cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(4 - 5/cos(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((4 - 5*sec(x)**2)**(-3/2), x)
```

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{8}\cos(x)\sqrt{5\tan^2(x)+1}-\frac{\cos(x)}{4\sqrt{5\tan^2(x)+1}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot}{2\sqrt{5\tan^2}}$$

Rubi [A] time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4377, 12, 3670, 472, 583, 377, 206, 3664, 271, 191}

$$-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]]/4 + Cos[x]/(4*Sqrt[-4 + 5*Sec[x]^
2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan[x
]^2]) + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = \int -\frac{2\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx + \int \frac{\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$$

$$= -\left(2\int \frac{\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx\right) + \text{Subst}\left(\int \frac{1}{x^2\left(-4+5x^2\right)^{3/2}}\,dx, x, \sec(x)\right)$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - 2\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(1+5x^2\right)^{3/2}}\,dx, x, \tan(x)\right) + \frac{5}{2}\,\text{Subst}$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{}{x^2\left(1+\right.}\right.$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{9}{2}\cot(x)\sqrt{1+5\tan}$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{9}{2}\cot(x)\sqrt{1+5\tan}$$

$$= -\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right) + \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5c}{2\sqrt{1+}}$$

Mathematica [A] time = 0.52, size = 131, normalized size = 1.39

$$-\frac{\sin^2(x)(2\cos(2x)-3)^{3/2}\tan(x)\left(2\cot^2(x)\csc(x)-1\right)\left(\sqrt{4\sin^2(x)+1}\,\left(16\csc^3(x)-3\csc^2(x)+164\csc(x)-\right.\right.}{2\sqrt{-(3-2\cos(2x))^2}\,\sqrt{5\tan^2(x)+1}\,\left(\cot^2(x)+5\right)(-3\sin(x)+\sin(3x)+4\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSinh
[2*Sin[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*Sq
rt[1 + 4*Sin[x]^2])*Tan[x])/(Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4 +
4*Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 97, normalized size = 1.03

$$\frac{2\left(4\cos(x)^2-5\right)\log\left(\sqrt{-\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)-2\sin(x)\right)\sin(x)+\left(164\cos(x)^3-\left(2\cos(x)^3-5\cos(x)\right)\sin(x)\right.}{8\left(4\cos(x)^2-5\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

[Out] 1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*sin(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x))*sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,\cot(x)^2 - \sin(x)}{\left(5\,\tan(x)^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="giac")

[Out] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)

maple [C] time = 1.14, size = 975, normalized size = 10.37

method	result
default	$\frac{i\left(6i\cos(x)\sin(x)\sqrt{-\frac{4(\cos^2(x))-5}{(1+\cos(x))^2}}\ \operatorname{arctanh}\left(\frac{\sqrt{-16}\ \cos(x)(-1+\cos(x))}{2\sin(x)^2\sqrt{-\frac{4(\cos^2(x))-5}{(1+\cos(x))^2}}}\right)+4i\cos(x)\sin(x)\sqrt{2}\ \sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\ \sqrt{-\frac{2(2\cos(x)\sqrt{5}+}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/8*I/(-9+4*5^(1/2))^(1/2)/(5^(1/2)+2)^2/(-2+5^(1/2))^2/(4*cos(x)^2-5)^2*(-8*I*2^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticPi((-9+4*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+4*5^(1/2))^(1/2))-3*I*sin(x)*arctanh(1/2*(-16)^(1/2)*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*5^(1/2)+4*I*cos(x)*sin(x)*2^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*EllipticF(I*(-1+cos(x))*(-2+5^(1/2))/sin(x),9+4*5^(1/2))-8*I*cos(x)*2^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticPi((-9+4*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+4*5^(1/2))^(1/2))+6*I*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arctanh(1/2*(-16)^(1/2)*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))-3*cos(x)*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arctan(2*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))-3*I*cos(x)*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)*arctanh(1/2*(-16)^(1/2)*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))+6*I*cos(x)*sin(x)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*arctanh(1/2*(-16)^(1/2)*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))+2*cos(x)^2*sin(x)*5^(1/2)+6*cos(x)*sin(x)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*arctan(2*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))-3*sin(x)*arctan(2*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*5^(1/2)+4*I*2^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(x)*EllipticF(I*(-1+cos(x))*(-2+5^(1/2))/sin(x),9+4*5^(1/2))-4*cos(x)^2*sin(x)-164*cos(x)^2*5^(1/2)+6*arctan(2*cos(x)*(-1+cos(x))/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)+328*cos(x)^2-5*sin(x)*5^(1/2)+10*sin(x)+180*5^(1/2)-360)*cos(x)^3*(-(4*cos(x)^2-5)/cos(x)^2)^(3/2)/sin(x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x) - 2\cot(x)^2}{\left(5\tan(x)^2 + 1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2),x)
[Out] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sin(x)}{5\sqrt{5\tan^2(x)+1}\,\tan^2(x)+\sqrt{5\tan^2(x)+1}}\right)dx-\int \frac{2\cot^2(x)}{5\sqrt{5\tan^2(x)+1}\,\tan^2(x)+\sqrt{5\tan^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
[Out] -Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 + 1
)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*t
an(x)**2 + 1)), x)
```

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

Rubi [A] time = 0.15, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {12, 434, 453, 191}

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

```
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx &= \text{Subst}\left(\int \frac{2\left(-1-2x^2\right)}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{-1-2x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{\left(-2-\frac{1}{x^2}\right)x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)\\ &= -\frac{1}{3}\sqrt{4-\cot^2(x)}\,\tan^3(x)-\frac{8}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)\\ &= -\frac{2}{3}\sqrt{4-\cot^2(x)}\,\tan(x)-\frac{1}{3}\sqrt{4-\cot^2(x)}\,\tan^3(x)\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.92

$$\frac{(\cos(2x)+3)(5\cos(2x)-3)\csc(x)\sec^3(x)}{12\sqrt{4-\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
[Out] ((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
[Out] Could not integrate
```

fricas [A] time = 1.55, size = 33, normalized size = 0.85

$$-\frac{\left(\cos(x)^2+1\right)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2-1}}\,\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/cos(x)^3
```

giac [C] time = 1.14, size = 340, normalized size = 8.72

$$-\frac{2\left(15948\sqrt{5}-49185i\right)\text{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}{-15296i\sqrt{5}+98560}+\frac{82\sqrt{5}+\frac{939\sqrt{5}\left(4\sqrt{5}-\sqrt{-\tan\left(\frac{1}{2}x\right)^4+18\tan\left(\frac{1}{2}x\right)^2-1}\right)^2}{\left(\tan\left(\frac{1}{2}x\right)^2-9\right)^2}+\frac{537\sqrt{5}\left(4\sqrt{5}-\sqrt{-\tan\cdots}\right.}{\left(\tan\cdots\right.}}{96\left(\frac{\sqrt{5}\left(4\sqrt{5}-\cdots\right.}{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(15948*sqrt(5) - 49185*I)*sgn(tan(1/2*x))/(-15296*I*sqrt(5) + 98560) + 1
/96*(82*sqrt(5) + 939*sqrt(5)*(4*sqrt(5) - sqrt(-tan(1/2*x)^4 + 18*tan(1/2*
x)^2 - 1))^2/(tan(1/2*x)^2 - 9)^2 + 537*sqrt(5)*(4*sqrt(5) - sqrt(-tan(1/2*
x)^4 + 18*tan(1/2*x)^2 - 1))^4/(tan(1/2*x)^2 - 9)^4 + 975*(4*sqrt(5) - sqrt
(-tan(1/2*x)^4 + 18*tan(1/2*x)^2 - 1))/(tan(1/2*x)^2 - 9) + 2255*(4*sqrt(5)
 - sqrt(-tan(1/2*x)^4 + 18*tan(1/2*x)^2 - 1))^3/(tan(1/2*x)^2 - 9)^3 + 255*
(4*sqrt(5) - sqrt(-tan(1/2*x)^4 + 18*tan(1/2*x)^2 - 1))^5/(tan(1/2*x)^2 - 9
)^5)/((sqrt(5)*(4*sqrt(5) - sqrt(-tan(1/2*x)^4 + 18*tan(1/2*x)^2 - 1))/(tan
(1/2*x)^2 - 9) + (4*sqrt(5) - sqrt(-tan(1/2*x)^4 + 18*tan(1/2*x)^2 - 1))^2/
(tan(1/2*x)^2 - 9)^2 + 1)^3*sgn(tan(1/2*x)))
```

maple [B] time = 0.76, size = 64, normalized size = 1.64

method	result	size
default	$-\frac{\left(5\left(\cos^{2}\left(x\right)\right)+2\right)\sqrt{-\frac{5\left(\cos^{2}\left(x\right)\right)-4}{\sin\left(x\right)^{2}}}\,\sin\left(x\right)\sqrt{4}}{12\cos\left(x\right)^{3}}+\frac{\sqrt{4}\,\sin\left(x\right)\sqrt{-\frac{5\left(\cos^{2}\left(x\right)\right)-4}{\sin\left(x\right)^{2}}}}{4\cos\left(x\right)}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(5*cos(x)^2+2)*(-(5*cos(x)^2-4)/sin(x)^2)^(1/2)*sin(x)*4^(1/2)/cos(x)
^3+1/4*4^(1/2)*sin(x)*(-(5*cos(x)^2-4)/sin(x)^2)^(1/2)/cos(x)
```

maxima [B] time = 0.48, size = 63, normalized size = 1.62

$$-\frac{1}{48}\left(-\frac{1}{\tan\left(x\right)^{2}}+4\right)^{\frac{3}{2}}\tan\left(x\right)^{3}+\frac{3}{16}\sqrt{-\frac{1}{\tan\left(x\right)^{2}}+4}\,\tan\left(x\right)-\frac{8\,\tan\left(x\right)^{4}+26\,\tan\left(x\right)^{2}-7}{8\,\sqrt{2\,\tan\left(x\right)+1}\,\sqrt{2\,\tan\left(x\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 + 4)*tan(x)
- 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1)*sqrt(2*tan(x) - 1)
)
```

mupad [B] time = 0.76, size = 20, normalized size = 0.51

$$-\frac{\tan\left(x\right)\,\left({\tan\left(x\right)}^2+2\right)\,\sqrt{4-\frac{1}{{\tan\left(x\right)}^2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x) - 3)/(cos(x)^4*(4 - cot(x)^2)^(1/2)),x)
```

```
[Out] -(tan(x)*(tan(x)^2 + 2)*(4 - 1/tan(x)^2)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(2x\right)-3}{\sqrt{-\left(\cot\left(x\right)-2\right)\left(\cot\left(x\right)+2\right)}\,\cos^{4}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
```

```
[Out] Integral((cos(2*x) - 3)/(sqrt(-(cot(x) - 2)*(cot(x) + 2))*cos(x)**4), x)
```

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Rubi [A] time = 1.23, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4373, 6725, 261, 266, 51, 63, 206, 514, 446, 85, 156, 207}

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

```
[Out] -ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[Sqrt[5 - 4*Sec
[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(1-x^2)(4-x^2)}{\left(5-\frac{4}{x^2}\right)^{3/2}x^3(-2+x^2)}\,dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{2}{\left(5-\frac{4}{x^2}\right)^{3/2}x^3} + \frac{3}{2\left(5-\frac{4}{x^2}\right)^{3/2}x} - \frac{x}{2\left(5-\frac{4}{x^2}\right)^{3/2}(-2+\right.\right.$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(5-\frac{4}{x^2}\right)^{3/2}(-2+x^2)}\,dx, x, \cos(x)\right) - \frac{3}{2}\text{Subst}\left(\int \frac{}{\left(5-\right.}\right.$$

$$= -\frac{1}{2\sqrt{5-4\sec^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}\left(1-\frac{2}{x^2}\right)x}\,dx, x, \cos(x)\right.$$

$$= -\frac{1}{5\sqrt{5-4\sec^2(x)}} + \frac{3}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}\,x}\,dx, x, \sec^2(x)\right) - \frac{1}{4}\text{Su}$$

$$= -\frac{2}{15\sqrt{5-4\sec^2(x)}} + \frac{1}{120}\text{Subst}\left(\int \frac{-6-8x}{\sqrt{5-4x}\,(1-2x)x}\,dx, x, \sec^2(x\right.$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} - \frac{1}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-\right.}}\right.$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} + \frac{1}{40}\text{Subst}\left(\int \frac{1}{\frac{5}{4}-\frac{x^2}{4}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

Mathematica [B] time = 2.57, size = 255, normalized size = 3.49

$$\frac{15\sin^2(x)\sqrt{15\cos(2x)-9}\,\tanh^{-1}\left(\frac{\sqrt{5\cos(2x)-3}}{\sqrt{6}\sqrt{\cos^2(x)}}\right) - 2\left(15\sqrt{2}\sqrt{\sin^2(x)}\sqrt{\sin^2(2x)} + 9\sqrt{5}\sin^2(x)\sqrt{5\cos(2x)-3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)
),x]
```

```
[Out] (15*ArcTanh[Sqrt[-3 + 5*Cos[2*x]]/(Sqrt[6]*Sqrt[Cos[x]^2])]*Sqrt[-9 + 15*Co
s[2*x]]*Sin[x]^2 - 2*(9*Sqrt[5]*Sqrt[-3 + 5*Cos[2*x]]*(Log[10*Sin[x]^2] - L
og[5*(-Sqrt[-3 + 5*Cos[2*x]] + Cos[2*x]*Sqrt[-3 + 5*Cos[2*x]] + Sqrt[10]*Sq
rt[Sin[x]^2*Sqrt[Sin[2*x]^2]])])*Sin[x]^2 + 15*Sqrt[2]*Sqrt[Sin[x]^2]*Sqrt[
Sin[2*x]^2] + 10*ArcTanh[(Sqrt[6]*Cos[x])/Sqrt[-3 + 5*Cos[2*x]]]*Sqrt[-9 +
15*Cos[2*x]]*Sec[x]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2]))/(225*Sqrt[10 - 8*Sec[
x]^2]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]
^2)^(3/2)),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.82, size = 257, normalized size = 3.52

$$-\frac{480\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}\cos(x)^2-18\left(5\sqrt{5}\cos(x)^2-4\sqrt{5}\right)\log\left(625\cos(x)^8-1000\cos(x)^6+500\cos(x)^4-80\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(x
)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos(x
)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4 -
4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3)*co
s(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4 - 41
6*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3)*cos(
x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(cos(x)^8
- 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)
```

giac [C] time = 0.93, size = 171, normalized size = 2.34

$$-\frac{1}{4500}\sqrt{15}\sqrt{5}\left(6i\sqrt{15}\pi+12\sqrt{15}\log(2)-25\log\left(-\frac{\sqrt{15}+5}{\sqrt{15}-5}\right)\right)\mathrm{sgn}\left(\cos(x)\right)-\frac{\sqrt{15}\sqrt{5}\log\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5}\cos\right.\right.}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5}\cos\right.\right.}\right.}{180\,\mathrm{sgn}\left(\cos(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/4500*sqrt(15)*sqrt(5)*(6*I*sqrt(15)*pi + 12*sqrt(15)*log(2) - 25*log(-(s
qrt(15) + 5)/(sqrt(15) - 5)))*sgn(cos(x)) - 1/180*sqrt(15)*sqrt(5)*log(-2*(
(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)
*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*sqrt(15) - 32))/sgn(cos(x)) + 1/50*sq
rt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2)/sgn(cos(x)) - 2/15*cos
(x)/(sqrt(5*cos(x)^2 - 4)*sgn(cos(x)))
```

maple [B] time = 0.91, size = 1615, normalized size = 22.12

method	result	size
default	Expression too large to display	1615

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -3/5/(-5+2*5^(1/2))/(5+2*5^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(-6+2*5^(1/2)-2^(1
/2))/(6+2*5^(1/2)+2^(1/2))/(2*3^(1/2)+6^(1/2))/(6+2*5^(1/2)-2^(1/2))/(2*3^(
1/2)-6^(1/2))*(5*cos(x)^2-4)*(50*cos(x)*3^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(1+
cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos
```

```
(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(
1/2))+50*cos(x)*3^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh
(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*
2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-25*cos(x)*2^(1/2)*
6^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))
*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*
cos(x)^2-4)/(1+cos(x))^2)^(1/2))+25*cos(x)*2^(1/2)*6^(1/2)*((5*cos(x)^2-4)/
(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-1+cos(x))*(5*
cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2
)^(1/2))+100*cos(x)*3^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(-1+cos
(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+c
os(x))^2)^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)-100*cos(x)*3^(1/2)*((5
*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-
1+cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)
/(1+cos(x))^2)^(1/2))+72*cos(x)*5^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)
*arctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(-1+cos(x))/sin(x)^2/((5*cos(x)^2-4)/(1
+cos(x))^2)^(1/2))-50*cos(x)*6^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2
)*(-1+cos(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^
2-4)/(1+cos(x))^2)^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)-50*cos(x)*6^(
1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^
(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos
(x)^2-4)/(1+cos(x))^2)^(1/2))+50*3^(1/2)*2^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(
1/2))*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2
/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)+5
0*3^(1/2)*2^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-1+cos(x))*(5*co
s(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^
(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)-25*2^(1/2)*6^(1/2)*arctanh(1/2/(
2*3^(1/2)+6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2
)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))*((5*cos(x)^2-4)/(1+cos(x
))^2)^(1/2)+25*2^(1/2)*6^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-1+
cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(
1+cos(x))^2)^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)+100*3^(1/2)*((5*cos
(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(-1+co
s(x))*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+
cos(x))^2)^(1/2))-100*3^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1
/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^
(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))+72*5^(1/2)*((5*cos(x
)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(-1+cos(x))/s
in(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-50*6^(1/2)*((5*cos(x)^2-4)/(1+
cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos
(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(
1/2))-50*6^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)
-6^(1/2))*4^(1/2)*(-1+cos(x))*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(
x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-240*cos(x))/cos(x)^3/((5*cos(x)^2
-4)/cos(x)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sin(x)^2+3\right)\tan(x)^3}{\left(\cos(x)^2-2\right)\left(-4\sec(x)^2+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3 \left(\sin(x)^2 + 3\right)}{\left(\cos(x)^2 - 2\right) \left(5 - \frac{4}{\cos(x)^2}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)),x)
[Out] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
[Out] Timed out
```

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

Rubi [A] time = 0.85, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {6742, 191, 271, 266, 36, 29, 31}

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x
]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[x
]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1+x^2-3x\sqrt{4+9x^2}}{x^2\left(4+9x^2\right)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int\left(\frac{1}{\left(4+9x^2\right)^{3/2}}+\frac{1}{x^2\left(4+9x^2\right)^{3/2}}-\frac{3}{x\left(4+9\right.}\right.\right.$$

$$= -\left(3\,\text{Subst}\left(\int\frac{1}{x\left(4+9x^2\right)}\,dx, x, \tan(x)\right)\right)+\text{Subst}\left(\right.$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}+\frac{\tan(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{3}{2}\,\text{Subst}\left(\int\right.$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}-\frac{3}{8}\,\text{Subst}\left(\int\right.$$

$$= -\frac{3}{4}\log(\tan(x))+\frac{3}{8}\log\left(4+9\tan^2(x)\right)-\frac{\cot(x}{4\sqrt{4+9\tan}}$$

Mathematica [B] time = 0.92, size = 116, normalized size = 2.04

$$\frac{-5\tan(x)+5\cot(x)-9\csc(x)\sec(x)-6\sqrt{2}\,\log\left(\tan\left(\frac{x}{2}\right)\right)\sqrt{5\tan^2(x)+13\sec^2(x)-5}+6\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}\,\log\left(\tan\right.}{16\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4
*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2 +
 Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[-5
 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x
]^2]))/(4*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.58, size = 84, normalized size = 1.47

$$\frac{3\left(5\cos(x)^2-9\right)\log\left(-\frac{5}{4}\cos(x)^2+\frac{9}{4}\right)\sin(x)-6\left(5\cos(x)^2-9\right)\log\left(\frac{1}{2}\sin(x)\right)\sin(x)-\left(5\cos(x)^3-7\cos(x)\right)}{8\left(5\cos(x)^2-9\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 - 9
)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)/c
os(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)^2 - 3\sqrt{4\sec(x)^2 + 5\tan(x)^2}\,\tan(x)}{\left(4\sec(x)^2 + 5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2
+ 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

maple [B] time = 0.97, size = 117, normalized size = 2.05

method	result
default	$-\frac{6\left(\cos^3(x)\right)\sin(x)\left(-\frac{5\left(\cos^2(x)\right)-9}{\cos(x)^2}\right)^{\frac{3}{2}}\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)-3\left(\cos^3(x)\right)\sin(x)\left(-\frac{5\left(\cos^2(x)\right)-9}{\cos(x)^2}\right)^{\frac{3}{2}}\ln\left(-\frac{5\left(\cos^2(x)\right)-9}{(1+\cos(x))^2}\right)+25\left(\cos^4(x)\right)-80\left(\cos^2(x\right.}{8\cos(x)^3\sin(x)\left(-\frac{5\left(\cos^2(x)\right)-9}{\cos(x)^2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+
5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(6*cos(x)^3*sin(x)*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(-1+cos(x))/si
n(x))-3*cos(x)^3*sin(x)*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(5*cos(x)^2-9)
/(1+cos(x))^2)+25*cos(x)^4-80*cos(x)^2+63)/cos(x)^3/sin(x)/(-(5*cos(x)^2-9)
/cos(x)^2)^(3/2)
```

maxima [A] time = 0.98, size = 47, normalized size = 0.82

$$-\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}} - \frac{1}{4\sqrt{9\tan(x)^2+4}\,\tan(x)} + \frac{3}{8}\log\left(9\tan(x)^2+4\right) - \frac{3}{4}\log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8*
log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

mupad [B] time = 1.47, size = 113, normalized size = 1.98

$$\frac{3\ln\left(\left(\cos(2x)+\sin(2x)\,1\mathrm{i}\right)\,\left(5\cos(2x)-13\right)\right)}{8} - \frac{3\ln\left(\cos(2x)\,852930\mathrm{i} - 852930\sin(2x) - 852930\mathrm{i}\right)}{4} - \frac{18\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2 - 3*tan(x)*(4/cos(x)^2 + 5*tan(x)^2)^(1/2))/(sin(x)^2*(4/co
s(x)^2 + 5*tan(x)^2)^(3/2)),x)
```

```
[Out] (3*log((cos(2*x) + sin(2*x)*1i)*(5*cos(2*x) - 13)))/8 - (3*log(cos(2*x)*852
930i - 852930*sin(2*x) - 852930i))/4 - ((18*sin(2*x)*(13 - 5*cos(2*x))^(1/2
))/(cos(2*x) + 1)^(1/2) - (5*sin(4*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x) +
1)^(1/2))/(80*cos(2*x)^2 - 288*cos(2*x) + 208)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/(
4*sec(x)**2+5*tan(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.440 $\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 203}

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4*(1 + 5*Ta
n[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}dx = \text{Subst}\left(\int \frac{x\left(1+5x^2\right)^{5/2}}{1+x^2}dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+5x)^{5/2}}{1+x}dx, x, \tan^2(x)\right)$$
$$= \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - 2\,\text{Subst}\left(\int \frac{(1+5x)^{3/2}}{1+x}dx, x, \tan^2(x)\right)$$
$$= -\frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} + 8\,\text{Subst}\left(\int \frac{\sqrt{1+5x}}{1+x}dx, x, \tan^2(\right.$$
$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - 32\,\text{Subst}\left(\int \right.$$
$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - \frac{64}{5}\text{Subst}\left(\int \right.$$
$$= -32\tan^{-1}\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) + 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(\right.$$

Mathematica [C] time = 0.07, size = 49, normalized size = 0.74

$$\frac{5\sqrt{5}\left(5\tan^2(x)+1\right)^{5/2}{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{4\cos^2(x)}{5}\right)}{(3-2\cos(2x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] (5*Sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*Cos[x]^2)/5]*(1 + 5*Tan[x
]^2)^(5/2))/(3 - 2*Cos[2*x])^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.56, size = 50, normalized size = 0.76

$$\frac{1}{15}\left(75\tan(x)^4 - 70\tan(x)^2 + 223\right)\sqrt{5\tan(x)^2+1} - 16\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/4
*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

giac [A] time = 0.62, size = 52, normalized size = 0.79

$$\frac{1}{5}\left(5\tan(x)^2+1\right)^{\frac{5}{2}} - \frac{4}{3}\left(5\tan(x)^2+1\right)^{\frac{3}{2}} + 16\sqrt{5\tan(x)^2+1} - 32\arctan\left(\frac{1}{2}\sqrt{5\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)^
2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

maple [A] time = 0.08, size = 61, normalized size = 0.92

method	result
derivativedivides	$5\left(\tan^4(x)\right)\sqrt{1+5\left(\tan^2(x)\right)}-\frac{14\left(\tan^2(x)\right)\sqrt{1+5\left(\tan^2(x)\right)}}{3}+\frac{223\sqrt{1+5\left(\tan^2(x)\right)}}{15}-32\arctan\left(\frac{\sqrt{}}{}\right.$
default	$5\left(\tan^4(x)\right)\sqrt{1+5\left(\tan^2(x)\right)}-\frac{14\left(\tan^2(x)\right)\sqrt{1+5\left(\tan^2(x)\right)}}{3}+\frac{223\sqrt{1+5\left(\tan^2(x)\right)}}{15}-32\arctan\left(\frac{\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*(1+5*tan(x)^2)^(1/2)+223/15*(
1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(5\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

mupad [B] time = 1.57, size = 90, normalized size = 1.36

$$\frac{\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}\left(25\tan(x)^4-\frac{70\tan(x)^2}{3}+\frac{223}{3}\right)}{5}-\ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)16\mathrm{i}-\ln\left(\tan(x)+\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(5*tan(x)^2 + 1)^(5/2),x)
```

```
[Out] log(tan(x) - 1i)*16i - log(tan(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 -
1i/5)*16i - log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*16i +
 log(tan(x) + 1i)*16i + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2)*(25*tan(x)^4 - (70*
tan(x)^2)/3 + 223/3))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(5\tan^2(x)+1\right)^{\frac{5}{2}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

3.441 $\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}+\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 203}

$$\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2)) + 1/(16*S
qrt[1 + 5*Tan[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(1+5x^2\right)^{5/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{5/2}}\,dx, x, \tan^2(x)\right)$$
$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{3/2}}\,dx, x, \tan^2(x)\right)$$
$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{32}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+5x}}\,dx, x, \text{t}$$
$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{80}\text{Subst}\left(\int \frac{1}{\frac{4}{5}+\frac{x^2}{5}}\,dx, x, \sqrt{1+5\tan}\right.$$
$$= \frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}$$

Mathematica [A] time = 0.41, size = 71, normalized size = 1.31

$$\frac{(2\cos(2x)-3)\sec^5(x)\left(-6\cos(x)+8\cos(3x)-3(2\cos(2x)-3)^{3/2}\log\left(2\cos(x)+\sqrt{2\cos(2x)-3}\right)\right)}{96\left(5\tan^2(x)+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
[Out] ((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log[
2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
[Out] Could not integrate
```

fricas [A] time = 1.36, size = 76, normalized size = 1.41

$$\frac{3\left(25\tan(x)^4+10\tan(x)^2+1\right)\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)+4\left(15\tan(x)^2-1\right)\sqrt{5\tan(x)^2+1}}{192\left(25\tan(x)^4+10\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5
*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4 +
10*tan(x)^2 + 1)
```

giac [A] time = 0.61, size = 36, normalized size = 0.67

$$\frac{15\,\tan(x)^2-1}{48\left(5\,\tan(x)^2+1\right)^{\frac{3}{2}}}+\frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{5\,\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan(
x)^2 + 1))
```

maple [A] time = 0.09, size = 41, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\left(\tan^2(x)\right)}}-\frac{1}{12\left(1+5\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$	41
default	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\left(\tan^2(x)\right)}}-\frac{1}{12\left(1+5\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*t
an(x)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(5\,\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

mupad [B] time = 0.51, size = 172, normalized size = 3.19

$$\frac{\ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)1\mathrm{i}}{64}+\frac{\ln\left(\tan(x)+\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)1\mathrm{i}}{64}-\frac{\ln\left(\tan(x)-\mathrm{i}\right)1\mathrm{i}}{64}-\frac{\ln\left(\tan(x)+1\mathrm{i}\right)1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(5*tan(x)^2 + 1)^(5/2),x)
```

```
[Out] (log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*1i)/64 + (log(ta
n(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 - 1i/5)*1i)/64 - (log(tan(x) -
1i)*1i)/64 - (log(tan(x) + 1i)*1i)/64 - ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(ta
n(x) - (5^(1/2)*1i)/5)) + ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(tan(x) + (5^(1/2
)*1i)/5)) + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*(tan(x)^2 + (5^(1/2)*tan(
x)*2i)/5 - 1/5)) - (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*((5^(1/2)*tan(x)*2
i)/5 - tan(x)^2 + 1/5))
```

sympy [A] time = 8.11, size = 46, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^{2}(x)+1}}{2}\right)}{32}+\frac{1}{16\sqrt{5\tan^{2}(x)+1}}-\frac{1}{12\left(5\tan^{2}(x)+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5*
tan(x)**2 + 1)**(3/2))
```

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

Rubi [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3670, 444, 55, 617, 204, 31}

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt
[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3)^(1/3)) + (3*Log[(a
^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(4*(a^3 - b^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\sqrt[3]{a^3+b^3x^2}}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{a^3+b^3x}}\,dx,x,\tan^2(x)\right)\\
&= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}\,x+x^2}\,dx,x,\sqrt[3]{a^3+b^3\tan^2(x)}\right)\\
&= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2}{}\right.}{2\sqrt[3]{a^3-b^3}}\\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 105, normalized size = 0.79

$$
\frac{2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)+3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)+2\log(\cos(x))}{4\sqrt[3]{a^3-b^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sq
rt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/
3)])/(4*(a^3 - b^3)^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.76, size = 186, normalized size = 1.40

$$\frac{3\left(a^3-b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}+\left(a^3-b^3\right)^{\frac{1}{3}}\right)}{3\left(a^3-b^3\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}\,a^3-\sqrt{3}\,b^3\right)}-\frac{\log\left(\left(b^3\tan(x)^2+a^3\right)^{\frac{2}{3}}+\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}\left(a^3-b^3\right)^{\frac{1}{3}}+\left(a^3\right.\right.}{4\left(a^3-b^3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="giac")
[Out] 3/2*(a^3 - b^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b^3*tan(x)^2 + a^3)^(1/3) + (a
^3 - b^3)^(1/3))/(a^3 - b^3)^(1/3))/(sqrt(3)*a^3 - sqrt(3)*b^3) - 1/4*log((
b^3*tan(x)^2 + a^3)^(2/3) + (b^3*tan(x)^2 + a^3)^(1/3)*(a^3 - b^3)^(1/3) +
(a^3 - b^3)^(2/3))/(a^3 - b^3)^(1/3) + 1/2*log(abs((b^3*tan(x)^2 + a^3)^(1/
3) - (a^3 - b^3)^(1/3)))/(a^3 - b^3)^(1/3)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a^3+b^3\left(\tan^2(x)\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
[Out] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")
[Out] integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)
```

mupad [B] time = 1.45, size = 250, normalized size = 1.88

$$\frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{9\,a^3-9\,b^3}{4\,(a-b)^{2/3}\left(a^2+a\,b+b^2\right)^{2/3}}\right)}{2\,(a-b)^{1/3}\left(a^2+a\,b+b^2\right)^{1/3}}+\frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(9\,a^3-9\,b^3\right)}{16\,(a-b)^{2/3}\left(a^2+a\,b+b^2\right)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(a-b)^{1/3}\left(a^2+a\,b+b^2\right)^{1/3}}-\frac{\ln\left(\frac{9\left(a^3+b\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x)
```

```
[Out] log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - (9*a^3 - 9*b^3)/(4*(a - b)^(2/3)*(a*
b + a^2 + b^2)^(2/3)))/(2*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3)) + (log((9*
(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*1i - 1)^2*(9*a^3 - 9*b^3))/(16*(a
 - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(1/2)*1i - 1))/(4*(a - b)^(1/3)*(a
*b + a^2 + b^2)^(1/3)) - (log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*
1i + 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(
1/2)*1i + 1))/(4*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

```
[Out] Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

3.443 $\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx$

Optimal. Leaf size=69

$$\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 444, 50, 55, 618, 204, 31}

$$2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*
Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx &= \text{Subst}\left(\int \frac{x\left(1-7x^2\right)^{2/3}}{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-7x)^{2/3}}{1+x}\,dx,x,\tan^2(x)\right)\\ &= \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+4\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-7x}\,(1+x)}\,dx,x,\tan^2(x)\right)\\ &= 2\log(\cos(x))+\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}-3\,\text{Subst}\left(\int \frac{1}{2-x}\,dx,x,\sqrt[3]{1-7\tan^2(x}\right.\\ &= 2\log(\cos(x))+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}-12\,\text{Subst}\\ &= 2\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right)+2\log(\cos(x))+3\log\left(2-\sqrt[3]{1-7\tan^2(x}\right.\end{aligned}$$

Mathematica [C] time = 0.09, size = 42, normalized size = 0.61

$$-\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{8}(4\cos(2x)-3)\sec^2(x)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] (-3*(-1 + Hypergeometric2F1[2/3, 1, 5/3, ((-3 + 4*Cos[2*x])*Sec[x]^2)/8])*(
1 - 7*Tan[x]^2)^(2/3))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 3.76, size = 117, normalized size = 1.70

$$2\sqrt{3}\arctan\left(\frac{7\sqrt{3}\tan(x)^2+4\sqrt{3}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}-16\sqrt{3}\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}-\sqrt{3}}{7\tan(x)^2-65}\right)+\frac{3}{4}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) -
16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-7*
tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(-7*
tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))
```

giac [A] time = 0.66, size = 79, normalized size = 1.14

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{4}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}-\log\left(\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}+2\left(-7\tan(x)^2+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4*(-7*tan(x
)^2 + 1)^(2/3) - log((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan(x)^2 + 1)^(1/3) +
4) + 2*log(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \tan(x)\left(1-7\left(\tan^2(x)\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

```
[Out] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-7\tan(x)^2+1\right)^{\frac{2}{3}}\tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)
```

mupad [B] time = 0.72, size = 101, normalized size = 1.46

$$2\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-288\right)+\frac{3\left(1-7\tan(x)^2\right)^{2/3}}{4}+\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-72\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\right)\left(-1+\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1 - 7*tan(x)^2)^(2/3),x)
```

```
[Out] 2*log(144*(1 - 7*tan(x)^2)^(1/3) - 288) + (3*(1 - 7*tan(x)^2)^(2/3))/4 + lo
g(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i - 1)^2)*(3^(1/2)*1i - 1) - lo
g(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i + 1)^2)*(3^(1/2)*1i + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-7\tan^2(x)\right)^{\frac{2}{3}}\tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

```
[Out] Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)
```

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4 \csc^2(x)}} \, dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4139, 266, 63, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x^2}}\,dx, x, \csc(x)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x}}\,dx, x, \csc^2(x)\right)\right)\\ &= -\frac{2\operatorname{Subst}\left(\int \frac{x^2}{-\frac{a^4}{b^4}+\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)}{b^4}\\ &= \operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)-\operatorname{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}\end{aligned}$$

Mathematica [B] time = 0.30, size = 256, normalized size = 4.92

$$\frac{\sqrt[4]{a^4\cos(2x)-a^4-2b^4}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{a^4\left(-\sin^2(x)\right)-b^4}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{a^4\left(-\sin^2(x)\right)-b^4}}+1\right)-\log\left(-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{a^4\left(-\sin^2(x)\right)-b^4}}\right.\right.}{2\,2^{3/4}a\sqrt{\sin(x)}\sqrt[4]{a^4+b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 - 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(-b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^
4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[-b^4 - a^4*Sin[x]^2] -
 (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x
])/Sqrt[-b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^
2)^(1/4)]))/(2*2^(3/4)*a*(a^4 + b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.35, size = 73, normalized size = 1.40

$$-\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(\left|a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}-\frac{\log\left(\left|-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

```
[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 + b^4/sin(x)
^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 + b^4/sin(x)^2)^(1/4)))/a
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a^4+b^4\left(\csc^2(x)\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

maxima [A] time = 0.98, size = 71, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

mupad [B] time = 0.41, size = 46, normalized size = 0.88

$$-\frac{\mathrm{atan}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right)-\mathrm{atanh}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b^4/sin(x)^2 + a^4)^(1/4),x)
```

```
[Out] -(atan((b^4/sin(x)^2 + a^4)^(1/4)/a) - atanh((b^4/sin(x)^2 + a^4)^(1/4)/a))
/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot{(x)}}{\sqrt[4]{a^4+b^4\csc^2{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4139, 266, 63, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x^2}}\,dx, x, \csc(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x}}\,dx, x, \csc^2(x)\right)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\frac{a^4}{b^4}-\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)}{b^4}$$
$$= \text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right) - \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Mathematica [B] time = 0.28, size = 245, normalized size = 4.54

$$\frac{\sqrt[4]{a^4\cos(2x)-a^4+2b^4}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}+1\right)-\log\left(-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}+\frac{a^2 \sin(x)}{\sqrt{b^4-a^4\sin^2(x)}}\right.\right.}{2\,2^{3/4}a\sqrt{\sin(x)}\sqrt[4]{a^4-b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4
- a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] - (S
qrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x])/S
qrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/
4)]))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 1.08, size = 76, normalized size = 1.41

$$-\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(\left|a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}-\frac{\log\left(\left|-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

```
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 - b^4/sin(x)
^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 - b^4/sin(x)^2)^(1/4)))/a
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a^4-b^4\left(\csc^2(x)\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

maxima [A] time = 0.97, size = 74, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)}{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot{\left(x \right)}}{\sqrt[4]{\left(a^2-b^2\csc{\left(x \right)}\right)\left(a^2+b^2\csc{\left(x \right)}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)
```

3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\,\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}+\frac{1}{4}\log\left(\sec^2(x)\right)-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\mathrm{lc}$$

Rubi [A] time = 5.11, antiderivative size = 174, normalized size of antiderivative = 1.31, number of steps used = 29, number of rules used = 16, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {4361, 6742, 6684, 261, 6697, 341, 57, 618, 204, 31, 6688, 266, 47, 63, 206, 25}

$$\frac{\cos^2(x)}{6}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)+\frac{1}{6}\cos^2(x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + ArcTanh[Sqrt[1 - 3
*Sec[x]^2]]/2 + Cos[x]^2/6 + Log[1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 -
(3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (
1 - 3*Sec[x]^2)^(1/6) + (Cos[x]^2*Sqrt[1 - 3*Sec[x]^2])/6 - (1 - 3*Sec[x]^2
)^(2/3)/4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6697

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\ \sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = -\text{Subst}\left(\int \frac{\left(1-x^2\right)\left(3+\sqrt[3]{1-\frac{3}{x^2}}\,x^2\right)}{\left(1-\sqrt{1-\frac{3}{x^2}}\right)\left(1-\frac{3}{x^2}\right)^{5/6}x^5}\,dx, x, \text{co}\right.$$

$$= -\text{Subst}\left(\int \left(\frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}+\frac{}{(1-}\right.\right.$$

$$= -\text{Subst}\left(\int \frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}\,dx, x, \text{c}\right.$$

$$= -\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-\frac{3}{x^2}}\,x^3\left(1-\sqrt{\frac{-3+x^2}{x^2}}\right)}-\frac{}{\left(1-\frac{3}{x^2}\right.}\right.\right.$$

$$= 3\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}\,dx, x, \text{c}\right.$$

$$= \frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right)-\frac{1}{2}\,\text{Subst}\left(\right.$$

$$= \frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right)-\text{Subst}\left(\int\right.$$

$$= \frac{\cos^2(x)}{6}+\frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right)-$$

$$= \frac{\cos^2(x)}{6}-\frac{3}{2}\log\left(1-\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}\right)+$$

$$= \sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right)+\frac{\text{cos}}{}$$

$$= \sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right)+\frac{\text{cos}}{}$$

$$= \sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right)+\frac{1}{2}\,\text{ta}$$

Mathematica [C] time = 50.44, size = 4397, normalized size = 33.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/
((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] (-3*(6 + ((-5 + Cos[2*x])/(1 + Cos[2*x]))^(1/3) + Cos[2*x]*((-5 + Cos[2*x])
/(1 + Cos[2*x]))^(1/3))*(3*Sec[x]^2 + (1 - 3*Sec[x]^2)^(1/3))*Sin[x]^2*Tan[
x]*(-2 - 3*Tan[x]^2)^(5/6)*(1 + Tan[x]^2)*(2 + 3*Tan[x]^2)*(-8*AppellF1[1,
1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2] + 4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x
]^2)/2, -Tan[x]^2]*Tan[x]^2 + 3*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Ta
n[x]^2]*Tan[x]^2)^2*((4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]
+ 3*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2*(30*3^(2/3
)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^(-1)]*Sqrt[-2 - 3*Tan[x
]^2]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^2))^(1/3) + 12*3^(1/6)*Hy
pergeometric2F1[5/6, 5/6, 11/6, (3 + 3*Tan[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 +
 3*Tan[x]^2)/(1 + Tan[x]^2))^(5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^
2)^(5/6)*(1 + Tan[x]^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + 3*Tan[x]
^2*(20 - 2*(-2 - 3*Tan[x]^2)^(1/3) + 5*Sqrt[-2 - 3*Tan[x]^2]) + 2*(12 - 2*(
-2 - 3*Tan[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3*Tan[x]^2)^(5/6))
)) - 8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*(30*3^(2/3)*Hyper
geometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^(-1)]*Sqrt[-2 - 3*Tan[x]^2]*(1
 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^2))^(1/3) + 12*3^(1/6)*Hypergeom
etric2F1[5/6, 5/6, 11/6, (3 + 3*Tan[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 + 3*Tan[
x]^2)/(1 + Tan[x]^2))^(5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^2)^(5/6
)*(1 + Tan[x]^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + Tan[x]^2*(60 -
7*(-2 - 3*Tan[x]^2)^(1/3) + 15*Sqrt[-2 - 3*Tan[x]^2]) + 2*(12 - 2*(-2 - 3*T
an[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3*Tan[x]^2)^(5/6))))))/(10
*2^(1/6)*(-1 + Sqrt[(-5 + Cos[2*x])/(1 + Cos[2*x])])*(1 - 3*Sec[x]^2)^(5/6)
*(6 + (1 - 3*Sec[x]^2)^(1/3) + Cos[2*x]*(1 - 3*Sec[x]^2)^(1/3))*(-4 - 6*Tan
[x]^2)^(5/6)*(-8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2] + (4*Ap
pellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2] + 3*AppellF1[2, 3/2, 1, 3,
 (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2)*(1152*AppellF1[1, 1/2, 1, 2, (-3*Ta
n[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3 + 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2
)/2, -Tan[x]^2]^2*Tan[x]^5 - 1152*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -
Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5 - 864
*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3,
(-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5 + 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[
x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/
2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7 +
 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 2160*A
ppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-
3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7 + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^
2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^
7 + 162*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 172
8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 720*AppellF1[2, 1/2, 2, 3, (-3*Tan[
x]^2)/2, -Tan[x]^2]^2*Tan[x]^9 - 1296*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/
2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 +
 1080*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1
, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 405*AppellF1[2, 3/2, 1, 3, (-3*
Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9 + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^
2)/2, -Tan[x]^2]^2*Tan[x]^11 + 648*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2,
-Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^11 + 2
43*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11 + 720*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2
)^(1/3) - 192*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2
, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) -
 144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1,
 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) + 1008*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5*(-2 - 3*Tan[x]^2
)^(1/3) - 1032*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[
2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3)
- 774*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1
, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 128*App
```

ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 96*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 108*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) - 1080*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 96*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) - 810*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 54*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 320*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 240*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 270*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 216*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 81*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 192*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 162*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 1152*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*Sqrt[-2 - 3*Tan[x]^2] + 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 1152*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 864*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] + 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 2160*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 162*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 720*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] - 1296*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 1080*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 405*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 648*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 243*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 384*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6) + 576*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5*(-2

```
 - 3*Tan[x]^2)^(5/6) - 384*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^
2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[
x]^2)^(5/6) - 288*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*Appell
F1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/
6) - 576*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2
, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 96*A
ppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]
^2)^(5/6) - 432*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1
[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6)
 + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2,
1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 54*App
ellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2
)^(5/6) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9
*(-2 - 3*Tan[x]^2)^(5/6) + 216*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan
[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*
Tan[x]^2)^(5/6) + 81*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*T
an[x]^9*(-2 - 3*Tan[x]^2)^(5/6)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\,\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Ta
n[x]^2))/((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)\left(\left(1-3\left(\sec^2(x)\right)\right)^{\frac{1}{3}}\left(\sin^2(x)\right)+3\left(\tan^2(x)\right)\right)}{\cos(x)^2\left(1-3\left(\sec^2(x)\right)\right)^{\frac{5}{6}}\left(1-\sqrt{1-3\left(\sec^2(x)\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

```
[Out] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\tan(x)\left(\sin(x)^2\left(1-\frac{3}{\cos(x)^2}\right)^{1/3}+3\tan(x)^2\right)}{\cos(x)^2\left(\sqrt{1-\frac{3}{\cos(x)^2}}-1\right)\left(1-\frac{3}{\cos(x)^2}\right)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
- 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)),x)
```

```
[Out] -int((tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
- 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/(
1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

3.447 $\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}}+\frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}+\frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}}+2\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)-\frac{11\tanh^{-1}\left(\frac{\sqrt{2}\,\text{ta}}{\sqrt{\tan(x)}}\right.}{4\sqrt{2}}$$

Rubi [B] time = 1.23, antiderivative size = 208, normalized size of antiderivative = 2.08, number of steps used = 21, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {4397, 12, 6719, 6725, 266, 47, 50, 63, 203, 444}

$$\frac{\left(1-\tan^2(x)\right)\tan(x)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}+\frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}-\frac{11\tan^{-1}\left(\sqrt{\tan^2(x)-1}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{2}}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{\left(1-\tan^2(\right.}{4\sqrt{2}\sqrt{\frac{}{1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] (3*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Cot[x]*(1 - Tan[x]^
2))/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Tan[x]*(1 - Tan[x]^2))/(3*
Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) - (11*ArcTan[Sqrt[-1 + Tan[x]^2]]*Ta
n[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^2]) + (2*Ar
cTan[Sqrt[-1 + Tan[x]^2]/Sqrt[2]]*Tan[x])/(Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sq
rt[-1 + Tan[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(-1+\sec(2x))^{3/2}}\,dx$$

$$= \text{Subst}\left(\int \frac{\left(1-x^2\right)\left(-1+3x^2+2x^4\right)}{2\sqrt{2}x^2\sqrt{\frac{x^2}{1-x^2}}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{x^2\sqrt{\frac{x^2}{1-x^2}}(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}}$$

$$= \frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-1+3x^2+2x^4)}{x^3(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{\tan(x)\,\text{Subst}\left(\int \left(-\frac{(1-x^2)^{3/2}}{x^3}+\frac{4(1-x^2)^{3/2}}{x}-\frac{2x(1-x^2)^{3/2}}{1+x^2}\right)dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= -\frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^3}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\,\text{Subst}\left(\int \frac{x(1-x^2)^{3/2}}{1+x^2}\,dx, x\right.}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2}}$$

$$= -\frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}}{1+x}\,dx, x,\right.}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan}}$$

$$= \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} +$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan}{3}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan}{3}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{3\tanh^{-1}\left(\sqrt{1-\tan^2(x)}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{1-\right.}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1}}$$

Mathematica [A] time = 4.83, size = 168, normalized size = 1.68

$$\frac{\tan^2(2x)\left(2\tan^2(x)-\cos(2x)\right)\left(-3\sin(x)\cos(x)\tan^{-1}\left(\sqrt{\tan^2(x)-1}\right)\sqrt{\tan^2(x)-1} + \frac{4\sqrt{2}\cos(2x)\tan(x)\left(\sqrt{2}\tanh^{-1}\left(\right.\right.}{\sqrt{}}\right.}{2(6\cos(2x)+\cos(4x)-3)(\tan(x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] ((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[2 - 2*Tan[x]^2]/2] +
 Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Tan[x]^2] -
 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2] + (-3*Cot[
x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)/(2*(-3 + 6
*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3
/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.49, size = 271, normalized size = 2.71

$$-\frac{24\left(\cos(x)^5-\cos(x)^3\right)\log\left(-\frac{4\sqrt{2}\left(8\cos(x)^5-6\cos(x)^3+\cos(x)\right)\sqrt{-\frac{\cos(x)^2-1}{2\cos(x)^2-1}}-\left(32\cos(x)^4-16\cos(x)^2+1\right)\sin(x)}{\sin(x)}\right)\sin(x)-33\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="fricas")
```

```
[Out] -1/48*(24*(cos(x)^5 - cos(x)^3)*log(-(4*sqrt(2)*(8*cos(x)^5 - 6*cos(x)^3 +
cos(x))*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - (32*cos(x)^4 - 16*cos(x)^2
 + 1)*sin(x))/sin(x))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*log
(4*(sqrt(2)*(2*(3*sqrt(2) - 4)*cos(x)^3 - (3*sqrt(2) - 4)*cos(x))*sqrt(-(co
s(x)^2 - 1)/(2*cos(x)^2 - 1)) + (3*(2*sqrt(2) - 3)*cos(x)^2 - 2*sqrt(2) + 3
)*sin(x))/((cos(x)^2 - 1)*sin(x)))*sin(x) - 2*sqrt(2)*(22*cos(x)^6 - 47*cos
(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - 44*(cos(x
)^5 - cos(x)^3)*sin(x))/((cos(x)^5 - cos(x)^3)*sin(x))
```

giac [B] time = 1.87, size = 193, normalized size = 1.93

$$-\frac{\sqrt{2}\left(2\left(-\tan(x)^2+1\right)^{\frac{3}{2}}+3\sqrt{-\tan(x)^2+1}\right)}{12\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}\left(\tan(x)\right)}+\frac{11\sqrt{2}\log\left(\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}\left(\tan(x)\right)}-\frac{11\sqrt{2}\log\left(-\sqrt{-\tan(x)^2+\cdots}\right.}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}\left(\mathrm{t}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="giac")
```

```
[Out] -1/12*sqrt(2)*(2*(-tan(x)^2 + 1)^(3/2) + 3*sqrt(-tan(x)^2 + 1))/(sgn(tan(x)
^2 - 1)*sgn(tan(x))) + 11/16*sqrt(2)*log(sqrt(-tan(x)^2 + 1) + 1)/(sgn(tan(
x)^2 - 1)*sgn(tan(x))) - 11/16*sqrt(2)*log(-sqrt(-tan(x)^2 + 1) + 1)/(sgn(t
an(x)^2 - 1)*sgn(tan(x))) + log((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqrt(2) +
sqrt(-tan(x)^2 + 1)))/(sgn(tan(x)^2 - 1)*sgn(tan(x))) - 1/8*sqrt(2)*sqrt(-t
an(x)^2 + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x))*tan(x)^2)
```

maple [B] time = 1.02, size = 559, normalized size = 5.59

method	result
default	$-\frac{\sqrt{2}\,\sqrt{4}\,(-1+\cos(x))^2\left(-48\left(\cos^4(x)\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\cos(x)\sqrt{4}\,(-1+\cos(x))}{2\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(1+\cos(x))^2}}\,\sin(x)^2}\right)+22\left(\cos^4(x)\right)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(1+\cos(x))^2}}-168\left(\cos^4(x)\right)\ln\left(-\frac{4\left(\left(\cos^2(x)\right)\cdots\right.}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/96*2^(1/2)*4^(1/2)*(-1+cos(x))^2*(-48*cos(x)^4*2^(1/2)*arctanh(1/2*2^(1/
2)*cos(x)*4^(1/2)*(-1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)
+22*cos(x)^4*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-168*cos(x)^4*ln(-4*(cos(x)
^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x)^2-1)/(1
+cos(x))^2)^(1/2)+1)/sin(x)^2)+33*cos(x)^4*arctanh(1/2*4^(1/2)*(2*cos(x)^2-
3*cos(x)+1)/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)+201*cos(x)^4*ln(-
2*(cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x
)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)+48*cos(x)^3*2^(1/2)*arctanh(1/2*2^(
1/2)*cos(x)*4^(1/2)*(-1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)/sin(x)^
2)+168*cos(x)^3*ln(-4*(cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x
)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)-33*cos(x)^3*arc
tanh(1/2*4^(1/2)*(2*cos(x)^2-3*cos(x)+1)/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2
)/sin(x)^2)-201*cos(x)^3*ln(-2*(cos(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2
)-2*cos(x)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)-36*cos
(x)^2*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+8*((2*cos(x)^2-1)/(1+cos(x))^2)^(
1/2))/cos(x)^3/sin(x)^3/((2*cos(x)^2-1)/(1+cos(x))^2)^(3/2)/(sin(x)^2/(2*co
s(x)^2-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,\tan(x)^2-\cos(2\,x)}{(\tan(2\,x)\tan(x))^{\frac{3}{2}}\cos(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cos(2\,x)-2\tan(x)^2}{\cos(x)^2\,(\tan(2\,x)\,\tan(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)),x)
```

```
[Out] -int((cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

```
[Out] Timed out
```

3.448 $\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx$

Optimal. Leaf size=112

$$\frac{\log(\cos(x))}{2a^4}-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}$$

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3230, 266, 51, 55, 617, 204, 31}

$$-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}+\frac{\log(\cos(x))}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)])/(a^4*n))
 - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(2*a^4) - (3*Log[a -
(a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(a^3-b^3x^n\right)^{4/3}}\,dx, x, \cos(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x\left(a^3-b^3x\right)^{4/3}}\,dx, x, \cos^n(x)\right)}{n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx, x, \cos^n(x)\right)}{a^3n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}+\frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\ldots\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}+\frac{3\,\text{Subst}\left(\int \ldots\right.}{\ldots}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}}{\sqrt{3}}\right)}{a^4n}-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}-\frac{3\log\left(a-\sqrt[3]{a\ldots}\right.}{2\ldots}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 47, normalized size = 0.42

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-\frac{b^3\cos^n(x)}{a^3}\right)}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 -
b^3*Cos[x]^n)^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
[Out] Could not integrate
```

fricas [A] time = 1.36, size = 185, normalized size = 1.65

$$-\frac{2\left(\sqrt{3}\,b^3\cos(x)^n-\sqrt{3}\,a^3\right)\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}}{3\,a}\right)-\left(b^3\cos(x)^n-a^3\right)\log\left(a^2+\left(-b^3\cos(x)^n+a^3\right)\right.}{2\left(a^4b^3n\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")
[Out] -1/2*(2*(sqrt(3)*b^3*cos(x)^n - sqrt(3)*a^3)*arctan(1/3*(sqrt(3)*a + 2*sqrt
(3)*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (b^3*cos(x)^n - a^3)*log(a^2 + (-b^3*
cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)) + 2*(b^3*cos(x)^n -
a^3)*log(-a + (-b^3*cos(x)^n + a^3)^(1/3)) - 6*(-b^3*cos(x)^n + a^3)^(2/3)*
a)/(a^4*b^3*n*cos(x)^n - a^7*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\left(-b^3\cos(x)^n+a^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="giac")
[Out] integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

maple [A] time = 0.51, size = 127, normalized size = 1.13

method	result
derivativedivides	$-\frac{\frac{\ln\left(a-\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}+\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^4}+\frac{}{a^3\left(a^3\right.}}{n}$
default	$-\frac{\frac{\ln\left(a-\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}+\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3\left(\cos^n(x)\right)\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^4}+\frac{}{a^3\left(a^3\right.}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x,method=_RETURNVERBOSE)
[Out] -1/n*(1/a^4*ln(a-(a^3-b^3*cos(x)^n)^(1/3))+1/a^4*(-1/2*ln(a^2+a*(a^3-b^3*co
s(x)^n)^(1/3)+(a^3-b^3*cos(x)^n)^(2/3))+3^(1/2)*arctan(1/3*(a+2*(a^3-b^3*co
s(x)^n)^(1/3))/a*3^(1/2)))+3/a^3/(a^3-b^3*cos(x)^n)^(1/3))
```

maxima [A] time = 0.98, size = 136, normalized size = 1.21

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^4n}+\frac{\log\left(a^2+\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}a+\left(-b^3\cos(x)^n+a^3\right)^{\frac{2}{3}}\right)}{2a^4n}-\frac{\log\left(-a+\left(-b^3\cos\right.\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a)/(a^4*n)
+ 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)
)/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1/3))/(a^4*n) - 3/((-b^3*cos(x)
^n + a^3)^(1/3)*a^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{\left(a^3-b^3\cos(x)^n\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3),x)
```

```
[Out] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

```
[Out] Integral(tan(x)/(a**3 - b**3*cos(x)**n)**(4/3), x)
```

3.449 $\int \left(1+2\cos^9(x)\right)^{5/6} \tan(x)\,dx$

Optimal. Leaf size=95

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}+\frac{\tan^{-1}\left(\frac{1-\sqrt[3]{2\cos^9(x)+1}}{\sqrt{3}\sqrt[6]{2\cos^9(x)+1}}\right)}{3\sqrt{3}}+\frac{1}{3}\tanh^{-1}\left(\sqrt[6]{2\cos^9(x)+1}\right)-\frac{1}{9}\tanh^{-1}\left(\sqrt{2\cos^9(x)+1}\right)$$

Rubi [A] time = 0.26, antiderivative size = 162, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 266, 50, 63, 296, 634, 618, 204, 628, 206}

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}-\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}-\sqrt[6]{2\cos^9(x)+1}+1\right)+\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}+\sqrt[6]{2\cos^9(x)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 + 2*
(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) + (2*ArcTanh[(1 + 2*Cos[x]^9)^
(1/6)])/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15 - Log[1 - (1 + 2*Cos[x]^9)^(1/6)
+ (1 + 2*Cos[x]^9)^(1/3)]/18 + Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[
x]^9)^(1/3)]/18
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = -\text{Subst}\left(\int \frac{\left(1+2x^9\right)^{5/6}}{x}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{9}\text{Subst}\left(\int \frac{(1+2x)^{5/6}}{x}\,dx, x, \cos^9(x)\right)\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt[6]{1+2x}}\,dx, x, \cos^9(x)\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{-\frac{1}{2}+\frac{x^6}{2}}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right) + \frac{2}{9}\text{Subs}$$
$$= \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\right.$$
$$= \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{18}\log\left(1-\sqrt[6]{1+2\cos^9}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right.$$

Mathematica [A] time = 0.14, size = 154, normalized size = 1.62

$$\frac{1}{90}\left(-12\left(2\cos^9(x)+1\right)^{5/6} - 5\log\left(\sqrt[3]{2\cos^9(x)+1} - \sqrt[6]{2\cos^9(x)+1}+1\right) + 5\log\left(\sqrt[3]{2\cos^9(x)+1} + \sqrt[6]{2\cos^9(}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*Arc
Tan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)^(
1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (1 +
 2*Cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(
1/3)])/90
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.24, size = 146, normalized size = 1.54

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right)-\frac{2}{15}\left(2\cos(x)^9+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(1+2\left(\cos^{9}(x)\right)\right)^{\frac{5}{6}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

```
[Out] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

maxima [B] time = 0.97, size = 145, normalized size = 1.53

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right)-\frac{2}{15}\left(2\cos(x)^9+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)\left(2\cos(x)^9+1\right)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(2*cos(x)^9 + 1)^(5/6),x)
```

```
[Out] int(tan(x)*(2*cos(x)^9 + 1)^(5/6), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

```
[Out] Timed out
```

3.450 $\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4334, 266, 43}

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 - (2 - 5*Si
n[x]^3)^(5/3)/625
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx &= \text{Subst}\left(\int \frac{x^8}{\left(2-5x^3\right)^{4/3}}\,dx,x,\sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(2-5x)^{4/3}}\,dx,x,\sin^3(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{4}{25(2-5x)^{4/3}}-\frac{4}{25\sqrt[3]{2-5x}}+\frac{1}{25}(2-5x)^{2/3}\right)dx,x,\sin^3(x)\right)\\
&= \frac{4}{125\sqrt[3]{2-5\sin^3(x)}}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 30, normalized size = 0.61

$$\frac{-25\sin^6(x)-30\sin^3(x)+36}{625\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
[Out] (36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
[Out] Could not integrate
```

fricas [A] time = 1.27, size = 46, normalized size = 0.94

$$\frac{25\cos(x)^6-75\cos(x)^4+75\cos(x)^2+30\left(\cos(x)^2-1\right)\sin(x)+11}{625\left(5\left(\cos(x)^2-1\right)\sin(x)+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")
[Out] 1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x) +
 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)
```

giac [A] time = 0.91, size = 37, normalized size = 0.76

$$-\frac{1}{625}\left(-5\sin(x)^3+2\right)^{\frac{5}{3}}+\frac{2}{125}\left(-5\sin(x)^3+2\right)^{\frac{2}{3}}+\frac{4}{125\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="giac")
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*
sin(x)^3 + 2)^(1/3)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int\frac{\cot(x)\left(\sin^9(x)\right)}{\left(2-5\left(\sin^3(x)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
[Out] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

maxima [A] time = 0.43, size = 37, normalized size = 0.76

$$-\frac{1}{625}\left(-5\sin(x)^3+2\right)^{\frac{5}{3}}+\frac{2}{125}\left(-5\sin(x)^3+2\right)^{\frac{2}{3}}+\frac{4}{125\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*
sin(x)^3 + 2)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)\sin(x)^9}{\left(2-5\sin(x)^3\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3),x)
```

```
[Out] int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

```
[Out] Timed out
```

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=20

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

Rubi [A] time = 0.23, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {4342, 6686}

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x
]
```

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx &= \text{Subst}\left(\int \frac{x\left(1+\sqrt[3]{1-8x^2}\right)}{\left(1-8x^2\right)^{2/3}}\,dx, x, \tan(x)\right)\\ &= -\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2\end{aligned}$$

Mathematica [B] time = 0.21, size = 42, normalized size = 2.10

$$-\frac{3(9\cos(2x)-7)\left(\sqrt[3]{1-8\tan^2(x)}+2\right)\sec^2(x)}{64\left(1-8\tan^2(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(
2/3),x]
```

```
[Out] (-3*(-7 + 9*Cos[2*x])*Sec[x]^2*(2 + (1 - 8*Tan[x]^2)^(1/3)))/(64*(1 - 8*Tan
[x]^2)^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Ta
n[x]^2)^(2/3),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.56, size = 35, normalized size = 1.75

$$-\frac{3}{32}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}-\frac{3}{16}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")
```

```
[Out] -3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/cos(x)^2)^
(1/3)
```

giac [A] time = 0.88, size = 25, normalized size = 1.25

$$-\frac{3}{32}\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

maple [A] time = 0.10, size = 26, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{3\left(1-8\left(\tan^2(x)\right)\right)^{\frac{1}{3}}}{16}-\frac{3\left(1-8\left(\tan^2(x)\right)\right)^{\frac{2}{3}}}{32}$	26
default	$-\frac{3\left(1-8\left(\tan^2(x)\right)\right)^{\frac{1}{3}}}{16}-\frac{3\left(1-8\left(\tan^2(x)\right)\right)^{\frac{2}{3}}}{32}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,method=
_RETURNVERBOSE)
```

```
[Out] -3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

maxima [B] time = 0.69, size = 86, normalized size = 4.30

$$-\frac{3\left(\frac{\left(9\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\sin(x)+1)^{\frac{1}{3}}}+\frac{2\left(9\sin(x)^2-1\right)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\sin(x)+1)^{\frac{2}{3}}}\right)}{32\left(\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
```

```
[Out] -3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(sin(x) - 1
)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(x) + 1)^(2/3)*(sin(x
) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2 - 1)*(3*sin(x) - 1)^(2/3))
```

mupad [B] time = 0.62, size = 43, normalized size = 2.15

$$-\frac{3\left(\left(18\cos(x)^2-16\right)^{1/3}+2\left(2\cos(x)^2\right)^{1/3}\right)\left(18\cos(x)^2-16\right)^{1/3}}{32\left(2\cos(x)^2\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
,x)
```

```
[Out] -(3*((18*cos(x)^2 - 16)^(1/3) + 2*(2*cos(x)^2)^(1/3))*(18*cos(x)^2 - 16)^(1
/3))/(32*(2*cos(x)^2)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\tan(x)}{\left(1-8\tan^2(x)\right)^{\frac{2}{3}}\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
```

```
[Out] Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*tan(x)/((1 - 8*tan(x)**2)**(2/3)*co
s(x)**2), x)
```

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=27

$$\frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)-\log(\tan(x))$$

Rubi [A] time = 0.97, antiderivative size = 35, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {4366, 6725, 514, 444, 57, 618, 204, 31, 55}

$$\frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

```
[Out] -Log[1 - Sec[x]^2]/2 + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\text{Subst}\left(\int \frac{1+\sqrt[3]{9-\frac{8}{x^2}}}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(1-x^2\right)}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(-1+x^2\right)}-\frac{1}{\sqrt[3]{9-\frac{8}{x^2}}\,x\left(-1+x^2\right)}\right)dx, x, \right.$$
$$= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(-1+x^2\right)}\,dx, x, \cos(x)\right)+\text{Subst}\left(\int \frac{}{\sqrt[3]{9-\frac{8}{x^2}}}\right.$$
$$= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}\left(1-\frac{1}{x^2}\right)x^3}\,dx, x, \cos(x)\right)+\text{Subst}\left(\int \frac{}{\sqrt[3]{9-\frac{8}{x^2}}}\right.$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(9-8x)^{2/3}(1-x)}\,dx, x, \sec^2(x)\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{}{\sqrt[3]{9-}}\right.$$
$$= -\log(\tan(x))-2\left(\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{9-8\sec^2(x)}\right)\right)$$
$$= \frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\log(\tan(x))$$

Mathematica [B] time = 4.30, size = 58, normalized size = 2.15

$$\frac{1}{4}\left(5\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)-\log\left(\left(1-8\tan^2(x)\right)^{2/3}+\sqrt[3]{1-8\tan^2(x)}+1\right)-2\log(\tan(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/
3),x]
```

```
[Out] (-2*Log[Tan[x]] + 5*Log[1 - (1 - 8*Tan[x]^2)^(1/3)] - Log[1 + (1 - 8*Tan[x]
^2)^(1/3) + (1 - 8*Tan[x]^2)^(2/3)])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[
x]^2)^(2/3),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 7.06, size = 93, normalized size = 3.44

$$-\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-16\cos(x\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")
```

```
[Out] -1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((9
*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)^2
 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))
```

giac [A] time = 0.84, size = 40, normalized size = 1.48

$$-\frac{1}{2}\log\left(\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}+\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)+\log\left(\left|\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -1/2*log((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) + log(abs((
-8*tan(x)^2 + 1)^(1/3) - 1))
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\left(1+\left(1-8\left(\tan^2(x)\right)\right)^{\frac{1}{3}}\right)}{\cos(x)^2\left(1-8\left(\tan^2(x)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

```
[Out] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(-8 \tan(x)^2 + 1\right)^{\frac{1}{3}} + 1\right)\cot(x)}{\left(-8 \tan(x)^2 + 1\right)^{\frac{2}{3}} \cos(x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
```

```
[Out] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos
(x)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cot(x)\left(\left(1 - 8\tan(x)^2\right)^{1/3} + 1\right)}{\cos(x)^2 \left(1 - 8\tan(x)^2\right)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
,x)
```

```
[Out] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt[3]{1 - 8\tan^2(x)} + 1\right)\cot(x)}{\left(1 - 8\tan^2(x)\right)^{\frac{2}{3}} \cos^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
```

```
[Out] Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*cot(x)/((1 - 8*tan(x)**2)**(2/3)*co
s(x)**2), x)
```

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

Optimal. Leaf size=101

$$2\sqrt[4]{5\sin^2(x)-1}-\frac{\sqrt[4]{5\sin^2(x)-1}}{2\left(\sqrt{5\sin^2(x)-1}+2\right)}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 1.41, antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 10, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4361, 6742, 6697, 341, 50, 63, 203, 470, 522, 207}

$$2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)*
(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan[(4 - 5*Co
s[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[2]
) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^2)^(1/4)/(2*(2 + Sqrt[4 - 5*Co
s[x]^2]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6697

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = -\operatorname{Subst}\left(\int \frac{5x^2-\sqrt{4-5x^2}}{\sqrt[4]{4-5x^2}\left(2x+x\sqrt{4-5x^2}\right)}\,dx,x,\cos(x)\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{5x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)}-\frac{\sqrt[4]{4-5x^2}}{x\left(2+\sqrt{4-5x^2}\right)}\right.\right.$$

$$= -\left(5\operatorname{Subst}\left(\int \frac{x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)}\,dx,x,\cos(x)\right)\right)+\operatorname{Su}$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[4]{4-5x}}{\left(2+\sqrt{4-5x}\right)x}\,dx,x,\cos^2(x)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \right.$$

$$= 2\operatorname{Subst}\left(\int \frac{x^4}{\left(-2+x^2\right)\left(2+x^2\right)^2}\,dx,x,\sqrt[4]{4-5\cos^2(x)}\right)+\operatorname{Sul}$$

$$= 2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \right.$$

$$= 2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)}+\frac{1}{2}\operatorname{Subst}\left(\int \right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)-\frac{\tanh^{-}}{}$$

Mathematica [A] time = 0.43, size = 89, normalized size = 0.88

$$\frac{1}{4}\left(-2\sqrt[4]{4-5\cos^2(x)}\left(\frac{1}{\sqrt{4-5\cos^2(x)}+2}-4\right)-6\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^
(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] (-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 - 5
*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 - 5*
Cos[x]^2])^(-1)))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*S
in[x]^2)^(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5 \cos(x)^2 - \sqrt{5 \sin(x)^2 - 1}\right)\tan(x)}{\left(5 \sin(x)^2 - 1\right)^{\frac{1}{4}}\left(\sqrt{5 \sin(x)^2 - 1} + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/4
)*(sqrt(5*sin(x)^2 - 1) + 2)), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\left(5\left(\cos^2(x)\right) - \sqrt{-1 + 5\left(\sin^2(x)\right)}\right)\tan(x)}{\left(-1 + 5\left(\sin^2(x)\right)\right)^{\frac{1}{4}}\left(2 + \sqrt{-1 + 5\left(\sin^2(x)\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

```
[Out] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

maxima [A] time = 0.99, size = 100, normalized size = 0.99

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin(x)^2 - 1\right)^{\frac{1}{4}}\right) + \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2} - \left(5\sin(x)^2 - 1\right)^{\frac{1}{4}}}{\sqrt{2} + \left(5\sin(x)^2 - 1\right)^{\frac{1}{4}}}\right) + 2\left(5\sin(x)^2 - 1\right)^{\frac{1}{4}} - \frac{\left(5\sin(x)^2\right.}{2\left(\sqrt{5\sin(x)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(-
(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) + 2*
(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1) +
 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)\left(5\cos(x)^2 - \sqrt{5\sin(x)^2 - 1}\right)}{\left(5\sin(x)^2 - 1\right)^{1/4}\left(\sqrt{5\sin(x)^2 - 1} + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*
((5*sin(x)^2 - 1)^(1/2) + 2)),x)
```

```
[Out] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*
((5*sin(x)^2 - 1)^(1/2) + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sqrt{5\sin^2(x)-1}+5\cos^2(x)\right)\tan(x)}{\left(\sqrt{5\sin^2(x)-1}+2\right)\sqrt[4]{5\sin^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(1
/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

```
[Out] Integral((-sqrt(5*sin(x)**2 - 1) + 5*cos(x)**2)*tan(x)/((sqrt(5*sin(x)**2 -
 1) + 2)*(5*sin(x)**2 - 1)**(1/4)), x)
```

3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$

Optimal. Leaf size=25

$$-\frac{3}{64}\cos^{\frac{8}{3}}(2x)-\frac{3}{40}\cos^{\frac{5}{3}}(2x)$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4357, 266, 43}

$$-\frac{3}{64}\cos^{\frac{8}{3}}(2x)-\frac{3}{40}\cos^{\frac{5}{3}}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx &= -\text{Subst}\left(\int x^3\left(-1+2x^2\right)^{2/3}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int x(-1+2x)^{2/3}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{2}(-1+2x)^{2/3}+\frac{1}{2}(-1+2x)^{5/3}\right)\,dx,x,\cos^2(x)\right)\right)\\
&= -\frac{3}{40}\left(-1+2\cos^2(x)\right)^{5/3}-\frac{3}{64}\left(-1+2\cos^2(x)\right)^{8/3}
\end{aligned}$$

Mathematica [C] time = 0.32, size = 140, normalized size = 5.60

$$-\frac{3}{40}\cos^{\frac{5}{3}}(2x)-\frac{3e^{-6ix}\sqrt[3]{1+e^{4ix}}\left(2e^{4ix}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{4ix}\right)+e^{8ix}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{4ix}\right)+\left(1+e^{4ix}\right)^{2/3}\left(1+e^{8ix}\right)\right)}{256\,2^{2/3}\sqrt[3]{e^{-2ix}+e^{2ix}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*(1 + E^((4*I)*x))^(1/3)*((1 + E^((4*I)*x))^(2/3
)*(1 + E^((8*I)*x)) + 2*E^((4*I)*x)*Hypergeometric2F1[-1/3, 1/3, 2/3, -E^((
4*I)*x)] + E^((8*I)*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((4*I)*x)]))/(25
6*2^(2/3)*E^((6*I)*x)*(E^((-2*I)*x) + E^((2*I)*x))^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.71, size = 26, normalized size = 1.04

$$-\frac{3}{320}\left(20\cos(x)^4-4\cos(x)^2-3\right)\left(2\cos(x)^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")
```

```
[Out] -3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

giac [A] time = 0.64, size = 25, normalized size = 1.00

$$-\frac{3}{64}\left(2\cos(x)^2-1\right)^{\frac{8}{3}}-\frac{3}{40}\left(2\cos(x)^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="giac")
```

```
[Out] -3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^4(x)\right)\left(\cos^{\frac{2}{3}}(2x)\right)\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

```
[Out] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(2x)^{\frac{2}{3}}\cos(x)^4\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos(2x)^{2/3}\cos(x)^4\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(2/3)*cos(x)^4*tan(x),x)
```

```
[Out] int(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
```

```
[Out] Timed out
```

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\tan^{-1}\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\cos(2x)}+1}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4361, 446, 88, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\log\left(\sqrt{\cos(2x)}-\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{\cos(2x)}+\sqrt{2}\sqrt[4]{\cos(2x)}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Cos[2*x]^(1
/4)]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36
+ Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2]) - Log[1 + Sq
rt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = -\text{Subst}\left(\int \frac{(1-x^2)^3}{x(-1+2x^2)^{3/4}}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^3}{x(-1+2x)^{3/4}}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int\left(-\frac{7}{4(-1+2x)^{3/4}} + \frac{1}{x(-1+2x)^{3/4}} + \sqrt[4]{-1+2x} - \frac{1}{4}(-1+2x)^{5/4}\right)dx, x, c\right.\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\text{Subst}\left(\int \frac{}{x(-}\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1-}{\frac{1}{2}+}\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\text{Subst}\left(\int \frac{}{1-}\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-\right.}{}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{-1+2\cos^2}$$

Mathematica [A] time = 0.13, size = 153, normalized size = 1.50

$$\frac{1}{360}\left(-72\cos^{\frac{5}{4}}(2x) + 5\cos(4x)\sqrt[4]{\cos(2x)} + 635\sqrt[4]{\cos(2x)} + 90\sqrt{2}\log\left(\sqrt{\cos(2x)} - \sqrt{2}\sqrt[4]{\cos(2x)} + 1\right) - 90\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] (180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + Sq
rt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*x]
^(1/4)*Cos[4*x] + 90*Sqrt[2]*Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]
]] - 90*Sqrt[2]*Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]])/360
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] Could not integrate
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.68, size = 120, normalized size = 1.18

$$\frac{1}{36}\cos(2x)^{\frac{9}{4}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\cos(2x)^{\frac{1}{4}}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\cos(2x)^{\frac{1}{4}}\right)\right) - \frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

```
[Out] 1/36*cos(2*x)^(9/4) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*cos(2*x)^
(1/4))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*cos(2*x)^(1/4))) - 1
/4*sqrt(2)*log(sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) + 1/4*sqrt(2)*l
og(-sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) - 1/5*cos(2*x)^(5/4) + 7/4
*cos(2*x)^(1/4)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(\sin^6(x)\right)\tan(x)}{\cos(2x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

```
[Out] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^6*tan(x))/cos(2*x)^(3/4),x)
```

```
[Out] int((sin(x)^6*tan(x))/cos(2*x)^(3/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

```
[Out] Timed out
```

3.456 $\int \sqrt{\tan(x)\tan(2x)}\, dx$

Optimal. Leaf size=17

$$-\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4397, 3774, 207}

$$-\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[x]*Tan[2*x]],x]
```

```
[Out] -ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\tan(x)\tan(2x)}\, dx &= \int \sqrt{-1+\sec(2x)}\, dx\\ &= -\text{Subst}\left(\int \frac{1}{-1+x^2}\, dx, x, -\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right)\\ &= -\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.07, size = 45, normalized size = 2.65

$$-\frac{\sqrt{\cos(2x)}\sqrt{\tan(x)\tan(2x)}\csc(x)\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan[
x]*Tan[2*x]])/Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[Tan[x]*Tan[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.41, size = 50, normalized size = 2.94

$$\frac{1}{2}\log\left(-\frac{\tan(x)^3-2\sqrt{2}\left(\tan(x)^2-1\right)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2-1}}-3\tan(x)}{\tan(x)^3+\tan(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(tan(x)^3 - 2*sqrt(2)*(tan(x)^2 - 1)*sqrt(-tan(x)^2/(tan(x)^2 - 1)
) - 3*tan(x))/(tan(x)^3 + tan(x)))
```

giac [B] time = 0.77, size = 85, normalized size = 5.00

$$\frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan(x)^2+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan(x)^2+1}\right)\right)\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}\left(\tan(x)\right)+\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)*log(sqrt
(2) - sqrt(-tan(x)^2 + 1)))*sgn(tan(x)^2 - 1)*sgn(tan(x)) + (sqrt(2)*log(sq
rt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(x)))
```

maple [B] time = 0.43, size = 88, normalized size = 5.18

method	result	size
default	$-\frac{\sqrt{4}\,\sqrt{\frac{1-\left(\cos^2(x)\right)}{2\left(\cos^2(x)\right)-1}}\,\sin(x)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(1+\cos(x))^2}}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\cos(x)\sqrt{4}\,(-1+\cos(x))}{2\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(1+\cos(x))^2}}\,\sin(x)^2}\right)}{2(-1+\cos(x))}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*tan(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*4^(1/2)*((1-cos(x)^2)/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(x)^2-1)/(1+
cos(x))^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(x)*4^(1/2)*(-1+cos(x))/((2*cos(x)^
2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)/(-1+cos(x))
```

maxima [B] time = 1.06, size = 259, normalized size = 15.24

$$\frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan\left(\sin(4x),\cos(4x)+1\right)\right)^2+4\sqrt{\cos(4x)^2+\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 8*(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 4) - 1/4*l
og(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*x), co
s(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2
*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\tan(2\,x)\ \tan(x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(2*x)*tan(x))^(1/2),x)
```

```
[Out] int((tan(2*x)*tan(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x)\tan(2x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)*tan(2*x)), x)
```

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

Optimal. Leaf size=32

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {12, 402, 216, 377, 203}

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{2}\,(1+x^2)}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{2}}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)}{\sqrt{2}} + \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \sqrt{2}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$
$$= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$

Mathematica [A] time = 0.07, size = 52, normalized size = 1.62

$$\frac{\cos(x)\sqrt{\tan(x)\cot(2x)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)}{\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] ((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*Sq
rt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(2x)\tan(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.25, size = 115, normalized size = 3.59

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(3\cos(2x)^2 + 2\cos(2x) - 1\right)\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}}{4\cos(2x)\sin(2x)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos(2x)^2 + \sqrt{2}\cos(2x)\right.}{4\cos(2x)\sin(2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*cos(2*x)^2 + 2*cos(2*x) - 1)*sqrt(cos(2*x
)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x))) - 1/2*arctan(1/4*sqrt(2)*(2*sqrt(2)*
cos(2*x)^2 + sqrt(2)*cos(2*x) - sqrt(2))*sqrt(cos(2*x)/(cos(2*x) + 1))/(cos
(2*x)*sin(2*x)))
```

giac [C] time = 0.83, size = 138, normalized size = 4.31

$$\frac{1}{2}\left(\pi - \sqrt{2}\arctan(-i) - \sqrt{2}\arctan\left(\sqrt{2}\right) - i\log\left(2\sqrt{2}+3\right)\right)\text{sgn}(\sin(2x)) - \frac{\sqrt{2}\left(-i\sqrt{2}\log\left(2i\sqrt{2}+3i\right) - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) +
3))*sgn(sin(2*x)) - 1/4*(sqrt(2)*(-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*arc
tan(-I))*sgn(cos(x)) + 2*(sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(-2*
cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x)^2 - 3) - 1)) + arcsin(4*cos(x)^2
- 3))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))
```

maple [C] time = 0.76, size = 242, normalized size = 7.56

method	result
default	$\frac{\sqrt{2}\left(4\,\mathrm{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{2}}\,(-1+\cos(x))}{\sin(x)},-\frac{1}{3+2\sqrt{2}},\frac{\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}}\right)-2\,\mathrm{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{2}}\,(-1+\cos(x))}{\sin(x)},\frac{1}{3+2\sqrt{2}},\frac{\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}}\right)-\mathrm{EllipticF}\left(\frac{(-1+\cos(x))(1}{\sin(x)}\right.\right.}{2\sqrt{3+2\sqrt{2}}\,\left(1+\sqrt{2}\right)(-1+\cos(x))(2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(2*x)/cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)/(3+2*2^(1/2))^(1/2)/(1+2^(1/2))*(4*EllipticPi((3+2*2^(1/2))^(1/
2)*(-1+cos(x))/sin(x),-1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1
/2))-2*EllipticPi((3+2*2^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(3+2*2^(1/2)),(3
-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-EllipticF((-1+cos(x))*(1+2^(1/2))/si
n(x),3-2*2^(1/2)))*(2+2^(1/2))*cos(x)*sin(x)^2*((2*cos(x)^2-1)/cos(x)^2)^(1
/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*((cos(x)*2^(1
/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)/(-1+cos(x))/(2*cos(x)^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\frac{\cot(2\,x)}{\cot(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(2*x)/cot(x))^(1/2),x)
```

```
[Out] int((cot(2*x)/cot(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(2*x)/cot(x)), x)
```

3.458 $\int \frac{1}{x^5(5+x^2)} dx$

Optimal. Leaf size=31

$$-\frac{1}{20x^4}+\frac{1}{50x^2}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{50x^2}-\frac{1}{20x^4}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(5+x^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(5+x)} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{5x^3}-\frac{1}{25x^2}+\frac{1}{125x}-\frac{1}{125(5+x)}\right) dx, x, x^2\right)\\ &= -\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\log(x)}{125}-\frac{1}{250}\log\left(5+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{20x^4}+\frac{1}{50x^2}-\frac{1}{250}\log\left(x^2+5\right)+\frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

IntegrateAlgebraic [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{250}\log\left(x^2+5\right)+\frac{2x^2-5}{100x^4}+\frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(5 + x^2)),x]
[Out] (-5 + 2*x^2)/(100*x^4) + Log[x]/125 - Log[5 + x^2]/250
```

fricas [A] time = 1.28, size = 30, normalized size = 0.97

$$-\frac{2x^4\log\left(x^2+5\right)-4x^4\log(x)-10x^2+25}{500x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="fricas")
[Out] -1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

giac [A] time = 0.59, size = 32, normalized size = 1.03

$$-\frac{3x^4-10x^2+25}{500x^4}-\frac{1}{250}\log\left(x^2+5\right)+\frac{1}{250}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="giac")
[Out] -1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

maple [A] time = 0.32, size = 24, normalized size = 0.77

method	result	size
default	$-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln\left(x^2+5\right)}{250}$	24
norman	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln\left(x^2+5\right)}{250}$	25
risch	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln\left(x^2+5\right)}{250}$	25
meijerg	$-\frac{\ln\left(1+\frac{x^2}{5}\right)}{250}+\frac{\ln(x)}{125}-\frac{\ln(5)}{250}-\frac{1}{20x^4}+\frac{1}{50x^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^2+5),x,method=_RETURNVERBOSE)
[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

maxima [A] time = 0.41, size = 27, normalized size = 0.87

$$\frac{2x^2-5}{100x^4}-\frac{1}{250}\log\left(x^2+5\right)+\frac{1}{250}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="maxima")
[Out] 1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

mupad [B] time = 0.31, size = 24, normalized size = 0.77

$$\frac{\ln(x)}{125}-\frac{\ln\left(x^2+5\right)}{250}+\frac{\frac{x^2}{50}-\frac{1}{20}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^2 + 5)),x)
```

```
[Out] log(x)/125 - log(x^2 + 5)/250 + (x^2/50 - 1/20)/x^4
```

sympy **[A]** time = 0.12, size = 24, normalized size = 0.77

$$\frac{\log(x)}{125} - \frac{\log(x^2+5)}{250} + \frac{2x^2-5}{100x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**2+5),x)
```

```
[Out] log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {325, 203}

$$\frac{1}{75x^3}-\frac{1}{25x^5}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(5+x^2\right)}\,dx &= -\frac{1}{25x^5}-\frac{1}{5}\int \frac{1}{x^4\left(5+x^2\right)}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}+\frac{1}{25}\int \frac{1}{x^2\left(5+x^2\right)}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{1}{125}\int \frac{1}{5+x^2}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(5 + x^2)),x]

[Out] -1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])

IntegrateAlgebraic [A] time = 0.02, size = 37, normalized size = 0.95

$$\frac{-3x^4+5x^2-15}{375x^5}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(5 + x^2)),x]

[Out] (-15 + 5*x^2 - 3*x^4)/(375*x^5) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])

fricas [A] time = 1.36, size = 32, normalized size = 0.82

$$-\frac{3\sqrt{5}\,x^5\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)+15\,x^4-25\,x^2+75}{1875\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+5),x, algorithm="fricas")

[Out] -1/1875*(3*sqrt(5)*x^5*arctan(1/5*sqrt(5)*x) + 15*x^4 - 25*x^2 + 75)/x^5

giac [A] time = 0.58, size = 30, normalized size = 0.77

$$-\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+5),x, algorithm="giac")

[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5

maple [A] time = 0.32, size = 29, normalized size = 0.74

method	result	size
default	$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	29
risch	$\frac{-\frac{1}{125}x^4+\frac{1}{75}x^2-\frac{1}{25}}{x^5}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	30
meijerg	$\frac{\sqrt{5}\left(-\frac{2\sqrt{5}}{x}+\frac{10\sqrt{5}}{3x^3}-\frac{10\sqrt{5}}{x^5}-2\arctan\left(\frac{x\sqrt{5}}{5}\right)\right)}{1250}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^2+5),x,method=_RETURNVERBOSE)

[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)

maxima [A] time = 0.95, size = 30, normalized size = 0.77

$$-\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="maxima")
```

```
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

mupad [B] time = 0.03, size = 30, normalized size = 0.77

$$-\frac{\frac{x^4}{125}-\frac{x^2}{75}+\frac{1}{25}}{x^5}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^2 + 5)),x)
```

```
[Out] - (x^4/125 - x^2/75 + 1/25)/x^5 - (5^(1/2)*atan((5^(1/2)*x)/5))/625
```

sympy [A] time = 0.13, size = 32, normalized size = 0.82

$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{625}+\frac{-3x^4+5x^2-15}{375x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**2+5),x)
```

```
[Out] -sqrt(5)*atan(sqrt(5)*x/5)/625 + (-3*x**4 + 5*x**2 - 15)/(375*x**5)
```

3.460 $\int \frac{1}{x(-4+x^2)^4} \, dx$

Optimal. Leaf size=58

$$\frac{1}{128(4-x^2)} + \frac{1}{64(4-x^2)^2} + \frac{1}{24(4-x^2)^3} - \frac{1}{512}\log(4-x^2) + \frac{\log(x)}{256}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{128(4-x^2)} + \frac{1}{64(4-x^2)^2} + \frac{1}{24(4-x^2)^3} - \frac{1}{512}\log(4-x^2) + \frac{\log(x)}{256}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-4 + x^2)^4),x]
```

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log[x]/256 -
Log[4 - x^2]/512
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(-4+x^2)^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-4+x)^4 x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{4(-4+x)^4} - \frac{1}{16(-4+x)^3} + \frac{1}{64(-4+x)^2} - \frac{1}{256(-4+x)} + \frac{1}{256x}\right) dx, x, x^2\right) \\
&= \frac{1}{24(4-x^2)^3} + \frac{1}{64(4-x^2)^2} + \frac{1}{128(4-x^2)} + \frac{\log(x)}{256} - \frac{1}{512}\log(4-x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.69

$$\frac{-3\log(4-x^2) - \frac{4(3x^4-30x^2+88)}{(x^2-4)^3} + 6\log(x)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-4 + x^2)^4),x]
```

```
[Out] ((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 0.69

$$-\frac{1}{512}\log\left(x^2-4\right)+\frac{-3x^4+30x^2-88}{384\left(x^2-4\right)^3}+\frac{\log(x)}{256}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-4 + x^2)^4),x]
[Out] (-88 + 30*x^2 - 3*x^4)/(384*(-4 + x^2)^3) + Log[x]/256 - Log[-4 + x^2]/512
```

fricas [A] time = 1.14, size = 73, normalized size = 1.26

$$-\frac{12\,x^4-120\,x^2+3\left(x^6-12\,x^4+48\,x^2-64\right)\log\left(x^2-4\right)-6\left(x^6-12\,x^4+48\,x^2-64\right)\log(x)+352}{1536\left(x^6-12\,x^4+48\,x^2-64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="fricas")
[Out] -1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^2 - 4) - 6
*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)
```

giac [A] time = 0.61, size = 42, normalized size = 0.72

$$\frac{11\,x^6-156\,x^4+768\,x^2-1408}{3072\left(x^2-4\right)^3}+\frac{1}{512}\log\left(x^2\right)-\frac{1}{512}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="giac")
[Out] 1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*log(x^2) - 1
/512*log(abs(x^2 - 4))
```

maple [A] time = 0.35, size = 34, normalized size = 0.59

method	result	s
risch	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{\left(x^2-4\right)^3}+\frac{\ln(x)}{256}-\frac{\ln\left(x^2-4\right)}{512}$	3
norman	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{\left(x^2-4\right)^3}+\frac{\ln(x)}{256}-\frac{\ln(-2+x)}{512}-\frac{\ln(2+x)}{512}$	3
meijerg	$\frac{x^2\left(\frac{11}{16}x^4-\frac{27}{4}x^2+18\right)}{12288\left(1-\frac{x^2}{4}\right)^3}-\frac{\ln\left(1-\frac{x^2}{4}\right)}{512}+\frac{11}{3072}+\frac{\ln(x)}{256}-\frac{\ln(2)}{256}+\frac{i\pi}{512}$	5
default	$\frac{\ln(x)}{256}-\frac{1}{1536(-2+x)^3}+\frac{3}{2048(-2+x)^2}-\frac{11}{4096(-2+x)}-\frac{\ln(-2+x)}{512}+\frac{1}{1536(2+x)^3}+\frac{3}{2048(2+x)^2}+\frac{11}{4096(2+x)}-\frac{\ln(2+x)}{512}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-4)^4,x,method=_RETURNVERBOSE)
[Out] (-1/128*x^4+5/64*x^2-11/48)/(x^2-4)^3+1/256*ln(x)-1/512*ln(x^2-4)
```

maxima [A] time = 0.42, size = 46, normalized size = 0.79

$$-\frac{3\,x^4-30\,x^2+88}{384\left(x^6-12\,x^4+48\,x^2-64\right)}-\frac{1}{512}\log\left(x^2-4\right)+\frac{1}{512}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*log(x^2 -
 4) + 1/512*log(x^2)
```

mupad [B] time = 0.08, size = 44, normalized size = 0.76

$$\frac{\ln(x)}{256}-\frac{\ln\left(x^2-4\right)}{512}-\frac{\frac{x^4}{128}-\frac{5\,x^2}{64}+\frac{11}{48}}{x^6-12\,x^4+48\,x^2-64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 4)^4),x)
```

```
[Out] log(x)/256 - log(x^2 - 4)/512 - (x^4/128 - (5*x^2)/64 + 11/48)/(48*x^2 - 12
*x^4 + x^6 - 64)
```

sympy [A] time = 0.16, size = 41, normalized size = 0.71

$$\frac{-3x^4+30x^2-88}{384x^6-4608x^4+18432x^2-24576}+\frac{\log\left(x\right)}{256}-\frac{\log\left(x^2-4\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-4)**4,x)
```

```
[Out] (-3*x**4 + 30*x**2 - 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + log(
x)/256 - log(x**2 - 4)/512
```

3.461 $\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=52

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] -1/(6*(-2 + x^2)^(3/2)) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 + x^2]/Sqrt
[2]]/(4*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-2+x)^{5/2}x}\,dx,x,x^2\right)\\ &= -\frac{1}{6\left(-2+x^2\right)^{3/2}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(-2+x)^{3/2}x}\,dx,x,x^2\right)\\ &= -\frac{1}{6\left(-2+x^2\right)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,x}\,dx,x,x^2\right)\\ &= -\frac{1}{6\left(-2+x^2\right)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-2+x^2}\right)\\ &= -\frac{1}{6\left(-2+x^2\right)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{\tan^{-1}\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.58

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{x^2}{2}\right)}{6\left(x^2-2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + x^2)^(5/2)),x]
[Out] -1/6*Hypergeometric2F1[-3/2, 1, -1/2, 1 - x^2/2]/(-2 + x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.88

$$\frac{3x^2-8}{12\left(x^2-2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-2 + x^2)^(5/2)),x]
[Out] (-8 + 3*x^2)/(12*(-2 + x^2)^(3/2)) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqrt
[2])
```

fricas [A] time = 0.74, size = 65, normalized size = 1.25

$$\frac{3\sqrt{2}\left(x^4-4\,x^2+4\right)\arctan\left(-\frac{1}{2}\sqrt{2}\,x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\left(3\,x^2-8\right)\sqrt{x^2-2}}{12\left(x^4-4\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="fricas")
[Out] 1/12*(3*sqrt(2)*(x^4 - 4*x^2 + 4)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(
x^2 - 2)) + (3*x^2 - 8)*sqrt(x^2 - 2))/(x^4 - 4*x^2 + 4)
```

giac [A] time = 0.61, size = 35, normalized size = 0.67

$$\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\frac{3\,x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)^
(3/2)
```

maple [A] time = 0.33, size = 35, normalized size = 0.67

method	result	size
risch	$\frac{3x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	35
default	$-\frac{1}{6\left(x^2-2\right)^{\frac{3}{2}}}+\frac{1}{4\sqrt{x^2-2}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	37
trager	$\frac{3x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\sqrt{x^2-2}-\mathrm{RootOf}\left(\_Z^2+2\right)}{x}\right)}{8}$	47
meijerg	$\frac{\sqrt{2}\,\left(-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{5}{2}}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(-6x^2+16\right)}{8\left(-\frac{x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{2}+1}}{2}\right)}{2}+\frac{3\left(\frac{8}{3}-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{4}\right)}{12\sqrt{\pi}\,\mathrm{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{5}{2}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(3*x^2-8)/(x^2-2)^(3/2)-1/8*2^(1/2)*arctan(2^(1/2)/(x^2-2)^(1/2))
```

maxima [A] time = 0.96, size = 33, normalized size = 0.63

$$-\frac{1}{8}\sqrt{2}\,\arcsin\left(\frac{\sqrt{2}}{|x|}\right)+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^2 - 2)^(3/
2)
```

mupad [B] time = 0.47, size = 34, normalized size = 0.65

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x^2-2}}{2}\right)}{8}+\frac{\frac{x^2}{4}-\frac{2}{3}}{\left(x^2-2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 2)^(5/2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/8 + (x^2/4 - 2/3)/(x^2 - 2)^(3/
2)
```

sympy [C] time = 3.83, size = 986, normalized size = 18.96

$$\begin{cases} -\frac{6ix^4\log(x)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} + \frac{3ix^4\log(x^2)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} + \frac{6x^4\operatorname{asin}\left(\frac{\sqrt{2}}{x}\right)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} - \frac{6\sqrt{2}x^2\sqrt{x^2-2}}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} + \frac{24ix^2\log \cdots}{-24\sqrt{2}x^4+96\sqrt{\cdots}} \\ \frac{3ix^4\log(x^2)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} - \frac{6ix^4\log\left(\sqrt{1-\frac{x^2}{2}}+1\right)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} + \frac{3\pi x^4}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} - \frac{3ix^4\log(2)}{-24\sqrt{2}x^4+96\sqrt{2}x^2-96\sqrt{2}} - \frac{6\sqrt{2}ix^2\sqrt{2-\cdots}}{-24\sqrt{2}x^4+96\sqrt{2}\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-2)**(5/2),x)
```

```
[Out] Piecewise((-6*I*x**4*log(x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2
)) + 3*I*x**4*log(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) +
 6*x**4*asin(sqrt(2)/x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) -
 6*sqrt(2)*x**2*sqrt(x**2 - 2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqr
t(2)) + 24*I*x**2*log(x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2))
- 12*I*x**2*log(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 2
4*x**2*asin(sqrt(2)/x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) +
16*sqrt(2)*sqrt(x**2 - 2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2))
 - 24*I*log(x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 12*I*log
(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 24*asin(sqrt(2)/
x)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)), Abs(x**2)/2 > 1), (3*
I*x**4*log(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 6*I*x*
*4*log(sqrt(1 - x**2/2) + 1)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(
2)) + 3*pi*x**4/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 3*I*x**
4*log(2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 6*sqrt(2)*I*x*
*2*sqrt(2 - x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 12*I*
x**2*log(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 24*I*x**
2*log(sqrt(1 - x**2/2) + 1)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2
)) - 12*pi*x**2/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 12*I*x*
*2*log(2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 16*sqrt(2)*I*
sqrt(2 - x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 12*I*log
(x**2)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 24*I*log(sqrt(1
- x**2/2) + 1)/(-24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) + 12*pi/(-
24*sqrt(2)*x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)) - 12*I*log(2)/(-24*sqrt(2)*
x**4 + 96*sqrt(2)*x**2 - 96*sqrt(2)), True))
```

3.462 $\int \frac{\left(-10+x^2\right)^{5/2}}{x} \, dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 203}

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2)/5 - 100*
Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-10+x^2\right)^{5/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-10+x)^{5/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{5}\left(-10+x^2\right)^{5/2}-5\operatorname{Subst}\left(\int \frac{(-10+x)^{3/2}}{x}\,dx,x,x^2\right)\\
&= -\frac{10}{3}\left(-10+x^2\right)^{3/2}+\frac{1}{5}\left(-10+x^2\right)^{5/2}+50\operatorname{Subst}\left(\int \frac{\sqrt{-10+x}}{x}\,dx,x,x^2\right)\\
&= 100\sqrt{-10+x^2}-\frac{10}{3}\left(-10+x^2\right)^{3/2}+\frac{1}{5}\left(-10+x^2\right)^{5/2}-500\operatorname{Subst}\left(\int \frac{1}{\sqrt{-10+x}\,x}\,dx,x,\right.\\
&= 100\sqrt{-10+x^2}-\frac{10}{3}\left(-10+x^2\right)^{3/2}+\frac{1}{5}\left(-10+x^2\right)^{5/2}-1000\operatorname{Subst}\left(\int \frac{1}{10+x^2}\,dx,x,\sqrt{\cdot}\right.\\
&= 100\sqrt{-10+x^2}-\frac{10}{3}\left(-10+x^2\right)^{3/2}+\frac{1}{5}\left(-10+x^2\right)^{5/2}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.77

$$\frac{1}{15}\sqrt{x^2-10}\left(3x^4-110x^2+2300\right)-100\sqrt{10}\tan^{-1}\left(\sqrt{\frac{x^2}{10}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-1
 + x^2/10]]
```

IntegrateAlgebraic [A] time = 0.05, size = 49, normalized size = 0.80

$$\frac{1}{15}\sqrt{x^2-10}\left(3x^4-110x^2+2300\right)-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-1
0 + x^2]/Sqrt[10]]
```

fricas [A] time = 1.06, size = 47, normalized size = 0.77

$$\frac{1}{15}\left(3\,x^4-110\,x^2+2300\right)\sqrt{x^2-10}-200\,\sqrt{10}\,\arctan\left(-\frac{1}{10}\,\sqrt{10}\,x+\frac{1}{10}\,\sqrt{10}\,\sqrt{x^2-10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*sq
rt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))
```

giac [A] time = 0.63, size = 46, normalized size = 0.75

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}}-100\,\sqrt{10}\,\arctan\left(\frac{1}{10}\,\sqrt{10}\,\sqrt{x^2-10}\right)+100\,\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sqr
t(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

maple [A] time = 0.35, size = 46, normalized size = 0.75

method	result	si
default	$\frac{(x^2-10)^{\frac{5}{2}}}{5}-\frac{10(x^2-10)^{\frac{3}{2}}}{3}+100\sqrt{x^2-10}+100\sqrt{10}\ \arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$	4
trager	$\left(\frac{1}{5}x^4-\frac{22}{3}x^2+\frac{460}{3}\right)\sqrt{x^2-10}-100\,\mathrm{RootOf}\left(\_Z^2+10\right)\ln\left(\frac{\sqrt{x^2-10}+\mathrm{RootOf}\left(\_Z^2+10\right)}{x}\right)$	4
meijerg	$-\frac{375\sqrt{2}\ \sqrt{5}\ \mathrm{signum}\left(-1+\frac{x^2}{10}\right)^{\frac{5}{2}}\left(\frac{368\sqrt{\pi}}{225}-\frac{4\sqrt{\pi}\left(\frac{3}{25}x^4-\frac{22}{5}x^2+92\right)\sqrt{1-\frac{x^2}{10}}}{225}+\frac{16\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{x^2}{10}}}{2}\right)}{15}-\frac{8\left(\frac{46}{15}-3\ln(2)+2\ln(x)-\ln(5)+i\pi\right)\sqrt{\pi}}{15}\right)}{4\sqrt{\pi}\left(-\mathrm{signum}\left(-1+\frac{x^2}{10}\right)\right)^{\frac{5}{2}}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(x^2-10)^(5/2)-10/3*(x^2-10)^(3/2)+100*(x^2-10)^(1/2)+100*10^(1/2)*arct
an(10^(1/2)/(x^2-10)^(1/2))
```

maxima [A] time = 0.95, size = 42, normalized size = 0.69

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}}+100\sqrt{10}\ \arcsin\left(\frac{\sqrt{10}}{|x|}\right)+100\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10)
/abs(x)) + 100*sqrt(x^2 - 10)
```

mupad [B] time = 0.47, size = 46, normalized size = 0.75

$$100\sqrt{x^2-10}-100\sqrt{10}\ \mathrm{atan}\left(\frac{\sqrt{10}\ \sqrt{x^2-10}}{10}\right)-\frac{10\left(x^2-10\right)^{3/2}}{3}+\frac{\left(x^2-10\right)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 10)^(5/2)/x,x)
```

```
[Out] 100*(x^2 - 10)^(1/2) - 100*10^(1/2)*atan((10^(1/2)*(x^2 - 10)^(1/2))/10) -
(10*(x^2 - 10)^(3/2))/3 + (x^2 - 10)^(5/2)/5
```

sympy [C] time = 5.55, size = 167, normalized size = 2.74

$$\begin{cases} \frac{x^4\sqrt{x^2-10}}{5}-\frac{22x^2\sqrt{x^2-10}}{3}+\frac{460\sqrt{x^2-10}}{3}-100\sqrt{10}\,i\log\left(x\right)+50\sqrt{10}\,i\log\left(x^2\right)+100\sqrt{10}\ \mathrm{asin}\left(\frac{\sqrt{10}}{x}\right) & \text{for}\ \frac{|x^2|}{10}>1 \\ \frac{ix^4\sqrt{10-x^2}}{5}-\frac{22ix^2\sqrt{10-x^2}}{3}+\frac{460i\sqrt{10-x^2}}{3}+50\sqrt{10}\,i\log\left(x^2\right)-100\sqrt{10}\,i\log\left(\sqrt{1-\frac{x^2}{10}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)**(5/2)/x,x)
```

```
[Out] Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x*
*2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(10)
*asin(sqrt(10)/x), Abs(x**2)/10 > 1), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**2
*sqrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) - 10
0*sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))
```

3.463 $\int x^{1+2n} dx$

Optimal. Leaf size=16

$$\frac{x^{2(n+1)}}{2(n+1)}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {30}

$$\frac{x^{2(n+1)}}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + 2*n),x]
[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{1+2n} dx = \frac{x^{2(1+n)}}{2(1+n)}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^{2n+2}}{2n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + 2*n),x]
[Out] x^(2 + 2*n)/(2 + 2*n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{1+2n} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^(1 + 2*n),x]
[Out] Could not integrate
```

fricas [A] time = 0.86, size = 15, normalized size = 0.94

$$\frac{xx^{2n+1}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="fricas")
[Out] 1/2*x*x^(2*n + 1)/(n + 1)
```

giac [A] time = 0.58, size = 14, normalized size = 0.88

$$\frac{x^{2n+2}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="giac")
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

method	result	size
gosper	$\frac{x^{2+2n}}{2+2n}$	15
default	$\frac{x^{2+2n}}{2+2n}$	16
risch	$\frac{x\,x^{1+2n}}{2+2n}$	16
norman	$\frac{x\,e^{(1+2n)\ln(x)}}{2+2n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+2*n),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(2+2*n)/(1+n)
```

maxima [A] time = 0.42, size = 14, normalized size = 0.88

$$\frac{x^{2n+2}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="maxima")
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

mupad [B] time = 0.46, size = 24, normalized size = 1.50

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{2n+2}}{2(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n + 1),x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(2*n + 2)/(2*(n + 1)))
```

sympy [A] time = 0.06, size = 19, normalized size = 1.19

$$\begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } 2n+1 \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*n),x)
[Out] Piecewise((x**(2*n + 2)/(2*n + 2), Ne(2*n + 1, -1)), (log(x), True))
```

3.464 $\int \frac{x^7}{(-5+x^2)^3} dx$

Optimal. Leaf size=46

$$\frac{x^2}{2}+\frac{75}{2(5-x^2)}-\frac{125}{4(5-x^2)^2}+\frac{15}{2}\log(5-x^2)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 43}

$$\frac{x^2}{2}+\frac{75}{2(5-x^2)}-\frac{125}{4(5-x^2)^2}+\frac{15}{2}\log(5-x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(-5 + x^2)^3,x]
```

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(-5+x^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(-5+x)^3} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{125}{(-5+x)^3}+\frac{75}{(-5+x)^2}+\frac{15}{-5+x}\right) dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{125}{4(5-x^2)^2}+\frac{75}{2(5-x^2)}+\frac{15}{2}\log(5-x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{1}{4}\left(2x^2-\frac{150}{x^2-5}-\frac{125}{(x^2-5)^2}+30\log(x^2-5)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(-5 + x^2)^3,x]
```

```
[Out] (2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.85

$$\frac{15}{2}\log\left(x^2-5\right)+\frac{2x^6-20x^4-100x^2+625}{4\left(x^2-5\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(-5 + x^2)^3,x]
[Out] (625 - 100*x^2 - 20*x^4 + 2*x^6)/(4*(-5 + x^2)^2) + (15*Log[-5 + x^2])/2
```

fricas [A] time = 0.76, size = 49, normalized size = 1.07

$$\frac{2\,x^6-20\,x^4-100\,x^2+30\left(x^4-10\,x^2+25\right)\log\left(x^2-5\right)+625}{4\left(x^4-10\,x^2+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="fricas")
[Out] 1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)/
(x^4 - 10*x^2 + 25)
```

giac [A] time = 0.60, size = 36, normalized size = 0.78

$$\frac{1}{2}\,x^2-\frac{5\left(9\,x^4-60\,x^2+100\right)}{4\left(x^2-5\right)^2}+\frac{15}{2}\,\log\left(\left|x^2-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="giac")
[Out] 1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))
```

maple [A] time = 0.33, size = 30, normalized size = 0.65

method	result	size
norman	$\frac{-75x^2+\frac{1}{2}x^6+\frac{1125}{4}}{\left(x^2-5\right)^2}+\frac{15\ln\left(x^2-5\right)}{2}$	30
risch	$\frac{x^2}{2}+\frac{-\frac{75x^2}{2}+\frac{625}{4}}{\left(x^2-5\right)^2}+\frac{15\ln\left(x^2-5\right)}{2}$	30
default	$\frac{x^2}{2}-\frac{125}{4\left(x^2-5\right)^2}-\frac{75}{2\left(x^2-5\right)}+\frac{15\ln\left(x^2-5\right)}{2}$	33
meijerg	$\frac{x^2\left(\frac{4}{25}x^4-\frac{18}{5}x^2+12\right)}{8\left(-\frac{x^2}{5}+1\right)^2}+\frac{15\ln\left(-\frac{x^2}{5}+1\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^2-5)^3,x,method=_RETURNVERBOSE)
[Out] (-75*x^2+1/2*x^6+1125/4)/(x^2-5)^2+15/2*ln(x^2-5)
```

maxima [A] time = 0.47, size = 35, normalized size = 0.76

$$\frac{1}{2}\,x^2-\frac{25\left(6\,x^2-25\right)}{4\left(x^4-10\,x^2+25\right)}+\frac{15}{2}\,\log\left(x^2-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)
```

mupad [B] time = 0.06, size = 35, normalized size = 0.76

$$\frac{15\ln\left(x^2-5\right)}{2}-\frac{\frac{75\,x^2}{2}-\frac{625}{4}}{x^4-10\,x^2+25}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^2 - 5)^3,x)
```

```
[Out] (15*log(x^2 - 5))/2 - ((75*x^2)/2 - 625/4)/(x^4 - 10*x^2 + 25) + x^2/2
```

sympy [A] time = 0.12, size = 32, normalized size = 0.70

$$\frac{x^2}{2}+\frac{625-150x^2}{4x^4-40x^2+100}+\frac{15\log\left(x^2-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**2-5)**3,x)
```

```
[Out] x**2/2 + (625 - 150*x**2)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2
```

3.465 $\int \frac{-4x^3+3x^5}{\left(-1+x^2\right)^5}\,dx$

Optimal. Leaf size=40

$$-\frac{3}{4\left(1-x^2\right)^2}+\frac{1}{3\left(1-x^2\right)^3}+\frac{1}{8\left(1-x^2\right)^4}$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 77}

$$-\frac{3}{4\left(1-x^2\right)^2}+\frac{1}{3\left(1-x^2\right)^3}+\frac{1}{8\left(1-x^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4x^3+3x^5}{\left(-1+x^2\right)^5}\,dx &= \int \frac{x^3\left(-4+3x^2\right)}{\left(-1+x^2\right)^5}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(-4+3x)}{(-1+x)^5}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{3}{(-1+x)^3}\right)dx, x, x^2\right)\\
&= \frac{1}{8\left(1-x^2\right)^4}+\frac{1}{3\left(1-x^2\right)^3}-\frac{3}{4\left(1-x^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.58

$$\frac{-18x^4+28x^2-7}{24\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 0.58

$$\frac{-18x^4+28x^2-7}{24\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

fricas [A] time = 0.95, size = 36, normalized size = 0.90

$$-\frac{18\,x^4-28\,x^2+7}{24\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

giac [A] time = 0.58, size = 21, normalized size = 0.52

$$-\frac{18\,x^4-28\,x^2+7}{24\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="giac")
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

maple [A] time = 0.34, size = 21, normalized size = 0.52

method	result	size
norman	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{\left(x^2-1\right)^4}$	21
risch	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{\left(x^2-1\right)^4}$	21
gosper	$-\frac{18x^4-28x^2+7}{24\left(x^2-1\right)^4}$	22
meijerg	$-\frac{x^6\left(-x^2+4\right)}{8\left(-x^2+1\right)^4}+\frac{x^4\left(x^4-4x^2+6\right)}{6\left(-x^2+1\right)^4}$	47
default	$\frac{1}{128(-1+x)^4}-\frac{11}{192(-1+x)^3}-\frac{27}{256(-1+x)^2}+\frac{27}{256(-1+x)}+\frac{1}{128(1+x)^4}+\frac{11}{192(1+x)^3}-\frac{27}{256(1+x)^2}-\frac{27}{256(1+x)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-4*x^3)/(x^2-1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*x^4+7/6*x^2-7/24)/(x^2-1)^4
```

maxima [A] time = 0.43, size = 36, normalized size = 0.90

$$-\frac{18\,x^4-28\,x^2+7}{24\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

mupad [B] time = 0.07, size = 36, normalized size = 0.90

$$-\frac{\frac{3\,x^4}{4}-\frac{7\,x^2}{6}+\frac{7}{24}}{x^8-4\,x^6+6\,x^4-4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x^3 - 3*x^5)/(x^2 - 1)^5,x)
```

```
[Out] -((3*x^4)/4 - (7*x^2)/6 + 7/24)/(6*x^4 - 4*x^2 - 4*x^6 + x^8 + 1)
```

sympy [A] time = 0.13, size = 32, normalized size = 0.80

$$\frac{-18x^4+28x^2-7}{24x^8-96x^6+144x^4-96x^2+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

```
[Out] (-18*x**4 + 28*x**2 - 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)
```

3.466 $\int x^3 \left(1+x^2\right)^{9/14} dx$

Optimal. Leaf size=27

$$\frac{7}{37}\left(x^2+1\right)^{37/14}-\frac{7}{23}\left(x^2+1\right)^{23/14}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{37}\left(x^2+1\right)^{37/14}-\frac{7}{23}\left(x^2+1\right)^{23/14}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(1+x^2\right)^{9/14} dx &= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{9/14}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{9/14}+(1+x)^{23/14}\right)dx,x,x^2\right)\\ &= -\frac{7}{23}\left(1+x^2\right)^{23/14}+\frac{7}{37}\left(1+x^2\right)^{37/14}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{7}{851}\left(x^2+1\right)^{23/14}\left(23x^2-14\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (7*(1 + x^2)^(23/14)*(-14 + 23*x^2))/851
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.93

$$\frac{7}{851}\left(x^2+1\right)^{9/14}\left(23x^4+9x^2-14\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^2)^(9/14),x]
[Out] (7*(1 + x^2)^(9/14)*(-14 + 9*x^2 + 23*x^4))/851
```

fricas [A] time = 1.00, size = 21, normalized size = 0.78

$$\frac{7}{851}\left(23\,x^4+9\,x^2-14\right)\left(x^2+1\right)^{\frac{9}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")
[Out] 7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

giac [A] time = 0.60, size = 19, normalized size = 0.70

$$\frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}}-\frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="giac")
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

maple [A] time = 0.29, size = 17, normalized size = 0.63

method	result	size
gosper	$\frac{7\left(x^2+1\right)^{\frac{23}{14}}\left(23x^2-14\right)}{851}$	17
meijerg	$\frac{x^4\,\mathrm{hypergeom}\left(\left[-\frac{9}{14},2\right],[3],-x^2\right)}{4}$	17
trager	$\left(\frac{7}{37}x^4+\frac{63}{851}x^2-\frac{98}{851}\right)\left(x^2+1\right)^{\frac{9}{14}}$	21
risch	$\frac{7\left(x^2+1\right)^{\frac{9}{14}}\left(23x^4+9x^2-14\right)}{851}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(9/14),x,method=_RETURNVERBOSE)
[Out] 7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

maxima [A] time = 0.42, size = 19, normalized size = 0.70

$$\frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}}-\frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

mupad [B] time = 0.38, size = 20, normalized size = 0.74

$$\left(x^2+1\right)^{9/14}\left(\frac{7\,x^4}{37}+\frac{63\,x^2}{851}-\frac{98}{851}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)^(9/14),x)
```

```
[Out] (x^2 + 1)^(9/14)*((63*x^2)/851 + (7*x^4)/37 - 98/851)
```

sympy [**A**]   time = 5.54, size = 41, normalized size = 1.52

$$\frac{7x^4 \left(x^2+1\right)^{\frac{9}{14}}}{37}+\frac{63x^2 \left(x^2+1\right)^{\frac{9}{14}}}{851}-\frac{98 \left(x^2+1\right)^{\frac{9}{14}}}{851}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(9/14),x)
```

```
[Out] 7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 +
1)**(9/14)/851
```

3.467 $\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5/6))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-4+x)^{13/6}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{16}{(-4+x)^{13/6}}+\frac{8}{(-4+x)^{7/6}}+\frac{1}{\sqrt[6]{-4+x}}\right)dx,x,x^2\right)\\
&= -\frac{48}{7\left(-4+x^2\right)^{7/6}}-\frac{24}{\sqrt[6]{-4+x^2}}+\frac{3}{5}\left(-4+x^2\right)^{5/6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.66

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 0.66

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(-4 + x^2)^(13/6),x]
[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

fricas [A] time = 1.10, size = 33, normalized size = 0.87

$$\frac{3\left(7x^4-336x^2+1152\right)\left(x^2-4\right)^{\frac{5}{6}}}{35\left(x^4-8x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")
[Out] 3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^(5/6)/(x^4 - 8*x^2 + 16)
```

giac [A] time = 0.61, size = 26, normalized size = 0.68

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24\left(7x^2-26\right)}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="giac")
[Out] 3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

maple [A] time = 0.33, size = 22, normalized size = 0.58

method	result	size
trager	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{\left(x^2-4\right)^{\frac{7}{6}}}$	22
risch	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{\left(x^2-4\right)^{\frac{7}{6}}}$	22
gosper	$\frac{3(-2+x)(2+x)\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{\frac{13}{6}}}$	28
meijerg	$\frac{2^{\frac{2}{3}}\left(-\mathrm{signum}\left(-1+\frac{x^2}{4}\right)\right)^{\frac{13}{6}}x^6\,\mathrm{hypergeom}\left(\left[\frac{13}{6},3\right],[4],\frac{x^2}{4}\right)}{192\,\mathrm{signum}\left(-1+\frac{x^2}{4}\right)^{\frac{13}{6}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2-4)^(13/6),x,method=_RETURNVERBOSE)
[Out] 3/35*(7*x^4-336*x^2+1152)/(x^2-4)^(7/6)
```

maxima [A] time = 0.42, size = 28, normalized size = 0.74

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24}{\left(x^2-4\right)^{\frac{1}{6}}}-\frac{48}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")
```

```
[Out] 3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

mupad [B] time = 0.45, size = 21, normalized size = 0.55

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2 - 4)^(13/6),x)
```

```
[Out] (3*(7*x^4 - 336*x^2 + 1152))/(35*(x^2 - 4)^(7/6))
```

sympy [B] time = 5.06, size = 82, normalized size = 2.16

$$\frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}-\frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}+\frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2-4)**(13/6),x)
```

```
[Out] 21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(35
*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 - 4)
**(1/6) - 140*(x**2 - 4)**(1/6))
```

3.468 $\int \frac{1}{\left(1+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)^(-5/2),x]
[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+2x^2\right)^{5/2}}\,dx &= \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2}{3}\int \frac{1}{\left(1+2x^2\right)^{3/2}}\,dx\\
&= \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2x}{3\sqrt{1+2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.70

$$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)^(-5/2),x]
[Out] (x*(3 + 4*x^2))/(3*(1 + 2*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 23, normalized size = 0.70

$$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^2)^(-5/2),x]
[Out] (x*(3 + 4*x^2))/(3*(1 + 2*x^2)^(3/2))
```

fricas [A] time = 0.68, size = 34, normalized size = 1.03

$$\frac{\left(4 x^3 + 3 x\right)\sqrt{2 x^2 + 1}}{3 \left(4 x^4 + 4 x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")
[Out] 1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)
```

giac [A] time = 0.63, size = 19, normalized size = 0.58

$$\frac{\left(4 x^2 + 3\right)x}{3 \left(2 x^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

maple [A] time = 0.30, size = 20, normalized size = 0.61

method	result	size
gosper	$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{\frac{3}{2}}}$	20
trager	$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{\frac{3}{2}}}$	20
meijerg	$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{\frac{3}{2}}}$	20
risch	$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{\frac{3}{2}}}$	20
default	$\frac{x}{3\left(2x^2+1\right)^{\frac{3}{2}}} + \frac{2x}{3\sqrt{2x^2+1}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

maxima [A] time = 0.43, size = 25, normalized size = 0.76

$$\frac{2 x}{3 \sqrt{2 x^2 + 1}} + \frac{x}{3 \left(2 x^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

mupad [B] time = 0.04, size = 99, normalized size = 3.00

$$\frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\left(-x^2+1\mathrm{i}\,\sqrt{2}\,x+\frac{1}{2}\right)}+\frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\left(x^2+1\mathrm{i}\,\sqrt{2}\,x-\frac{1}{2}\right)}+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 1)^(5/2),x)
```

```
[Out] ((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i - x^2 + 1/2)) + ((x^2 + 1/2)^(1/2)
*1i)/(24*(2^(1/2)*x*1i + x^2 - 1/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x -
(2^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x + (2^(1/2)*1i)/2))
```

sympy [B] time = 2.40, size = 61, normalized size = 1.85

$$\frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}+\frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+1)**(5/2),x)
```

```
[Out] 4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2*
x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

3.469 $\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)^(-5/2),x]
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(-1-2x+x^2\right)^{3/2}}\,dx\\ &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1-x}{6\sqrt{-1-2x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.60

$$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)^(-5/2),x]
[Out] (2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 27, normalized size = 0.63

$$\frac{(x-1)\left(x^2-2x-2\right)}{6\left(x^2-2x-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + x^2)^(-5/2),x]
```

```
[Out] ((-1 + x)*(-2 - 2*x + x^2))/(6*(-1 - 2*x + x^2)^(3/2))
```

fricas [A] time = 1.15, size = 61, normalized size = 1.42

$$\frac{x^4-4\,x^3+2\,x^2+\left(x^3-3\,x^2+2\right)\sqrt{x^2-2\,x-1}+4\,x+1}{6\left(x^4-4\,x^3+2\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1)
/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)
```

giac [A] time = 0.66, size = 21, normalized size = 0.49

$$\frac{(x-3)x^2+2}{6\left(x^2-2\,x-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

maple [A] time = 0.39, size = 23, normalized size = 0.53

method	result	size
gosper	$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}$	23
trager	$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}$	23
risch	$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}$	23
default	$-\frac{2x-2}{12\left(x^2-2x-1\right)^{\frac{3}{2}}}+\frac{2x-2}{12\sqrt{x^2-2x-1}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

maxima [A] time = 0.42, size = 51, normalized size = 1.19

$$\frac{x}{6\sqrt{x^2-2\,x-1}}-\frac{1}{6\sqrt{x^2-2\,x-1}}-\frac{x}{6\left(x^2-2\,x-1\right)^{\frac{3}{2}}}+\frac{1}{6\left(x^2-2\,x-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1)
^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

mupad [B] time = 0.28, size = 22, normalized size = 0.51

$$\frac{x^3 - 3\,x^2 + 2}{6\left(x^2 - 2\,x - 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x - 1)^(5/2),x)
```

```
[Out] (x^3 - 3*x^2 + 2)/(6*(x^2 - 2*x - 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - 2x - 1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-1)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 1)**(-5/2), x)
```

3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}\,x}+\frac{1}{24\sqrt{x^2-8}\,x^3}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 191}

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}\,x}+\frac{1}{24\sqrt{x^2-8}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt[-8 + x^2]
)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(-8+x^2\right)^{3/2}}\,dx &= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{6}\int \frac{1}{x^2\left(-8+x^2\right)^{3/2}}\,dx\\
&= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}+\frac{1}{24}\int \frac{1}{\left(-8+x^2\right)^{3/2}}\,dx\\
&= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}-\frac{x}{192\sqrt{-8+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.60

$$\frac{-x^4+4x^2+8}{192x^3\sqrt{x^2-8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 28, normalized size = 0.60

$$\frac{-x^4+4x^2+8}{192x^3\sqrt{x^2-8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

fricas [A] time = 1.04, size = 40, normalized size = 0.85

$$-\frac{x^5-8\,x^3+\left(x^4-4\,x^2-8\right)\sqrt{x^2-8}}{192\,\left(x^5-8\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*sqrt(x^2 - 8))/(x^5 - 8*x^3)
```

giac [A] time = 0.65, size = 62, normalized size = 1.32

$$-\frac{x}{512\,\sqrt{x^2-8}}-\frac{3\,\left(x-\sqrt{x^2-8}\right)^4+96\,\left(x-\sqrt{x^2-8}\right)^2+320}{96\,\left(\left(x-\sqrt{x^2-8}\right)^2+8\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="giac")
```

```
[Out] -1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x - sqrt(x^2 -
 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

maple [A] time = 0.34, size = 23, normalized size = 0.49

method	result	size
gosper	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
trager	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
risch	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
default	$\frac{1}{24x^3\sqrt{x^2-8}}+\frac{1}{48x\sqrt{x^2-8}}-\frac{x}{192\sqrt{x^2-8}}$	36
meijerg	$-\frac{\sqrt{2}\left(-\mathrm{signum}\left(-1+\frac{x^2}{8}\right)\right)^{\frac{3}{2}}\left(-\frac{1}{8}x^4+\frac{1}{2}x^2+1\right)}{96\mathrm{signum}\left(-1+\frac{x^2}{8}\right)^{\frac{3}{2}}x^3\sqrt{1-\frac{x^2}{8}}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^2-8)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

maxima [A] time = 1.00, size = 35, normalized size = 0.74

$$-\frac{x}{192\,\sqrt{x^2-8}}+\frac{1}{48\,\sqrt{x^2-8}\,x}+\frac{1}{24\,\sqrt{x^2-8}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2 - 8)*x^3)
```

mupad [B] time = 0.45, size = 24, normalized size = 0.51

$$\frac{-x^4+4\,x^2+8}{192\,x^3\,\sqrt{x^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^2 - 8)^(3/2)),x)
```

```
[Out] (4*x^2 - x^4 + 8)/(192*x^3*(x^2 - 8)^(1/2))
```

sympy [A] time = 2.67, size = 151, normalized size = 3.21

$$\begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } \frac{8}{|x^2|}>1 \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**2-8)**(3/2),x)
```

```
[Out] Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqrt
(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 - 15
36*x**2), 8/Abs(x**2) > 1), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2)
+ 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)/(192*
x**4 - 1536*x**2), True))
```

3.471 $\int \frac{(5+x^2)^2}{x^{13/3}} \, dx$

Optimal. Leaf size=28

$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^2)^2/x^(13/3),x]
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(5+x^2)^2}{x^{13/3}} \, dx &= \int \left(\frac{25}{x^{13/3}}+\frac{10}{x^{7/3}}+\frac{1}{\sqrt[3]{x}}\right) dx\\ &= -\frac{15}{2x^{10/3}}-\frac{15}{2x^{4/3}}+\frac{3x^{2/3}}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.68

$$\frac{3(x^4-5x^2-5)}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^2)^2/x^(13/3),x]
[Out] (3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

IntegrateAlgebraic [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 + x^2)^2/x^(13/3),x]
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

fricas [A] time = 0.89, size = 15, normalized size = 0.54

$$\frac{3\left(x^4-5\,x^2-5\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

giac [A] time = 0.63, size = 16, normalized size = 0.57

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\left(x^2+1\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

maple [A] time = 0.29, size = 16, normalized size = 0.57

method	result	size
gosper	$\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$	16
trager	$\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$	16
risch	$\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$	16
derivativedivides	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17
default	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)^2/x^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(x^4-5*x^2-5)/x^(10/3)
```

maxima [A] time = 0.44, size = 16, normalized size = 0.57

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\left(x^2+1\right)}{2\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

mupad [B] time = 0.27, size = 17, normalized size = 0.61

$$-\frac{-3\,x^4+15\,x^2+15}{2\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 5)^2/x^(13/3),x)
```

```
[Out] -(15*x^2 - 3*x^4 + 15)/(2*x^(10/3))
```

sympy [A] time = 7.85, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{2}{3}}}{2} - \frac{15}{2x^{\frac{4}{3}}} - \frac{15}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)**2/x**(13/3),x)
```

```
[Out] 3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4\left(x^2+1\right)^2}+5\log\left(x^2+1\right)-10\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4\left(x^2+1\right)^2}+\frac{3}{4x^4}-\frac{1}{6x^6}+5\log\left(x^2+1\right)-10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -1/(6*x^6) + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2) - 10*Log[x
] + 5*Log[1 + x^2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+x^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(1+x)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{3}{x^3}+\frac{6}{x^2}-\frac{10}{x}+\frac{1}{(1+x)^3}+\frac{4}{(1+x)^2}+\frac{10}{1+x}\right)dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4\left(1+x^2\right)^2}-\frac{2}{1+x^2}-10\log(x)+5\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$5\log\left(x^2+1\right)-\frac{60x^8+90x^6+20x^4-5x^2+2}{12x^6\left(x^2+1\right)^2}-10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -1/12*(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(x^6*(1 + x^2)^2) - 10*Log[x]
+ 5*Log[1 + x^2]
```

IntegrateAlgebraic [A] time = 0.03, size = 49, normalized size = 0.94

$$5\log\left(x^2+1\right)+\frac{-60x^8-90x^6-20x^4+5x^2-2}{12x^6\left(x^2+1\right)^2}-10\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] (-2 + 5*x^2 - 20*x^4 - 90*x^6 - 60*x^8)/(12*x^6*(1 + x^2)^2) - 10*Log[x] +
5*Log[1 + x^2]
```

fricas [A] time = 1.11, size = 74, normalized size = 1.42

$$-\frac{60\,x^8+90\,x^6+20\,x^4-5\,x^2-60\left(x^{10}+2\,x^8+x^6\right)\log\left(x^2+1\right)+120\left(x^{10}+2\,x^8+x^6\right)\log(x)+2}{12\left(x^{10}+2\,x^8+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)*log(x^2 +
 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^8 + x^6)
```

giac [A] time = 0.62, size = 58, normalized size = 1.12

$$-\frac{30\,x^4+68\,x^2+39}{4\left(x^2+1\right)^2}+\frac{110\,x^6-36\,x^4+9\,x^2-2}{12\,x^6}+5\,\log\left(x^2+1\right)-5\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 -
2)/x^6 + 5*log(x^2 + 1) - 5*log(x^2)
```

maple [A] time = 0.33, size = 47, normalized size = 0.90

method	result	size
default	$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4\left(x^2+1\right)^2}-\frac{2}{x^2+1}-10\ln(x)+5\ln\left(x^2+1\right)$	47
norman	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6\left(x^2+1\right)^2}-10\ln(x)+5\ln\left(x^2+1\right)$	47
risch	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6\left(x^2+1\right)^2}-10\ln(x)+5\ln\left(x^2+1\right)$	47
meijerg	$\frac{x^2\left(9x^2+10\right)}{4\left(x^2+1\right)^2}+5\ln\left(x^2+1\right)-\frac{9}{4}-10\ln(x)-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

maxima [A] time = 0.45, size = 53, normalized size = 1.02

$$-\frac{60\,x^8+90\,x^6+20\,x^4-5\,x^2+2}{12\,\left(x^{10}+2\,x^8+x^6\right)}+5\,\log\left(x^2+1\right)-5\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6) + 5*log(x
^2 + 1) - 5*log(x^2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.98

$$5\,\ln\left(x^2+1\right)-10\,\ln(x)-\frac{5\,x^8+\frac{15\,x^6}{2}+\frac{5\,x^4}{3}-\frac{5\,x^2}{12}+\frac{1}{6}}{x^{10}+2\,x^8+x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^2 + 1)^3),x)
```

```
[Out] 5*log(x^2 + 1) - 10*log(x) - ((5*x^4)/3 - (5*x^2)/12 + (15*x^6)/2 + 5*x^8 +
 1/6)/(x^6 + 2*x^8 + x^10)
```

sympy [A] time = 0.17, size = 49, normalized size = 0.94

$$-10\log\left(x\right)+5\log\left(x^2+1\right)+\frac{-60x^8-90x^6-20x^4+5x^2-2}{12x^{10}+24x^8+12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**2+1)**3,x)
```

```
[Out] -10*log(x) + 5*log(x**2 + 1) + (-60*x**8 - 90*x**6 - 20*x**4 + 5*x**2 - 2)/
(12*x**10 + 24*x**8 + 12*x**6)
```

3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=25

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9} x}{10\sqrt{x^2+2}}$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1991, 435, 264}

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9} x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 1991

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && BinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}} \, dx &= \int \frac{\left(1+\frac{2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}} \, dx\\
&= \frac{\left(\left(1+\frac{2}{x^2}\right)^{7/9} x^{14/9}\right) \int \frac{1}{x^{14/9}\left(2+x^2\right)^{13/18}} \, dx}{\left(2+x^2\right)^{7/9}}\\
&= -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

IntegrateAlgebraic [A] time = 8.54, size = 25, normalized size = 1.00

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

fricas [A] time = 1.28, size = 21, normalized size = 0.84

$$-\frac{9\,x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\,\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")
[Out] -9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{\left(x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

maple [A] time = 0.28, size = 22, normalized size = 0.88

method	result	size
gosper	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22
risch	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{\left(x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

mupad [B] time = 0.45, size = 15, normalized size = 0.60

$$-\frac{9\,x\,{\left(x^2+2\right)}^{5/18}\,{\left(\frac{1}{x^2}\right)}^{7/9}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x)
```

```
[Out] -(9*x*(x^2 + 2)^(5/18)*(1/x^2)^(7/9))/10
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}+\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {271, 264}

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
[Out] x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{1}{5}\sqrt{\frac{2}{5}}\int \frac{x^6}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx\\ &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.70

$$\frac{7\sqrt{10}\,x^5-2x^7}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]
[Out] (7*Sqrt[10]*x^5 - 2*x^7)/(350*(Sqrt[10] - x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.29, size = 35, normalized size = 0.70

$$-\frac{x^5\left(2x^2-7\sqrt{10}\right)}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[10] - x^2)^(9/2),x]
[Out] -1/350*(x^5*(-7*Sqrt[10] + 2*x^2))/(Sqrt[10] - x^2)^(7/2)
```

fricas [A] time = 1.24, size = 69, normalized size = 1.38

$$-\frac{\left(2\,x^{15}-160\,x^{11}-2600\,x^{7}+\sqrt{10}\left(x^{13}-340\,x^{9}-700\,x^{5}\right)\right)\sqrt{-x^2+\sqrt{10}}}{350\left(x^{16}-40\,x^{12}+600\,x^{8}-4000\,x^{4}+10000\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")
[Out] -1/350*(2*x^15 - 160*x^11 - 2600*x^7 + sqrt(10)*(x^13 - 340*x^9 - 700*x^5))
*sqrt(-x^2 + sqrt(10))/(x^16 - 40*x^12 + 600*x^8 - 4000*x^4 + 10000)
```

giac [B] time = 0.80, size = 98, normalized size = 1.96

$$-\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="giac")
[Out] -16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + sqrt(10)) -
 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 +
sqrt(10)) - 10^(1/4))/x)^7
```

maple [A] time = 0.35, size = 28, normalized size = 0.56

method	result
gosper	$\frac{x^5\left(-2x^2+7\sqrt{10}\right)}{350\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$
meijerg	$\frac{10^{\frac{3}{4}}x^5\left(-\frac{\sqrt{2}\,\sqrt{5}\,x^2}{5}+7\right)}{35000\left(1-\frac{\sqrt{10}\,x^2}{10}\right)^{\frac{7}{2}}}$
risch	$\frac{2x^7-7\sqrt{10}\,x^5}{350\left(x^2-\sqrt{10}\right)^3\sqrt{-x^2+\sqrt{10}}}$
trager	$-\frac{2\sqrt{10}\,\left(\sqrt{10}\,x^2-35\right)x^5\sqrt{-x^2+\sqrt{10}}}{35\left(\sqrt{10}\,x^2-10\right)^4}$

default	$\frac{x^{3}}{4\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{3\sqrt{10}\left(\frac{x}{6\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{\sqrt{10}\left(\frac{x\sqrt{10}}{70\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\sqrt{10}\left(\frac{x\sqrt{10}}{50\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}+\frac{2\sqrt{10}\left(\frac{x\sqrt{10}}{30\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{x}{15\sqrt{-x^{2}+\sqrt{10}}}\right)}{25}\right)}{35}\right)}{6}\right)}{4}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+10^(1/2))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)
```

maxima [B] time = 0.98, size = 79, normalized size = 1.58

$$\frac{x}{175\sqrt{-x^{2}+\sqrt{10}}}+\frac{\sqrt{10}\,x}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{x^{3}}{4\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\,x}{140\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}-\frac{3\sqrt{10}\,x}{28\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")
```

```
[Out] 1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10))^(3/2) +
1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(10))^(5/2) - 3/28*sq
rt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4}}{\left(\sqrt{10}-x^{2}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(10^(1/2) - x^2)^(9/2),x)
```

```
[Out] int(x^4/(10^(1/2) - x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

```
[Out] Timed out
```

3.475 $\int \frac{x^2}{(3-x^2)^{3/2}} dx$

Optimal. Leaf size=24

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(3-x^2)^{3/2}} dx &= \frac{x}{\sqrt{3-x^2}} - \int \frac{1}{\sqrt{3-x^2}} dx\\ &= \frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

IntegrateAlgebraic [C] time = 0.06, size = 44, normalized size = 1.83

$$-\frac{\sqrt{3-x^2}\,x}{x^2-3} - i\log\left(\sqrt{3-x^2} - ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(3 - x^2)^(3/2),x]
[Out] -((x*Sqrt[3 - x^2])/(-3 + x^2)) - I*Log[(-I)*x + Sqrt[3 - x^2]]
```

fricas [A] time = 0.84, size = 41, normalized size = 1.71

$$\frac{\left(x^2-3\right)\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)-\sqrt{-x^2+3}\,x}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")
[Out] ((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)
```

giac [A] time = 0.66, size = 29, normalized size = 1.21

$$-\frac{\sqrt{-x^2+3}\,x}{x^2-3}-\arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

maple [A] time = 0.31, size = 22, normalized size = 0.92

method	result	size
default	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
risch	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
meijerg	$\frac{i\left(-\frac{i\sqrt{\pi}\,x\sqrt{3}}{3\sqrt{-\frac{x^2}{3}+1}}+i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-x^2+3}}{x^2-3}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+3}+x\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

maxima [A] time = 0.96, size = 21, normalized size = 0.88

$$\frac{x}{\sqrt{-x^2+3}}-\arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

mupad [B] time = 0.30, size = 54, normalized size = 2.25

$$-\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right)-\frac{\sqrt{3-x^2}}{2\left(x-\sqrt{3}\right)}-\frac{\sqrt{3-x^2}}{2\left(x+\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3 - x^2)^(3/2),x)
```

```
[Out] - asin((3^(1/2)*x)/3) - (3 - x^2)^(1/2)/(2*(x - 3^(1/2))) - (3 - x^2)^(1/2)
/(2*(x + 3^(1/2)))
```

sympy [B] time = 0.53, size = 49, normalized size = 2.04

$$-\frac{x^2 \operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}-\frac{x\sqrt{3-x^2}}{x^2-3}+\frac{3 \operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+3)**(3/2),x)
```

```
[Out] -x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(s
qrt(3)*x/3)/(x**2 - 3)
```

3.476 $\int \frac{\left(25-x^2\right)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=40

$$\frac{\sqrt{25-x^2}}{x}-\frac{\left(25-x^2\right)^{3/2}}{3 x^3}+\sin^{-1}\left(\frac{x}{5}\right)$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{\left(25-x^2\right)^{3/2}}{3 x^3}+\frac{\sqrt{25-x^2}}{x}+\sin^{-1}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(25 - x^2)^(3/2)/x^4,x]
```

```
[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(25-x^2\right)^{3/2}}{x^4} \, dx &= -\frac{\left(25-x^2\right)^{3/2}}{3 x^3}-\int \frac{\sqrt{25-x^2}}{x^2} \, dx\\ &= \frac{\sqrt{25-x^2}}{x}-\frac{\left(25-x^2\right)^{3/2}}{3 x^3}+\int \frac{1}{\sqrt{25-x^2}} \, dx\\ &= \frac{\sqrt{25-x^2}}{x}-\frac{\left(25-x^2\right)^{3/2}}{3 x^3}+\sin^{-1}\left(\frac{x}{5}\right)\\ \end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.60

$$-\frac{125 \, _2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{x^2}{25}\right)}{3 x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(25 - x^2)^(3/2)/x^4,x]
```

```
[Out] (-125*Hypergeometric2F1[-3/2, -3/2, -1/2, x^2/25])/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.07, size = 46, normalized size = 1.15

$$\frac{\sqrt{25-x^2}\,(4x^2-25)}{3x^3}-2\tan^{-1}\left(\frac{\sqrt{25-x^2}}{x+5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(25 - x^2)^(3/2)/x^4,x]
[Out] (Sqrt[25 - x^2]*(-25 + 4*x^2))/(3*x^3) - 2*ArcTan[Sqrt[25 - x^2]/(5 + x)]
```

fricas [A] time = 0.80, size = 45, normalized size = 1.12

$$-\frac{6\,x^3\arctan\left(\frac{\sqrt{-x^2+25}-5}{x}\right)-\left(4\,x^2-25\right)\sqrt{-x^2+25}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")
[Out] -1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25))
/x^3
```

giac [B] time = 0.66, size = 77, normalized size = 1.92

$$-\frac{x^3\left(\frac{15\left(\sqrt{-x^2+25}-5\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+25}-5\right)^3}+\frac{5\left(\sqrt{-x^2+25}-5\right)}{8\,x}-\frac{\left(\sqrt{-x^2+25}-5\right)^3}{24\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="giac")
[Out] -1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) - 5)^3 + 5/
8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^3/x^3 + arcsin(1/5*x
)
```

maple [A] time = 0.33, size = 32, normalized size = 0.80

method	result	size
risch	$-\frac{4x^4-125x^2+625}{3x^3\sqrt{-x^2+25}}+\arcsin\left(\frac{x}{5}\right)$	32
meijerg	$-\frac{3i\left(-\frac{1000i\sqrt{\pi}\left(-\frac{4x^2}{25}+1\right)\sqrt{-\frac{x^2}{25}+1}}{9x^3}+\frac{8i\sqrt{\pi}\arcsin\left(\frac{x}{5}\right)}{3}\right)}{8\sqrt{\pi}}$	43
trager	$\frac{(4x^2-25)\sqrt{-x^2+25}}{3x^3}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+25}\right)$	50
default	$-\frac{(-x^2+25)^{\frac{5}{2}}}{75x^3}+\frac{2(-x^2+25)^{\frac{5}{2}}}{1875x}+\frac{2x(-x^2+25)^{\frac{3}{2}}}{1875}+\frac{\sqrt{-x^2+25}\,x}{25}+\arcsin\left(\frac{x}{5}\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+25)^(3/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(4*x^4-125*x^2+625)/x^3/(-x^2+25)^(1/2)+arcsin(1/5*x)
```

maxima [A] time = 0.97, size = 45, normalized size = 1.12

$$\frac{1}{25}\sqrt{-x^2+25}\,x+\frac{2\left(-x^2+25\right)^{\frac{3}{2}}}{75\,x}-\frac{\left(-x^2+25\right)^{\frac{5}{2}}}{75\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)/
x^3 + arcsin(1/5*x)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.82

$$\mathrm{asin}\left(\frac{x}{5}\right)+\frac{4\,\sqrt{25-x^2}}{3\,x}-\frac{25\,\sqrt{25-x^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((25 - x^2)^(3/2)/x^4,x)
```

```
[Out] asin(x/5) + (4*(25 - x^2)^(1/2))/(3*x) - (25*(25 - x^2)^(1/2))/(3*x^3)
```

sympy [A] time = 2.23, size = 32, normalized size = 0.80

$$\mathrm{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{25-x^2}}{3x}-\frac{25\sqrt{25-x^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+25)**(3/2)/x**4,x)
```

```
[Out] asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)
```

3.477 $\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=49

$$\frac{8x}{15\sqrt{1-2x^2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{x}{5\left(1-2x^2\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{15\sqrt{1-2x^2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{x}{5\left(1-2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(15*Sqrt[1 -
 2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx &= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4}{5}\int \frac{1}{\left(1-2x^2\right)^{5/2}}\,dx\\
&= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{8}{15}\int \frac{1}{\left(1-2x^2\right)^{3/2}}\,dx\\
&= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{8x}{15\sqrt{1-2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4-40x^2+15\right)}{15\left(1-2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] (x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 37, normalized size = 0.76

$$-\frac{x\sqrt{1-2x^2}\left(32x^4-40x^2+15\right)}{15\left(2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x^2)^(-7/2),x]
[Out] -1/15*(x*Sqrt[1 - 2*x^2]*(15 - 40*x^2 + 32*x^4))/(-1 + 2*x^2)^3
```

fricas [A] time = 1.19, size = 44, normalized size = 0.90

$$-\frac{\left(32\,x^5-40\,x^3+15\,x\right)\sqrt{-2\,x^2+1}}{15\left(8\,x^6-12\,x^4+6\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")
[Out] -1/15*(32*x^5 - 40*x^3 + 15*x)*sqrt(-2*x^2 + 1)/(8*x^6 - 12*x^4 + 6*x^2 - 1
)
```

giac [A] time = 0.66, size = 35, normalized size = 0.71

$$-\frac{\left(8\left(4\,x^2-5\right)x^2+15\right)\sqrt{-2\,x^2+1}\,x}{15\left(2\,x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="giac")
[Out] -1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

maple [A] time = 0.31, size = 25, normalized size = 0.51

method	result	size
gosper	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$	25
meijerg	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$	25
trager	$-\frac{\left(32x^4-40x^2+15\right)x\sqrt{-2x^2+1}}{15\left(2x^2-1\right)^3}$	34
risch	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(2x^2-1\right)^2\sqrt{-2x^2+1}}$	34
default	$\frac{x}{5\left(-2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(-2x^2+1\right)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{-2x^2+1}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/15*x*(32*x^4-40*x^2+15)/(-2*x^2+1)^(5/2)
```

maxima [A] time = 0.42, size = 37, normalized size = 0.76

$$\frac{8\,x}{15\sqrt{-2\,x^2+1}}+\frac{4\,x}{15\left(-2\,x^2+1\right)^{\frac{3}{2}}}+\frac{x}{5\left(-2\,x^2+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x^2 + 1)^(5
/2)
```

mupad [B] time = 0.28, size = 179, normalized size = 3.65

$$\frac{19\sqrt{\frac{1}{2}-x^2}}{480\left(x^2-\sqrt{2}\,x+\frac{1}{2}\right)}-\frac{19\sqrt{\frac{1}{2}-x^2}}{480\left(x^2+\sqrt{2}\,x+\frac{1}{2}\right)}-\frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\left(x^3-\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}-\frac{\sqrt{2}}{4}\right)}-\frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\left(x^3+\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}+\frac{\sqrt{2}}{4}\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - 2*x^2)^(7/2),x)
```

```
[Out] (19*(1/2 - x^2)^(1/2))/(480*(x^2 - 2^(1/2)*x + 1/2)) - (19*(1/2 - x^2)^(1/2
))/(480*(2^(1/2)*x + x^2 + 1/2)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x)/
2 - 2^(1/2)/4 - (3*2^(1/2)*x^2)/2 + x^3)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(16
0*((3*x)/2 + 2^(1/2)/4 + (3*2^(1/2)*x^2)/2 + x^3)) - (2*2^(1/2)*(1/2 - x^2)
^(1/2))/(15*(x - 2^(1/2)/2)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x + 2^(1/
2)/2))
```

sympy [B] time = 12.35, size = 291, normalized size = 5.94

$$\begin{cases} -\frac{32ix^5}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}}+\frac{40ix^3}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}}-\frac{15ix}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} & \text{for } 2 \cdots \\ \frac{32x^5}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}}-\frac{40x^3}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}}+\frac{15x}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} & \text{other}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+1)**(7/2),x)
```

```
[Out] Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt(2*x**2 - 1)
+ 15*sqrt(2*x**2 - 1)) + 40*I*x**3/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt
(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)) - 15*I*x/(60*x**4*sqrt(2*x**2 - 1) - 60
*x**2*sqrt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)), 2*Abs(x**2) > 1), (32*x**5/(
60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1 - 2*x**2))
- 40*x**3/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1
- 2*x**2)) + 15*x/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15
*sqrt(1 - 2*x**2)), True))
```

3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -(3 - x)/(6*(-7 + 6*x - x^2)^(3/2)) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-7+6x-x^2\right)^{5/2}}\,dx &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(-7+6x-x^2\right)^{3/2}}\,dx\\ &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}-\frac{3-x}{6\sqrt{-7+6x-x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.62

$$-\frac{(x-3)\left(x^2-6x+6\right)}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -1/6*((-3 + x)*(6 - 6*x + x^2))/(-7 + 6*x - x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.32, size = 55, normalized size = 1.17

$$\frac{\sqrt{-x^2+6x-7}\left(-x^3+9x^2-24x+18\right)}{6\left(-x+\sqrt{2}+3\right)^2\left(x+\sqrt{2}-3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] (Sqrt[-7 + 6*x - x^2]*(18 - 24*x + 9*x^2 - x^3))/(6*(3 + Sqrt[2] - x)^2*(-3
 + Sqrt[2] + x)^2)
```

fricas [A] time = 1.32, size = 47, normalized size = 1.00

$$-\frac{\left(x^3-9\,x^2+24\,x-18\right)\sqrt{-x^2+6\,x-7}}{6\left(x^4-12\,x^3+50\,x^2-84\,x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3 + 50*x^2
- 84*x + 49)
```

giac [A] time = 0.65, size = 35, normalized size = 0.74

$$-\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6\,x-7}}{6\left(x^2-6\,x+7\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x + 7)^2
```

maple [A] time = 0.44, size = 28, normalized size = 0.60

method	result	size
gosper	$-\frac{x^3-9x^2+24x-18}{6\left(-x^2+6x-7\right)^{\frac{3}{2}}}$	28
trager	$-\frac{\left(x^3-9x^2+24x-18\right)\sqrt{-x^2+6x-7}}{6\left(x^2-6x+7\right)^2}$	38
risch	$\frac{x^3-9x^2+24x-18}{6\left(x^2-6x+7\right)\sqrt{-x^2+6x-7}}$	38
default	$-\frac{-2x+6}{12\left(-x^2+6x-7\right)^{\frac{3}{2}}}-\frac{-2x+6}{12\sqrt{-x^2+6x-7}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+6*x-7)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(x^3-9*x^2+24*x-18)/(-x^2+6*x-7)^(3/2)
```

maxima [A] time = 0.46, size = 59, normalized size = 1.26

$$\frac{x}{6\sqrt{-x^2+6\,x-7}}-\frac{1}{2\sqrt{-x^2+6\,x-7}}+\frac{x}{6\left(-x^2+6\,x-7\right)^{\frac{3}{2}}}-\frac{1}{2\left(-x^2+6\,x-7\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x^2 + 6*x -
 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

mupad [B] time = 0.29, size = 29, normalized size = 0.62

$$-\frac{(4\,x-12)\,\left(8\,x^2-48\,x+48\right)}{192\,{\left(-x^2+6\,x-7\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x - x^2 - 7)^(5/2),x)
```

```
[Out] -((4*x - 12)*(8*x^2 - 48*x + 48))/(192*(6*x - x^2 - 7)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+6x-7\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+6*x-7)**(5/2),x)
```

```
[Out] Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

3.479 $\int \left(1-2x-2x^2\right)^3 \, dx$

Optimal. Leaf size=36

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {611}

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(1-2x-2x^2\right)^3 \, dx &= \int \left(1-6x+6x^2+16x^3-12x^4-24x^5-8x^6\right) \, dx\\ &= x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 36, normalized size = 1.00

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

IntegrateAlgebraic [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{1}{35}\left(-40x^7-140x^6-84x^5+140x^4+70x^3-105x^2+35x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] (35*x - 105*x^2 + 70*x^3 + 140*x^4 - 84*x^5 - 140*x^6 - 40*x^7)/35
```

fricas [A] time = 0.89, size = 32, normalized size = 0.89

$$-\frac{8}{7}x^7-4x^6-\frac{12}{5}x^5+4x^4+2x^3-3x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

giac [A] time = 0.61, size = 32, normalized size = 0.89

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="giac")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

maple [A] time = 0.36, size = 33, normalized size = 0.92

method	result	size
default	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
norman	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
risch	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
gosper	$-\frac{x(40x^6+140x^5+84x^4-140x^3-70x^2+105x-35)}{35}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2-2*x+1)^3,x,method=_RETURNVERBOSE)
[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

maxima [A] time = 0.43, size = 32, normalized size = 0.89

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

mupad [B] time = 0.03, size = 32, normalized size = 0.89

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2*x^2 - 1)^3,x)
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

sympy [A] time = 0.06, size = 34, normalized size = 0.94

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2-2*x+1)**3,x)
[Out] -8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

3.480 $\int (-1+5x)\left(-1-x+x^2\right)^2 \, dx$

Optimal. Leaf size=39

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {631}

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (-1+5x)\left(-1-x+x^2\right)^2 \, dx &= \int \left(-1+3x+11x^2-3x^3-11x^4+5x^5\right) dx\\ &= -x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

IntegrateAlgebraic [A] time = 0.01, size = 30, normalized size = 0.77

$$\frac{1}{60}x\left(50x^5-132x^4-45x^3+220x^2+90x-60\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] (x*(-60 + 90*x + 220*x^2 - 45*x^3 - 132*x^4 + 50*x^5))/60
```

fricas [A] time = 0.99, size = 29, normalized size = 0.74

$$\frac{5}{6}x^6-\frac{11}{5}x^5-\frac{3}{4}x^4+\frac{11}{3}x^3+\frac{3}{2}x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

giac [A] time = 0.58, size = 29, normalized size = 0.74

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

maple [A] time = 0.36, size = 30, normalized size = 0.77

method	result	size
gosper	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
default	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
norman	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
risch	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+5*x)*(x^2-x-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

maxima [A] time = 0.46, size = 29, normalized size = 0.74

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

mupad [B] time = 0.03, size = 29, normalized size = 0.74

$$\frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 1)*(x - x^2 + 1)^2,x)
```

```
[Out] (3*x^2)/2 - x + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

sympy [A] time = 0.06, size = 34, normalized size = 0.87

$$\frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x**2-x-1)**2,x)
```

```
[Out] 5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{1-2x}{6(2x^2-8x+1)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {638, 613}

$$\frac{1-2x}{6(2x^2-8x+1)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x^
2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx &= \frac{1-2x}{6(1-8x+2x^2)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-8x+2x^2)^{3/2}}\,dx\\ &= \frac{1-2x}{6(1-8x+2x^2)^{3/2}}-\frac{2(2-x)}{21\sqrt{1-8x+2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 0.70

$$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.37, size = 33, normalized size = 0.70

$$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

fricas [A] time = 0.78, size = 73, normalized size = 1.55

$$-\frac{4\,x^4-32\,x^3+68\,x^2-\left(8\,x^3-48\,x^2+54\,x-1\right)\sqrt{2\,x^2-8\,x+1}-16\,x+1}{42\left(4\,x^4-32\,x^3+68\,x^2-16\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*sqrt(2*x^2 - 8
*x + 1) - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)
```

giac [A] time = 0.70, size = 27, normalized size = 0.57

$$\frac{2\,(4\,(x-6)x+27)x-1}{42\left(2\,x^2-8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

maple [A] time = 0.45, size = 30, normalized size = 0.64

method	result	size
gosper	$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{\frac{3}{2}}}$	30
trager	$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{\frac{3}{2}}}$	30
risch	$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{\frac{3}{2}}}$	30
default	$-\frac{4x-8}{12\left(2x^2-8x+1\right)^{\frac{3}{2}}}+\frac{4x-8}{42\sqrt{2x^2-8x+1}}-\frac{1}{2\left(2x^2-8x+1\right)^{\frac{3}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+3*x)/(2*x^2-8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

maxima [A] time = 0.47, size = 59, normalized size = 1.26

$$\frac{2\,x}{21\,\sqrt{2\,x^2-8\,x+1}}-\frac{4}{21\,\sqrt{2\,x^2-8\,x+1}}-\frac{x}{3\left(2\,x^2-8\,x+1\right)^{\frac{3}{2}}}+\frac{1}{6\left(2\,x^2-8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x/(2*x^2 -
8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

mupad [B] time = 0.36, size = 29, normalized size = 0.62

$$\frac{8\,x^3-48\,x^2+54\,x-1}{42\,{\left(2\,x^2-8\,x+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x)
```

```
[Out] (54*x - 48*x^2 + 8*x^3 - 1)/(42*(2*x^2 - 8*x + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+1}{\left(2x^2-8x+1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

```
[Out] Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1660, 636}

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
4*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx &= -\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{1}{30}\int \frac{46+60x}{\left(1+2x-4x^2\right)^{3/2}}\,dx\\ &= -\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{7+122x}{75\sqrt{1+2x-4x^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 33, normalized size = 0.73

$$-\frac{-488x^3+216x^2+156x+27}{75\left(-4x^2+2x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] -1/75*(27 + 156*x + 216*x^2 - 488*x^3)/(1 + 2*x - 4*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.29, size = 45, normalized size = 1.00

$$\frac{\sqrt{-4x^2+2x+1}\left(488x^3-216x^2-156x-27\right)}{75\left(4x^2-2x-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 + 2*x - 4*x^2]*(-27 - 156*x - 216*x^2 + 488*x^3))/(75*(-1 - 2*x + 4
*x^2)^2)
```

fricas [A] time = 1.10, size = 73, normalized size = 1.62

$$-\frac{432\,x^4-432\,x^3-108\,x^2-\left(488\,x^3-216\,x^2-156\,x-27\right)\sqrt{-4\,x^2+2\,x+1}+108\,x+27}{75\left(16\,x^4-16\,x^3-4\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/75*(432*x^4 - 432*x^3 - 108*x^2 - (488*x^3 - 216*x^2 - 156*x - 27)*sqrt(
-4*x^2 + 2*x + 1) + 108*x + 27)/(16*x^4 - 16*x^3 - 4*x^2 + 4*x + 1)
```

giac [A] time = 0.66, size = 41, normalized size = 0.91

$$\frac{(4\,(2\,(61\,x-27)x-39)x-27)\sqrt{-4\,x^2+2\,x+1}}{75\left(4\,x^2-2\,x-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x
- 1)^2
```

maple [A] time = 0.40, size = 30, normalized size = 0.67

method	result	size
gosper	$\frac{488x^3-216x^2-156x-27}{75\left(-4x^2+2x+1\right)^{\frac{3}{2}}}$	30
trager	$\frac{\left(488x^3-216x^2-156x-27\right)\sqrt{-4x^2+2x+1}}{75\left(4x^2-2x-1\right)^2}$	42
risch	$-\frac{488x^3-216x^2-156x-27}{75\left(4x^2-2x-1\right)\sqrt{-4x^2+2x+1}}$	42
default	$\frac{2x^2}{\left(-4x^2+2x+1\right)^{\frac{3}{2}}}-\frac{x}{4\left(-4x^2+2x+1\right)^{\frac{3}{2}}}-\frac{49}{48\left(-4x^2+2x+1\right)^{\frac{3}{2}}}+\frac{\frac{61}{240}-\frac{61x}{60}}{\left(-4x^2+2x+1\right)^{\frac{3}{2}}}+\frac{\frac{61}{150}-\frac{122x}{75}}{\sqrt{-4x^2+2x+1}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/75*(488*x^3-216*x^2-156*x-27)/(-4*x^2+2*x+1)^(3/2)
```

maxima [B] time = 0.56, size = 76, normalized size = 1.69

$$-\frac{122\,x}{75\,\sqrt{-4\,x^2+2\,x+1}}+\frac{2\,x^2}{\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}+\frac{61}{150\,\sqrt{-4\,x^2+2\,x+1}}-\frac{19\,x}{15\,\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}-\frac{23}{30\,\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2) + 61/150/
sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/2) - 23/30/(-4*x^2 +
 2*x + 1)^(3/2)
```

mupad [B] time = 0.19, size = 29, normalized size = 0.64

$$-\frac{-488\,x^3+216\,x^2+156\,x+27}{75\,\left(-4\,x^2+2\,x+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x - 8*x^3 + 1)/(2*x - 4*x^2 + 1)^(5/2),x)
```

```
[Out] -(156*x + 216*x^2 - 488*x^3 + 27)/(75*(2*x - 4*x^2 + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{8x^3-8x-1}{\left(-4x^2+2x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

```
[Out] Integral((8*x**3 - 8*x - 1)/(-4*x**2 + 2*x + 1)**(5/2), x)
```

3.483 $\int x^2 \cos^5(x) \, dx$

Optimal. Leaf size=83

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\,\mathrm{c}$$

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 3296, 2637, 2633}

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^5,x]
```

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*Sin[x])/225
 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^2*Cos[x]^4*Sin[x])/5
 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\cos^5(x)\,dx = \frac{2}{25}x\cos^5(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)-\frac{2}{25}\int\cos^5(x)\,dx+\frac{4}{5}\int x^2\cos^3(x)\,dx$$
$$=\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{2}{25}\text{Subst}\left(\int\left(1-2x\right.\right.$$
$$=\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{2\sin(x)}{25}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin($$
$$=\frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{58\sin(x)}{225}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}$$
$$=\frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{298\sin(x)}{225}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.81

$$\frac{5}{8}\left(x^2-2\right)\sin(x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{\left(25x^2-2\right)\sin(5x)}{2000}+\frac{5}{4}x\cos(x)+\frac{5}{72}x\cos(3x)+\frac{1}{200}x\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^5,x]
```

```
[Out] (5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[x
])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Cos[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 57, normalized size = 0.69

$$\frac{2}{25}x\cos(x)^5+\frac{8}{45}x\cos(x)^3+\frac{16}{15}x\cos(x)+\frac{1}{3375}\left(27\left(25x^2-2\right)\cos(x)^4+4\left(225x^2-68\right)\cos(x)^2+1800x^2-414$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

```
[Out] 2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(25*x^2 - 2
)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 4144)*sin(x)
```

giac [A] time = 0.58, size = 55, normalized size = 0.66

$$\frac{1}{200}x\cos(5x)+\frac{5}{72}x\cos(3x)+\frac{5}{4}x\cos(x)+\frac{1}{2000}\left(25x^2-2\right)\sin(5x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{5}{8}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

maple [A] time = 0.38, size = 56, normalized size = 0.67

method	result
risch	$\frac{5x\cos(x)}{4}+\frac{5(x^2-2)\sin(x)}{8}+\frac{x\cos(5x)}{200}+\frac{(25x^2-2)\sin(5x)}{2000}+\frac{5x\cos(3x)}{72}+\frac{5(9x^2-2)\sin(3x)}{432}$
default	$\frac{x^2\left(\frac{8}{3}+\cos^4(x)+\frac{4(\cos^2(x))}{3}\right)\sin(x)}{5}-\frac{16\sin(x)}{15}+\frac{16x\cos(x)}{15}+\frac{2x(\cos^5(x))}{25}-\frac{2\left(\frac{8}{3}+\cos^4(x)+\frac{4(\cos^2(x))}{3}\right)\sin(x)}{125}+\frac{8x(\cos^3(x))}{45}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x*cos(x)+5/8*(x^2-2)*sin(x)+1/200*x*cos(5*x)+1/2000*(25*x^2-2)*sin(5*x)
+5/72*x*cos(3*x)+5/432*(9*x^2-2)*sin(3*x)
```

maxima [A] time = 0.51, size = 55, normalized size = 0.66

$$\frac{1}{200}\,x\cos\left(5\,x\right)+\frac{5}{72}\,x\cos\left(3\,x\right)+\frac{5}{4}\,x\cos(x)+\frac{1}{2000}\left(25\,x^2-2\right)\sin\left(5\,x\right)+\frac{5}{432}\left(9\,x^2-2\right)\sin\left(3\,x\right)+\frac{5}{8}\left(x^2-2\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

mupad [B] time = 0.40, size = 69, normalized size = 0.83

$$\frac{8\,x\cos(x)^3}{45}-\frac{4144\,\sin(x)}{3375}+\frac{2\,x\cos(x)^5}{25}+\frac{8\,x^2\,\sin(x)}{15}-\frac{272\cos(x)^2\,\sin(x)}{3375}-\frac{2\cos(x)^4\,\sin(x)}{125}+\frac{16\,x\,\cos(x)}{15}+\frac{4\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^5,x)
```

```
[Out] (8*x*cos(x)^3)/45 - (4144*sin(x))/3375 + (2*x*cos(x)^5)/25 + (8*x^2*sin(x))
/15 - (272*cos(x)^2*sin(x))/3375 - (2*cos(x)^4*sin(x))/125 + (16*x*cos(x))/
15 + (4*x^2*cos(x)^2*sin(x))/15 + (x^2*cos(x)^4*sin(x))/5
```

sympy [A] time = 3.49, size = 112, normalized size = 1.35

$$\frac{8x^2\sin^5(x)}{15}+\frac{4x^2\sin^3(x)\cos^2(x)}{3}+x^2\sin(x)\cos^4(x)+\frac{16x\sin^4(x)\cos(x)}{15}+\frac{104x\sin^2(x)\cos^3(x)}{45}+\frac{298x\cos^5}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**5,x)
```

```
[Out] 8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*cos(x)**4
+ 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/45 + 298*x*cos(x)**5
/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*cos(x)**2/675 - 298*sin(x)*cos(
x)**4/225
```

3.484 $\int x^3 \sin^3(x)\,dx$

Optimal. Leaf size=73

$$-\frac{2}{3}x^3\cos(x)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+2x^2\sin(x)-\frac{2\sin^3(x)}{27}-\frac{40\sin(x)}{9}+\frac{40}{9}x\cos(x)+\frac{2}{9}x\sin^2(x)\cos(x)$$

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 3296, 2637, 3310}

$$\frac{1}{3}x^2\sin^3(x)+2x^2\sin(x)-\frac{2}{3}x^3\cos(x)-\frac{1}{3}x^3\sin^2(x)\cos(x)-\frac{2\sin^3(x)}{27}-\frac{40\sin(x)}{9}+\frac{40}{9}x\cos(x)+\frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x]^3,x]
```

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x] + (2*x*Co
s[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^3)/27 + (x^2*Sin[x]^
3)/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\sin^3(x)\,dx = -\frac{1}{3}x^3\cos(x)\sin^2(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}\int x^3\sin(x)\,dx-\frac{2}{3}\int x\sin^3(x)\,dx$$
$$= -\frac{2}{3}x^3\cos(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin^3(x)-\frac{4}{9}\int x\,\mathrm{si}$$
$$= \frac{4}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+$$
$$= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{4\sin(x)}{9}+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-$$
$$= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{40\sin(x)}{9}+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.70

$$\frac{1}{108}\left(243\left(x^2-2\right)\sin(x)-\left(9x^2-2\right)\sin(3x)-81x\left(x^2-6\right)\cos(x)+3x\left(3x^2-2\right)\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x]^3,x]
```

```
[Out] (-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[x
] - (-2 + 9*x^2)*Sin[3*x])/108
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sin^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*Sin[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.83, size = 52, normalized size = 0.71

$$\frac{1}{9}\left(3x^3-2x\right)\cos(x)^3-\frac{1}{3}\left(3x^3-14x\right)\cos(x)-\frac{1}{27}\left(\left(9x^2-2\right)\cos(x)^2-63x^2+122\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)*
cos(x)^2 - 63*x^2 + 122)*sin(x)
```

giac [A] time = 0.61, size = 49, normalized size = 0.67

$$\frac{1}{36}\left(3x^3-2x\right)\cos(3x)-\frac{3}{4}\left(x^3-6x\right)\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{9}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

maple [A] time = 0.38, size = 50, normalized size = 0.68

method	result
risch	$\left(-\frac{3}{4}x^3+\frac{9}{2}x\right)\cos(x)+\frac{9\left(x^2-2\right)\sin(x)}{4}+\left(\frac{1}{12}x^3-\frac{1}{18}x\right)\cos(3x)-\frac{\left(9x^2-2\right)\sin(3x)}{108}$
default	$-\frac{x^3\left(2+\sin^2(x)\right)\cos(x)}{3}+2x^2\sin(x)-\frac{40\sin(x)}{9}+4x\cos(x)+\frac{x^2\left(\sin^3(x)\right)}{3}+\frac{2x\left(2+\sin^2(x)\right)\cos(x)}{9}-\frac{2\left(\sin^3(x)\right)}{27}$
norman	$\frac{\frac{40x}{9}-\frac{2x^3}{3}-\frac{496\left(\tan^3\left(\frac{x}{2}\right)\right)}{27}-\frac{80\left(\tan^5\left(\frac{x}{2}\right)\right)}{9}+\frac{16x\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{16x\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{40x\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+4x^2\tan\left(\frac{x}{2}\right)+\frac{32x^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+4x^2\left(\tan^5\left(\frac{x}{2}\right)\right)-2x^3\left(\tan^2\left(\right.\right.}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*x^3+9/2*x)*cos(x)+9/4*(x^2-2)*sin(x)+(1/12*x^3-1/18*x)*cos(3*x)-1/108
*(9*x^2-2)*sin(3*x)
```

maxima [A] time = 0.46, size = 49, normalized size = 0.67

$$\frac{1}{36}\left(3x^3-2x\right)\cos(3x)-\frac{3}{4}\left(x^3-6x\right)\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{9}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

mupad [B] time = 0.33, size = 59, normalized size = 0.81

$$\frac{7x^2\sin(x)}{3}-\frac{2x\cos(x)^3}{9}-x^3\cos(x)-\frac{122\sin(x)}{27}+\frac{x^3\cos(x)^3}{3}+\frac{2\cos(x)^2\sin(x)}{27}+\frac{14x\cos(x)}{3}-\frac{x^2\cos(x)^2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x)^3,x)
```

```
[Out] (7*x^2*sin(x))/3 - (2*x*cos(x)^3)/9 - x^3*cos(x) - (122*sin(x))/27 + (x^3*c
os(x)^3)/3 + (2*cos(x)^2*sin(x))/27 + (14*x*cos(x))/3 - (x^2*cos(x)^2*sin(x
))/3
```

sympy [A] time = 2.04, size = 92, normalized size = 1.26

$$-x^3\sin^2(x)\cos(x)-\frac{2x^3\cos^3(x)}{3}+\frac{7x^2\sin^3(x)}{3}+2x^2\sin(x)\cos^2(x)+\frac{14x\sin^2(x)\cos(x)}{3}+\frac{40x\cos^3(x)}{9}-\frac{122\sin^3(}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x)**3,x)
```

```
[Out] -x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3 + 2*x**2*s
in(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)**3/9 - 122*sin(x)**
3/27 - 40*sin(x)*cos(x)**2/9
```

3.485 $\int x^2 \sin^6(x)\,dx$

Optimal. Leaf size=105

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2($$

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 30, 2635, 8}

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2($$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^6,x]
```

```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Cos[x]*Sin[x
])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 - (5*x^2*Cos[x]*Sin[x
]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^5)/108 - (x^2*Cos[x]*Sin[x]^5)
/6 + (x*Sin[x]^6)/18
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2 \sin^6(x)\,dx = -\frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x) - \frac{1}{18}\int \sin^6(x)\,dx + \frac{5}{6}\int x^2\sin^4(x)\,dx$$
$$= -\frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) + \frac{1}{108}\cos(x)\sin^5(x) - \frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x) -$$
$$= -\frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) +$$
$$= \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x$$
$$= -\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{2}$$

Mathematica [A] time = 0.09, size = 70, normalized size = 0.67

$$\frac{1440x^3 - 1620\left(2x^2-1\right)\sin(2x) + 81\left(8x^2-1\right)\sin(4x) - 4\left(18x^2-1\right)\sin(6x) - 3240x\cos(2x) + 324x\cos(4x) -}{13824}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^6,x]
```

```
[Out] (1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 + 2
*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/13824
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sin^6(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Sin[x]^6,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.26, size = 72, normalized size = 0.69

$$-\frac{1}{18}\,x\cos(x)^6 + \frac{13}{48}\,x\cos(x)^4 + \frac{5}{48}\,x^3 - \frac{11}{16}\,x\cos(x)^2 - \frac{1}{3456}\left(32\left(18\,x^2-1\right)\cos(x)^5 - 2\left(936\,x^2-97\right)\cos(x)^3 + 3\left(79\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2 - 1/3456*
(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 + 3*(792*x^2 - 299)*c
os(x))*sin(x) + 299/1152*x
```

giac [A] time = 0.59, size = 66, normalized size = 0.63

$$\frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2-1\right)\sin(6\,x) + \frac{3}{512}\left(8\,x^2-1\right)\sin(4\,x) - \frac{15}{128}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

maple [A] time = 0.42, size = 67, normalized size = 0.64

method	result
risch	$\frac{5x^3}{48}-\frac{x\cos(6x)}{576}-\frac{(18x^2-1)\sin(6x)}{3456}+\frac{3x\cos(4x)}{128}+\frac{3(8x^2-1)\sin(4x)}{512}-\frac{15x\cos(2x)}{64}-\frac{15(2x^2-1)\sin(2x)}{128}$
default	$x^2\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+\frac{x(\sin^6(x))}{18}+\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{108}+\frac{115x}{1152}+\frac{5x(\sin^4($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 5/48*x^3-1/576*x*cos(6*x)-1/3456*(18*x^2-1)*sin(6*x)+3/128*x*cos(4*x)+3/512
*(8*x^2-1)*sin(4*x)-15/64*x*cos(2*x)-15/128*(2*x^2-1)*sin(2*x)
```

maxima [A] time = 0.56, size = 66, normalized size = 0.63

$$\frac{5}{48}x^3-\frac{1}{576}x\cos(6x)+\frac{3}{128}x\cos(4x)-\frac{15}{64}x\cos(2x)-\frac{1}{3456}\left(18x^2-1\right)\sin(6x)+\frac{3}{512}\left(8x^2-1\right)\sin(4x)-\frac{15}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

mupad [B] time = 0.40, size = 88, normalized size = 0.84

$$\frac{15\sin(2x)}{128}-\frac{3\sin(4x)}{512}+\frac{\sin(6x)}{3456}-\frac{3x\left(2\sin(2x)^2-1\right)}{128}+\frac{x\left(2\sin(3x)^2-1\right)}{576}-\frac{15x^2\sin(2x)}{64}+\frac{3x^2\sin(4x)}{64}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^6,x)
```

```
[Out] (15*sin(2*x))/128 - (3*sin(4*x))/512 + sin(6*x)/3456 - (3*x*(2*sin(2*x)^2 -
 1))/128 + (x*(2*sin(3*x)^2 - 1))/576 - (15*x^2*sin(2*x))/64 + (3*x^2*sin(4
*x))/64 - (x^2*sin(6*x))/192 + (5*x^3)/48 + (15*x*(2*sin(x)^2 - 1))/64
```

sympy [A] time = 5.90, size = 192, normalized size = 1.83

$$\frac{5x^3\sin^6(x)}{48}+\frac{5x^3\sin^4(x)\cos^2(x)}{16}+\frac{5x^3\sin^2(x)\cos^4(x)}{16}+\frac{5x^3\cos^6(x)}{48}-\frac{11x^2\sin^5(x)\cos(x)}{16}-\frac{5x^2\sin^3(x)\cos^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**6,x)
```

```
[Out] 5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos(
x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin(x
)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 35*x
*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)**6/
1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin(x)*c
os(x)**5/1152
```

3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3443, 3310, 3296, 2637}

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[x]^3)/27 +
 (x^2*Sin[x]^3)/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos(x)\sin^2(x)\,dx &= \frac{1}{3}x^2\sin^3(x) - \frac{2}{3}\int x\sin^3(x)\,dx \\
&= \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int x\sin(x)\,dx \\
&= \frac{4}{9}x\cos(x) + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int \cos(x)\,dx \\
&= \frac{4}{9}x\cos(x) - \frac{4\sin(x)}{9} + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\sin(x)\left(9x^2+\left(2-9x^2\right)\cos(2x)-26\right)+27x\cos(x)-3x\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])/
54
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos(x)\sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.20, size = 36, normalized size = 0.82

$$-\frac{2}{9}\,x\cos(x)^3+\frac{2}{3}\,x\cos(x)-\frac{1}{27}\left(\left(9\,x^2-2\right)\cos(x)^2-9\,x^2+14\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x^2 + 14)*s
in(x)
```

giac [A] time = 0.62, size = 35, normalized size = 0.80

$$-\frac{1}{18}\,x\cos\left(3\,x\right)+\frac{1}{2}\,x\cos(x)-\frac{1}{108}\left(9\,x^2-2\right)\sin\left(3\,x\right)+\frac{1}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

maple [A] time = 0.38, size = 32, normalized size = 0.73

method	result	size
default	$\frac{x^2\left(\sin^3(x)\right)}{3}+\frac{2x\left(2+\sin^2(x)\right)\cos(x)}{9}-\frac{2\left(\sin^3(x)\right)}{27}-\frac{4\sin(x)}{9}$	32
risch	$\frac{x\cos(x)}{2}+\frac{\left(x^2-2\right)\sin(x)}{4}-\frac{x\cos(3x)}{18}-\frac{\left(9x^2-2\right)\sin(3x)}{108}$	36
norman	$\frac{\frac{4x}{9}-\frac{64\left(\tan^3\left(\frac{x}{2}\right)\right)}{27}-\frac{8\left(\tan^5\left(\frac{x}{2}\right)\right)}{9}+\frac{4x\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+\frac{8x^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\frac{8\tan\left(\frac{x}{2}\right)}{9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(x)
```

maxima [A] time = 0.61, size = 35, normalized size = 0.80

$$-\frac{1}{18}\,x\cos\left(3\,x\right)+\frac{1}{2}\,x\cos\left(x\right)-\frac{1}{108}\,\left(9\,x^{2}-2\right)\sin\left(3\,x\right)+\frac{1}{4}\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

mupad [B] time = 0.07, size = 40, normalized size = 0.91

$$\frac{x^{2}\sin(x)^{3}}{3}+\frac{4\,x\cos(x)^{3}}{9}+\frac{2\,x\,\cos(x)\sin(x)^{2}}{3}-\frac{4\cos(x)^{2}\,\sin(x)}{9}-\frac{14\sin(x)^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)*sin(x)^2,x)
```

```
[Out] (4*x*cos(x)^3)/9 - (14*sin(x)^3)/27 + (x^2*sin(x)^3)/3 - (4*cos(x)^2*sin(x)
)/9 + (2*x*cos(x)*sin(x)^2)/3
```

sympy [A] time = 1.23, size = 53, normalized size = 1.20

$$\frac{x^{2}\sin^{3}\left(x\right)}{3}+\frac{2x\sin^{2}\left(x\right)\cos\left(x\right)}{3}+\frac{4x\cos^{3}\left(x\right)}{9}-\frac{14\sin^{3}\left(x\right)}{27}-\frac{4\sin\left(x\right)\cos^{2}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*sin(x)**3/
27 - 4*sin(x)*cos(x)**2/9
```

3.487 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cos^2(x)\cot^2(x)\,dx = -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx$$
$$= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx$$
$$= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$-\frac{3x^2}{4} - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^2(x)\cot^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Cos[x]^2*Cot[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.09, size = 45, normalized size = 1.36

$$\frac{4\,x\cos(x)^3 - 12\,x\cos(x) - \left(6\,x^2 + 2\,\cos(x)^2 - 1\right)\sin(x) + 8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

giac [B] time = 0.70, size = 206, normalized size = 6.24

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan\left(\frac{1}{2}\,x\right)^4 + \mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

maple [C] time = 0.07, size = 60, normalized size = 1.82

method	result
risch	$-\frac{3x^2}{4}+\frac{i(2x+i)e^{2ix}}{16}-\frac{i(-i+2x)e^{-2ix}}{16}-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$
norman	$\frac{-\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}-\frac{x}{2}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3x^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}-\frac{3x^2\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^4/sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*x^2+1/16*I*(2*x+I)*exp(2*I*x)-1/16*I*(-I+2*x)*exp(-2*I*x)-2*I*x-2*I*x/
(exp(2*I*x)-1)+ln(exp(2*I*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 0.49, size = 56, normalized size = 1.70

$$\ln\left(e^{x\,2i}-1\right)-e^{-x\,2i}\left(\frac{1}{16}+\frac{x\,1i}{8}\right)+e^{x\,2i}\left(-\frac{1}{16}+\frac{x\,1i}{8}\right)-\frac{3\,x^2}{4}-x\,2i-\frac{x\,2i}{e^{x\,2i}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x)^4)/sin(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i
)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

sympy [B] time = 2.01, size = 507, normalized size = 15.36

$$-\frac{3x^2\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}-\frac{6x^2\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**4/sin(x)**2,x)
```

```
[Out] -3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*t
an(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)/(
4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2)**
5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*tan(x/
2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*ta
n(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)
**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 8*log(t
an(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) -
4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) +
 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

3.488 $\int x \sec(x) \tan^3(x) \, dx$

Optimal. Leaf size=30

$$\frac{1}{3}x\sec^3(x) - x\sec(x) + \frac{5}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tan(x)\sec(x)$$

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2606, 4417, 3770, 3768}

$$\frac{1}{3}x\sec^3(x) - x\sec(x) + \frac{5}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4417

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || Int
egerQ[(p - 1)/2])
```

Rubi steps

$$\begin{aligned}
\int x\sec(x)\tan^3(x)\,dx &= -x\sec(x) + \frac{1}{3}x\sec^3(x) - \int\left(-\sec(x) + \frac{\sec^3(x)}{3}\right)dx \\
&= -x\sec(x) + \frac{1}{3}x\sec^3(x) - \frac{1}{3}\int\sec^3(x)\,dx + \int\sec(x)\,dx \\
&= \tanh^{-1}(\sin(x)) - x\sec(x) + \frac{1}{3}x\sec^3(x) - \frac{1}{6}\sec(x)\tan(x) - \frac{1}{6}\int\sec(x)\,dx \\
&= \frac{5}{6}\tanh^{-1}(\sin(x)) - x\sec(x) + \frac{1}{3}x\sec^3(x) - \frac{1}{6}\sec(x)\tan(x)
\end{aligned}$$

Mathematica [B] time = 0.13, size = 104, normalized size = 3.47

$$-\frac{1}{24}\sec^3(x)\left(4x+2\sin(2x)+12x\cos(2x)+5\cos(3x)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+15\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\text{si}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] -1/24*(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]]
+ 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos[3
*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec(x)\tan^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.20, size = 47, normalized size = 1.57

$$\frac{5\cos(x)^3\log(\sin(x)+1)-5\cos(x)^3\log(-\sin(x)+1)-12\,x\cos(x)^2-2\cos(x)\sin(x)+4\,x}{12\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(x
)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

giac [B] time = 1.22, size = 341, normalized size = 11.37

$$\frac{8\,x\tan\left(\frac{1}{2}\,x\right)^6+5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6-5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6-24\,x\,\text{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2
*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x)^2
- 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5 - 24*
x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x)^2 - 1)
```

maple [B] time = 0.10, size = 76, normalized size = 2.53

method	result	size

norman	$\frac{\frac{2x}{3}-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)-2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{\tan\left(\frac{x}{2}\right)}{3}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{5\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{6}$	76
risch	$-\frac{6x\,e^{5ix}+4x\,e^{3ix}-i e^{5ix}+6x\,e^{ix}+i e^{ix}}{3\left(1+e^{2ix}\right)^3}-\frac{5\ln\left(e^{ix}-i\right)}{6}+\frac{5\ln\left(e^{ix}+i\right)}{6}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*x-1/3*tan(1/2*x)^5-2*x*tan(1/2*x)^2-2*x*tan(1/2*x)^4+2/3*x*tan(1/2*x)^
6+1/3*tan(1/2*x))/(tan(1/2*x)^2-1)^3-5/6*ln(tan(1/2*x)-1)+5/6*ln(tan(1/2*x)
+1)
```

maxima [B] time = 1.47, size = 619, normalized size = 20.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) + 6*x*cos(x)
 + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x*cos(2*x) + 2*x - si
n(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x) + 6*x*cos(x) - sin(x))*cos(4
*x) + 16*(3*x*cos(2*x) + x)*cos(3*x) + 12*(6*x*cos(x) - sin(x))*cos(2*x) +
24*x*cos(x) - 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*
(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin
(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(
2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 5*(2*(3*
cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4
*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(
6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) +
1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) + 4*x*sin(3*x)
 + 6*x*sin(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x) + 18*x*sin(2
*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x) + 6*x*sin(x)
 + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x) - 4*sin(x))/(2*(3*c
os(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*
x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6
*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1
)
```

mupad [B] time = 0.50, size = 35, normalized size = 1.17

$$-\frac{x\cos(x)^2-\frac{x}{3}+\frac{\sin(2\,x)}{12}}{\cos(x)^3}-\frac{\mathrm{atan}\left(\cos(x)+\sin(x)\,1\mathrm{i}\right)\,5\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)^3)/cos(x)^4,x)
```

```
[Out] - (atan(cos(x) + sin(x)*1i)*5i)/3 - (sin(2*x)/12 - x/3 + x*cos(x)^2)/cos(x)
^3
```

sympy [B] time = 1.69, size = 551, normalized size = 18.37

$$\frac{4x\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6}-\frac{12x\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6}-\frac{12x\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x)**4,x)
```

```
[Out] 4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*
x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*x*
tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/(6*
tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1)*ta
n(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(ta
n(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 -
6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*
tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18
*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan
(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4/(6*tan(x/
2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) + 1)*tan(x/2
)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2)
 + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 2*tan(x/2)**5
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*tan(x/2)/(6*tan(
x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

3.489 $\int x\sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3757, 3767, 8}

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]^2*Tan[x],x]
```

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(x)\tan(x)\,dx &= \frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \sec^2(x)\,dx\\ &= \frac{1}{2}x\sec^2(x)+\frac{1}{2}\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\ &= \frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]^2*Tan[x],x]
```

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2(x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sec[x]^2*Tan[x],x]
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 15, normalized size = 0.94

$$-\frac{\cos(x)\sin(x) - x}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
[Out] -1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

giac [B] time = 0.60, size = 53, normalized size = 3.31

$$\frac{x\tan\left(\frac{1}{2}x\right)^4 + 2x\tan\left(\frac{1}{2}x\right)^2 + 2\tan\left(\frac{1}{2}x\right)^3 + x - 2\tan\left(\frac{1}{2}x\right)}{2\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
[Out] 1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))
/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

maple [A] time = 0.34, size = 13, normalized size = 0.81

method	result	size
default	$\frac{x}{2\cos(x)^2} - \frac{\tan(x)}{2}$	13
risch	$\frac{2x\,e^{2ix} - ie^{2ix} - i}{\left(1+e^{2ix}\right)^2}$	30
norman	$\frac{\tan^3\left(\frac{x}{2}\right) + x\left(\tan^2\left(\frac{x}{2}\right)\right) + \frac{x}{2} + \frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2} - \tan\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right) - 1\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*x/cos(x)^2-1/2*tan(x)
```

maxima [B] time = 0.60, size = 132, normalized size = 8.25

$$\frac{4x\cos(2x)^2 + 4x\sin(2x)^2 + (2x\cos(2x) + \sin(2x))\cos(4x) + 2x\cos(2x) + (2x\sin(2x) - \cos(2x) - 1)}{2(2\cos(2x) + 1)\cos(4x) + \cos(4x)^2 + 4\cos(2x)^2 + \sin(4x)^2 + 4\sin(4x)\sin(2x) + 4\sin(2x)^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
[Out] (4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*x
*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(2*
x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

mupad [B] time = 0.35, size = 16, normalized size = 1.00

$$\frac{2\,x-\sin\left(2\,x\right)}{4\,{\cos\left(x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x))/cos(x)^3,x)
```

```
[Out] (2*x - sin(2*x))/(4*cos(x)^2)
```

sympy [B] time = 1.03, size = 128, normalized size = 8.00

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{x}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2\tan^3\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)**3,x)
```

```
[Out] x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan(
x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*ta
n(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**4 -
 4*tan(x/2)**2 + 2)
```

3.490 $\int x \sin^2(x) \tan(x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{2} i \operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4407, 3443, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{1}{2} i \operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2*Tan[x],x]
```

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] -
 (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\tan(x)\,dx &= -\int x\cos(x)\sin(x)\,dx+\int x\tan(x)\,dx\\ &= \frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)-2i\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx+\frac{1}{2}\int\sin^2(x)\,dx\\ &= \frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)+\frac{\int 1\,dx}{4}+\int\log\left(1+e^{2ix}\right)dx\\ &= \frac{x}{4}+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)-\frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x\right.\\ &= \frac{x}{4}+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.92

$$\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{8}\sin(2x)+\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2*Tan[x],x]
[Out] (I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^(
(2*I)*x)] - Sin[2*x]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^2(x)\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sin[x]^2*Tan[x],x]
[Out] Could not integrate
```

fricas [B] time = 1.41, size = 113, normalized size = 1.82

$$\frac{1}{2}x\cos(x)^2-\frac{1}{2}x\log(i\cos(x)+\sin(x)+1)-\frac{1}{2}x\log(i\cos(x)-\sin(x)+1)-\frac{1}{2}x\log(-i\cos(x)+\sin(x)+1)-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)^2 - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - si
n(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(x
```

```
) + 1) - 1/4*cos(x)*sin(x) - 1/4*x - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I
*dilog(I*cos(x) - sin(x)) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-
I*cos(x) - sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x), x)
```

maple [A] time = 0.09, size = 57, normalized size = 0.92

method	result	size
risch	$\frac{ix^2}{2}+\frac{(2x+i)\mathrm{e}^{2ix}}{16}+\frac{(-i+2x)\mathrm{e}^{-2ix}}{16}-x\ln\left(1+\mathrm{e}^{2ix}\right)+\frac{i\,\mathrm{polylog}\left(2,-\mathrm{e}^{2ix}\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2+1/16*(2*x+I)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(1+exp(2*I*
x))+1/2*I*polylog(2,-exp(2*I*x))
```

maxima [A] time = 1.47, size = 66, normalized size = 1.06

$$\frac{1}{2}i\,x^2-i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+\frac{1}{4}\,x\cos\left(2\,x\right)-\frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)+\frac{1}{2}i\,\mathrm{I}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*lo
g(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1/8
*sin(2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)^3)/cos(x),x)
```

```
[Out] int((x*sin(x)^3)/cos(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin^3\left(x\right)}{\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x),x)
```

```
[Out] Integral(x*sin(x)**3/cos(x), x)
```

3.491 $\int x\tan^3(x)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^3,x]

[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] -
 Tan[x]/2 + (x*Tan[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tan^3(x)\,dx &= \frac{1}{2}x\tan^2(x)-\frac{1}{2}\int\tan^2(x)\,dx-\int x\tan(x)\,dx\\ &= -\frac{ix^2}{2}-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)+2i\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)-\int\log\left(1+e^{2ix}\right)\,dx\\ &= \frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\ &= \frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.92

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^3,x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] + (x
*Sec[x]^2)/2 - Tan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Tan[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.47, size = 138, normalized size = 2.34

$$\frac{x\cos(x)^2\log\left(i\cos(x)+\sin(x)+1\right)+x\cos(x)^2\log\left(i\cos(x)-\sin(x)+1\right)+x\cos(x)^2\log\left(-i\cos(x)+\sin(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(I*cos(x) - sin(
x) + 1) + x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(-I*cos(x)
 - sin(x) + 1) + I*cos(x)^2*dilog(I*cos(x) + sin(x)) - I*cos(x)^2*dilog(I*c
os(x) - sin(x)) - I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + I*cos(x)^2*dilog(-
I*cos(x) - sin(x)) - cos(x)*sin(x) + x)/cos(x)^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x)^3, x)
```

maple [A] time = 0.07, size = 59, normalized size = 1.00

method	result	size
risch	$-\frac{i x^{2}}{2}+\frac{2 x \,{\mathrm e}^{2 i x}-i {\mathrm e}^{2 i x}-i}{\left(1+{\mathrm e}^{2 i x}\right)^{2}}+x \ln \left(1+{\mathrm e}^{2 i x}\right)-\frac{i \operatorname{polylog}\left(2, -{\mathrm e}^{2 i x}\right)}{2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2+(2*x*exp(2*I*x)-I*exp(2*I*x)-I)/(1+exp(2*I*x))^2+x*ln(1+exp(2*I*
x))-1/2*I*polylog(2,-exp(2*I*x))
```

maxima [B] time = 1.30, size = 213, normalized size = 3.61

$$-\frac{x^2\cos(4x) + i\,x^2\sin(4x) + x^2 - (2\,x\cos(4x) + 4\,x\cos(2x) + 2i\,x\sin(4x) + 4i\,x\sin(2x) + 2\,x)\arctan(\sin(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

```
[Out] -(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*
x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(2*x), cos(2*x) + 1) + 2*(x^2
 + 2*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
 + 1)*dilog(-e^(2*I*x)) - (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*
x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - (-2*I*x^2
 + 4*x - 2*I)*sin(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*
sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\sin\left(x\right)}^3}{{\cos\left(x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)^3)/cos(x)^3,x)
```

```
[Out] int((x*sin(x)^3)/cos(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin^{3}{\left(x \right)}}{\cos^{3}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x)**3,x)
```

```
[Out] Integral(x*sin(x)**3/cos(x)**3, x)
```

3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

Rubi [A] time = 0.10, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6711, 32}

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] 2/(1 + Cot[x]/x)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6711

```
Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] :> With[{c
= Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[c*p, Subst[Int[(b + a*x^p)
^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q}, x]
&& EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \frac{\cot(x)}{x}\right)\right)\\ &= \frac{2}{1+\frac{\cot(x)}{x}}\end{aligned}$$

Mathematica [A] time = 0.23, size = 14, normalized size = 1.17

$$\frac{2x\sin(x)}{x\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] (2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.25, size = 13, normalized size = 1.08

$$-\frac{2 \cos(x)}{x \sin(x) + \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -2*cos(x)/(x*sin(x) + cos(x))
```

giac [A] time = 0.63, size = 10, normalized size = 0.83

$$-\frac{2}{x \tan(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/(x*tan(x) + 1)
```

maple [C] time = 0.83, size = 44, normalized size = 3.67

method	result	size
risch	$-\frac{2i}{x+i} - \frac{4ix}{(x+i)\left(x\,e^{2ix}-x+ie^{2ix}+i\right)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

maxima [B] time = 1.18, size = 78, normalized size = 6.50

$$-\frac{2 \left(\cos(2x)^2 + 2x \sin(2x) + \sin(2x)^2 + 2 \cos(2x) + 1\right)}{\left(x^2+1\right) \cos(2x)^2 + \left(x^2+1\right) \sin(2x)^2 + x^2 - 2\left(x^2-1\right) \cos(2x) + 4x \sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*cos
(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x)
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{2x + \sin(2x)}{(\cos(x) + x \sin(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2,x)
```

```
[Out] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + \sin(2x)}{(x\sin(x) + \cos(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

```
[Out] Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$

Optimal. Leaf size=20

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4594, 3767, 8}

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x*Cos[x] - Sin[x])^2,x]
```

```
[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4594

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx &= \frac{x\csc(x)}{x\cos(x)-\sin(x)}+\int \csc^2(x)\,dx\\
&= \frac{x\csc(x)}{x\cos(x)-\sin(x)}-\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\
&= -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 19, normalized size = 0.95

$$\frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
```

```
[Out] (Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(x*Cos[x] - Sin[x])^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 19, normalized size = 0.95

$$\frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")
[Out] (x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

giac [A] time = 0.63, size = 39, normalized size = 1.95

$$-\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^2+1}{x\tan\left(\frac{1}{2}\,x\right)^2-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="giac")
[Out] -(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(1/2*x))
```

maple [C] time = 0.25, size = 29, normalized size = 1.45

method	result	size
risch	$\frac{2i(x-i)}{i\mathrm{e}^{2ix}+x\,\mathrm{e}^{2ix}-i+x}$	29
norman	$\frac{-1+\tan^2\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)}{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x*cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*I*(x-I)/(I*exp(2*I*x)+x*exp(2*I*x)-I+x)
```

maxima [B] time = 0.70, size = 69, normalized size = 3.45

$$\frac{2\left(2\,x\cos\left(2\,x\right)+\left(x^2-1\right)\sin\left(2\,x\right)\right)}{\left(x^2+1\right)\cos\left(2\,x\right)^2+\left(x^2+1\right)\sin\left(2\,x\right)^2+x^2+2\left(x^2-1\right)\cos\left(2\,x\right)-4\,x\sin\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")
[Out] 2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin
(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{(\sin(x) - x\ \cos(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(sin(x) - x*cos(x))^2,x)
```

```
[Out] int(x^2/(sin(x) - x*cos(x))^2, x)
```

sympy [B] time = 1.47, size = 66, normalized size = 3.30

$$-\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right) - x + 2\tan\left(\frac{x}{2}\right)} + \frac{\tan^2\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right) - x + 2\tan\left(\frac{x}{2}\right)} - \frac{1}{x\tan^2\left(\frac{x}{2}\right) - x + 2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x*cos(x)-sin(x))**2,x)
```

```
[Out] -2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**2
 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

3.494 $\int a^{mx}b^{nx}\,dx$

Optimal. Leaf size=22

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2287, 2194}

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int a^{mx}b^{nx}\,dx &= \int e^{x(m\log(a)+n\log(b))}\,dx\\ &= \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a^{mx}b^{nx}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[a^(m*x)*b^(n*x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.08, size = 22, normalized size = 1.00

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
```

```
[Out] a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

giac [C] time = 0.68, size = 325, normalized size = 14.77

$$2\left(\frac{2\left(m\log\left(|a|\right)+n\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi mx\mathrm{sgn}(a)-\frac{1}{2}\pi nx\mathrm{sgn}(b)+\frac{1}{2}\pi mx+\frac{1}{2}\pi nx\right)}{\left(\pi m\mathrm{sgn}(a)+\pi n\mathrm{sgn}(b)-\pi m-\pi n\right)^2+4\left(m\log\left(|a|\right)+n\log\left(|b|\right)\right)^2}-\frac{\left(\pi m\mathrm{sgn}(a)+\pi n\mathrm{sgn}(b)-\right.}{\left(\pi m\mathrm{sgn}(a)+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

```
[Out] 2*(2*(m*log(abs(a)) + n*log(abs(b)))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sg
n(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^
2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) - pi*
m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2*pi*
n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) + n*lo
g(abs(b)))^2))*e^((m*log(abs(a)) + n*log(abs(b)))*x) - 1/2*I*(-2*I*e^(1/2*I
*pi*m*x*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m
*sgn(a) + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(b
))) + 2*I*e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/
2*I*pi*n*x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(abs
(a)) + 2*n*log(abs(b))))*e^((m*log(abs(a)) + n*log(abs(b)))*x)
```

maple [A] time = 0.06, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$	23
risch	$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$	23
norman	$\frac{e^{mx\ln(a)}e^{nx\ln(b)}}{m\ln(a)+n\ln(b)}$	25
meijerg	$-\frac{1-e^{xn\ln(b)\left(1+\frac{m\ln(a)}{n\ln(b)}\right)}}{n\ln(b)\left(1+\frac{m\ln(a)}{n\ln(b)}\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(m*x)*b^(n*x),x,method=_RETURNVERBOSE)
```

```
[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume((log(b)*n)/(log(a)*m)>0)', see `ass
ume?` for more details)Is (log(b)*n)/(log(a)*m) equal to -1?
```

mupad [B] time = 0.33, size = 22, normalized size = 1.00

$$\frac{a^{m\,x}\,b^{n\,x}}{m\,\ln(a)+n\,\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(m*x)*b^(n*x),x)
```

```
[Out] (a^(m*x)*b^(n*x))/(m*log(a) + n*log(b))
```

sympy [A] time = 0.70, size = 42, normalized size = 1.91

$$\begin{cases} \frac{a^{mx}b^{nx}}{m\log{(a)}+n\log{(b)}} & \text{for } m \neq -\frac{n\log{(b)}}{\log{(a)}} \\ b^{nx}xe^{-nx\log{(b)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(m*x)*b**(n*x),x)
```

```
[Out] Piecewise(((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a))
), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

3.495 $\int a^{-x}b^{-x}\left(a^x-b^x\right)^2\,dx$

Optimal. Leaf size=34

$$\frac{a^x b^{-x}-a^{-x}b^x}{\log(a)-\log(b)}-2x$$

Rubi [A] time = 0.21, antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2287, 6742, 2194, 8}

$$-\frac{a^{-x}b^x}{\log(a)-\log(b)}+\frac{a^x b^{-x}}{\log(a)-\log(b)}-2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int a^{-x}b^{-x}\left(a^x-b^x\right)^2\,dx &= \int \left(a^x-b^x\right)^2 e^{-x(\log(a)+\log(b))}\,dx\\
&= \int \left(a^{2x}e^{-x(\log(a)+\log(b))}-2a^x b^x e^{-x(\log(a)+\log(b))}+b^{2x}e^{-x(\log(a)+\log(b))}\right)dx\\
&= -\left(2\int a^x b^x e^{-x(\log(a)+\log(b))}\,dx\right)+\int a^{2x}e^{-x(\log(a)+\log(b))}\,dx+\int b^{2x}e^{-x(\log(a)+\log(b))}\,dx\\
&= -(2\int 1\,dx)+\int e^{-x(\log(a)-\log(b))}\,dx+\int e^{x(\log(a)-\log(b))}\,dx\\
&= -2x+\frac{a^x b^{-x}}{\log(a)-\log(b)}-\frac{a^{-x}b^x}{\log(a)-\log(b)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 1.35

$$\frac{e^{x(\log(a)-\log(b))}}{\log(a)-\log(b)}+\frac{e^{x(\log(b)-\log(a))}}{\log(b)-\log(a)}-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b]))
/(-Log[a] + Log[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a^{-x}b^{-x}\left(a^x-b^x\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.85, size = 52, normalized size = 1.53

$$-\frac{2\left(x\log\left(a\right)-x\log\left(b\right)\right)a^xb^x-a^{2x}+b^{2x}}{a^xb^x\left(\log\left(a\right)-\log\left(b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
```

```
[Out] -(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) - l
og(b)))
```

giac [C] time = 0.94, size = 436, normalized size = 12.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) -
1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*s
gn(b) + 2*log(abs(a)) - 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*
pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*
e^(x*(log(abs(a)) - log(abs(b)))) - 2*(2*(log(abs(a)) - log(abs(b)))*cos(1/
2*pi*x*sgn(a) - 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)
) - log(abs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(1/2*pi*x*sgn(a) - 1/2*pi*
x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^
(-x*(log(abs(a)) - log(abs(b)))) - 1/2*I*(2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*
pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) - 2*log(abs(a)) + 2*log(abs(b))) -
2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b)
- 2*log(abs(a)) + 2*log(abs(b))))*e^(-x*(log(abs(a)) - log(abs(b)))) - 2*x
```

maple [A] time = 0.05, size = 42, normalized size = 1.24

method	result	size
risch	$-2x+\frac{a^xb^{-x}}{\ln(a)-\ln(b)}-\frac{b^xa^{-x}}{\ln(a)-\ln(b)}$	42
norman	$\left(\frac{e^{2x\ln(a)}}{\ln(a)-\ln(b)}-\frac{e^{2x\ln(b)}}{\ln(a)-\ln(b)}-2x\,e^{x\ln(a)}e^{x\ln(b)}\right)e^{-x\ln(a)}e^{-x\ln(b)}$	65
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x-b^x)^2/(a^x)/(b^x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+a^x/(b^x)/(ln(a)-ln(b))-b^x/(a^x)/(ln(a)-ln(b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?` f
or more details)Is -log(b)/log(a) equal to -1?
```

mupad [B] time = 0.49, size = 34, normalized size = 1.00

$$\frac{\frac{a^x}{b^x}-\frac{b^x}{a^x}}{\ln(a)-\ln(b)}-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x - b^x)^2/(a^x*b^x),x)
```

```
[Out] (a^x/b^x - b^x/a^x)/(log(a) - log(b)) - 2*x
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

```
[Out] Exception raised: TypeError
```

3.496 $\int (-e^{-x}+e^x)\,dx$

Optimal. Leaf size=9

$$e^{-x}+e^x$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2194}

$$e^{-x}+e^x$$

Antiderivative was successfully verified.

```
[In] Int[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (-e^{-x}+e^x)\,dx &= -\int e^{-x}\,dx+\int e^x\,dx\\ &= e^{-x}+e^x\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$e^{-x}+e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-e^{-x}+e^x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-E^(-x) + E^x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.11, size = 11, normalized size = 1.22

$$\left(e^{(2x)}+1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
```

```
[Out] (e^(2*x) + 1)*e^(-x)
```

giac [A] time = 0.60, size = 7, normalized size = 0.78

$$e^{(-x)}+e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="giac")
```

```
[Out] e^(-x) + e^x
```

maple [A] time = 0.03, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$e^{-x}+e^{x}$	8
default	$e^{-x}+e^{x}$	8
risch	$e^{-x}+e^{x}$	8
meijerg	$-2+e^{-x}+e^{x}$	9
norman	$\left(1+e^{2x}\right)e^{-x}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/exp(x)+exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/exp(x)+exp(x)
```

maxima [A] time = 0.55, size = 7, normalized size = 0.78

$$e^{(-x)}+e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
```

```
[Out] e^(-x) + e^x
```

mupad [B] time = 0.05, size = 4, normalized size = 0.44

$$2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x) - exp(-x),x)
```

```
[Out] 2*cosh(x)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.78

$$e^{x}+e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x)
```

```
[Out] exp(x) + exp(-x)
```

3.497 $\int (-e^{-x} + e^x)^2 \, dx$

Optimal. Leaf size=22

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 266, 43}

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (-e^{-x} + e^x)^2 \, dx &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3} \, dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2} \, dx, x, e^{2x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1 + \frac{1}{x^2} - \frac{2}{x}\right) dx, x, e^{2x}\right)\\
&= -\frac{1}{2}e^{-2x} + \frac{e^{2x}}{2} - 2x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/2*1/E^(2*x) + E^(2*x)/2 - 2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-e^{-x} + e^x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-E^(-x) + E^x)^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.75, size = 21, normalized size = 0.95

$$-\frac{1}{2}\left(4\,xe^{(2\,x)} - e^{(4\,x)} + 1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

giac [A] time = 0.58, size = 24, normalized size = 1.09

$$\frac{1}{2}\left(2\,e^{(2\,x)} - 1\right)e^{(-2\,x)} - 2\,x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

maple [A] time = 0.03, size = 17, normalized size = 0.77

method	result	size
risch	$-2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$	17
derivativedivides	$\frac{e^{2x}}{2} - 2\ln\left(e^{x}\right) - \frac{e^{-2x}}{2}$	19
default	$\frac{e^{2x}}{2} - 2\ln\left(e^{x}\right) - \frac{e^{-2x}}{2}$	19
norman	$\left(-\frac{1}{2} + \frac{e^{4x}}{2} - 2x\,e^{2x}\right)e^{-2x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+1/2*exp(2*x)-1/2*exp(-2*x)
```

maxima [A] time = 0.52, size = 16, normalized size = 0.73

$$-2\,x + \frac{1}{2}\,e^{(2\,x)} - \frac{1}{2}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

mupad [B] time = 0.06, size = 8, normalized size = 0.36

$$\sinh(2x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x) - exp(x))^2,x)
```

```
[Out] sinh(2*x) - 2*x
```

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$-2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**2,x)
```

```
[Out] -2*x + exp(2*x)/2 - exp(-2*x)/2
```

3.498 $\int (-e^{-x}+e^x)^3\,dx$

Optimal. Leaf size=31

$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 270}

$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^3,x]
```

```
[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (-e^{-x}+e^x)^3\,dx &= \text{Subst}\left(\int \frac{(-1+x^2)^3}{x^4}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(-3-\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx,x,e^x\right)\\
&= \frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{1}{3}e^{-3x}\left(-9e^{2x}-9e^{4x}+e^{6x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^3,x]
```

```
[Out] (1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-e^{-x}+e^x)^3\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-E^(-x) + E^x)^3,x]
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 24, normalized size = 0.77

$$\frac{1}{3} \left(e^{(6x)} - 9e^{(4x)} - 9e^{(2x)} + 1\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")
[Out] 1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

giac [A] time = 0.62, size = 25, normalized size = 0.81

$$-\frac{1}{3} \left(9e^{(2x)} - 1\right)e^{(-3x)} + \frac{1}{3} e^{(3x)} - 3e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="giac")
[Out] -1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

maple [A] time = 0.04, size = 24, normalized size = 0.77

method	result	size
derivativedivides	$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$	24
default	$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$	24
risch	$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$	24
meijerg	$\frac{16}{3} + \frac{e^{-3x}}{3} - 3e^{-x} - 3e^{x} + \frac{e^{3x}}{3}$	25
norman	$\left(\frac{1}{3} - 3e^{2x} - 3e^{4x} + \frac{e^{6x}}{3}\right)e^{-3x}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/3*exp(x)^3-3*exp(x)-3/exp(x)+1/3/exp(x)^3
```

maxima [A] time = 0.54, size = 23, normalized size = 0.74

$$\frac{1}{3} e^{(3x)} - 3e^{(-x)} + \frac{1}{3} e^{(-3x)} - 3e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")
[Out] 1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

mupad [B] time = 0.31, size = 23, normalized size = 0.74

$$\frac{e^{-3x}}{3} - 3e^{-x} + \frac{e^{3x}}{3} - 3e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(-x) - exp(x))^3,x)
```

```
[Out] exp(-3*x)/3 - 3*exp(-x) + exp(3*x)/3 - 3*exp(x)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.77

$$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**3,x)
```

```
[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

3.499 $\int (-e^{-x}+e^x)^4\,dx$

Optimal. Leaf size=36

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 266, 43}

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^4,x]
```

```
[Out] -1/(4*E^(4*x)) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int (-e^{-x}+e^x)^4\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx,x,e^{2x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx,x,e^{2x}\right)\\ &= -\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{1}{4}\left(24x-e^{-4x}+8e^{-2x}-8e^{2x}+e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^4,x]
```

```
[Out] (-E^(-4*x) + 8/E^(2*x) - 8*E^(2*x) + E^(4*x) + 24*x)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-e^{-x} + e^x)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-E^(-x) + E^x)^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 31, normalized size = 0.86

$$\frac{1}{4} \left(24 \, x e^{(4 \, x)} + e^{(8 \, x)} - 8 \, e^{(6 \, x)} + 8 \, e^{(2 \, x)} - 1\right) e^{(-4 \, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

giac [A] time = 0.59, size = 36, normalized size = 1.00

$$-\frac{1}{4} \left(18 \, e^{(4 \, x)} - 8 \, e^{(2 \, x)} + 1\right) e^{(-4 \, x)} + 6 \, x + \frac{1}{4} \, e^{(4 \, x)} - 2 \, e^{(2 \, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="giac")
```

```
[Out] -1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) - 2*e^(2*x)
```

maple [A] time = 0.04, size = 29, normalized size = 0.81

method	result	size
risch	$6x + \frac{e^{4x}}{4} - 2\,e^{2x} + 2\,e^{-2x} - \frac{e^{-4x}}{4}$	29
derivativedivides	$\frac{e^{4x}}{4} - 2\,e^{2x} + 2\,e^{-2x} - \frac{e^{-4x}}{4} + 6\ln\left(e^{x}\right)$	31
default	$\frac{e^{4x}}{4} - 2\,e^{2x} + 2\,e^{-2x} - \frac{e^{-4x}}{4} + 6\ln\left(e^{x}\right)$	31
norman	$\left(-\frac{1}{4} + 2\,e^{2x} - 2\,e^{6x} + \frac{e^{8x}}{4} + 6x\,e^{4x}\right) e^{-4x}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 6*x+1/4*exp(4*x)-2*exp(2*x)+2*exp(-2*x)-1/4*exp(-4*x)
```

maxima [A] time = 0.50, size = 28, normalized size = 0.78

$$6 \, x + \frac{1}{4} \, e^{(4 \, x)} - 2 \, e^{(2 \, x)} + 2 \, e^{(-2 \, x)} - \frac{1}{4} \, e^{(-4 \, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")
```

```
[Out] 6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

mupad [B] time = 0.34, size = 28, normalized size = 0.78

$$6\,x+2\,{\mathrm{e}}^{-2\,x}-2\,{\mathrm{e}}^{2\,x}-\frac{{\mathrm{e}}^{-4\,x}}{4}+\frac{{\mathrm{e}}^{4\,x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x) - exp(x))^4,x)
[Out] 6*x + 2*exp(-2*x) - 2*exp(2*x) - exp(-4*x)/4 + exp(4*x)/4
```

sympy [A] time = 0.14, size = 31, normalized size = 0.86

$$6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**4,x)
[Out] 6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

3.500 $\int (-e^{-x}+e^{x})^n \, dx$

Optimal. Leaf size=48

$$-\frac{\left(1-e^{2x}\right)(e^x-e^{-x})^n \, {}_2F_1\left(1,\frac{n+2}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2282, 2032, 365, 364}

$$-\frac{(e^x-e^{-x})^n \left(1-e^{2x}\right)^{-n} \text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^n,x]
```

```
[Out] -(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^(2*x)])/((1 - E
^(2*x))^n*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \left(-e^{-x}+e^x\right)^n\,dx = \text{Subst}\left(\int \frac{\left(-\frac{1}{x}+x\right)^n}{x}\,dx, x, e^x\right)$$
$$= \left((e^x)^n\left(-e^{-x}+e^x\right)^n\left(-1+e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(-1+x^2\right)^n\,dx, x, e^x\right)$$
$$= \left((e^x)^n\left(-e^{-x}+e^x\right)^n\left(1-e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(1-x^2\right)^n\,dx, x, e^x\right)$$
$$= -\frac{\left(-e^{-x}+e^x\right)^n\left(1-e^{2x}\right)^{-n}\,{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.94

$$\frac{\left(e^{2x}-1\right)\left(e^x-e^{-x}\right)^n\,{}_2F_1\left(1,\frac{n}{2}+1;1-\frac{n}{2};e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^n,x]
[Out] ((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E^
(2*x)])/n
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^{-x}+e^x\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-E^(-x) + E^x)^n,x]
[Out] Could not integrate
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-e^{(-x)}+e^x\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")
[Out] integral((-e^(-x) + e^x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^{(-x)}+e^x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="giac")
[Out] integrate((-e^(-x) + e^x)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-e^{-x}+e^x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^n,x)
```

```
[Out] int((-1/exp(x)+exp(x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^{(-x)} + e^{x}\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-e^(-x) + e^x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\mathrm{e}^{x} - \mathrm{e}^{-x})^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x) - exp(-x))^n,x)
```

```
[Out] int((exp(x) - exp(-x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e^{x} - e^{-x})^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**n,x)
```

```
[Out] Integral((exp(x) - exp(-x))**n, x)
```

3.501 $\int \left(a^{-4x}-a^{2x}\right)^3 \, dx$

Optimal. Leaf size=43

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2282, 266, 43}

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*Log[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{-4x}-a^{2x}\right)^3 \, dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^3\right)^3}{x^7} \, dx,x,a^{2x}\right)}{2\log(a)}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3} \, dx,x,a^{6x}\right)}{6\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right) \, dx,x,a^{6x}\right)}{6\log(a)}\\
&= 3x-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.77

$$-\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36x\log(a)}{12\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-4*x) - a^(2*x))^3,x]
[Out] -1/12*(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*x*Log[a])/Log[a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(-4*x) - a^(2*x))^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.14, size = 39, normalized size = 0.91

$$\frac{36\,a^{12\,x}x\log(a)-2\,a^{18\,x}+6\,a^{6\,x}-1}{12\,a^{12\,x}\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")
[Out] 1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*log(a))
```

giac [A] time = 0.60, size = 46, normalized size = 1.07

$$-\frac{2\,a^{6\,x}+\frac{9\,a^{12\,x}-6\,a^{6\,x}+1}{a^{12\,x}}-6\,\log\left(a^{6\,x}\right)}{12\,\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="giac")
[Out] -1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*log(a^(6*x)))/
log(a)
```

maple [A] time = 0.07, size = 44, normalized size = 1.02

method	result	size
risch	$3x-\frac{a^{6x}}{6\ln(a)}+\frac{a^{-6x}}{2\ln(a)}-\frac{a^{-12x}}{12\ln(a)}$	44
norman	$\left(-\frac{1}{12\ln(a)}+3x\,\mathrm{e}^{12x\ln(a)}+\frac{\mathrm{e}^{6x\ln(a)}}{2\ln(a)}-\frac{\mathrm{e}^{18x\ln(a)}}{6\ln(a)}\right)\mathrm{e}^{-12x\ln(a)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a^(4*x))-a^(2*x))^3,x,method=_RETURNVERBOSE)
[Out] 3*x-1/6/ln(a)*(a^(2*x))^3+1/2/ln(a)/(a^(2*x))^3-1/12/ln(a)/(a^(2*x))^6
```

maxima [A] time = 0.56, size = 41, normalized size = 0.95

$$3\,x-\frac{a^{6\,x}}{6\,\log(a)}-\frac{1}{12\,a^{12\,x}\log(a)}+\frac{1}{2\,a^{6\,x}\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")
```

```
[Out] 3*x - 1/6*a^(6*x)/log(a) - 1/12/(a^(12*x)*log(a)) + 1/2/(a^(6*x)*log(a))
```

mupad [B] time = 0.41, size = 41, normalized size = 0.95

$$3\,x+\frac{1}{2\,a^{6\,x}\,\ln(a)}-\frac{a^{6\,x}}{6\,\ln(a)}-\frac{1}{12\,a^{12\,x}\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^(2*x) - 1/a^(4*x))^3,x)
```

```
[Out] 3*x + 1/(2*a^(6*x)*log(a)) - a^(6*x)/(6*log(a)) - 1/(12*a^(12*x)*log(a))
```

sympy [A] time = 0.22, size = 54, normalized size = 1.26

$$3x+\begin{cases}\frac{-24a^{6x}\log{(a)}^2+72a^{-6x}\log{(a)}^2-12a^{-12x}\log{(a)}^2}{144\log{(a)}^3} & \text{for}\ 144\log{(a)}^3\neq 0\\ -3x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

```
[Out] 3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*a**(-6*x)*log(a)**2 - 12*a**(
-12*x)*log(a)**2)/(144*log(a)**3), Ne(144*log(a)**3, 0)), (-3*x, True))
```

3.502 $\int (a^{kx} + a^{lx}) \, dx$

Optimal. Leaf size=27

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2194}

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx}) \, dx &= \int a^{kx} \, dx + \int a^{lx} \, dx \\ &= \frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{kx} + a^{lx}) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[a^(k*x) + a^(l*x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.92, size = 26, normalized size = 0.96

$$\frac{a^{lx}k + a^{kx}l}{kl \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")
```

```
[Out] (a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

giac [A] time = 0.63, size = 27, normalized size = 1.00

$$\frac{a^{kx}}{k\log(a)} + \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

maple [A] time = 0.04, size = 28, normalized size = 1.04

method	result	size
default	$\frac{a^{kx}}{k\ln(a)} + \frac{a^{lx}}{l\ln(a)}$	28
risch	$\frac{a^{kx}}{k\ln(a)} + \frac{a^{lx}}{l\ln(a)}$	28
norman	$\frac{e^{kx\ln(a)}}{k\ln(a)} + \frac{e^{lx\ln(a)}}{l\ln(a)}$	30
meijerg	$-\frac{1-e^{kx\ln(a)}}{k\ln(a)} - \frac{1-e^{lx\ln(a)}}{l\ln(a)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x)+a^(l*x),x,method=_RETURNVERBOSE)
```

```
[Out] a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

maxima [A] time = 0.63, size = 27, normalized size = 1.00

$$\frac{a^{kx}}{k\log(a)} + \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

mupad [B] time = 0.36, size = 26, normalized size = 0.96

$$\frac{a^{k\,x}\,l + a^{l\,x}\,k}{k\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x) + a^(l*x),x)
```

```
[Out] (a^(k*x)*l + a^(l*x)*k)/(k*l*log(a))
```

sympy [A] time = 0.26, size = 29, normalized size = 1.07

$$\begin{cases} \frac{a^{kx}}{k\log{(a)}} & \text{for}\: k\log{(a)} \neq 0 \\ x & \text{otherwise} \end{cases} + \begin{cases} \frac{a^{lx}}{l\log{(a)}} & \text{for}\: l\log{(a)} \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(k*x)+a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

3.503 $\int \left(a^{kx}+a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right)^2 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^2 \, dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}+2e^{(k+l)x}\right) \, dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{2kx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{2\,\text{Subst}\left(\int e^{(k+l)x} \, dx, x, x\log\right.}{\log(a)}\\ &= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.77, size = 62, normalized size = 1.17

$$\frac{4\, a^{kx} a^{lx} kl + \left(kl + l^2\right) a^{2\,kx} + \left(k^2 + kl\right) a^{2\,lx}}{2\,\left(k^2 l + kl^2\right) \log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x))
/((k^2*l + k*l^2)*log(a))
```

giac [C] time = 0.73, size = 691, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) + 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) - 1/2*I*(-4*I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/
2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l
 + 2*k*log(abs(a)) + 2*l*log(abs(a))) + 4*I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I
*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(
a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a)
) + l*log(abs(a)))*x)
```

maple [A] time = 0.04, size = 55, normalized size = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln\left(a\right)}+\frac{a^{2lx}}{2l\ln\left(a\right)}+\frac{2a^{kx}a^{lx}}{\ln\left(a\right)\left(k+l\right)}$	55
norman	$\frac{{\mathrm e}^{2kx\ln\left(a\right)}}{2k\ln\left(a\right)}+\frac{{\mathrm e}^{2lx\ln\left(a\right)}}{2l\ln\left(a\right)}+\frac{2\,{\mathrm e}^{kx\ln\left(a\right)}{\mathrm e}^{lx\ln\left(a\right)}}{\ln\left(a\right)\left(k+l\right)}$	59

meijerg	error in int/gbinthm/express: improper op or subscript selector\	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2+2/ln(a)/(k+l)*a^(k*x)*a^(l*
x)
```

maxima [A] time = 0.45, size = 51, normalized size = 0.96

$$\frac{2\,a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2\,kx}}{2\,k\log(a)}+\frac{a^{2\,lx}}{2\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] 2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x)
/(l*log(a))
```

mupad [B] time = 0.38, size = 68, normalized size = 1.28

$$\frac{a^{2\,k\,x}}{2\,k\,\ln(a)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}+l\,\left(2\,a^{k\,x+l\,x}\,k+\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln(a)\,(k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) + a^(l*x))^2,x)
```

```
[Out] a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 + l*(2*a^(k*x + l*x)*k + (a^(2*
l*x)*k)/2))/(k*l*log(a)*(k + l))
```

sympy [A] time = 2.07, size = 250, normalized size = 4.72

$$\begin{cases} 4x & \text{for}\ a = \\ \frac{a^{2lx}}{2l\log(a)}+\frac{2a^{lx}}{l\log(a)}+x & \text{for}\ k = \\ \frac{a^{2lx}}{2l\log(a)}+2x-\frac{a^{-2lx}}{2l\log(a)} & \text{for}\ k = \\ \frac{a^{2kx}}{2k\log(a)}+\frac{2a^{kx}}{k\log(a)}+x & \text{for}\ l = \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**2,x)
```

```
[Out] Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x
)/(2*l*log(a)) + 2*x - a**(-2*l*x)/(2*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(2
*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*
l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*l
og(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(
2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*
log(a) + 2*k*l**2*log(a)), True))
```

3.504 $\int \left(a^{kx}+a^{lx}\right)^3 \, dx$

Optimal. Leaf size=79

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right)^3 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^3 \, dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{3kx}+e^{3lx}+3e^{(2k+l)x}+3e^{(k+2l)x}\right) dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{3kx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{3lx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{3\,\text{Subst}\left(\int e^{(2k+l)x}\,dx,x,x\right.}{\log(a)}\\ &= \frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 65, normalized size = 0.82

$$\frac{\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx}+a^{lx}\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 130, normalized size = 1.65

$$\frac{9\left(2k^2l+kl^2\right)a^{kx}a^{2lx}+9\left(k^2l+2kl^2\right)a^{2kx}a^{lx}+\left(2k^2l+5kl^2+2l^3\right)a^{3kx}+\left(2k^3+5k^2l+2kl^2\right)a^{3lx}}{3\left(2k^3l+5k^2l^2+2kl^3\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

giac [C] time = 0.82, size = 1033, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) + 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
(3*l*x) - 1/2*I*abs(a)^(3*k*x)*(-2*I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) + 2*I*e^(-3/2*I*pi*k*x*sgn(
a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) - 1/2*I
*abs(a)^(3*l*x)*(-2*I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(
a) - 3*I*pi*l + 6*l*log(abs(a))) + 2*I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l
*x)/(-3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) + 6*(2*(2*k*log(abs(a)
) + l*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi
*l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a))
 + l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-p
i*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + p
i*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^(
(2*k*log(abs(a)) + l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + 1/2
*I*pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a
) - 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) + 6*I*e^(-I*pi*k
*x*sgn(a) - 1/2*I*pi*l*x*sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a
) - I*pi*l*sgn(a) + 2*I*pi*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))
*e^((2*k*log(abs(a)) + l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k
*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs
(a)))^2) - (pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sg
n(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) -
 pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a
)) + 2*l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sg
```

```
n(a) - 1/2*I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k -
 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))) + 6*I*e^(-1/2*I*pi*k*x*sgn(a
) - I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*s
gn(a) + I*pi*k + 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(a
bs(a)) + 2*l*log(abs(a)))*x)
```

maple [A] time = 0.07, size = 84, normalized size = 1.06

method	result	size
risch	$\frac{a^{3kx}}{3k\ln(a)}+\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}+\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	84
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)}+\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}+\frac{3\,e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	90
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/k/ln(a)*(a^(k*x))^3+1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a^
(l*x))^2+3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

maxima [A] time = 0.55, size = 77, normalized size = 0.97

$$\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log(a)}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log(a)}+\frac{a^{3\,kx}}{3\,k\log(a)}+\frac{a^{3\,lx}}{3\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] 3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a))
 + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

mupad [B] time = 0.39, size = 81, normalized size = 1.03

$$\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln(a)+2\,l\,\ln(a)}+\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln(a)+l\,\ln(a)}+\frac{a^{3\,k\,x}}{3\,k\,\ln(a)}+\frac{a^{3\,l\,x}}{3\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) + a^(l*x))^3,x)
```

```
[Out] (3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) + (3*a^(2*k*x)*a^(l*x))/(2*k*
log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) + a^(3*l*x)/(3*l*log(a))
```

sympy [A] time = 23.44, size = 665, normalized size = 8.42

$$\begin{cases} 8x \\ \frac{a^{3lx}}{3l\log(a)}+\frac{3a^{2lx}}{2l\log(a)}+\frac{3a^{lx}}{l\log(a)}+x \\ \frac{a^{3lx}}{3l\log(a)}+3x-\frac{a^{-3lx}}{l\log(a)}-\frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)}+\frac{a^{3lx}}{3l\log(a)}+3x-\frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)}+\frac{3a^{2kx}}{2k\log(a)}+\frac{3a^{kx}}{k\log(a)}+x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{9}{6k^3l\log(a)+15}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**3,x)
```

```
[Out] Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(a)
) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - a**(-3*l*x)/(l*log(a)) -
 a**(-6*l*x)/(6*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) + a**(3
*l*x)/(3*l*log(a)) + 3*x - 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**
(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) +
x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*
k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*
l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l
**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(
a) + 6*k*l**3*log(a)), True))
```

3.505 $\int \left(a^{kx}+a^{lx}\right)^4 \, dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) + (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}+a^{lx}\right)^4 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^4 \, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}+4e^{(3k+l)x}+4e^{(k+3l)x}\right) dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{4kx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{4lx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{4\,\text{Subst}\left(\int e^{(3k+l)x} \, dx, x, x\right.}{\log(a)}\\
&= \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}+\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}+\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}+\frac{16a^{x(3k+l)}}{3k+l}+\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k + l
)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.34, size = 205, normalized size = 2.09

$$\frac{16\left(3\,k^3l+4\,k^2l^2+kl^3\right)a^{kx}a^{3\,lx}+12\left(3\,k^3l+10\,k^2l^2+3\,kl^3\right)a^{2\,kx}a^{2\,lx}+16\left(k^3l+4\,k^2l^2+3\,kl^3\right)a^{3\,kx}a^{lx}+\left(3\,k^3l+1\right.}{4\left(3\,k^4l+13\,k^3l^2+13\,k^2l^3+3\,kl^4\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k^3*l + 10*
k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*a
^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) + (3
*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2 +
 13*k^2*l^3 + 3*k*l^4)*log(a))
```

giac [C] time = 0.94, size = 1359, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) + 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) - 1/2*I*(-8*I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3
/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I
*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) + 8*I*e^(-3/2*I*pi*k*x*sgn(a) -
1/2*I*pi*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi
*l*sgn(a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k
*log(abs(a)) + l*log(abs(a)))*x) + 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*c
os(-1/2*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k
*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs
(a)))^2) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sg
n(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*
```

```
sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*l
og(abs(a)) + 3*l*log(abs(a)))*x) - 1/2*I*(-8*I*e^(1/2*I*pi*k*x*sgn(a) + 3/2
*I*pi*l*x*sgn(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*s
gn(a) - I*pi*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) + 8*I*e^(-1/
2*I*pi*k*x*sgn(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*
pi*k*sgn(a) - 3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*l
og(abs(a))))*e^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a))
+ l*log(abs(a)))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi
*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a))
)^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*
x*sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 +
4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*
x) - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x
)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*
log(abs(a))) + 6*I*e^(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*
l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) +
2*l*log(abs(a))))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

maple [A] time = 0.03, size = 109, normalized size = 1.11

method	result	size
risch	$\frac{a^{4kx}}{4k\ln(a)}+\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}+\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$	109
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

maxima [A] time = 0.55, size = 99, normalized size = 1.01

$$\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)}+\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)}+\frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)}+\frac{a^{4\,kx}}{4\,k\log(a)}+\frac{a^{4\,lx}}{4\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] 4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a))
 + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a^
(4*l*x)/(l*log(a))
```

mupad [B] time = 0.42, size = 106, normalized size = 1.08

$$\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln(a)+l\,\ln(a)}+\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln(a)+3\,l\,\ln(a)}+\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln(a)+l\,\ln(a)}+\frac{a^{4\,k\,x}}{4\,k\,\ln(a)}+\frac{a^{4\,l\,x}}{4\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) + a^(l*x))^4,x)
```

```
[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) + (4*a^(k*x)*a^(3*l*x))/(k*lo
g(a) + 3*l*log(a)) + (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k
*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**4,x)
```

```
[Out] Timed out
```

3.506 $\int \left(a^{kx}+a^{lx}\right)^n\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^{x(k-l)}+1\right)\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;-a^{(k-l)x}\right)}{ln\log(a)}$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2285, 2251}

$$\frac{\left(a^{x(-(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},-a^{x(-(k-l))}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right)^n\,dx &= \left(a^{-knx}\left(1+a^{-(k-l)x}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n\right)\int a^{knx}\left(1+a^{-(k-l)x}\right)^n\,dx\\ &= \frac{\left(1+a^{-(k-l)x}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};-a^{-(k-l)x}\right)}{kn\log(a)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.01

$$\frac{\left(a^{x(l-k)}+1\right)\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{l-k}+n+1;\frac{kn}{l-k}+1;-a^{(l-k)x}\right)}{kn\log(a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*(1 + a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), -a^((-k + l)*x)])/(k*n*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^n dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) + a^(l*x))^n,x]
[Out] Could not integrate
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^{kx} + a^{lx}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")
[Out] integral((a^(k*x) + a^(l*x))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="giac")
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^n,x)
[Out] int((a^(k*x)+a^(l*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^{k\,x} + a^{l\,x}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) + a^(l*x))^n,x)
[Out] int((a^(k*x) + a^(l*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**n,x)
[Out] Integral((a**(k*x) + a**(l*x))**n, x)
```

3.507 $\int \left(a^{kx}-a^{lx}\right) dx$

Optimal. Leaf size=28

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2194}

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(k*x) - a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right) dx &= \int a^{kx}\,dx-\int a^{lx}\,dx\\ &= \frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(k*x) - a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx}-a^{lx}\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[a^(k*x) - a^(l*x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.66, size = 28, normalized size = 1.00

$$-\frac{a^{lx}k-a^{kx}l}{kl\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")
```

```
[Out] -(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

giac [A] time = 0.58, size = 28, normalized size = 1.00

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

maple [A] time = 0.03, size = 29, normalized size = 1.04

method	result	size
default	$\frac{a^{kx}}{k\ln(a)} - \frac{a^{lx}}{l\ln(a)}$	29
risch	$\frac{a^{kx}}{k\ln(a)} - \frac{a^{lx}}{l\ln(a)}$	29
norman	$\frac{e^{kx\ln(a)}}{k\ln(a)} - \frac{e^{lx\ln(a)}}{l\ln(a)}$	31
meijerg	$-\frac{1-e^{kx\ln(a)}}{k\ln(a)} + \frac{1-e^{lx\ln(a)}}{l\ln(a)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x)-a^(l*x),x,method=_RETURNVERBOSE)
```

```
[Out] a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

maxima [A] time = 0.45, size = 28, normalized size = 1.00

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

mupad [B] time = 0.31, size = 27, normalized size = 0.96

$$\frac{a^{k\,x}\,l - a^{l\,x}\,k}{k\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x) - a^(l*x),x)
```

```
[Out] (a^(k*x)*l - a^(l*x)*k)/(k*l*log(a))
```

sympy [A] time = 0.27, size = 29, normalized size = 1.04

$$\begin{cases} \frac{a^{kx}}{k\log{(a)}} & \text{for}\: k\log{(a)} \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l\log{(a)}} & \text{for}\: l\log{(a)} \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(k*x)-a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

3.508 $\int \left(a^{kx}-a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right)^2 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^2 \, dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}-2e^{(k+l)x}\right) \, dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{2kx} \, dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx} \, dx,x,x\log(a)\right)}{\log(a)}-\frac{2\,\text{Subst}\left(\int e^{(k+l)x} \, dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.00

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.38, size = 64, normalized size = 1.21

$$-\frac{4 a^{kx} a^{lx} kl - \left(kl + l^2\right) a^{2 kx} - \left(k^2 + kl\right) a^{2 lx}}{2 \left(k^2 l + kl^2\right) \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x)
)/((k^2*l + k*l^2)*log(a))
```

giac [C] time = 0.75, size = 691, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) - 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) - 1/2*I*(4*I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/2
*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l
+ 2*k*log(abs(a)) + 2*l*log(abs(a))) - 4*I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I*
pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a
) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a))
 + l*log(abs(a)))*x)
```

maple [A] time = 0.04, size = 55, normalized size = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln(a)} + \frac{a^{2lx}}{2l\ln(a)} - \frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx\ln(a)}}{2k\ln(a)} + \frac{e^{2lx\ln(a)}}{2l\ln(a)} - \frac{2\,e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$	59

meijerg	error in int/gbinthm/express: improper op or subscript selector\	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2-2/ln(a)/(k+l)*a^(k*x)*a^(l*
x)
```

maxima [A] time = 0.55, size = 51, normalized size = 0.96

$$-\frac{2\,a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2\,kx}}{2\,k\log(a)}+\frac{a^{2\,lx}}{2\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] -2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x
)/(l*log(a))
```

mupad [B] time = 0.36, size = 69, normalized size = 1.30

$$\frac{a^{2\,k\,x}}{2\,k\,\ln(a)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}-l\,\left(2\,a^{k\,x+l\,x}\,k-\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln(a)\,\left(k+l\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) - a^(l*x))^2,x)
```

```
[Out] a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 - l*(2*a^(k*x + l*x)*k - (a^(2*
l*x)*k)/2))/(k*l*log(a)*(k + l))
```

sympy [A] time = 1.98, size = 248, normalized size = 4.68

$$\begin{cases} 0 & \text{for } a = \\ \frac{a^{2lx}}{2l\log(a)} - \frac{2a^{lx}}{l\log(a)} + x & \text{for } k = \\ \frac{a^{2lx}}{2l\log(a)} - 2x - \frac{a^{-2lx}}{2l\log(a)} & \text{for } k = \\ \frac{a^{2kx}}{2k\log(a)} - \frac{2a^{kx}}{k\log(a)} + x & \text{for } l = \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} - \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**2,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a
**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x)/
(2*l*log(a)) - 2*x - a**(-2*l*x)/(2*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(2*k
*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*
log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log
(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*
l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*lo
g(a) + 2*k*l**2*log(a)), True))
```

3.509 $\int \left(a^{kx}-a^{lx}\right)^3 \, dx$

Optimal. Leaf size=79

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}-a^{lx}\right)^3 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^3 \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{3kx}-e^{3lx}-3e^{(2k+l)x}+3e^{(k+2l)x}\right) dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{3kx}\,dx,x,x\log(a)\right)}{\log(a)}-\frac{\text{Subst}\left(\int e^{3lx}\,dx,x,x\log(a)\right)}{\log(a)}-\frac{3\,\text{Subst}\left(\int e^{(2k+l)x}\,dx,x,x\right.}{\log(a)}\\
&= \frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}-\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.84

$$\frac{-\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}-\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.34, size = 131, normalized size = 1.66

$$\frac{9\left(2\,k^2l+kl^2\right)a^{kx}a^{2\,lx}-9\left(k^2l+2\,kl^2\right)a^{2\,kx}a^{lx}+\left(2\,k^2l+5\,kl^2+2\,l^3\right)a^{3\,kx}-\left(2\,k^3+5\,k^2l+2\,kl^2\right)a^{3\,lx}}{3\left(2\,k^3l+5\,k^2l^2+2\,kl^3\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

giac [C] time = 1.28, size = 1033, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) - 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
(3*l*x) - 1/2*I*abs(a)^(3*k*x)*(-2*I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) + 2*I*e^(-3/2*I*pi*k*x*sgn(
a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) - 1/2*I
*abs(a)^(3*l*x)*(2*I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a
) - 3*I*pi*l + 6*l*log(abs(a))) - 2*I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*
x)/(-3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) - 6*(2*(2*k*log(abs(a))
 + l*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*
l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a))
+ l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi
*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi
*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((
2*k*log(abs(a)) + l*log(abs(a)))*x) - 1/2*I*(6*I*e^(I*pi*k*x*sgn(a) + 1/2*I
*pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) - 6*I*e^(-I*pi*k*x
*sgn(a) - 1/2*I*pi*l*x*sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a)
- I*pi*l*sgn(a) + 2*I*pi*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))*e
^((2*k*log(abs(a)) + l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(abs(
a)))*cos(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*s
gn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a
)))^2) - (pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sgn(
a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) - p
i*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a))
 + 2*l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sgn(
```

```
a) - 1/2*I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k - 2
*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))) + 6*I*e^(-1/2*I*pi*k*x*sgn(a)
- I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*sgn
(a) + I*pi*k + 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(abs
(a)) + 2*l*log(abs(a)))*x)
```

maple [A] time = 0.07, size = 84, normalized size = 1.06

method	result	size
risch	$\frac{a^{3kx}}{3k\ln(a)}-\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}-\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	84
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)}-\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}-\frac{3\,e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	90
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/k/ln(a)*(a^(k*x))^3-1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a^
(l*x))^2-3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

maxima [A] time = 0.53, size = 77, normalized size = 0.97

$$-\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log(a)}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log(a)}+\frac{a^{3\,kx}}{3\,k\log(a)}-\frac{a^{3\,lx}}{3\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] -3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)
) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

mupad [B] time = 0.36, size = 81, normalized size = 1.03

$$\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln(a)+2\,l\,\ln(a)}-\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln(a)+l\,\ln(a)}+\frac{a^{3\,k\,x}}{3\,k\,\ln(a)}-\frac{a^{3\,l\,x}}{3\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) - a^(l*x))^3,x)
```

```
[Out] (3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) - (3*a^(2*k*x)*a^(l*x))/(2*k*
log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) - a^(3*l*x)/(3*l*log(a))
```

sympy [A] time = 23.59, size = 663, normalized size = 8.39

$$\begin{cases} 0 \\ -\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{2lx}}{2l\log(a)}-\frac{3a^{lx}}{l\log(a)}+x \\ -\frac{a^{3lx}}{3l\log(a)}+3x+\frac{a^{-3lx}}{l\log(a)}-\frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)}-\frac{a^{3lx}}{3l\log(a)}-3x-\frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)}-\frac{3a^{2kx}}{2k\log(a)}+\frac{3a^{kx}}{k\log(a)}-x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}-\frac{9}{6k^3l\log(a)+15} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**3,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (-
a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a))
 + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + a**(-3*l*x)/(l*log(a)) -
 a**(-6*l*x)/(6*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) - a**(3
*l*x)/(3*l*log(a)) - 3*x - 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**
(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) -
x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*
k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*
l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l
**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(
a) + 6*k*l**3*log(a)), True))
```

3.510 $\int \left(a^{kx}-a^{lx}\right)^4 \, dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) - (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}-a^{lx}\right)^4 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^4 \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}-4e^{(3k+l)x}-4e^{(k+3l)x}\right) dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{4kx} \, dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{4lx} \, dx,x,x\log(a)\right)}{\log(a)}-\frac{4\,\text{Subst}\left(\int e^{(3k+l)x} \, dx,x,x\right.}{\log(a)}\\
&= \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}-\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}-\frac{16a^{x(3k+l)}}{3k+l}-\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k + l
)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.29, size = 207, normalized size = 2.11

$$-\frac{16\left(3k^3l+4k^2l^2+kl^3\right)a^{kx}a^{3lx}-12\left(3k^3l+10k^2l^2+3kl^3\right)a^{2kx}a^{2lx}+16\left(k^3l+4k^2l^2+3kl^3\right)a^{3kx}a^{lx}-\left(3k^3l+\right.}{4\left(3k^4l+13k^3l^2+13k^2l^3+3kl^4\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*k^3*l + 10
*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*
a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) - (
3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2
+ 13*k^2*l^3 + 3*k*l^4)*log(a))
```

giac [C] time = 1.21, size = 1359, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) - 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) - 1/2*I*(8*I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3/
2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I*
pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) - 8*I*e^(-3/2*I*pi*k*x*sgn(a) - 1
/2*I*pi*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi*
l*sgn(a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k*
log(abs(a)) + l*log(abs(a)))*x) - 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*co
s(-1/2*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*
sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(
a)))^2) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sgn
(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*s
```

```
gn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*lo
g(abs(a)) + 3*l*log(abs(a)))*x) - 1/2*I*(8*I*e^(1/2*I*pi*k*x*sgn(a) + 3/2*I
*pi*l*x*sgn(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*sgn
(a) - I*pi*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) - 8*I*e^(-1/2*
I*pi*k*x*sgn(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*pi
*k*sgn(a) - 3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log
(abs(a))))*e^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) +
l*log(abs(a)))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi*k
*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^
2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*x*
sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*
(k*log(abs(a)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
 - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x)/
(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*lo
g(abs(a))) + 6*I*e^(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*l*
x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*
l*log(abs(a))))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

maple [A] time = 0.02, size = 109, normalized size = 1.11

method	result	size
risch	$\frac{a^{4kx}}{4k\ln(a)}-\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}-\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$	109
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

maxima [A] time = 0.49, size = 99, normalized size = 1.01

$$-\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)}-\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)}+\frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)}+\frac{a^{4\,kx}}{4\,k\log(a)}+\frac{a^{4\,lx}}{4\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] -4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a)
) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a
^(4*l*x)/(l*log(a))
```

mupad [B] time = 0.35, size = 106, normalized size = 1.08

$$\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln(a)+l\,\ln(a)}-\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln(a)+3\,l\,\ln(a)}-\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln(a)+l\,\ln(a)}+\frac{a^{4\,k\,x}}{4\,k\,\ln(a)}+\frac{a^{4\,l\,x}}{4\,l\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) - a^(l*x))^4,x)
```

```
[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) - (4*a^(k*x)*a^(3*l*x))/(k*lo
g(a) + 3*l*log(a)) - (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k
*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**4,x)
```

```
[Out] Timed out
```

3.511 $\int \left(a^{kx}-a^{lx}\right)^n\,dx$

Optimal. Leaf size=74

$$\frac{\left(1-a^{x(k-l)}\right)\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;a^{(k-l)x}\right)}{ln\log(a)}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2285, 2251}

$$\frac{\left(1-a^{x(-(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{x(-(k-l))}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right)^n\,dx &= \left(a^{-knx}\left(1-a^{-(k-l)x}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n\right)\int a^{knx}\left(1-a^{-(k-l)x}\right)^n\,dx\\ &= \frac{\left(1-a^{-(k-l)x}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};a^{-(k-l)x}\right)}{kn\log(a)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.01

$$\frac{\left(1-a^{x(l-k)}\right)\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{l-k}+n+1;\frac{kn}{l-k}+1;a^{(l-k)x}\right)}{kn\log(a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*(1 - a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), a^((-k + l)*x)])/(k*n*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(k*x) - a^(l*x))^n,x]
[Out] Could not integrate
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^{kx} - a^{lx}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")
[Out] integral((a^(k*x) - a^(l*x))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="giac")
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^n,x)
[Out] int((a^(k*x)-a^(l*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="maxima")
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^{k\,x} - a^{l\,x}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x) - a^(l*x))^n,x)
[Out] int((a^(k*x) - a^(l*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**n,x)
```

```
[Out] Integral((a**(k*x) - a**(l*x))**n, x)
```

3.512 $\int (1 + a^{mx})\,dx$

Optimal. Leaf size=15

$$\frac{a^{mx}}{m\log(a)} + x$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2194}

$$\frac{a^{mx}}{m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (1 + a^{mx})\,dx &= x + \int a^{mx}\,dx\\ &= x + \frac{a^{mx}}{m\log(a)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{a^{mx}}{m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + a^{mx})\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1 + a^(m*x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.33, size = 19, normalized size = 1.27

$$\frac{mx\log(a) + a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="fricas")
```

```
[Out] (m*x*log(a) + a^(m*x))/(m*log(a))
```

giac [A] time = 0.58, size = 15, normalized size = 1.00

$$x + \frac{a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="giac")
[Out] x + a^(m*x)/(m*log(a))
```

maple [A] time = 0.04, size = 16, normalized size = 1.07

method	result	size
default	$x+\frac{a^{mx}}{m\ln(a)}$	16
risch	$x+\frac{a^{mx}}{m\ln(a)}$	16
norman	$x+\frac{e^{mx\ln(a)}}{m\ln(a)}$	17
derivativedivides	$\frac{a^{mx}+\ln(a^{mx})}{m\ln(a)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+a^(m*x),x,method=_RETURNVERBOSE)
[Out] x+a^(m*x)/m/ln(a)
```

maxima [A] time = 0.57, size = 15, normalized size = 1.00

$$x + \frac{a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="maxima")
[Out] x + a^(m*x)/(m*log(a))
```

mupad [B] time = 0.32, size = 15, normalized size = 1.00

$$x + \frac{a^{m\,x}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(m*x) + 1,x)
[Out] x + a^(m*x)/(m*log(a))
```

sympy [A] time = 0.09, size = 15, normalized size = 1.00

$$x + \begin{cases} \frac{a^{mx}}{m\log(a)} & \text{for}\ m\log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a**(m*x),x)
[Out] x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

3.513 $\int (1 + a^{mx})^2 \, dx$

Optimal. Leaf size=33

$$\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^2,x]
```

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1 + a^{mx})^2 \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x} \, dx, x, a^{mx}\right)}{m\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(2 + \frac{1}{x} + x\right) dx, x, a^{mx}\right)}{m\log(a)} \\
&= x + \frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$\frac{2a^{mx} + \frac{1}{2}a^{2mx} + mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^2,x]
```

```
[Out] (2*a^(m*x) + a^(2*m*x)/2 + m*x*Log[a])/(m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1+a^{mx})^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + a^(m*x))^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.17, size = 29, normalized size = 0.88

$$\frac{2\,mx\log(a)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="fricas")
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

giac [A] time = 0.58, size = 30, normalized size = 0.91

$$\frac{2\,mx\log\left(|a|\right)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="giac")
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

maple [A] time = 0.04, size = 32, normalized size = 0.97

method	result	si
derivativedivides	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x+\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x+\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$	35
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/m/ln(a)*(1/2*(a^(m*x))^2+2*a^(m*x)+ln(a^(m*x)))
```

maxima [A] time = 0.53, size = 31, normalized size = 0.94

$$x+\frac{a^{2\,mx}}{2\,m\log(a)}+\frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="maxima")
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

mupad [B] time = 0.32, size = 26, normalized size = 0.79

$$x+\frac{2\,a^{m\,x}+\frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) + 1)^2,x)
```

```
[Out] x + (2*a^(m*x) + a^(2*m*x)/2)/(m*log(a))
```

sympy [A] time = 0.12, size = 46, normalized size = 1.39

$$x+\begin{cases}\frac{a^{2mx}m\log{(a)}+4a^{mx}m\log{(a)}}{2m^2\log{(a)}^2} & \text{for } 2m^2\log{(a)}^2\neq 0\\ 3x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(2*m**2*log(a)**2, 0)), (3*x, True))
```

3.514 $\int (1+a^{mx})^3\,dx$

Optimal. Leaf size=50

$$\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}+\frac{a^{3mx}}{3m\log(a)}+x$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}+\frac{a^{3mx}}{3m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^3,x]
```

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1+a^{mx})^3\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x}\,dx,x,a^{mx}\right)}{m\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x}+3x+x^2\right)dx,x,a^{mx}\right)}{m\log(a)}\\
&= x+\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}+\frac{a^{3mx}}{3m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.70

$$\frac{\left(9a^{mx}+2a^{2mx}+18\right)a^{mx}}{6m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^3,x]
```

```
[Out] x + (a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + a^{mx})^3 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + a^(m*x))^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.27, size = 39, normalized size = 0.78

$$\frac{6\,mx\log(a) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="fricas")
[Out] 1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

giac [A] time = 0.62, size = 40, normalized size = 0.80

$$\frac{6\,mx\log\left(|a|\right) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="giac")
[Out] 1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

maple [A] time = 0.04, size = 41, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
risch	$x+\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}+\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x+\frac{3\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{2m\ln(a)}+\frac{e^{3mx\ln(a)}}{3m\ln(a)}$	52
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^3,x,method=_RETURNVERBOSE)
[Out] 1/m/ln(a)*(1/3*(a^(m*x))^3+3/2*(a^(m*x))^2+3*a^(m*x)+ln(a^(m*x)))
```

maxima [A] time = 0.57, size = 46, normalized size = 0.92

$$x + \frac{a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{2\,m\log(a)} + \frac{3\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="maxima")
```

```
[Out] x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log(
a))
```

mupad [B] time = 0.33, size = 34, normalized size = 0.68

$$x+\frac{3\,a^{m\,x}+\frac{3\,a^{2\,m\,x}}{2}+\frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) + 1)^3,x)
```

```
[Out] x + (3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

sympy [A] time = 0.14, size = 71, normalized size = 1.42

$$x+\begin{cases}\frac{2a^{3mx}m^2\log{(a)}^2+9a^{2mx}m^2\log{(a)}^2+18a^{mx}m^2\log{(a)}^2}{6m^3\log{(a)}^3} & \text{for } 6m^3\log{(a)}^3\neq 0\\ 7x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2 +
 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*m**3*log(a)**3, 0)),
(7*x, True))
```

3.515 $\int (1 + a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$\frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)} + x$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^4,x]
```

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1 + a^{mx})^4 \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x} \, dx, x, a^{mx}\right)}{m\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(4 + \frac{1}{x} + 6x + 4x^2 + x^3\right) dx, x, a^{mx}\right)}{m\log(a)} \\
&= x + \frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.75

$$\frac{4a^{mx} + 3a^{2mx} + \frac{4}{3}a^{3mx} + \frac{1}{4}a^{4mx} + mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^4,x]
```

[Out] (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4 + m*x*Log[a])/(m*Log[a])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1+a^{mx})^4 \, dx$$

Verification is Not applicable to the result.

[In] IntegrateAlgebraic[(1 + a^(m*x))^4,x]

[Out] Could not integrate

fricas [A] time = 1.38, size = 47, normalized size = 0.72

$$\frac{12\,mx\log(a) + 3\,a^{4\,mx} + 16\,a^{3\,mx} + 36\,a^{2\,mx} + 48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^4,x, algorithm="fricas")

[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x))/(m*log(a))

giac [A] time = 0.58, size = 48, normalized size = 0.74

$$\frac{12\,mx\log(|a|) + 3\,a^{4\,mx} + 16\,a^{3\,mx} + 36\,a^{2\,mx} + 48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^4,x, algorithm="giac")

[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x))/(m*log(a))

maple [A] time = 0.04, size = 50, normalized size = 0.77

method	result	si
derivativedivides	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
risch	$x+\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}+\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x+\frac{4\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{m\ln(a)}+\frac{4\,e^{3mx\ln(a)}}{3m\ln(a)}+\frac{e^{4mx\ln(a)}}{4m\ln(a)}$	69
meijerg	error in int/gbinthm/express: improper op or subscript selector\	N

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+a^(m*x))^4,x,method=_RETURNVERBOSE)

[Out] 1/m/ln(a)*(1/4*(a^(m*x))^4+4/3*(a^(m*x))^3+3*(a^(m*x))^2+4*a^(m*x)+ln(a^(m*x)))

maxima [A] time = 0.51, size = 61, normalized size = 0.94

$$x + \frac{a^{4\,mx}}{4\,m\log(a)} + \frac{4\,a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{m\log(a)} + \frac{4\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) + 4*a^(m*x)/(m*log(a))
```

mupad [B] time = 0.33, size = 42, normalized size = 0.65

$$x + \frac{4\,a^{m\,x} + 3\,a^{2\,m\,x} + \frac{4\,a^{3\,m\,x}}{3} + \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) + 1)^4,x)
```

```
[Out] x + (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4)/(m*log(a))
```

sympy [A] time = 0.16, size = 88, normalized size = 1.35

$$x + \begin{cases} \frac{3a^{4mx}m^3\log{(a)}^3+16a^{3mx}m^3\log{(a)}^3+36a^{2mx}m^3\log{(a)}^3+48a^{mx}m^3\log{(a)}^3}{12m^4\log{(a)}^4} & \text{for}\ 12m^4\log{(a)}^4 \neq 0 \\ 15x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(12*m**4*log(a)**4, 0)), (15*x, True))
```

3.516 $\int (1 + a^{mx})^n \, dx$

Optimal. Leaf size=40

$$-\frac{(a^{mx}+1)^{n+1} \, {}_2F_1\left(1,n+1;n+2;a^{mx}+1\right)}{m(n+1)\log(a)}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 65}

$$-\frac{(a^{mx}+1)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,a^{mx}+1\right)}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1+a^{mx})^n \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^n}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\
&= -\frac{(1+a^{mx})^{1+n} \, {}_2F_1\left(1,1+n;2+n;1+a^{mx}\right)}{m(1+n)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{(a^{mx}+1)^{n+1} \, {}_2F_1\left(1,n+1;n+2;a^{mx}+1\right)}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + a^{mx})^n \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + a^(m*x))^n,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left((a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(m*x) + 1)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="giac")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (1 + a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^n,x)
```

```
[Out] int((1+a^(m*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

mupad [B] time = 0.31, size = 55, normalized size = 1.38

$$\frac{(a^{m\,x} + 1)^n \, {}_2F_1\left(-n, -n;\ 1 - n;\ -\frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\left(\frac{1}{a^{m\,x}} + 1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) + 1)^n,x)
```

```
[Out] ((a^(m*x) + 1)^n*hypergeom([-n, -n], 1 - n, -1/a^(m*x)))/(m*n*log(a)*(1/a^(
m*x) + 1)^n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**n,x)
```

```
[Out] Integral((a**(m*x) + 1)**n, x)
```

3.517 $\int (1 - a^{mx})\, dx$

Optimal. Leaf size=16

$$x - \frac{a^{mx}}{m \log(a)}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2194}

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[1 - a^(m*x),x]
```

```
[Out] x - a^(m*x)/(m*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (1 - a^{mx})\, dx &= x - \int a^{mx}\, dx \\ &= x - \frac{a^{mx}}{m \log(a)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - a^(m*x),x]
```

```
[Out] x - a^(m*x)/(m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1 - a^(m*x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.98, size = 21, normalized size = 1.31

$$\frac{mx \log(a) - a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="fricas")
```

```
[Out] (m*x*log(a) - a^(m*x))/(m*log(a))
```

giac [A] time = 0.59, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="giac")
[Out] x - a^(m*x)/(m*log(a))
```

maple [A] time = 0.03, size = 17, normalized size = 1.06

method	result	size
default	$x - \frac{a^{mx}}{m\ln(a)}$	17
risch	$x - \frac{a^{mx}}{m\ln(a)}$	17
norman	$x - \frac{e^{mx\ln(a)}}{m\ln(a)}$	18
derivativedivides	$\frac{-a^{mx}+\ln(a^{mx})}{m\ln(a)}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-a^(m*x),x,method=_RETURNVERBOSE)
[Out] x-a^(m*x)/m/ln(a)
```

maxima [A] time = 0.68, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="maxima")
[Out] x - a^(m*x)/(m*log(a))
```

mupad [B] time = 0.30, size = 16, normalized size = 1.00

$$x - \frac{a^{m\,x}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - a^(m*x),x)
[Out] x - a^(m*x)/(m*log(a))
```

sympy [A] time = 0.10, size = 19, normalized size = 1.19

$$x + \begin{cases} -\frac{a^{mx}}{m\log(a)} & \text{for}\ m\log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a**(m*x),x)
[Out] x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))
```

3.518 $\int (1-a^{mx})^2\,dx$

Optimal. Leaf size=33

$$-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^2,x]
```

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int (1-a^{mx})^2\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx,x,a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx,x,a^{mx}\right)}{m\log(a)}\\ &= x-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.76

$$\frac{(a^{mx}-4)\,a^{mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^2,x]
```

```
[Out] x + (a^(m*x)*(-4 + a^(m*x)))/(2*m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^2 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - a^(m*x))^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.43, size = 29, normalized size = 0.88

$$\frac{2\,mx\log(a) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="fricas")
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

giac [A] time = 0.61, size = 30, normalized size = 0.91

$$\frac{2\,mx\log(|a|) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="giac")
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

maple [A] time = 0.03, size = 32, normalized size = 0.97

method	result	si
derivativedivides	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x-\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x-\frac{2\,e^{mx\ln(a)}}{m\ln(a)}+\frac{e^{2mx\ln(a)}}{2m\ln(a)}$	35
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/m/ln(a)*(1/2*(a^(m*x))^2-2*a^(m*x)+ln(a^(m*x)))
```

maxima [A] time = 0.54, size = 31, normalized size = 0.94

$$x + \frac{a^{2\,mx}}{2\,m\log(a)} - \frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="maxima")
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

mupad [B] time = 0.30, size = 27, normalized size = 0.82

$$x-\frac{2\,a^{m\,x}-\frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) - 1)^2,x)
```

```
[Out] x - (2*a^(m*x) - a^(2*m*x)/2)/(m*log(a))
```

sympy [A] time = 0.12, size = 46, normalized size = 1.39

$$x+\begin{cases}\frac{a^{2mx}m\log{(a)}-4a^{mx}m\log{(a)}}{2m^2\log{(a)}^2} & \text{for } 2m^2\log{(a)}^2\neq 0\\ -x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(2*m**2*log(a)**2, 0)), (-x, True))
```

3.519 $\int (1-a^{mx})^3 \, dx$

Optimal. Leaf size=50

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^3,x]
```

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1-a^{mx})^3 \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(-3+\frac{1}{x}+3x-x^2\right) dx, x, a^{mx}\right)}{m\log(a)}\\
&= x-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.70

$$x-\frac{a^{mx}\left(-9a^{mx}+2a^{2mx}+18\right)}{6m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^3,x]
```

```
[Out] x - (a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/(6*m*Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1-a^{mx})^3 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - a^(m*x))^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.32, size = 39, normalized size = 0.78

$$\frac{6\,mx\log(a) - 2\,a^{3\,mx} + 9\,a^{2\,mx} - 18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="fricas")
[Out] 1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

giac [A] time = 0.57, size = 40, normalized size = 0.80

$$\frac{6\,mx\log\left(|a|\right) - 2\,a^{3\,mx} + 9\,a^{2\,mx} - 18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="giac")
[Out] 1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

maple [A] time = 0.04, size = 41, normalized size = 0.82

method	result	size
derivativedivides	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
risch	$x-\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}-\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x-\frac{3\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{2m\ln(a)}-\frac{e^{3mx\ln(a)}}{3m\ln(a)}$	52
meijerg	error in int/gbinthm/express: improper op or subscript selector\	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^3,x,method=_RETURNVERBOSE)
[Out] 1/m/ln(a)*(-1/3*(a^(m*x))^3+3/2*(a^(m*x))^2-3*a^(m*x)+ln(a^(m*x)))
```

maxima [A] time = 0.50, size = 46, normalized size = 0.92

$$x - \frac{a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{2\,m\log(a)} - \frac{3\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="maxima")
```

```
[Out] x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log(
a))
```

mupad [B]   time = 0.32, size = 35, normalized size = 0.70

$$x - \frac{3\,a^{m\,x} - \frac{3\,a^{2\,m\,x}}{2} + \frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^(m*x) - 1)^3,x)
```

```
[Out] x - (3*a^(m*x) - (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

sympy [A]   time = 0.14, size = 71, normalized size = 1.42

$$x + \begin{cases} \frac{-2a^{3mx}m^2\log{(a)}^2 + 9a^{2mx}m^2\log{(a)}^2 - 18a^{mx}m^2\log{(a)}^2}{6m^3\log{(a)}^3} & \text{for } 6m^3\log{(a)}^3 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
- 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*m**3*log(a)**3, 0)),
 (-x, True))
```

3.520 $\int (1-a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^4,x]
```

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (1-a^{mx})^4 \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x}+6x-4x^2+x^3\right) \, dx, x, a^{mx}\right)}{m\log(a)}\\
&= x-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.75

$$\frac{-4a^{mx}+3a^{2mx}-\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^4,x]
```

```
[Out] (-4*a^(m*x) + 3*a^(2*m*x) - (4*a^(3*m*x))/3 + a^(4*m*x)/4 + m*x*Log[a])/(m*
Log[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^4 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - a^(m*x))^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.18, size = 47, normalized size = 0.72

$$\frac{12\, mx \log(a) + 3\, a^{4\, mx} - 16\, a^{3\, mx} + 36\, a^{2\, mx} - 48\, a^{mx}}{12\, m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*x
))/(m*log(a))
```

giac [A] time = 0.61, size = 48, normalized size = 0.74

$$\frac{12\, mx \log\left(|a|\right) + 3\, a^{4\, mx} - 16\, a^{3\, mx} + 36\, a^{2\, mx} - 48\, a^{mx}}{12\, m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a
^(m*x))/(m*log(a))
```

maple [A] time = 0.04, size = 50, normalized size = 0.77

method	result	si
derivativedivides	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
risch	$x-\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}-\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x-\frac{4\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{m\ln(a)}-\frac{4\,e^{3mx\ln(a)}}{3m\ln(a)}+\frac{e^{4mx\ln(a)}}{4m\ln(a)}$	69
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/m/ln(a)*(1/4*(a^(m*x))^4-4/3*(a^(m*x))^3+3*(a^(m*x))^2-4*a^(m*x)+ln(a^(m*
x)))
```

maxima [A] time = 0.51, size = 61, normalized size = 0.94

$$x + \frac{a^{4\, mx}}{4\, m \log(a)} - \frac{4\, a^{3\, mx}}{3\, m \log(a)} + \frac{3\, a^{2\, mx}}{m \log(a)} - \frac{4\, a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) - 4*a^(m*x)/(m*log(a))
```

mupad [B] time = 0.31, size = 43, normalized size = 0.66

$$x - \frac{4\,a^{m\,x} - 3\,a^{2\,m\,x} + \frac{4\,a^{3\,m\,x}}{3} - \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(m*x) - 1)^4,x)
```

```
[Out] x - (4*a^(m*x) - 3*a^(2*m*x) + (4*a^(3*m*x))/3 - a^(4*m*x)/4)/(m*log(a))
```

sympy [A] time = 0.16, size = 88, normalized size = 1.35

$$x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3 - 16a^{3mx}m^3\log(a)^3 + 36a^{2mx}m^3\log(a)^3 - 48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for}\ 12m^4\log(a)^4 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(12*m**4*log(a)**4, 0)), (-x, True))
```

3.521 $\int (1-a^{mx})^n \, dx$

Optimal. Leaf size=44

$$-\frac{(1-a^{mx})^{n+1} \,_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 65}

$$-\frac{(1-a^{mx})^{n+1} \text{Hypergeometric2F1}(1,n+1,n+2,1-a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int (1-a^{mx})^n \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^n}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\ &= -\frac{(1-a^{mx})^{1+n} \,_2F_1(1,1+n;2+n;1-a^{mx})}{m(1+n)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{(1-a^{mx})^{n+1} \,_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^n \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - a^(m*x))^n,x]
[Out] Could not integrate
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left((-a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="fricas")
[Out] integral((-a^(m*x) + 1)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="giac")
[Out] integrate((-a^(m*x) + 1)^n, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^n,x)
[Out] int((1-a^(m*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="maxima")
[Out] integrate((-a^(m*x) + 1)^n, x)
```

mupad [B] time = 0.32, size = 57, normalized size = 1.30

$$\frac{(1 - a^{m\,x})^n \, {}_2F_1\left(-n, -n;\ 1 - n;\ \frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\left(1 - \frac{1}{a^{m\,x}}\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^(m*x))^n,x)
```

```
[Out] ((1 - a^(m*x))^n*hypergeom([-n, -n], 1 - n, 1/a^(m*x)))/(m*n*log(a)*(1 - 1/
a^(m*x))^n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**n,x)
```

```
[Out] Integral((1 - a**(m*x))**n, x)
```

3.522 $\int \frac{1}{b+ae^{nx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 36, 29, 31}

$$\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*E^(n*x))^(-1),x]
```

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{b+ae^{nx}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{nx}\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{nx}\right)}{bn}-\frac{a\,\text{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{nx}\right)}{bn}\\
&= \frac{x}{b}-\frac{\log\left(b+ae^{nx}\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*E^(n*x))^(-1),x]
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b + ae^{nx}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(b + a*E^(n*x))^(-1),x]
[Out] Could not integrate
```

fricas [A] time = 1.31, size = 22, normalized size = 0.92

$$\frac{nx - \log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")
[Out] (n*x - log(a*e^(n*x) + b))/(b*n)
```

giac [A] time = 0.58, size = 26, normalized size = 1.08

$$\frac{\frac{nx}{b} - \frac{\log\left(\left|ae^{(nx)}+b\right|\right)}{b}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="giac")
[Out] (n*x/b - log(abs(a*e^(n*x) + b))/b)/n
```

maple [A] time = 0.03, size = 24, normalized size = 1.00

method	result	size
norman	$\frac{x}{b}-\frac{\ln(b+a\,\mathrm{e}^{nx})}{bn}$	24
risch	$\frac{x}{b}-\frac{\ln\left(\mathrm{e}^{nx}+\frac{b}{a}\right)}{bn}$	26
derivativedivides	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{nx}\right)}{b}+\frac{\ln\left(\mathrm{e}^{nx}\right)}{b}}{n}$	29
default	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{nx}\right)}{b}+\frac{\ln\left(\mathrm{e}^{nx}\right)}{b}}{n}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b+a*exp(n*x)),x,method=_RETURNVERBOSE)
[Out] x/b-ln(b+a*exp(n*x))/b/n
```

maxima [A] time = 0.45, size = 23, normalized size = 0.96

$$\frac{x}{b} - \frac{\log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")
```

```
[Out] x/b - log(a*e^(n*x) + b)/(b*n)
```

mupad [B] time = 0.33, size = 22, normalized size = 0.92

$$-\frac{\ln\left(b+a\,\mathrm{e}^{n\,x}\right)-n\,x}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + a*exp(n*x)),x)
```

```
[Out] -(log(b + a*exp(n*x)) - n*x)/(b*n)
```

sympy [A] time = 0.12, size = 15, normalized size = 0.62

$$\frac{x}{b}-\frac{\log\left(e^{nx}+\frac{b}{a}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x)
```

```
[Out] x/b - log(exp(n*x) + b/a)/(b*n)
```

3.523 $\int \frac{e^x}{b+ae^{3x}}\,dx$

Optimal. Leaf size=100

$$\frac{\log\left(\sqrt[3]{a}\,e^x+\sqrt[3]{b}\right)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(ae^{3x}+b\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,e^x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2249, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\,\sqrt[3]{b}\,e^x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\,e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,e^x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2
/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) - a^(1
/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x}{b+ae^{3x}}\,dx &= \text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, e^x\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, e^x\right)}{3b^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{3b^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{2\sqrt[3]{b}}\\
&= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 97, normalized size = 0.97

$$
-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)-2\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{6\sqrt[3]{a}b^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/
3) + a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])/(
a^(1/3)*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{e^x}{b+ae^{3x}}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(b + a*E^(3*x)),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.34, size = 311, normalized size = 3.11

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2abe^{(3x)}-3\left(ab^2\right)^{\frac{1}{3}}be^x-b^2+3\sqrt{\frac{1}{3}}\left(2abe^{(2x)}+\left(ab^2\right)^{\frac{2}{3}}e^x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ae^{(3x)}+b}\right)-\left(ab^2\right)^{\frac{2}{3}}\log\left(abe^{(2x)}-\left(ab\right.\right.}{6ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2)
^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*b^
2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b)) - (a*b^2)^(2/3)*log(a*
b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*
e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*a
rctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/
a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)
*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]
```

giac [A] time = 0.63, size = 116, normalized size = 1.16

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right|\right)}{3b}+\frac{\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2e^x\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2x)}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-a^2*b)^(1
/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/3))/(a*b) + 1/6*(-a
^2*b)^(1/3)*log((-b/a)^(1/3)*e^x + (-b/a)^(2/3) + e^(2*x))/(a*b)
```

maple [C] time = 0.06, size = 26, normalized size = 0.26

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(27b^2a\_Z^3-1\right)}\_R\ln\left(3b\_R+\mathrm{e}^x\right)$	26
default	$\frac{\ln\left(\mathrm{e}^x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\mathrm{e}^{2x}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{e}^x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\mathrm{e}^x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(b+a*exp(3*x)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(3*b*_R+exp(x)),_R=RootOf(27*_Z^3*a*b^2-1))
```

maxima [A] time = 1.37, size = 100, normalized size = 1.00

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2e^{x}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}e^{x}+\left(\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2x)}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+e^{x}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*e^x)/(b/a)^(1/3))/(a*(b/a)
^(2/3)) - 1/6*log(-(b/a)^(1/3)*e^x + (b/a)^(2/3) + e^(2*x))/(a*(b/a)^(2/3))
 + 1/3*log((b/a)^(1/3) + e^x)/(a*(b/a)^(2/3))
```

mupad [B] time = 1.51, size = 104, normalized size = 1.04

$$\frac{\ln\left(\frac{b^{1/3}}{a^{7/3}}+\frac{e^{x}}{a^2}\right)}{3a^{1/3}b^{2/3}}+\frac{\ln\left(\frac{e^{x}}{a^2}+\frac{b^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2a^{7/3}}\right)\left(-1+\sqrt{3}\,1i\right)}{6a^{1/3}b^{2/3}}-\frac{\ln\left(\frac{e^{x}}{a^2}-\frac{b^{1/3}\left(1+\sqrt{3}\,1i\right)}{2a^{7/3}}\right)\left(1+\sqrt{3}\,1i\right)}{6a^{1/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(b + a*exp(3*x)),x)
```

```
[Out] log(b^(1/3)/a^(7/3) + exp(x)/a^2)/(3*a^(1/3)*b^(2/3)) + (log(exp(x)/a^2 + (
b^(1/3)*(3^(1/2)*1i - 1))/(2*a^(7/3)))*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)
) - (log(exp(x)/a^2 - (b^(1/3)*(3^(1/2)*1i + 1))/(2*a^(7/3)))*(3^(1/2)*1i +
 1))/(6*a^(1/3)*b^(2/3))
```

sympy [A] time = 0.17, size = 22, normalized size = 0.22

$$\operatorname{RootSum}\left(27z^3ab^2-1,\left(i\mapsto i\log\left(3ib+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

3.524 $\int \frac{-1+e^x}{1+e^x}\,dx$

Optimal. Leaf size=12

$$2\log\left(e^x+1\right)-x$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 72}

$$2\log\left(e^x+1\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + E^x)/(1 + E^x),x]
```

```
[Out] -x + 2*Log[1 + E^x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+e^x}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{-1+x}{x(1+x)}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{1+x}\right)dx,x,e^x\right)\\
&= -x+2\log\left(1+e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$2\log\left(e^x+1\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + E^x)/(1 + E^x),x]
```

```
[Out] -x + 2*Log[1 + E^x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+e^x}{1+e^x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + E^x)/(1 + E^x),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 11, normalized size = 0.92

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")
[Out] -x + 2*log(e^x + 1)
```

giac [A] time = 0.57, size = 11, normalized size = 0.92

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="giac")
[Out] -x + 2*log(e^x + 1)
```

maple [A] time = 0.02, size = 12, normalized size = 1.00

method	result	size
norman	$-x + 2\ln(1 + e^x)$	12
risch	$-x + 2\ln(1 + e^x)$	12
derivativedivides	$2\ln(1 + e^x) - \ln(e^x)$	14
default	$2\ln(1 + e^x) - \ln(e^x)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(x))/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] -x+2*ln(1+exp(x))
```

maxima [A] time = 0.48, size = 11, normalized size = 0.92

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")
[Out] -x + 2*log(e^x + 1)
```

mupad [B] time = 0.05, size = 11, normalized size = 0.92

$$2 \ln(e^x + 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x) - 1)/(exp(x) + 1),x)
[Out] 2*log(exp(x) + 1) - x
```

sympy [A] time = 0.08, size = 8, normalized size = 0.67

$$-x + 2\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x)
[Out] -x + 2*log(exp(x) + 1)
```

3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2282, 634, 618, 204, 628}

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] -ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*E^(2*x) + 3*E^(4*x
)]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{12}\,\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, e^{2x}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+6e^{2x}\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.94

$$
\frac{1}{12}\left(\log\left(-2e^{2x}+3e^{4x}+1\right)+\sqrt{2}\tan^{-1}\left(\frac{3e^{2x}-1}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*E^(2*x))/Sqrt[2]] + Log[1 - 2*E^(2*x) + 3*E^(4*x)])
/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 39, normalized size = 0.83

$$
\frac{1}{12}\sqrt{2}\arctan\left(\frac{3}{2}\sqrt{2}e^{(2x)}-\frac{1}{2}\sqrt{2}\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x)
 - 2*e^(2*x) + 1)
```

giac [A] time = 0.60, size = 37, normalized size = 0.79

$$
\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

maple [A] time = 0.05, size = 38, normalized size = 0.81

method	result	size
default	$\frac{\ln\left(1-2\,\mathrm{e}^{2x}+3\,\mathrm{e}^{4x}\right)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(6\,\mathrm{e}^{2x}-2\right)\sqrt{2}}{4}\right)}{12}$	38
risch	$\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)}{12}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)}{12}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^(
1/2))
```

maxima [A] time = 1.34, size = 37, normalized size = 0.79

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3\,e^{(2\,x)}-1\right)\right)+\frac{1}{12}\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

mupad [B] time = 0.33, size = 39, normalized size = 0.83

$$\frac{\ln\left(3\,\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,\mathrm{e}^{2\,x}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4*x)/(3*exp(4*x) - 2*exp(2*x) + 1),x)
```

```
[Out] log(3*exp(4*x) - 2*exp(2*x) + 1)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*
exp(2*x))/2))/12
```

sympy [A] time = 0.14, size = 22, normalized size = 0.47

$$\mathrm{RootSum}\left(96z^2-16z+1,\left(i\mapsto i\log\left(8i+e^{2x}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))
```

3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx$

Optimal. Leaf size=39

$$e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^{2x}+1\right)-\tan^{-1}\left(e^x\right)$$

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2282, 2074, 635, 203, 260}

$$e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^{2x}+1\right)-\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = \text{Subst}\left(\int \frac{-1-x^4}{1-x+x^2-x^3}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right)dx, x, e^x\right)$$
$$= e^x+\frac{e^{2x}}{2}+\log(1-e^x)+\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx, x, e^x\right)$$
$$= e^x+\frac{e^{2x}}{2}+\log(1-e^x)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)-\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^x\right)$$
$$= e^x+\frac{e^{2x}}{2}-\tan^{-1}(e^x)+\log(1-e^x)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

Mathematica [C] time = 0.04, size = 51, normalized size = 1.31

$$\frac{1}{2}\left(2e^x+e^{2x}+(-1+i)\log(-e^x+i)+2\log(1-e^x)-(1+i)\log(e^x+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
[Out] (2*E^x + E^(2*x) - (1 - I)*Log[I - E^x] + 2*Log[1 - E^x] - (1 + I)*Log[I +
E^x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
[Out] Could not integrate
```

fricas [A] time = 1.29, size = 28, normalized size = 0.72

$$-\arctan(e^x)+\frac{1}{2}e^{(2x)}+e^x-\frac{1}{2}\log\left(e^{(2x)}+1\right)+\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fri
cas")
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

giac [A] time = 0.63, size = 29, normalized size = 0.74

$$-\arctan(e^x)+\frac{1}{2}e^{(2x)}+e^x-\frac{1}{2}\log\left(e^{(2x)}+1\right)+\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="gia
c")
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(abs(e^x - 1))
```

maple [A] time = 0.09, size = 29, normalized size = 0.74

method	result	size
default	$-\frac{\ln(1+e^{2x})}{2}-\arctan(e^x)+\ln(-1+e^x)+e^x+\frac{e^{2x}}{2}$	29
risch	$\frac{e^{2x}}{2}+e^x+\ln(-1+e^x)-\frac{\ln(e^x-i)}{2}+\frac{i\ln(e^x-i)}{2}-\frac{\ln(e^x+i)}{2}-\frac{i\ln(e^x+i)}{2}$	49
meijerg	$\frac{\left(\sum_{\_k1=0}^{\infty}\frac{1-e^{-x(3+\_k1)\left(1-\frac{1}{3+\_k1}\right)}}{(3+\_k1)\left(1-\frac{1}{3+\_k1}\right)}\right)}{2}-\frac{\left(\sum_{\_k1=0}^{\infty}\frac{1-e^{x(2-\_k1)}}{2-\_k1}\right)}{2}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*ln(exp(x)^2+1)-arctan(exp(x))+ln(-1+exp(x))+exp(x)+1/2*exp(x)^2
```

maxima [A] time = 1.16, size = 28, normalized size = 0.72

$$-\arctan(e^x)+\frac{1}{2}e^{(2x)}+e^x-\frac{1}{2}\log\left(e^{(2x)}+1\right)+\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="max
ima")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

mupad [B] time = 0.09, size = 28, normalized size = 0.72

$$\frac{e^{2x}}{2}-\frac{\ln\left(e^{2x}+1\right)}{2}-\mathrm{atan}(e^x)+\ln(e^x-1)+e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(5*x) + exp(x))/(exp(2*x) - exp(3*x) - exp(x) + 1),x)
```

```
[Out] exp(2*x)/2 - log(exp(2*x) + 1)/2 - atan(exp(x)) + log(exp(x) - 1) + exp(x)
```

sympy [A] time = 0.21, size = 48, normalized size = 1.23

$$\frac{e^{2x}}{2}+e^x+\log(e^x-1)+\mathrm{RootSum}\left(2z^2+2z+1,\left(i\mapsto i\log\left(\frac{4i^2}{5}-\frac{6i}{5}+e^x-\frac{3}{5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z + 1, Lambda(
_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

3.527 $\int e^{nx} (a + be^{nx})^{r/s} \, dx$

Optimal. Leaf size=30

$$\frac{s (a + be^{nx})^{\frac{r+s}{s}}}{bn(r + s)}$$

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2246, 32}

$$\frac{s (a + be^{nx})^{\frac{r+s}{s}}}{bn(r + s)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{nx} (a + be^{nx})^{r/s} \, dx &= \frac{\text{Subst}\left(\int (a + bx)^{r/s} \, dx, x, e^{nx}\right)}{n} \\
&= \frac{(a + be^{nx})^{\frac{r+s}{s}} s}{bn(r + s)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 1.00

$$\frac{s (a + be^{nx})^{\frac{r}{s}+1}}{bnr + bns}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{nx} (a + be^{nx})^{r/s} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.30, size = 37, normalized size = 1.23

$$\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")
[Out] (b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

giac [A] time = 0.64, size = 32, normalized size = 1.07

$$\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="giac")
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

maple [A] time = 0.04, size = 33, normalized size = 1.10

method	result	size
derivativedivides	$\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
default	$\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
risch	$\frac{s(a+b\,e^{nx})(a+b\,e^{nx})^{\frac{r}{s}}}{bn(r+s)}$	36
norman	$\frac{s\,e^{nx}\,e^{\frac{r\ln\left(a+b\,e^{nx}\right)}{s}}}{n(r+s)}+\frac{as\,e^{\frac{r\ln\left(a+b\,e^{nx}\right)}{s}}}{bn(r+s)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*x)*(a+b*exp(n*x))^(r/s),x,method=_RETURNVERBOSE)
[Out] 1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

maxima [A] time = 0.59, size = 32, normalized size = 1.07

$$\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

mupad [B] time = 0.35, size = 29, normalized size = 0.97

$$\frac{s\,(a+b\,e^{n\,x})^{\frac{r}{s}+1}}{b\,n\,(r+s)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*x)*(a + b*exp(n*x))^(r/s),x)
```

```
[Out] (s*(a + b*exp(n*x))^(r/s + 1))/(b*n*(r + s))
```

sympy [A] time = 1.32, size = 94, normalized size = 3.13

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge n = 0 \wedge r = -s \\ \frac{a^{\frac{r}{s}} e^{nx}}{n} & \text{for } b = 0 \\ x (a + b)^{\frac{r}{s}} & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b} + e^{nx}\right)}{bn} & \text{for } r = -s \\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns} + \frac{bs(a+be^{nx})^{\frac{r}{s}} e^{nx}}{bnr+bns} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq(
b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -s)
), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**(r/
s)*exp(n*x)/(b*n*r + b*n*s), True))
```

3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

Optimal. Leaf size=54

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2282, 50, 63, 212, 206, 203}

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{1-2e^{x/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{\sqrt[4]{1-2x}}{x}\,dx, x, e^{x/3}\right)\\ &= 12\sqrt[4]{1-2e^{x/3}} + 3\,\text{Subst}\left(\int \frac{1}{(1-2x)^{3/4}x}\,dx, x, e^{x/3}\right)\\ &= 12\sqrt[4]{1-2e^{x/3}} - 6\,\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)\\ &= 12\sqrt[4]{1-2e^{x/3}} - 6\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right) - 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)\\ &= 12\sqrt[4]{1-2e^{x/3}} - 6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.00

$$12\sqrt[4]{1-2e^{x/3}} - 6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*E^(x/3))^(1/4),x]
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{1-2e^{x/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 2*E^(x/3))^(1/4),x]
[Out] Could not integrate
```

fricas [A] time = 1.28, size = 56, normalized size = 1.04

$$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}} - 6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right) - 3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

giac [A] time = 0.59, size = 57, normalized size = 1.06

$$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}} - 6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right) - 3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left|\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="giac")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log(abs((-2*e^(1/3*x) + 1)^(1/4) - 1))
```

maple [A] time = 0.08, size = 57, normalized size = 1.06

method	result
derivativedivides	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}\right)$
default	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}\right)$
meijerg	error in int/gbinthm/express: improper op or subscript selector\

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*exp(1/3*x))^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 12*(1-2*exp(1/3*x))^(1/4)+3*ln((1-2*exp(1/3*x))^(1/4)-1)-3*ln((1-2*exp(1/3*
x))^(1/4)+1)-6*arctan((1-2*exp(1/3*x))^(1/4))
```

maxima [A] time = 1.31, size = 56, normalized size = 1.04

$$12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

mupad [B] time = 0.35, size = 33, normalized size = 0.61

$$\frac{12\left(2-4\,\mathrm{e}^{x/3}\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{\mathrm{e}^{-\frac{x}{3}}}{2}\right)}{\left(2-\mathrm{e}^{-\frac{x}{3}}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 2*exp(x/3))^(1/4),x)
```

```
[Out] (12*(2 - 4*exp(x/3))^(1/4)*hypergeom([-1/4, -1/4], 3/4, exp(-x/3)/2))/(2 -
exp(-x/3))^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{1-2e^{\frac{x}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))**(1/4),x)
```

```
[Out] Integral((1 - 2*exp(x/3))**(1/4), x)
```

3.529 $\int (a+be^{nx})^{r/s}\,dx$

Optimal. Leaf size=59

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\,{}_2F_1\left(1,\frac{r+s}{s};\frac{r}{s}+2;\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2282, 65}

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},\frac{r}{s}+2,\frac{be^{nx}}{a}+1\right)}{an(r+s)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \left(a+be^{nx}\right)^{r/s}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{r/s}}{x}\,dx,x,e^{nx}\right)}{n}\\ &= -\frac{\left(a+be^{nx}\right)^{\frac{r+s}{s}}s\,{}_2F_1\left(1,\frac{r+s}{s};2+\frac{r}{s};1+\frac{be^{nx}}{a}\right)}{an(r+s)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\,{}_2F_1\left(1,\frac{r+s}{s};\frac{r}{s}+2;\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + be^{nx})^{r/s}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*E^(n*x))^(r/s),x]
[Out] Could not integrate
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(be^{(nx)} + a\right)^{\frac{r}{s}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")
[Out] integral((b*e^(n*x) + a)^(r/s), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(be^{(nx)} + a\right)^{\frac{r}{s}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="giac")
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (a + b\,e^{nx})^{\frac{r}{s}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*exp(n*x))^(r/s),x)
[Out] int((a+b*exp(n*x))^(r/s),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(be^{(nx)} + a\right)^{\frac{r}{s}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

mupad [B] time = 0.40, size = 75, normalized size = 1.27

$$\frac{s\,(a + b\,e^{n\,x})^{r/s}\,{}_2F_1\left(-\frac{r}{s}, -\frac{r}{s};\,1 - \frac{r}{s};\,-\frac{a\,e^{-n\,x}}{b}\right)}{n\,r\left(\frac{a\,e^{-n\,x}}{b} + 1\right)^{r/s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*exp(n*x))^(r/s),x)
```

```
[Out] (s*(a + b*exp(n*x))^(r/s)*hypergeom([-r/s, -r/s], 1 - r/s, -(a*exp(-n*x))/b
))/(n*r*((a*exp(-n*x))/b + 1)^(r/s))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + be^{nx})^{\frac{r}{s}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))**(r/s),x)
```

```
[Out] Integral((a + b*exp(n*x))**(r/s), x)
```

3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 217, 206}

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/Sqrt[a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}} \, dx, x, e^x\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{e^x}{\sqrt{a^2+e^{2x}}}\right) \\
&= \tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\sqrt{a^2 + e^{2x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/Sqrt[a^2 + E^(2*x)],x]
[Out] Could not integrate
```

fricas [A] time = 1.26, size = 18, normalized size = 1.00

$$-\log\left(\sqrt{a^2 + e^{(2\,x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

giac [A] time = 0.58, size = 18, normalized size = 1.00

$$-\log\left(\sqrt{a^2 + e^{(2\,x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="giac")
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

maple [A] time = 0.04, size = 15, normalized size = 0.83

method	result	size
default	$\ln\left(e^x + \sqrt{a^2 + e^{2x}}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

maxima [A] time = 0.55, size = 7, normalized size = 0.39

$$\operatorname{arsinh}\left(\frac{e^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] arcsinh(e^x/a)
```

mupad [B] time = 0.41, size = 14, normalized size = 0.78

$$\ln\left(e^x + \sqrt{a^2 + e^{2\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) + a^2)^(1/2),x)
```

```
[Out] log(exp(x) + (exp(2*x) + a^2)^(1/2))
```

sympy [**A**] time = 0.73, size = 31, normalized size = 1.72

$$\begin{cases} \operatorname{asinh}\left(\sqrt{\frac{1}{a^2}}\, e^x\right) & \text{for } a^2 > 0 \\ \operatorname{acosh}\left(\sqrt{-\frac{1}{a^2}}\, e^x\right) & \text{for } a^2 < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(a**(-2))*exp(x)), a**2 > 0), (acosh(sqrt(-1/a**2)*exp
(x)), a**2 < 0))
```

3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2249, 217, 206}

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)\\ &= \tanh^{-1}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\sqrt{-a^2 + e^{2x}}}\, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/Sqrt[-a^2 + E^(2*x)],x]
[Out] Could not integrate
```

fricas [A] time = 1.30, size = 20, normalized size = 1.00

$$-\log\left(\sqrt{-a^2 + e^{(2x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(-a^2 + e^(2*x)) - e^x)
```

giac [A] time = 0.61, size = 20, normalized size = 1.00

$$-\log\left(-\sqrt{-a^2 + e^{(2x)}} + e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="giac")
[Out] -log(-sqrt(-a^2 + e^(2*x)) + e^x)
```

maple [A] time = 0.04, size = 17, normalized size = 0.85

method	result	size
default	$\ln\left(e^x + \sqrt{-a^2 + e^{2x}}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

maxima [A] time = 0.61, size = 20, normalized size = 1.00

$$\log\left(2\sqrt{-a^2 + e^{(2x)}} + 2e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

mupad [B] time = 0.41, size = 16, normalized size = 0.80

$$\ln\left(e^x + \sqrt{e^{2x} - a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) - a^2)^(1/2),x)
```

```
[Out] log(exp(x) + (exp(2*x) - a^2)^(1/2))
```

sympy [**A**] time = 0.73, size = 31, normalized size = 1.55

$$\begin{cases} \operatorname{asinh}\left(\sqrt{-\frac{1}{a^2}}e^x\right) & \text{for } a^2 < 0 \\ \operatorname{acosh}\left(\sqrt{\frac{1}{a^2}}e^x\right) & \text{for } a^2 > 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(-1/a**2)*exp(x)), a**2 < 0), (acosh(sqrt(a**(-2))*exp
(x)), a**2 > 0))
```

3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2282, 724, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),x
]
```

```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx &= \frac{4}{3}\text{Subst}\left(\int \frac{1}{(-2+x)\sqrt{-2+x+x^2}}\,dx, x, e^{3x/4}\right)\\
&= -\left(\frac{8}{3}\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \frac{-2+5e^{3x/4}}{\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)\right)\\
&= \frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\frac{5e^{3x/4}-2}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/
2)]),x]
```

```
[Out] (-2*ArcTanh[(-2 + 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])
/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) +
E^((3*x)/2)]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 46, normalized size = 1.15

$$-\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e^
(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

giac [A] time = 0.82, size = 48, normalized size = 1.20

$$-\frac{2}{3}\log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right|\right)+\frac{2}{3}\log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="giac")
```

```
[Out] -2/3*log(abs(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4)) + 2/3*log(ab
s(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x)))
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\frac{3x}{4}}}{\left(-2+\mathrm{e}^{\frac{3x}{4}}\right)\sqrt{-2+\mathrm{e}^{\frac{3x}{4}}+\mathrm{e}^{\frac{3x}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

```
[Out] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

maxima [A] time = 1.30, size = 39, normalized size = 0.98

$$-\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="maxima")
```

```
[Out] -2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3/
4*x) - 2) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{2}}+e^{\frac{3x}{4}}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2
)),x)
```

```
[Out] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

```
[Out] Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)), x
)
```

3.533 $\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}e^{-2x}\left(e^{7x}-3\right)^{5/3}\,{}_2F_1\left(1,\frac{29}{21};\frac{5}{7};\frac{e^{7x}}{3}\right)$$

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2249, 335, 365, 364}

$$-\frac{3^{2/3}e^{-2x}\left(e^{7x}-3\right)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\left(3-e^{7x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -(3^(2/3)*(-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3
])/(2*E^(2*x)*(3 - E^(7*x))^(2/3))
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=-\mathrm{Subst}\left(\int\left(-3+\frac{1}{x^7}\right)^{2/3}x\,dx,x,e^{-x}\right)$$
$$=\mathrm{Subst}\left(\int\frac{\left(-3+x^7\right)^{2/3}}{x^3}\,dx,x,e^x\right)$$
$$=\frac{\left(-3+e^{7x}\right)^{2/3}\mathrm{Subst}\left(\int\frac{\left(1-\frac{x^7}{3}\right)^{2/3}}{x^3}\,dx,x,e^x\right)}{\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$
$$=-\frac{3^{2/3}e^{-2x}\left(-3+e^{7x}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};\frac{e^{7x}}{3}\right)}{2\left(3-e^{7x}\right)^{2/3}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.46

$$-\frac{e^{-2x}\left(e^{7x}-3\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};\frac{e^{7x}}{3}\right)}{2\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
[Out] -1/2*((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/(
E^(2*x)*(1 - E^(7*x)/3)^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
[Out] Could not integrate
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")
[Out] integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="giac")
```

```
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

maple [F] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-3+e^{7x}\right)^{\frac{2}{3}} e^{-2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

```
[Out] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}} e^{(-2\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")
```

```
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int e^{-2\,x}\,\left(e^{7\,x}-3\right)^{2/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)*(exp(7*x) - 3)^(2/3),x)
```

```
[Out] int(exp(-2*x)*(exp(7*x) - 3)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e^{7x}-3\right)^{\frac{2}{3}} e^{-2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
```

```
[Out] Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$

Optimal. Leaf size=73

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2248, 43}

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E^(x/2))^(9
/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(3-x)^{3/4}}\,dx, x, e^{x/2}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{27}{(3-x)^{3/4}}-27\sqrt[4]{3-x}+9(3-x)^{5/4}-(3-x)^{9/4}\right)dx, x, e^{x/2}\right)\\
&= -216\sqrt[4]{3-e^{x/2}}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.60

$$-\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(96e^{x/2}+20e^{x}+5e^{3x/2}+1152\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] (-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.25, size = 30, normalized size = 0.41

$$-\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)}+96\,e^{\left(\frac{1}{2}\,x\right)}+20\,e^{x}+1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")
```

```
[Out] -8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)
```

giac [A] time = 0.66, size = 65, normalized size = 0.89

$$-\frac{8}{13}\left(e^{\left(\frac{1}{2}\,x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}-8\left(e^{\left(\frac{1}{2}\,x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="giac")
```

```
[Out] -8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(1
/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4
)
```

maple [A] time = 0.03, size = 37, normalized size = 0.51

method	result	size
risch	$\frac{8\left(5\,\mathrm{e}^{\frac{3x}{2}}+20\,\mathrm{e}^{x}+96\,\mathrm{e}^{\frac{x}{2}}+1152\right)\left(-3+\mathrm{e}^{\frac{x}{2}}\right)}{65\left(3-\mathrm{e}^{\frac{x}{2}}\right)^{\frac{3}{4}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(3-exp(1/2*x))^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+e
xp(1/2*x))
```

maxima [A] time = 0.57, size = 49, normalized size = 0.67

$$\frac{8}{13}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{13}{4}}-8\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{9}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")
```

```
[Out] 8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x)
 + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

mupad [B] time = 0.11, size = 30, normalized size = 0.41

$$-\left(3-e^{x/2}\right)^{1/4}\left(\frac{768\,e^{x/2}}{65}+\frac{8\,e^{\frac{3x}{2}}}{13}+\frac{32\,e^{x}}{13}+\frac{9216}{65}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(3 - exp(x/2))^(3/4),x)
```

```
[Out] -(3 - exp(x/2))^(1/4)*((768*exp(x/2))/65 + (8*exp((3*x)/2))/13 + (32*exp(x)
)/13 + 9216/65)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\left(3-e^{\frac{x}{2}}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

```
[Out] Integral(exp(2*x)/(3 - exp(x/2))**(3/4), x)
```

3.535 $\int e^{-x/2}x^3\,dx$

Optimal. Leaf size=44

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2176, 2194}

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(x/2),x]
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x/2}x^3\,dx &= -2e^{-x/2}x^3+6\int e^{-x/2}x^2\,dx\\ &= -12e^{-x/2}x^2-2e^{-x/2}x^3+24\int e^{-x/2}x\,dx\\ &= -48e^{-x/2}x-12e^{-x/2}x^2-2e^{-x/2}x^3+48\int e^{-x/2}\,dx\\ &= -96e^{-x/2}-48e^{-x/2}x-12e^{-x/2}x^2-2e^{-x/2}x^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.52

$$e^{-x/2}\left(-2x^3-12x^2-48x-96\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(x/2),x]
[Out] (-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-x/2}x^3\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/E^(x/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.27, size = 19, normalized size = 0.43

$$-2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="fricas")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

giac [A] time = 0.60, size = 19, normalized size = 0.43

$$-2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="giac")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

maple [A] time = 0.04, size = 21, normalized size = 0.48

method	result	size
risch	$\left(-2x^3-12x^2-48x-96\right)\mathrm{e}^{-\frac{x}{2}}$	21
gosper	$-2\left(x^3+6x^2+24x+48\right)\mathrm{e}^{-\frac{x}{2}}$	22
norman	$\left(-2x^3-12x^2-48x-96\right)\mathrm{e}^{-\frac{x}{2}}$	23
meijerg	$96-4\left(\frac{1}{2}x^3+3x^2+12x+24\right)\mathrm{e}^{-\frac{x}{2}}$	24
derivativedivides	$-96\,\mathrm{e}^{-\frac{x}{2}}-48x\,\mathrm{e}^{-\frac{x}{2}}-12x^2\mathrm{e}^{-\frac{x}{2}}-2x^3\mathrm{e}^{-\frac{x}{2}}$	41
default	$-96\,\mathrm{e}^{-\frac{x}{2}}-48x\,\mathrm{e}^{-\frac{x}{2}}-12x^2\mathrm{e}^{-\frac{x}{2}}-2x^3\mathrm{e}^{-\frac{x}{2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/exp(1/2*x),x,method=_RETURNVERBOSE)
[Out] (-2*x^3-12*x^2-48*x-96)*exp(-1/2*x)
```

maxima [A] time = 0.51, size = 19, normalized size = 0.43

$$-2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="maxima")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.48

$$-16\,\mathrm{e}^{-\frac{x}{2}}\left(\frac{x^3}{8}+\frac{3\,x^2}{4}+3\,x+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(-x/2),x)
```

```
[Out] -16*exp(-x/2)*(3*x + (3*x^2)/4 + x^3/8 + 6)
```

sympy [A] time = 0.09, size = 20, normalized size = 0.45

$$\left(-2x^3 - 12x^2 - 48x - 96\right)e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/exp(1/2*x),x)
[Out] (-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

3.536 $\int \frac{e^{-x/2}}{x^3}\,dx$

Optimal. Leaf size=39

$$\frac{\mathrm{Ei}\left(-\frac{x}{2}\right)}{8}-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2177, 2178}

$$\frac{1}{8}\text{ExpIntegralEi}\left(-\frac{x}{2}\right)-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(x/2)*x^3),x]
```

```
[Out] -1/(2*E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-x/2]/8
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-x/2}}{x^3}\,dx &= -\frac{e^{-x/2}}{2x^2}-\frac{1}{4}\int \frac{e^{-x/2}}{x^2}\,dx\\ &= -\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}+\frac{1}{8}\int \frac{e^{-x/2}}{x}\,dx\\ &= -\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}+\frac{\mathrm{Ei}\left(-\frac{x}{2}\right)}{8}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.67

$$\frac{1}{8}\left(\mathrm{Ei}\left(-\frac{x}{2}\right)+\frac{2e^{-x/2}(x-2)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(x/2)*x^3),x]
```

```
[Out] ((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-1/2*x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-x/2}}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(E^(x/2)*x^3),x]
[Out] Could not integrate
```

fricas [A] time = 1.40, size = 23, normalized size = 0.59

$$\frac{x^2 \mathrm{Ei}\left(-\frac{1}{2}x\right) + 2\,(x-2)e^{\left(-\frac{1}{2}x\right)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

giac [A] time = 0.60, size = 27, normalized size = 0.69

$$\frac{x^2 \mathrm{Ei}\left(-\frac{1}{2}x\right) + 2\,xe^{\left(-\frac{1}{2}x\right)} - 4\,e^{\left(-\frac{1}{2}x\right)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="giac")
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

maple [A] time = 0.07, size = 27, normalized size = 0.69

method	result	size
risch	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{expIntegralEi}\left(1,\frac{x}{2}\right)}{8}$	27
derivativedivides	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{expIntegralEi}\left(1,\frac{x}{2}\right)}{8}$	31
default	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{expIntegralEi}\left(1,\frac{x}{2}\right)}{8}$	31
meijerg	$\frac{\frac{9}{4}x^2-6x+6}{12x^2}-\frac{\left(-\frac{3x}{2}+3\right)e^{-\frac{x}{2}}}{6x^2}-\frac{\ln\left(\frac{x}{2}\right)}{8}-\frac{\mathrm{expIntegralEi}\left(1,\frac{x}{2}\right)}{8}-\frac{3}{16}+\frac{\ln(x)}{8}-\frac{\ln(2)}{8}-\frac{1}{2x^2}+\frac{1}{2x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(1/2*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*exp(-1/2*x)/x^2+1/4*exp(-1/2*x)/x-1/8*Ei(1,1/2*x)
```

maxima [A] time = 0.58, size = 7, normalized size = 0.18

$$-\frac{1}{4}\Gamma\left(-2,\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")
[Out] -1/4*gamma(-2, 1/2*x)
```

mupad [B] time = 0.27, size = 22, normalized size = 0.56

$$\frac{e^{-\frac{x}{2}}\left(\frac{1}{x}-\frac{2}{x^2}\right)}{4}-\frac{\mathrm{expint}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x/2)/x^3,x)
```

```
[Out] (exp(-x/2)*(1/x - 2/x^2))/4 - expint(x/2)/8
```

sympy [C] time = 1.34, size = 32, normalized size = 0.82

$$\frac{\mathrm{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{e^{-\frac{x}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x**3,x)
```

```
[Out] Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

3.537 $\int a^{3x}x^2\,dx$

Optimal. Leaf size=44

$$\frac{x^2a^{3x}}{3\log(a)}+\frac{2a^{3x}}{27\log^3(a)}-\frac{2xa^{3x}}{9\log^2(a)}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$\frac{x^2a^{3x}}{3\log(a)}-\frac{2xa^{3x}}{9\log^2(a)}+\frac{2a^{3x}}{27\log^3(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(3*x)*x^2,x]
```

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)*x^2)/(3*L
og[a])
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int a^{3x}x^2\,dx&=\frac{a^{3x}x^2}{3\log(a)}-\frac{2\int a^{3x}x\,dx}{3\log(a)}\\&=-\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}+\frac{2\int a^{3x}\,dx}{9\log^2(a)}\\&=\frac{2a^{3x}}{27\log^3(a)}-\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$\frac{a^{3x}\left(9x^2\log^2(a)-6x\log(a)+2\right)}{27\log^3(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(3*x)*x^2,x]
```

```
[Out] (a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a^{3x}x^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[a^(3*x)*x^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.13, size = 27, normalized size = 0.61

$$\frac{\left(9\,x^2\log(a)^2 - 6\,x\log(a) + 2\right)a^{3\,x}}{27\,\log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

giac [C] time = 0.70, size = 826, normalized size = 18.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="giac")
```

```
[Out] -1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a)
 + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))
^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^
2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2) -
 (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs(a))
+ 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)/((p
i^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*
pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))*cos(-3/
2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(a
bs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) -
 pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^
3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a))
 + 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(
a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) +
 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*l
og(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(a
bs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*x) + 1/2*I*abs(a)
^(3*x)*((36*I*pi^2*x^2*sgn(a) - 72*pi*x^2*log(abs(a))*sgn(a) - 36*I*pi^2*x^
2 + 72*pi*x^2*log(abs(a)) + 72*I*x^2*log(abs(a))^2 + 24*pi*x*sgn(a) - 24*pi
*x - 48*I*x*log(abs(a)) + 16*I)*e^(3/2*I*pi*x*sgn(a) - 3/2*I*pi*x)/(-108*I*
pi^3*sgn(a) + 324*pi^2*log(abs(a))*sgn(a) + 324*I*pi*log(abs(a))^2*sgn(a) +
 108*I*pi^3 - 324*pi^2*log(abs(a)) - 324*I*pi*log(abs(a))^2 + 216*log(abs(a
))^3) - (36*I*pi^2*x^2*sgn(a) + 72*pi*x^2*log(abs(a))*sgn(a) - 36*I*pi^2*x^
2 - 72*pi*x^2*log(abs(a)) + 72*I*x^2*log(abs(a))^2 - 24*pi*x*sgn(a) + 24*pi
*x - 48*I*x*log(abs(a)) + 16*I)*e^(-3/2*I*pi*x*sgn(a) + 3/2*I*pi*x)/(108*I*
pi^3*sgn(a) + 324*pi^2*log(abs(a))*sgn(a) - 324*I*pi*log(abs(a))^2*sgn(a) -
 108*I*pi^3 - 324*pi^2*log(abs(a)) + 324*I*pi*log(abs(a))^2 + 216*log(abs(a
))^3))
```

maple [A] time = 0.04, size = 28, normalized size = 0.64

method	result	size
gosper	$\frac{\left(9x^2\ln(a)^2-6x\ln(a)+2\right)a^{3x}}{27\ln(a)^3}$	28
risch	$\frac{\left(9x^2\ln(a)^2-6x\ln(a)+2\right)a^{3x}}{27\ln(a)^3}$	28

meijerg	$-\frac{2-\frac{(27x^2\ln(a)^2-18x\ln(a)+6)e^{3x\ln(a)}}{3}}{27\ln(a)^3}$	33
norman	$\frac{2\,e^{3x\ln(a)}}{27\ln(a)^3}-\frac{2x\,e^{3x\ln(a)}}{9\ln(a)^2}+\frac{x^2e^{3x\ln(a)}}{3\ln(a)}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(3*x)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

maxima [A] time = 0.46, size = 27, normalized size = 0.61

$$\frac{\left(9\,x^2\log(a)^2-6\,x\log(a)+2\right)a^{3\,x}}{27\,\log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="maxima")
```

```
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

mupad [B] time = 0.06, size = 27, normalized size = 0.61

$$\frac{a^{3\,x}\,\left(9\,x^2\,\ln(a)^2-6\,x\,\ln(a)+2\right)}{27\ln(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(3*x)*x^2,x)
```

```
[Out] (a^(3*x)*(9*x^2*log(a)^2 - 6*x*log(a) + 2))/(27*log(a)^3)
```

sympy [A] time = 0.11, size = 39, normalized size = 0.89

$$\begin{cases} \frac{a^{3x}\left(9x^2\log{(a)}^2-6x\log{(a)}+2\right)}{27\log{(a)}^3} & \text{for}\ 27\log{(a)}^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(3*x)*x**2,x)
```

```
[Out] Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne(
27*log(a)**3, 0)), (x**3/3, True))
```

3.538 $\int e^{x^2} x \left(1+x^2\right) \, dx$

Optimal. Leaf size=12

$$\frac{1}{2} e^{x^2} x^2$$

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2226, 2209, 2212}

$$\frac{1}{2} e^{x^2} x^2$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x*(1 + x^2),x]
```

```
[Out] (E^x^2*x^2)/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2} x \left(1+x^2\right) \, dx &= \int \left(e^{x^2} x+e^{x^2} x^3\right) \, dx \\
&= \int e^{x^2} x \, dx+\int e^{x^2} x^3 \, dx \\
&= \frac{e^{x^2}}{2}+\frac{1}{2} e^{x^2} x^2-\int e^{x^2} x \, dx \\
&= \frac{1}{2} e^{x^2} x^2
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{2} e^{x^2} x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x*(1 + x^2),x]
[Out] (E^x^2*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2} x \left(1 + x^2\right) dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x^2*x*(1 + x^2),x]
[Out] Could not integrate
```

fricas [A] time = 1.24, size = 9, normalized size = 0.75

$$\frac{1}{2} x^2 e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2*e^(x^2)
```

giac [A] time = 0.57, size = 18, normalized size = 1.50

$$\frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)} + \frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

maple [A] time = 0.03, size = 10, normalized size = 0.83

method	result	size
gosper	$\frac{e^{x^2} x^2}{2}$	10
derivativedivides	$\frac{e^{x^2} x^2}{2}$	10
default	$\frac{e^{x^2} x^2}{2}$	10
norman	$\frac{e^{x^2} x^2}{2}$	10
risch	$\frac{e^{x^2} x^2}{2}$	10
meijerg	$-\frac{\left(-2x^2+2\right) e^{x^2}}{4} + \frac{e^{x^2}}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x*(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)*x^2
```

maxima [A] time = 0.56, size = 18, normalized size = 1.50

$$\frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)} + \frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

mupad [B] time = 0.05, size = 9, normalized size = 0.75

$$\frac{x^2\,\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2)*(x^2 + 1),x)
[Out] (x^2*exp(x^2))/2
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\frac{x^2e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x*(x**2+1),x)
[Out] x**2*exp(x**2)/2
```

3.539 $\int \frac{x}{(e^{-x}+e^x)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{x}{2\left(e^{2x}+1\right)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{x}{2\left(e^{2x}+1\right)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(E^(-x) + E^x)^2,x]
```

```
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2283

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \int \frac{e^{2x}x}{\left(1+e^{2x}\right)^2}\,dx$$
$$= -\frac{x}{2\left(1+e^{2x}\right)} + \frac{1}{2}\int \frac{1}{1+e^{2x}}\,dx$$
$$= -\frac{x}{2\left(1+e^{2x}\right)} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, e^{2x}\right)$$
$$= -\frac{x}{2\left(1+e^{2x}\right)} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, e^{2x}\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^{2x}\right)$$
$$= \frac{x}{2} - \frac{x}{2\left(1+e^{2x}\right)} - \frac{1}{4}\log\left(1+e^{2x}\right)$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.97

$$\frac{e^{2x}x}{2e^{2x}+2} - \frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(E^(-x) + E^x)^2,x]
[Out] (E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x/(E^(-x) + E^x)^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.36, size = 33, normalized size = 1.03

$$\frac{2\,xe^{(2\,x)} - \left(e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}+1\right)}{4\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")
[Out] 1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

giac [A] time = 0.60, size = 40, normalized size = 1.25

$$\frac{2\,xe^{(2\,x)} - e^{(2\,x)}\log\left(e^{(2\,x)}+1\right) - \log\left(e^{(2\,x)}+1\right)}{4\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="giac")
[Out] 1/4*(2*x*e^(2*x) - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) +
1)
```

maple [A] time = 0.04, size = 25, normalized size = 0.78

method	result	size
risch	$\frac{x}{2}-\frac{x}{2\left(1+e^{2x}\right)}-\frac{\ln\left(1+e^{2x}\right)}{4}$	25
default	$-\frac{\ln\left(1+e^{2x}\right)}{4}+\frac{x\,e^{2x}}{2+2\,e^{2x}}$	26
norman	$-\frac{\ln\left(1+e^{2x}\right)}{4}+\frac{x\,e^{2x}}{2+2\,e^{2x}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

maxima [A] time = 1.16, size = 25, normalized size = 0.78

$$\frac{xe^{(2x)}}{2\left(e^{(2x)}+1\right)}-\frac{1}{4}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")
[Out] 1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

mupad [B] time = 0.33, size = 26, normalized size = 0.81

$$\frac{x\,e^{2\,x}}{2\,\left(e^{2\,x}+1\right)}-\frac{\ln\left(e^{2\,x}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(exp(-x) + exp(x))^2,x)
[Out] (x*exp(2*x))/(2*(exp(2*x) + 1)) - log(exp(2*x) + 1)/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.69

$$\frac{x}{2}-\frac{x}{2e^{2x}+2}-\frac{\log\left(e^{2x}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))**2,x)
[Out] x/2 - x/(2*exp(2*x) + 2) - log(exp(2*x) + 1)/4
```

3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=15

$$e^x\sqrt{1-x^2}$$

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2288}

$$e^x\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] E^x*Sqrt[1 - x^2]
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}} \, dx = e^x\sqrt{1-x^2}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$e^x\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] E^x*Sqrt[1 - x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.05, size = 12, normalized size = 0.80

$$\sqrt{-x^2+1}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1)*e^x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(x^2 + x - 1\right)e^x}{\sqrt{-x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

maple [A] time = 0.30, size = 20, normalized size = 1.33

method	result	size
gosper	$-\frac{e^x(1+x)(-1+x)}{\sqrt{-x^2+1}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)*(1+x)*(-1+x)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.65, size = 21, normalized size = 1.40

$$-\frac{\left(x^2 - 1\right)e^x}{\sqrt{x + 1}\,\sqrt{-x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

mupad [B] time = 0.45, size = 12, normalized size = 0.80

$$e^x\,\sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x)*(x + x^2 - 1))/(1 - x^2)^(1/2),x)
```

```
[Out] exp(x)*(1 - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^x}{\sqrt{1 - x^2}}\right)dx - \int \frac{xe^x}{\sqrt{1 - x^2}}\,dx - \int \frac{x^2e^x}{\sqrt{1 - x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x)
 - Integral(x**2*exp(x)/sqrt(1 - x**2), x)
```

3.541 $\int e^{-3x}\cos(2x)\,dx$

Optimal. Leaf size=27

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]/E^(3*x),x]
[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(2x)\,dx=-\frac{3}{13}e^{-3x}\cos(2x)+\frac{2}{13}e^{-3x}\sin(2x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{-3x}(2\sin(2x)-3\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]/E^(3*x),x]
[Out] (-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-3x}\cos(2x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[2*x]/E^(3*x),x]
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 21, normalized size = 0.78

$$-\frac{3}{13}\cos(2x)e^{(-3x)}+\frac{2}{13}e^{(-3x)}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

giac [A] time = 0.64, size = 19, normalized size = 0.70

$$-\frac{1}{13}\,(3\,\cos(2\,x) - 2\,\sin(2\,x))e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="giac")
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

maple [A] time = 0.05, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{3\,\mathrm{e}^{-3x}\cos(2x)}{13}+\frac{2\,\mathrm{e}^{-3x}\sin(2x)}{13}$	22
norman	$\frac{\left(-\frac{3}{13}+\frac{3\left(\tan^{2}(x)\right)}{13}+\frac{4\tan(x)}{13}\right)\mathrm{e}^{-3x}}{1+\tan^{2}(x)}$	28
risch	$-\frac{3\,\mathrm{e}^{(-3+2i)x}}{26}-\frac{i\mathrm{e}^{(-3+2i)x}}{13}-\frac{3\,\mathrm{e}^{(-3-2i)x}}{26}+\frac{i\mathrm{e}^{(-3-2i)x}}{13}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/exp(3*x),x,method=_RETURNVERBOSE)
[Out] -3/13*exp(-3*x)*cos(2*x)+2/13*exp(-3*x)*sin(2*x)
```

maxima [A] time = 0.55, size = 19, normalized size = 0.70

$$-\frac{1}{13}\,(3\,\cos(2\,x) - 2\,\sin(2\,x))e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{-3\,x}\,\left(3\,\cos(2\,x) - 2\,\sin(2\,x)\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*exp(-3*x),x)
[Out] -(exp(-3*x)*(3*cos(2*x) - 2*sin(2*x)))/13
```

sympy [A] time = 0.47, size = 26, normalized size = 0.96

$$\frac{2e^{-3x}\sin(2x)}{13} - \frac{3e^{-3x}\cos(2x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x)
[Out] 2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$

Optimal. Leaf size=35

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Rubi [A] time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2281, 6742, 4433, 4432}

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

```
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = \frac{e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)dx}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}(\cos(3x)+\sin(3x))\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int\left(e^{-2x}\cos(3x)+e^{-2x}\sin(3x)\right)dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\cos(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}} + \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\sin(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= -\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} + \frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Mathematica [**A**] time = 0.06, size = 26, normalized size = 0.74

$$\frac{6\left(\sin\left(\frac{x}{2}\right)-5\cos\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
[Out] (6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
[Out] Could not integrate
```

fricas [**A**] time = 1.35, size = 21, normalized size = 0.60

$$-\frac{30}{13}\cos\left(\frac{1}{2}x\right)e^{\left(-\frac{1}{3}x\right)} + \frac{6}{13}e^{\left(-\frac{1}{3}x\right)}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")
[Out] -30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

giac [**A**] time = 0.60, size = 39, normalized size = 1.11

$$-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)} - \frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="giac")
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin
(1/2*x))*e^(-1/3*x)
```

maple [A] time = 0.10, size = 22, normalized size = 0.63

method	result	size
default	$-\frac{30\,\mathrm{e}^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$	22
risch	$\frac{\left(-\frac{15}{169}-\frac{3i}{169}\right)\left((25-5i)\cos\left(\frac{x}{2}\right)+(-5+i)\sin\left(\frac{x}{2}\right)\right)}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -30/13*exp(-1/3*x)*cos(1/2*x)+6/13*exp(-1/3*x)*sin(1/2*x)
```

maxima [A] time = 0.66, size = 39, normalized size = 1.11

$$-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}-\frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")
```

```
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin
(1/2*x))*e^(-1/3*x)
```

mupad [B] time = 0.10, size = 19, normalized size = 0.54

$$-\frac{6\,\mathrm{e}^{-\frac{x}{3}}\,\left(5\,\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x/2) + sin(x/2))/exp(x)^(1/3),x)
```

```
[Out] -(6*exp(-x/3)*(5*cos(x/2) - sin(x/2)))/13
```

sympy [A] time = 0.85, size = 29, normalized size = 0.83

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^{x}}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

```
[Out] 6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$

Optimal. Leaf size=57

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2281, 4433}

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*Sin[(3*x)/
2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx &= \frac{3^{3x/4}\int 3^{-3x/4}\cos\left(\frac{3x}{2}\right)\,dx}{\sqrt[4]{3^{3x}}}\\ &= -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}+\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.65

$$-\frac{4\left(\log(3)\cos\left(\frac{3x}{2}\right)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] Could not integrate
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [A] time = 0.63, size = 39, normalized size = 0.68

$$-\frac{4\left(\frac{\cos\left(\frac{3}{2}x\right)\log(3)}{\log(3)^2+4} - \frac{2\sin\left(\frac{3}{2}x\right)}{\log(3)^2+4}\right)}{3\cdot 3^{\frac{3}{4}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

```
[Out] -4/3*(cos(3/2*x)*log(3)/(log(3)^2 + 4) - 2*sin(3/2*x)/(log(3)^2 + 4))/3^(3/
4*x)
```

maple [C] time = 0.09, size = 37, normalized size = 0.65

method	result	size
risch	$-\frac{2\left(2\cos\left(\frac{3x}{2}\right)\ln(3)-4\sin\left(\frac{3x}{2}\right)\right)}{3(2i+\ln(3))(-2i+\ln(3))(27^{x})^{\frac{1}{4}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3/2*x)/(3^(3*x))^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*I+ln(3))/(-2*I+ln(3))/(27^x)^(1/4)*(2*cos(3/2*x)*ln(3)-4*sin(3/2*x)
)
```

maxima [A] time = 1.35, size = 31, normalized size = 0.54

$$-\frac{4\left(\cos\left(\frac{3}{2}x\right)\log(3) - 2\sin\left(\frac{3}{2}x\right)\right)}{3\left(\log(3)^2+4\right)3^{\frac{3}{4}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
```

```
[Out] -4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))/((log(3)^2 + 4)*3^(3/4*x))
```

mupad [B] time = 0.04, size = 33, normalized size = 0.58

$$\frac{\frac{3\sin\left(\frac{3x}{2}\right)}{2}-\frac{3\cos\left(\frac{3x}{2}\right)\ln(3)}{4}}{3^{\frac{3x}{4}}\left(\frac{9\ln(3)^2}{16}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((3*x)/2)/(3^(3*x))^(1/4),x)
```

```
[Out] ((3*sin((3*x)/2))/2 - (3*cos((3*x)/2)*log(3))/4)/(3^((3*x)/4)*((9*log(3)^2)
/16 + 9/4))
```

sympy [A] time = 2.45, size = 70, normalized size = 1.23

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2+12\sqrt[4]{3^{3x}}}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2+12\sqrt[4]{3^{3x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

```
[Out] 8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*log
(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4))
```

3.544 $\int e^{mx}\cos^2(x)\,dx$

Optimal. Leaf size=54

$$\frac{2e^{mx}}{m(m^2+4)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4435, 2194}

$$\frac{2e^{mx}}{m(m^2+4)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Cos
[x]*Sin[x])/(4 + m^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\cos^2(x)\,dx&=\frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}+\frac{2\int e^{mx}\,dx}{4+m^2}\\&=\frac{2e^{mx}}{m(4+m^2)}+\frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.72

$$\frac{e^{mx}\left(m^2\cos(2x)+m^2+2m\sin(2x)+4\right)}{2m(m^2+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx}\cos^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)*Cos[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 37, normalized size = 0.69

$$\frac{2\,m\cos(x)e^{(mx)}\sin(x) + \left(m^2\cos(x)^2 + 2\right)e^{(mx)}}{m^3 + 4\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")
[Out] (2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4*m)
```

giac [A] time = 0.60, size = 43, normalized size = 0.80

$$\frac{1}{2}\left(\frac{m\cos\left(2\,x\right)}{m^2+4} + \frac{2\,\sin\left(2\,x\right)}{m^2+4}\right)e^{(mx)} + \frac{e^{(mx)}}{2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="giac")
[Out] 1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e^(m*x)/m
```

maple [C] time = 0.08, size = 41, normalized size = 0.76

method	result	size
risch	$\frac{e^{mx}}{2m}+\frac{e^{(2i+m)x}}{8i+4m}+\frac{e^{x(m-2i)}}{4m-8i}$	41
norman	$\frac{\frac{(m^2+2)e^{mx}}{m(m^2+4)}+\frac{(m^2+2)e^{mx}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m(m^2+4)}+\frac{4\,e^{mx}\tan\left(\frac{x}{2}\right)}{m^2+4}-\frac{4\,e^{mx}\left(\tan^3\left(\frac{x}{2}\right)\right)}{m^2+4}-\frac{2(m^2-2)e^{mx}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m(m^2+4)}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(m*x)/m+1/4/(2*I+m)*exp((2*I+m)*x)+1/4/(m-2*I)*exp(x*(m-2*I))
```

maxima [A] time = 0.62, size = 45, normalized size = 0.83

$$\frac{m^2\cos\left(2\,x\right)e^{(mx)} + 2\,me^{(mx)}\sin\left(2\,x\right) + \left(m^2+4\right)e^{(mx)}}{2\left(m^3 + 4\,m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")
[Out] 1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m*x))/(m^3
+ 4*m)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.69

$$\frac{e^{m\,x}}{2\,m} + \frac{e^{m\,x}\,\left(2\,\sin\left(2\,x\right) + m\,\cos\left(2\,x\right)\right)}{2\,\left(m^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*cos(x)^2,x)
```

```
[Out] exp(m*x)/(2*m) + (exp(m*x)*(2*sin(2*x) + m*cos(2*x)))/(2*(m^2 + 4))
```

sympy [A] time = 2.81, size = 265, normalized size = 4.91

$$\begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = 0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4} + \frac{ixe^{-2ix}\sin(x)\cos(x)}{2} + \frac{xe^{-2ix}\cos^2(x)}{4} - \frac{e^{-2ix}\sin(x)\cos(x)}{4} + \frac{ie^{-2ix}\cos^2(x)}{2} & \text{for } m = -2i \\ -\frac{xe^{2ix}\sin^2(x)}{4} - \frac{ixe^{2ix}\sin(x)\cos(x)}{2} + \frac{xe^{2ix}\cos^2(x)}{4} - \frac{e^{2ix}\sin(x)\cos(x)}{4} - \frac{ie^{2ix}\cos^2(x)}{2} & \text{for } m = 2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m} + \frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m} + \frac{2e^{mx}\sin^2(x)}{m^3+4m} + \frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)**2,x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x*
exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)*c
os(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq(m,
 -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 + x*ex
p(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x)**2/
2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*sin(x)
*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x)*cos(x
)**2/(m**3 + 4*m), True))
```

3.545 $\int e^{mx}\sin^3(x)\,dx$

Optimal. Leaf size=82

$$\frac{me^{mx}\sin^3(x)}{m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4434, 4432}

$$\frac{me^{mx}\sin^3(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9 + 10*m^2 +
 m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m
^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\sin^3(x)\,dx&=-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}+\frac{6\int e^{mx}\sin(x)\,dx}{9+m^2}\\&=-\frac{6e^{mx}\cos(x)}{9+10m^2+m^4}+\frac{6e^{mx}m\sin(x)}{9+10m^2+m^4}-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}\end{aligned}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 0.78

$$\frac{e^{mx}\left(-3\left(m^2+9\right)\cos(x)+3\left(m^2+1\right)\cos(3x)-2m\sin(x)\left(\left(m^2+1\right)\cos(2x)-m^2-13\right)\right)}{4\left(m^4+10m^2+9\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1
+ m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx}\sin^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)*Sin[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 65, normalized size = 0.79

$$\frac{\left(m^3-\left(m^3+m\right)\cos(x)^2+7\,m\right)e^{(mx)}\sin(x)+3\left(\left(m^2+1\right)\cos(x)^3-\left(m^2+3\right)\cos(x)\right)e^{(mx)}}{m^4+10\,m^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")
```

```
[Out] ((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*cos(x)^3 -
(m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

giac [A] time = 0.61, size = 63, normalized size = 0.77

$$-\frac{1}{4}\left(\frac{m\sin(3\,x)}{m^2+9}-\frac{3\cos(3\,x)}{m^2+9}\right)e^{(mx)}+\frac{3}{4}\left(\frac{m\sin(x)}{m^2+1}-\frac{\cos(x)}{m^2+1}\right)e^{(mx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*(m*sin(x)/
(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

maple [C] time = 0.11, size = 66, normalized size = 0.80

method	result
risch	$\frac{i e^{(3i+m)x}}{24i+8m}-\frac{3i e^{(i+m)x}}{8(i+m)}+\frac{3i e^{x(m-i)}}{8(m-i)}-\frac{i e^{x(m-3i)}}{8(m-3i)}$
norman	$\frac{-\frac{6\,e^{mx}}{m^4+10m^2+9}+\frac{6\,e^{mx}\left(\tan^6\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{12m\,e^{mx}\tan\left(\frac{x}{2}\right)}{m^4+10m^2+9}+\frac{12m\,e^{mx}\left(\tan^5\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}-\frac{6(2m^2+3)e^{mx}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{6(2m^2+3)e^{mx}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{8m(m^2+4)e^{mx}\left(\tan^3\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/(3*I+m)*exp((3*I+m)*x)-3/8*I/(I+m)*exp((I+m)*x)+3/8*I/(m-I)*exp(x*(m-
I))-1/8*I/(m-3*I)*exp(x*(m-3*I))
```

maxima [A] time = 0.55, size = 73, normalized size = 0.89

$$\frac{3\left(m^2+1\right)\cos(3\,x)\,e^{(mx)}-3\left(m^2+9\right)\cos(x)e^{(mx)}-\left(m^3+m\right)e^{(mx)}\sin(3\,x)+3\left(m^3+9\,m\right)e^{(mx)}\sin(x)}{4\left(m^4+10\,m^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) - (m^3 + m)*
e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 + 10*m^2 + 9)
```

mupad [B] time = 0.06, size = 47, normalized size = 0.57

$$-\frac{e^{m\,x}\,\left(\frac{3\,(\cos(x)-m\,\sin(x))}{m^2+1}-\frac{3\,\cos(3\,x)-m\,\sin(3\,x)}{m^2+9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*sin(x)^3,x)
```

```
[Out] -(exp(m*x)*((3*(cos(x) - m*sin(x)))/(m^2 + 1) - (3*cos(3*x) - m*sin(3*x))/(
m^2 + 9)))/4
```

sympy [A] time = 11.65, size = 638, normalized size = 7.78

$$\begin{cases} \frac{xe^{-3ix}\sin^3(x)}{8} - \frac{3ixe^{-3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{-3ix}\sin(x)\cos^2(x)}{8} + \frac{ixe^{-3ix}\cos^3(x)}{8} + \frac{7ie^{-3ix}\sin^3(x)}{24} + \frac{ie^{-3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{-3ix}\cos^3}{8} \\ \frac{3xe^{-ix}\sin^3(x)}{8} - \frac{3ixe^{-ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{-ix}\sin(x)\cos^2(x)}{8} - \frac{3ixe^{-ix}\cos^3(x)}{8} + \frac{5ie^{-ix}\sin^3(x)}{8} + \frac{3ie^{-ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{-ix}\cos^3(x}{8} \\ \frac{3xe^{ix}\sin^3(x)}{8} + \frac{3ixe^{ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{ix}\sin(x)\cos^2(x)}{8} + \frac{3ixe^{ix}\cos^3(x)}{8} - \frac{5ie^{ix}\sin^3(x)}{8} - \frac{3ie^{ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{ix}\cos^3(x)}{8} \\ \frac{xe^{3ix}\sin^3(x)}{8} + \frac{3ixe^{3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{3ix}\sin(x)\cos^2(x)}{8} - \frac{ixe^{3ix}\cos^3(x)}{8} - \frac{7ie^{3ix}\sin^3(x)}{24} - \frac{ie^{3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{3ix}\cos^3(x)}{8} \\ \frac{m^3e^{mx}\sin^3(x)}{m^4+10m^2+9} - \frac{3m^2e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} + \frac{7me^{mx}\sin^3(x)}{m^4+10m^2+9} + \frac{6me^{mx}\sin(x)\cos^2(x)}{m^4+10m^2+9} - \frac{9e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} - \frac{6e^{mx}\cos^3(x)}{m^4+10m^2+9} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)**3,x)
```

```
[Out] Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**2*cos(x)/8
 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)*cos(x)**3/8 + 7*I*e
xp(-3*I*x)*sin(x)**3/24 + I*exp(-3*I*x)*sin(x)*cos(x)**2/4 + exp(-3*I*x)*co
s(x)**3/8, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*sin(x
)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos(x)**
3/8 + 5*I*exp(-I*x)*sin(x)**3/8 + 3*I*exp(-I*x)*sin(x)*cos(x)**2/4 + 3*exp(
-I*x)*cos(x)**3/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*exp(I*x)*s
in(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I*x)*cos(x)
**3/8 - 5*I*exp(I*x)*sin(x)**3/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/4 + 3*exp(
I*x)*cos(x)**3/8, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*exp(3*I*x)*s
in(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(3*I*x)*cos(
x)**3/8 - 7*I*exp(3*I*x)*sin(x)**3/24 - I*exp(3*I*x)*sin(x)*cos(x)**2/4 + e
xp(3*I*x)*cos(x)**3/8, Eq(m, 3*I)), (m**3*exp(m*x)*sin(x)**3/(m**4 + 10*m**
2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) + 7*m*exp(m*
x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)*cos(x)**2/(m**4 + 1
0*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) - 6*exp(m*x)
*cos(x)**3/(m**4 + 10*m**2 + 9), True))
```

3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$

Optimal. Leaf size=79

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2281, 4435, 4433}

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (32*Sin[x/3]
)/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x])
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx &= \frac{e^{x/2}\int e^{-x/2}\cos^3\left(\frac{x}{3}\right)\,dx}{\sqrt{e^x}}\\ &= -\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{\left(8e^{x/2}\right)\int e^{-x/2}\cos\left(\frac{x}{3}\right)\,dx}{15\sqrt{e^x}}\\ &= -\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.46

$$\frac{90\sin\left(\frac{x}{3}\right)+26\sin(x)-135\cos\left(\frac{x}{3}\right)-13\cos(x)}{130\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x/3]^3/Sqrt[E^x],x]
[Out] (-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x/3]^3/Sqrt[E^x],x]
[Out] Could not integrate
```

fricas [A] time = 1.38, size = 42, normalized size = 0.53

$$\frac{4}{65}\left(13\cos\left(\frac{1}{3}x\right)^2+8\right)e^{\left(-\frac{1}{2}x\right)}\sin\left(\frac{1}{3}x\right)-\frac{2}{65}\left(13\cos\left(\frac{1}{3}x\right)^3+24\cos\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")
[Out] 4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1/3*x)^3 +
24*cos(1/3*x))*e^(-1/2*x)
```

giac [A] time = 0.63, size = 33, normalized size = 0.42

$$-\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right)-2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}-\frac{1}{10}\left(\cos(x)-2\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="giac")
[Out] -9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2*sin(x))*e
^(-1/2*x)
```

maple [C] time = 0.10, size = 48, normalized size = 0.61

method	result	size
risch	$\frac{\left(-\frac{1}{1300}-\frac{i}{650}\right)\left(-52i\mathrm{e}^{-ix}+65\,\mathrm{e}^{ix}-39\,\mathrm{e}^{-ix}+(270-540i)\cos\left(\frac{x}{3}\right)+(-180+360i)\sin\left(\frac{x}{3}\right)\right)}{\sqrt{\mathrm{e}^{x}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/3*x)^3/exp(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1/1300-1/650*I)/exp(x)^(1/2)*(-52*I*exp(-I*x)+65*exp(I*x)-39*exp(-I*x)+(2
70-540*I)*cos(1/3*x)+(-180+360*I)*sin(1/3*x))
```

maxima [A] time = 0.63, size = 27, normalized size = 0.34

$$-\frac{1}{130}\left(135\cos\left(\frac{1}{3}x\right)+13\cos(x)-90\sin\left(\frac{1}{3}x\right)-26\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e^(-1/2*x)
```

mupad [B] time = 0.30, size = 39, normalized size = 0.49

$$-\frac{e^{-\frac{x}{2}}\left(\frac{8\cos\left(\frac{x}{3}\right)^3}{5}-\frac{16\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)^2}{5}+\frac{192\cos\left(\frac{x}{3}\right)}{65}-\frac{128\sin\left(\frac{x}{3}\right)}{65}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x/3)^3/exp(x)^(1/2),x)
```

```
[Out] -(exp(-x/2)*((192*cos(x/3))/65 - (128*sin(x/3))/65 - (16*cos(x/3)^2*sin(x/3
))/5 + (8*cos(x/3)^3)/5))/4
```

sympy [A] time = 2.47, size = 76, normalized size = 0.96

$$\frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

```
[Out] 32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sqrt(exp(x))
) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(x/3)**3/(65*sqrt(exp
(x)))
```

3.547 $\int e^{2x} \cos^2(x) \sin^2(x) \, dx$

Optimal. Leaf size=36

$$\frac{e^{2x}}{16}-\frac{1}{40}e^{2x}\sin(4x)-\frac{1}{80}e^{2x}\cos(4x)$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4469, 2194, 4433}

$$\frac{e^{2x}}{16}-\frac{1}{40}e^{2x}\sin(4x)-\frac{1}{80}e^{2x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\cos^2(x)\sin^2(x)\,dx&=\int\left(\frac{e^{2x}}{8}-\frac{1}{8}e^{2x}\cos(4x)\right)dx\\&=\frac{1}{8}\int e^{2x}\,dx-\frac{1}{8}\int e^{2x}\cos(4x)\,dx\\&=\frac{e^{2x}}{16}-\frac{1}{80}e^{2x}\cos(4x)-\frac{1}{40}e^{2x}\sin(4x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.58

$$-\frac{1}{80}e^{2x}(2\sin(4x)+\cos(4x)-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] -1/80*(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.41, size = 40, normalized size = 1.11

$$-\frac{1}{10}\left(2\cos(x)^3-\cos(x)\right)e^{(2x)}\sin(x)-\frac{1}{20}\left(2\cos(x)^4-2\cos(x)^2-1\right)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2*cos(x)^2
- 1)*e^(2*x)
```

giac [A] time = 0.62, size = 24, normalized size = 0.67

$$-\frac{1}{80}\left(\cos(4x)+2\sin(4x)\right)e^{(2x)}+\frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

maple [A] time = 0.08, size = 28, normalized size = 0.78

method	result	size
default	$\frac{e^{2x}}{16}-\frac{e^{2x}\cos(4x)}{80}-\frac{e^{2x}\sin(4x)}{40}$	28
risch	$\frac{e^{2x}}{16}-\frac{e^{(2+4i)x}}{160}+\frac{i e^{(2+4i)x}}{80}-\frac{e^{(2-4i)x}}{160}-\frac{i e^{(2-4i)x}}{80}$	42
norman	$\frac{-\frac{e^{2x}\tan\left(\frac{x}{2}\right)}{5}+\frac{3e^{2x}\left(\tan^2\left(\frac{x}{2}\right)\right)}{5}+\frac{7e^{2x}\left(\tan^3\left(\frac{x}{2}\right)\right)}{5}-\frac{e^{2x}\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\frac{7e^{2x}\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{3e^{2x}\left(\tan^6\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}\left(\tan^7\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}\left(\tan^8\left(\frac{x}{2}\right)\right)}{20}+\frac{e^{2x}}{20}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)+1/16*exp(x)^2
```

maxima [A] time = 0.53, size = 27, normalized size = 0.75

$$-\frac{1}{80}\cos(4x)e^{(2x)}-\frac{1}{40}e^{(2x)}\sin(4x)+\frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

mupad [B] time = 0.41, size = 18, normalized size = 0.50

$$-\frac{e^{2x}\left(\cos(4x)+2\sin(4x)-5\right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

```
[Out] -(exp(2*x)*(cos(4*x) + 2*sin(4*x) - 5))/80
```

sympy [B] time = 5.06, size = 70, normalized size = 1.94

$$\frac{e^{2x}\sin^4(x)}{20}+\frac{e^{2x}\sin^3(x)\cos(x)}{10}+\frac{e^{2x}\sin^2(x)\cos^2(x)}{5}-\frac{e^{2x}\sin(x)\cos^3(x)}{10}+\frac{e^{2x}\cos^4(x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

```
[Out] exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*sin(x)**2*c
os(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*cos(x)**4/20
```

3.548 $\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) dx$

Optimal. Leaf size=36

$$\frac{e^{3x}}{24} - \frac{1}{60} e^{3x} \sin(6x) - \frac{1}{120} e^{3x} \cos(6x)$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4469, 2194, 4433}

$$\frac{e^{3x}}{24} - \frac{1}{60} e^{3x} \sin(6x) - \frac{1}{120} e^{3x} \cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) dx &= \int \left(\frac{e^{3x}}{8} - \frac{1}{8} e^{3x} \cos(6x)\right) dx \\
&= \frac{1}{8} \int e^{3x} dx - \frac{1}{8} \int e^{3x} \cos(6x) dx \\
&= \frac{e^{3x}}{24} - \frac{1}{120} e^{3x} \cos(6x) - \frac{1}{60} e^{3x} \sin(6x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.58

$$-\frac{1}{120} e^{3x} (2 \sin(6x) + \cos(6x) - 5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] -1/120*(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.37, size = 50, normalized size = 1.39

$$-\frac{1}{15}\left(2\cos\left(\frac{3}{2}x\right)^3-\cos\left(\frac{3}{2}x\right)\right)e^{(3x)}\sin\left(\frac{3}{2}x\right)-\frac{1}{30}\left(2\cos\left(\frac{3}{2}x\right)^4-2\cos\left(\frac{3}{2}x\right)^2-1\right)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x)
^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

giac [A] time = 0.65, size = 24, normalized size = 0.67

$$-\frac{1}{120}\left(\cos\left(6x\right)+2\sin\left(6x\right)\right)e^{(3x)}+\frac{1}{24}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="giac")
```

```
[Out] -1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)
```

maple [C] time = 0.08, size = 42, normalized size = 1.17

method	result
risch	$\frac{e^{3x}}{24}-\frac{e^{(3+6i)x}}{240}+\frac{i e^{(3+6i)x}}{120}-\frac{e^{(3-6i)x}}{240}-\frac{i e^{(3-6i)x}}{120}$
norman	$\frac{-\frac{2e^{3x}\tan\left(\frac{3x}{4}\right)}{15}+\frac{2e^{3x}\left(\tan^2\left(\frac{3x}{4}\right)\right)}{5}+\frac{14e^{3x}\left(\tan^3\left(\frac{3x}{4}\right)\right)}{15}-\frac{e^{3x}\left(\tan^4\left(\frac{3x}{4}\right)\right)}{3}-\frac{14e^{3x}\left(\tan^5\left(\frac{3x}{4}\right)\right)}{15}+\frac{2e^{3x}\left(\tan^6\left(\frac{3x}{4}\right)\right)}{5}+\frac{2e^{3x}\left(\tan^7\left(\frac{3x}{4}\right)\right)}{15}+\frac{e^{3x}\left(\tan^8\left(\frac{3x}{4}\right)\right)}{30}}{\left(1+\tan^2\left(\frac{3x}{4}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(3*x)-1/240*exp((3+6*I)*x)+1/120*I*exp((3+6*I)*x)-1/240*exp((3-6*I)
*x)-1/120*I*exp((3-6*I)*x)
```

maxima [A] time = 0.52, size = 27, normalized size = 0.75

$$-\frac{1}{120}\cos\left(6x\right)e^{(3x)}-\frac{1}{60}e^{(3x)}\sin\left(6x\right)+\frac{1}{24}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

mupad [B] time = 0.39, size = 18, normalized size = 0.50

$$-\frac{e^{3x}\left(\cos\left(6x\right)+2\sin\left(6x\right)-5\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((3*x)/2)^2*sin((3*x)/2)^2*exp(3*x),x)
```

```
[Out] -(exp(3*x)*(cos(6*x) + 2*sin(6*x) - 5))/120
```

sympy [B] time = 4.87, size = 99, normalized size = 2.75

$$\frac{e^{3x}\sin^{4}\left(\frac{3x}{2}\right)}{30}+\frac{e^{3x}\sin^{3}\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15}+\frac{2e^{3x}\sin^{2}\left(\frac{3x}{2}\right)\cos^{2}\left(\frac{3x}{2}\right)}{15}-\frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^{3}\left(\frac{3x}{2}\right)}{15}+\frac{e^{3x}\cos^{4}\left(\frac{3x}{2}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

```
[Out] exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3*
x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15 +
exp(3*x)*cos(3*x/2)**4/30
```

3.549 $\int e^{mx} \tan^2(x)\,dx$

Optimal. Leaf size=58

$$-\frac{e^{mx}}{m}+\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i}$$

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4442, 2194, 2251}

$$\frac{4e^{mx}\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{4e^{mx}\text{Hypergeometric2F1}\left(2,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Tan[x]^2,x]
```

```
[Out] -(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-I/2)*m, 1 - (I/2)*m, -E^((
2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-I/2)*m, 1 - (I/2)*m, -E^((2
*I)*x)])/m
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{mx}\tan^2(x)\,dx &= -\int\left(e^{mx}+\frac{4e^{mx}}{\left(1+e^{2ix}\right)^2}-\frac{4e^{mx}}{1+e^{2ix}}\right)dx\\
&= -\left(4\int\frac{e^{mx}}{\left(1+e^{2ix}\right)^2}\,dx\right)+4\int\frac{e^{mx}}{1+e^{2ix}}\,dx-\int e^{mx}\,dx\\
&= -\frac{e^{mx}}{m}+\frac{4e^{mx}\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m}-\frac{4e^{mx}\,{}_2F_1\left(2,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 97, normalized size = 1.67

$$\frac{e^{mx}\left(\frac{im^2e^{2ix}{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i}-im\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)+m\tan(x)-1\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Tan[x]^2,x]
```

```
[Out] (E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/
2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (
I/2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx}\tan^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)*Tan[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(mx)}\tan(x)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(m*x)*tan(x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(mx)}\tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)*tan(x)^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{mx}\left(\tan^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*tan(x)^2,x)
```

```
[Out] int(exp(m*x)*tan(x)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{m x} \tan(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*tan(x)^2,x)
```

```
[Out] int(exp(m*x)*tan(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx} \tan^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)**2,x)
```

```
[Out] Integral(exp(m*x)*tan(x)**2, x)
```

3.550 $\int e^{mx}\csc^2(x)\,dx$

Optimal. Leaf size=45

$$-\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{m+2i}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4453}

$$-\frac{4e^{(m+2i)x}\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{m+2i}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I)
*x)])/(2*I + m)
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int e^{mx}\csc^2(x)\,dx=-\frac{4e^{(2i+m)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{2i+m}$$

Mathematica [A] time = 0.20, size = 90, normalized size = 2.00

$$\frac{e^{mx}\left(me^{2ix}\,{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)+(m+2i)\left({}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};e^{2ix}\right)-i\cot(x)\right)\right)}{-2+im}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^((
2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I
/2)*m, E^((2*I)*x)])))/(-2 + I*m)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx}\csc^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)*Csc[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{e^{(mx)}}{\cos(x)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")
[Out] integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="giac")
[Out] integrate(e^(m*x)/sin(x)^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/sin(x)^2,x)
[Out] int(exp(m*x)/sin(x)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{m\,x}}{{\sin(x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/sin(x)^2,x)
[Out] int(exp(m*x)/sin(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)**2,x)
[Out] Integral(exp(m*x)/sin(x)**2, x)
```

3.551 $\int e^{mx} \sec^3(x)\,dx$

Optimal. Leaf size=51

$$\frac{8e^{(m+3i)x}\,{}_2F_1\left(3,\frac{1}{2}(3-im);\frac{1}{2}(5-im);-e^{2ix}\right)}{m+3i}$$

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4448, 4451}

$$(-m+i)\left(-e^{(m+i)x}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-im),\frac{1}{2}(3-im),-e^{2ix}\right)-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Sec[x]^3,x]
```

```
[Out] -(E^((I + m)*x)*(I - m)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*m)/2, -E^(
(2*I)*x)]) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan[x])/2
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\sec^3(x)\,dx &= -\frac{1}{2}e^{mx}m\sec(x)+\frac{1}{2}e^{mx}\sec(x)\tan(x)+\frac{1}{2}\left(1+m^2\right)\int e^{mx}\sec(x)\,dx\\ &= -e^{(i+m)x}(i-m)\,{}_2F_1\left(1,\frac{1}{2}(1-im);\frac{1}{2}(3-im);-e^{2ix}\right)-\frac{1}{2}e^{mx}m\sec(x)+\frac{1}{2}e^{mx}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.29

$$\frac{1}{2}e^{mx}\left(\sec(x)(\tan(x)-m)+2(m-i)e^{ix}\,{}_2F_1\left(1,\frac{1}{2}-\frac{im}{2};\frac{3}{2}-\frac{im}{2};-e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Sec[x]^3,x]
```

```
[Out] (E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/2
)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx}\sec^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)*Sec[x]^3,x]
[Out] Could not integrate
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(mx)}}{\cos(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")
[Out] integral(e^(m*x)/cos(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="giac")
[Out] integrate(e^(m*x)/cos(x)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/cos(x)^3,x)
[Out] int(exp(m*x)/cos(x)^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{m\,x}}{{\cos(x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/cos(x)^3,x)
[Out] int(exp(m*x)/cos(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)**3,x)
```

```
[Out] Integral(exp(m*x)/cos(x)**3, x)
```

3.552 $\int \frac{e^x}{1+\cos(x)} \, dx$

Optimal. Leaf size=28

$$(1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4457, 4451}

$$(1-i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + Cos[x]),x]
```

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4457

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+\cos(x)}\,dx &= \frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}\right)dx\\ &= (1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$(1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + Cos[x]),x]
```

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1+\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(1 + Cos[x]),x]
```

```
[Out] Could not integrate
```

fricas [F]   time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{x}}{\cos(x)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] integral(e^x/(cos(x) + 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="giac")
```

```
[Out] integrate(e^x/(cos(x) + 1), x)
```

maple [F]   time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{1+\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+cos(x)),x)
```

```
[Out] int(exp(x)/(1+cos(x)),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1\right)\int \frac{e^{x}\sin(x)}{\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1}\,dx-e^{x}\sin(x)\right)}{\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e^{x}}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cos(x) + 1),x)
```

```
[Out] int(exp(x)/(cos(x) + 1), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{\cos{\left(x \right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x)
```

```
[Out] Integral(exp(x)/(cos(x) + 1), x)
```

3.553 $\int \frac{e^x}{1-\cos(x)}\,dx$

Optimal. Leaf size=26

$$(-1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4458, 4453}

$$(-1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Cos[x]),x]
```

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rule 4458

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Sin[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1-\cos(x)}\,dx &= \frac{1}{2}\int e^x\csc^2\left(\frac{x}{2}\right)dx\\ &= (-1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)\end{aligned}$$

Mathematica [B] time = 0.08, size = 84, normalized size = 3.23

$$\frac{(1+i)e^x\sin\left(\frac{x}{2}\right)\left((1+i)\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1-i)\cos\left(\frac{x}{2}\right)\right)}{\cos(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Cos[x]),x]
```

```
[Out] ((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1,
1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*
x)]*Sin[x/2]))/(-1 + Cos[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1-\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(1 - Cos[x]),x]
[Out] Could not integrate
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{e^{x}}{\cos(x)-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")
[Out] integral(-e^x/(cos(x) - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{e^{x}}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="giac")
[Out] integrate(-e^x/(cos(x) - 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{1-\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-cos(x)),x)
[Out] int(exp(x)/(1-cos(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\left(\cos(x)^{2}+\sin(x)^{2}-2\,\cos(x)+1\right)\int \frac{e^{x}\sin(x)}{\cos(x)^{2}+\sin(x)^{2}-2\,\cos(x)+1}\,dx-e^{x}\sin(x)\right)}{\cos(x)^{2}+\sin(x)^{2}-2\,\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")
[Out] 2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) +
1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{e^{x}}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(x)/(cos(x) - 1),x)
[Out] -int(exp(x)/(cos(x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^x}{\cos(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x)
```

3.554 $\int \frac{e^x}{1+\sin(x)}\,dx$

Optimal. Leaf size=30

$$(-1+i)e^{(1-i)x}\,{}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4456, 4450}

$$(-1+i)e^{(1-i)x}\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + Sin[x]),x]
```

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^(I*x)]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+\sin(x)}\,dx &= \frac{1}{2}\int e^x\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx\\ &= (-1+i)e^{(1-i)x}\,{}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)\end{aligned}$$

Mathematica [B] time = 0.62, size = 61, normalized size = 2.03

$$\frac{2e^x\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}-(1-i)(\sinh(x)+\cosh(x))(1-(1-i)\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometri
c2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(1 + Sin[x]),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{x}}{\sin(x)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] integral(e^x/(sin(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="giac")
```

```
[Out] integrate(e^x/(sin(x) + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{x}}{1+\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+sin(x)),x)
```

```
[Out] int(exp(x)/(1+sin(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\cos(x)e^{x}-\left(\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1\right)\int\frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}\,dx\right)}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x/
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) +
 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{e^{x}}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(sin(x) + 1),x)
```

```
[Out] int(exp(x)/(sin(x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x)
```

```
[Out] Integral(exp(x)/(sin(x) + 1), x)
```

3.555 $\int \frac{e^x}{1-\sin(x)}\,dx$

Optimal. Leaf size=30

$$(1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4456, 4450}

$$(1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Sin[x]),x]
```

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1-\sin(x)}\,dx &= \frac{1}{2}\int e^x \sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)dx\\ &= (1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)\end{aligned}$$

Mathematica [B] time = 0.68, size = 61, normalized size = 2.03

$$\frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+(1+i)(\sinh(x)+\cosh(x))(1-(1+i)\,{}_2F_1(-i,1;1-i;\sin(x)-i\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometri
c2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1-\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(1 - Sin[x]),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{e^x}{\sin(x)-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] integral(-e^x/(sin(x) - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{e^x}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="giac")
```

```
[Out] integrate(-e^x/(sin(x) - 1), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-sin(x)),x)
```

```
[Out] int(exp(x)/(1-sin(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\cos(x)e^x - \left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\int \frac{\cos(x)e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}\,dx\right)}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/(
cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) +
1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{e^x}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(x)/(sin(x) - 1),x)
```

```
[Out] -int(exp(x)/(sin(x) - 1), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^{x}}{\sin(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x)
```

```
[Out] -Integral(exp(x)/(sin(x) - 1), x)
```

3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=15

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2288}

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.23, size = 11, normalized size = 0.73

$$-e^x\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -(E^x*Cot[x/2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 12, normalized size = 0.80

$$-\frac{(\cos(x)+1)e^x}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)*e^x/sin(x)
```

giac [A] time = 0.64, size = 10, normalized size = 0.67

$$-\frac{e^x}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="giac")
[Out] -e^x/tan(1/2*x)
```

maple [C] time = 0.10, size = 21, normalized size = 1.40

method	result	size
risch	$-ie^x - \frac{2ie^x}{e^{ix}-1}$	21
norman	$\frac{-e^x\left(\tan^2\left(\frac{x}{2}\right)\right)-e^x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sin(x))/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -I*exp(x)-2*I*exp(x)/(exp(I*x)-1)
```

maxima [A] time = 0.72, size = 22, normalized size = 1.47

$$-\frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")
[Out] -2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 0.39, size = 8, normalized size = 0.53

$$-\cot\left(\frac{x}{2}\right)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(sin(x) - 1))/(cos(x) - 1),x)
[Out] -cot(x/2)*exp(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(\sin(x)-1)\,e^x}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
[Out] Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=41

$$\frac{e^x\sin(x)}{1-\cos(x)}-(2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

Rubi [A] time = 0.11, antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4463, 4461, 4443, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)+2ie^x-\frac{e^x\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)] - (E^x*Sin[x
])/(1 - Cos[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 4461

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4463

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
```

```
f*i, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx &= 2\int \frac{e^x\sin(x)}{1-\cos(x)}\,dx+\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}+2\int e^x\cot\left(\frac{x}{2}\right)dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}-2i\int\left(-e^x-\frac{2e^x}{-1+e^{ix}}\right)dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}+2i\int e^x\,dx+4i\int\frac{e^x}{-1+e^{ix}}\,dx\\
&= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)-\frac{e^x\sin(x)}{1-\cos(x)}
\end{aligned}
$$

Mathematica [B] time = 0.22, size = 100, normalized size = 2.44

$$
\frac{2e^x\sin\left(\frac{x}{2}\right)(\sin(x)+1)\left(2i\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{(\cos(x)-1)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
[Out] (2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]*
Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Sin[
x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
[Out] Could not integrate
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{e^x\sin(x)+e^x}{\cos(x)-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
[Out] integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{x} \left(1+\sin \left(x\right)\right)}{1-\cos \left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left(2 \left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \cos \left(x\right)+1\right) \int \frac{e^{x} \sin \left(x\right)}{\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \cos \left(x\right)+1}\, dx - e^{x} \sin \left(x\right)\right)}{\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \cos \left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] 2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{e^{x} \left(\sin \left(x\right)+1\right)}{\cos \left(x\right)-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1),x)
```

```
[Out] int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^{x}}{\cos{\left(x \right)} - 1}\, dx - \int \frac{e^{x} \sin{\left(x \right)}}{\cos{\left(x \right)} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x)
```

3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2288}

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x \sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.19, size = 10, normalized size = 0.83

$$e^x \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
[Out] E^x*Tan[x/2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
[Out] Could not integrate
```

fricas [A] time = 1.30, size = 11, normalized size = 0.92

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] e^x*sin(x)/(cos(x) + 1)
```

giac [A] time = 0.63, size = 7, normalized size = 0.58

$$e^{x} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="giac")
[Out] e^x*tan(1/2*x)
```

maple [A] time = 0.08, size = 8, normalized size = 0.67

method	result	size
norman	$e^{x} \tan\left(\frac{x}{2}\right)$	8
risch	$-ie^{x} + \frac{2ie^{x}}{e^{ix}+1}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sin(x))/(1+cos(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)*tan(1/2*x)
```

maxima [A] time = 0.72, size = 22, normalized size = 1.83

$$\frac{2 e^{x} \sin(x)}{\cos(x)^{2} + \sin(x)^{2} + 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")
[Out] 2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

mupad [B] time = 0.37, size = 7, normalized size = 0.58

$$\tan\left(\frac{x}{2}\right) e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(sin(x) + 1))/(cos(x) + 1),x)
[Out] tan(x/2)*exp(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sin(x) + 1) e^{x}}{\cos(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
[Out] Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{e^x \sin(x)}{\cos(x)+1}+(2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

Rubi [A] time = 0.11, antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4463, 4460, 4442, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)+2ie^x+\frac{e^x\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)] + (E^x*Sin[
x])/(1 + Cos[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4460

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4463

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
```

```
f*i, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx &= -\left(2\int \frac{e^x\sin(x)}{1+\cos(x)}\,dx\right)+\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx\\
&= \frac{e^x\sin(x)}{1+\cos(x)}-2\int e^x\tan\left(\frac{x}{2}\right)dx\\
&= \frac{e^x\sin(x)}{1+\cos(x)}-2i\int\left(-e^x+\frac{2e^x}{1+e^{ix}}\right)dx\\
&= \frac{e^x\sin(x)}{1+\cos(x)}+2i\int e^x\,dx-4i\int \frac{e^x}{1+e^{ix}}\,dx\\
&= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)+\frac{e^x\sin(x)}{1+\cos(x)}
\end{aligned}$$

Mathematica [B] time = 0.23, size = 87, normalized size = 2.07

$$-\frac{2e^x\cos\left(\frac{x}{2}\right)\left(2i\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
[Out] (-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
- (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] - S
in[x/2]))/(1 + Cos[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
[Out] Could not integrate
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{e^x\sin(x)-e^x}{\cos(x)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
[Out] integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

maple [F]   time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{e^{x} (1 - \sin(x))}{1 + \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(2\left(\cos(x)^{2} + \sin(x)^{2} + 2\cos(x) + 1\right)\int \frac{e^{x}\sin(x)}{\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1}\, dx - e^{x}\sin(x)\right)}{\cos(x)^{2} + \sin(x)^{2} + 2\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{e^{x}\,(\sin(x) - 1)}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x)*(sin(x) - 1))/(cos(x) + 1),x)
```

```
[Out] -int((exp(x)*(sin(x) - 1))/(cos(x) + 1), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^{x}}{\cos(x) + 1}\right) dx - \int \frac{e^{x}\sin(x)}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] -Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1), x
)
```

3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{e^x\cos(x)}{1-\sin(x)}+(2+2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

Rubi [A] time = 0.13, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4462, 4459, 4442, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)+2ie^x+\frac{e^x\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)] + (E^x*
Cos[x])/(1 - Sin[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4462

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
```

```
f*i, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx &= -\left(2\int \frac{e^x\cos(x)}{1-\sin(x)}\,dx\right)+\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2\int e^x\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2i\int \left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\right)dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}+2i\int e^x\,dx-4i\int \frac{e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\,dx\\
&= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;-ie^{ix}\right)+\frac{e^x\cos(x)}{1-\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.76, size = 72, normalized size = 1.57

$$
\frac{1}{2}(\cos(x)-1)\csc^2\left(\frac{x}{2}\right)\left(4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;\sin(x)-i\cos(x))-\frac{e^x\left((1+2i)\cot\left(\frac{x}{2}\right)+(1-2i)\right)}{\cot\left(\frac{x}{2}\right)-1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] ((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]))/(-1 + C
ot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]]*(Co
sh[x] + Sinh[x])))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{(\cos(x)-1)e^x}{\sin(x)-1},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{e^{x} (1-\cos(x))}{1-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\cos(x)e^{x}-2\left(\cos(x)^{2}+\sin(x)^{2}-2\,\sin(x)+1\right)\int\frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}-2\,\sin(x)+1}\,dx\right)}{\cos(x)^{2}+\sin(x)^{2}-2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x
/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{x}\,(\cos(x)-1)}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(cos(x) - 1))/(sin(x) - 1),x)
```

```
[Out] int((exp(x)*(cos(x) - 1))/(sin(x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(x)-1)\,e^{x}}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{e^x\cos(x)}{1-\sin(x)}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{e^x\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx=\frac{e^x\cos(x)}{1-\sin(x)}$$

Mathematica [A] time = 0.08, size = 23, normalized size = 1.64

$$-\frac{e^x\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan\left(\frac{x}{2}\right)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] -((E^x*(1 + Tan[x/2]))/(-1 + Tan[x/2]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 24, normalized size = 1.71

$$\frac{(\cos(x)+1)e^x+e^x\sin(x)}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
[Out] ((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

giac [A] time = 0.61, size = 20, normalized size = 1.43

$$-\frac{e^{x}\tan\left(\frac{1}{2}x\right)+e^{x}}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="giac")
[Out] -(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

maple [C] time = 0.12, size = 21, normalized size = 1.50

method	result	size
risch	$-ie^{x}+\frac{2\,e^{x}}{e^{ix}-i}$	21
norman	$\frac{-e^{x}\tan\left(\frac{x}{2}\right)-e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-e^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-e^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+cos(x))/(1-sin(x)),x,method=_RETURNVERBOSE)
[Out] -I*exp(x)+2*exp(x)/(exp(I*x)-I)
```

maxima [A] time = 0.74, size = 22, normalized size = 1.57

$$\frac{2\,\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}-2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
[Out] 2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 0.37, size = 24, normalized size = 1.71

$$-\frac{e^{x}\left(-1+e^{x\,1i}\,1i\right)}{e^{x\,1i}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x)*(cos(x) + 1))/(sin(x) - 1),x)
[Out] -(exp(x)*(exp(x*1i)*1i - 1))/(exp(x*1i) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{e^{x}}{\sin\left(x\right)-1}\,dx-\int\frac{e^{x}\cos\left(x\right)}{\sin\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
[Out] -Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x)
```

3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)} dx$

Optimal. Leaf size=43

$$\frac{e^x \cos(x)}{\sin(x)+1} - (2+2i)e^{(1+i)x} {}_2F_1\left(1-i,2;2-i;ie^{ix}\right)$$

Rubi [A] time = 0.13, antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4462, 4459, 4442, 2194, 2251, 2288}

$$4ie^x \text{Hypergeometric2F1}\left(i,1,1+i,-ie^{-ix}\right) - 2ie^x - \frac{e^x \cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] (-2*I)*E^x + (4*I)*E^x*Hypergeometric2F1[I, 1, 1 + I, (-I)/E^(I*x)] - (E^x*
Cos[x])/(1 + Sin[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4462

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
```

```
f*i, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx &= 2\int \frac{e^x\cos(x)}{1+\sin(x)}\,dx+\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)}+2\int e^x\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)}+2i\int \left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right)dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)}-2i\int e^x\,dx+4i\int \frac{e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\,dx \\
&= -2ie^x+4ie^x\,{}_2F_1\left(i,1;1+i;-ie^{-ix}\right)-\frac{e^x\cos(x)}{1+\sin(x)}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 73, normalized size = 1.70

$$\frac{1}{2}(\cos(x)+1)\sec^2\left(\frac{x}{2}\right)\left(\frac{e^x\left((1+2i)\tan\left(\frac{x}{2}\right)-(1-2i)\right)}{\tan\left(\frac{x}{2}\right)+1}-4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
[Out] ((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x] -
 Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2]))/(1 +
 Tan[x/2])))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
[Out] Could not integrate
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(\cos(x)+1)e^x}{\sin(x)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
[Out] integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{x} (1+\cos(x))}{1+\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] int(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\cos(x)e^{x}-2\left(\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1\right)\int\frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}\,dx\right)}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{x}\,\left(\cos(x)+1\right)}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(cos(x) + 1))/(sin(x) + 1),x)
```

```
[Out] int((exp(x)*(cos(x) + 1))/(sin(x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos{\left(x \right)} + 1\right) e^{x}}{\sin{\left(x \right)} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx$

Optimal. Leaf size=13

$$-\frac{e^x \cos(x)}{\sin(x)+1}$$

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$-\frac{e^x \cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx = -\frac{e^x \cos(x)}{1+\sin(x)}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 1.77

$$-\frac{e^x \left(\cot\left(\frac{x}{2}\right)-1\right)}{\cot\left(\frac{x}{2}\right)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
[Out] -((E^x*(-1 + Cot[x/2]))/(1 + Cot[x/2]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
[Out] Could not integrate
```

fricas [A] time = 1.30, size = 24, normalized size = 1.85

$$-\frac{(\cos(x)+1)e^x - e^x \sin(x)}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

giac [A] time = 0.63, size = 21, normalized size = 1.62

$$\frac{e^{x} \tan\left(\frac{1}{2}\,x\right) - e^{x}}{\tan\left(\frac{1}{2}\,x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] (e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

maple [C] time = 0.11, size = 21, normalized size = 1.62

method	result	size
risch	$-i e^{x} - \frac{2\,e^{x}}{e^{i x}+i}$	21
norman	$\frac{e^{x}\tan\left(\frac{x}{2}\right)+e^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-e^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-cos(x))/(1+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*exp(x)-2*exp(x)/(exp(I*x)+I)
```

maxima [A] time = 0.72, size = 22, normalized size = 1.69

$$-\frac{2\,\cos\left(x\right)e^{x}}{\cos\left(x\right)^{2} + \sin\left(x\right)^{2} + 2\,\sin\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

mupad [B] time = 0.42, size = 20, normalized size = 1.54

$$-e^{x}\,1{}\mathrm{i} - \frac{2\,e^{x}}{e^{x\,1{}\mathrm{i}} + 1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x)*(cos(x) - 1))/(sin(x) + 1),x)
```

```
[Out] - exp(x)*1i - (2*exp(x))/(exp(x*1i) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^{x}}{\sin\left(x\right) + 1}\right)\,dx - \int \frac{e^{x}\cos\left(x\right)}{\sin\left(x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

```
[Out] -Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1), x
)
```

3.564 $\int e^x x \cos(x) \, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)+\frac{1}{2}e^x x\cos(x)$$

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)+\frac{1}{2}e^x x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x\cos(x)\,dx &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\
&= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\
&= \frac{1}{2}e^x x\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.60

$$\frac{1}{2}e^x((x-1)\sin(x)+x\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x x \cos(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x*x*Cos[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.38, size = 17, normalized size = 0.57

$$\frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x-1) e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

giac [A] time = 0.62, size = 15, normalized size = 0.50

$$\frac{1}{2} (x \cos(x) + (x-1) \sin(x)) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

maple [A] time = 0.06, size = 20, normalized size = 0.67

method	result	size
default	$\frac{e^x x \cos(x)}{2} - \left(-\frac{x}{2} + \frac{1}{2}\right) e^x \sin(x)$	20
risch	$\left(\frac{1}{8} - \frac{i}{8}\right) (-1 + i + 2x) e^{(1+i)x} + \left(\frac{1}{8} + \frac{i}{8}\right) (-1 - i + 2x) e^{(1-i)x}$	36
norman	$\frac{e^x x \tan\left(\frac{x}{2}\right) + \frac{e^x x}{2} - e^x \tan\left(\frac{x}{2}\right) - \frac{e^x x \left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.57

$$\frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x-1) e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.57

$$\frac{e^x \left(x \cos(x) - \sin(x) + x \sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

sympy [**A**] time = 0.83, size = 27, normalized size = 0.90

$$\frac{xe^{x}\sin(x)}{2}+\frac{xe^{x}\cos(x)}{2}-\frac{e^{x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

3.565 $\int e^x x^2 \sin(x)\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x
*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int e^x x^2 \sin(x)\,dx &= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int x\left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)dx \\ &= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int\left(-\frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x)\right)dx \\ &= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) + \int e^x x \cos(x)\,dx - \int e^x x \sin(x)\,dx \\ &= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) + \int\left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)dx - \int\left(\frac{1}{2}e^x \cos(x \right. \\ &= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\left(\frac{1}{2}\int e^x \cos(x)\,dx\right) \\ &= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\left(\frac{1}{4}e^x \cos(x) + \frac{1}{4}e^x \sin(x)\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.50

$$\frac{1}{2}e^x\left(\left(x^2-1\right)\sin(x) - (x-1)^2\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Sin[x],x]
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x x^2 \sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x*x^2*Sin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.40, size = 26, normalized size = 0.52

$$-\frac{1}{2}\left(x^2 - 2x + 1\right)\cos(x)e^x + \frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

giac [A] time = 0.60, size = 25, normalized size = 0.50

$$-\frac{1}{2}\left(\left(x^2-2x+1\right)\cos(x) - \left(x^2-1\right)\sin(x)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

maple [A] time = 0.06, size = 27, normalized size = 0.54

method	result	size
default	$\left(-\frac{1}{2}x^2+x-\frac{1}{2}\right)\mathrm{e}^x\cos(x)+\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\sin(x)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^2+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^2-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^x x+\mathrm{e}^x x^2\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^x x^2}{2}-\mathrm{e}^x\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\mathrm{e}^x x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\mathrm{e}^x x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\mathrm{e}^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)
```

maxima [A] time = 0.46, size = 26, normalized size = 0.52

$$-\frac{1}{2}\left(x^2-2\,x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

mupad [B] time = 0.33, size = 21, normalized size = 0.42

$$\frac{\mathrm{e}^x\,\left(x-1\right)\,\left(\cos(x)+\sin(x)-x\,\cos(x)+x\,\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x)*sin(x),x)
```

```
[Out] (exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

sympy [A] time = 2.06, size = 48, normalized size = 0.96

$$\frac{x^2e^x\sin\left(x\right)}{2}-\frac{x^2e^x\cos\left(x\right)}{2}+xe^x\cos\left(x\right)-\frac{e^x\sin\left(x\right)}{2}-\frac{e^x\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

3.566 $\int e^{-3x}x^2\sin(x)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4432, 4465, 14, 4433, 4466}

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos[x])/(10*E
^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E^(3*x)) - (3*x^2*Sin
[x])/(10*E^(3*x))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-3x}x^2\sin(x)\,dx &= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-2\int x\left(-\frac{1}{10}e^{-3x}\cos(x)-\frac{3}{10}e^{-3x}\sin(x)\right)dx\\ &= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-2\int\left(-\frac{1}{10}e^{-3x}x\cos(x)-\frac{3}{10}e^{-3x}x\sin(x)\right)dx\\ &= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)+\frac{1}{5}\int e^{-3x}x\cos(x)\,dx+\frac{3}{5}\int e^{-3x}x\sin(x)\,dx\\ &= -\frac{3}{25}e^{-3x}x\cos(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{5}\int\left(-\frac{3}{10}e^{-3x}\right.\\ &= -\frac{3}{25}e^{-3x}x\cos(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{50}\int e^{-3x}\sin(\\ &= -\frac{2}{125}e^{-3x}\cos(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{6}{125}e^{-3x}\sin(x)-\frac{4}{25}e^{-3x}x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.51

$$\frac{1}{250}e^{-3x}\left(-\left(75x^2+40x+9\right)\sin(x)-\left(\left(25x^2+30x+13\right)\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[x])/E^(3*x),x]
[Out] (-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-3x}x^2\sin(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*Sin[x])/E^(3*x),x]
[Out] Could not integrate
```

fricas [A] time = 1.11, size = 37, normalized size = 0.49

$$-\frac{1}{250}\left(25\,x^2+30\,x+13\right)\cos(x)e^{(-3\,x)}-\frac{1}{250}\left(75\,x^2+40\,x+9\right)e^{(-3\,x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")
[Out] -1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*x + 9)*e^(
-3*x)*sin(x)
```

giac [A] time = 0.57, size = 33, normalized size = 0.44

$$-\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos(x)+\left(75\,x^2+40\,x+9\right)\sin(x)\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="giac")
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

maple [A] time = 0.08, size = 36, normalized size = 0.48

method	result
default	$\left(-\frac{1}{10}x^2-\frac{3}{25}x-\frac{13}{250}\right)\mathrm{e}^{-3x}\cos(x)+\left(-\frac{3}{10}x^2-\frac{4}{25}x-\frac{9}{250}\right)\mathrm{e}^{-3x}\sin(x)$
risch	$\left(-\frac{1}{500}+\frac{3i}{500}\right)\left(25x^2+5ix+15x+3i+4\right)\mathrm{e}^{(-3+i)x}+\left(-\frac{1}{500}-\frac{3i}{500}\right)\left(25x^2-5ix+15x-3i+4\right)\mathrm{e}^{(-3-i)x}$
norman	$\frac{\left(-\frac{13}{250}-\frac{3x}{25}-\frac{x^2}{10}+\frac{13\left(\tan^2\left(\frac{x}{2}\right)\right)}{250}-\frac{8x\tan\left(\frac{x}{2}\right)}{25}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{25}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{5}+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{10}-\frac{9\tan\left(\frac{x}{2}\right)}{125}\right)\mathrm{e}^{-3x}}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)/exp(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/10*x^2-3/25*x-13/250)*exp(-3*x)*cos(x)+(-3/10*x^2-4/25*x-9/250)*exp(-3*
x)*sin(x)
```

maxima [A] time = 0.46, size = 33, normalized size = 0.44

$$-\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos(x)+\left(75\,x^2+40\,x+9\right)\sin(x)\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

mupad [B] time = 0.11, size = 39, normalized size = 0.52

$$-\frac{\mathrm{e}^{-3\,x}\,\left(13\,\cos(x)+9\,\sin(x)+25\,x^2\,\cos(x)+75\,x^2\,\sin(x)+30\,x\,\cos(x)+40\,x\,\sin(x)\right)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-3*x)*sin(x),x)
```

```
[Out] -(exp(-3*x)*(13*cos(x) + 9*sin(x) + 25*x^2*cos(x) + 75*x^2*sin(x) + 30*x*co
s(x) + 40*x*sin(x)))/250
```

sympy [A] time = 2.28, size = 80, normalized size = 1.07

$$-\frac{3x^2e^{-3x}\sin{(x)}}{10}-\frac{x^2e^{-3x}\cos{(x)}}{10}-\frac{4xe^{-3x}\sin{(x)}}{25}-\frac{3xe^{-3x}\cos{(x)}}{25}-\frac{9e^{-3x}\sin{(x)}}{250}-\frac{13e^{-3x}\cos{(x)}}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)/exp(3*x),x)
```

```
[Out] -3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin(
x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*cos
(x)/250
```

3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$

Optimal. Leaf size=187

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{24}{125}e^{x/2}\sin(x)-\frac{24}{25}e^{x/2}x\sin(x)-$$

Rubi [A] time = 0.48, antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 31, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4435, 4433, 4466, 14, 4432, 4469, 4465, 4470}

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{1218672e^{x/2}\sin(x)}{6331625}-\frac{32556e^{x/2}}{3422}$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (-6687696*E^(x/2)*Cos[x])/6331625 + (24792*E^(x/2)*x*Cos[x])/34225 + (48*E^
(x/2)*x^2*Cos[x])/185 + (16*E^(x/2)*Cos[x]^3)/50653 - (8*E^(x/2)*x*Cos[x]^3
)/1369 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (432*E^(x/2)*Cos[3*x])/50653 + (72*E
^(x/2)*x*Cos[3*x])/1369 - (1218672*E^(x/2)*Sin[x])/6331625 - (32556*E^(x/2)
*x*Sin[x])/34225 + (96*E^(x/2)*x^2*Sin[x])/185 + (96*E^(x/2)*Cos[x]^2*Sin[x
])/50653 - (48*E^(x/2)*x*Cos[x]^2*Sin[x])/1369 + (12*E^(x/2)*x^2*Cos[x]^2*S
in[x])/37 - (816*E^(x/2)*Sin[3*x])/50653 - (12*E^(x/2)*x*Sin[3*x])/1369
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
```

```
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4470

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x/2}x^2\cos^3(x)\,dx &= \frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-2\int x\Big(\\ &= \frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-2\int\Big(\frac{4}{1}\\ &= \frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-\frac{4}{37}\int e\\ &= \frac{20352e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)-\frac{8e^{x/2}x\cos^3(x)}{1369}+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{30336e^{x/2}x\sin}{34225}\\ &= \frac{20352e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)-\frac{8e^{x/2}x\cos^3(x)}{1369}+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{30336e^{x/2}x\sin}{34225}\\ &= -\frac{48384e^{x/2}\cos(x)}{171125}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos^3(}{1369}\\ &= -\frac{1780608e^{x/2}\cos(x)}{6331625}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos}{1369}\\ &= -\frac{2482128e^{x/2}\cos(x)}{6331625}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos}{1369}\end{aligned}$$

Mathematica [A] time = 0.16, size = 72, normalized size = 0.39

$$\frac{e^{x/2}\left(303918\left(25x^2-40x-8\right)\sin(x)+750\left(1369x^2-296x-264\right)\sin(3x)+151959\left(25x^2+60x-88\right)\cos(x)+1\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*x
^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 136
9*x^2)*Sin[3*x]))/12663250
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x/2}x^2\cos^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.33, size = 72, normalized size = 0.39

$$\frac{12}{6331625}\left(125\left(1369\,x^2-296\,x-264\right)\cos(x)^2+273800\,x^2-497280\,x-93056\right)e^{\left(\frac{1}{2}\,x\right)}\sin(x)+\frac{2}{6331625}\left(125\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")
```

```
[Out] 12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x -
 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)^
3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)
```

giac [A] time = 0.60, size = 73, normalized size = 0.39

$$\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{3}{250}\left(\left(25\,x^2+60\,x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*s
in(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
- 8)*sin(x))*e^(1/2*x)
```

maple [C] time = 0.11, size = 106, normalized size = 0.57

method	result
risch	$\left(\frac{1}{202612}-\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{3}{500}-\frac{3i}{250}\right)\left(25x^2+40ix-20x-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)*x^2*cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/202612-3/101306*I)*(888*I*x+1369*x^2-280-96*I-148*x)*exp((1/2+3*I)*x)+(3
/500-3/250*I)*(40*I*x+25*x^2-24-32*I-20*x)*exp((1/2+I)*x)+(3/500+3/250*I)*(
-40*I*x+25*x^2-24+32*I-20*x)*exp((1/2-I)*x)+(1/202612+3/101306*I)*(-888*I*x
+1369*x^2-280+96*I-148*x)*exp((1/2-3*I)*x)
```

maxima [A] time = 0.48, size = 77, normalized size = 0.41

$$\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{3}{250}\left(25\,x^2+60\,x-88\right)\cos(x)e^{\left(\frac{1}{2}\,x\right)}+\frac{3}{50653}\left(1369\,x^2-296\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")
```

```
[Out] 1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60*
x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(3
*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

mupad [B] time = 0.31, size = 83, normalized size = 0.44

$$-\frac{e^{x/2}\left(107000\cos(3x)+198000\sin(3x)+13372392\cos(x)+2431344\sin(x)-647500x\cos(3x)-3798975\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x/2)*cos(x)^3,x)
```

```
[Out] -(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) + 13372392*cos(x) + 2431344*s
in(x) - 647500*x*cos(3*x) - 3798975*x^2*cos(x) + 222000*x*sin(3*x) - 759795
0*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) - 9117540*x*cos(x
) + 12156720*x*sin(x)))/12663250
```

sympy [A] time = 11.77, size = 202, normalized size = 1.08

$$\frac{96x^2e^{\frac{x}{2}}\sin^3(x)}{185}+\frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185}+\frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185}+\frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185}-\frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225}+\frac{19392xe}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

```
[Out] 96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 15
6*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 322
56*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 3
4656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/34225 -
 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x)/633
1625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*cos(x)*
*3/6331625
```

3.568 $\int e^{2x} x^2 \sin(4x)\, dx$

Optimal. Leaf size=87

$$\frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{3}{50} e^{2x} x \sin(4x) - \frac{11}{500} e^{2x} \sin(4x) + \frac{2}{25} e^{2x} x \cos(4x) + \frac{1}{250} e^{2x} \cos(4x)$$

Rubi [A] time = 0.16, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{3}{50} e^{2x} x \sin(4x) - \frac{11}{500} e^{2x} \sin(4x) + \frac{2}{25} e^{2x} x \cos(4x) + \frac{1}{250} e^{2x} \cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2*Cos[4*x])
/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])/50 + (E^(2*x)*x^2*S
in[4*x])/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{5}e^{2x}x^2\cos(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) - 2\int x\left(-\frac{1}{5}e^{2x}\cos(4x) + \frac{1}{10}e^{2x}\sin(4x)\right)dx$$
$$= -\frac{1}{5}e^{2x}x^2\cos(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) - 2\int\left(-\frac{1}{5}e^{2x}x\cos(4x) + \frac{1}{10}e^{2x}x\sin(4x)\right)dx$$
$$= -\frac{1}{5}e^{2x}x^2\cos(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) - \frac{1}{5}\int e^{2x}x\sin(4x)\,dx + \frac{2}{5}\int e^{2x}x\cos(4x)\,dx$$
$$= \frac{2}{25}e^{2x}x\cos(4x) - \frac{1}{5}e^{2x}x^2\cos(4x) + \frac{3}{50}e^{2x}x\sin(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) + \frac{1}{5}\int\left(-\frac{1}{5}e^{2x}\cos(4x\right.$$
$$= \frac{2}{25}e^{2x}x\cos(4x) - \frac{1}{5}e^{2x}x^2\cos(4x) + \frac{3}{50}e^{2x}x\sin(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) + \frac{1}{50}\int e^{2x}\sin(4x)\,d$$
$$= \frac{3}{250}e^{2x}\cos(4x) + \frac{2}{25}e^{2x}x\cos(4x) - \frac{1}{5}e^{2x}x^2\cos(4x) - \frac{3}{500}e^{2x}\sin(4x) + \frac{3}{50}e^{2x}x\sin(4x) + \frac{1}{10}$$

Mathematica [A] time = 0.08, size = 40, normalized size = 0.46

$$\frac{1}{500}e^{2x}\left(\left(50x^2+30x-11\right)\sin(4x)+\left(-100x^2+40x+2\right)\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))/
500
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{2x}x^2\sin(4x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.27, size = 41, normalized size = 0.47

$$-\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)}+\frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

giac [A] time = 0.57, size = 39, normalized size = 0.45

$$-\frac{1}{500}\left(2\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)-\left(50\,x^2+30\,x-11\right)\sin\left(4\,x\right)\right)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^(
2*x)
```

maple [A] time = 0.07, size = 40, normalized size = 0.46

method	result
default	$\left(-\frac{1}{5}x^2+\frac{2}{25}x+\frac{1}{250}\right)e^{2x}\cos(4x)+\left(\frac{1}{10}x^2+\frac{3}{50}x-\frac{11}{500}\right)e^{2x}\sin(4x)$
risch	$\left(-\frac{1}{500}-\frac{i}{1000}\right)\left(50x^2+20ix-10x-4i-3\right)e^{(2+4i)x}+\left(-\frac{1}{500}+\frac{i}{1000}\right)\left(50x^2-20ix-10x+4i-3\right)e$
norman	$\frac{\frac{2xe^{2x}}{25}-\frac{e^{2x}x^2}{5}-\frac{11e^{2x}\tan(2x)}{250}-\frac{e^{2x}(\tan^2(2x))}{250}+\frac{3xe^{2x}\tan(2x)}{25}-\frac{2xe^{2x}(\tan^2(2x))}{25}+\frac{e^{2x}x^2\tan(2x)}{5}+\frac{e^{2x}x^2(\tan^2(2x))}{5}+\frac{e^{2x}}{250}}{1+\tan^2(2x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*x^2*sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x)
*sin(4*x)
```

maxima [A] time = 0.45, size = 41, normalized size = 0.47

$$-\frac{1}{250}\left(50x^2-20x-1\right)\cos(4x)e^{(2x)}+\frac{1}{500}\left(50x^2+30x-11\right)e^{(2x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

mupad [B] time = 0.35, size = 51, normalized size = 0.59

$$\frac{e^{2x}\left(2\cos(4x)-11\sin(4x)+40x\cos(4x)+30x\sin(4x)-100x^2\cos(4x)+50x^2\sin(4x)\right)}{500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(4*x)*exp(2*x),x)
```

```
[Out] (exp(2*x)*(2*cos(4*x) - 11*sin(4*x) + 40*x*cos(4*x) + 30*x*sin(4*x) - 100*x
^2*cos(4*x) + 50*x^2*sin(4*x)))/500
```

sympy [A] time = 2.11, size = 85, normalized size = 0.98

$$\frac{x^2e^{2x}\sin(4x)}{10}-\frac{x^2e^{2x}\cos(4x)}{5}+\frac{3xe^{2x}\sin(4x)}{50}+\frac{2xe^{2x}\cos(4x)}{25}-\frac{11e^{2x}\sin(4x)}{500}+\frac{e^{2x}\cos(4x)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x**2*sin(4*x),x)
```

```
[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*x
)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos(4
*x)/250
```

3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=185

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin$$

Rubi [A] time = 0.36, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4470, 4433, 4466, 14, 4432, 4465}

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/1
0 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2)*
x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E^(x
/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/
1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4470

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \int\left(\frac{1}{4}e^{x/2}x^2\cos(x)-\frac{1}{4}e^{x/2}x^2\cos(3x)\right)dx$$
$$=\frac{1}{4}\int e^{x/2}x^2\cos(x)\,dx-\frac{1}{4}\int e^{x/2}x^2\cos(3x)\,dx$$
$$=\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)-\frac{1}{2}\int x\Big($$
$$=\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)-\frac{1}{2}\int\Big(\frac{2}{5}$$
$$=\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{37}\int e$$
$$=\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x$$
$$=\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x$$
$$=-\frac{12}{125}e^{x/2}\cos(x)+\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)+\frac{140e^{x/2}\cos(3x)}{50653}-\frac{70e^{x/2}x\,c}{136}$$

Mathematica [A] time = 0.22, size = 76, normalized size = 0.41

$$\frac{e^{x/2}\left(50653\left(2\left(25x^2-40x-8\right)\sin(x)+\left(25x^2+60x-88\right)\cos(x)\right)-125\left(6\left(1369x^2-296x-264\right)\sin(3x)+\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin[
x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x^2
)*Sin[3*x])))/12663250
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 72, normalized size = 0.39

$$-\frac{4}{6331625}\left(375\left(1369\,x^2-296\,x-264\right)\cos(x)^2-444925\,x^2+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin(x)-\frac{2}{6331625}\left(12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x +
 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)
^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)
```

giac [A] time = 0.60, size = 73, normalized size = 0.39

$$-\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{1}{250}\left(\left(25\,x^2+60\,x-88\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*
sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
 - 8)*sin(x))*e^(1/2*x)
```

maple [C] time = 0.09, size = 106, normalized size = 0.57

method	result
risch	$\left(-\frac{1}{202612}+\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{1}{500}-\frac{i}{250}\right)\left(25x^2+40ix-20x-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/202612+3/101306*I)*(888*I*x+1369*x^2-280-96*I-148*x)*exp((1/2+3*I)*x)+(
1/500-1/250*I)*(40*I*x+25*x^2-24-32*I-20*x)*exp((1/2+I)*x)+(1/500+1/250*I)*
(-40*I*x+25*x^2-24+32*I-20*x)*exp((1/2-I)*x)-(1/202612+3/101306*I)*(-888*I*
x+1369*x^2-280+96*I-148*x)*exp((1/2-3*I)*x)
```

maxima [A] time = 0.49, size = 77, normalized size = 0.42

$$-\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{1}{250}\left(25\,x^2+60\,x-88\right)\cos\left(x\right)e^{\left(\frac{1}{2}\,x\right)}-\frac{3}{50653}\left(1369\,x^2-296\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 60
*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(
3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

mupad [B] time = 0.53, size = 83, normalized size = 0.45

$$e^{x/2}\left(107000\,\cos\left(3\,x\right)+198000\,\sin\left(3\,x\right)-4457464\,\cos\left(x\right)-810448\,\sin\left(x\right)-647500\,x\,\cos\left(3\,x\right)+1266325\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x/2)*cos(x)*sin(x)^2,x)
```

```
[Out] (exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) - 4457464*cos(x) - 810448*sin(
x) - 647500*x*cos(3*x) + 1266325*x^2*cos(x) + 222000*x*sin(3*x) + 2532650*x
^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) + 3039180*x*cos(x) -
 4052240*x*sin(x)))/12663250
```

sympy [A] time = 11.81, size = 202, normalized size = 1.09

$$\frac{52x^2e^{\frac{x}{2}}\sin^3\left(x\right)}{185}+\frac{26x^2e^{\frac{x}{2}}\sin^2\left(x\right)\cos\left(x\right)}{185}-\frac{8x^2e^{\frac{x}{2}}\sin\left(x\right)\cos^2\left(x\right)}{185}+\frac{16x^2e^{\frac{x}{2}}\cos^3\left(x\right)}{185}-\frac{11552xe^{\frac{x}{2}}\sin^3\left(x\right)}{34225}+\frac{13464xe^{\frac{x}{2}}\,s}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

```
[Out] 52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8*
x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 11552
*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 915
2*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 - 504
224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/6331625
- 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)**3/633
1625
```

3.570 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2637}

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x],x]
[Out] Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(x)\,dx = \sinh(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x],x]
[Out] Sinh[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cosh[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.20, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="fricas")
[Out] sinh(x)
```

giac [B] time = 0.63, size = 11, normalized size = 5.50

$$-\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="giac")
[Out] -1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.05, size = 3, normalized size = 1.50

method	result	size
lookup	$\sinh(x)$	3
default	$\sinh(x)$	3
meijerg	$\sinh(x)$	3
risch	$-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x,method=_RETURNVERBOSE)
[Out] sinh(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="maxima")
[Out] sinh(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x)
[Out] sinh(x)
```

sympy [A] time = 0.13, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x)
[Out] sinh(x)
```

3.571 $\int \sinh(x)\,dx$

Optimal. Leaf size=2

$$\cosh(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2638}

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x],x]
[Out] Cosh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(x)\,dx = \cosh(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x],x]
[Out] Cosh[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sinh[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.17, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="fricas")
[Out] cosh(x)
```

giac [B] time = 0.62, size = 11, normalized size = 5.50

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.03, size = 3, normalized size = 1.50

method	result	size
lookup	$\cosh(x)$	3
default	$\cosh(x)$	3
risch	$\frac{e^{-x}}{2}+\frac{e^{x}}{2}$	12
meijerg	$-\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] cosh(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x)
```

```
[Out] cosh(x)
```

sympy [A] time = 0.13, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x)
```

```
[Out] cosh(x)
```

3.572 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x],x]
[Out] Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x],x]
[Out] Log[Cosh[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tanh[x],x]
[Out] Could not integrate
```

fricas [B] time = 1.09, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.63, size = 11, normalized size = 3.67

$$-x + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

maple [A] time = 0.02, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln\left(1+e^{2x}\right)$	12
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh(x))}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(cosh(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
```

```
[Out] log(cosh(x))
```

mupad [B] time = 0.02, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
```

```
[Out] log(cosh(x))
```

sympy [B] time = 0.13, size = 7, normalized size = 2.33

$$x-\log(\tanh(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
```

```
[Out] x - log(tanh(x) + 1)
```

3.573 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(x)\,dx = \log(\sinh(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Coth[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.12, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2\,\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.62, size = 12, normalized size = 4.00

$$-x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="giac")
[Out] -x + log(abs(e^(2*x) - 1))
```

maple [A] time = 0.02, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln\left(-1+e^{2x}\right)$	12
derivativedivides	$-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(\coth(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x,method=_RETURNVERBOSE)
[Out] ln(sinh(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
[Out] log(sinh(x))
```

mupad [B] time = 0.03, size = 3, normalized size = 1.00

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
[Out] log(sinh(x))
```

sympy [B] time = 0.31, size = 12, normalized size = 4.00

$$x-\log(\tanh(x)+1)+\log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

3.574 $\int \operatorname{sech}(x)\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sinh(x))$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3770}

$$\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x],x]
[Out] ArcTan[Sinh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(x)\,dx = \tan^{-1}(\sinh(x))$$

Mathematica [B] time = 0.00, size = 9, normalized size = 3.00

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x],x]
[Out] 2*ArcTan[Tanh[x/2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sech[x],x]
[Out] Could not integrate
```

fricas [B] time = 1.24, size = 8, normalized size = 2.67

$$2\arctan\left(\cosh(x)+\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="fricas")
[Out] 2*arctan(cosh(x) + sinh(x))
```

giac [A] time = 0.62, size = 5, normalized size = 1.67

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

maple [A] time = 0.03, size = 4, normalized size = 1.33

method	result	size
lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(e^x+i)-i\ln(e^x-i)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(x))
```

maxima [A] time = 0.43, size = 3, normalized size = 1.00

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x))
```

mupad [B] time = 0.02, size = 5, normalized size = 1.67

$$2\,\mathrm{atan}\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x),x)
```

```
[Out] 2*atan(exp(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x)
```

```
[Out] Integral(sech(x), x)
```

3.575 $\int \operatorname{csch}(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cosh(x))$$

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3770}

$$-\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(x)\,dx = -\tanh^{-1}(\cosh(x))$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.40

$$\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x],x]
```

```
[Out] Log[Tanh[x/2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csch[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.34, size = 17, normalized size = 3.40

$$-\log\left(\cosh(x) + \sinh(x) + 1\right) + \log\left(\cosh(x) + \sinh(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="fricas")
```

```
[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

giac [B] time = 0.59, size = 14, normalized size = 2.80

$$-\log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="giac")
[Out] -log(e^x + 1) + log(abs(e^x - 1))
```

maple [A] time = 0.02, size = 6, normalized size = 1.20

method	result	size
lookup	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
risch	$\ln\left(-1+\mathrm{e}^{x}\right)-\ln\left(1+\mathrm{e}^{x}\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x),x,method=_RETURNVERBOSE)
[Out] ln(tanh(1/2*x))
```

maxima [A] time = 0.43, size = 5, normalized size = 1.00

$$\log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="maxima")
[Out] log(tanh(1/2*x))
```

mupad [B] time = 0.01, size = 5, normalized size = 1.00

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(x),x)
[Out] log(tanh(x/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x)
[Out] Integral(csch(x), x)
```

3.576 $\int \cosh^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2,x]
```

```
[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(x)\,dx &= \frac{1}{2}\cosh(x)\sinh(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2,x]
```

```
[Out] x/2 + Sinh[2*x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cosh[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 10, normalized size = 0.71

$$\frac{1}{2}\cosh(x)\sinh(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="fricas")
[Out] 1/2*cosh(x)*sinh(x) + 1/2*x
```

giac [B] time = 0.61, size = 24, normalized size = 1.71

$$-\frac{1}{8}\left(2e^{(2x)}+1\right)e^{(-2x)}+\frac{1}{2}x+\frac{1}{8}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="giac")
[Out] -1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

maple [A] time = 0.03, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cosh(x)\sinh(x)}{2}$	11
risch	$\frac{x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}$	17
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cosh(x)*sinh(x)
```

maxima [A] time = 0.43, size = 16, normalized size = 1.14

$$\frac{1}{2}x+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sinh(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2,x)
[Out] x/2 + sinh(2*x)/4
```

sympy [B] time = 0.20, size = 24, normalized size = 1.71

$$-\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2,x)
[Out] -x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

3.577 $\int \sinh^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5,x]
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(x)\,dx &= \text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cosh(x)\right)\\ &= \cosh(x)-\frac{2\cosh^3(x)}{3}+\frac{\cosh^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.21

$$\frac{5\cosh(x)}{8}-\frac{5}{48}\cosh(3x)+\frac{1}{80}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5,x]
[Out] (5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sinh[x]^5,x]
[Out] Could not integrate
```

fricas [B] time = 0.95, size = 42, normalized size = 2.21

$$\frac{1}{80}\cosh(x)^5+\frac{1}{16}\cosh(x)\sinh(x)^4-\frac{5}{48}\cosh(x)^3+\frac{1}{16}\left(2\cosh(x)^3-5\cosh(x)\right)\sinh(x)^2+\frac{5}{8}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(2*cosh(x)^
3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

giac [B] time = 0.59, size = 37, normalized size = 1.95

$$\frac{1}{480} \left(150 e^{(4x)} - 25 e^{(2x)} + 3\right)e^{(-5x)} + \frac{1}{160} e^{(5x)} - \frac{5}{96} e^{(3x)} + \frac{5}{16} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="giac")
```

```
[Out] 1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5/96*e^(3*x
) + 5/16*e^x
```

maple [A] time = 0.34, size = 18, normalized size = 0.95

method	result	size
default	$\left(\frac{8}{15}+\frac{\left(\sinh^4(x)\right)}{5}-\frac{4\left(\sinh^2(x)\right)}{15}\right)\cosh(x)$	18
risch	$\frac{e^{5x}}{160}-\frac{5\,e^{3x}}{96}+\frac{5\,e^{x}}{16}+\frac{5\,e^{-x}}{16}-\frac{5\,e^{-3x}}{96}+\frac{e^{-5x}}{160}$	36
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

maxima [B] time = 0.43, size = 35, normalized size = 1.84

$$\frac{1}{160} e^{(5x)} - \frac{5}{96} e^{(3x)} + \frac{5}{16} e^{(-x)} - \frac{5}{96} e^{(-3x)} + \frac{1}{160} e^{(-5x)} + \frac{5}{16} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x)
 + 5/16*e^x
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$\frac{\cosh(x)^5}{5} - \frac{2\cosh(x)^3}{3} + \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5,x)
```

```
[Out] cosh(x) - (2*cosh(x)^3)/3 + cosh(x)^5/5
```

sympy [A] time = 1.12, size = 29, normalized size = 1.53

$$\sinh^4(x)\cosh(x) - \frac{4\sinh^2(x)\cosh^3(x)}{3} + \frac{8\cosh^5(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5,x)
```

```
[Out] sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

3.578 $\int \tanh^4(x)\,dx$

Optimal. Leaf size=14

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4,x]
[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^4(x)\,dx&=-\frac{1}{3}\tanh^3(x)+\int \tanh^2(x)\,dx\\&=-\tanh(x)-\frac{\tanh^3(x)}{3}+\int 1\,dx\\&=x-\tanh(x)-\frac{\tanh^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.29

$$x-\frac{4\tanh(x)}{3}+\frac{1}{3}\tanh(x)\text{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4,x]
[Out] x - (4*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Tanh[x]^4,x]
[Out] Could not integrate
```

fricas [B] time = 1.28, size = 68, normalized size = 4.86

$$\frac{(3x+4)\cosh(x)^3+3(3x+4)\cosh(x)\sinh(x)^2-12\cosh(x)^2\sinh(x)-4\sinh(x)^3+3(3x+4)\cosh(x)}{3\left(\cosh(x)^3+3\cosh(x)\sinh(x)^2+3\cosh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*sin
h(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
+ 3*cosh(x))
```

giac [B] time = 0.63, size = 26, normalized size = 1.86

$$x+\frac{4\left(3e^{(4x)}+3e^{(2x)}+2\right)}{3\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="giac")
```

```
[Out] x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

maple [B] time = 0.02, size = 26, normalized size = 1.86

method	result	size
derivativedivides	$-\frac{\left(\tanh^3(x)\right)}{3}-\tanh(x)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}$	26
default	$-\frac{\left(\tanh^3(x)\right)}{3}-\tanh(x)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}$	26
risch	$x+\frac{4e^{4x}+4e^{2x}+\frac{8}{3}}{\left(1+e^{2x}\right)^3}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)^3-tanh(x)-1/2*ln(tanh(x)-1)+1/2*ln(1+tanh(x))
```

maxima [B] time = 0.43, size = 38, normalized size = 2.71

$$x-\frac{4\left(3e^{(-2x)}+3e^{(-4x)}+2\right)}{3\left(3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="maxima")
```

```
[Out] x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

mupad [B] time = 0.07, size = 12, normalized size = 0.86

$$-\frac{\tanh(x)^3}{3}-\tanh(x)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4,x)
```

```
[Out] x - tanh(x) - tanh(x)^3/3
```

sympy [A] time = 0.23, size = 10, normalized size = 0.71

$$x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4,x)
```

```
[Out] x - tanh(x)**3/3 - tanh(x)
```

3.579 $\int \operatorname{csch}^3(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(x)\,dx &= -\frac{1}{2}\coth(x)\operatorname{csch}(x)-\frac{1}{2}\int \operatorname{csch}(x)\,dx\\ &= \frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 36, normalized size = 2.25

$$-\frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3,x]
```

```
[Out] -1/8*Csch[x/2]^2 - Log[Tanh[x/2]]/2 - Sech[x/2]^2/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Csch[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.26, size = 211, normalized size = 13.19

$$-\frac{2\,\cosh(x)^3 + 6\,\cosh(x)\sinh(x)^2 + 2\,\sinh(x)^3 - \left(\cosh(x)^4 + 4\,\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\,\cosh(x)^2 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4
*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh
(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2
*(3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - co
sh(x))*sinh(x) + 1)
```

giac [B] time = 0.60, size = 45, normalized size = 2.81

$$-\frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^{2} - 4} + \frac{1}{4}\,\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{4}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="giac")
```

```
[Out] -(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*log(e^(-x) + e^x + 2) - 1/4*lo
g(e^(-x) + e^x - 2)
```

maple [A] time = 0.34, size = 11, normalized size = 0.69

method	result	size
default	$-\frac{\coth(x)\operatorname{csch}(x)}{2} + \operatorname{arctanh}(e^{x})$	11
risch	$-\frac{e^{x}(1+e^{2x})}{(-1+e^{2x})^{2}} + \frac{\ln(1+e^{x})}{2} - \frac{\ln(-1+e^{x})}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*coth(x)*csch(x)+arctanh(exp(x))
```

maxima [B] time = 0.43, size = 45, normalized size = 2.81

$$\frac{e^{(-x)} + e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x) + 1) - 1/2
*log(e^(-x) - 1)
```

mupad [B] time = 0.29, size = 16, normalized size = 1.00

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\cosh(x)}{2\,\sinh(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(x)^3,x)
[Out] - log(tanh(x/2))/2 - cosh(x)/(2*sinh(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3,x)
[Out] Integral(csch(x)**3, x)
```

3.580 $\int \text{sech}^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^5(x)\,dx &= \frac{1}{4}\text{sech}^3(x)\tanh(x)+\frac{3}{4}\int \text{sech}^3(x)\,dx\\ &= \frac{3}{8}\text{sech}(x)\tanh(x)+\frac{1}{4}\text{sech}^3(x)\tanh(x)+\frac{3}{8}\int \text{sech}(x)\,dx\\ &= \frac{3}{8}\tan^{-1}(\sinh(x))+\frac{3}{8}\text{sech}(x)\tanh(x)+\frac{1}{4}\text{sech}^3(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.15

$$\frac{3}{4}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{sech}^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sech[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.27, size = 461, normalized size = 17.73

$$\frac{3\cosh(x)^7 + 21\cosh(x)\sinh(x)^6 + 3\sinh(x)^7 + \left(63\cosh(x)^2 + 11\right)\sinh(x)^5 + 11\cosh(x)^5 + 5\left(21\cosh(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sin
h(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x
)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(
x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 +
 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + co
sh(x))*sinh(x) + 1)
```

giac [B] time = 0.64, size = 60, normalized size = 2.31

$$\frac{3}{16}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^3 + 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^2} + \frac{3}{8}\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="giac")
```

```
[Out] 3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2 +
 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

maple [A] time = 0.35, size = 21, normalized size = 0.81

method	result	size
default	$\left(\frac{\operatorname{sech}(x)^3}{4} + \frac{3\operatorname{sech}(x)}{8}\right)\tanh(x) + \frac{3\arctan(e^x)}{4}$	21
risch	$\frac{e^x\left(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3\right)}{4\left(1+e^{2x}\right)^4} + \frac{3i\ln(e^x+i)}{8} - \frac{3i\ln(e^x-i)}{8}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

maxima [B] time = 1.01, size = 61, normalized size = 2.35

$$\frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} - \frac{3}{4}\,\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

mupad [B] time = 0.08, size = 22, normalized size = 0.85

$$\frac{3\,\mathrm{atan}\left(e^{x}\right)}{4}+\frac{3\sinh(x)}{8\cosh(x)^{2}}+\frac{\sinh(x)}{4\cosh(x)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x)^5,x)
```

```
[Out] (3*atan(exp(x)))/4 + (3*sinh(x))/(8*cosh(x)^2) + sinh(x)/(4*cosh(x)^4)
```

sympy [B] time = 2.83, size = 422, normalized size = 16.23

$$\frac{3\tanh^{8}\left(\frac{x}{2}\right)\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^{8}\left(\frac{x}{2}\right)+16\tanh^{6}\left(\frac{x}{2}\right)+24\tanh^{4}\left(\frac{x}{2}\right)+16\tanh^{2}\left(\frac{x}{2}\right)+4}-\frac{5\tanh^{7}\left(\frac{x}{2}\right)}{4\tanh^{8}\left(\frac{x}{2}\right)+16\tanh^{6}\left(\frac{x}{2}\right)+24\tanh^{4}\left(\frac{x}{2}\right)+16\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)**5,x)
```

```
[Out] 3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(
x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 12*tanh(x/2)**6*atan(tanh
(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**
2 + 4) + 3*tanh(x/2)**5/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4
 + 16*tanh(x/2)**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2)**8 +
 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x/2)**3/
(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
+ 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*ta
nh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*tanh(x/2)/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/2))/(4*tanh
(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
```

3.581 $\int \sinh^4(x)\tanh(x)\,dx$

Optimal. Leaf size=18

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2590, 266, 43}

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4*Tanh[x],x]
```

```
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sinh^4(x)\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \cosh^2(x)\right)\\
&= -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4*Tanh[x],x]
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^4(x)\tanh(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sinh[x]^4*Tanh[x],x]
[Out] Could not integrate
```

fricas [B] time = 1.09, size = 257, normalized size = 14.28

$$\frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 3\right)\sinh(x)^6 - 12\cosh(x)^6 + 8\left(7\cosh(x)^3 - 9\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
[Out] 1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sin
h(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 - 64*x*cosh(x
)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*sinh(x)^4 + 8*(7*cosh(x)^5 - 3
0*cosh(x)^3 - 32*x*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 - 96*
x*cosh(x)^2 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^3*sin
h(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^3 - 3*c
osh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

giac [B] time = 0.61, size = 43, normalized size = 2.39

$$\frac{1}{64}\left(48\,e^{(4\,x)} - 12\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - x + \frac{1}{64}\,e^{(4\,x)} - \frac{3}{16}\,e^{(2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
[Out] 1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) - 3/16*e^(2*
x) + log(e^(2*x) + 1)
```

maple [A] time = 0.04, size = 17, normalized size = 0.94

method	result	size
default	$\frac{\left(\sinh^4(x)\right)}{4} - \frac{\left(\sinh^2(x)\right)}{2} + \ln\left(\cosh(x)\right)$	17
risch	$-x + \frac{e^{4x}}{64} - \frac{3\,e^{2x}}{16} - \frac{3\,e^{-2x}}{16} + \frac{e^{-4x}}{64} + \ln\left(1 + e^{2x}\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/sech(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

maxima [B] time = 0.96, size = 35, normalized size = 1.94

$$-\frac{1}{64}\left(12\,e^{(-2\,x)}-1\right)e^{(4\,x)}+x-\frac{3}{16}\,e^{(-2\,x)}+\frac{1}{64}\,e^{(-4\,x)}+\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*x) + log(e
^(-2*x) + 1)
```

mupad [B] time = 0.36, size = 35, normalized size = 1.94

$$\ln\left(e^{2\,x}+1\right)-x-\frac{3\,e^{-2\,x}}{16}-\frac{3\,e^{2\,x}}{16}+\frac{e^{-4\,x}}{64}+\frac{e^{4\,x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4*tanh(x)^5,x)
```

```
[Out] log(exp(2*x) + 1) - x - (3*exp(-2*x))/16 - (3*exp(2*x))/16 + exp(-4*x)/64 +
 exp(4*x)/64
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^{5}(x)}{\operatorname{sech}^{4}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/sech(x)**4,x)
```

```
[Out] Integral(tanh(x)**5/sech(x)**4, x)
```

3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$

Optimal. Leaf size=31

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2622, 270}

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx &= -\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{\sqrt[4]{x}}\,dx,x,\operatorname{sech}(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt[4]{x}}-2x^{7/4}+x^{15/4}\right)dx,x,\operatorname{sech}(x)\right)\\ &= -\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.87

$$\operatorname{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{19}\operatorname{sech}^4(x)+\frac{8\operatorname{sech}^2(x)}{11}-\frac{4}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.67, size = 359, normalized size = 11.58

$$-\frac{4 \cdot 2^{\frac{3}{4}} \left(209 \cosh(x)^{8} + 1672 \cosh(x) \sinh(x)^{7} + 209 \sinh(x)^{8} + 76 \left(77 \cosh(x)^{2} + 5\right) \sinh(x)^{6} + 380 \cosh(x\right.}{627 \left(\cosh(x)^{8} + 8 \cosh(x) \sinh(x)^{7} + \sinh(x)^{8} + 4 \left(7 c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh(x)^8 + 76
*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77*cosh(x)^3 + 15*cosh
(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cosh(x)^2 + 87)*sinh(x)^4 + 870*c
osh(x)^4 + 8*(1463*cosh(x)^5 + 950*cosh(x)^3 + 435*cosh(x))*sinh(x)^3 + 4*(
1463*cosh(x)^6 + 1425*cosh(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380*cosh
(x)^2 + 8*(209*cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x))*sinh
(x) + 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 +
 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35
*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 1
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh
(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)
^3 + cosh(x))*sinh(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \left(\tanh^{5}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^(3/4)*tanh(x)^5,x)
```

```
[Out] int(sech(x)^(3/4)*tanh(x)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

mupad [B] time = 0.17, size = 120, normalized size = 3.87

$$\frac{32\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{11\left(e^{2x}+1\right)}-\frac{1312\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{209\left(e^{2x}+1\right)^{2}}+\frac{128\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\left(e^{2x}+1\right)^{3}}-\frac{64\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\left(e^{2x}+1\right)^{4}}-\frac{4\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5*(1/cosh(x))^(3/4),x)
```

```
[Out] (32*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(11*(exp(2*x) + 1)) - (1312*(1/(exp(-
x)/2 + exp(x)/2))^(3/4))/(209*(exp(2*x) + 1)^2) + (128*(1/(exp(-x)/2 + exp(
x)/2))^(3/4))/(19*(exp(2*x) + 1)^3) - (64*(1/(exp(-x)/2 + exp(x)/2))^(3/4))
/(19*(exp(2*x) + 1)^4) - (4*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

```
[Out] Timed out
```

3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-1),x]
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-1),x]
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\cosh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*Cosh[x])^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 175, normalized size = 4.27

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{\sqrt{a^2-b^2}}, -\frac{2\,\sqrt{-a^2+b^2}\,\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))/sqr
t(a^2 - b^2), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]
```

giac [A] time = 0.59, size = 32, normalized size = 0.78

$$\frac{2\,\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

maple [A] time = 0.07, size = 36, normalized size = 0.88

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$\frac{\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.16, size = 43, normalized size = 1.05

$$\frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{b^2-a^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)),x)
```

```
[Out] (2*atan(a/(b^2 - a^2)^(1/2) + (b*exp(x))/(b^2 - a^2)^(1/2)))/(b^2 - a^2)^(1
/2)
```

sympy [A] time = 4.41, size = 126, normalized size = 3.07

$$\begin{cases} \tilde{\infty}\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a=0 \wedge b=0 \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (-1/(b*tanh(x/2)), Eq
(a, -b)), (tanh(x/2)/b, Eq(a, b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tan
h(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b))) + l
og(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))
- b*sqrt(a/(a - b) + b/(a - b))), True))
```

3.584 $\int \frac{1}{(1+\cosh(x))^2} dx$

Optimal. Leaf size=25

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2650, 2648}

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x])^(-2),x]
```

```
[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\cosh(x))^2} dx &= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{1}{3}\int \frac{1}{1+\cosh(x)} dx \\
&= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.64

$$\frac{\sinh(x)(\cosh(x)+2)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x])^(-2),x]
```

```
[Out] ((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+\cosh(x))^2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Cosh[x])^(-2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.13, size = 58, normalized size = 2.32

$$-\frac{2\,(3\,\cosh(x)+3\,\sinh(x)+1)}{3\,\left(\cosh(x)^3+3\,(\cosh(x)+1)\,\sinh(x)^2+\sinh(x)^3+3\,\cosh(x)^2+3\,\left(\cosh(x)^2+2\,\cosh(x)+1\right)\sinh(x)+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sinh(x)^2 + s
inh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x) + 3*cosh(x)
+ 1)
```

giac [A] time = 0.61, size = 14, normalized size = 0.56

$$-\frac{2\,(3\,e^x+1)}{3\,(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^x + 1)/(e^x + 1)^3
```

maple [A] time = 0.04, size = 15, normalized size = 0.60

method	result	size
risch	$-\frac{2(1+3\,e^x)}{3(1+e^x)^3}$	15
default	$-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+3*exp(x))/(1+exp(x))^3
```

maxima [B] time = 0.43, size = 49, normalized size = 1.96

$$\frac{2\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*x
) + e^(-3*x) + 1)
```

mupad [B] time = 0.29, size = 14, normalized size = 0.56

$$-\frac{2\,\left(3\,e^x+1\right)}{3\,\left(e^x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x) + 1)^2,x)
```

```
[Out] -(2*(3*exp(x) + 1))/(3*(exp(x) + 1)^3)
```

sympy [A] time = 0.38, size = 14, normalized size = 0.56

$$-\frac{\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))**2,x)
```

```
[Out] -tanh(x/2)**3/6 + tanh(x/2)/2
```

3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\tanh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*Tanh[x])^(-1),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.33, size = 42, normalized size = 1.08

$$\frac{(a+b)x - b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

giac [A] time = 0.58, size = 43, normalized size = 1.10

$$-\frac{b\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^2-b^2} + \frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

maple [A] time = 0.06, size = 55, normalized size = 1.41

method	result	size
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b} - \frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)} - \frac{\ln(\tanh(x)-1)}{2a+2b}$	55
default	$\frac{\ln(1+\tanh(x))}{2a-2b} - \frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)} - \frac{\ln(\tanh(x)-1)}{2a+2b}$	55
risch	$\frac{x}{a+b} + \frac{2bx}{a^2-b^2} - \frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-b/(a+b)/(a-b)*ln(a+b*tanh(x))-1/(2*a+2*b)*ln(tanh
(x)-1)
```

maxima [A] time = 0.44, size = 41, normalized size = 1.05

$$-\frac{b\log\left(-(a-b)e^{(-2\,x)} - a - b\right)}{a^2-b^2} + \frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

mupad [B] time = 0.13, size = 35, normalized size = 0.90

$$\frac{a\,x - b\,\left(x - \ln\left(\tanh(x)+1\right) + \ln\left(a+b\tanh(x)\right)\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(x)),x)
```

```
[Out] (a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

sympy [A] time = 0.61, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\left(x - \log\left(\tanh\left(x\right) + 1\right) + \log\left(\tanh\left(x\right)\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ -\frac{x\tanh\left(x\right)}{2b\tanh\left(x\right) - 2b} + \frac{x}{2b\tanh\left(x\right) - 2b} + \frac{1}{2b\tanh\left(x\right) - 2b} & \text{for } a = -b \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right) + 2b} + \frac{x}{2b\tanh\left(x\right) + 2b} - \frac{1}{2b\tanh\left(x\right) + 2b} & \text{for } a = b \\ \frac{ax}{a^2 - b^2} - \frac{bx}{a^2 - b^2} - \frac{b\log\left(\frac{a}{b} + \tanh\left(x\right)\right)}{a^2 - b^2} + \frac{b\log\left(\tanh\left(x\right) + 1\right)}{a^2 - b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

3.586 $\int \frac{1}{a^2+b^2 \cosh^2(x)} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a^2+b^2\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-\left(a^2+b^2\right)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.40, size = 288, normalized size = 9.29

$$\frac{\sqrt{a^2+b^2}\log\left(\frac{b^4\cosh(x)^4+4b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8a^4+8a^2b^2+b^4+2\left(2a^2b^2+b^4\right)\cosh(x)^2+2\left(3b^4\cosh(x)^2+2a^2b^2+b^4\right)\sinh(x)^2}{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2a^2+b^2\right)\cosh(x)^2+2\left(3b^2\cosh(x)^2\right.}\right)}{2\left(a^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh
(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^4*
cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 + b^
4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*
sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)*
sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
+ 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*cosh(x))*
sinh(x)))/(a^3 + a*b^2)
```

giac [B] time = 0.60, size = 79, normalized size = 2.55

$$\frac{\log\left(\frac{b^2e^{(2x)}+2a^2+b^2-2\sqrt{a^2+b^2}|a|}{b^2e^{(2x)}+2a^2+b^2+2\sqrt{a^2+b^2}|a|}\right)}{2\sqrt{a^2+b^2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^2*e^(2*x)
 + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*abs(a))
```

maple [B] time = 0.17, size = 98, normalized size = 3.16

method	result	size
default	$\frac{\ln\left(\sqrt{a^2+b^2}\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(\sqrt{a^2+b^2}\left(\tanh^2\left(\frac{x}{2}\right)\right)-2a\tanh\left(\frac{x}{2}\right)+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}$	98
risch	$\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}-2a^3-2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}+2a^3+2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^2
+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*tan
h(1/2*x)+(a^2+b^2)^(1/2))
```

maxima [B] time = 0.98, size = 76, normalized size = 2.45

$$-\frac{\log\left(\frac{b^2e^{(-2x)}+2a^2+b^2-2\sqrt{a^2+b^2}a}{b^2e^{(-2x)}+2a^2+b^2+2\sqrt{a^2+b^2}a}\right)}{2\sqrt{a^2+b^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b^2*e^(-2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*a)/(b^2*e^(-2*x) +
 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*a))/(sqrt(a^2 + b^2)*a)
```

mupad [B] time = 0.65, size = 109, normalized size = 3.52

$$\frac{\operatorname{atan}\left(\frac{2\,a^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\mathrm{e}^{2\,x}\,\left(-a^4-a^2\,b^2\right)^{3/2}}{2\,a^8+4\,a^6\,b^2+2\,a^4\,b^4}\right)}{\sqrt{-a^4-a^2\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cosh(x)^2 + a^2),x)
```

```
[Out] atan((2*a^2*(- a^4 - a^2*b^2)^(3/2) + b^2*(- a^4 - a^2*b^2)^(3/2) + b^2*exp
(2*x)*(- a^4 - a^2*b^2)^(3/2))/(2*a^8 + 2*a^4*b^4 + 4*a^6*b^2))/(- a^4 - a^
2*b^2)^(1/2)
```

sympy [A] time = 46.11, size = 605, normalized size = 19.52

$$\begin{cases} \frac{\tilde{\infty}\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} \\ -\frac{\tanh\left(\frac{x}{2}\right)}{2b^2}-\frac{1}{2b^2\tanh\left(\frac{x}{2}\right)} \\ \frac{2\tanh\left(\frac{x}{2}\right)}{b^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)} \\ \frac{a\log\left(-\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}+\tanh\left(\frac{x}{2}\right)\right)}{-2a^3\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}-2ab^2\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}}-\frac{a\log\left(\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}+\tanh\left(\frac{x}{2}\right)\right)}{-2a^3\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}-2ab^2\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}}-\frac{ib\log\left(-\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}+\tanh\left(\frac{x}{2}\right)\right)}{-2a^3\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}-2ab^2\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}}+\frac{ib\log\left(\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}+\right.}{-2a^3\sqrt{\frac{a}{a+ib}-\frac{ib}{a+ib}}-2ab^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(x
/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, I*b) | Eq(a, -I*b)), (2*tanh(x/2
)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (a*log(-sqrt(a/(a + I*b) - I*b/(a +
 I*b)) + tanh(x/2))/(-2*a**3*sqrt(a/(a + I*b) - I*b/(a + I*b)) - 2*a*b**2*s
qrt(a/(a + I*b) - I*b/(a + I*b))) - a*log(sqrt(a/(a + I*b) - I*b/(a + I*b))
 + tanh(x/2))/(-2*a**3*sqrt(a/(a + I*b) - I*b/(a + I*b)) - 2*a*b**2*sqrt(a/
(a + I*b) - I*b/(a + I*b))) - I*b*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) +
tanh(x/2))/(-2*a**3*sqrt(a/(a + I*b) - I*b/(a + I*b)) - 2*a*b**2*sqrt(a/(a
+ I*b) - I*b/(a + I*b))) + I*b*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh
(x/2))/(-2*a**3*sqrt(a/(a + I*b) - I*b/(a + I*b)) - 2*a*b**2*sqrt(a/(a + I*
b) - I*b/(a + I*b))) + sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) -
 I*b/(a + I*b))*log(-sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(-2*a**
2*sqrt(a/(a + I*b) - I*b/(a + I*b)) - 2*I*a*b*sqrt(a/(a + I*b) - I*b/(a + I
*b))) - sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))
*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(-2*a**2*sqrt(a/(a + I*
b) - I*b/(a + I*b)) - 2*I*a*b*sqrt(a/(a + I*b) - I*b/(a + I*b))), True))
```

3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-\left(a^2-b^2\right)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.37, size = 388, normalized size = 11.09

$$\left[\frac{\sqrt{a^2-b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4-8\,a^2b^2+b^4-2\left(2\,a^2b^2-b^4\right)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2-2\,a^2b^2+b^4\right)\sinh(x)^2+4}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\left(2\,a^2-b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(\right.}\right.}{2\left(a^3-ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sin
h(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b^4
*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2 - b
^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2
*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)
*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*cosh(x))
*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)^2 + 2*b
^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)/(a^3 - a
*b^2))/(a^3 - a*b^2)]
```

giac [A] time = 0.59, size = 50, normalized size = 1.43

$$-\frac{\arctan\left(\frac{b^2e^{(2\,x)}-2\,a^2+b^2}{2\,\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 +
b^2)*a)
```

maple [B] time = 0.13, size = 74, normalized size = 2.11

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$	74
risch	$\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}-2a^3+2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}-\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}+2a^3-2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}$	166

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a/
((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.38, size = 106, normalized size = 3.03

$$-\frac{\operatorname{atan}\left(\frac{b^2\left(a^2 b^2-a^4\right)^{3/2}-2 a^2\left(a^2 b^2-a^4\right)^{3/2}+b^2 e^{2 x}\left(a^2 b^2-a^4\right)^{3/2}}{2 a^8-4 a^6 b^2+2 a^4 b^4}\right)}{\sqrt{a^2 b^2-a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b^2*cosh(x)^2 - a^2),x)
```

```
[Out] -atan((b^2*(a^2*b^2 - a^4)^(3/2) - 2*a^2*(a^2*b^2 - a^4)^(3/2) + b^2*exp(2*
x)*(a^2*b^2 - a^4)^(3/2))/(2*a^8 + 2*a^4*b^4 - 4*a^6*b^2))/(a^2*b^2 - a^4)^
(1/2)
```

sympy [A] time = 40.60, size = 892, normalized size = 25.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (tanh(x/
2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, b)), (-2*tanh(x/2)/(b**2*(tanh(x/
2)**2 + 1)), Eq(a, 0)), (tanh(x/2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, -
b)), (-a*sqrt(a/(a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(a + b)) + tan
h(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2
*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + a*sqrt(a
/(a - b) + b/(a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*
sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(
a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a
+ b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b)
+ b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a -
 b))*sqrt(a/(a + b) - b/(a + b))) + a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(
a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqr
t(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a +
 b) - b/(a + b))) + b*sqrt(a/(a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(
a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b
/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)
)) - b*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x
/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*
b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a
 + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sq
rt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a
- b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(a + b) - b/(a +
b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b
/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b)
)*sqrt(a/(a + b) - b/(a + b))), True))
```

3.588 $\int \frac{1}{1-\sinh^4(x)} dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^4(x)}\,dx&=\text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx,x,\tanh(x)\right)\\&=\frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)\\&=\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-\sinh^4(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.32, size = 113, normalized size = 4.52

$$\frac{\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\right.}{\cosh(x)^2+\sinh(x)^2-3}\right.}{8\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.59, size = 48, normalized size = 1.92

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right) - \frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

maple [B] time = 0.08, size = 46, normalized size = 1.84

method	result	size
risch	$-\frac{1}{1+e^{2x}}+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{8}$	46
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{4}+\frac{\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+exp(2*x))+1/8*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/8*2^(1/2)*ln(exp(2*x
)-3-2*2^(1/2))
```

maxima [B] time = 0.96, size = 69, normalized size = 2.76

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) + \frac{1}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

mupad [B] time = 0.40, size = 63, normalized size = 2.52

$$\frac{\sqrt{2}\ \ln\left(2\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{8}\right)}{8}-\frac{\sqrt{2}\ \ln\left(2\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{8}\right)}{8}-\frac{1}{e^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log(
2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

sympy [B] time = 7.44, size = 908, normalized size = 36.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] 3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt
(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**
2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(tanh
(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 + sqrt(
2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(
2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1225881
6*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 1733658
4) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) +
 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh
(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) -
 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1
2258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2
)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) -
 3064704*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh
(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584
*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**
2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2)
+ 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) + 1)/(12258816*sqrt(2)*ta
nh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 1225881
6*sqrt(2)*tanh(x/2)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
+ 12258816*sqrt(2) + 17336584) + 17336584*tanh(x/2)/(12258816*sqrt(2)*tanh(
x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584)
```

3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.14, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2074, 618, 204}

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3\left(1-x+x^2\right)}\right)dx,x,\tanh(x)\right)\\
&= -\frac{1}{3(1+\tanh(x))}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\tanh(x)\right)\\
&= -\frac{1}{3(1+\tanh(x))}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\tanh(x)\right)\\
&= -\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(1+\tanh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 37, normalized size = 1.12

$$\frac{1}{18}\left(3\sinh(2x)-3\cosh(2x)+8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.30, size = 74, normalized size = 2.24

$$-\frac{8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+3}{18\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas"
)
```

```
[Out] -1/18*(8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2
)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.60, size = 22, normalized size = 0.67

$$\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)-\frac{1}{6}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

maple [C] time = 0.21, size = 44, normalized size = 1.33

method	result
risch	$-\frac{e^{-2x}}{6}+\frac{2i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\right)}{9}$
default	$\frac{2i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^2}+\frac{2}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)}$
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*exp(-2*x)+2/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-2/9*I*3^(1/2)*ln(exp(2*
x)-I*3^(1/2))
```

maxima [B] time = 1.03, size = 70, normalized size = 2.12

$$\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{6}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima"
)
```

```
[Out] 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) - 1/6*e^(-2*x)
```

mupad [B] time = 0.37, size = 22, normalized size = 0.67

$$\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2x}}{3}\right)}{9}-\frac{e^{-2x}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x)
```

```
[Out] (4*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9 - exp(-2*x)/6
```

sympy [B] time = 1.83, size = 102, normalized size = 3.09

$$\frac{4\sqrt{3}\sinh(x)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)}-\frac{\sqrt{3}}{3}\right)}{9\sinh(x)+9\cosh(x)}+\frac{3\sinh(x)}{9\sinh(x)+9\cosh(x)}+\frac{4\sqrt{3}\cosh(x)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)}-\frac{\sqrt{3}}{3}\right)}{9\sinh(x)+9\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x
) + 9*cosh(x)) + 3*sinh(x)/(9*sinh(x) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*atan
(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x) + 9*cosh(x))
```

3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\cosh(2x)\cosh(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(4x)+\frac{1}{4}\cosh(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cosh(2x)\,dx+\frac{1}{4}\int \cosh(4x)\,dx+\frac{1}{4}\int \cosh(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 44, normalized size = 1.47

$$\frac{1}{4}\cosh(x)\sinh(x)^5+\frac{1}{12}\left(10\cosh(x)^3+3\cosh(x)\right)\sinh(x)^3+\frac{1}{4}\left(\cosh(x)^5+\cosh(x)^3+\cosh(x)\right)\sinh(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(co
sh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

giac [B] time = 0.63, size = 48, normalized size = 1.60

$$-\frac{1}{96}\left(22e^{(6x)}+6e^{(4x)}+3e^{(2x)}+2\right)e^{(-6x)}+\frac{1}{4}x+\frac{1}{48}e^{(6x)}+\frac{1}{32}e^{(4x)}+\frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] -1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x + 1/48*e^(6
*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

maple [A] time = 0.13, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{e^{6x}}{48}+\frac{e^{4x}}{32}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}-\frac{e^{-4x}}{32}-\frac{e^{-6x}}{48}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(2*x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

maxima [A] time = 0.44, size = 42, normalized size = 1.40

$$\frac{1}{96}\left(3e^{(-2x)}+6e^{(-4x)}+2\right)e^{(6x)}+\frac{1}{4}x-\frac{1}{16}e^{(-2x)}-\frac{1}{32}e^{(-4x)}-\frac{1}{48}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*e
^(-4*x) - 1/48*e^(-6*x)
```

mupad [B] time = 0.40, size = 40, normalized size = 1.33

$$\frac{x}{4}-\frac{e^{-2x}}{16}+\frac{e^{2x}}{16}-\frac{e^{-4x}}{32}+\frac{e^{4x}}{32}-\frac{e^{-6x}}{48}+\frac{e^{6x}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*cosh(3*x)*cosh(x),x)
```

```
[Out] x/4 - exp(-2*x)/16 + exp(2*x)/16 - exp(-4*x)/32 + exp(4*x)/32 - exp(-6*x)/4
8 + exp(6*x)/48
```

sympy [B] time = 13.12, size = 116, normalized size = 3.87

$$\frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4}-\frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4}-\frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4}+\frac{x\cosh(x)\cosh(2x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh(
2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - 3*sinh(x)*sinh
(2*x)*sinh(3*x)/8 + sinh(x)*cosh(2*x)*cosh(3*x)/3 + 5*sinh(2*x)*cosh(x)*cos
h(3*x)/24
```

3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$

Optimal. Leaf size=30

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4355, 2637}

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx &= -\int \left(\frac{1}{4}-\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(3x)-\frac{1}{4}\cosh(5x)\right)dx\\
&= -\frac{x}{4}+\frac{1}{4}\int \cosh(2x)\,dx-\frac{1}{4}\int \cosh(3x)\,dx+\frac{1}{4}\int \cosh(5x)\,dx\\
&= -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.23, size = 111, normalized size = 3.70

$$6 \cosh\left(\frac{1}{2}x\right)^{3} \sinh\left(\frac{1}{2}x\right)^{7} + \frac{1}{2} \cosh\left(\frac{1}{2}x\right) \sinh\left(\frac{1}{2}x\right)^{9} + \frac{1}{10}\left(126 \cosh\left(\frac{1}{2}x\right)^{5} - 5 \cosh\left(\frac{1}{2}x\right)\right) \sinh\left(\frac{1}{2}x\right)^{5} + \frac{1}{6}\left(36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")
```

```
[Out] 6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1/10*(126*c
osh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*cosh(1/2*x)^7 - 10*co
sh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 + 1/2*(cosh(1/2*x)^9 - cosh(1/2*
x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) - 1/4*x
```

giac [B] time = 0.64, size = 48, normalized size = 1.60

$$\frac{1}{240}\left(137 e^{(5x)} - 15 e^{(3x)} + 10 e^{(2x)} - 6\right)e^{(-5x)} - \frac{1}{4}x + \frac{1}{40}e^{(5x)} - \frac{1}{24}e^{(3x)} + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="giac")
```

```
[Out] 1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*x + 1/40*e
^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

maple [A] time = 0.20, size = 23, normalized size = 0.77

method	result	size
default	$-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$	23
risch	$-\frac{x}{4}+\frac{e^{5x}}{40}-\frac{e^{3x}}{24}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}+\frac{e^{-3x}}{24}-\frac{e^{-5x}}{40}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

maxima [A] time = 0.45, size = 42, normalized size = 1.40

$$-\frac{1}{240}\left(10 e^{(-2x)} - 15 e^{(-3x)} - 6\right)e^{(5x)} - \frac{1}{4}x - \frac{1}{16}e^{(-2x)} + \frac{1}{24}e^{(-3x)} - \frac{1}{40}e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")
```

```
[Out] -1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1/
24*e^(-3*x) - 1/40*e^(-5*x)
```

mupad [B] time = 0.42, size = 40, normalized size = 1.33

$$\frac{e^{2x}}{16} - \frac{e^{-2x}}{16} - \frac{x}{4} + \frac{e^{-3x}}{24} - \frac{e^{3x}}{24} - \frac{e^{-5x}}{40} + \frac{e^{5x}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((3*x)/2)*sinh((5*x)/2)*sinh(x),x)
```

```
[Out] exp(2*x)/16 - exp(-2*x)/16 - x/4 + exp(-3*x)/24 - exp(3*x)/24 - exp(-5*x)/4
0 + exp(5*x)/40
```

sympy [B] time = 12.14, size = 138, normalized size = 4.60

$$-\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4}+\frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4}+\frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4}-\frac{x\cosh(x)\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh(3*x/2)/4
+ x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(3*x/2)*cosh(5*x/2)/4
 - sinh(x)*sinh(3*x/2)*sinh(5*x/2)/12 + 7*sinh(x)*cosh(3*x/2)*cosh(5*x/2)/2
0 - sinh(3*x/2)*cosh(x)*cosh(5*x/2)/15
```

3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right)+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\sqrt{2}\tan^{-1}\left(\text{sech}(x)\sqrt{\sinh(x)\cosh(x)}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\text{sech}(x)\sqrt{\sinh(x)\cosh}\right.$$

Rubi [A] time = 0.97, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4390, 6725, 207, 203}

$$-\frac{2\sinh(x)\tanh^{-1}\left(\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\frac{2\sinh(x)\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\sinh(x)\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x
])),x]
```

```
[Out] Cosh[x]/Sqrt[Sinh[2*x]] + (2*ArcTan[Sqrt[Tanh[x]]]*Sinh[x])/(Sqrt[Sinh[2*x]
]*Sqrt[Tanh[x]]) + (ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]*Sinh[x])/(3*Sqrt[2]*Sqrt[
Sinh[2*x]]*Sqrt[Tanh[x]]) - (2*ArcTanh[Sqrt[Tanh[x]]]*Sinh[x])/(3*Sqrt[Sinh
[2*x]]*Sqrt[Tanh[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = \frac{\sinh(x)\int \frac{-\cosh(2x)+\tanh(x)}{\left(\sinh^2(x)+\sinh(2x)\right)\sqrt{\tanh(x)}}\,dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\sinh(x)\,\text{Subst}\left(\int \frac{-1+x-x^2-x^3}{x^{3/2}(2+x)(1-x^2)}\,dx, x, \tanh(x)\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\,\text{Subst}\left(\int \frac{1-x^2+x^4+x^6}{x^2(2+x^2)(-1+x^4)}\,dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\,\text{Subst}\left(\int\left(-\frac{1}{2x^2}+\frac{1}{3(-1+x^2)}+\frac{1}{1+x^2}+\frac{1}{6(2+x^2)}\right)dx, x, \sqrt{\tan}\right.}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{\sinh(x)\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{(2\sinh(x}{}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{2\tan^{-1}\left(\sqrt{\tanh(x)}\right)\sinh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)\sin}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tan}}$$

Mathematica [C] time = 30.38, size = 392, normalized size = 5.68

$$\frac{\sqrt{\sinh(2x)}(\tanh(x)-\cosh(2x))\left(-3\coth(x)+\frac{\sqrt[4]{-1}\cosh(x)\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\left(\frac{8\sqrt[6]{-1}\left(2\left(\sqrt[3]{-1}-1\right)\Pi\left(i;\sin^{-1}\left((-1)^{3/4}\sqrt{\tanh\left(\frac{x}{2}\right)}\right)\right|-1\right.}{}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Si
nh[2*x])),x]
```

```
[Out] (Sqrt[Sinh[2*x]]*(-3*Coth[x] + ((-1)^(1/4)*Cosh[x]*Sqrt[Tanh[x/2] + Tanh[x/
2]^3]*((-9*Coth[x/2]*(EllipticF[I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1]
- EllipticPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1] - Elli
pticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1]))/Sqrt[1 + C
oth[x/2]^2] + (8*(-1)^(1/6)*((3 - (3*I)*Sqrt[3])*EllipticPi[-I, I*ArcSinh[(
-1)^(1/4)*Sqrt[Tanh[x/2]]], -1] + 2*(-1 + (-1)^(1/3))*EllipticPi[I, ArcSin[
(-1)^(3/4)*Sqrt[Tanh[x/2]]], -1] + I*(I + Sqrt[3])*EllipticPi[-(-1)^(1/6),
I*ArcSinh[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1] + 2*(-1 + (-1)^(1/3))*EllipticPi
[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1]))/((-I + Sqrt[3])*
Sqrt[1 + Tanh[x/2]^2])))/((1 + Cosh[x])*Sqrt[Sinh[2*x]/(1 + Cosh[x])^2]*Sqr
t[Tanh[x/2]]))*(-Cosh[2*x] + Tanh[x]))/(3*(Cosh[x] + Cosh[3*x] - 2*Sinh[x])
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[
x]^2 + Sinh[2*x])),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.15, size = 376, normalized size = 5.45

$$-\frac{\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\arctan\left(\frac{\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+3\sqrt{2}\right)\sqrt{\frac{\cos}{\cosh(x)^2-2\,\mathrm{co}}}}{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="fricas")
```

```
[Out] -1/12*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(1/2*(sqrt(2)*
cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + 3*sqrt(2))*sqrt
(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 +
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4 - 1)) + 6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*arctan(2*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 +
4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh
(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(2*cosh(x)^4 + 8*cosh(x)^3*si
nh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*cosh(x)*sinh(x)^3 + 2*sinh(x)^4 - 4*(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) - 1) - 12*sqrt(2)*sqrt(cosh(x)*sinh(x)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -1/2*
(((-i)*pi+sqrt(2)*atan((1+2*i)/(1-i)/sqrt(2))+12*atan(i)+6-6*i)/3/sqrt(2)+2
*(-sqrt(2)/(-exp(x)^2+sqrt(exp(x)^4-1)+1)-1/6*atan(1/2*(3*(-exp(x)^2+sqrt(e
xp(x)^4-1))-1)/sqrt(2))-1/3*ln(exp(x)^2-sqrt(exp(x)^4-1))/sqrt(2)-sqrt(2)*a
tan(-exp(x)^2+sqrt(exp(x)^4-1))))
```

maple [C] time = 0.48, size = 987, normalized size = 14.30

method	result
default	$-\frac{\sqrt{\frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)-1\right)^2}}\left(\tanh^2\left(\frac{x}{2}\right)-1\right)\left(\sqrt{3}\sqrt{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}\sqrt{2}\sqrt{-i\left(\tanh\left(\frac{x}{2}\right)+i\right)}\sqrt{-i\left(-\tanh\left(\frac{x}{2}\right)+i\right)}\sqrt{i\tanh\left(\frac{x}{2}\right)}\ \mathrm{Elliptic}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/24*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x
)^2-1)*(3^(1/2)*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*2^(1/2)*(-I*(tanh(1/2
*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi(
(-I*(tanh(1/2*x)+I))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*2^(1/2))-3^(1/2)*((tanh(
1/2*x)^2+1)*tanh(1/2*x))^(1/2)*2^(1/2)*(-I*(tanh(1/2*x)+I))^(1/2)*(-I*(-tan
h(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1
/2),1/2-I-1/2*I*3^(1/2),1/2*2^(1/2))+I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(
-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x
)+I))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x)
)^(1/2)+24*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)
*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-8*I*(-I*(tanh(1/2*x)+I))^(1/
2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I
*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*
x))^(1/2)+I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*
(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I-1/2*I*3^(
1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-18*I*(-I*(tanh(1/2*
x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*Elli
pticF((-I*(tanh(1/2*x)+I))^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x
))^(1/2)-2*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*2^(1/2)*(-I*(tanh(1/2*x)+I
))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(
tanh(1/2*x)+I))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*2^(1/2))-24*(-I*(tanh(1/2*x)+
I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*Ellipti
cPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*ta
nh(1/2*x))^(1/2)-8*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))
^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-2*((tanh(1/2*x)^2+1)*t
anh(1/2*x))^(1/2)*2^(1/2)*(-I*(tanh(1/2*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^
(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I-1/2
*I*3^(1/2),1/2*2^(1/2))+12*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)*tanh(1/2*x)^2+
12*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/(tanh(1/2*x)^2+1)/tanh(1/2*x)/(tanh(1
/2*x)^3+tanh(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(\cosh(2x) - \tanh(x))\cosh(x)}{\left(\sinh(x)^2 + \sinh(2x)\right)\sqrt{\sinh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh
(2*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cosh(x)\,\left(\cosh(2x) - \tanh(x)\right)}{\sqrt{\sinh(2x)}\,\left(\sinh(x)^2 + \sinh(2x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^
2)),x)
```

```
[Out] -int((cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^
2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh(x)\cosh(2x)}{\sinh^2(x)\sqrt{\sinh(2x)} + \sinh^{\frac{3}{2}}(2x)}\,dx - \int \left(-\frac{\cosh(x)\tanh(x)}{\sinh^2(x)\sqrt{\sinh(2x)} + \sinh^{\frac{3}{2}}(2x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(1
/2),x)
```

```
[Out] -Integral(cosh(x)*cosh(2*x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2))
, x) - Integral(-cosh(x)*tanh(x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(
3/2)), x)
```

3.593 $\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 192, 191}

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] -Cosh[x]/(27*(-9 + 4*Cosh[x]^2)^(3/2)) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh[
x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(-9+4x^2\right)^{5/2}}\,dx, x, \cosh(x)\right)\\
&= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}}-\frac{2}{27}\text{Subst}\left(\int \frac{1}{\left(-9+4x^2\right)^{3/2}}\,dx, x, \cosh(x)\right)\\
&= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}}+\frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 26, normalized size = 0.70

$$\frac{\cosh(x)(4\cosh(2x)-23)}{243(2\cosh(2x)-7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.94, size = 474, normalized size = 12.81

$$\frac{2\cosh(x)^8+16\cosh(x)\sinh(x)^7+2\sinh(x)^8+28\left(2\cosh(x)^2-1\right)\sinh(x)^6-28\cosh(x)^6+56\left(2\cosh(x)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/486*(2*cosh(x)^8 + 16*cosh(x)*sinh(x)^7 + 2*sinh(x)^8 + 28*(2*cosh(x)^2 -
 1)*sinh(x)^6 - 28*cosh(x)^6 + 56*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(
70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 102*cosh(x)^4 + 8*(14*cosh(x
)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 4*(14*cosh(x)^6 - 105*cosh(x)^
4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 28*cosh(x)^2 + 8*(2*cosh(x)^7 - 21*cosh(
x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + (2*cosh(x)^6 + 12*cosh(x)*sinh(x
)^5 + 2*sinh(x)^6 + 3*(10*cosh(x)^2 - 7)*sinh(x)^4 - 21*cosh(x)^4 + 4*(10*c
osh(x)^3 - 21*cosh(x))*sinh(x)^3 + 3*(10*cosh(x)^4 - 42*cosh(x)^2 - 7)*sinh
(x)^2 - 21*cosh(x)^2 + 6*(2*cosh(x)^5 - 14*cosh(x)^3 - 7*cosh(x))*sinh(x) +
 2)*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 2)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 14*(2*cosh(x
)^2 - 1)*sinh(x)^6 - 14*cosh(x)^6 + 28*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5
+ (70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 51*cosh(x)^4 + 4*(14*cosh
(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 105*cosh(x
)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(2*cosh(x)^7 - 21*cos
h(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.73, size = 40, normalized size = 1.08

$$-\frac{\left(\left(2\,e^{(2\,x)}-21\right)e^{(2\,x)}-21\right)e^{(2\,x)}+2}{486\left(e^{(4\,x)}-7\,e^{(2\,x)}+1\right)^{\frac{3}{2}}}+\frac{1}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*e^(2*x) +
 1)^(3/2) + 1/243
```

maple [A] time = 0.05, size = 30, normalized size = 0.81

method	result	size

derivativedivides	$-\frac{\cosh(x)}{27\left(-9+4\left(\cosh^2(x)\right)\right)^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4\left(\cosh^2(x)\right)}}$	30
default	$-\frac{\cosh(x)}{27\left(-9+4\left(\cosh^2(x)\right)\right)^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4\left(\cosh^2(x)\right)}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

maxima [B] time = 0.58, size = 125, normalized size = 3.38

$$-\frac{1855\,e^{(-2\,x)}-8485\,e^{(-4\,x)}+5285\,e^{(-6\,x)}-980\,e^{(-8\,x)}+56\,e^{(-10\,x)}-106}{12150\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}+\frac{980\,e^{(-2\,x)}-5285\,e^{(-4\,x)}+8485\,e^{(-6\,x)}}{12150\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(-8*x) + 56
*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x) +
1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-4*x) + 8485*e^(-6*x) - 1855*e^
(-8*x) + 106*e^(-10*x) - 56)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) +
e^(-2*x) + 1)^(5/2))
```

mupad [B] time = 0.14, size = 57, normalized size = 1.54

$$-\frac{\mathrm{e}^x\,\sqrt{4\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^x}{2}\right)}^2-9}\,\left(21\,\mathrm{e}^{2\,x}+21\,\mathrm{e}^{4\,x}-2\,\mathrm{e}^{6\,x}-2\right)}{486\,{\left(\mathrm{e}^{4\,x}-7\,\mathrm{e}^{2\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x)
```

```
[Out] -(exp(x)*(4*(exp(-x)/2 + exp(x)/2)^2 - 9)^(1/2)*(21*exp(2*x) + 21*exp(4*x)
- 2*exp(6*x) - 2))/(486*(exp(4*x) - 7*exp(2*x) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 266, 43}

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx &= i\,\text{Subst}\left(\int -\frac{2ix^3}{\left(1-x^2\right)^{3/2}}\,dx,x,\sinh(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)^{3/2}}\,dx,x,\sinh(x)\right)\\
&= \text{Subst}\left(\int \frac{x}{(1-x)^{3/2}}\,dx,x,\sinh^2(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{(1-x)^{3/2}}-\frac{1}{\sqrt{1-x}}\right)dx,x,\sinh^2(x)\right)\\
&= \frac{2}{\sqrt{1-\sinh^2(x)}}+2\sqrt{1-\sinh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 21, normalized size = 0.72

$$\frac{5-\cosh(2x)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
[Out] (5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
[Out] Could not integrate
```

fricas [B] time = 1.34, size = 161, normalized size = 5.55

$$\frac{\sqrt{2}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-5\right)\sinh(x)^2-10\cosh(x)^2+4\left(\cosh(x\right.\right.}{\cosh(x)^5+5\cosh(x)\sinh(x)^4+\sinh(x)^5+2\left(5\cosh(x)^2-3\right)\sinh(x)^3-6\cosh(x)^3+2\left(5\cosh(x)^3-9\text{ co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)*sqrt(-(co
sh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh
(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3 - 6
*cosh(x)^3 + 2*(5*cosh(x)^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18*cosh
(x)^2 + 1)*sinh(x) + cosh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(2x)\sinh(x)^2}{\left(-\sinh(x)^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

maple [C] time = 0.13, size = 28, normalized size = 0.97

method	result	size
default	$`int/indef0`\left(-\frac{2\left(\sinh^3(x)\right)}{\left(\sinh^2(x)-1\right)\sqrt{1-\left(\sinh^2(x)\right)}},\sinh(x)\right)$	28
meijerg	`error in int/gbinthm/express: improper op or subscript selector\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(-2*sinh(x)^3/(sinh(x)^2-1)/(1-sinh(x)^2)^(1/2),sinh(x))
```

maxima [B] time = 1.21, size = 177, normalized size = 6.10

$$-\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}+\frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}-\frac{16\,e^{(}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2)
) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^
(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x)
+ 1)^(3/2)) + e^(-7*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x
) + 1)^(3/2)) + e^x/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) +
 1)^(3/2))
```

mupad [B] time = 0.49, size = 47, normalized size = 1.62

$$\frac{2\sqrt{1-\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^2}\,\left(\mathrm{e}^{4\,x}-10\,\mathrm{e}^{2\,x}+1\right)}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(2*x)*sinh(x)^2)/(1 - sinh(x)^2)^(3/2),x)
```

```
[Out] (2*(1 - (exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x) - 10*exp(2*x) + 1))/(exp(
4*x) - 6*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^2(x)\sinh(2x)}{(-(\sinh(x)-1)(\sinh(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

```
[Out] Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)
```

3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4356, 215}

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[Cosh[2*x]],x]
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, \sinh(x)\right)\\ &= \frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.97, size = 482, normalized size = 32.13

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\cosh(x)^{8}+8\cosh(x)\sinh(x)^{7}+\sinh(x)^{8}+\left(28\cosh(x)^{2}-3\right)\sinh(x)^{6}-3\cosh(x)^{6}+2\left(28\cosh(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)
^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 +
5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^
5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30
*cosh(x)^2 - 4)*sinh(x)^2 + sqrt(2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*co
sh(x))*sinh(x)^3 + (15*cosh(x)^4 - 18*cosh(x)^2 + 4)*sinh(x)^2 + 4*cosh(x)^
2 + 2*(3*cosh(x)^5 - 6*cosh(x)^3 + 4*cosh(x))*sinh(x) - 4)*sqrt((cosh(x)^2
+ sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*cosh(x)^2 + 2
*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + 4)/(cosh(
x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^
3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2
)*log((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt((cosh(
x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + cosh(x)^2
+ 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2))
```

giac [B] time = 0.66, size = 58, normalized size = 3.87

$$-\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
- e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

maple [B] time = 0.18, size = 63, normalized size = 4.20

method	result	size
default	$\frac{\sqrt{\left(2\left(\cosh^{2}(x)\right)-1\right)\left(\sinh^{2}(x)\right)}\,\ln\left(\sqrt{2}\,\left(\sinh^{2}(x)\right)+\frac{\sqrt{2}}{4}+\sqrt{2\left(\sinh^{4}(x)\right)+\sinh^{2}(x)}\right)\sqrt{2}}{4\sinh(x)\sqrt{2\left(\cosh^{2}(x)\right)-1}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+1/4*2^(1/2)+(2*s
inh(x)^4+sinh(x)^2)^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(2*x)^(1/2),x)
```

```
[Out] int(cosh(x)/cosh(2*x)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

3.596 $\int x \tanh^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3475, 30}

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tanh^2(x)\,dx &= -x\tanh(x) + \int x\,dx + \int \tanh(x)\,dx\\ &= \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\tanh^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Tanh[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.29, size = 93, normalized size = 5.81

$$\frac{\left(x^2-4\,x\right)\cosh(x)^2+2\left(x^2-4\,x\right)\cosh(x)\sinh(x)+\left(x^2-4\,x\right)\sinh(x)^2+x^2+2\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)\right.}{2\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cos
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.63, size = 51, normalized size = 3.19

$$\frac{x^2e^{(2\,x)}+x^2-4\,xe^{(2\,x)}+2\,e^{(2\,x)}\log\left(e^{(2\,x)}+1\right)+2\,\log\left(e^{(2\,x)}+1\right)}{2\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) + 1) + 2*log(e
^(2*x) + 1))/(e^(2*x) + 1)
```

maple [A] time = 0.03, size = 28, normalized size = 1.75

method	result	size
risch	$\frac{x^2}{2}-2x+\frac{2x}{1+e^{2x}}+\ln\left(1+e^{2x}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

maxima [B] time = 1.12, size = 49, normalized size = 3.06

$$-\frac{xe^{(2\,x)}}{e^{(2\,x)}+1}+\frac{x^2+\left(x^2-2\,x\right)e^{(2\,x)}}{2\left(e^{(2\,x)}+1\right)}+\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) +
log(e^(2*x) + 1)
```

mupad [B] time = 0.31, size = 21, normalized size = 1.31

$$\ln\left(e^{2\,x}+1\right)-x-x\tanh(x)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(x)^2,x)
```

```
[Out] log(exp(2*x) + 1) - x - x*tanh(x) + x^2/2
```

sympy [A]   time = 0.19, size = 17, normalized size = 1.06

$$\frac{x^2}{2} - x\tanh(x) + x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)**2,x)
```

```
[Out] x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

3.597 $\int x \coth^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3475, 30}

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\coth^2(x)\,dx &= -x\coth(x) + \int x\,dx + \int \coth(x)\,dx\\ &= \frac{x^2}{2} - x\coth(x) + \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{x^2}{2} - x\coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Coth[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.32, size = 95, normalized size = 5.94

$$\frac{\left(x^{2}-4\,x\right)\cosh(x)^{2}+2\left(x^{2}-4\,x\right)\cosh(x)\sinh(x)+\left(x^{2}-4\,x\right)\sinh(x)^{2}-x^{2}+2\left(\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\right.}{2\left(\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\sinh(x)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sin
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.59, size = 53, normalized size = 3.31

$$\frac{x^{2}e^{(2\,x)}-x^{2}-4\,xe^{(2\,x)}+2\,e^{(2\,x)}\log\left(e^{(2\,x)}-1\right)-2\,\log\left(e^{(2\,x)}-1\right)}{2\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(e
^(2*x) - 1))/(e^(2*x) - 1)
```

maple [A] time = 0.03, size = 28, normalized size = 1.75

method	result	size
risch	$\frac{x^{2}}{2}-2x-\frac{2x}{-1+{\rm e}^{2x}}+\ln\left(-1+{\rm e}^{2x}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-2*x-2*x/(-1+exp(2*x))+ln(-1+exp(2*x))
```

maxima [B] time = 0.64, size = 53, normalized size = 3.31

$$-\frac{xe^{(2\,x)}}{e^{(2\,x)}-1}-\frac{x^{2}-\left(x^{2}-2\,x\right)e^{(2\,x)}}{2\left(e^{(2\,x)}-1\right)}+\log\left(e^{x}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) +
log(e^x + 1) + log(e^x - 1)
```

mupad [B] time = 0.29, size = 27, normalized size = 1.69

$$\ln\left({\mathrm{e}}^{2\,x}-1\right)-2\,x-\frac{2\,x}{{\mathrm{e}}^{2\,x}-1}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(x)^2,x)
```

```
[Out] log(exp(2*x) - 1) - 2*x - (2*x)/(exp(2*x) - 1) + x^2/2
```

sympy [**A**] time = 0.66, size = 22, normalized size = 1.38

$$\frac{x^2}{2} + x - \frac{x}{\tanh(x)} - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)**2,x)
```

```
[Out] x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=20

$$e^x x - e^x + \frac{e^{2x}}{2}$$

Rubi [A] time = 0.07, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5648, 6742, 2176, 2194, 2282, 12, 14}

$$e^x x - e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \int e^x(x+\cosh(x)+\sinh(x))\,dx$$
$$= \int \left(e^x x+e^x\cosh(x)+e^x\sinh(x)\right)\,dx$$
$$= \int e^x x\,dx+\int e^x\cosh(x)\,dx+\int e^x\sinh(x)\,dx$$
$$= e^x x-\int e^x\,dx+\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx,x,e^x\right)$$
$$= -e^x+e^x x+\frac{1}{2}\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,e^x\right)$$
$$= -e^x+e^x x+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx,x,e^x\right)+\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx,x,e^x\right)$$
$$= -e^x+\frac{e^{2x}}{2}+e^x x$$

Mathematica [A] time = 0.10, size = 23, normalized size = 1.15

$$(x-1)\sinh(x)+\frac{1}{2}\cosh(2x)+(x+\sinh(x)-1)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
[Out] Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
[Out] Could not integrate
```

fricas [A] time = 1.36, size = 20, normalized size = 1.00

$$\frac{2\,x+\cosh(x)+\sinh(x)-2}{2\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

giac [A] time = 0.63, size = 11, normalized size = 0.55

$$\frac{1}{2}\,(2\,x+e^x-2)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(2*x + e^x - 2)*e^x
```

maple [A] time = 0.09, size = 14, normalized size = 0.70

method	result	size
risch	$(-1+x)\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
[Out] (-1+x)*exp(x)+1/2*exp(2*x)
```

maxima [A] time = 0.49, size = 13, normalized size = 0.65

$$(x-1)e^{x}+\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] (x - 1)*e^x + 1/2*e^(2*x)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.80

$$\mathrm{e}^{x}\,\left(x+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
[Out] exp(x)*(x + exp(-x)/2 + exp(x)/2 - 1)
```

sympy [A] time = 0.42, size = 26, normalized size = 1.30

$$\frac{x}{-\sinh\left(x\right)+\cosh\left(x\right)}+\frac{\sinh\left(x\right)}{-\sinh\left(x\right)+\cosh\left(x\right)}-\frac{1}{-\sinh\left(x\right)+\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
[Out] x/(-sinh(x) + cosh(x)) + sinh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh(
x))
```

3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} dx$

Optimal. Leaf size=15

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.13, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 3318, 4184, 3475}

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] x - (1 - x)*Tanh[x/2]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx &= \int \left(\frac{x+\cosh(x)}{1+\cosh(x)}+\tanh\left(\frac{x}{2}\right)\right)dx\\
&= \int \frac{x+\cosh(x)}{1+\cosh(x)}\,dx+\int \tanh\left(\frac{x}{2}\right)dx\\
&= 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \left(1+\frac{-1+x}{1+\cosh(x)}\right)dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \frac{-1+x}{1+\cosh(x)}\,dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\int (-1+x)\operatorname{sech}^2\left(\frac{x}{2}\right)dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)-(1-x)\tanh\left(\frac{x}{2}\right)-\int \tanh\left(\frac{x}{2}\right)dx\\
&= x-(1-x)\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 20, normalized size = 1.33

$$\frac{\sinh(x)\left(x+x\coth\left(\frac{x}{2}\right)-1\right)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
[Out] ((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
[Out] Could not integrate
```

fricas [A] time = 1.40, size = 20, normalized size = 1.33

$$\frac{2\,(x\cosh(x)+x\sinh(x)+1)}{\cosh(x)+\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
[Out] 2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.61, size = 14, normalized size = 0.93

$$\frac{2\,(xe^{x}+1)}{e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="giac")
[Out] 2*(x*e^x + 1)/(e^x + 1)
```

maple [A] time = 0.06, size = 16, normalized size = 1.07

method	result	size
risch	$2x-\frac{2(-1+x)}{1+e^{x}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x)+sinh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)
[Out] 2*x-2*(-1+x)/(1+exp(x))
```

maxima [B] time = 0.45, size = 35, normalized size = 2.33

$$x+\frac{2\,xe^{x}}{e^{x}+1}-\frac{2}{e^{(-x)}+1}+\log\left(\cosh(x)+1\right)-2\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log(e^x + 1)
```

mupad [B] time = 0.29, size = 17, normalized size = 1.13

$$2\,x-\frac{2\,x-2}{{\mathrm{e}}^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cosh(x) + sinh(x))/(cosh(x) + 1),x)
[Out] 2*x - (2*x - 2)/(exp(x) + 1)
```

sympy [A] time = 0.47, size = 12, normalized size = 0.80

$$x\tanh\left(\frac{x}{2}\right)+x-\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
[Out] x*tanh(x/2) + x - tanh(x/2)
```

3.600 $\int e^{2x}\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=20

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 264}

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{2x}\text{csch}^4(x)\,dx &= \text{Subst}\left(\int \frac{16x^5}{\left(1-x^2\right)^4}\,dx,x,e^x\right)\\
&= 16\,\text{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)^4}\,dx,x,e^x\right)\\
&= \frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{2x}\operatorname{csch}^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(2*x)*Csch[x]^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.31, size = 75, normalized size = 3.75

$$-\frac{8\left(4\cosh(x)^2+4\cosh(x)\sinh(x)+4\sinh(x)^2-3\right)}{3\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-2\right)\sinh(x)^2-4\cosh(x)^2+4\left(\cosh(x)^3-\text{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2 +
4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

giac [A] time = 0.62, size = 24, normalized size = 1.20

$$-\frac{8\left(3\,e^{(4\,x)}-3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] -8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

maple [A] time = 0.10, size = 25, normalized size = 1.25

method	result	size
risch	$-\frac{8\left(3\,e^{4x}-3\,e^{2x}+1\right)}{3\left(-1+e^{2x}\right)^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/sinh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*(3*exp(4*x)-3*exp(2*x)+1)/(-1+exp(2*x))^3
```

maxima [A] time = 0.43, size = 22, normalized size = 1.10

$$\frac{8}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] 8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

mupad [B] time = 0.33, size = 24, normalized size = 1.20

$$-\frac{8\left(3\,e^{4\,x}-3\,e^{2\,x}+1\right)}{3\left(e^{2\,x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/sinh(x)^4,x)
[Out] -(8*(3*exp(4*x) - 3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\sinh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)**4,x)
[Out] Integral(exp(2*x)/sinh(x)**4, x)
```

3.601 $\int e^{-2x}\textbf{sech}^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 261}

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/E^(2*x),x]
```

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}\text{sech}^4(x)\,dx &= \text{Subst}\left(\int \frac{16x}{\left(1+x^2\right)^4}\,dx, x, e^x\right)\\ &= 16\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^4}\,dx, x, e^x\right)\\ &= -\frac{8}{3\left(1+e^{2x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/E^(2*x),x]
[Out] -8/(3*(1 + E^(2*x))^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-2x}\operatorname{sech}^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sech[x]^4/E^(2*x),x]
[Out] Could not integrate
```

fricas [B] time = 0.77, size = 102, normalized size = 7.85

$$-\frac{}{3\left(\cosh(x)^6+6\,\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\,\cosh(x)^2+1\right)\sinh(x)^4+3\,\cosh(x)^4+4\left(5\,\cosh(x)^3+3\,\mathrm{c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")
[Out] -8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sin
h(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)
^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3
+ cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.59, size = 10, normalized size = 0.77

$$-\frac{8}{3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="giac")
[Out] -8/3/(e^(2*x) + 1)^3
```

maple [A] time = 0.35, size = 11, normalized size = 0.85

method	result	size
risch	$-\frac{8}{3\left(1+e^{2x}\right)^3}$	11
default	$2\tanh(x)+\frac{1}{\cosh(x)^2}-\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*x)/cosh(x)^4,x,method=_RETURNVERBOSE)
[Out] -8/3/(1+exp(2*x))^3
```

maxima [B] time = 0.46, size = 75, normalized size = 5.77

$$\frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")
```

```
[Out] 8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/(3*e^(-2*x
) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
1)
```

mupad [B] time = 0.31, size = 19, normalized size = 1.46

$$-\frac{e^{-3x}}{3\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)/cosh(x)^4,x)
```

```
[Out] -exp(-3*x)/(3*(exp(-x)/2 + exp(x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)**4,x)
```

```
[Out] Integral(exp(-2*x)/cosh(x)**4, x)
```

3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=9

$$\frac{e^{2x}}{2}$$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 30}

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx &= \text{Subst}\left(\int x\,dx, x, e^x\right)\\ &= \frac{e^{2x}}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.93, size = 16, normalized size = 1.78

$$\frac{\cosh(x) + \sinh(x)}{2 (\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

giac [A] time = 0.59, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
[Out] 1/2*e^(2*x)
```

maple [A] time = 0.10, size = 7, normalized size = 0.78

method	result	size
risch	$\frac{e^{2x}}{2}$	7
gosper	$\frac{e^{x}}{2\cosh(x)-2\sinh(x)}$	14
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*x)
```

maxima [A] time = 0.44, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] 1/2*e^(2*x)
```

mupad [B] time = 0.32, size = 6, normalized size = 0.67

$$\frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x) - sinh(x)),x)
[Out] exp(2*x)/2
```

sympy [B] time = 0.49, size = 12, normalized size = 1.33

$$\frac{e^{x}}{-2\sinh(x) + 2\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] exp(x)/(-2*sinh(x) + 2*cosh(x))
```

3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=13

$$\frac{e^{(m-1)x}}{m-1}$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5648, 2227, 2194}

$$-\frac{e^{-(1-m)x}}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -(1/(E^((1 - m)*x)*(1 - m)))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} \, dx &= \int e^{-x+mx} \, dx \\
&= \int e^{-(1-m)x} \, dx \\
&= -\frac{e^{-(1-m)x}}{1-m}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.38

$$\frac{e^{mx}(\cosh(x)-\sinh(x))}{m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] (E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.97, size = 25, normalized size = 1.92

$$\frac{\cosh(mx) + \sinh(mx)}{(m-1)\cosh(x) + (m-1)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] (cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

giac [A] time = 0.58, size = 16, normalized size = 1.23

$$\frac{e^{(mx)}}{me^x - e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] e^(m*x)/(m*e^x - e^x)
```

maple [A] time = 0.14, size = 13, normalized size = 1.00

method	result	size
risch	$\frac{e^{(-1+m)x}}{-1+m}$	13
gosper	$\frac{e^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$	18
default	$\frac{\sinh((-1+m)x)}{-1+m}+\frac{\cosh((-1+m)x)}{-1+m}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp((-1+m)*x)/(-1+m)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-m>0)', see `assume?` for more deta
ils)Is -m equal to -1?
```

mupad [B] time = 0.11, size = 14, normalized size = 1.08

$$\frac{e^{m\,x-x}}{m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/(cosh(x) + sinh(x)),x)
```

```
[Out] exp(m*x - x)/(m - 1)
```

sympy [A] time = 0.56, size = 32, normalized size = 2.46

$$\begin{cases} \frac{e^{mx}}{m \sinh(x) + m \cosh(x) - \sinh(x) - \cosh(x)} & \text{for } m \neq 1 \\ \frac{xe^{x}}{\sinh(x) + \cosh(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1)),
 (x*exp(x)/(sinh(x) + cosh(x)), True))
```

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=1

$$x$$

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 29}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx &= \text{Subst}\left(\int \frac{1}{x} \, dx, x, e^x\right) \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.78, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 0.59, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.06, size = 2, normalized size = 2.00

method	result	size
default	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x
```

maxima [A] time = 0.43, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 0.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x) + sinh(x)),x)
```

```
[Out] x
```

sympy [B] time = 0.41, size = 10, normalized size = 10.00

$$\frac{xe^{x}}{\sinh\left(x\right)+\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

```
[Out] x*exp(x)/(sinh(x) + cosh(x))
```

3.605 $\int \frac{e^x}{1-\cosh(x)} \, dx$

Optimal. Leaf size=22

$$-\frac{2}{1-e^x}-2\log\left(1-e^x\right)$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 43}

$$-\frac{2}{1-e^x}-2\log\left(1-e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Cosh[x]),x]
```

```
[Out] -2/(1 - E^x) - 2*Log[1 - E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{1-\cosh(x)} \, dx &= \text{Subst}\left(\int -\frac{2x}{(1-x)^2} \, dx, x, e^x\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{x}{(1-x)^2} \, dx, x, e^x\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right) dx, x, e^x\right)\right)\\
&= -\frac{2}{1-e^x}-2\log\left(1-e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 1.64

$$\frac{4\left(\frac{1}{1-e^x}+\log\left(1-e^x\right)\right)\sinh^2\left(\frac{x}{2}\right)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Cosh[x]),x]
[Out] (4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1-\cosh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x/(1 - Cosh[x]),x]
[Out] Could not integrate
```

fricas [A] time = 0.96, size = 26, normalized size = 1.18

$$-\frac{2\left((\cosh(x)+\sinh(x)-1)\log(\cosh(x)+\sinh(x)-1)-1\right)}{\cosh(x)+\sinh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")
[Out] -2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh
(x) - 1)
```

giac [A] time = 0.61, size = 17, normalized size = 0.77

$$\frac{2}{e^x-1}-2\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="giac")
[Out] 2/(e^x - 1) - 2*log(abs(e^x - 1))
```

maple [A] time = 0.05, size = 17, normalized size = 0.77

method	result	size
risch	$\frac{2}{-1+e^x}-2\ln\left(-1+e^x\right)$	17
default	$2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(-1+exp(x))-2*ln(-1+exp(x))
```

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{2}{e^x-1}-2\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x - 1) - 2*log(e^x - 1)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.73

$$\frac{2}{e^x-1}-2\ln\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(x)/(cosh(x) - 1),x)
```

```
[Out] 2/(exp(x) - 1) - 2*log(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^{x}}{\cosh(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x)
```

```
[Out] -Integral(exp(x)/(cosh(x) - 1), x)
```

3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$

Optimal. Leaf size=13

$$e^x+\frac{2}{e^x+1}$$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2282, 683}

$$e^x+\frac{2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

```
[Out] E^x + 2/(1 + E^x)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx &= \text{Subst}\left(\int \frac{-1+2x+x^2}{(1+x)^2}\,dx,x,e^x\right)\\ &= \text{Subst}\left(\int \left(1-\frac{2}{(1+x)^2}\right)dx,x,e^x\right)\\ &= e^x+\frac{2}{1+e^x}\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.38

$$\frac{e^x+e^{2x}+2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

```
[Out] (2 + E^x + E^(2*x))/(1 + E^x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
[Out] Could not integrate
```

fricas [A] time = 0.69, size = 21, normalized size = 1.62

$$\frac{3 \cosh(x) - \sinh(x) + 1}{\cosh(x) - \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
[Out] (3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

giac [A] time = 0.60, size = 11, normalized size = 0.85

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="giac")
[Out] 2/(e^x + 1) + e^x
```

maple [A] time = 0.05, size = 12, normalized size = 0.92

method	result	size
risch	$e^x + \frac{2}{1+e^x}$	12
default	$-\tanh\left(\frac{x}{2}\right) - \frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sinh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)+2/(1+exp(x))
```

maxima [A] time = 0.44, size = 11, normalized size = 0.85

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x + 1) + e^x
```

mupad [B] time = 0.30, size = 11, normalized size = 0.85

$$e^x + \frac{2}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(sinh(x) + 1))/(cosh(x) + 1),x)
[Out] exp(x) + 2/(exp(x) + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sinh(x)+1)\,e^{x}}{\cosh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

```
[Out] Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} dx$

Optimal. Leaf size=15

$$e^x - \frac{2}{1-e^x}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 683}

$$e^x - \frac{2}{1-e^x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] E^x - 2/(1 - E^x)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx &= \text{Subst}\left(\int \frac{-1-2x+x^2}{(1-x)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1-\frac{2}{(-1+x)^2}\right)dx, x, e^x\right)\\
&= e^x - \frac{2}{1-e^x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.33

$$\frac{-e^x + e^{2x} + 2}{e^x - 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] (2 - E^x + E^(2*x))/(-1 + E^x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
[Out] Could not integrate
```

fricas [A] time = 1.71, size = 22, normalized size = 1.47

$$-\frac{3 \cosh(x) - \sinh(x) - 1}{\cosh(x) - \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")
[Out] -(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

giac [A] time = 0.59, size = 11, normalized size = 0.73

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="giac")
[Out] 2/(e^x - 1) + e^x
```

maple [A] time = 0.07, size = 12, normalized size = 0.80

method	result	size
risch	$e^x + \frac{2}{-1+e^x}$	12
default	$\frac{1}{\tanh\left(\frac{x}{2}\right)} - \frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)+2/(-1+exp(x))
```

maxima [A] time = 0.44, size = 11, normalized size = 0.73

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x - 1) + e^x
```

mupad [B] time = 0.04, size = 11, normalized size = 0.73

$$e^x + \frac{2}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)*(sinh(x) - 1))/(cosh(x) - 1),x)
[Out] exp(x) + 2/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sinh(x) - 1)\, e^{x}}{\cosh(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

3.608 $\int x^m \log(x)\,dx$

Optimal. Leaf size=26

$$\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[x],x]
```

```
[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log(x)\,dx=-\frac{x^{1+m}}{(1+m)^2}+\frac{x^{1+m}\log(x)}{1+m}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^{m+1}((m+1)\log(x)-1)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[x],x]
```

```
[Out] (x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*Log[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 25, normalized size = 0.96

$$\frac{\left((m+1)x\log(x)-x\right)x^m}{m^2+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="fricas")
```

```
[Out] ((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^m*log(x), x)
```

maple [A] time = 0.03, size = 19, normalized size = 0.73

method	result	size
risch	$\frac{x(m\ln(x)+\ln(x)-1)x^m}{(1+m)^2}$	19
norman	$\frac{x\ln(x)e^{m\ln(x)}}{1+m} - \frac{x\, e^{m\ln(x)}}{m^2+2m+1}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*(m*ln(x)+ln(x)-1)/(1+m)^2*x^m
```

maxima [A] time = 0.42, size = 26, normalized size = 1.00

$$\frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="maxima")
```

```
[Out] x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2
```

mupad [B] time = 0.40, size = 32, normalized size = 1.23

$$\begin{cases} \frac{\ln(x)^2}{2} & \text{if } m = -1 \\ \frac{x^{m+1}\,(\ln(x)\,(m+1)-1)}{(m+1)^2} & \text{if } m \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(x),x)
```

```
[Out] piecewise(m == -1, log(x)^2/2, m ~= -1, (x^(m + 1)*(log(x)*(m + 1) - 1))/(m
 + 1)^2)
```

sympy [A] time = 0.76, size = 56, normalized size = 2.15

$$\begin{cases} \frac{mxx^m \log(x)}{m^2+2m+1} + \frac{xx^m \log(x)}{m^2+2m+1} - \frac{xx^m}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(x),x)
```

```
[Out] Piecewise((m*x*x**m*log(x)/(m**2 + 2*m + 1) + x*x**m*log(x)/(m**2 + 2*m + 1
) - x*x**m/(m**2 + 2*m + 1), Ne(m, -1)), (log(x)**2/2, True))
```

3.609 $\int x^m \log^2(x)\,dx$

Optimal. Leaf size=42

$$\frac{2x^{m+1}}{(m+1)^3}+\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2x^{m+1}\log(x)}{(m+1)^2}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2305, 2304}

$$\frac{2x^{m+1}}{(m+1)^3}+\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2x^{m+1}\log(x)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[x]^2,x]
```

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1 + m)*Log[x
]^2)/(1 + m)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \log^2(x)\,dx &= \frac{x^{1+m}\log^2(x)}{1+m}-\frac{2\int x^m\log(x)\,dx}{1+m}\\ &= \frac{2x^{1+m}}{(1+m)^3}-\frac{2x^{1+m}\log(x)}{(1+m)^2}+\frac{x^{1+m}\log^2(x)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.71

$$\frac{x^{m+1}\left((m+1)^2\log^2(x)-2(m+1)\log(x)+2\right)}{(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[x]^2,x]
```

```
[Out] (x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log^2(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*Log[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.92, size = 45, normalized size = 1.07

$$\frac{\left(\left(m^{2}+2\,m+1\right)x\log(x)^{2}-2\,(m+1)x\log(x)+2\,x\right)x^{m}}{m^{3}+3\,m^{2}+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="fricas")
```

```
[Out] ((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 +
3*m + 1)
```

giac [A] time = 0.64, size = 84, normalized size = 2.00

$$-\frac{2\,mxx^{m}\log(x)}{\left(m^{2}+2\,m+1\right)(m+1)}+\frac{x^{m+1}\log(x)^{2}}{m+1}-\frac{2\,xx^{m}\log(x)}{\left(m^{2}+2\,m+1\right)(m+1)}+\frac{2\,xx^{m}}{\left(m^{2}+2\,m+1\right)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="giac")
```

```
[Out] -2*m*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*log(x)^2/(m + 1) -
2*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + 2*x*x^m/((m^2 + 2*m + 1)*(m + 1)
)
```

maple [A] time = 0.04, size = 41, normalized size = 0.98

method	result	size
risch	$\frac{x\left(m^{2}\ln(x)^{2}+2m\ln(x)^{2}-2m\ln(x)+\ln(x)^{2}-2\ln(x)+2\right)x^{m}}{(1+m)^{3}}$	41
norman	$\frac{x\ln(x)^{2}e^{m\ln(x)}}{1+m}+\frac{2x\,e^{m\ln(x)}}{m^{3}+3m^{2}+3m+1}-\frac{2x\ln(x)e^{m\ln(x)}}{m^{2}+2m+1}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(m^2*ln(x)^2+2*m*ln(x)^2-2*m*ln(x)+ln(x)^2-2*ln(x)+2)/(1+m)^3*x^m
```

maxima [A] time = 0.42, size = 42, normalized size = 1.00

$$\frac{x^{m+1}\log(x)^{2}}{m+1}-\frac{2\,x^{m+1}\log(x)}{(m+1)^{2}}+\frac{2\,x^{m+1}}{(m+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="maxima")
```

```
[Out] x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/(m
+ 1)^3
```

mupad [B] time = 0.35, size = 43, normalized size = 1.02

$$\begin{cases}\frac{\ln(x)^{3}}{3} & \text{if } m=-1\\ \frac{x^{m+1}\left(\ln(x)^{2}\,(m+1)^{2}-2\,\ln(x)\,(m+1)+2\right)}{(m+1)^{3}} & \text{if } m\neq -1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(x)^2,x)
```

```
[Out] piecewise(m == -1, log(x)^3/3, m ~= -1, (x^(m + 1)*(- 2*log(x)*(m + 1) + lo
g(x)^2*(m + 1)^2 + 2))/(m + 1)^3)
```

sympy [A] time = 1.57, size = 155, normalized size = 3.69

$$\begin{cases} \frac{m^2 x x^m \log(x)^2}{m^3+3m^2+3m+1} + \frac{2mxx^m \log(x)^2}{m^3+3m^2+3m+1} - \frac{2mxx^m \log(x)}{m^3+3m^2+3m+1} + \frac{xx^m \log(x)^2}{m^3+3m^2+3m+1} - \frac{2xx^m \log(x)}{m^3+3m^2+3m+1} + \frac{2xx^m}{m^3+3m^2+3m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(x)**2,x)
```

```
[Out] Piecewise((m**2*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) + 2*m*x*x**m*log
(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*m*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m +
 1) + x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*x*x**m*log(x)/(m**3 +
3*m**2 + 3*m + 1) + 2*x*x**m/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (log(x)
**3/3, True))
```

3.610 $\int \frac{\log^2(x)}{x^{5/2}} dx$

Optimal. Leaf size=34

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2305, 2304}

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2/x^(5/2),x]
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/2))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(x)}{x^{5/2}} dx &= -\frac{2\log^2(x)}{3x^{3/2}}+\frac{4}{3}\int \frac{\log(x)}{x^{5/2}} dx\\ &= -\frac{16}{27x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\left(9\log^2(x)+12\log(x)+8\right)}{27x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2/x^(5/2),x]
[Out] (-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^2(x)}{x^{5/2}} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[x]^2/x^(5/2),x]
[Out] Could not integrate
```

fricas [A] time = 1.16, size = 17, normalized size = 0.50

$$-\frac{2\left(9\,\log(x)^2+12\,\log(x)+8\right)}{27\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
[Out] -2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

giac [A] time = 0.63, size = 22, normalized size = 0.65

$$-\frac{2\,\log(x)^2}{3\,x^{\frac{3}{2}}}-\frac{8\,\log(x)}{9\,x^{\frac{3}{2}}}-\frac{16}{27\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="giac")
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

maple [A] time = 0.10, size = 23, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
default	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

maxima [A] time = 0.42, size = 22, normalized size = 0.65

$$-\frac{2\,\log(x)^2}{3\,x^{\frac{3}{2}}}-\frac{8\,\log(x)}{9\,x^{\frac{3}{2}}}-\frac{16}{27\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.50

$$-\frac{18\,{\ln(x)}^2+24\,\ln(x)+16}{27\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2/x^(5/2),x)
```

```
[Out] -(24*log(x) + 18*log(x)^2 + 16)/(27*x^(3/2))
```

sympy [A] time = 6.09, size = 34, normalized size = 1.00

$$-\frac{2\log{(x)}^2}{3x^{\frac{3}{2}}}-\frac{8\log{(x)}}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2/x**(5/2),x)
[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))
```

3.611 $\int (a+bx)\log(x)\,dx$

Optimal. Leaf size=28

$$-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2313}

$$\frac{1}{2}\log(x)\left(2ax+bx^2\right)-ax-\frac{bx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[x],x]
[Out] -(a*x) - (b*x^2)/4 + ((2*a*x + b*x^2)*Log[x])/2
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(x)\,dx&=\frac{1}{2}\left(2ax+bx^2\right)\log(x)-\int\left(a+\frac{bx}{2}\right)dx\\&=-ax-\frac{bx^2}{4}+\frac{1}{2}\left(2ax+bx^2\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[x],x]
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\log(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*Log[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.05, size = 25, normalized size = 0.89

$$-\frac{1}{4}bx^2-ax+\frac{1}{2}\left(bx^2+2\,ax\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="fricas")
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

giac [A] time = 0.58, size = 24, normalized size = 0.86

$$\frac{1}{2}\,bx^2\log(x) - \frac{1}{4}\,bx^2 + ax\log(x) - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="giac")
[Out] 1/2*b*x^2*log(x) - 1/4*b*x^2 + a*x*log(x) - a*x
```

maple [A] time = 0.02, size = 25, normalized size = 0.89

method	result	size
norman	$-ax-\frac{b\,x^2}{4}+ax\ln(x)+\frac{b\,x^2\ln(x)}{2}$	25
risch	$\left(\frac{1}{2}b\,x^2+ax\right)\ln(x)-\frac{b\,x^2}{4}-ax$	25
default	$b\left(\frac{x^2\ln(x)}{2}-\frac{x^2}{4}\right)+a\,(x\ln(x)-x)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(x),x,method=_RETURNVERBOSE)
[Out] -a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

maxima [A] time = 0.42, size = 25, normalized size = 0.89

$$-\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\,\left(bx^2 + 2\,ax\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="maxima")
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

mupad [B] time = 0.33, size = 21, normalized size = 0.75

$$-\frac{x\,\left(4\,a+b\,x-4\,a\,\ln(x)-2\,b\,x\,\ln(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(a + b*x),x)
[Out] -(x*(4*a + b*x - 4*a*log(x) - 2*b*x*log(x)))/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.79

$$-ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(x),x)
[Out] -a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

3.612 $\int (a+bx)^3 \log(x)\,dx$

Optimal. Leaf size=67

$$-\frac{a^4\log(x)}{4b}-a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {32, 2313, 12, 43}

$$-\frac{3}{4}a^2bx^2-\frac{a^4\log(x)}{4b}-a^3x-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*Log[x])/(4
*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3\log(x)\,dx&=\frac{(a+bx)^4\log(x)}{4b}-\int\frac{(a+bx)^4}{4bx}\,dx\\&=\frac{(a+bx)^4\log(x)}{4b}-\frac{\int\frac{(a+bx)^4}{x}\,dx}{4b}\\&=\frac{(a+bx)^4\log(x)}{4b}-\frac{\int\left(4a^3b+\frac{a^4}{x}+6a^2b^2x+4ab^3x^2+b^4x^3\right)dx}{4b}\\&=-a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{b^3x^4}{16}-\frac{a^4\log(x)}{4b}+\frac{(a+bx)^4\log(x)}{4b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.21

$$-a^3x+a^3x\log(x)-\frac{3}{4}a^2bx^2+\frac{3}{2}a^2bx^2\log(x)-\frac{1}{3}ab^2x^3+ab^2x^3\log(x)-\frac{1}{16}b^3x^4+\frac{1}{4}b^3x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
(3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3\log(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*Log[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.69, size = 69, normalized size = 1.03

$$-\frac{1}{16}\,b^3x^4-\frac{1}{3}\,ab^2x^3-\frac{3}{4}\,a^2bx^2-a^3x+\frac{1}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

giac [A] time = 0.63, size = 71, normalized size = 1.06

$$\frac{1}{4}\,b^3x^4\log(x)-\frac{1}{16}\,b^3x^4+ab^2x^3\log(x)-\frac{1}{3}\,ab^2x^3+\frac{3}{2}\,a^2bx^2\log(x)-\frac{3}{4}\,a^2bx^2+a^3x\log(x)-a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(x) - 1/16*b^3*x^4 + a*b^2*x^3*log(x) - 1/3*a*b^2*x^3 + 3/2*
a^2*b*x^2*log(x) - 3/4*a^2*b*x^2 + a^3*x*log(x) - a^3*x
```

maple [A] time = 0.32, size = 58, normalized size = 0.87

method	result	size
risch	$-a^3x-\frac{3a^2b\,x^2}{4}-\frac{a\,b^2x^3}{3}-\frac{b^3x^4}{16}-\frac{a^4\ln(x)}{4b}+\frac{(bx+a)^4\ln(x)}{4b}$	58
default	$b^3\left(\frac{x^4\ln(x)}{4}-\frac{x^4}{16}\right)+3b^2a\left(\frac{x^3\ln(x)}{3}-\frac{x^3}{9}\right)+3a^2b\left(\frac{x^2\ln(x)}{2}-\frac{x^2}{4}\right)+a^3\left(x\ln(x)-x\right)$	69
norman	$a^3x\ln(x)+a\,b^2x^3\ln(x)-a^3x-\frac{b^3x^4}{16}-\frac{a\,b^2x^3}{3}-\frac{3a^2b\,x^2}{4}+\frac{b^3x^4\ln(x)}{4}+\frac{3a^2b\,x^2\ln(x)}{2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a)
^4*ln(x)/b
```

maxima [A] time = 0.43, size = 69, normalized size = 1.03

$$-\frac{1}{16}\,b^{3}x^{4}-\frac{1}{3}\,ab^{2}x^{3}-\frac{3}{4}\,a^{2}bx^{2}-a^{3}x+\frac{1}{4}\left(b^{3}x^{4}+4\,ab^{2}x^{3}+6\,a^{2}bx^{2}+4\,a^{3}x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

mupad [B] time = 0.38, size = 71, normalized size = 1.06

$$a^{3}\,x\,\ln(x)-\frac{b^{3}\,x^{4}}{16}-\frac{3\,a^{2}\,b\,x^{2}}{4}-\frac{a\,b^{2}\,x^{3}}{3}-a^{3}\,x+\frac{b^{3}\,x^{4}\,\ln(x)}{4}+\frac{3\,a^{2}\,b\,x^{2}\,\ln(x)}{2}+a\,b^{2}\,x^{3}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(a + b*x)^3,x)
```

```
[Out] a^3*x*log(x) - (b^3*x^4)/16 - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - a^3*x + (b^
3*x^4*log(x))/4 + (3*a^2*b*x^2*log(x))/2 + a*b^2*x^3*log(x)
```

sympy [A] time = 0.15, size = 71, normalized size = 1.06

$$-a^{3}x-\frac{3a^{2}bx^{2}}{4}-\frac{ab^{2}x^{3}}{3}-\frac{b^{3}x^{4}}{16}+\left(a^{3}x+\frac{3a^{2}bx^{2}}{2}+ab^{2}x^{3}+\frac{b^{3}x^{4}}{4}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(x),x)
```

```
[Out] -a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**2
*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

3.613 $\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx$

Optimal. Leaf size=23

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2296, 2295}

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx &= -x+3 \int \log^3(x) \, dx-8 \int \log^2(x) \, dx\\ &= -x-8 x \log^2(x)+3 x \log^3(x)-9 \int \log^2(x) \, dx+16 \int \log(x) \, dx\\ &= -17 x+16 x \log(x)-17 x \log^2(x)+3 x \log^3(x)+18 \int \log(x) \, dx\\ &= -35 x+34 x \log(x)-17 x \log^2(x)+3 x \log^3(x)\\ \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 23, normalized size = 1.00

$$3 x \log(x)^3 - 17 x \log(x)^2 + 34 x \log(x) - 35 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

giac [A] time = 0.58, size = 23, normalized size = 1.00

$$3 x \log(x)^3 - 17 x \log(x)^2 + 34 x \log(x) - 35 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="giac")
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

maple [A] time = 0.02, size = 24, normalized size = 1.04

method	result	size
default	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
norman	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
risch	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1-8*ln(x)^2+3*ln(x)^3,x,method=_RETURNVERBOSE)
[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

maxima [A] time = 0.47, size = 36, normalized size = 1.57

$$3 \left(\log(x)^3 - 3 \log(x)^2 + 6 \log(x) - 6\right)x - 8 \left(\log(x)^2 - 2 \log(x) + 2\right)x - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")
[Out] 3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x
- x
```

mupad [B] time = 0.29, size = 20, normalized size = 0.87

$$x \left(3 \ln(x)^3 - 17 \ln(x)^2 + 34 \ln(x) - 35\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*log(x)^3 - 8*log(x)^2 - 1,x)
[Out] x*(34*log(x) - 17*log(x)^2 + 3*log(x)^3 - 35)
```

sympy [A] time = 0.11, size = 26, normalized size = 1.13

$$3x \log{\left(x \right)}^{3} - 17x \log{\left(x \right)}^{2} + 34x \log{\left(x \right)} - 35x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

3.614 $\int \left(1+x^4\right)\left(1-2\log(x)+\log^3(x)\right)\,dx$

Optimal. Leaf size=60

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x)-3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6742, 2313, 12, 2330, 2296, 2295, 2305, 2304}

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)+\frac{6}{125}x^5\log(x)-\frac{2}{5}\left(x^5+5x\right)\log(x)-3x+x\log^3(x)-3x\log^2(x)+6x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 6*x*Log[x] + (6*x^5*Log[x])/125 - (2*(5*x + x^5)*Log
[x])/5 - 3*x*Log[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
```

```
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (1+x^4)\left(1-2\log(x)+\log^3(x)\right)dx &= \int \left(1+x^4-2(1+x^4)\log(x)+(1+x^4)\log^3(x)\right)dx\\
&= x+\frac{x^5}{5}-2\int (1+x^4)\log(x)\,dx+\int (1+x^4)\log^3(x)\,dx\\
&= x+\frac{x^5}{5}-\frac{2}{5}(5x+x^5)\log(x)+2\int \frac{1}{5}(5+x^4)\,dx+\int \left(\log^3(x)+x^4\right.\\
&= x+\frac{x^5}{5}-\frac{2}{5}(5x+x^5)\log(x)+\frac{2}{5}\int (5+x^4)\,dx+\int \log^3(x)\,dx+\int\\
&= 3x+\frac{7x^5}{25}-\frac{2}{5}(5x+x^5)\log(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)-\frac{3}{5}\int x^4 \mathrm{l}\\
&= 3x+\frac{7x^5}{25}-\frac{2}{5}(5x+x^5)\log(x)-3x\log^2(x)-\frac{3}{25}x^5\log^2(x)+x\log^3\\
&= -3x+\frac{169x^5}{625}+6x\log(x)+\frac{6}{125}x^5\log(x)-\frac{2}{5}(5x+x^5)\log(x)-3x\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x)-3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3
*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1+x^4)\left(1-2\log(x)+\log^3(x)\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 48, normalized size = 0.80

$$\frac{169}{625}x^5+\frac{1}{5}\left(x^5+5x\right)\log(x)^3-\frac{3}{25}\left(x^5+25x\right)\log(x)^2-\frac{4}{125}\left(11x^5-125x\right)\log(x)-3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")
```

```
[Out] 169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)^2 - 4/125
*(11*x^5 - 125*x)*log(x) - 3*x
```

giac [A] time = 0.62, size = 52, normalized size = 0.87

$$\frac{1}{5}\,x^{5}\log(x)^{3}-\frac{3}{25}\,x^{5}\log(x)^{2}-\frac{44}{125}\,x^{5}\log(x)+\frac{169}{625}\,x^{5}+x\log(x)^{3}-3\,x\log(x)^{2}+4\,x\log(x)-3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(x)^3 - 3/25*x^5*log(x)^2 - 44/125*x^5*log(x) + 169/625*x^5 + x*
log(x)^3 - 3*x*log(x)^2 + 4*x*log(x) - 3*x
```

maple [A] time = 0.02, size = 48, normalized size = 0.80

method	result	size
risch	$\left(\frac{1}{5}x^{5}+x\right)\ln(x)^{3}+\left(-\frac{3}{25}x^{5}-3x\right)\ln(x)^{2}+\left(-\frac{44}{125}x^{5}+4x\right)\ln(x)+\frac{169x^{5}}{625}-3x$	48
default	$-3x+\frac{169x^{5}}{625}+4x\ln(x)-\frac{44x^{5}\ln(x)}{125}-3x\ln(x)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln(x)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
norman	$-3x+\frac{169x^{5}}{625}+4x\ln(x)-\frac{44x^{5}\ln(x)}{125}-3x\ln(x)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln(x)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(1-2*ln(x)+ln(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] (1/5*x^5+x)*ln(x)^3+(-3/25*x^5-3*x)*ln(x)^2+(-44/125*x^5+4*x)*ln(x)+169/625
*x^5-3*x
```

maxima [A] time = 0.44, size = 66, normalized size = 1.10

$$\frac{1}{625}\left(125\,\log(x)^{3}-75\,\log(x)^{2}+30\,\log(x)-6\right)x^{5}-\frac{2}{25}\,x^{5}\left(5\,\log(x)-1\right)+\frac{1}{5}\,x^{5}+\left(\log(x)^{3}-3\,\log(x)^{2}+6\,\log(x)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="maxima")
```

```
[Out] 1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5*(5*log(x)
 - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 2*x*(log(x) -
1) + x
```

mupad [B] time = 0.37, size = 51, normalized size = 0.85

$$\frac{x\,\left(125\,x^4\,{\ln(x)}^3-75\,x^4\,{\ln(x)}^2-220\,x^4\,\ln(x)+169\,x^4+625\,{\ln(x)}^3-1875\,{\ln(x)}^2+2500\,\ln(x)-1875\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x)
```

```
[Out] (x*(2500*log(x) - 220*x^4*log(x) - 1875*log(x)^2 + 625*log(x)^3 - 75*x^4*lo
g(x)^2 + 125*x^4*log(x)^3 + 169*x^4 - 1875))/625
```

sympy [A] time = 0.16, size = 51, normalized size = 0.85

$$\frac{169x^{5}}{625}-3x+\left(-\frac{44x^{5}}{125}+4x\right)\log\left(x\right)+\left(-\frac{3x^{5}}{25}-3x\right)\log\left(x\right)^{2}+\left(\frac{x^{5}}{5}+x\right)\log\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

```
[Out] 169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 - 3*x)*log(x
)**2 + (x**5/5 + x)*log(x)**3
```

3.615 $\int \frac{1}{x^3 \log^4(x)} \, dx$

Optimal. Leaf size=43

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}$$

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2306, 2309, 2178}

$$-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x)) + \frac{1}{3x^2\log^2(x)} - \frac{1}{3x^2\log^3(x)} - \frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Log[x]^4),x]

[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\log^4(x)}\,dx &= -\frac{1}{3x^2\log^3(x)} - \frac{2}{3}\int \frac{1}{x^3\log^3(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} + \frac{2}{3}\int \frac{1}{x^3\log^2(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\int \frac{1}{x^3\log(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(x)\right) \\
&= -\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$-\frac{4}{3}\text{Ei}(-2\log(x))-\frac{1}{3x^2\log^3(x)}+\frac{1}{3x^2\log^2(x)}-\frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Log[x]^4),x]
```

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\log^4(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*Log[x]^4),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.96, size = 34, normalized size = 0.79

$$-\frac{4\,x^2\log(x)^3\,\text{log\_integral}\left(\frac{1}{x^2}\right)+2\,\log(x)^2-\log(x)+1}{3\,x^2\log(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) + 1)/(x^2*l
og(x)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\log(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log(x)^4), x)
```

maple [A] time = 0.03, size = 31, normalized size = 0.72

method	result	size
risch	$-\frac{2\ln(x)^2-\ln(x)+1}{3x^2\ln(x)^3}+\frac{4\,\text{expIntegralEi}(1,2\ln(x))}{3}$	31
default	$-\frac{1}{3x^2\ln(x)^3}+\frac{1}{3x^2\ln(x)^2}-\frac{2}{3x^2\ln(x)}+\frac{4\,\text{expIntegralEi}(1,2\ln(x))}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*ln(x)^2-ln(x)+1)/x^2/ln(x)^3+4/3*Ei(1,2*ln(x))
```

maxima [A] time = 0.54, size = 8, normalized size = 0.19

$$-8\,\Gamma\left(-3,2\,\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="maxima")
[Out] -8*gamma(-3, 2*log(x))
```

mupad [B] time = 0.28, size = 29, normalized size = 0.67

$$-\frac{4\,\mathrm{ei}\left(-2\,\ln(x)\right)}{3}-\frac{\frac{2\ln(x)^2}{3}-\frac{\ln(x)}{3}+\frac{1}{3}}{x^2\ln(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(x)^4),x)
[Out] - (4*ei(-2*log(x)))/3 - ((2*log(x)^2)/3 - log(x)/3 + 1/3)/(x^2*log(x)^3)
```

sympy [A] time = 0.72, size = 32, normalized size = 0.74

$$-\frac{4\,\mathrm{Ei}\left(-2\log\left(x\right)\right)}{3}+\frac{-2\log\left(x\right)^2+\log\left(x\right)-1}{3x^2\log\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(x)**4,x)
[Out] -4*Ei(-2*log(x))/3 + (-2*log(x)**2 + log(x) - 1)/(3*x**2*log(x)**3)
```

3.616 $\int \frac{\log(x)}{a+bx} dx$

Optimal. Leaf size=29

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2317, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(a + b*x),x]
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{a+bx}\,dx &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b}-\frac{\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx}{b}\\ &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b}+\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.03

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x),x]
[Out] (Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{a+bx}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[x]/(a + b*x),x]
[Out] Could not integrate
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(x)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="fricas")
[Out] integral(log(x)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="giac")
[Out] integrate(log(x)/(b*x + a), x)
```

maple [A] time = 0.31, size = 32, normalized size = 1.10

method	result	size
default	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}$	32
risch	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

maxima [A] time = 0.43, size = 25, normalized size = 0.86

$$\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="maxima")
[Out] (log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a + b*x),x)
[Out] int(log(x)/(a + b*x), x)
```

sympy [C] time = 6.44, size = 151, normalized size = 5.21

$$\begin{cases} \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b} + \frac{i\pi\log\left(\frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} - \frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)}{b} + \frac{G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} + \frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(b*x+a),x)
```

```
[Out] Piecewise((log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(a/
b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log(1/
(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meijerg(
((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1, 1))
, ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)*l
og(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - polylog(2
, b*(a/b + x)/a)/b, True))
```

3.617 $\int \frac{\log(x)}{(a+bx)^2} dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2314, 31}

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(a + b*x)^2,x]
```

```
[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{(a+bx)^2} dx &= \frac{x\log(x)}{a(a+bx)} - \frac{\int \frac{1}{a+bx} dx}{a}\\ &= \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x)^2,x]
```

```
[Out] ((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[x]/(a + b*x)^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 34, normalized size = 1.17

$$\frac{bx\log(x) - (bx+a)\log(bx+a)}{ab^2x + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

giac [B] time = 0.61, size = 138, normalized size = 4.76

$$b^2\left(\frac{\log\left(\frac{(bx+a)^2|b|\left|\frac{a}{bx+a}-1\right|}{b^2|bx+a|}\right)}{ab^3} + \frac{\log\left(-\frac{a+\frac{(bx+a)b\left(\frac{a}{bx+a}-1\right)-ab}{b}}{b}\right)}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)b^3} - \frac{\log\left(\left|-(bx+a)\left(\frac{a}{bx+a}-1\right)+a\right|\right)}{ab^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] b^2*(log((b*x + a)^2*abs(b)*abs(a/(b*x + a) - 1)/(b^2*abs(b*x + a)))/(a*b^3
) + log(-(a + ((b*x + a)*b*(a/(b*x + a) - 1) - a*b)/b)/b)/(((b*x + a)*(a/(b
*x + a) - 1) - a)*b^3) - log(abs(-(b*x + a)*(a/(b*x + a) - 1) + a))/(a*b^3)
)
```

maple [A] time = 0.31, size = 30, normalized size = 1.03

method	result	size
default	$\frac{x\ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
norman	$\frac{x\ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
risch	$-\frac{\ln(x)}{b(bx+a)} + \frac{\ln(-x)}{ab} - \frac{\ln(bx+a)}{ab}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

maxima [A] time = 0.43, size = 38, normalized size = 1.31

$$-\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

mupad [B] time = 0.39, size = 35, normalized size = 1.21

$$\frac{x^2\ln(x)}{a\left(b\,x^2+a\,x\right)} - \frac{\ln\left(a+b\,x\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a + b*x)^2,x)
```

```
[Out] (x^2*log(x))/(a*(a*x + b*x^2)) - log(a + b*x)/(a*b)
```

sympy [A] time = 0.35, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{ab+b^2x}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(b*x+a)**2,x)
```

```
[Out] -log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

3.618 $\int \frac{\log^n(x)}{x} \, dx$

Optimal. Leaf size=12

$$\frac{\log^{n+1}(x)}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 30}

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^n(x)}{x} \, dx &= \text{Subst}\left(\int x^n \, dx, x, \log(x)\right) \\ &= \frac{\log^{1+n}(x)}{1+n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^n(x)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[x]^n/x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.05, size = 12, normalized size = 1.00

$$\frac{\log(x)^n \log(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="fricas")
[Out] log(x)^n*log(x)/(n + 1)
```

giac [A] time = 0.61, size = 12, normalized size = 1.00

$$\frac{\log(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="giac")
[Out] log(x)^(n + 1)/(n + 1)
```

maple [A] time = 0.03, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+n}}{1+n}$	13
default	$\frac{\ln(x)^{1+n}}{1+n}$	13
risch	$\frac{\ln(x)\ln(x)^n}{1+n}$	13
norman	$\frac{\ln(x)e^{n\ln(\ln(x))}}{1+n}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^n/x,x,method=_RETURNVERBOSE)
[Out] ln(x)^(1+n)/(1+n)
```

maxima [A] time = 0.42, size = 12, normalized size = 1.00

$$\frac{\log(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="maxima")
[Out] log(x)^(n + 1)/(n + 1)
```

mupad [B] time = 0.33, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\ln(x)) & \text{if } n = -1 \\ \frac{\ln(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^n/x,x)
[Out] piecewise(n == -1, log(log(x)), n ~= -1, log(x)^(n + 1)/(n + 1))
```

sympy [A] time = 0.89, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**n/x,x)
```

```
[Out] Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))
```

3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 30}

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(x))^n}{x}\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{(a+b\log(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*Log[x])^n/x,x]
```

[Out] Could not integrate

fricas [A] time = 0.96, size = 22, normalized size = 1.16

$$\frac{\left(b\log(x)+a\right)\left(b\log(x)+a\right)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="fricas")
```

```
[Out] (b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

giac [A] time = 0.61, size = 19, normalized size = 1.00

$$\frac{\left(b\log(x)+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="giac")
```

```
[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.04, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
risch	$\frac{(a+b\ln(x))(a+b\ln(x))^{n}}{b(1+n)}$	24
norman	$\frac{\ln(x)e^{n\ln(a+b\ln(x))}}{1+n}+\frac{a\,e^{n\ln(a+b\ln(x))}}{b(1+n)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(x))^n/x,x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*ln(x))^(1+n)/b/(1+n)
```

maxima [A] time = 0.43, size = 19, normalized size = 1.00

$$\frac{\left(b\log(x)+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="maxima")
```

```
[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 0.45, size = 19, normalized size = 1.00

$$\frac{(a+b\,\ln(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(x))^n/x,x)
```

```
[Out] (a + b*log(x))^(n + 1)/(b*(n + 1))
```

sympy [A] time = 1.27, size = 36, normalized size = 1.89

$$-\begin{cases} -a^{n}\log(x) & \text{for } b = 0 \\ -\frac{\begin{cases} \frac{(a+b\log(x))^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log\left(a + b\log(x)\right) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(x))**n/x,x)
```

```
[Out] -Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/(
n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))
```

3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\log(x))}{b}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 29}

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Log[x])),x]
```

```
[Out] Log[a + b*Log[x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+b\log(x))}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{\log(a+b\log(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Log[x])),x]
```

```
[Out] Log[a + b*Log[x]]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*Log[x])),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 11, normalized size = 1.00

$$\frac{\log\left(b\log(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="fricas")
```

```
[Out] log(b*log(x) + a)/b
```

giac [B] time = 0.60, size = 30, normalized size = 2.73

$$\frac{\log\left(\frac{1}{4}\pi^2b^2\left(\operatorname{sgn}(x)-1\right)^2+\left(b\log\left(|x|\right)+a\right)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*pi^2*b^2*(sgn(x) - 1)^2 + (b*log(abs(x)) + a)^2)/b
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\ln(x))}{b}$	12
default	$\frac{\ln(a+b\ln(x))}{b}$	12
norman	$\frac{\ln(a+b\ln(x))}{b}$	12
risch	$\frac{\ln\left(\ln(x)+\frac{a}{b}\right)}{b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*ln(x))/b
```

maxima [A] time = 0.42, size = 11, normalized size = 1.00

$$\frac{\log\left(b\log(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="maxima")
```

```
[Out] log(b*log(x) + a)/b
```

mupad [B] time = 0.29, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\,\ln(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*log(x))),x)
```

```
[Out] log(a + b*log(x))/b
```

sympy [A] time = 0.13, size = 8, normalized size = 0.73

$$\frac{\log\left(\frac{a}{b}+\log\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*ln(x)),x)
```

```
[Out] log(a/b + log(x))/b
```

3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$

Optimal. Leaf size=23

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 30}

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Log[x])^n),x]
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(x))^{-n}}{x}\,dx &= \frac{\text{Subst}\left(\int x^{-n}\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{(a+b\log(x))^{1-n}}{b(1-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Log[x])^n),x]
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*Log[x])^n),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.69, size = 27, normalized size = 1.17

$$-\frac{b\log(x)+a}{(bn-b)\left(b\log(x)+a\right)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")
[Out] -(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

giac [A] time = 0.63, size = 22, normalized size = 0.96

$$-\frac{\left(b\log(x)+a\right)^{-n+1}}{b(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="giac")
[Out] -(b*log(x) + a)^(-n + 1)/(b*(n - 1))
```

maple [A] time = 0.04, size = 24, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
default	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
risch	$-\frac{(a+b\ln(x))(a+b\ln(x))^{-n}}{b(-1+n)}$	27
norman	$\left(-\frac{\ln(x)}{-1+n}-\frac{a}{b(-1+n)}\right)e^{-n\ln(a+b\ln(x))}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a+b*ln(x))^n),x,method=_RETURNVERBOSE)
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-n>0)', see `assume?` for more deta
ils)Is -n equal to -1?
```

mupad [B] time = 0.43, size = 22, normalized size = 0.96

$$-\frac{(a+b\,\ln(x))^{1-n}}{b\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*log(x))^n),x)
```

```
[Out] -(a + b*log(x))^(1 - n)/(b*(n - 1))
```

sympy [**A**] time = 20.65, size = 71, normalized size = 3.09

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 1 \\ a^{-n}\log(x) & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b} & \text{for } n = 1 \\ -\frac{a}{bn\left(a+b\log(x)\right)^{n}-b\left(a+b\log(x)\right)^{n}} - \frac{b\log(x)}{bn\left(a+b\log(x)\right)^{n}-b\left(a+b\log(x)\right)^{n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*ln(x))**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (a**(-n)*log(x), Eq(b, 0)), (log
(a/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))*
*n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))
```

3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)\\
&= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.03, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -1/2*Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[a^2 + Log[x
]^2]]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2 + \log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + Log[x]^2]),x]
[Out] Could not integrate
```

fricas [A] time = 0.80, size = 18, normalized size = 1.12

$$-\log\left(\sqrt{a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

giac [A] time = 0.60, size = 18, normalized size = 1.12

$$-\log\left(\sqrt{a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

maple [A] time = 0.02, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\ln\left(\ln(x) + \sqrt{a^2 + \ln(x)^2}\right)$	15
default	$\ln\left(\ln(x) + \sqrt{a^2 + \ln(x)^2}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

maxima [A] time = 0.42, size = 7, normalized size = 0.44

$$\operatorname{arsinh}\left(\frac{\log(x)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(log(x)/a)
```

mupad [B] time = 0.43, size = 14, normalized size = 0.88

$$\ln\left(\ln(x) + \sqrt{a^2 + {\ln(x)}^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + a^2)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 + a^2)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2 + \log(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)\\
&= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.04, size = 50, normalized size = 2.78

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] -1/2*Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-a^2 + Log
[x]^2]]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a^2 + \log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 20, normalized size = 1.11

$$-\log\left(\sqrt{-a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(-a^2 + log(x)^2) - log(x))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.02, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\ln\left(\ln(x) + \sqrt{-a^2 + \ln(x)^2}\right)$	17
default	$\ln\left(\ln(x) + \sqrt{-a^2 + \ln(x)^2}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

maxima [A] time = 0.43, size = 20, normalized size = 1.11

$$\log\left(2\sqrt{-a^2 + \log(x)^2} + 2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

mupad [B] time = 0.39, size = 16, normalized size = 0.89

$$\ln\left(\ln(x) + \sqrt{\ln(x)^2 - a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 - a^2)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)\\
&= \tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2 - \log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 - Log[x]^2]),x]
[Out] Could not integrate
```

fricas [A] time = 0.91, size = 25, normalized size = 1.39

$$-2\arctan\left(-\frac{a - \sqrt{a^2 - \log(x)^2}}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

giac [A] time = 0.77, size = 10, normalized size = 0.56

$$\arcsin\left(\frac{\log(x)}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
[Out] arcsin(log(x)/a)*sgn(a)
```

maple [A] time = 0.02, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17
default	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

maxima [A] time = 1.00, size = 7, normalized size = 0.39

$$\arcsin\left(\frac{\log(x)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(log(x)/a)
```

mupad [**B**] time = 0.61, size = 16, normalized size = 0.89

$$\operatorname{atan}\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 - log(x)^2)^(1/2)),x)
```

```
[Out] atan(log(x)/(a^2 - log(x)^2)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(a-\log(x))\,(a+\log(x))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

3.625 $\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Rubi [A] time = 0.09, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, \log^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+\log^2(x)}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
[Out] Could not integrate
```

fricas [B] time = 0.90, size = 44, normalized size = 2.00

$$-\frac{\log\left(a+\sqrt{a^2+\log(x)^2}-\log(x)\right)-\log\left(-a+\sqrt{a^2+\log(x)^2}-\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + log(x)^2) -
log(x)))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.02, size = 37, normalized size = 1.68

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x))
```

maxima [A] time = 0.42, size = 13, normalized size = 0.59

$$-\frac{\operatorname{arsinh}\left(\frac{a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/abs(log(x)))/a
```

mupad [B] time = 0.58, size = 27, normalized size = 1.23

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a^2+\ln(x)^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)*(log(x)^2 + a^2)^(1/2)),x)
```

```
[Out] atan((log(x)^2 + a^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2 + \log(x)^2}\,\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

3.626 $\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Rubi [A] time = 0.10, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {266, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2-x\,x}}\,dx, x, \log^2(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt{a^2-\log^2(x)}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
[Out] Could not integrate
```

fricas [A] time = 0.77, size = 27, normalized size = 1.12

$$\frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.02, size = 39, normalized size = 1.62

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x))
```

maxima [A] time = 0.43, size = 37, normalized size = 1.54

$$-\frac{\log\left(\frac{2\,a^2}{|\log(x)|}+\frac{2\,\sqrt{a^2-\log(x)^2}\,a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] -log(2*a^2/abs(log(x)) + 2*sqrt(a^2 - log(x)^2)*a/abs(log(x)))/a
```

mupad [B] time = 0.61, size = 22, normalized size = 0.92

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-\ln(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)*(a^2 - log(x)^2)^(1/2)),x)
[Out] -atanh((a^2 - log(x)^2)^(1/2)/a)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}\,\log\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

3.627 $\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Rubi [A] time = 0.10, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {266, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx,x,\log(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx,x,\log^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx,x,\sqrt{-a^2+\log^2(x)}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.87, size = 27, normalized size = 1.17

$$\frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

giac [A] time = 1.63, size = 21, normalized size = 0.91

$$\frac{\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a^2 + log(x)^2)/a)/a
```

maple [A] time = 0.02, size = 43, normalized size = 1.87

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/ln(x))
```

maxima [A] time = 0.95, size = 13, normalized size = 0.57

$$-\frac{\arcsin\left(\frac{a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(a/abs(log(x)))/a
```

mupad [B] time = 0.52, size = 25, normalized size = 1.09

$$\frac{\operatorname{atan}\left(\frac{\sqrt{\ln(x)^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)*(log(x)^2 - a^2)^(1/2)),x)
[Out] atan((log(x)^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(a-\log(x)\right)\left(a+\log(x)\right)}\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

3.628 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal. Leaf size=11

$$\log(x)\log(\log(x)) - \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2521}

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]/x,x]
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]/x,x]
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(\log(x))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Log[x]]/x,x]
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 11, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="fricas")
[Out] log(x)*log(log(x)) - log(x)
```

giac [A] time = 1.65, size = 11, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
default	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
norman	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
risch	$-\ln(x)+\ln(x)\ln(\ln(x))$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

maxima [A] time = 0.42, size = 11, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

mupad [B] time = 0.32, size = 8, normalized size = 0.73

$$\ln(x)\ \left(\ln\left(\ln(x)\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))/x,x)
```

```
[Out] log(x)*(log(log(x)) - 1)
```

sympy [A] time = 0.26, size = 10, normalized size = 0.91

$$\log\left(x\right)\log\left(\log\left(x\right)\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))/x,x)
```

```
[Out] log(x)*log(log(x)) - log(x)
```

3.629 $\int \frac{\log^2(\log(x))}{x}\,dx$

Optimal. Leaf size=20

$$\log(x)\log^2(\log(x))-2\log(x)\log(\log(x))+2\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^2(\log(x))-2\log(x)\log(\log(x))+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^2/x,x]
```

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(\log(x))}{x}\,dx &= \text{Subst}\left(\int \log^2(x)\,dx,x,\log(x)\right)\\ &= \log(x)\log^2(\log(x))-2\,\text{Subst}\left(\int \log(x)\,dx,x,\log(x)\right)\\ &= 2\log(x)-2\log(x)\log(\log(x))+\log(x)\log^2(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\log(x)\log^2(\log(x))-2\log(x)\log(\log(x))+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^2/x,x]
```

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^2(\log(x))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Log[x]]^2/x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.65, size = 20, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^{2} - 2\log(x)\log\left(\log(x)\right) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="fricas")
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

giac [A] time = 1.21, size = 20, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^{2} - 2\log(x)\log\left(\log(x)\right) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="giac")
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^{2}$	21
default	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^{2}$	21
norman	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^{2}$	21
risch	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

maxima [A] time = 0.42, size = 15, normalized size = 0.75

$$\left(\log\left(\log(x)\right)^{2} - 2\log\left(\log(x)\right) + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="maxima")
[Out] (log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

mupad [B] time = 0.38, size = 15, normalized size = 0.75

$$\ln(x)\left(\ln(\ln(x))^{2} - 2\ln(\ln(x)) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))^2/x,x)
[Out] log(x)*(log(log(x))^2 - 2*log(log(x)) + 2)
```

sympy [A] time = 0.31, size = 24, normalized size = 1.20

$$\log(x)\log\left(\log(x)\right)^{2} - 2\log(x)\log\left(\log(x)\right) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**2/x,x)
[Out] log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

3.630 $\int \frac{\log^3(\log(x))}{x} \, dx$

Optimal. Leaf size=29

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^3(\log(x))}{x} \, dx &= \text{Subst}\left(\int \log^3(x) \, dx, x, \log(x)\right)\\
&= \log(x)\log^3(\log(x)) - 3\,\text{Subst}\left(\int \log^2(x) \, dx, x, \log(x)\right)\\
&= -3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x)) + 6\,\text{Subst}(\int \log(x) \, dx, x, \log(x))\\
&= -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^3(\log(x))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Log[x]]^3/x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 29, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^3 - 3\,\log(x)\log\left(\log(x)\right)^2 + 6\,\log(x)\log\left(\log(x)\right) - 6\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

giac [A] time = 0.79, size = 29, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^3 - 3\,\log(x)\log\left(\log(x)\right)^2 + 6\,\log(x)\log\left(\log(x)\right) - 6\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

maple [A] time = 0.04, size = 30, normalized size = 1.03

method	result	size
derivativedivides	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
default	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
norman	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
risch	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

maxima [A] time = 0.42, size = 22, normalized size = 0.76

$$\left(\log\left(\log(x)\right)^3 - 3\,\log\left(\log(x)\right)^2 + 6\,\log\left(\log(x)\right) - 6\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

mupad [B] time = 0.36, size = 29, normalized size = 1.00

$$\ln(x)\ln(\ln(x))^3 - 3\,\ln(x)\ln(\ln(x))^2 + 6\,\ln(x)\,\ln(\ln(x)) - 6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))^3/x,x)
```

```
[Out] 6*log(log(x))*log(x) - 6*log(x) - 3*log(log(x))^2*log(x) + log(log(x))^3*lo
g(x)
```

sympy [**A**] time = 0.35, size = 36, normalized size = 1.24

$$\log(x)\log\left(\log(x)\right)^{3} - 3\log(x)\log\left(\log(x)\right)^{2} + 6\log(x)\log\left(\log(x)\right) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**3/x,x)
```

```
[Out] log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6*
log(x)
```

3.631 $\int \frac{\log^4(\log(x))}{x} \, dx$

Optimal. Leaf size=38

$$\log(x)\log^4(\log(x))-4\log(x)\log^3(\log(x))+12\log(x)\log^2(\log(x))-24\log(x)\log(\log(x))+24\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^4(\log(x))-4\log(x)\log^3(\log(x))+12\log(x)\log^2(\log(x))-24\log(x)\log(\log(x))+24\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^4(\log(x))}{x} \, dx &= \text{Subst}\left(\int \log^4(x) \, dx, x, \log(x)\right)\\
&= \log(x)\log^4(\log(x)) - 4\,\text{Subst}\left(\int \log^3(x) \, dx, x, \log(x)\right)\\
&= -4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x)) + 12\,\text{Subst}\left(\int \log^2(x) \, dx, x, \log(x)\right)\\
&= 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x)) - 24\,\text{Subst}(\int \log(\\
&= 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\log(x)\log^4(\log(x))-4\log(x)\log^3(\log(x))+12\log(x)\log^2(\log(x))-24\log(x)\log(\log(x))+24\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^4(\log(x))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Log[x]]^4/x,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 38, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^{4}-4\log(x)\log\left(\log(x)\right)^{3}+12\log(x)\log\left(\log(x)\right)^{2}-24\log(x)\log\left(\log(x)\right)+24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

giac [A] time = 0.95, size = 38, normalized size = 1.00

$$\log(x)\log\left(\log(x)\right)^{4}-4\log(x)\log\left(\log(x)\right)^{3}+12\log(x)\log\left(\log(x)\right)^{2}-24\log(x)\log\left(\log(x)\right)+24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

maple [A] time = 0.04, size = 39, normalized size = 1.03

method	result
derivativedivides	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^{2}-4\ln(x)\ln(\ln(x))^{3}+\ln(x)\ln(\ln(x))^{4}$
default	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^{2}-4\ln(x)\ln(\ln(x))^{3}+\ln(x)\ln(\ln(x))^{4}$
norman	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^{2}-4\ln(x)\ln(\ln(x))^{3}+\ln(x)\ln(\ln(x))^{4}$
risch	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^{2}-4\ln(x)\ln(\ln(x))^{3}+\ln(x)\ln(\ln(x))^{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*
ln(ln(x))^4
```

maxima [A] time = 0.42, size = 29, normalized size = 0.76

$$\left(\log\left(\log(x)\right)^{4}-4\log\left(\log(x)\right)^{3}+12\log\left(\log(x)\right)^{2}-24\log\left(\log(x)\right)+24\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(x)) + 24)*
log(x)
```

mupad [B] time = 0.37, size = 38, normalized size = 1.00

$$\ln(x)\ln(\ln(x))^{4}-4\ln(x)\ln(\ln(x))^{3}+12\ln(x)\ln(\ln(x))^{2}-24\ln(x)\ln(\ln(x))+24\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))^4/x,x)
```

```
[Out] 24*log(x) - 24*log(log(x))*log(x) + 12*log(log(x))^2*log(x) - 4*log(log(x))
^3*log(x) + log(log(x))^4*log(x)
```

sympy [A] time = 0.39, size = 48, normalized size = 1.26

$$\log(x)\log\left(\log(x)\right)^4 - 4\log(x)\log\left(\log(x)\right)^3 + 12\log(x)\log\left(\log(x)\right)^2 - 24\log(x)\log\left(\log(x)\right) + 24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**4/x,x)
```

```
[Out] log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(log(x))**2
- 24*log(x)*log(log(x)) + 24*log(x)
```

3.632 $\int \frac{\log^n(\log(x))}{x} dx$

Optimal. Leaf size=24

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\Gamma(n+1,-\log(\log(x)))$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2299, 2181}

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\text{Gamma}(n+1,-\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^n/x,x]
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^n(\log(x))}{x} dx &= \text{Subst}\left(\int \log^n(x)\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int e^x x^n\,dx, x, \log(\log(x))\right)\\ &= \Gamma(1+n,-\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\Gamma(n+1,-\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^n/x,x]
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log^n(\log(x))}{x} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Log[x]]^n/x,x]
```

[Out] Could not integrate

fricas [A] time = 1.05, size = 14, normalized size = 0.58

$$\cos(\pi n)\Gamma\left(n+1,-\log\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="fricas")
[Out] cos(pi*n)*gamma(n + 1, -log(log(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log(x)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="giac")
[Out] integrate(log(log(x))^n/x, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\ln(x)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^n/x,x)
[Out] int(ln(ln(x))^n/x,x)
```

maxima [A] time = 0.55, size = 29, normalized size = 1.21

$$-\left(-\log\left(\log(x)\right)\right)^{-n-1}\log\left(\log(x)\right)^{n+1}\Gamma\left(n+1,-\log\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="maxima")
[Out] -(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))
```

mupad [B] time = 0.45, size = 24, normalized size = 1.00

$$\frac{\ln\left(\ln(x)\right)^n\,\Gamma\left(n+1,-\ln\left(\ln(x)\right)\right)}{\left(-\ln\left(\ln(x)\right)\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))^n/x,x)
[Out] (log(log(x))^n*igamma(n + 1, -log(log(x))))/(-log(log(x)))^n
```

sympy [A] time = 1.85, size = 24, normalized size = 1.00

$$\left(-\log\left(\log(x)\right)\right)^{-n}\log\left(\log(x)\right)^n\Gamma\left(n+1,-\log\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**n/x,x)
[Out] (-log(log(x)))**(-n)*log(log(x))**n*uppergamma(n + 1, -log(log(x)))
```

3.633 $\int \frac{\cot(x)}{\log(\sin(x))}\,dx$

Optimal. Leaf size=4

$$\log(\log(\sin(x)))$$

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4338, 2302, 29}

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[Sin[x]],x]
```

```
[Out] Log[Log[Sin[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\log(\sin(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x\log(x)}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(\sin(x))\right)\\ &= \log(\log(\sin(x)))\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[Sin[x]],x]
```

```
[Out] Log[Log[Sin[x]]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cot[x]/Log[Sin[x]],x]
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 4, normalized size = 1.00

$$\log\left(\log\left(\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
[Out] log(log(sin(x)))
```

giac [A] time = 0.95, size = 5, normalized size = 1.25

$$\log\left(\left|\log\left(\sin(x)\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="giac")
[Out] log(abs(log(sin(x))))
```

maple [A] time = 0.18, size = 5, normalized size = 1.25

method	result
derivativedivides	$\ln\left(\ln\left(\sin(x)\right)\right)$
default	$\ln\left(\ln\left(\sin(x)\right)\right)$
risch	$\ln\left(-\frac{i\pi \operatorname{csgn}(\sin(x))\operatorname{csgn}(i\sin(x))}{2}-\frac{i\pi\operatorname{csgn}(i\sin(x))^2}{2}-\frac{i\pi \operatorname{csgn}\left(i\left(e^{2ix}-1\right)\right)\operatorname{csgn}(\sin(x))^2}{2}-\frac{i\pi \operatorname{csgn}\left(i\left(e^{2ix}-1\right)\right)}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(ln(sin(x)))
```

maxima [A] time = 0.42, size = 4, normalized size = 1.00

$$\log\left(\log\left(\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
[Out] log(log(sin(x)))
```

mupad [B] time = 0.40, size = 4, normalized size = 1.00

$$\ln\left(\ln\left(\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/log(sin(x)),x)
[Out] log(log(sin(x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(sin(x)),x)
```

```
[Out] Integral(cot(x)/log(sin(x)), x)
```

3.634 $\int (\cos(x)+\sec(x))\tan(x)\,dx$

Optimal. Leaf size=7

$$\sec(x)-\cos(x)$$

Rubi [A] time = 0.05, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4236}

$$\sec(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sec[x])*Tan[x],x]
```

```
[Out] -Cos[x] + Sec[x]
```

Rule 4236

```
Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)]*(B_.) + (C_.)*sec[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Int[(ActivateTrig[u]*(C + A*Cos[a + b*x] + B*Cos[a + b*
x]^2))/Cos[a + b*x], x] /; FreeQ[{a, b, A, B, C}, x]
```

Rubi steps

$$\begin{aligned}\int (\cos(x)+\sec(x))\tan(x)\,dx &= \int \left(1+\cos^2(x)\right)\sec(x)\tan(x)\,dx\\ &= -\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \cos(x)\right)\\ &= -\cos(x)+\sec(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\sec(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sec[x])*Tan[x],x]
```

```
[Out] -Cos[x] + Sec[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x)+\sec(x))\tan(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x] + Sec[x])*Tan[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 12, normalized size = 1.71

$$-\frac{\cos(x)^2-1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")
```

```
[Out] -(cos(x)^2 - 1)/cos(x)
```

giac [A] time = 0.97, size = 9, normalized size = 1.29

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="giac")
[Out] 1/cos(x) - cos(x)
```

maple [A] time = 0.07, size = 10, normalized size = 1.43

method	result	size
derivativedivides	$-\cos(x) + \frac{1}{\cos(x)}$	10
default	$-\cos(x) + \frac{1}{\cos(x)}$	10
risch	$-\frac{e^{ix}}{2} - \frac{e^{-ix}}{2} + \frac{2\,e^{ix}}{1+e^{2ix}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)+cos(x))*tan(x),x,method=_RETURNVERBOSE)
[Out] -cos(x)+1/cos(x)
```

maxima [A] time = 0.43, size = 9, normalized size = 1.29

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")
[Out] 1/cos(x) - cos(x)
```

mupad [B] time = 0.39, size = 9, normalized size = 1.29

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(cos(x) + 1/cos(x)),x)
[Out] 1/cos(x) - cos(x)
```

sympy [A] time = 1.73, size = 7, normalized size = 1.00

$$-\cos{\left(x \right)} + \frac{1}{\cos{\left(x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x)
[Out] -cos(x) + 1/cos(x)
```

3.635 $\int \log(\cosh(x)) \sinh(x) \, dx$

Optimal. Leaf size=11

$$\cosh(x) \log(\cosh(x)) - \cosh(x)$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2638, 2554}

$$\cosh(x) \log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]]*Sinh[x],x]
```

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \log(\cosh(x)) \sinh(x) \, dx &= \cosh(x) \log(\cosh(x)) - \int \sinh(x) \, dx \\ &= -\cosh(x) + \cosh(x) \log(\cosh(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\cosh(x) \log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]]*Sinh[x],x]
```

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\cosh(x)) \sinh(x) \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Cosh[x]]*Sinh[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.12, size = 46, normalized size = 4.18

$$-\frac{\cosh(x)^2 - \left(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1\right) \log\left(\cosh(x)\right) + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1}{2 \left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(cosh(
x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

giac [B] time = 1.12, size = 32, normalized size = 2.91

$$\frac{1}{2}\left(e^{(-x)} + e^{x}\right)\log\left(\frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}\right) - \frac{1}{2}e^{(-x)} - \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)*log(1/2*e^(-x) + 1/2*e^x) - 1/2*e^(-x) - 1/2*e^x
```

maple [A] time = 0.16, size = 12, normalized size = 1.09

method	result
derivativedivides	$-\cosh(x) + \cosh(x)\ln(\cosh(x))$
default	$-\cosh(x) + \cosh(x)\ln(\cosh(x))$
risch	$-\frac{(1+e^{2x})e^{-x}\ln(e^{x})}{2} - \frac{\left(-i\pi\,\mathrm{csgn}(ie^{-x})\mathrm{csgn}\left(ie^{-x}(1+e^{2x})\right)^{2} + i\pi\,\mathrm{csgn}(ie^{-x})\mathrm{csgn}\left(i(1+e^{2x})\right)\mathrm{csgn}\left(ie^{-x}(1+e^{2x})\right)e^{2x} + i\mathrm{csgr}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x))*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(x)+cosh(x)*ln(cosh(x))
```

maxima [A] time = 0.43, size = 11, normalized size = 1.00

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)*log(cosh(x)) - cosh(x)
```

mupad [B] time = 0.36, size = 8, normalized size = 0.73

$$\cosh(x)\ (\ln(\cosh(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(x))*sinh(x),x)
```

```
[Out] cosh(x)*(log(cosh(x)) - 1)
```

sympy [A] time = 0.95, size = 10, normalized size = 0.91

$$\log(\cosh(x))\cosh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x))*sinh(x),x)
```

```
[Out] log(cosh(x))*cosh(x) - cosh(x)
```

3.636 $\int \log(\cosh(x))\tanh(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\cosh(x))$$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475, 4341, 2301}

$$\frac{1}{2}\log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]]*Tanh[x],x]
[Out] Log[Cosh[x]]^2/2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \log(\cosh(x))\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cosh(x)\right)\\ &= \frac{1}{2}\log^2(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]]*Tanh[x],x]
[Out] Log[Cosh[x]]^2/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\cosh(x))\tanh(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Cosh[x]]*Tanh[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.78, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\cosh(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(cosh(x))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\cosh(x)\right)\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(log(cosh(x))*tanh(x), x)
```

maple [A] time = 0.15, size = 8, normalized size = 0.89

method	result
derivativedivides	$\frac{\ln(\cosh(x))^2}{2}$
default	$\frac{\ln(\cosh(x))^2}{2}$
risch	$\left(x-\ln\left(1+e^{2x}\right)\right)\ln\left(e^{x}\right)+\frac{\ln\left(1+e^{2x}\right)^2}{2}-\frac{x^2}{2}+\frac{i\ln\left(1+e^{2x}\right)\pi\,\mathrm{csgn}\left(i\left(1+e^{2x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^2}{2}+\frac{i\ln(1+e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x))*tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(cosh(x))^2
```

maxima [A] time = 0.43, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\cosh(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(x))^2
```

mupad [B] time = 0.44, size = 16, normalized size = 1.78

$$\frac{\ln\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(x))*tanh(x),x)
```

```
[Out] log(exp(-x)/2 + exp(x)/2)^2/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\cosh\left(x\right)\right)\tanh\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x))*tanh(x),x)
[Out] Integral(log(cosh(x))*tanh(x), x)
```

3.637 $\int \log \left(x-\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=26

$$\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2534, 261}

$$\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[x - Sqrt[1 + x^2]],x]

[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned} \int \log \left(x-\sqrt{1+x^2}\right) dx &= x \log \left(x-\sqrt{1+x^2}\right)+\int \frac{x}{\sqrt{1+x^2}} dx \\ &= \sqrt{1+x^2}+x \log \left(x-\sqrt{1+x^2}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x - Sqrt[1 + x^2]],x]

[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(x-\sqrt{1+x^2}\right) dx$$

Verification is Not applicable to the result.

[In] IntegrateAlgebraic[Log[x - Sqrt[1 + x^2]],x]

[Out] Could not integrate

fricas [A] time = 1.06, size = 22, normalized size = 0.85

$$x \log \left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

giac [A] time = 1.13, size = 22, normalized size = 0.85

$$x \log \left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

method	result	size
default	$x \ln \left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \log \left(x - \sqrt{x^2 + 1}\right) - x + \arctan(x) + \int -\frac{x}{x^3 - \left(x^2 + 1\right)^{\frac{3}{2}} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(3
/2) + x), x)
```

mupad [B] time = 0.08, size = 22, normalized size = 0.85

$$x\,\ln\left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x - (x^2 + 1)^(1/2)),x)
[Out] x*log(x - (x^2 + 1)^(1/2)) + (x^2 + 1)^(1/2)
```

sympy [A] time = 6.26, size = 20, normalized size = 0.77

$$x \log \left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x-(x**2+1)**(1/2)),x)
[Out] x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

3.638 $\int \frac{\log(-1+x)}{x^3} dx$

Optimal. Leaf size=35

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 44}

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + x]/x^3,x]
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(-1+x)}{x^3}\,dx &= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \frac{1}{(-1+x)x^2}\,dx\\ &= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx\\ &= \frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(-1+x)}{2x^2}-\frac{\log(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.77

$$\frac{1}{2}\left(-\frac{\log(x-1)}{x^2}+\frac{1}{x}+\log(1-x)-\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + x]/x^3,x]
[Out] (x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(-1+x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[-1 + x]/x^3,x]
[Out] Could not integrate
```

fricas [A] time = 0.78, size = 26, normalized size = 0.74

$$-\frac{x^2\log(x)-\left(x^2-1\right)\log\left(x-1\right)-x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="fricas")
[Out] -1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

giac [A] time = 0.79, size = 27, normalized size = 0.77

$$\frac{1}{2\,x}-\frac{\log\left(x-1\right)}{2\,x^2}+\frac{1}{2}\,\log\left(|x-1|\right)-\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="giac")
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(abs(x - 1)) - 1/2*log(abs(x))
```

maple [A] time = 0.04, size = 26, normalized size = 0.74

method	result	size
derivativedivides	$\frac{1}{2x}-\frac{\ln(x)}{2}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
default	$\frac{1}{2x}-\frac{\ln(x)}{2}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
norman	$\frac{\frac{x}{2}+\frac{x^2\ln(-1+x)}{2}-\frac{\ln(-1+x)}{2}}{x^2}-\frac{\ln(x)}{2}$	29
risch	$-\frac{\ln(-1+x)}{2x^2}+\frac{\ln(-1+x)x-x\ln(x)+1}{2x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+x)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x-1/2*ln(x)+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

maxima [A] time = 0.43, size = 25, normalized size = 0.71

$$\frac{1}{2\,x}-\frac{\log\left(x-1\right)}{2\,x^2}+\frac{1}{2}\,\log\left(x-1\right)-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="maxima")
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

mupad [B] time = 0.06, size = 25, normalized size = 0.71

$$\frac{x-\ln\left(x-1\right)+x^2\,\ln\left(1-\frac{1}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x - 1)/x^3,x)
```

```
[Out] (x - log(x - 1) + x^2*log(1 - 1/x))/(2*x^2)
```

sympy [A] time = 0.14, size = 26, normalized size = 0.74

$$-\frac{\log(x)}{2}+\frac{\log(x-1)}{2}+\frac{1}{2x}-\frac{\log(x-1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+x)/x**3,x)
```

```
[Out] -log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

3.639 $\int (-e^{-x}+e^x)\log\left(1+e^{2x}\right)\,dx$

Optimal. Leaf size=32

$$-2e^x+e^{-x}\log\left(e^{2x}+1\right)+e^x\log\left(e^{2x}+1\right)$$

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 2476, 2448, 321, 203, 2455}

$$-2e^x+e^{-x}\log\left(e^{2x}+1\right)+e^x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\int (-e^{-x}+e^x)\log\left(1+e^{2x}\right)\,dx &= \text{Subst}\left(\int \frac{\left(-1+x^2\right)\log\left(1+x^2\right)}{x^2}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(\log\left(1+x^2\right)-\frac{\log\left(1+x^2\right)}{x^2}\right)dx,x,e^x\right)\\
&= \text{Subst}\left(\int \log\left(1+x^2\right)\,dx,x,e^x\right)-\text{Subst}\left(\int \frac{\log\left(1+x^2\right)}{x^2}\,dx,x,e^x\right)\\
&= e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)-2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)-2\,\text{Subst}\left(\int\right.\\
&= -2e^x-2\tan^{-1}\left(e^x\right)+e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)+2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.\\
&= -2e^x+e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.75

$$(e^{-x}+e^x)\log\left(e^{2x}+1\right)-2e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
[Out] -2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-e^{-x}+e^x)\log\left(1+e^{2x}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
[Out] Could not integrate
```

fricas [A] time = 1.21, size = 26, normalized size = 0.81

$$\left(\left(e^{(2x)}+1\right)\log\left(e^{(2x)}+1\right)-2\,e^{(2x)}\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")
[Out] ((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

giac [A] time = 0.94, size = 20, normalized size = 0.62

$$\left(e^{(-x)}+e^x\right)\log\left(e^{(2x)}+1\right)-2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="giac")
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

maple [A] time = 0.03, size = 24, normalized size = 0.75

method	result	size
risch	$\left(1+e^{2x}\right) e^{-x} \ln\left(1+e^{2x}\right)-2\,e^{x}$	24
norman	$\left(e^{2x} \ln\left(1+e^{2x}\right)-2\,e^{2x}+\ln\left(1+e^{2x}\right)\right) e^{-x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x,method=_RETURNVERBOSE)
[Out] (1+exp(2*x))*exp(-x)*ln(1+exp(2*x))-2*exp(x)
```

maxima [A] time = 0.44, size = 20, normalized size = 0.62

$$\left(e^{(-x)}+e^{x}\right)\log\left(e^{(2x)}+1\right)-2\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

mupad [B] time = 0.38, size = 24, normalized size = 0.75

$$2\,\ln\left(e^{2x}+1\right)\,\cosh(x)-\frac{e^{2x}+1}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(exp(2*x) + 1)*(exp(-x) - exp(x)),x)
[Out] 2*log(exp(2*x) + 1)*cosh(x) - (exp(2*x) + 1)/cosh(x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ShapeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
[Out] Exception raised: ShapeError
```

3.640 $\int e^{3x/2} \log(-1+e^x)\, dx$

Optimal. Leaf size=52

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2194, 2554, 12, 2248, 302, 207}

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*E^((3*x)/2
)*Log[-1 + E^x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{3x/2}\log(-1+e^x)\,dx &= \frac{2}{3}e^{3x/2}\log(-1+e^x)-\int\frac{2e^{5x/2}}{3(-1+e^x)}\,dx\\
&=\frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{2}{3}\int\frac{e^{5x/2}}{-1+e^x}\,dx\\
&=\frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\operatorname{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&=\frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\operatorname{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,e^{x/2}\right)\\
&=-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\operatorname{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&=-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)+\frac{2}{3}e^{3x/2}\log(-1+e^x)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.81

$$\frac{2}{9}\left(e^{x/2}\left(3e^x\log(e^x-1)-2(e^x+3)\right)+6\tanh^{-1}\left(e^{x/2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/2)*Log[-1 + E^x],x]
[Out] (2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x])))/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{3x/2}\log(-1+e^x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^((3*x)/2)*Log[-1 + E^x],x]
[Out] Could not integrate
```

fricas [A] time = 0.88, size = 42, normalized size = 0.81

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log(e^x-1)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

giac [A] time = 0.86, size = 43, normalized size = 0.83

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log(e^x-1)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(\left|e^{\left(\frac{1}{2}x\right)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="giac")
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))
```

maple [A] time = 0.03, size = 43, normalized size = 0.83

method	result	size
risch	$\frac{2\,\mathrm{e}^{\frac{3x}{2}}\ln(-1+\mathrm{e}^{x})}{3}-\frac{4\,\mathrm{e}^{\frac{3x}{2}}}{9}-\frac{4\,\mathrm{e}^{\frac{x}{2}}}{3}-\frac{2\ln\left(-1+\mathrm{e}^{\frac{x}{2}}\right)}{3}+\frac{2\ln\left(\mathrm{e}^{\frac{x}{2}}+1\right)}{3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3/2*x)*ln(-1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*exp(3/2*x)*ln(-1+exp(x))-4/9*exp(3/2*x)-4/3*exp(1/2*x)-2/3*ln(-1+exp(1/
2*x))+2/3*ln(exp(1/2*x)+1)
```

maxima [A] time = 0.45, size = 42, normalized size = 0.81

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

mupad [B] time = 0.55, size = 31, normalized size = 0.60

$$\frac{4\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^{x}}\right)}{3}-\frac{4\,\mathrm{e}^{\frac{3\,x}{2}}}{9}-\frac{4\,\mathrm{e}^{x/2}}{3}+\frac{2\,\mathrm{e}^{\frac{3\,x}{2}}\,\ln\left(\mathrm{e}^{x}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp((3*x)/2)*log(exp(x) - 1),x)
```

```
[Out] (4*atanh(exp(x)^(1/2)))/3 - (4*exp((3*x)/2))/9 - (4*exp(x/2))/3 + (2*exp((3
*x)/2)*log(exp(x) - 1))/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

```
[Out] Timed out
```

3.641 $\int \cos^3(x)\log(\sin(x))\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2633, 2554, 12, 4356}

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\int \frac{1}{6}\cos(x)(5+\cos(2x))\,dx\\
&= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\frac{1}{6}\int \cos(x)(5+\cos(2x))\,dx\\
&= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\frac{1}{6}\text{Subst}\left(\int \left(6-2x^2\right)\,dx,x,\sin(x)\right)\\
&= -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3(x)\log(\sin(x))\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Cos[x]^3*Log[Sin[x]],x]
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 24, normalized size = 0.80

$$\frac{1}{3}\left(\cos(x)^2+2\right)\log\left(\sin(x)\right)\sin(x)-\frac{1}{9}\left(\cos(x)^2+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

giac [A] time = 1.08, size = 26, normalized size = 0.87

$$-\frac{1}{3}\log\left(\sin(x)\right)\sin(x)^3+\frac{1}{9}\sin(x)^3+\log\left(\sin(x)\right)\sin(x)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
[Out] -1/3*log(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + log(sin(x))*sin(x) - sin(x)
```

maple [C] time = 0.38, size = 197, normalized size = 6.57

method	result
default	$-\frac{i\left(\frac{e^{3ix}\ln\left(i\left(-e^{2ix}+1\right)e^{-ix}\right)}{3}-\frac{e^{3ix}}{9}-\frac{11\,e^{ix}}{3}+3\,e^{ix}\ln\left(i\left(-e^{2ix}+1\right)e^{-ix}\right)-3\,e^{-ix}\ln\left(i\left(-e^{2ix}+1\right)e^{-ix}\right)+\frac{11\,e^{-ix}}{3}-\frac{e^{-3ix}\ln\left(i\left(-e^{2ix}+1\right)e^{-ix}\right)}{3}+\frac{e^{-3ix}}{9}-\frac{\ln(2}{}\right.}{8}$
risch	$\frac{3ie^{-ix}\ln\left(e^{2ix}-1\right)}{8}+2i\left(\frac{i\ln(2)}{24}-\frac{i\ln\left(e^{2ix}-1\right)}{24}-\frac{\pi}{48}+\frac{i}{72}+\frac{\mathrm{csgn}(\sin(x))^2\mathrm{csgn}\left(ie^{2ix}-i\right)\pi}{48}+\frac{\mathrm{csgn}(\sin(x))^3\pi}{48}+\frac{\pi\,\mathrm{csgn}(\sin(x))c}{48}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] -1/8*I*(1/3*exp(3*I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))-1/9*exp(I*x)^3-11/3*e
xp(I*x)+3*exp(I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))-3*exp(-I*x)*ln(I*(-exp(I*
x)^2+1)/exp(I*x))+11/3/exp(I*x)-1/3*exp(-3*I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*
x))+1/9/exp(I*x)^3-1/3*ln(2)*exp(I*x)^3-3*ln(2)*exp(I*x)+3*ln(2)/exp(I*x)+1
/3*ln(2)/exp(I*x)^3)
```

maxima [A] time = 0.42, size = 25, normalized size = 0.83

$$\frac{1}{9}\sin(x)^3-\frac{1}{3}\left(\sin(x)^3-3\sin(x)\right)\log\left(\sin(x)\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
```

```
[Out] 1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln(\sin(x))\,\cos(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x)^3,x)
```

```
[Out] int(log(sin(x))*cos(x)^3, x)
```

sympy [A] time = 5.24, size = 42, normalized size = 1.40

$$\frac{2\log(\sin(x))\sin^3(x)}{3} + \log(\sin(x))\sin(x)\cos^2(x) - \frac{8\sin^3(x)}{9} - \sin(x)\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*ln(sin(x)),x)
```

```
[Out] 2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(x)**3/9 -
sin(x)*cos(x)**2
```

3.642 $\int \log(\tan(x))\sec^4(x)\,dx$

Optimal. Leaf size=30

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3767, 2554, 12}

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \log(\tan(x))\sec^4(x)\,dx &= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\int \frac{1}{3}(2+\cos(2x))\sec^4(x)\,dx\\ &= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\frac{1}{3}\int (2+\cos(2x))\sec^4(x)\,dx\\ &= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\frac{1}{3}\text{Subst}\left(\int \left(3+x^2\right)\,dx,x,\tan(x)\right)\\ &= -\tan(x)+\log(\tan(x))\tan(x)-\frac{\tan^3(x)}{9}+\frac{1}{3}\log(\tan(x))\tan^3(x)\end{aligned}$$

Mathematica [A] time = 0.09, size = 29, normalized size = 0.97

$$\frac{1}{9}\tan(x)\left(\sec^2(x)(6\log(\tan(x))+3\cos(2x)\log(\tan(x))-1)-8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] ((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\tan(x))\sec^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.28, size = 39, normalized size = 1.30

$$\frac{3\left(2\cos(x)^2+1\right)\log\left(\frac{\sin(x)}{\cos(x)}\right)\sin(x)-\left(8\cos(x)^2+1\right)\sin(x)}{9\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 + 1)*sin(x)
)/cos(x)^3
```

giac [A] time = 0.95, size = 26, normalized size = 0.87

$$\frac{1}{3}\log(\tan(x))\tan(x)^3-\frac{1}{9}\tan(x)^3+\log(\tan(x))\tan(x)-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)
```

maple [B] time = 0.52, size = 55, normalized size = 1.83

method	result
default	$\frac{\left(6\left(\cos^2(x)\right)\ln(2)+6\left(\cos^2(x)\right)\ln\left(\frac{\sin(x)}{2\cos(x)}\right)-8\left(\cos^2(x)\right)+3\ln(2)+3\ln\left(\frac{\sin(x)}{2\cos(x)}\right)-1\right)\sin(x)}{9\cos(x)^3}$
risch	$-\frac{4i\left(3\,e^{2ix}+1\right)\ln\left(1+e^{2ix}\right)}{3\left(1+e^{2ix}\right)^3}+\frac{\frac{2\pi}{3}+\frac{2\pi\,\mathrm{csgn}\left(\frac{i\left(e^{2ix}-1\right)}{1+e^{2ix}}\right)^3}{3}-\frac{2\,\mathrm{csgn}\left(\frac{e^{2ix}-1}{1+e^{2ix}}\right)^2\pi}{3}+2\pi\,e^{2ix}+\frac{2\pi\,\mathrm{csgn}\left(\frac{e^{2ix}-1}{1+e^{2ix}}\right)^3}{3}-\frac{2\,\mathrm{csgn}\left(\frac{i\left(e^{2ix}-1\right)}{1+e^{2ix}}\right)^2\mathrm{csgn}\left(i\left(e^{2ix}-1\right)\right)\cdots}{3}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tan(x))/cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(6*cos(x)^2*ln(2)+6*cos(x)^2*ln(1/2*sin(x)/cos(x))-8*cos(x)^2+3*ln(2)+3
*ln(1/2*sin(x)/cos(x))-1)*sin(x)/cos(x)^3
```

maxima [A] time = 0.43, size = 25, normalized size = 0.83

$$-\frac{1}{9}\tan(x)^3+\frac{1}{3}\left(\tan(x)^3+3\tan(x)\right)\log(\tan(x))-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

mupad [B] time = 1.80, size = 148, normalized size = 4.93

$$\frac{\ln\left(-\frac{8\,e^{x\,2i}}{3}-\frac{8}{3}\right)\,2i}{3}-\frac{\ln\left(\frac{8}{3}-\frac{8\,e^{x\,2i}}{3}\right)\,2i}{3}+\frac{8i}{9\,\left(3\,e^{x\,2i}+3\,e^{x\,4i}+e^{x\,6i}+1\right)}-\frac{4i}{3\,\left(2\,e^{x\,2i}+e^{x\,4i}+1\right)}-\frac{4i}{3\,\left(e^{x\,2i}+1\right)}+\frac{\ln\left(-\frac{e^{x}}{e}\right.}{3\,e^{x\,2i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/cos(x)^4,x)
```

```
[Out] (log(- (8*exp(x*2i))/3 - 8/3)*2i)/3 - (log(8/3 - (8*exp(x*2i))/3)*2i)/3 + 8
i/(9*(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)) - 4i/(3*(2*exp(x*2i) + ex
p(x*4i) + 1)) - 4i/(3*(exp(x*2i) + 1)) + (log(-(exp(x*2i)*1i - 1i)/(exp(x*2
i) + 1))*(exp(x*2i)*4i + 4i/3))/(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)
```

sympy [A] time = 20.46, size = 46, normalized size = 1.53

$$\frac{\log\left(\tan\left(x\right)\right)\tan^{3}\left(x\right)}{3}+\log\left(\tan\left(x\right)\right)\tan\left(x\right)-\frac{\sin^{3}\left(x\right)}{9\cos^{3}\left(x\right)}+\frac{\sin\left(x\right)}{3\cos\left(x\right)}-\frac{4\tan\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tan(x))/cos(x)**4,x)
```

```
[Out] log(tan(x))*tan(x)**3/3 + log(tan(x))*tan(x) - sin(x)**3/(9*cos(x)**3) + si
n(x)/(3*cos(x)) - 4*tan(x)/3
```

3.643 $\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2648, 2554, 12, 3473, 8}

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

```
[Out] -x/2 + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} - \int -\frac{1}{2}\tan^2\left(\frac{x}{2}\right)dx$$
$$= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \frac{1}{2}\int \tan^2\left(\frac{x}{2}\right)dx$$
$$= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right) - \frac{\int 1\,dx}{2}$$
$$= -\frac{x}{2} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.10, size = 32, normalized size = 1.14

$$-\frac{\sin(x)\left(x\cot\left(\frac{x}{2}\right)-2\left(\log\left(\cos\left(\frac{x}{2}\right)\right)+1\right)\right)}{2(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
[Out] -1/2*((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(1 + Cos[x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Log[Cos[x/2]]/(1 + Cos[x]),x]
[Out] Could not integrate
```

fricas [A] time = 0.90, size = 32, normalized size = 1.14

$$-\frac{x\cos\left(\frac{1}{2}x\right)-2\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)-2\sin\left(\frac{1}{2}x\right)}{2\cos\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")
[Out] -1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))/cos(1/2*x
)
```

giac [A] time = 0.92, size = 43, normalized size = 1.54

$$-\frac{1}{2}x - \frac{2\log\left(\cos\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="giac")
[Out] -1/2*x - 2*log(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
 + tan(1/2*x)
```

maple [C] time = 0.25, size = 164, normalized size = 5.86

method	result
risch	$-\frac{2i\ln\left(e^{\frac{ix}{2}}\right)}{e^{ix}+1}+\frac{-i\ln\left(e^{ix}+1\right)e^{ix}+\pi\,\mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\mathrm{csgn}\left(ie^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)-\pi\,\mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)^{2}-\pi\,\mathrm{csgn}\left(ie^{-\frac{ix}{2}}\right)\mathrm{cs}}{e^{ix}+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(1/2*x))/(1+cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(exp(I*x)+1)*ln(exp(1/2*I*x))+(-I*ln(exp(I*x)+1)*exp(I*x)+Pi*csgn(I*(e
xp(I*x)+1))*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))-Pi*csgn(I*(exp(I*x)+1)
)*csgn(I*cos(1/2*x))^2-Pi*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))^2+Pi*csg
n(I*cos(1/2*x))^3-x*exp(I*x)+I*ln(exp(I*x)+1)-2*I*ln(2)+2*I-x)/(exp(I*x)+1)
```

maxima [B] time = 0.44, size = 56, normalized size = 2.00

$$\frac{\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin(x)}{\cos(x)+1}-\frac{x\cos(x)^{2}+x\sin(x)^{2}+2\,x\cos(x)+x-4\,\sin(x)}{2\left(\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^2 + 2*x*co
s(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

mupad [B] time = 0.58, size = 39, normalized size = 1.39

$$\tan\left(\frac{x}{2}\right)-x+\tan\left(\frac{x}{2}\right)\,\ln\left(\cos\left(\frac{x}{2}\right)\right)+\ln\left(\cos\left(\frac{x}{2}\right)\right)\,1\mathrm{i}-\ln\left(\cos(x)+1+\sin(x)\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x/2))/(cos(x) + 1),x)
```

```
[Out] tan(x/2) - x + log(cos(x/2))*1i - log(cos(x) + sin(x)*1i + 1)*1i + tan(x/2)
*log(cos(x/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] Integral(log(cos(x/2))/(cos(x) + 1), x)
```

3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

Rubi [A] time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2750, 2648, 2554, 12, 2968, 3019, 2735}

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])) - (Log[S
in[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx &= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \int \frac{\cos(x)(1+2\cos(x))}{3(1+\cos(x))^2}\,dx \\
&= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \frac{1}{3}\int \frac{\cos(x)(1+2\cos(x))}{(1+\cos(x))^2}\,dx \\
&= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \frac{1}{3}\int \frac{\cos(x)+2\cos^2(x)}{(1+\cos(x))^2}\,dx \\
&= -\frac{\sin(x)}{9(1+\cos(x))^2} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} + \frac{1}{9}\int \frac{2-6\cos(x)}{1+\cos(x)}\,dx \\
&= -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} + \frac{8}{9}\int \frac{1}{1+\cos(x)}\,dx \\
&= -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} + \frac{8\sin(x)}{9(1+\cos(x))} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 56, normalized size = 0.93

$$-\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right)+3x\cos\left(\frac{3x}{2}\right)-\sin\left(\frac{x}{2}\right)(3\log(\sin(x))+\cos(x)(6\log(\sin(x))+8)+7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] -1/18*(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] + C
os[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.86, size = 53, normalized size = 0.88

$$-\frac{6\,x\cos(x)^2-3\,(2\,\cos(x)+1)\log(\sin(x))\sin(x)+12\,x\cos(x)-(8\,\cos(x)+7)\sin(x)+6\,x}{9\left(\cos(x)^2+2\,\cos(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*cos(x) - (8
*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

giac [A] time = 1.09, size = 36, normalized size = 0.60

$$-\frac{1}{18}\tan\left(\frac{1}{2}x\right)^{3}-\frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^{3}-3\tan\left(\frac{1}{2}x\right)\right)\log\left(\sin\left(x\right)\right)-\frac{2}{3}x+\frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] -1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*log(sin(x)) - 2/3*x
+ 5/6*tan(1/2*x)
```

maple [B] time = 0.44, size = 106, normalized size = 1.77

method	result
default	$\frac{6(\cos^3(x))\ln(2)+6(\cos^3(x))\ln\left(\frac{\sin(x)}{2}\right)-12(\cos^2(x))\sin(x)\arctan\left(\frac{-1+\cos(x)}{\sin(x)}\right)+8(\cos^3(x))-9(\cos^2(x))\ln(2)-9(\cos^2(x))\ln\left(\frac{\sin(x)}{2}\right)-9(\mathrm{c}}{9\sin(x)^3}$
risch	$-\frac{2i(3e^{2ix}+3e^{ix}+2)\ln(e^{ix})}{3(e^{ix}+1)^3}+\frac{6\pi-12x+9\pi e^{2ix}+16i-9e^{ix}\pi\mathrm{csgn}(\sin(x))^2\mathrm{csgn}(ie^{-ix})+9e^{ix}\pi\,\mathrm{csgn}(\sin(x))\mathrm{csgn}(i\sin(x))^2-9\pi\,\mathrm{csgn}(i(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))/(1+cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(6*cos(x)^3*ln(2)+6*cos(x)^3*ln(1/2*sin(x))-12*cos(x)^2*sin(x)*arctan((
-1+cos(x))/sin(x))+8*cos(x)^3-9*cos(x)^2*ln(2)-9*cos(x)^2*ln(1/2*sin(x))-9*
cos(x)^2+12*arctan((-1+cos(x))/sin(x))*sin(x)-6*cos(x)+3*ln(2)+3*ln(1/2*sin
(x))+7)/sin(x)^3
```

maxima [A] time = 0.96, size = 86, normalized size = 1.43

$$\frac{1}{6}\left(\frac{3\sin(x)}{\cos(x)+1}-\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)+\frac{5\sin(x)}{6(\cos(x)+1)}-\frac{\sin(x)^3}{18(\cos(x)+1)^3}-\frac{4}{3}\arctan\left(\frac{\ }{\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x)/((sin(x)
^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos(x) + 1) - 1/18*sin(
x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.79, size = 164, normalized size = 2.73

$$\frac{\frac{4\sin(2x)}{9}-\frac{\ln(-2\sin(x)^2+\sin(2x)\,1i)\,7i}{3}-\frac{14x}{3}+\frac{\ln(\sin(x))\,7i}{3}+\frac{7\sin(x)}{9}+\frac{\sin(2x)\ln(\sin(x))}{3}-\frac{\sin(x)^2\,8i}{9}+\sin\left(\frac{x}{2}\right)^2\left(\frac{16x}{3}+\frac{\ln(-2\sin}{}\right.}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(sin(x))*cos(x))/(cos(x) + 1)^2,x)
```

```
[Out] ((4*sin(2*x))/9 - (log(sin(2*x)*1i - 2*sin(x)^2)*7i)/3 - (14*x)/3 + (log(si
n(x))*7i)/3 + (7*sin(x))/9 + (sin(2*x)*log(sin(x)))/3 - (sin(x)^2*8i)/9 + s
in(x/2)^2*((16*x)/3 + (log(sin(2*x)*1i - 2*sin(x)^2)*8i)/3 - (log(sin(x))*8
i)/3 - 32i/9) + (log(sin(2*x)*1i - 2*sin(x)^2)*(2*sin(x)^2 - 1)*1i)/3 + (lo
g(sin(x))*sin(x))/3 - (log(sin(x))*(2*sin(x)^2 - 1)*1i)/3 + (2*x*(2*sin(x)^
2 - 1))/3 + 32i/9)/(2*sin(x/2)^2 - 2)^2
```

sympy [A] time = 8.00, size = 107, normalized size = 1.78

$$-\frac{2x}{3}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^3\left(\frac{x}{2}\right)}{6}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{2}-\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^3\left(\frac{x}{2}\right)}{6}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

```
[Out] -2*x/3 + log(tan(x/2)**2 + 1)*tan(x/2)**3/6 - log(tan(x/2)**2 + 1)*tan(x/2)
/2 - log(tan(x/2))*tan(x/2)**3/6 + log(tan(x/2))*tan(x/2)/2 - log(2)*tan(x/
2)**3/6 - tan(x/2)**3/18 + log(2)*tan(x/2)/2 + 5*tan(x/2)/6
```

3.645 $\int \frac{\cos^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=65

$$-\frac{\cos^{-1}(x)^2}{4x^4}-\frac{1}{12x^2}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\log(x)}{3}$$

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4628, 4702, 4682, 29, 30}

$$-\frac{1}{12x^2}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(x)^2}{x^5}\,dx &= -\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{2}\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx \\ &= \frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{6x^3} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{1}{6}\int \frac{1}{x^3}\,dx - \frac{1}{3}\int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{6x^3} + \frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{3x} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{1}{3}\int \frac{1}{x}\,dx \\ &= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{6x^3} + \frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{3x} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 52, normalized size = 0.80

$$-\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{\sqrt{1-x^2}\left(2x^2+1\right)\cos^{-1}(x)}{6x^3} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/(
4*x^4) + Log[x]/3
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{-1}(x)^2}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCos[x]^2/x^5,x]
```

```
[Out] Could not integrate
```

fricas [**A**] time = 1.07, size = 44, normalized size = 0.68

$$\frac{4\,x^4\log(x) + 2\left(2\,x^3+x\right)\sqrt{-x^2+1}\,\arccos(x) - x^2 - 3\,\arccos(x)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2 - 3*arcco
s(x)^2)/x^4
```

giac [**B**] time = 0.99, size = 104, normalized size = 1.60

$$-\frac{1}{48}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) - \frac{2\,x^2+1}{12\,x^2} - \frac{\arccos(x)^2}{4\,x^4} + \frac{1}{6}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) - 1/12*(2*x^2
+ 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*log(x^2)
```

maple [A] time = 0.08, size = 52, normalized size = 0.80

method	result	size
default	$-\frac{1}{12x^2}-\frac{\arccos(x)^2}{4x^4}+\frac{\ln(x)}{3}+\frac{\arccos(x)\sqrt{-x^2+1}}{6x^3}+\frac{\arccos(x)\sqrt{-x^2+1}}{3x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1/
3*arccos(x)*(-x^2+1)^(1/2)/x
```

maxima [A] time = 0.96, size = 51, normalized size = 0.78

$$\frac{1}{6}\left(\frac{2\sqrt{-x^2+1}}{x}+\frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x)-\frac{1}{12\,x^2}-\frac{\arccos(x)^2}{4\,x^4}+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^2 - 1/4*ar
ccos(x)^2/x^4 + 1/3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x)^2/x^5,x)
```

```
[Out] int(acos(x)^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{acos}^2(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)**2/x**5,x)
```

```
[Out] Integral(acos(x)**2/x**5, x)
```

3.646 $\int x^2 \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=61

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}\,x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\,\sin^{-1}(x)-\frac{4x}{9}$$

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4627, 4707, 4677, 8, 30}

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}\,x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\,\sin^{-1}(x)-\frac{4x}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x]^2,x]
```

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sqrt[1 - x^2
]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sin^{-1}(x)^2\,dx=\frac{1}{3}x^3\sin^{-1}(x)^2-\frac{2}{3}\int\frac{x^3\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$=\frac{2}{9}x^2\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{1}{3}x^3\sin^{-1}(x)^2-\frac{2\int x^2\,dx}{9}-\frac{4}{9}\int\frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$=-\frac{2x^3}{27}+\frac{4}{9}\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{2}{9}x^2\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{1}{3}x^3\sin^{-1}(x)^2-\frac{4\int 1\,dx}{9}$$
$$=-\frac{4x}{9}-\frac{2x^3}{27}+\frac{4}{9}\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{2}{9}x^2\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{1}{3}x^3\sin^{-1}(x)^2$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.69

$$\frac{1}{27}\left(9x^3\sin^{-1}(x)^2-2\left(x^2+6\right)x+6\sqrt{1-x^2}\,\left(x^2+2\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x]^2,x]
```

```
[Out] (-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)/
27
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sin^{-1}(x)^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*ArcSin[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.05, size = 36, normalized size = 0.59

$$\frac{1}{3}\,x^3\arcsin(x)^2-\frac{2}{27}\,x^3+\frac{2}{9}\left(x^2+2\right)\sqrt{-x^2+1}\,\arcsin(x)-\frac{4}{9}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arcsin(x) - 4
/9*x
```

giac [A] time = 0.90, size = 57, normalized size = 0.93

$$\frac{1}{3}\left(x^2-1\right)x\arcsin(x)^2+\frac{1}{3}\,x\arcsin(x)^2-\frac{2}{9}\left(-x^2+1\right)^{\frac{3}{2}}\arcsin(x)-\frac{2}{27}\left(x^2-1\right)x+\frac{2}{3}\,\sqrt{-x^2+1}\,\arcsin(x)-\frac{14}{27}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^(3/2)*arcs
in(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x) - 14/27*x
```

maple [A] time = 0.36, size = 37, normalized size = 0.61

method	result	size
default	$\frac{x^3 \arcsin(x)^2}{3} + \frac{2 \arcsin(x)\left(x^2+2\right) \sqrt{-x^2+1}}{9} - \frac{2x^3}{27} - \frac{4x}{9}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3-4/9*x
```

maxima [A] time = 0.97, size = 47, normalized size = 0.77

$$\frac{1}{3}\, x^3 \arcsin(x)^2 - \frac{2}{27}\, x^3 + \frac{2}{9}\left(\sqrt{-x^2+1}\, x^2 + 2\, \sqrt{-x^2+1}\right)\arcsin(x) - \frac{4}{9}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1)
)*arcsin(x) - 4/9*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\, \mathrm{asin}(x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x)^2,x)
```

```
[Out] int(x^2*asin(x)^2, x)
```

sympy [A] time = 0.66, size = 54, normalized size = 0.89

$$\frac{x^3 \operatorname{asin}^2(x)}{3} - \frac{2x^3}{27} + \frac{2x^2\sqrt{1-x^2}\, \operatorname{asin}(x)}{9} - \frac{4x}{9} + \frac{4\sqrt{1-x^2}\, \operatorname{asin}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)**2,x)
```

```
[Out] x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(1 - x**2)*asin(x)/9 - 4*x/9 + 4
*sqrt(1 - x**2)*asin(x)/9
```

3.647 $\int x^3 \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=53

$$\frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{6}x^3 \tan^{-1}(x) + \frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{2}x\tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

Rubi [A] time = 0.12, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{6}x^3 \tan^{-1}(x) + \frac{1}{2}x\tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[x]^2,x]
```

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x^4*ArcTan[
x]^2)/4 - Log[1 + x^2]/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tan^{-1}(x)^2 \, dx &= \frac{1}{4} x^4 \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{x^4 \tan^{-1}(x)}{1+x^2} \, dx \\
&= \frac{1}{4} x^4 \tan^{-1}(x)^2 - \frac{1}{2} \int x^2 \tan^{-1}(x) \, dx + \frac{1}{2} \int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx \\
&= -\frac{1}{6} x^3 \tan^{-1}(x) + \frac{1}{4} x^4 \tan^{-1}(x)^2 + \frac{1}{6} \int \frac{x^3}{1+x^2} \, dx + \frac{1}{2} \int \tan^{-1}(x) \, dx - \frac{1}{2} \int \frac{\tan^{-1}(x)}{1+x^2} \, dx \\
&= \frac{1}{2} x \tan^{-1}(x) - \frac{1}{6} x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4} x^4 \tan^{-1}(x)^2 + \frac{1}{12} \text{Subst}\left(\int \frac{x}{1+x} \, dx, x, x^2 \right. \\
&= \frac{1}{2} x \tan^{-1}(x) - \frac{1}{6} x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4} x^4 \tan^{-1}(x)^2 - \frac{1}{4} \log\left(1+x^2\right) + \frac{1}{12} \text{Subst}\left(\right. \\
&= \frac{x^2}{12} + \frac{1}{2} x \tan^{-1}(x) - \frac{1}{6} x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4} x^4 \tan^{-1}(x)^2 - \frac{1}{3} \log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.70

$$\frac{1}{12}\left(3\left(x^4-1\right)\tan^{-1}(x)^2+x^2-4\log\left(x^2+1\right)-2\left(x^2-3\right)x\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[x]^2,x]
[Out] (x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2]
)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^{-1}(x)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*ArcTan[x]^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.01, size = 36, normalized size = 0.68

$$\frac{1}{4}\left(x^4-1\right)\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x\right)\arctan(x)-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="fricas")
[Out] 1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x) - 1/3*log(
x^2 + 1)
```

giac [A] time = 1.09, size = 41, normalized size = 0.77

$$\frac{1}{4}x^4\arctan(x)^2-\frac{1}{6}x^3\arctan(x)+\frac{1}{12}x^2+\frac{1}{2}x\arctan(x)-\frac{1}{4}\arctan(x)^2-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan(x) - 1/4*
arctan(x)^2 - 1/3*log(x^2 + 1)
```

maple [A] time = 0.10, size = 42, normalized size = 0.79

method	result
default	$\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{\arctan(x)^2}{4}+\frac{x^4\arctan(x)^2}{4}-\frac{\ln\left(x^2+1\right)}{3}$
risch	$-\frac{\left(\frac{x^4}{4}-\frac{1}{4}\right)\ln(ix+1)^2}{4}-\frac{\left(-\frac{x^4\ln(-ix+1)}{2}-\frac{ix^3}{3}+ix+\frac{\ln(-ix+1)}{2}\right)\ln(ix+1)}{4}-\frac{x^4\ln(-ix+1)^2}{16}+\frac{\ln(-ix+1)^2}{16}-\frac{ix^3\ln(-ix+1)}{12}+\frac{i\ln(-ix+1)x}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x
)^2-1/3*ln(x^2+1)
```

maxima [A] time = 1.00, size = 44, normalized size = 0.83

$$\frac{1}{4}x^4\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x+3\arctan(x)\right)\arctan(x)+\frac{1}{4}\arctan(x)^2-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) +
1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

mupad [B] time = 0.34, size = 41, normalized size = 0.77

$$\frac{x^4\operatorname{atan}(x)^2}{4}-\frac{x^3\operatorname{atan}(x)}{6}-\frac{\operatorname{atan}(x)^2}{4}-\frac{\ln\left(x^2+1\right)}{3}+\frac{x\operatorname{atan}(x)}{2}+\frac{x^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(x)^2,x)
```

```
[Out] (x^4*atan(x)^2)/4 - (x^3*atan(x))/6 - atan(x)^2/4 - log(x^2 + 1)/3 + (x*ata
n(x))/2 + x^2/12
```

sympy [A] time = 0.62, size = 44, normalized size = 0.83

$$\frac{x^4\operatorname{atan}^2(x)}{4}-\frac{x^3\operatorname{atan}(x)}{6}+\frac{x^2}{12}+\frac{x\operatorname{atan}(x)}{2}-\frac{\log\left(x^2+1\right)}{3}-\frac{\operatorname{atan}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)**2,x)
```

```
[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)/
3 - atan(x)**2/4
```

3.648 $\int \frac{\tan^{-1}(x)^2}{x^5} \, dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)-\frac{2\log(x)}{3}+\frac{1}{4}\tan^{-1}(x)^2+\frac{\tan^{-1}(x)}{2x}$$

Rubi [A] time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4852, 4918, 266, 44, 36, 29, 31, 4884}

$$-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}-\frac{2\log(x)}{3}+\frac{1}{4}\tan^{-1}(x)^2+\frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4 - ArcTan[
x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)^2}{x^5}\,dx &= -\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4\left(1+x^2\right)}\,dx \\
&= -\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4}\,dx-\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx \\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{6}\int \frac{1}{x^3\left(1+x^2\right)}\,dx-\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2}\,dx+\frac{1}{2}\int \frac{\tan^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx,x,x^2\right)-\frac{1}{2}\int \frac{}{x} \\
&= -\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx,x,x^2\right) \\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\log(x)}{6}+\frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\operatorname{Sub} \\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.92

$$\frac{\left(x^4-1\right)\tan^{-1}(x)^2}{4x^4}-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)+\frac{\left(3x^2-1\right)\tan^{-1}(x)}{6x^3}-\frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/(
4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(x)^2}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[x]^2/x^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.14, size = 53, normalized size = 0.87

$$\frac{4\,x^4\log\left(x^2+1\right)-8\,x^4\log(x)+3\left(x^4-1\right)\arctan(x)^2-x^2+2\left(3\,x^3-x\right)\arctan(x)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2 - x^2 + 2
*(3*x^3 - x)*arctan(x))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^5, x)
```

maple [A] time = 0.14, size = 48, normalized size = 0.79

method	result
default	$-\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\ln(x)}{3}+\frac{\ln(x^2+1)}{3}$
risch	$-\frac{(x^4-1)\ln(ix+1)^2}{16x^4}+\frac{(3x^4\ln(-ix+1)-6ix^3+2ix-3\ln(-ix+1))\ln(ix+1)}{24x^4}-\frac{3x^4\ln(-ix+1)^2+32x^4\ln(x)-16\ln(x^2+1)x^4-12ix^3\ln(-}{48x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^2
/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

maxima [A] time = 0.98, size = 64, normalized size = 1.05

$$\frac{1}{6}\left(\frac{3x^2-1}{x^3}+3\arctan(x)\right)\arctan(x)-\frac{3x^2\arctan(x)^2-4x^2\log\left(x^2+1\right)+8x^2\log(x)+1}{12x^2}-\frac{\arctan(x)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arctan(x)^2 - 4
*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(x)^2/x^4
```

mupad [B] time = 0.11, size = 44, normalized size = 0.72

$$\frac{\ln\left(x^2+1\right)}{3}-\frac{2\ln(x)}{3}-\mathrm{atan}(x)^2\left(\frac{1}{4x^4}-\frac{1}{4}\right)-\frac{1}{12x^2}+\frac{\mathrm{atan}(x)\left(\frac{x^2}{2}-\frac{1}{6}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)^2/x^5,x)
```

```
[Out] log(x^2 + 1)/3 - (2*log(x))/3 - atan(x)^2*(1/(4*x^4) - 1/4) - 1/(12*x^2) +
(atan(x)*(x^2/2 - 1/6))/x^3
```

sympy [A] time = 0.88, size = 53, normalized size = 0.87

$$-\frac{2\log(x)}{3}+\frac{\log\left(x^2+1\right)}{3}+\frac{\mathrm{atan}^2(x)}{4}+\frac{\mathrm{atan}(x)}{2x}-\frac{1}{12x^2}-\frac{\mathrm{atan}(x)}{6x^3}-\frac{\mathrm{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2/x**5,x)
```

```
[Out] -2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/(12*x**2)
- atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

3.649 $\int x^3 \csc^{-1}(x)^2 \, dx$

Optimal. Leaf size=63

$$\frac{1}{4}x^4 \csc^{-1}(x)^2 + \frac{x^2}{12} + \frac{1}{3}\sqrt{1 - \frac{1}{x^2}}\, x \csc^{-1}(x) + \frac{1}{6}\sqrt{1 - \frac{1}{x^2}}\, x^3 \csc^{-1}(x) + \frac{\log(x)}{3}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5223, 3758, 4185, 4184, 3475}

$$\frac{x^2}{12} + \frac{1}{4}x^4 \csc^{-1}(x)^2 + \frac{1}{6}\sqrt{1 - \frac{1}{x^2}}\, x^3 \csc^{-1}(x) + \frac{1}{3}\sqrt{1 - \frac{1}{x^2}}\, x \csc^{-1}(x) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCsc[x]^2,x]
```

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x]
)/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3758

```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^p)/(b*n*p), x
] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int x^3\csc^{-1}(x)^2\,dx &= -\operatorname{Subst}\left(\int x^2\cot(x)\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\ &= \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{2}\operatorname{Subst}\left(\int x\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\ &= \frac{x^2}{12} + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\operatorname{Subst}\left(\int x\csc^2(x)\,dx, x, \csc^{-1}(x)\right)\\ &= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\operatorname{Subst}\left(\int \cot(x)\,dx\right.\\ &= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 + \frac{\log(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.67

$$\frac{1}{12}\left(3x^4\csc^{-1}(x)^2 + x^2 + 2\sqrt{1-\frac{1}{x^2}}\left(x^2+2\right)x\csc^{-1}(x) + 4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCsc[x]^2,x]
```

```
[Out] (x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Log
[x])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\csc^{-1}(x)^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*ArcCsc[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.85, size = 35, normalized size = 0.56

$$\frac{1}{4}x^4\operatorname{arccsc}(x)^2 + \frac{1}{6}\left(x^2+2\right)\sqrt{x^2-1}\,\operatorname{arccsc}(x) + \frac{1}{12}x^2 + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/12*x^2 + 1/
3*log(x)
```

giac [B] time = 0.96, size = 106, normalized size = 1.68

$$\frac{1}{4}x^4\arcsin\left(\frac{1}{x}\right)^2 + \frac{1}{12}x^2\left(\frac{2}{x^2}+1\right) + \frac{1}{48}\left(x^3\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^3 + 9x\left(\sqrt{-\frac{1}{x^2}+1}-1\right) - \frac{9x^2\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^2+1}{x^3\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^3}\right)\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arcsin(1/x)^2 + 1/12*x^2*(2/x^2 + 1) + 1/48*(x^3*(sqrt(-1/x^2 + 1)
- 1)^3 + 9*x*(sqrt(-1/x^2 + 1) - 1) - (9*x^2*(sqrt(-1/x^2 + 1) - 1)^2 + 1)/
(x^3*(sqrt(-1/x^2 + 1) - 1)^3))*arcsin(1/x) - 1/6*log(x^(-2))
```

maple [A] time = 0.08, size = 56, normalized size = 0.89

method	result	size
default	$\frac{x^4 \operatorname{arccsc}(x)^2}{4}+\frac{x^3 \operatorname{arccsc}(x)\sqrt{\frac{x^2-1}{x^2}}}{6}+\frac{x^2}{12}+\frac{\operatorname{arccsc}(x)\sqrt{\frac{x^2-1}{x^2}}\,x}{3}-\frac{\ln\left(\frac{1}{x}\right)}{3}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccsc(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arccsc(x)^2+1/6*x^3*arccsc(x)*((x^2-1)/x^2)^(1/2)+1/12*x^2+1/3*arcc
sc(x)*((x^2-1)/x^2)^(1/2)*x-1/3*ln(1/x)
```

maxima [A] time = 1.23, size = 95, normalized size = 1.51

$$\frac{1}{4}\,x^4 \operatorname{arccsc}(x)^2+\frac{2\,x^4 \arctan\left(1, \sqrt{x+1}\,\sqrt{x-1}\right)+2\,x^2 \arctan\left(1, \sqrt{x+1}\,\sqrt{x-1}\right)+\left(x^2+2\,\log\left(x^2\right)\right)\sqrt{x+1}\,\cdots}{12\,\sqrt{x+1}\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/12*(2*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 2*x
^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + (x^2 + 2*log(x^2))*sqrt(x + 1)*sqr
t(x - 1) - 4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{asin}\left(\frac{1}{x}\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asin(1/x)^2,x)
```

```
[Out] int(x^3*asin(1/x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acsc}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acsc(x)**2,x)
```

```
[Out] Integral(x**3*acsc(x)**2, x)
```

3.650 $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

Optimal. Leaf size=148

$$-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45}{128x^2}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{9\sec^{-1}(x)^2}{16x^2}-\frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}}{4x^3}$$

Rubi [A] time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5222, 3444, 3311, 30, 3310}

$$-\frac{45}{128x^2}-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}+\frac{9\sec^{-1}(x)^2}{16x^2}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45\sqrt{1-\frac{1}{x^2}}\sec}{64x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x]^4/x^5,x]
```

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*x^3) - (45
*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/128 + (3*ArcSec[x]^2
)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x
^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[
x]^4/(4*x^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
```

```
LtQ[m, -1])
```

Rubi steps

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \text{Subst}\left(\int x^4\cos^3(x)\sin(x)\,dx, x, \sec^{-1}(x)\right)$$
$$= -\frac{\sec^{-1}(x)^4}{4x^4} + \text{Subst}\left(\int x^3\cos^4(x)\,dx, x, \sec^{-1}(x)\right)$$
$$= \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{3}{8}\text{Subst}\left(\int x\cos^4(x)\,dx, x, \sec^{-1}(x)\right) + \frac{3}{4}$$
$$= -\frac{3}{128x^4} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-}}{}$$
$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2}$$
$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} - \frac{45}{128}\sec^{-1}(x)^2 + \frac{3\sec^{-1}(x)}{16x^4}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.62

$$\frac{4\left(3x^4-8\right)\sec^{-1}(x)^4 - 45x^2 + 16\sqrt{1-\frac{1}{x^2}}\,x\left(3x^2+2\right)\sec^{-1}(x)^3 - 6\sqrt{1-\frac{1}{x^2}}\,x\left(15x^2+2\right)\sec^{-1}(x) + \left(-45x^4+\right.}{128x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]^4/x^5,x]
[Out] (-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24 + 72*x^2 -
 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2)*ArcSec[x]^3 + 4*(-
8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSec[x]^4/x^5,x]
[Out] Could not integrate
```

fricas [A] time = 1.17, size = 77, normalized size = 0.52

$$\frac{4\left(3x^4-8\right)\operatorname{arcsec}(x)^4 - 3\left(15x^4-24x^2-8\right)\operatorname{arcsec}(x)^2 - 45x^2 + 2\left(8\left(3x^2+2\right)\operatorname{arcsec}(x)^3 - 3\left(15x^2+2\right)\operatorname{a}\right.}{128x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
[Out] 1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec(x)^2 - 45
*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*arcsec(x))*sqrt(x^2 -
1) - 3)/x^4
```

giac [A] time = 0.98, size = 137, normalized size = 0.93

$$\frac{3}{32}\arccos\left(\frac{1}{x}\right)^4+\frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{8x}-\frac{45}{128}\arccos\left(\frac{1}{x}\right)^2-\frac{45\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{64x}+\frac{\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{4x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

```
[Out] 3/32*arccos(1/x)^4 + 3/8*sqrt(-1/x^2 + 1)*arccos(1/x)^3/x - 45/128*arccos(1
/x)^2 - 45/64*sqrt(-1/x^2 + 1)*arccos(1/x)/x + 1/4*sqrt(-1/x^2 + 1)*arccos(
1/x)^3/x^3 + 9/16*arccos(1/x)^2/x^2 - 1/4*arccos(1/x)^4/x^4 - 3/32*sqrt(-1/
x^2 + 1)*arccos(1/x)/x^3 - 45/128/x^2 + 3/16*arccos(1/x)^2/x^4 - 3/128/x^4
+ 189/1024
```

maple [A] time = 0.38, size = 165, normalized size = 1.11

method	result
default	$-\frac{\operatorname{arcsec}(x)^4}{4x^4}+\frac{\operatorname{arcsec}(x)^3\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)}{8x^3}+\frac{3\operatorname{arcsec}(x)^2}{16x^4}-\frac{3\operatorname{arcsec}(x)\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-}{x^2}}\right.}{64x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)^4/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^(
1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/64*arcsec(x)*(3*arcs
ec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+45/128*arcse
c(x)^2-3/128/x^4-45/128/x^2+9/16*arcsec(x)^2/x^2-9/16*arcsec(x)*(arcsec(x)*
x+((x^2-1)/x^2)^(1/2))/x+9/32-9/32*arcsec(x)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{8x^4\int\frac{12\left(x^2-1\right)\log\left(x^2\right)^2\log(x)^2-16\left(x^2-1\right)\log\left(x^2\right)\log(x)^3+8\left(x^2-1\right)\log(x)^4+\left(x^2-4\left(x^2-1\right)\log(x)-1\right)\log\left(x^2\right)^3-12\left(4\left(x^2-1\right)\log(x)^2+\left(x^2-4\left(x^2-1\right.\right.\right.}{x^7-x^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*(64*x^4*integrate(1/8*(12*(x^2 - 1)*log(x^2)^2*log(x)^2 - 16*(x^2 - 1)
*log(x^2)*log(x)^3 + 8*(x^2 - 1)*log(x)^4 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*
log(x^2)^3 - 12*(4*(x^2 - 1)*log(x)^2 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*log(
x^2))*arctan(sqrt(x + 1)*sqrt(x - 1))^2 + 2*(4*arctan(sqrt(x + 1)*sqrt(x -
1))^3 - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x^2)^2)*sqrt(x + 1)*sqrt(x -
1))/(x^7 - x^5), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))^4 + 24*arctan(sqrt
(x + 1)*sqrt(x - 1))^2*log(x^2)^2 - log(x^2)^4)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^4}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x)^4/x^5,x)
```

```
[Out] int(acos(1/x)^4/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^4(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)**4/x**5,x)
```

```
[Out] Integral(asec(x)**4/x**5, x)
```

3.651 $\int \sqrt{1-x^2} \sin^{-1}(x) \, dx$

Optimal. Leaf size=34

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}\, x \sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4647, 4641, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}\, x \sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x^2} \sin^{-1}(x) \, dx &= \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x)-\frac{\int x \, dx}{2}+\frac{1}{2}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx \\
&= -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2\sqrt{1-x^2}\, x \sin^{-1}(x)+\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-x^2}\,\sin^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]*ArcSin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.33, size = 26, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arcsin(x)-\frac{1}{4}\,x^2+\frac{1}{4}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

giac [A] time = 0.85, size = 27, normalized size = 0.79

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arcsin(x)-\frac{1}{4}\,x^2+\frac{1}{4}\,\arcsin(x)^2+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

maple [A] time = 0.31, size = 31, normalized size = 0.91

method	result	size
default	$\frac{\arcsin(x)\left(\sqrt{-x^2+1}\;x+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)*((-x^2+1)^(1/2)*x+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2
```

maxima [A] time = 0.97, size = 30, normalized size = 0.88

$$-\frac{1}{4}\,x^2+\frac{1}{2}\left(\sqrt{-x^2+1}\,x+\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{asin}(x)\,\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)*(1 - x^2)^(1/2),x)
```

```
[Out] int(asin(x)*(1 - x^2)^(1/2), x)
```

sympy [A] time = 24.12, size = 48, normalized size = 1.41

$$\left(\begin{cases} \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}\right)\operatorname{asin}(x) - \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\operatorname{asin}^2(x)}{4} - \frac{\pi^2}{16} - \frac{1}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*asin(x) - P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (n
an, True))
```

3.652 $\int \sqrt{1-x^2} \cos^{-1}(x)\,dx$

Optimal. Leaf size=34

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}\,x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4648, 4642, 30}

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}\,x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2} \cos^{-1}(x)\,dx &= \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x)+\frac{\int x\,dx}{2}+\frac{1}{2}\int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(x^2+2\sqrt{1-x^2}\,x\cos^{-1}(x)-\cos^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-x^2}\cos^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]*ArcCos[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.03, size = 26, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arccos(x)+\frac{1}{4}\,x^2-\frac{1}{4}\,\arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

giac [A] time = 1.12, size = 27, normalized size = 0.79

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arccos(x)+\frac{1}{4}\,x^2-\frac{1}{4}\,\arccos(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

maple [A] time = 0.32, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-\sqrt{-x^2+1}\,x+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arccos(x)*(-(-x^2+1)^(1/2)*x+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

maxima [A] time = 0.97, size = 30, normalized size = 0.88

$$\frac{1}{4}\,x^2+\frac{1}{2}\left(\sqrt{-x^2+1}\,x+\arcsin(x)\right)\arccos(x)+\frac{1}{4}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x)*(1 - x^2)^(1/2),x)
```

```
[Out] int(acos(x)*(1 - x^2)^(1/2), x)
```

sympy [A] time = 24.58, size = 48, normalized size = 1.41

$$\left(\begin{cases} \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}\right)\operatorname{acos}(x) + \begin{cases} \text{NaN} & \text{for}\ x < -1 \\ \frac{x^2}{4} + \frac{\operatorname{asin}^2(x)}{4} - \frac{\pi^2}{16} - \frac{1}{4} & \text{for}\ x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (n
an, True))
```

3.653 $\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4678}

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x],x]
[Out] -x/3 + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx &= -\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{1}{3}\int\left(1-x^2\right)\,dx\\ &= -\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.87

$$\frac{1}{9}\left(x^3-3\left(1-x^2\right)^{3/2}\cos^{-1}(x)-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
[Out] (-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sqrt[1 - x^2]*ArcCos[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 27, normalized size = 0.90

$$\frac{1}{9}x^3+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}\arccos(x)-\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

giac [A] time = 1.16, size = 22, normalized size = 0.73

$$\frac{1}{9}\,x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x) - \frac{1}{3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

maple [C] time = 0.32, size = 134, normalized size = 4.47

method	result
default	$-\frac{(i+3\arccos(x))\left(4ix^3-4\sqrt{-x^2+1}\,x^2-3ix+\sqrt{-x^2+1}\right)}{72}+\frac{(\arccos(x)+i)\left(ix-\sqrt{-x^2+1}\right)}{8}-\frac{(\arccos(x)-i)\left(ix+\sqrt{-x^2+1}\right)}{8}+\frac{(-i+3\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/72*(I+3*arccos(x))*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+(-x^2+1)^(1/2))+1
/8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-1/8*(arccos(x)-I)*(I*x+(-x^2+1)^(1/2)
)+1/72*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 22, normalized size = 0.73

$$\frac{1}{9}\,x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x) - \frac{1}{3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\mathrm{acos}(x)\,\sqrt{1-x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(x)*(1 - x^2)^(1/2),x)
[Out] int(x*acos(x)*(1 - x^2)^(1/2), x)
```

sympy [A] time = 1.06, size = 37, normalized size = 1.23

$$\frac{x^3}{9} + \frac{x^2\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{3} - \frac{x}{3} - \frac{\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)*(-x**2+1)**(1/2),x)
[Out] x**3/9 + x**2*sqrt(1 - x**2)*acos(x)/3 - x/3 - sqrt(1 - x**2)*acos(x)/3
```

3.654 $\int \left(1-x^2\right)^{3/2} \sin^{-1}(x)\,dx$

Optimal. Leaf size=59

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2} x\sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2}\,x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4649, 4647, 4641, 30, 14}

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2} x\sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2}\,x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 - x^2)^(3/2)
*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4649

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x
] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rubi steps

$$\int \left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx = \frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x) - \frac{1}{4}\int x\left(1-x^2\right)\,dx + \frac{3}{4}\int \sqrt{1-x^2}\,\sin^{-1}(x)\,dx$$
$$= \frac{3}{8}x\sqrt{1-x^2}\,\sin^{-1}(x) + \frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x) - \frac{1}{4}\int \left(x-x^3\right)\,dx - \frac{3\int x\,dx}{8} + \frac{3}{8}\int$$
$$= -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2}\,\sin^{-1}(x) + \frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x) + \frac{3}{16}\sin^{-1}(x)^2$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.71

$$\frac{1}{16}\left(x^4-5x^2-2\sqrt{1-x^2}\left(2x^2-5\right)x\sin^{-1}(x)+3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
[Out] (-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2)^(3/2)*ArcSin[x],x]
[Out] Could not integrate
```

fricas [A] time = 1.21, size = 39, normalized size = 0.66

$$\frac{1}{16}x^4 - \frac{1}{8}\left(2x^3-5x\right)\sqrt{-x^2+1}\arcsin(x) - \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
[Out] 1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2 + 3/16*arc
sin(x)^2
```

giac [A] time = 1.05, size = 50, normalized size = 0.85

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin(x)+\frac{3}{8}\sqrt{-x^2+1}\,x\arcsin(x)+\frac{1}{16}\left(x^2-1\right)^2-\frac{3}{16}x^2+\frac{3}{16}\arcsin(x)^2+\frac{9}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

maple [A] time = 0.37, size = 58, normalized size = 0.98

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3+5\sqrt{-x^2+1}\,x+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{\left(x^2-1\right)^2}{16}-\frac{3x^2}{16}+\frac{3}{16}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*(-x^2+1)^(1/2)*x+3*arcsin(x))-3/16*a
rcsin(x)^2+1/16*(x^2-1)^2-3/16*x^2+3/16
```

maxima [A] time = 1.02, size = 50, normalized size = 0.85

$$\frac{1}{16}x^4 - \frac{5}{16}x^2 + \frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x + 3\sqrt{-x^2+1}\,x + 3\,\arcsin(x)\right)\arcsin(x) - \frac{3}{16}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1)*x + 3*ar
csin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}(x)\left(1-x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)*(1 - x^2)^(3/2),x)
```

```
[Out] int(asin(x)*(1 - x^2)^(3/2), x)
```

sympy [A] time = 2.08, size = 53, normalized size = 0.90

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\operatorname{asin}(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\,\operatorname{asin}(x)}{8} + \frac{3\operatorname{asin}^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*as
in(x)/8 + 3*asin(x)**2/16
```

3.655 $\int x \left(1-x^2\right)^{3/2} \sin^{-1}(x)\, dx$

Optimal. Leaf size=37

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4677, 194}

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{1}{5}\int\left(1-x^2\right)^2\,dx\\
&= -\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{1}{5}\int\left(1-2x^2+x^4\right)dx\\
&= \frac{x}{5}-\frac{2x^3}{15}+\frac{x^5}{25}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.95

$$\frac{1}{5}\left(\frac{x^5}{5}-\frac{2x^3}{3}-\left(1-x^2\right)^{5/2}\sin^{-1}(x)+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*(1 - x^2)^(3/2)*ArcSin[x],x]
[Out] Could not integrate
```

fricas [A] time = 0.73, size = 37, normalized size = 1.00

$$\frac{1}{25}\,x^{5} - \frac{2}{15}\,x^{3} - \frac{1}{5}\left(x^{4} - 2\,x^{2} + 1\right)\sqrt{-x^{2} + 1}\,\arcsin\left(x\right) + \frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
[Out] 1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5*
x
```

giac [A] time = 0.86, size = 34, normalized size = 0.92

$$\frac{1}{25}\,x^{5} - \frac{1}{5}\left(x^{2} - 1\right)^{2}\sqrt{-x^{2} + 1}\,\arcsin\left(x\right) - \frac{2}{15}\,x^{3} + \frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
[Out] 1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x
```

maple [A] time = 0.35, size = 37, normalized size = 1.00

method	result	size
default	$-\frac{\left(x^{2}-1\right)^{2}\sqrt{-x^{2}+1}\,\arcsin\left(x\right)}{5}+\frac{\left(3x^{4}-10x^{2}+15\right)x}{75}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
[Out] -1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arcsin(x)+1/75*(3*x^4-10*x^2+15)*x
```

maxima [A] time = 0.98, size = 27, normalized size = 0.73

$$\frac{1}{25}\,x^{5} - \frac{1}{5}\left(-x^{2} + 1\right)^{\frac{5}{2}}\arcsin\left(x\right) - \frac{2}{15}\,x^{3} + \frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
[Out] 1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\mathrm{asin}(x)\,{\left(1-x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asin(x)*(1 - x^2)^(3/2),x)
[Out] int(x*asin(x)*(1 - x^2)^(3/2), x)
```

sympy [B] time = 3.45, size = 63, normalized size = 1.70

$$\frac{x^{5}}{25} - \frac{x^{4}\sqrt{1 - x^{2}}\,\operatorname{asin}{\left(x \right)}}{5} - \frac{2x^{3}}{15} + \frac{2x^{2}\sqrt{1 - x^{2}}\,\operatorname{asin}{\left(x \right)}}{5} + \frac{x}{5} - \frac{\sqrt{1 - x^{2}}\,\operatorname{asin}{\left(x \right)}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2)
*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5
```

3.656 $\int x^3 \left(1-x^2\right)^{3/2} \cos^{-1}(x) \, dx$

Optimal. Leaf size=61

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 43, 4690, 12, 373}

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \left(1-x^2\right)^{3/2} \cos^{-1}(x)\,dx &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\int \frac{1}{35}\left(-2-5x^2\right)\left(1-x^2\right)^2 d \\
&= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\frac{1}{35}\int \left(-2-5x^2\right)\left(1-x^2\right)^2 d \\
&= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\frac{1}{35}\int \left(-2-x^2+8x^4-5x^6\right) \\
&= -\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49}-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.77

$$
-\frac{1}{35}\left(5x^2+2\right)\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{x\left(75x^6-168x^4+35x^2+210\right)}{3675}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] -1/3675*(x*(210 + 35*x^2 - 168*x^4 + 75*x^6)) - ((1 - x^2)^(5/2)*(2 + 5*x^2
)*ArcCos[x])/35
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int x^3 \left(1-x^2\right)^{3/2} \cos^{-1}(x)\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.97, size = 47, normalized size = 0.77

$$
-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5x^6-8x^4+x^2+2\right)\sqrt{-x^2+1}\arccos(x)-\frac{2}{35}x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2)*sqrt(-x^
2 + 1)*arccos(x) - 2/35*x
```

giac [A] time = 1.18, size = 60, normalized size = 0.98

$$
-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1}+7\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos(x)-\frac{2}{35}x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="giac")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7*
(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

maple [C] time = 0.49, size = 286, normalized size = 4.69

method	result

default	$\frac{(i+7\arccos(x))\left(64ix^7-64\sqrt{-x^2+1}\,x^6-112ix^5+80\sqrt{-x^2+1}\,x^4+56ix^3-24\sqrt{-x^2+1}\,x^2-7ix+\sqrt{-x^2+1}\right)}{6272}+\frac{3(\arccos(x)+i)\left(ix-\sqrt{-x^2+1}\right)}{128}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(3/2)*arccos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6272*(I+7*arccos(x))*(64*I*x^7-64*(-x^2+1)^(1/2)*x^6-112*I*x^5+80*(-x^2+1
)^(1/2)*x^4+56*I*x^3-24*(-x^2+1)^(1/2)*x^2-7*I*x+(-x^2+1)^(1/2))+3/128*(arc
cos(x)+I)*(I*x-(-x^2+1)^(1/2))-3/128*(arccos(x)-I)*(I*x+(-x^2+1)^(1/2))+1/3
84*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))-3/3
9200*cos(6*arccos(x))*(2*I+35*arccos(x))*(I*x+(-x^2+1)^(1/2))+1/78400*sin(6
*arccos(x))*(37*I+35*arccos(x))*(-I*(-x^2+1)^(1/2)+x)-1/2400*cos(4*arccos(x
))*(7*I+15*arccos(x))*(I*x+(-x^2+1)^(1/2))+1/4800*sin(4*arccos(x))*(11*I+45
*arccos(x))*(-I*(-x^2+1)^(1/2)+x)
```

maxima [A] time = 0.96, size = 49, normalized size = 0.80

$$-\frac{1}{49}\,x^7+\frac{8}{175}\,x^5-\frac{1}{105}\,x^3-\frac{1}{35}\left(5\left(-x^2+1\right)^{\frac{5}{2}}x^2+2\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos(x)-\frac{2}{35}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2 + 2*(-x^2
+ 1)^(5/2))*arccos(x) - 2/35*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\mathrm{acos}(x)\left(1-x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(x)*(1 - x^2)^(3/2),x)
```

```
[Out] int(x^3*acos(x)*(1 - x^2)^(3/2), x)
```

sympy [A] time = 162.01, size = 88, normalized size = 1.44

$$-\frac{x^7}{49}-\frac{x^6\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{7}+\frac{8x^5}{175}+\frac{8x^4\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{35}-\frac{x^3}{105}-\frac{x^2\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{35}-\frac{2x}{35}-\frac{2\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

```
[Out] -x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x**
2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sqrt
(1 - x**2)*acos(x)/35
```

3.657 $\int \frac{(1-x^2)^{3/2}\cos^{-1}(x)}{x}\,dx$

Optimal. Leaf size=95

$$-i\,\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos$$

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4700, 4698, 4710, 4181, 2279, 2391, 8}

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/3 +
 (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x]
)] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
```

```
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-x^2\right)^{3/2}\cos^{-1}(x)}{x}\,dx &= \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\frac{1}{3}\int\left(1-x^2\right)dx+\int\frac{\sqrt{1-x^2}\,\cos^{-1}(x)}{x}\,dx \\
&= \frac{x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\int 1\,dx+\int\frac{\cos^{-1}(x)}{x\sqrt{1-x^2}}\,dx \\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\text{Subst}\left(\int x\sec(x)\,dx,x,\cos\right. \\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right) \\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right) \\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)
\end{aligned}$$

Mathematica [A] time = 0.28, size = 119, normalized size = 1.25

$$-i\,\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)+\sqrt{1-x^2}\,\cos^{-1}(x)+\frac{1}{36}\left(12\left(1-x^2\right)^{3/2}\cos^{-1}(x)+9x-\cos\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x] - Cos[3*A
rcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] + ArcCos[x]*Log[1 + I*
E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I
*ArcCos[x])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^2\right)^{3/2}\cos^{-1}(x)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(x^2-1\right)\sqrt{-x^2+1}\,\arccos(x)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
[Out] integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^2+1\right)^{\frac{3}{2}} \arccos(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="giac")
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

maple [B] time = 0.44, size = 230, normalized size = 2.42

method	result
default	$\frac{(i+3\arccos(x))\left(4ix^3-4\sqrt{-x^2+1}\,x^2-3ix+\sqrt{-x^2+1}\right)}{72}-\frac{5(\arccos(x)+i)\left(ix-\sqrt{-x^2+1}\right)}{8}+\frac{5(\arccos(x)-i)\left(ix+\sqrt{-x^2+1}\right)}{8}-\frac{(-i+3\,\mathrm{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arccos(x)/x,x,method=_RETURNVERBOSE)
[Out] 1/72*(I+3*arccos(x))*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+(-x^2+1)^(1/2))-5/
8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))+5/8*(arccos(x)-I)*(I*x+(-x^2+1)^(1/2))
-1/72*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))-
I*(I*arccos(x)*ln(1+I*(x+I*(-x^2+1)^(1/2)))-I*arccos(x)*ln(1-I*(x+I*(-x^2+1
)^(1/2)))+dilog(1+I*(x+I*(-x^2+1)^(1/2)))-dilog(1-I*(x+I*(-x^2+1)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^2+1\right)^{\frac{3}{2}} \arccos(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}(x)\left(1-x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acos(x)*(1 - x^2)^(3/2))/x,x)
[Out] int((acos(x)*(1 - x^2)^(3/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*acos(x)/x,x)
[Out] Timed out
```

3.658 $\int \frac{(1-x^2)^{3/2}\sin^{-1}(x)}{x^6}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4681, 266, 43}

$$\frac{1}{5x^2}-\frac{1}{20x^4}-\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

```
[Out] -1/(20*x^4) + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + Log[x]/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4681

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcSin[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^2)^{3/2}\sin^{-1}(x)}{x^6}\,dx &= -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{1}{5}\int \frac{(1-x^2)^2}{x^5}\,dx\\
&= -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{1}{10}\operatorname{Subst}\left(\int \frac{(1-x)^2}{x^3}\,dx,x,x^2\right)\\
&= -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{1}{10}\operatorname{Subst}\left(\int \left(\frac{1}{x^3}-\frac{2}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\
&= -\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}
\end{aligned}$$

<u>**Mathematica**</u> [A] time = 0.04, size = 36, normalized size = 0.88

$$-\frac{-4x^5\log(x)-4x^3+4\left(1-x^2\right)^{5/2}\sin^{-1}(x)+x}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
[Out] -1/20*(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/x^5
```

<u>**IntegrateAlgebraic**</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^2\right)^{3/2}\sin^{-1}(x)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
[Out] Could not integrate
```

<u>**fricas**</u> [A] time = 1.09, size = 44, normalized size = 1.07

$$\frac{4\,x^5\log(x)+4\,x^3-4\left(x^4-2\,x^2+1\right)\sqrt{-x^2+1}\,\arcsin(x)-x}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
[Out] 1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) -
 x)/x^5
```

<u>**giac**</u> [B] time = 0.99, size = 135, normalized size = 3.29

$$-\frac{1}{160}\left(\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-\frac{10\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{\left(\sqrt{-x^2+1}-1\right)^5}+\frac{10\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{5\left(\sqrt{-x^2+1}-1\right)^3}{x^3}+\frac{\left(\sqrt{-x^2+1}-1\right)^5}{x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
[Out] -1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4 -
 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1)
- 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2 +
1)/x^4 + 1/10*log(x^2)
```

<u>**maple**</u> [C] time = 1.00, size = 201, normalized size = 4.90

method	result
default	$-\frac{2i\arcsin(x)}{5}+\frac{\left(-\sqrt{-x^2+1}\,x^4+ix^5+2\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\right)\left(20\arcsin(x)x^8-4ix^8-4\sqrt{-x^2+1}\,x^7-40\arcsin(x)x^6+ix^6+9\sqrt{-x^2+1}\right.}{20\left(5x^8-10x^6+10x^4-5x^2+1\right)x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arcsin(x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*arcsin(x)+1/20*(-(-x^2+1)^(1/2)*x^4+I*x^5+2*(-x^2+1)^(1/2)*x^2-(-x^2
+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x^7-40*arcsin(x)*x^6+
I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6*(-x^2+1)^(1/2)*x^3-20*arcsin(
x)*x^2+(-x^2+1)^(1/2)*x+4*arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+1/5*
ln((I*x+(-x^2+1)^(1/2))^2-1)
```

maxima [A] time = 0.97, size = 35, normalized size = 0.85

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}\arcsin(x)}{5\,x^5}+\frac{4\,x^2-1}{20\,x^4}+\frac{1}{10}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10*log(x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asin}(x)\left(1-x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((asin(x)*(1 - x^2)^(3/2))/x^6,x)
```

```
[Out] int((asin(x)*(1 - x^2)^(3/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)(x+1)\right)^{\frac{3}{2}}\operatorname{asin}(x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

```
[Out] Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

3.659 $\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=34

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}\,x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4707, 4641, 30}

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}\,x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x) + \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{1}{4}\left(x^2 - 2\sqrt{1-x^2}\,x\sin^{-1}(x) + \sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
[Out] Could not integrate
```

fricas [A] time = 0.65, size = 26, normalized size = 0.76

$$-\frac{1}{2}\sqrt{-x^2+1}\,x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

giac [A] time = 1.01, size = 27, normalized size = 0.79

$$-\frac{1}{2}\sqrt{-x^2+1}\,x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

maple [A] time = 0.32, size = 32, normalized size = 0.94

method	result	size
default	$\frac{\arcsin(x)\left(-\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)*(-(-x^2+1)^(1/2)*x+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2
```

maxima [A] time = 0.97, size = 32, normalized size = 0.94

$$\frac{1}{4}x^2-\frac{1}{2}\left(\sqrt{-x^2+1}\,x-\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{asin}(x)}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asin(x))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x^2*asin(x))/(1 - x^2)^(1/2), x)
```

sympy [A] time = 0.37, size = 26, normalized size = 0.76

$$\frac{x^2}{4} - \frac{x\sqrt{1-x^2}\,\operatorname{asin}(x)}{2} + \frac{\operatorname{asin}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4
```

3.660 $\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=61

$$\frac{x^4}{16}+\frac{3x^2}{16}-\frac{3}{8}\sqrt{1-x^2}\,x\sin^{-1}(x)-\frac{1}{4}\sqrt{1-x^2}\,x^3\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4707, 4641, 30}

$$\frac{x^4}{16}+\frac{3x^2}{16}-\frac{1}{4}\sqrt{1-x^2}\,x^3\sin^{-1}(x)-\frac{3}{8}\sqrt{1-x^2}\,x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt[1 - x^2]*
ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx &= -\frac{1}{4}x^3\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{\int x^3 \, dx}{4}+\frac{3}{4}\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\
&= \frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\,\sin^{-1}(x)-\frac{1}{4}x^3\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{3\int x \, dx}{8}+\frac{3}{8}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\
&= \frac{3x^2}{16}+\frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\,\sin^{-1}(x)-\frac{1}{4}x^3\sqrt{1-x^2}\,\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.70

$$\frac{1}{16}\left(\left(x^2+3\right)x^2-2\sqrt{1-x^2}\,\left(2x^2+3\right)x\sin^{-1}(x)+3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 39, normalized size = 0.64

$$\frac{1}{16}\,x^4 - \frac{1}{8}\left(2\,x^3 + 3\,x\right)\sqrt{-x^2+1}\,\arcsin(x) + \frac{3}{16}\,x^2 + \frac{3}{16}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2 + 3/16*arc
sin(x)^2
```

giac [A] time = 1.41, size = 50, normalized size = 0.82

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}} x \arcsin(x) - \frac{5}{8}\,\sqrt{-x^2+1}\,x \arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}\,x^2 + \frac{3}{16}\,\arcsin(x)^2 - \frac{23}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

maple [A] time = 0.38, size = 53, normalized size = 0.87

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3-3\sqrt{-x^2+1}\,x+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{x^4}{16}+\frac{3x^2}{16}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*(-x^2+1)^(1/2)*x+3*arcsin(x))-3/16*a
rcsin(x)^2+1/16*x^4+3/16*x^2
```

maxima [A] time = 0.97, size = 52, normalized size = 0.85

$$\frac{1}{16}\,x^4 + \frac{3}{16}\,x^2 - \frac{1}{8}\left(2\,\sqrt{-x^2+1}\,x^3 + 3\,\sqrt{-x^2+1}\,x - 3\,\arcsin(x)\right)\arcsin(x) - \frac{3}{16}\,\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

[Out] 1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1)*x - 3*arcsin(x))*arcsin(x) - 3/16*arcsin(x)^2

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4 \operatorname{asin}(x)}{\sqrt{1-x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asin(x))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x^4*asin(x))/(1 - x^2)^(1/2), x)
```

sympy [A] time = 1.32, size = 53, normalized size = 0.87

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\operatorname{asin}(x)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\,\operatorname{asin}(x)}{8} + \frac{3\operatorname{asin}^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(1 - x**2)*asin(x)/8 + 3*asin(x)**2/16
```

3.661 $\int \frac{x\sin^{-1}(x)}{(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4677, 206}

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin^{-1}(x)}{(1-x^2)^{3/2}}\,dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\int \frac{1}{1-x^2}\,dx\\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin^{-1}(x)}{(1-x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.14, size = 44, normalized size = 2.32

$$-\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsin
(x))/(x^2 - 1)
```

giac [A] time = 1.37, size = 27, normalized size = 1.42

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}}-\frac{1}{2}\log(|x+1|)+\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

maple [B] time = 0.32, size = 46, normalized size = 2.42

method	result	size
default	$-\frac{\sqrt{-x^2+1}\arcsin(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+\frac{x}{\sqrt{-x^2+1}}\right)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+x/(-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 25, normalized size = 1.32

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}}-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{x\,\mathrm{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asin(x))/(1 - x^2)^(3/2),x)
```

```
[Out] int((x*asin(x))/(1 - x^2)^(3/2), x)
```

sympy [A] time = 11.29, size = 20, normalized size = 1.05

$$-\begin{cases} \operatorname{acoth}(x) & \text{for } x^2 > 1 \\ \operatorname{atanh}(x) & \text{for } x^2 < 1 \end{cases} + \frac{\operatorname{asin}(x)}{\sqrt{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + asin(x)/sqrt(1 - x
**2)
```

3.662 $\int \frac{x\cos^{-1}(x)}{(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=17

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4678, 206}

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos^{-1}(x)}{(1-x^2)^{3/2}}\,dx &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\int \frac{1}{1-x^2}\,dx\\ &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 1.88

$$\frac{1}{2}\left(\frac{2\cos^{-1}(x)}{\sqrt{1-x^2}}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
[Out] ((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos^{-1}(x)}{(1-x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.19, size = 44, normalized size = 2.59

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\sqrt{-x^2+1}\arccos(x)}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos(
x))/(x^2 - 1)
```

giac [A] time = 1.30, size = 27, normalized size = 1.59

$$\frac{\arccos(x)}{\sqrt{-x^2+1}}+\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [B] time = 0.31, size = 47, normalized size = 2.76

method	result	size
default	$-\frac{\sqrt{-x^2+1}\arccos(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}-\frac{x}{\sqrt{-x^2+1}}\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(1/(-x^2+1)^(1/2)-x/(-x^2+1)^(1/2))
```

maxima [A] time = 0.99, size = 25, normalized size = 1.47

$$\frac{\arccos(x)}{\sqrt{-x^2+1}}+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{x\,\mathrm{acos}(x)}{\left(1-x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acos(x))/(1 - x^2)^(3/2),x)
```

```
[Out] int((x*acos(x))/(1 - x^2)^(3/2), x)
```

sympy [A] time = 11.91, size = 20, normalized size = 1.18

$$\begin{cases} \operatorname{acoth}(x) & \text{for } x^2 > 1 \\ \operatorname{atanh}(x) & \text{for } x^2 < 1 \end{cases} + \frac{\operatorname{acos}(x)}{\sqrt{1 - x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)/(-x**2+1)**(3/2),x)
```

```
[Out] Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + acos(x)/sqrt(1 - x*
*2)
```

3.663 $\int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{6(1-x^2)}+\frac{1}{3}\log(1-x^2)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3(1-x^2)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4655, 4651, 260, 261}

$$-\frac{1}{6(1-x^2)}+\frac{1}{3}\log(1-x^2)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] -1/(6*(1 - x^2)) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSin[x])/(3*S
qrt[1 - x^2]) + Log[1 - x^2]/3
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4651

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(x*(a + b*ArcSin[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist[(
b*c*n)/Sqrt[d], Int[(x*(a + b*ArcSin[c*x])^(n - 1))/(d + e*x^2), x], x] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[d, 0]
```

Rule 4655

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*d*(p + 1)
), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcSi
n[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*(p
 + 1)*(1 - c^2*x^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcS
in[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\sin^{-1}(x)}{\left(1-x^2\right)^{5/2}}\,dx = \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} - \frac{1}{3}\int \frac{x}{\left(1-x^2\right)^2}\,dx + \frac{2}{3}\int \frac{\sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx$$
$$= -\frac{1}{6\left(1-x^2\right)} + \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} + \frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}} - \frac{2}{3}\int \frac{x}{1-x^2}\,dx$$
$$= -\frac{1}{6\left(1-x^2\right)} + \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} + \frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}} + \frac{1}{3}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.73

$$\frac{1}{6}\left(\frac{1}{x^2-1} + 2\log\left(1-x^2\right) - \frac{2x\left(2x^2-3\right)\sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] ((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1 -
 x^2])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(x)}{\left(1-x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.16, size = 61, normalized size = 0.98

$$-\frac{2\left(2x^3-3x\right)\sqrt{-x^2+1}\,\arcsin(x) - x^2 - 2\left(x^4-2x^2+1\right)\log\left(x^2-1\right) + 1}{6\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2*x^2 + 1)*
log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

giac [A] time = 1.38, size = 54, normalized size = 0.87

$$-\frac{\left(2x^2-3\right)\sqrt{-x^2+1}\,x\arcsin(x)}{3\left(x^2-1\right)^2} - \frac{2x^2-3}{6\left(x^2-1\right)} + \frac{1}{3}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/(
x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

maple [A] time = 0.32, size = 63, normalized size = 1.02

method	result	size
default	$\frac{1}{6x^2-6}+\frac{x \arcsin(x)\sqrt{-x^2+1}}{3(x^2-1)^2}+\frac{\ln(-x^2+1)}{3}-\frac{2\sqrt{-x^2+1}\ \arcsin(x)x}{3(x^2-1)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(x^2-1)+1/3*x*arcsin(x)*(-x^2+1)^(1/2)/(x^2-1)^2+1/3*ln(-x^2+1)-2/3*(-x
^2+1)^(1/2)/(x^2-1)*arcsin(x)*x
```

maxima [A] time = 0.96, size = 48, normalized size = 0.77

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-x^2+1}}+\frac{x}{(-x^2+1)^{\frac{3}{2}}}\right)\arcsin(x)+\frac{1}{6(x^2-1)}+\frac{1}{3}\log\left(-3x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^2 - 1) + 1
/3*log(-3*x^2 + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asin}(x)}{(1-x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(1 - x^2)^(5/2),x)
```

```
[Out] int(asin(x)/(1 - x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(-x**2+1)**(5/2),x)
```

```
[Out] Timed out
```

3.664 $\int \frac{x^3 \sin^{-1}(x)}{(1-x^2)^{3/2}} \, dx$

Optimal. Leaf size=36

$$\sqrt{1-x^2} \sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 43, 4689, 388, 206}

$$\sqrt{1-x^2} \sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4689

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcSin[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3 \sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\,\sin^{-1}(x) - \int \frac{2-x^2}{1-x^2}\,dx$$
$$= -x + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\,\sin^{-1}(x) - \int \frac{1}{1-x^2}\,dx$$
$$= -x + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\,\sin^{-1}(x) - \tanh^{-1}(x)$$

Mathematica [A] time = 0.07, size = 40, normalized size = 1.11

$$\frac{1}{2}\left(-\frac{2\left(x^2-2\right)\sin^{-1}(x)}{\sqrt{1-x^2}} - 2x + \log(1-x) - \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
[Out] (-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
[Out] Could not integrate
```

fricas [A] time = 1.10, size = 57, normalized size = 1.58

$$-\frac{2\,x^3 - 2\left(x^2-2\right)\sqrt{-x^2+1}\,\arcsin(x) + \left(x^2-1\right)\log\left(x+1\right) - \left(x^2-1\right)\log\left(x-1\right) - 2\,x}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*log(x + 1) -
 (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

giac [A] time = 1.35, size = 40, normalized size = 1.11

$$\left(\sqrt{-x^2+1} + \frac{1}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\,\log\left(|x+1|\right) + \frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] (sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(abs(x + 1)) + 1
/2*log(abs(x - 1))
```

maple [A] time = 0.53, size = 61, normalized size = 1.69

method	result	size
default	$-x+\arcsin(x)\sqrt{-x^2+1}-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+\frac{x}{\sqrt{-x^2+1}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x+arcsin(x)*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^
(1/2)+x/(-x^2+1)^(1/2))
```

maxima [A] time = 0.97, size = 45, normalized size = 1.25

$$-\left(\frac{x^2}{\sqrt{-x^2+1}}-\frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x)-x-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(x + 1) + 1
/2*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\mathrm{asin}(x)}{\left(1-x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asin(x))/(1 - x^2)^(3/2),x)
```

```
[Out] int((x^3*asin(x))/(1 - x^2)^(3/2), x)
```

sympy [A] time = 18.48, size = 37, normalized size = 1.03

$$-x-\left(-\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)\mathrm{asin}(x)+\frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -x - (-sqrt(1 - x**2) - 1/sqrt(1 - x**2))*asin(x) + log(x - 1)/2 - log(x +
1)/2
```

3.665 $\int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$i\,\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\,\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4705, 4709, 4183, 2279, 2391, 206}

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x]
 + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4705

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSin[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1)),
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Dist[(b*c*n
*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 - c^2*x^2)^FracPart[
p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^{-1}(x)}{x\left(1-x^2\right)^{3/2}}\,dx = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2}\,dx + \int \frac{\sin^{-1}(x)}{x\sqrt{1-x^2}}\,dx$$

$$= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x) + \text{Subst}\left(\int x\csc(x)\,dx, x, \sin^{-1}(x)\right)$$

$$= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right) - \tanh^{-1}(x) - \text{Subst}\Big(\int \log\left(1-e^{ix}\right)\,dx, x, \sin^{-1}(x$$

$$= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right) - \tanh^{-1}(x) + i\,\text{Subst}\Big(\int \frac{\log(1-x)}{x}\,dx, x, e^{i\sin^{-1}(x)}$$

$$= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right) - \tanh^{-1}(x) + i\text{Li}_2\left(-e^{i\sin^{-1}(x)}\right) - i\text{Li}_2\left(e^{i\sin^{-1}(x)}\right)$$

Mathematica [A] time = 0.19, size = 112, normalized size = 1.81

$$i\,\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sin^{-1}(x)\log\left(1-e^{i\sin^{-1}(x)}\right) - \sin^{-1}(x)\log\Big(1+e^{i\sin^{-1}(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*Lo
g[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[Cos
[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*Poly
Log[2, E^(I*ArcSin[x])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(x)}{x\left(1-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^5-2\,x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{\left(-x^2+1\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.64, size = 97, normalized size = 1.56

method	result
default	$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1} + 2i\arctan\left(ix+\sqrt{-x^2+1}\right) + i\,\mathrm{dilog}\left(ix+\sqrt{-x^2+1}+1\right) - \arcsin(x)\ln\left(ix+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+2*I*arctan(I*x+(-x^2+1)^(1/2))+I*dilog(I*
x+(-x^2+1)^(1/2)+1)-arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)+I*dilog(I*x+(-x^2+1)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{\left(-x^2+1\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asin}(x)}{x\left(1-x^2\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(x*(1 - x^2)^(3/2)),x)
```

```
[Out] int(asin(x)/(x*(1 - x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}{\left(x \right)}}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

3.666 $\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4702, 4682, 29, 30}

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - (2*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{1}{3}\int \frac{1}{x^3}\,dx + \frac{2}{3}\int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x} - \frac{2}{3}\int \frac{1}{x}\,dx$$
$$= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x} - \frac{2\log(x)}{3}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.70

$$\frac{-4x^3\log(x) - 2\sqrt{1-x^2}\left(2x^2+1\right)\cos^{-1}(x) + x}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
[Out] (x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
[Out] Could not integrate
```

fricas [A] time = 1.08, size = 36, normalized size = 0.67

$$-\frac{4\,x^3\log(x) + 2\left(2\,x^2+1\right)\sqrt{-x^2+1}\,\arccos(x) - x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/x^3
```

giac [B] time = 1.01, size = 95, normalized size = 1.76

$$\frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) + \frac{2\,x^2+1}{6\,x^2} - \frac{1}{3}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(sq
rt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2 +
1)/x^2 - 1/3*log(x^2)
```

maple [A] time = 0.31, size = 43, normalized size = 0.80

method	result	size
default	$\frac{1}{6x^2}-\frac{2\ln(x)}{3}-\frac{\arccos(x)\sqrt{-x^2+1}}{3x^3}-\frac{2\arccos(x)\sqrt{-x^2+1}}{3x}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^(
1/2)/x
```

maxima [A] time = 0.95, size = 42, normalized size = 0.78

$$-\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x}+\frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x)+\frac{1}{6x^2}-\frac{2}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^2 - 2/3*lo
g(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}(x)}{x^4\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x)/(x^4*(1 - x^2)^(1/2)),x)
```

```
[Out] int(acos(x)/(x^4*(1 - x^2)^(1/2)), x)
```

sympy [A] time = 139.93, size = 60, normalized size = 1.11

$$\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1 \end{cases}\right)\mathrm{acos}(x)+\begin{cases} \mathrm{NaN} & \text{for}\ x<-1 \\ -\frac{2\log(x)}{3}-\frac{1}{6}+\frac{2i\pi}{3}+\frac{1}{6x^2} & \text{for}\ x<1 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1)))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 - 1/6 + 2*I*pi/3 + 1/(
6*x**2), x < 1), (nan, True))
```

3.667 $\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx$

Optimal. Leaf size=66

$$\frac{2}{9}x^3\cos^{-1}(x)+\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4678, 4646, 444, 43}

$$\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}+\frac{2}{9}x^3\cos^{-1}(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3 + (2*x^3*A
rcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4646

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[c*x], u, x] +
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx = -\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{2}{3}\int\left(1-x^2\right)\cos^{-1}(x)\,dx$$
$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{2}{3}\int\frac{x\left(1-\frac{x^2}{3}\right)}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int\frac{1-\frac{x}{3}}{\sqrt{1-x}}\,dx, x, x$$
$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1-x}} + \frac{\sqrt{}}{}\right.\right.$$
$$= \frac{4\sqrt{1-x^2}}{9} + \frac{2}{27}\left(1-x^2\right)^{3/2} - \frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.76

$$\frac{1}{27}\left(-2\sqrt{1-x^2}\left(x^2-7\right) - 9\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 + 6x\left(x^2-3\right)\cos^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2)
*ArcCos[x]^2)/27
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.15, size = 41, normalized size = 0.62

$$\frac{2}{9}\left(x^3-3x\right)\arccos(x) + \frac{1}{27}\left(9\left(x^2-1\right)\arccos(x)^2 - 2x^2 + 14\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2 + 14)*sqr
t(-x^2 + 1)
```

giac [A] time = 1.05, size = 53, normalized size = 0.80

$$\frac{2}{9}x^3\arccos(x) - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}\,x^2 - \frac{2}{3}x\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*
x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

maple [C] time = 0.32, size = 158, normalized size = 2.39

method	result
default	$-\frac{(6i\arccos(x)+9\arccos(x)^2-2)\left(4ix^3-4\sqrt{-x^2+1}\,x^2-3ix+\sqrt{-x^2+1}\right)}{216}+\frac{(\arccos(x)^2-2+2i\arccos(x))\left(ix-\sqrt{-x^2+1}\right)}{8}-\frac{(\arccos(x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(6*I*arccos(x)+9*arccos(x)^2-2)*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+
(-x^2+1)^(1/2))+1/8*(arccos(x)^2-2+2*I*arccos(x))*(I*x-(-x^2+1)^(1/2))-1/8*
(arccos(x)^2-2-2*I*arccos(x))*(I*x+(-x^2+1)^(1/2))+1/216*(-6*I*arccos(x)+9*
arccos(x)^2-2)*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 52, normalized size = 0.79

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2-\frac{2}{27}\sqrt{-x^2+1}\,x^2+\frac{2}{9}\left(x^3-3\,x\right)\arccos(x)+\frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9*(x^3 - 3*
x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{acos}(x)^2\,\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(x)^2*(1 - x^2)^(1/2),x)
```

```
[Out] int(x*acos(x)^2*(1 - x^2)^(1/2), x)
```

sympy [A] time = 3.04, size = 78, normalized size = 1.18

$$\frac{2x^3\,\mathrm{acos}\,(x)}{9}+\frac{x^2\sqrt{1-x^2}\,\mathrm{acos}^2\,(x)}{3}-\frac{2x^2\sqrt{1-x^2}}{27}-\frac{2x\,\mathrm{acos}\,(x)}{3}-\frac{\sqrt{1-x^2}\,\mathrm{acos}^2\,(x)}{3}+\frac{14\sqrt{1-x^2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

```
[Out] 2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2)
/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27
```

3.668 $\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=73

$$-\frac{3x^2}{8} - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{3}{4}x^2\sin^{-1}(x)^2 + \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{8}\sin^{-1}(x)^2$$

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4707, 4641, 4627, 30}

$$-\frac{3x^2}{8} - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{3}{4}x^2\sin^{-1}(x)^2 + \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{8}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8 + (3*x^2*A
rcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSin[x]^4/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x)^3 + \frac{1}{2}\int \frac{\sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx + \frac{3}{2}\int x\sin^{-1}(x)^2\,dx$$
$$= \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x)^3 + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{2}\int \frac{x^2\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$= \frac{3}{4}x\sqrt{1-x^2}\,\sin^{-1}(x) + \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x)^3 + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3\int x\,dx}{4} -$$
$$= -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\,\sin^{-1}(x) - \frac{3}{8}\sin^{-1}(x)^2 + \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\,\sin^{-1}(x)^3 + \frac{1}{8}\mathrm{s}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.82

$$\frac{1}{8}\left(-3x^2 - 4x\sqrt{1-x^2}\,\sin^{-1}(x)^3 + \left(6x^2-3\right)\sin^{-1}(x)^2 + 6x\sqrt{1-x^2}\,\sin^{-1}(x) + \sin^{-1}(x)^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqrt
[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.95, size = 49, normalized size = 0.67

$$\frac{1}{8}\arcsin(x)^4 + \frac{3}{8}\left(2x^2-1\right)\arcsin(x)^2 - \frac{3}{8}x^2 - \frac{1}{4}\left(2x\arcsin(x)^3 - 3x\arcsin(x)\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(x
)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

giac [A] time = 0.97, size = 60, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2+1}\,x\arcsin(x)^3 + \frac{1}{8}\arcsin(x)^4 + \frac{3}{4}\left(x^2-1\right)\arcsin(x)^2 + \frac{3}{4}\sqrt{-x^2+1}\,x\arcsin(x) - \frac{3}{8}x^2 + \frac{3}{8}\arcsin(x)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin(
x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16
```

maple [A] time = 0.32, size = 69, normalized size = 0.95

method	result
default	$\frac{\arcsin(x)^3\left(-\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{2}+\frac{3\arcsin(x)^2\left(x^2-1\right)}{4}+\frac{3\arcsin(x)\left(\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{4}-\frac{3\arcsin(x)^2}{8}-\frac{3x^2}{8}-\frac{3\arcsin}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x)^3*(-(-x^2+1)^(1/2)*x+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*a
rcsin(x)*((-x^2+1)^(1/2)*x+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(x)
^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{asin}(x)}^3}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asin(x)^3)/(1 - x^2)^(1/2),x)
```

```
[Out] int((x^2*asin(x)^3)/(1 - x^2)^(1/2), x)
```

sympy [A] time = 1.32, size = 66, normalized size = 0.90

$$\frac{3x^2\operatorname{asin}^2(x)}{4}-\frac{3x^2}{8}-\frac{x\sqrt{1-x^2}\operatorname{asin}^3(x)}{2}+\frac{3x\sqrt{1-x^2}\operatorname{asin}(x)}{4}+\frac{\operatorname{asin}^4(x)}{8}-\frac{3\operatorname{asin}^2(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(1
 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

3.669 $\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=32

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 199, 203}

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} \, dx &= -\frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{(1+x^2)^2} \, dx \\
&= \frac{x}{4(1+x^2)} - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4}\int \frac{1}{1+x^2} \, dx \\
&= \frac{x}{4(1+x^2)} + \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.66

$$\frac{(x^2-1)\tan^{-1}(x) + x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\tan^{-1}(x)}{(1+x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.10, size = 19, normalized size = 0.59

$$\frac{(x^2-1)\arctan(x)+x}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

giac [A] time = 1.06, size = 26, normalized size = 0.81

$$\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{2(x^2+1)}+\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

maple [A] time = 0.36, size = 27, normalized size = 0.84

method	result	size
default	$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$	27
risch	$\frac{i\ln(ix+1)}{4x^2+4}-\frac{i(2\ln(-ix+1)+\ln(x-i)x^2+\ln(x-i)-\ln(x+i)x^2-\ln(x+i)+2ix)}{8(x+i)(x-i)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

maxima [A] time = 0.96, size = 26, normalized size = 0.81

$$\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{2(x^2+1)}+\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

mupad [B] time = 0.08, size = 21, normalized size = 0.66

$$\frac{\operatorname{atan}(x)}{4}+\frac{\frac{x}{4}-\frac{\operatorname{atan}(x)}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x))/(x^2 + 1)^2,x)
[Out] atan(x)/4 + (x/4 - atan(x)/2)/(x^2 + 1)
```

sympy [A] time = 0.64, size = 31, normalized size = 0.97

$$\frac{x^2\operatorname{atan}(x)}{4x^2+4}+\frac{x}{4x^2+4}-\frac{\operatorname{atan}(x)}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(x**2+1)**2,x)
[Out] x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

3.670 $\int \frac{x\tan^{-1}(x)}{(1+x^2)^3}\,dx$

Optimal. Leaf size=44

$$\frac{3x}{32(x^2+1)}+\frac{x}{16(x^2+1)^2}-\frac{\tan^{-1}(x)}{4(x^2+1)^2}+\frac{3}{32}\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 199, 203}

$$\frac{3x}{32(x^2+1)}+\frac{x}{16(x^2+1)^2}-\frac{\tan^{-1}(x)}{4(x^2+1)^2}+\frac{3}{32}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - ArcTan[x]/(4
*(1 + x^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tan^{-1}(x)}{(1+x^2)^3}\,dx &= -\frac{\tan^{-1}(x)}{4(1+x^2)^2}+\frac{1}{4}\int \frac{1}{(1+x^2)^3}\,dx\\
&= \frac{x}{16(1+x^2)^2}-\frac{\tan^{-1}(x)}{4(1+x^2)^2}+\frac{3}{16}\int \frac{1}{(1+x^2)^2}\,dx\\
&= \frac{x}{16(1+x^2)^2}+\frac{3x}{32(1+x^2)}-\frac{\tan^{-1}(x)}{4(1+x^2)^2}+\frac{3}{32}\int \frac{1}{1+x^2}\,dx\\
&= \frac{x}{16(1+x^2)^2}+\frac{3x}{32(1+x^2)}+\frac{3}{32}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4(1+x^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.82

$$\frac{x\left(3x^2+5\right)+\left(3x^4+6x^2-5\right)\tan^{-1}(x)}{32\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]
[Out] (x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\tan^{-1}(x)}{\left(1+x^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x*ArcTan[x])/(1 + x^2)^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.09, size = 38, normalized size = 0.86

$$\frac{3\,x^3+\left(3\,x^4+6\,x^2-5\right)\arctan(x)+5\,x}{32\,\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.89, size = 34, normalized size = 0.77

$$\frac{3\,x^3+5\,x}{32\,\left(x^2+1\right)^2}-\frac{\arctan(x)}{4\,\left(x^2+1\right)^2}+\frac{3}{32}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="giac")
[Out] 1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x)
```

maple [A] time = 0.36, size = 37, normalized size = 0.84

method	result	size
default	$\frac{x}{16(x^2+1)^2}+\frac{3x}{32(x^2+1)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(x^2+1)^2}$	37
risch	$\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{i\left(8\ln(-ix+1)+3\ln(x-i)x^4+6\ln(x-i)x^2+3\ln(x-i)-3\ln(x+i)x^4-6\ln(x+i)x^2-3\ln(x+i)+6ix^3+10ix\right)}{64(x+i)^2(x-i)^2}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

maxima [A] time = 0.96, size = 39, normalized size = 0.89

$$\frac{3\,x^3+5\,x}{32\,\left(x^4+2\,x^2+1\right)}-\frac{\arctan(x)}{4\,\left(x^2+1\right)^2}+\frac{3}{32}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arc
tan(x)
```

mupad [B] time = 0.32, size = 26, normalized size = 0.59

$$\frac{3\,\mathrm{atan}(x)}{32}+\frac{\frac{5\,x}{32}-\frac{\mathrm{atan}(x)}{4}+\frac{3\,x^3}{32}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x))/(x^2 + 1)^3,x)
```

```
[Out] (3*atan(x))/32 + ((5*x)/32 - atan(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

sympy [B] time = 1.07, size = 88, normalized size = 2.00

$$\frac{3x^4\,\mathrm{atan}\,(x)}{32x^4+64x^2+32}+\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\,\mathrm{atan}\,(x)}{32x^4+64x^2+32}+\frac{5x}{32x^4+64x^2+32}-\frac{5\,\mathrm{atan}\,(x)}{32x^4+64x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(x**2+1)**3,x)
```

```
[Out] 3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32) +
 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) - 5
*atan(x)/(32*x**4 + 64*x**2 + 32)
```

3.671 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4916, 4846, 260, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tan^{-1}(x)}{1+x^2}\,dx &= \int \tan^{-1}(x)\,dx-\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= x\tan^{-1}(x)-\frac{1}{2}\tan^{-1}(x)^2-\int \frac{x}{1+x^2}\,dx\\
&= x\tan^{-1}(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] Could not integrate
```

fricas [A] time = 1.22, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

giac [A] time = 0.90, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

maple [A] time = 0.29, size = 20, normalized size = 0.87

method	result	size
default	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8} + \frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2} + \frac{\ln(-ix+1)^2}{8} + \frac{i\ln(-ix+1)x}{2} - \frac{\ln(x^2+1)}{2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

maxima [A] time = 1.00, size = 24, normalized size = 1.04

$$(x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.31, size = 19, normalized size = 0.83

$$-\frac{\mathrm{atan}(x)^2}{2}+x\,\mathrm{atan}(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(x))/(x^2 + 1),x)
[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

sympy [A] time = 0.38, size = 19, normalized size = 0.83

$$x\,\mathrm{atan}\,(x)-\frac{\log\left(x^2+1\right)}{2}-\frac{\mathrm{atan}^2\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

3.672 $\int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx$

Optimal. Leaf size=67

$$\frac{1}{2} i \operatorname{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2} x^2 \tan^{-1}(x)-\frac{x}{2}+\frac{1}{2} i \tan^{-1}(x)^2+\frac{1}{2} \tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4916, 4852, 321, 203, 4920, 4854, 2402, 2315}

$$\frac{1}{2} i \text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2} x^2 \tan^{-1}(x)-\frac{x}{2}+\frac{1}{2} i \tan^{-1}(x)^2+\frac{1}{2} \tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcTan[x]*Log[
2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tan^{-1}(x)}{1+x^2}\,dx &= \int x\tan^{-1}(x)\,dx - \int \frac{x\tan^{-1}(x)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}i\tan^{-1}(x)^2 - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx + \int \frac{\tan^{-1}(x)}{i-x}\,dx\\
&= -\frac{x}{2} + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}i\tan^{-1}(x)^2 + \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{\log\left(\frac{2}{1+i}\right.}{1+x^2}\\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}i\tan^{-1}(x)^2 + \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + i\,\mathrm{Subst}\left(\int \frac{\log}{1-}\right.\\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}i\tan^{-1}(x)^2 + \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \frac{1}{2}i\mathrm{Li}_2\left(1-\frac{2}{1+}\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.85

$$\frac{1}{2}\left(i\,\mathrm{PolyLog}\left(2,\frac{x+i}{x-i}\right) + \left(x^2 + 2\log\left(-\frac{2i}{x-i}\right)+1\right)\tan^{-1}(x) - x + i\tan^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] (-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*Poly
Log[2, (I + x)/(-I + x)])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\tan^{-1}(x)}{1+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^3\arctan(x)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

maple [B] time = 0.33, size = 113, normalized size = 1.69

method	result
risch	$\frac{i\ln(-ix+1)x^2}{4}+\frac{\arctan(x)}{2}-\frac{x}{2}-\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}-\frac{i\ln(-ix+1)^2}{8}-\frac{i\ln(ix+1)x^2}{4}+\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}$
default	$\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)\ln\left(x^2+1\right)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}-\frac{i\ln(x-i)\ln\left(x^2+1\right)}{4}+\frac{i\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)}{4}+\frac{i\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)}{4}+\frac{i\ln(x-i)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*ln(1-I*x)*x^2+1/2*arctan(x)-1/2*x-1/4*I*ln(1/2+1/2*I*x)*ln(1-I*x)+1/4
*I*dilog(1/2-1/2*I*x)-1/8*I*ln(1-I*x)^2-1/4*I*ln(1+I*x)*x^2+1/4*I*ln(1/2-1/
2*I*x)*ln(1+I*x)-1/4*I*dilog(1/2+1/2*I*x)+1/8*I*ln(1+I*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \,\mathrm{atan}(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(x))/(x^2 + 1),x)
```

```
[Out] int((x^3*atan(x))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \,\mathrm{atan}\,(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*atan(x)/(x**2 + 1), x)
```

3.673 $\int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx$

Optimal. Leaf size=34

$$-\frac{1}{4(x^2+1)} - \frac{x\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)^2$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4934, 4884}

$$-\frac{1}{4(x^2+1)} - \frac{x\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -1/(4*(1 + x^2)) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tan^{-1}(x)}{(1+x^2)^2}\,dx &= -\frac{1}{4(1+x^2)} - \frac{x\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{\tan^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{1}{4(1+x^2)} - \frac{x\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4}\tan^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.82

$$\frac{(x^2+1)\tan^{-1}(x)^2 - 2x\tan^{-1}(x) - 1}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \tan^{-1}(x)}{\left(1+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.73, size = 26, normalized size = 0.76

$$\frac{\left(x^2+1\right)\arctan(x)^2-2\,x\arctan(x)-1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arctan(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(x)/(x^2 + 1)^2, x)
```

maple [A] time = 0.39, size = 29, normalized size = 0.85

method	result	size
default	$-\frac{1}{4\left(x^2+1\right)}-\frac{x\arctan(x)}{2\left(x^2+1\right)}+\frac{\arctan(x)^2}{4}$	29
risch	$-\frac{\ln(ix+1)^2}{16}+\frac{\left(x^2\ln(-ix+1)+\ln(-ix+1)+2ix\right)\ln(ix+1)}{8x^2+8}-\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+4i\ln(-ix+1)x+4}{16(x+i)(x-i)}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

maxima [A] time = 0.98, size = 40, normalized size = 1.18

$$-\frac{1}{2}\left(\frac{x}{x^2+1}-\arctan(x)\right)\arctan(x)-\frac{\left(x^2+1\right)\arctan(x)^2+1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)/
(x^2 + 1)
```

mupad [B] time = 0.06, size = 23, normalized size = 0.68

$$\frac{\operatorname{atan}(x)^2}{4} - \frac{\frac{x\operatorname{atan}(x)}{2} + \frac{1}{4}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(x))/(x^2 + 1)^2,x)
[Out] atan(x)^2/4 - ((x*atan(x))/2 + 1/4)/(x^2 + 1)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1)**2,x)
[Out] Exception raised: RecursionError
```

3.674 $\int \frac{x^3 \tan^{-1}(x)}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=79

$$-\frac{1}{2} i \operatorname{PolyLog}\left(2,1-\frac{2}{1+i x}\right)-\frac{x}{4\left(x^2+1\right)}+\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{1}{2} i \tan^{-1}(x)^2-\frac{1}{4} \tan^{-1}(x)-\log\left(\frac{2}{1+i x}\right) \tan^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4964, 4920, 4854, 2402, 2315, 4930, 199, 203}

$$-\frac{1}{2} i \operatorname{PolyLog}\left(2,1-\frac{2}{1+i x}\right)-\frac{x}{4\left(x^2+1\right)}+\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{1}{2} i \tan^{-1}(x)^2-\frac{1}{4} \tan^{-1}(x)-\log\left(\frac{2}{1+i x}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/(4*(1 + x^2)) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTan[x]^
2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \tan^{-1}(x)}{\left(1+x^2\right)^2} dx &= -\int \frac{x \tan^{-1}(x)}{\left(1+x^2\right)^2} dx + \int \frac{x \tan^{-1}(x)}{1+x^2} dx \\
&= \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{1}{\left(1+x^2\right)^2} dx - \int \frac{\tan^{-1}(x)}{i-x} dx \\
&= -\frac{x}{4\left(1+x^2\right)} + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{4} \int \frac{1}{1+x^2} dx + \int \frac{\text{lo}}{} \\
&= -\frac{x}{4\left(1+x^2\right)} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - i\, \text{Subst}\Big(\\
&= -\frac{x}{4\left(1+x^2\right)} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \text{Li}_2\Big(1 -
\end{aligned}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.81

$$\frac{1}{2} i \,\text{PolyLog}\left(2, -e^{2 i \tan^{-1}(x)}\right) + \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(1+e^{2 i \tan^{-1}(x)}\right) - \frac{1}{8} \sin\left(2 \tan^{-1}(x)\right) + \frac{1}{4} \tan^{-1}(x) \cos\Big(2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^((
2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[x]
]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \tan^{-1}(x)}{\left(1+x^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \arctan(x)}{x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(x)}{\left(x^2 + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

maple [B] time = 0.35, size = 139, normalized size = 1.76

method	result
default	$\frac{\arctan(x)\ln\left(x^2+1\right)}{2}+\frac{\arctan(x)}{2x^2+2}-\frac{x}{4\left(x^2+1\right)}-\frac{\arctan(x)}{4}+\frac{i\ln(x-i)\ln\left(x^2+1\right)}{4}-\frac{i\,\text{dilog}\left(-\frac{i(x+i)}{2}\right)}{4}-\frac{i\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)}{4}-\frac{i\ln(}{8}$
risch	$\frac{i\ln(-ix+1)}{-8ix+8}+\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i}{8(ix+1)}-\frac{i\ln(ix+1)^2}{8}-\frac{i\ln(ix+1)}{8(ix+1)}-\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}-\frac{\ln(-ix+1)x}{16(-ix-1)}+\frac{i\ln(ix+1)}{16ix-16}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)*ln(x^2+1)+1/2*arctan(x)/(x^2+1)-1/4*x/(x^2+1)-1/4*arctan(x)+1
/4*I*ln(x-I)*ln(x^2+1)-1/4*I*dilog(-1/2*I*(x+I))-1/4*I*ln(x-I)*ln(-1/2*I*(x
+I))-1/8*I*ln(x-I)^2-1/4*I*ln(x+I)*ln(x^2+1)+1/4*I*dilog(1/2*I*(x-I))+1/4*I
*ln(x+I)*ln(1/2*I*(x-I))+1/8*I*ln(x+I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(x)}{\left(x^2 + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(x)}{\left(x^2 + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(x))/(x^2 + 1)^2,x)
```

```
[Out] int((x^3*atan(x))/(x^2 + 1)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

3.675 $\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=89

$$i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4\left(x^2+1\right)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-1}$$

Rubi [A] time = 0.23, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {4964, 4916, 4852, 321, 203, 4920, 4854, 2402, 2315, 4930, 199}

$$i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4\left(x^2+1\right)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/2 + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - ArcTan[x]/(2
*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*x)] + I*PolyLog[2, 1
 - 2/(1 + I*x)]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2}\,dx = -\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2}\,dx + \int \frac{x^3 \tan^{-1}(x)}{1+x^2}\,dx$$

$$= \int x \tan^{-1}(x)\,dx + \int \frac{x \tan^{-1}(x)}{(1+x^2)^2}\,dx - 2\int \frac{x \tan^{-1}(x)}{1+x^2}\,dx$$

$$= \frac{1}{2}x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx - 2\left(-\frac{1}{2}i \tan^{-1}(x)^2 - \int \frac{\tan}{i}\right.$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{1}{2}x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - 2\left(-\frac{1}{2}i \tan\right.$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4}\tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\right.$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4}\tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\right.$$

Mathematica [A] time = 0.23, size = 70, normalized size = 0.79

$$\frac{1}{8}\left(-8i\,\mathrm{PolyLog}\left(2, -e^{2i\tan^{-1}(x)}\right) + 4\left(x^2+1\right)\tan^{-1}(x) - 4x - 8i\tan^{-1}(x)^2 + 16\tan^{-1}(x)\log\left(1+e^{2i\tan^{-1}(x)}\right) + \sin\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
[Out] (-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcTa
n[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*I)*PolyLog[2, -E^((2
*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*ArcTan[x])/(1 + x^2)^2,x]
[Out] Could not integrate
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^5 \arctan(x)}{x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \arctan(x)}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

maple [A] time = 0.34, size = 149, normalized size = 1.67

method	result
default	$\frac{x^2\arctan(x)}{2} - \arctan(x)\ln\left(x^2+1\right) - \frac{\arctan(x)}{2\left(x^2+1\right)} - \frac{x}{2} + \frac{x}{4x^2+4} + \frac{3\arctan(x)}{4} - \frac{i\ln(x-i)\ln\left(x^2+1\right)}{2} + \frac{i\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right.}{2}$
risch	$-\frac{x}{2} + \frac{5\arctan(x)}{8} - \frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{2} - \frac{i\ln(ix+1)x^2}{4} + \frac{i}{8ix+8} - \frac{i\ln(-ix+1)}{8(-ix+1)} + \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{2} + \frac{i\ln(-ix+1)x^2}{4} - \frac{i\ln(-ix+1}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctan(x)-arctan(x)*ln(x^2+1)-1/2*arctan(x)/(x^2+1)-1/2*x+1/4*x/(x^
2+1)+3/4*arctan(x)-1/2*I*ln(x-I)*ln(x^2+1)+1/2*I*dilog(-1/2*I*(x+I))+1/2*I*
ln(x-I)*ln(-1/2*I*(x+I))+1/4*I*ln(x-I)^2+1/2*I*ln(x+I)*ln(x^2+1)-1/2*I*dilo
g(1/2*I*(x-I))-1/2*I*ln(x+I)*ln(1/2*I*(x-I))-1/4*I*ln(x+I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\arctan(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\mathrm{atan}(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(x))/(x^2 + 1)^2,x)
```

```
[Out] int((x^5*atan(x))/(x^2 + 1)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

3.676 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\log(x^2+1)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4950, 4852, 266, 36, 29, 31, 4846, 260}

$$-\log(x^2+1)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x])/x^2,x]
```

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^2\right)\tan^{-1}(x)}{x^2}\,dx &= \int \tan^{-1}(x)\,dx + \int \frac{\tan^{-1}(x)}{x^2}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) + \int \frac{1}{x\left(1+x^2\right)}\,dx - \int \frac{x}{1+x^2}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{}{1}\right. \\
&= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) + \log(x) - \log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-\log\left(x^2+1\right) + \log(x) + x\tan^{-1}(x) - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)\tan^{-1}(x)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*ArcTan[x])/x^2,x]
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 26, normalized size = 1.18

$$\frac{\left(x^2-1\right)\arctan(x) - x\log\left(x^2+1\right) + x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")
[Out] ((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

giac [A] time = 0.99, size = 25, normalized size = 1.14

$$\left(x-\frac{1}{x}\right)\arctan(x) - \log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="giac")
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)
```

maple [A] time = 0.13, size = 23, normalized size = 1.05

method	result	size
default	$-\frac{\arctan(x)}{x}+x\arctan(x)+\ln(x)-\ln\left(x^{2}+1\right)$	23
meijerg	$-\frac{\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}-\ln\left(x^{2}+1\right)+\ln(x)+\frac{x^{2}\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}$	40
risch	$-\frac{i\left(x^{2}-1\right)\ln(ix+1)}{2x}+\frac{i\left(-2i\ln(x)x+2i\ln\left(x^{2}+1\right)x+x^{2}\ln(-ix+1)-\ln(-ix+1)\right)}{2x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

maxima [A] time = 0.95, size = 21, normalized size = 0.95

$$\left(x-\frac{1}{x}\right)\arctan(x)-\log\left(x^{2}+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

mupad [B] time = 0.07, size = 22, normalized size = 1.00

$$\ln(x)-\ln\left(x^{2}+1\right)-\frac{\mathrm{atan}(x)}{x}+x\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x)*(x^2 + 1))/x^2,x)
[Out] log(x) - log(x^2 + 1) - atan(x)/x + x*atan(x)
```

sympy [A] time = 0.37, size = 19, normalized size = 0.86

$$x\,\mathrm{atan}\,(x)+\log\,(x)-\log\left(x^{2}+1\right)-\frac{\mathrm{atan}\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)/x**2,x)
[Out] x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

3.677 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{12x^3}-\frac{(x^2+1)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4944, 14}

$$-\frac{1}{12x^3}-\frac{(x^2+1)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx &= -\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}+\frac{1}{4}\int \frac{1+x^2}{x^4}\,dx\\ &= -\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}+\frac{1}{4}\int \left(\frac{1}{x^4}+\frac{1}{x^2}\right)dx\\ &= -\frac{1}{12x^3}-\frac{1}{4x}-\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}\end{aligned}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 1.90

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x^2\right)}{2x}-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x^2\right)}{12x^3}-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
```

```
[Out] -1/4*ArcTan[x]/x^4 - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2, -
x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*ArcTan[x])/x^5,x]
[Out] Could not integrate
```

fricas [A] time = 1.02, size = 26, normalized size = 0.84

$$-\frac{3\,x^3+3\,(x^4+2\,x^2+1)\arctan(x)+x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")
[Out] -1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

giac [A] time = 0.95, size = 31, normalized size = 1.00

$$-\frac{3\,x^2+1}{12\,x^3}-\frac{(2\,x^2+1)\arctan(x)}{4\,x^4}-\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="giac")
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

maple [A] time = 0.08, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4x^4}-\frac{1}{12x^3}-\frac{1}{4x}-\frac{\arctan(x)}{4}$	30
meijerg	$-\frac{1}{12x^3}-\frac{1}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{2x^2}$	47
risch	$\frac{i(2x^2+1)\ln(ix+1)}{8x^4}+\frac{i\left(3\ln(x-i)x^4-3\ln(x+i)x^4+6ix^3-6x^2\ln(-ix+1)+2ix-3\ln(-ix+1)\right)}{24x^4}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)/x^2-1/4*arctan(x)/x^4-1/12/x^3-1/4/x-1/4*arctan(x)
```

maxima [A] time = 0.95, size = 31, normalized size = 1.00

$$-\frac{3\,x^2+1}{12\,x^3}-\frac{(2\,x^2+1)\arctan(x)}{4\,x^4}-\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

mupad [B] time = 0.32, size = 30, normalized size = 0.97

$$-\frac{\mathrm{atan}(x)}{4}-\frac{\frac{x}{12}+\frac{\mathrm{atan}(x)}{4}+\frac{x^2\,\mathrm{atan}(x)}{2}+\frac{x^3}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x)*(x^2 + 1))/x^5,x)
```

```
[Out] - atan(x)/4 - (x/12 + atan(x)/4 + (x^2*atan(x))/2 + x^3/4)/x^4
```

sympy [A] time = 0.84, size = 34, normalized size = 1.10

$$-\frac{\operatorname{atan}(x)}{4}-\frac{1}{4x}-\frac{\operatorname{atan}(x)}{2x^2}-\frac{1}{12x^3}-\frac{\operatorname{atan}(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)/x**5,x)
```

```
[Out] -atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(4*x**4)
```

3.678 $\int \frac{(1+x^2)^2 \tan^{-1}(x)}{x^5} \, dx$

Optimal. Leaf size=63

$$\frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix) - \frac{\tan^{-1}(x)}{4x^4} - \frac{1}{12x^3} - \frac{\tan^{-1}(x)}{x^2} - \frac{3}{4x} - \frac{3}{4}\tan^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {4948, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2} i\operatorname{PolyLog}(2,-ix) - \frac{1}{2} i\operatorname{PolyLog}(2,ix) - \frac{1}{12x^3} - \frac{\tan^{-1}(x)}{x^2} - \frac{\tan^{-1}(x)}{4x^4} - \frac{3}{4x} - \frac{3}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^2
 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
```

```
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^2\right)^2 \tan^{-1}(x)}{x^5} dx &= \int \left(\frac{\tan^{-1}(x)}{x^5}+\frac{2\tan^{-1}(x)}{x^3}+\frac{\tan^{-1}(x)}{x}\right) dx\\ &= 2\int \frac{\tan^{-1}(x)}{x^3} dx+\int \frac{\tan^{-1}(x)}{x^5} dx+\int \frac{\tan^{-1}(x)}{x} dx\\ &= -\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\int \frac{\log(1-ix)}{x} dx-\frac{1}{2}i\int \frac{\log(1+ix)}{x} dx+\frac{1}{4}\int \frac{1}{x^4\left(1+\right.}\\ &= -\frac{1}{12x^3}-\frac{1}{x}-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)-\frac{1}{4}\int \frac{1}{x^2\left(1+x^2\right)} dx\\ &= -\frac{1}{12x^3}-\frac{3}{4x}-\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)+\frac{1}{4}\int \frac{}{1}\\ &= -\frac{1}{12x^3}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)\end{aligned}$$

Mathematica [C] time = 0.01, size = 81, normalized size = 1.29

$$\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x^2\right)}{x}-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x^2\right)}{12x^3}-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
[Out] -1/4*ArcTan[x]/x^4 - ArcTan[x]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x^2]
/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/x + (I/2)*PolyLog[2, (-I)
*x] - (I/2)*PolyLog[2, I*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)^2 \tan^{-1}(x)}{x^5} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)^2*ArcTan[x])/x^5,x]
[Out] Could not integrate
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+2\,x^2+1\right)\arctan(x)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")
[Out] integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2 \arctan(x)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

maple [A] time = 0.36, size = 79, normalized size = 1.25

method	result
default	$\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}-\frac{\arctan(x)}{4x^4}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{3\,\mathrm{arcta}}{4}$
meijerg	$-\frac{1}{12x^3}-\frac{3}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{ix\,\mathrm{polylog}\left(2,i\sqrt{x^2}\right)}{2\sqrt{x^2}}+\frac{ix\,\mathrm{polylog}\left(2,-i\sqrt{x^2}\right)}{2\sqrt{x^2}}-\frac{\left(x^2+1\right)\arctan(x)}{x^2}$
risch	$\frac{3i\ln(-ix)}{8}-\frac{3}{4x}-\frac{3\arctan(x)}{4}-\frac{i\ln(-ix+1)}{2x^2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{1}{12x^3}-\frac{i\ln(-ix+1)}{8x^4}-\frac{3i\ln(ix)}{8}+\frac{i\ln(ix+1)}{2x^2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2*arctan(x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)*ln(x)-arctan(x)/x^2-1/4*arctan(x)/x^4+1/2*I*ln(x)*ln(1+I*x)-1/2*I
*ln(x)*ln(1-I*x)+1/2*I*dilog(1+I*x)-1/2*I*dilog(1-I*x)-3/4*arctan(x)-1/12/x
^3-3/4/x
```

maxima [A] time = 1.12, size = 71, normalized size = 1.13

$$-\frac{3\,\pi x^4\log\left(x^2+1\right)-12\,x^4\arctan(x)\log(x)+6i\,x^4\mathrm{Li}_2\left(i\,x+1\right)-6i\,x^4\mathrm{Li}_2\left(-i\,x+1\right)+9\,x^3+3\left(3\,x^4+4\,x^2+1\right)\mathrm{a}}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*dilog(I*x
+ 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x^2 + 1)*arctan(x) +
x)/x^4
```

mupad [B] time = 0.50, size = 53, normalized size = 0.84

$$\frac{x^2-\frac{1}{3}}{4\,x^3}-\frac{\mathrm{atan}(x)}{x^2}-\frac{\mathrm{atan}(x)}{4\,x^4}-\frac{3\,\mathrm{atan}(x)}{4}-\frac{1}{x}-\frac{\mathrm{Li}_2\left(1-x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\mathrm{polylog}\left(2,-x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x)*(x^2 + 1)^2)/x^5,x)
```

```
[Out] (polylog(2, -x*1i)*1i)/2 - (3*atan(x))/4 - atan(x)/x^2 - atan(x)/(4*x^4) -
(dilog(1 - x*1i)*1i)/2 + (x^2 - 1/3)/(4*x^3) - 1/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2+1\right)^2\mathrm{atan}\left(x\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2*atan(x)/x**5,x)
```

```
[Out] Integral((x**2 + 1)**2*atan(x)/x**5, x)
```

3.679 $\int \frac{\tan^{-1}(x)}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {4918, 4852, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx &= \int \frac{\tan^{-1}(x)}{x^2}\,dx - \int \frac{\tan^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \int \frac{1}{x\left(1+x^2\right)}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right) + \log(x) - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[x]/(x^2*(1 + x^2)),x]
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 29, normalized size = 1.04

$$-\frac{x\arctan(x)^2 + x\log\left(x^2+1\right) - 2\,x\log(x) + 2\,\arctan(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

maple [A] time = 0.40, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}+\ln(x)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
risch	$\frac{\ln(ix+1)^{2}}{8}-\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-x\ln(-ix+1)^{2}-8x\ln(x)+4\ln\left(x^{2}+1\right)x+4i\ln(-ix+1)}{8x}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.99, size = 27, normalized size = 0.96

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^{2}-\frac{1}{2}\log\left(x^{2}+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 0.09, size = 24, normalized size = 0.86

$$\ln(x)-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\mathrm{atan}(x)}{x}-\frac{\mathrm{atan}(x)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x^2*(x^2 + 1)),x)
[Out] log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2/2
```

sympy [A] time = 0.57, size = 22, normalized size = 0.79

$$\log(x)-\frac{\log\left(x^{2}+1\right)}{2}-\frac{\mathrm{atan}^{2}(x)}{2}-\frac{\mathrm{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/x**2/(x**2+1),x)
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

3.680 $\int \frac{\tan^{-1}(x)^2}{x^3} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - Log[1 + x^2
]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(x)^2}{x^3}\,dx &= -\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{\tan^{-1}(x)}{x^2}\,dx-\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{1}{x\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^2\right)\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\right.\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.97

$$
-\frac{1}{2}\log\left(x^2+1\right)+\frac{\left(-x^2-1\right)\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{\tan^{-1}(x)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^2/x^3,x]
[Out] -(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\tan^{-1}(x)^2}{x^3}\,dx
$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[x]^2/x^3,x]
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 38, normalized size = 0.97

$$
-\frac{\left(x^2+1\right)\arctan(x)^2+x^2\log\left(x^2+1\right)-2\,x^2\log(x)+2\,x\arctan(x)}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2*x*arctan(
x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\arctan(x)^2}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^3, x)
```

maple [A] time = 0.11, size = 34, normalized size = 0.87

method	result
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}-\frac{\arctan(x)^{2}}{2x^{2}}+\ln(x)-\frac{\ln\left(x^{2}+1\right)}{2}$
risch	$\frac{\left(x^{2}+1\right)\ln(ix+1)^{2}}{8x^{2}}-\frac{\left(x^{2}\ln(-ix+1)-2ix+\ln(-ix+1)\right)\ln(ix+1)}{4x^{2}}+\frac{x^{2}\ln(-ix+1)^{2}-4i\ln(-ix+1)x+8x^{2}\ln(x)-4\ln\left(x^{2}+1\right)x^{2}+\ln(-ix+1)^{2}}{8x^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.96, size = 36, normalized size = 0.92

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^{2}-\frac{\arctan(x)^{2}}{2x^{2}}-\frac{1}{2}\log\left(x^{2}+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2*
log(x^2 + 1) + log(x)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.79

$$\ln(x)-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\mathrm{atan}(x)}{x}-\mathrm{atan}(x)^{2}\left(\frac{1}{2x^{2}}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)^2/x^3,x)
```

```
[Out] log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2*(1/(2*x^2) + 1/2)
```

sympy [A] time = 0.53, size = 32, normalized size = 0.82

$$\log(x)-\frac{\log\left(x^{2}+1\right)}{2}-\frac{\mathrm{atan}^{2}(x)}{2}-\frac{\mathrm{atan}(x)}{x}-\frac{\mathrm{atan}^{2}(x)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2/x**3,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/(2*x**2)
```

3.681 $\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{\tan^{-1}(x)}{6x^3}-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4944, 4950, 4852, 266, 44, 36, 29, 31}

$$-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*ArcTan[x]^
2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx &= -\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{(1+x^2)\tan^{-1}(x)}{x^4}\,dx\\
&= -\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4}\,dx+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2}\,dx\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{6}\int \frac{1}{x^3(1+x^2)}\,dx+\frac{1}{2}\int \frac{1}{x(1+x^2)}
\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx,x,x^2\right)+\frac{1}{4}
\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int\left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx,x,
\\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{1}{6}\log(1+x^2)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.93

$$\frac{-2(3x^3+x)\tan^{-1}(x)+x^2(4x^2\log(x)-2x^2\log(x^2+1)-1)-3(x^2+1)^2\tan^{-1}(x)^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] (-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Log
[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.00, size = 54, normalized size = 0.90

$$-\frac{2\,x^4\log\left(x^2+1\right)-4\,x^4\log(x)+3\left(x^4+2\,x^2+1\right)\arctan(x)^2+x^2+2\left(3\,x^3+x\right)\arctan(x)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*arctan(x)^2
+ x^2 + 2*(3*x^3 + x)*arctan(x))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2+1\right)\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

maple [A] time = 0.15, size = 57, normalized size = 0.95

method	result
default	$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{\arctan(x)^2}{4}-\frac{\ln\left(x^2+1\right)}{6}-\frac{1}{12x^2}+\frac{\ln(x)}{3}$
risch	$\frac{\left(x^4+2x^2+1\right)\ln(ix+1)^2}{16x^4}-\frac{\left(3x^4\ln(-ix+1)-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1)\right)\ln(ix+1)}{24x^4}+\frac{3x^4\ln(-ix+1)^2+16x^4\ln(x)-8\ln\left(x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(x)^2/x^2-1/4*arctan(x)^2/x^4-1/6*arctan(x)/x^3-1/2*arctan(x)/x-
1/4*arctan(x)^2-1/6*ln(x^2+1)-1/12/x^2+1/3*ln(x)
```

maxima [A] time = 0.97, size = 71, normalized size = 1.18

$$-\frac{1}{6}\left(\frac{3\,x^2+1}{x^3}+3\,\arctan(x)\right)\arctan(x)+\frac{3\,x^2\arctan(x)^2-2\,x^2\log\left(x^2+1\right)+4\,x^2\log(x)-1}{12\,x^2}-\frac{\left(2\,x^2+1\right)\text{arcta}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arctan(x)^2 -
2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*arctan(x)^2/x^
4
```

mupad [B] time = 0.12, size = 51, normalized size = 0.85

$$\frac{\ln(x)}{3}-\frac{\ln\left(x^2+1\right)}{6}-\mathrm{atan}(x)^2\left(\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}+\frac{1}{4}\right)-\frac{1}{12\,x^2}-\frac{\mathrm{atan}(x)\left(\frac{x^2}{2}+\frac{1}{6}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x)^2*(x^2 + 1))/x^5,x)
```

```
[Out] log(x)/3 - log(x^2 + 1)/6 - atan(x)^2*((x^2/2 + 1/4)/x^4 + 1/4) - 1/(12*x^2
) - (atan(x)*(x^2/2 + 1/6))/x^3
```

sympy [A] time = 0.90, size = 61, normalized size = 1.02

$$\frac{\log(x)}{3} - \frac{\log(x^2+1)}{6} - \frac{\operatorname{atan}^2(x)}{4} - \frac{\operatorname{atan}(x)}{2x} - \frac{\operatorname{atan}^2(x)}{2x^2} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)**2/x**5,x)
```

```
[Out] log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(x)**2/(2*x
**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

3.682 $\int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} dx$

Optimal. Leaf size=79

$$\frac{5}{32(x^2+1)} - \frac{1}{32(x^2+1)^2} + \frac{3x\tan^{-1}(x)}{16(x^2+1)} + \frac{x^4\tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3\tan^{-1}(x)}{8(x^2+1)^2} - \frac{3}{32}\tan^{-1}(x)^2$$

Rubi [A] time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4944, 4938, 4934, 4884}

$$-\frac{x^4}{32(x^2+1)^2} + \frac{3}{32(x^2+1)} + \frac{x^4\tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3\tan^{-1}(x)}{8(x^2+1)^2} + \frac{3x\tan^{-1}(x)}{16(x^2+1)} - \frac{3}{32}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] -x^4/(32*(1 + x^2)^2) + 3/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1 + x^2)^2)
+ (3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 + (x^4*ArcTan[x]^2)/(
4*(1 + x^2)^2)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(x)^2}{\left(1+x^2\right)^3} dx = \frac{x^4 \tan^{-1}(x)^2}{4\left(1+x^2\right)^2} - \frac{1}{2}\int \frac{x^4 \tan^{-1}(x)}{\left(1+x^2\right)^3} dx$$
$$= -\frac{x^4}{32\left(1+x^2\right)^2} + \frac{x^3 \tan^{-1}(x)}{8\left(1+x^2\right)^2} + \frac{x^4 \tan^{-1}(x)^2}{4\left(1+x^2\right)^2} - \frac{3}{8}\int \frac{x^2 \tan^{-1}(x)}{\left(1+x^2\right)^2} dx$$
$$= -\frac{x^4}{32\left(1+x^2\right)^2} + \frac{3}{32\left(1+x^2\right)} + \frac{x^3 \tan^{-1}(x)}{8\left(1+x^2\right)^2} + \frac{3x \tan^{-1}(x)}{16\left(1+x^2\right)} + \frac{x^4 \tan^{-1}(x)^2}{4\left(1+x^2\right)^2} - \frac{3}{16}\int \frac{\tan^{-1}(x)}{1+x^2}$$
$$= -\frac{x^4}{32\left(1+x^2\right)^2} + \frac{3}{32\left(1+x^2\right)} + \frac{x^3 \tan^{-1}(x)}{8\left(1+x^2\right)^2} + \frac{3x \tan^{-1}(x)}{16\left(1+x^2\right)} - \frac{3}{32}\tan^{-1}(x)^2 + \frac{x^4 \tan^{-1}(x)^2}{4\left(1+x^2\right)^2}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.59

$$\frac{5x^2 + 2\left(5x^2+3\right)x\tan^{-1}(x) + \left(5x^4-6x^2-3\right)\tan^{-1}(x)^2 + 4}{32\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] (4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)/
(32*(1 + x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \tan^{-1}(x)^2}{\left(1+x^2\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.94, size = 51, normalized size = 0.65

$$\frac{\left(5x^4-6x^2-3\right)\arctan(x)^2 + 5x^2 + 2\left(5x^3+3x\right)\arctan(x) + 4}{32\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*arctan(x) +
 4)/(x^4 + 2*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(x)^2}{\left(x^2+1\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="giac")
```

[Out] integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)

maple [A] time = 0.42, size = 78, normalized size = 0.99

method	result
default	$\frac{\arctan(x)^2}{4(x^2+1)^2} - \frac{\arctan(x)^2}{2(x^2+1)} + \frac{5x^3\arctan(x)}{16(x^2+1)^2} + \frac{3x\arctan(x)}{16(x^2+1)^2} + \frac{5\arctan(x)^2}{32} - \frac{1}{32(x^2+1)^2} + \frac{5}{32(x^2+1)}$
risch	$-\frac{(5x^4-6x^2-3)\ln(ix+1)^2}{128(x^2+1)^2} + \frac{(-6x^2\ln(-ix+1)-3\ln(-ix+1)+5x^4\ln(-ix+1)-10ix^3-6ix)\ln(ix+1)}{64(x+i)^2(x-i)^2} - \frac{5x^4\ln(-ix+1)^2-6x^2\ln(-ix+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(x)^2/(x^2+1)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan(x)/(x^2+1)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2-1/32/(x^2+1)^2+5/32/(x^2+1)

maxima [A] time = 1.04, size = 94, normalized size = 1.19

$$\frac{1}{16}\left(\frac{5x^3+3x}{x^4+2x^2+1}+5\arctan(x)\right)\arctan(x)-\frac{(2x^2+1)\arctan(x)^2}{4(x^4+2x^2+1)}-\frac{5(x^4+2x^2+1)\arctan(x)^2-5x^2-4}{32(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")

[Out] 1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) - 1/4*(2*x^2 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*x^2 + 1)*arctan(x)^2 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)

mupad [B] time = 0.36, size = 56, normalized size = 0.71

$$-\frac{-5x^4\operatorname{atan}(x)^2+4x^4-10x^3\operatorname{atan}(x)+6x^2\operatorname{atan}(x)^2+3x^2-6x\operatorname{atan}(x)+3\operatorname{atan}(x)^2}{32(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*atan(x)^2)/(x^2 + 1)^3,x)

[Out] -(3*atan(x)^2 - 10*x^3*atan(x) + 6*x^2*atan(x)^2 - 5*x^4*atan(x)^2 - 6*x*atan(x) + 3*x^2 + 4*x^4)/(32*(x^2 + 1)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\operatorname{atan}^2(x)}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atan(x)**2/(x**2+1)**3,x)

[Out] Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)

3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=107

$$\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2-1}\,\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\,\sec^{-1}(x)\tan^{-1}\left(e^{i\right.}}{x}$$

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5242, 4698, 4710, 4181, 2279, 2391, 8}

$$\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\,\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\,\sec^{-1}(x)\tan^{-}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcSec[x])/x - ((2*I)*Sqrt[x^2
]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*
ArcSec[x])])/x - (I*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
```

```
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\cos^{-1}(x)}{x}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x\sqrt{1-x^2}}\,dx, x,\right.}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} + \frac{\sqrt{x^2}\,\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(x)\right)}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\left(i\sqrt{x^2}\right)\text{Sub}}{}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{i\sqrt{x^2}\,\text{Li}_2\left(-i\right.}{x}$$

Mathematica [A] time = 0.20, size = 116, normalized size = 1.08

$$-\frac{\sqrt{1-\frac{1}{x^2}}\left(-ix\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+ix\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)+\sqrt{1-\frac{1}{x^2}}\,x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-\right.\right.}{\sqrt{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1 -
 I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] - I*x*PolyLog[
2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 + x^
2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] Could not integrate
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-1}\ \text{arcsec}(x)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}\ \text{arcsec}(x)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

maple [B] time = 0.74, size = 708, normalized size = 6.62

method	result
default	$\frac{\sqrt{x^2-1}\left(\sqrt{\frac{x^2-1}{x^2}}\,x^3-3ix^2-4\sqrt{\frac{x^2-1}{x^2}}\,x+4i\right)}{4\left(-i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)x}-\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^3-2i\sqrt{\frac{x^2-1}{x^2}}\,x+2x^2-2\right)\text{arcsec}(x)}{4\sqrt{x^2-1}\,x}-\frac{\sqrt{x^2-1}\left(\sqrt{\frac{x^2-1}{x^2}}\,x-i\right)x}{2\left(-i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)}+\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^3\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)*(x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2)*x+
4*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x-1/4/(x^2-1)^(1/2)/x*(I*((x^2-1)/x^2
)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*arcsec(x)-1/2*(x^2-1)^(1/2)*
(((x^2-1)/x^2)^(1/2)*x-I)*x/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)+1/4/(x^2-1)^(1
/2)*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*arcsec(x)
/x+I*(x^2-1)^(1/2)*x^3/(4*I*((x^2-1)/x^2)^(1/2)*x^3-8*I*((x^2-1)/x^2)^(1/2)
*x+8*x^2-8)-1/2/(x^2-1)^(1/2)*(I*x^2+((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(
1+I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*(I*x^2+((x^2-1)/x^2)^(1/2)*x
-I)*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))-1/2/(x^2-1)^(1/2)*(-I*((x^2-1
)/x^2)^(1/2)*x+x^2-1)*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*
(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^
2-1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2
)^(1/2)))-1/2/(x^2-1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(1-
I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1
)*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))-1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/
2)*x+x^2-1)*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}\ \text{arcsec}(x)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acos(1/x)*(x^2 - 1)^(1/2))/x^2,x)
```

```
[Out] int((acos(1/x)*(x^2 - 1)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}\,\operatorname{asec}(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)
```

3.684 $\int \frac{\left(-1+x^2\right)^{5/2} \csc^{-1}(x)}{x^3} \, dx$

Optimal. Leaf size=106

$$-\frac{7x\log(x)}{3\sqrt{x^2}}+\frac{\left(x^2-1\right)^{5/2}\csc^{-1}(x)}{3x^2}-\frac{5\left(x^2-1\right)^{3/2}\csc^{-1}(x)}{3x^2}-\frac{5\sqrt{x^2-1}\,\csc^{-1}(x)}{2x^2}-\frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}}+\frac{2x^4+3}{12x\sqrt{x^2}}$$

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5243, 4695, 4647, 4641, 30, 14, 266, 43}

$$\frac{x\sqrt{x^2}}{6}+\frac{\sqrt{x^2}}{4x^3}-\frac{7\sqrt{x^2}\log(x)}{3x}+\frac{1}{3}\left(x^2\right)^{3/2}\left(1-\frac{1}{x^2}\right)^{5/2}\csc^{-1}(x)-\frac{5}{3}\sqrt{x^2}\left(1-\frac{1}{x^2}\right)^{3/2}\csc^{-1}(x)-\frac{5\sqrt{1-\frac{1}{x^2}}\,\csc^{-1}(x)}{2\sqrt{x^2}}-5\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] Sqrt[x^2]/(4*x^3) + (x*Sqrt[x^2])/6 - (5*Sqrt[1 - x^(-2)]*ArcCsc[x])/(2*Sqr
t[x^2]) - (5*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcCsc[x])/3 + ((1 - x^(-2))^(5/2
)*(x^2)^(3/2)*ArcCsc[x])/3 - (5*Sqrt[x^2]*ArcCsc[x]^2)/(4*x) - (7*Sqrt[x^2]
*Log[x])/(3*x)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
```

```
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4695

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Dist[(b*c*n*d^IntPar
t[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f
*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5243

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcSin[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^2\right)^{5/2}\csc^{-1}(x)}{x^3}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}\sin^{-1}(x)}{x^4}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}(x) - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx, x, \frac{1}{x}\right)}{3x} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}$$

$$= -\frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}(x) - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{(1}{}\right.}{6x}$$

$$= -\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}($$

$$= \frac{\sqrt{x^2}}{4x^3} + \frac{x\sqrt{x^2}}{6} - \frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5}$$

Mathematica [A] time = 0.31, size = 86, normalized size = 0.81

$$\frac{\sqrt{x^2-1}\left(4x^2+\csc^{-1}(x)\left(8\sqrt{1-\frac{1}{x^2}}\,x\left(x^2-7\right)-6\sin\left(2\csc^{-1}(x)\right)\right)+48\log\left(\frac{1}{x}\right)-8\log(x)-30\csc^{-1}(x)^2-3\,c}{24\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] (Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-1
)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcCsc
[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.96, size = 51, normalized size = 0.48

$$\frac{2\,x^4-15\,x^2\operatorname{arccsc}(x)^2-28\,x^2\log(x)+2\left(2\,x^4-14\,x^2-3\right)\sqrt{x^2-1}\,\operatorname{arccsc}(x)+3}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*x^2 - 3)*s
qrt(x^2 - 1)*arccsc(x) + 3)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

maple [C] time = 0.59, size = 305, normalized size = 2.88

method	result
default	$-\frac{5\sqrt{\frac{x^2-1}{x^2}}\,x\operatorname{arccsc}(x)^2}{4\sqrt{x^2-1}}+\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^3-2i\sqrt{\frac{x^2-1}{x^2}}\,x-2x^2+2\right)(2\operatorname{arccsc}(x)+i)}{16\sqrt{x^2-1}\,x^2}-\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^3-2i\sqrt{\frac{x^2-1}{x^2}}\,x+2x^2-2\right)(-i+2\operatorname{arccsc}(x))}{16\sqrt{x^2-1}\,x^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(5/2)*arccsc(x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -5/4/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arccsc(x)^2+1/16/(x^2-1)^(1/2)/x^2
*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*(2*arccsc(x)
+I)-1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/
2)*x+2*x^2-2)*(-I+2*arccsc(x))-14/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*a
rccsc(x)+1/6/(x^2-1)^(1/2)*(x^4+7*I*((x^2-1)/x^2)^(1/2)*x-8*x^2+7)*(2*arccs
c(x)*x^4+((x^2-1)/x^2)^(1/2)*x^3-30*arccsc(x)*x^2-7*((x^2-1)/x^2)^(1/2)*x+1
26*arccsc(x)-7*I)/(x^4-15*x^2+63)+7/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*l
n((I/x+(1-1/x^2)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}\left(\frac{1}{x}\right)\left(x^2-1\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3,x)
[Out] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
[Out] Timed out
```

3.685 $\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} \, dx$

Optimal. Leaf size=41

$$\frac{1}{3\sqrt{x^2}} - \frac{1}{9\left(x^2\right)^{3/2}} + \frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {264, 5238, 12, 14}

$$\frac{1}{3\sqrt{x^2}} - \frac{1}{9\left(x^2\right)^{3/2}} + \frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

```
[Out] -1/(9*(x^2)^(3/2)) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{3x^4}\,dx}{\sqrt{x^2}}$$
$$= \frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{x^4}\,dx}{3\sqrt{x^2}}$$
$$= \frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \left(-\frac{1}{x^4}+\frac{1}{x^2}\right)dx}{3\sqrt{x^2}}$$
$$= -\frac{1}{9\left(x^2\right)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.17

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x\left(3x^2-1\right)+3\left(x^2-1\right)^2\sec^{-1}(x)}{9x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
[Out] (Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-1
 + x^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
[Out] Could not integrate
```

fricas [A] time = 1.14, size = 23, normalized size = 0.56

$$\frac{3\left(x^2-1\right)^{\frac{3}{2}}\operatorname{arcsec}(x)+3\,x^2-1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3
```

giac [B] time = 1.16, size = 75, normalized size = 1.83

$$-\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{3\operatorname{sgn}(x)} + \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^4+1\right)\arccos\left(\frac{1}{x}\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{3\,x^2-1}{9\,x^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)*
arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))
```

maple [C] time = 0.78, size = 329, normalized size = 8.02

method	result
default	$-\frac{\sqrt{x^2-1}\left(\sqrt{\frac{x^2-1}{x^2}}\,x^5-5ix^4-12\sqrt{\frac{x^2-1}{x^2}}\,x^3+20ix^2+16\sqrt{\frac{x^2-1}{x^2}}\,x-16i\right)}{144\left(-i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)x^3}+\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^5-8i\sqrt{\frac{x^2-1}{x^2}}\,x^3+4x^4+8i\sqrt{\frac{x^2-1}{x^2}}\,x-12x^2+8\right)\mathrm{arcsec}}{48\sqrt{x^2-1}\,x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/144*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5-5*I*x^4-12*((x^2-1)/x^2)^(1/2
)*x^3+20*I*x^2+16*((x^2-1)/x^2)^(1/2)*x-16*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2
-1)/x^3+1/48/(x^2-1)^(1/2)/x^3*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)
^(1/2)*x^3+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)+1/72/(x^2-1)
^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(-13*I+3*arcsec(x))/x-1/72*(5*I+3*ar
csec(x))*(I*((x^2-1)/x^2)^(1/2)*x-1)*(x^2-1)^(1/2)/x-1/144/(x^2-1)^(1/2)*(I
*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(7*I+9*arcsec(x))*cos(3*arcsec(x))-1/48/(x^2-
1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*(3*I+arcsec(x))*sin(3*arcsec(x))
```

maxima [A] time = 0.99, size = 27, normalized size = 0.66

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}\,\mathrm{arcsec}(x)}{3\,x^3}+\frac{3\,x^2-1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)\,\sqrt{x^2-1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4,x)
```

```
[Out] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.686 $\int \frac{\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right)$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {192, 191, 5228, 12, 385, 206}

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{5x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x*ArcSec[x])/(3*(-1 + x^2)^(3/2)) + (2*x*ArcSec[
x])/(3*Sqrt[-1 + x^2]) + (5*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 5228

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
```

```
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{x\int \frac{-3+2x^2}{3(1-x^2)^2}\,dx}{\sqrt{x^2}}$$

$$= -\frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{x\int \frac{-3+2x^2}{(1-x^2)^2}\,dx}{3\sqrt{x^2}}$$

$$= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{(5x)\int \frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}$$

$$= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{5x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 1.03

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x\left(-5\left(x^2-1\right)\log(1-x)+5\left(x^2-1\right)\log(x+1)-2x\right)+4x\left(2x^2-3\right)\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] (4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 + x^2)*Log[1
 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 0.93, size = 75, normalized size = 1.15

$$-\frac{2\,x^3-4\left(2\,x^3-3\,x\right)\sqrt{x^2-1}\,\operatorname{arcsec}(x)-5\left(x^4-2\,x^2+1\right)\log(x+1)+5\left(x^4-2\,x^2+1\right)\log(x-1)-2\,x}{12\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 - 2*x^2 + 1
)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

giac [A] time = 0.97, size = 58, normalized size = 0.89

$$\frac{\left(2\,x^{2}-3\right)x\arccos\left(\frac{1}{x}\right)}{3\left(x^{2}-1\right)^{\frac{3}{2}}}+\frac{5\,\log\left(\left|x+1\right|\right)}{12\,\mathrm{sgn}(x)}-\frac{5\,\log\left(\left|x-1\right|\right)}{12\,\mathrm{sgn}(x)}-\frac{x}{6\left(x^{2}-1\right)\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*log(abs(x + 1))/sgn(x)
 - 5/12*log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

maple [C] time = 0.57, size = 128, normalized size = 1.97

method	result	size
default	$\frac{\sqrt{x^{2}-1}\,x\left(4\,\mathrm{arcsec}(x)x^{2}-\sqrt{\frac{x^{2}-1}{x^{2}}}\,x-6\,\mathrm{arcsec}(x)\right)}{6x^{4}-12x^{2}+6}+\frac{5\sqrt{\frac{x^{2}-1}{x^{2}}}\,x\ln\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^{2}}}+1\right)}{6\sqrt{x^{2}-1}}-\frac{5\sqrt{\frac{x^{2}-1}{x^{2}}}\,x\ln\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^{2}}}-1\right)}{6\sqrt{x^{2}-1}}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(4*arcsec(x)*x^2-((x^2-1)/x^2)^(1/2)*x-6*
arcsec(x))+5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)
+1)-5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)-1)
```

maxima [A] time = 0.64, size = 48, normalized size = 0.74

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{x^{2}-1}}-\frac{x}{\left(x^{2}-1\right)^{\frac{3}{2}}}\right)\mathrm{arcsec}(x)-\frac{x}{6\left(x^{2}-1\right)}+\frac{5}{12}\,\log\left(x+1\right)-\frac{5}{12}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) + 5
/12*log(x + 1) - 5/12*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{\left(x^{2}-1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x)/(x^2 - 1)^(5/2),x)
```

```
[Out] int(acos(1/x)/(x^2 - 1)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

3.687 $\int \frac{x^2 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}} dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right)-\frac{x^3\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 5238, 12, 288, 207}

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}-\frac{x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x^3*ArcSec[x])/(3*(-1 + x^2)^(3/2)) - (x*ArcTanh
[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^2\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx = -\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} - \frac{x\int -\frac{x^2}{3\left(-1+x^2\right)^2}\,dx}{\sqrt{x^2}}$$

$$= -\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} + \frac{x\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx}{3\sqrt{x^2}}$$

$$= \frac{\sqrt{x^2}}{6\left(1-x^2\right)} - \frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} + \frac{x\int \frac{1}{-1+x^2}\,dx}{6\sqrt{x^2}}$$

$$= \frac{\sqrt{x^2}}{6\left(1-x^2\right)} - \frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} - \frac{x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 1.20

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x\left(\left(x^2-1\right)\log(1-x)-\left(x^2-1\right)\log(x+1)-2x\right)-4x^3\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
+ x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 68, normalized size = 1.33

$$-\frac{4\sqrt{x^2-1}\,x^3\,\mathrm{arcsec}(x)+2\,x^3+\left(x^4-2\,x^2+1\right)\log\left(x+1\right)-\left(x^4-2\,x^2+1\right)\log\left(x-1\right)-2\,x}{12\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*log(x + 1)
 - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

giac [A] time = 1.13, size = 53, normalized size = 1.04

$$-\frac{x^3\arccos\left(\frac{1}{x}\right)}{3\left(x^2-1\right)^{\frac{3}{2}}} - \frac{\log\left(|x+1|\right)}{12\,\mathrm{sgn}(x)} + \frac{\log\left(|x-1|\right)}{12\,\mathrm{sgn}(x)} - \frac{x}{6\left(x^2-1\right)\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*log(abs(x + 1))/sgn(x) + 1/12*l
og(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

maple [C] time = 0.62, size = 121, normalized size = 2.37

method	result	size
default	$-\frac{\sqrt{x^{2}-1}\, x^{2} \left(2 x \,\mathrm{arcsec}\left(x \right)+\sqrt{\frac{x^{2}-1}{x^{2}}}\right)}{6 \left(x^{4}-2 x^{2}+1\right)}+\frac{\sqrt{\frac{x^{2}-1}{x^{2}}}\, x \ln \left(\frac{1}{x}+i \sqrt{1-\frac{1}{x^{2}}}-1\right)}{6 \sqrt{x^{2}-1}}-\frac{\sqrt{\frac{x^{2}-1}{x^{2}}}\, x \ln \left(\frac{1}{x}+i \sqrt{1-\frac{1}{x^{2}}}+1\right)}{6 \sqrt{x^{2}-1}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(x^2-1)^(1/2)*x^2/(x^4-2*x^2+1)*(2*x*arcsec(x)+((x^2-1)/x^2)^(1/2))+1/
6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)-1)-1/6/(x^2-
1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)+1)
```

maxima [A] time = 0.64, size = 46, normalized size = 0.90

$$-\frac{1}{3} \left(\frac{x}{\sqrt{x^{2}-1}}+\frac{x}{\left(x^{2}-1\right)^{\frac{3}{2}}}\right) \mathrm{arcsec}\left(x\right)-\frac{x}{6 \left(x^{2}-1\right)}-\frac{1}{12} \log \left(x+1\right)+\frac{1}{12} \log \left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) - 1/
12*log(x + 1) + 1/12*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\mathrm{acos}\left(\frac{1}{x}\right)}{{\left(x^2-1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acos(1/x))/(x^2 - 1)^(5/2),x)
```

```
[Out] int((x^2*acos(1/x))/(x^2 - 1)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

3.688 $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(x^2-1)}{3\sqrt{x^2}} - \frac{\sec^{-1}(x)}{\sqrt{x^2-1}} - \frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 43, 5238, 12, 446, 77}

$$\frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(1-x^2)}{3\sqrt{x^2}} - \frac{\sec^{-1}(x)}{\sqrt{x^2-1}} - \frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcSec[x]/Sqrt
[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[1 - x^2])/(3*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx &= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\int\frac{2-3x^2}{3x\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\int\frac{2-3x^2}{x\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\,\mathrm{Subst}\left(\int\frac{2-3x}{(1-x)^2x}\,dx,x,x^2\right)}{6\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\,\mathrm{Subst}\left(\int\left(-\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{2}{x}\right)dx,x,x^2\right)}{6\sqrt{x^2}}\\
&= \frac{x}{6\sqrt{x^2}\left(1-x^2\right)}-\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(1-x^2\right)}{3\sqrt{x^2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 72, normalized size = 0.88

$$\frac{-\frac{\left(x^2-1\right)\left(4\left(x^2-1\right)\log(x)-2\left(x^2-1\right)\log\left(1-x^2\right)+1\right)}{\sqrt{1-\frac{1}{x^2}}\,x}-2\left(3x^2-2\right)\sec^{-1}(x)}{6\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-2*(-2 + 3*x^2)*ArcSec[x] - ((-1 + x^2)*(1 + 4*(-1 + x^2)*Log[x] - 2*(-1 +
 x^2)*Log[1 - x^2]))/(Sqrt[1 - x^(-2)]*x))/(6*(-1 + x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.18, size = 69, normalized size = 0.84

$$-\frac{2\left(3\,x^2-2\right)\sqrt{x^2-1}\,\mathrm{arcsec}(x)+x^2-2\left(x^4-2\,x^2+1\right)\log\left(x^2-1\right)+4\left(x^4-2\,x^2+1\right)\log(x)-1}{6\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*log
(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)
```

giac [A] time = 1.25, size = 64, normalized size = 0.78

$$-\frac{\left(3\,x^{2}-2\right)\arccos\left(\frac{1}{x}\right)}{3\,\left(x^{2}-1\right)^{\frac{3}{2}}}-\frac{\log\left(x^{2}\right)}{3\,\mathrm{sgn}(x)}+\frac{\log\left(\left|x^{2}-1\right|\right)}{3\,\mathrm{sgn}(x)}-\frac{2\,x^{2}-1}{6\,\left(x^{2}-1\right)\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*lo
g(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))
```

maple [C] time = 0.84, size = 197, normalized size = 2.40

method	result
default	$-\frac{4i\sqrt{\frac{x^2-1}{x^2}}\,x\,\mathrm{arcsec}(x)}{3\sqrt{x^2-1}}+\frac{\sqrt{x^2-1}\left(2i\sqrt{\frac{x^2-1}{x^2}}\,x^3-2i\sqrt{\frac{x^2-1}{x^2}}\,x-3x^2+2\right)\left(8x^4\mathrm{arcsec}(x)+2ix^4+3\sqrt{\frac{x^2-1}{x^2}}\,x^3-6\,\mathrm{arcsec}(x)x^2-4ix^2-2\sqrt{\right.}{6x^2\left(4x^6-11x^4+10x^2-3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)+1/6*(x^2-1)^(1/2)/x^2*
(2*I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-3*x^2+2)*(8*x^4*arcs
ec(x)+2*I*x^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*arcsec(x)*x^2-4*I*x^2-2*((x^2-1)/
x^2)^(1/2)*x+2*I)/(4*x^6-11*x^4+10*x^2-3)+2/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(
1/2)*x*ln((1/x+I*(1-1/x^2)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\,\mathrm{arcsec}(x)}{\left(x^{2}-1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\mathrm{acos}\left(\frac{1}{x}\right)}{\left(x^{2}-1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acos(1/x))/(x^2 - 1)^(5/2),x)
```

```
[Out] int((x^3*acos(1/x))/(x^2 - 1)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

3.689 $\int \frac{x^6 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=175

$$\frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}\left(2-3x^2\right)}{6\left(x^2-1\right)}-\frac{5x\sec^{-1}(x)}{2\sqrt{x^2-1}}-\frac{13}{6}\coth^{-1}\left(\sqrt{x^2}\right.$$

Rubi [A] time = 0.32, antiderivative size = 232, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {5242, 4702, 4706, 4710, 4181, 2279, 2391, 206, 199, 290, 325}

$$\frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)}-\frac{3\sqrt{x^2}}{4}-\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}}+\frac{x\sqrt{x^2}\,\text{se}}{2\left(1-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -5/(12*(1 - x^(-2))*Sqrt[x^2]) - (3*Sqrt[x^2])/4 + Sqrt[x^2]/(4*(1 - x^(-2)
)) - (13*Sqrt[x^2]*ArcCoth[x])/(6*x) - (5*Sqrt[x^2]*ArcSec[x])/(6*(1 - x^(-
2))^(3/2)*x) - (5*Sqrt[x^2]*ArcSec[x])/(2*Sqrt[1 - x^(-2)]*x) + (x*Sqrt[x^2
]*ArcSec[x])/(2*(1 - x^(-2))^(3/2)) - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(
I*ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x
 - (((5*I)/2)*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4706

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCos[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1)),
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Dist[(b*c*n
*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 - c^2*x^2)^FracPart[
p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{x^6 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x^3\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{x\sqrt{x^2}\,\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)^2}\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2x}$$

$$= \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} + \frac{x\sqrt{x^2}\,\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx, x, \frac{1}{x}\right)}{4x} - \ldots$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{x\sqrt{x^2}\,\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right.\ldots}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\,\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\,\ldots}{2\sqrt{1-\ldots}}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\,\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\,\ldots}{2\sqrt{1-\ldots}}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\,\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\,\ldots}{2\sqrt{1-\ldots}}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\,\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\,\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\,\ldots}{2\sqrt{1-\ldots}}$$

Mathematica [B] time = 1.99, size = 383, normalized size = 2.19

$$-\frac{x^5\left(-60i\sqrt{1-\frac{1}{x^2}}\,\sin^2\left(2\sec^{-1}(x)\right)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+60i\sqrt{1-\frac{1}{x^2}}\,\sin^2\left(2\sec^{-1}(x)\right)\text{PolyLog}\left(2,ie^{i\sec\ldots}\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -1/96*(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x]*Cos
[4*ArcSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])] +
30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*Sqrt[1 - x^(-
2)]*Log[Cos[ArcSec[x]/2]] - 26*Sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] + 16*
Sin[2*ArcSec[x]] - (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec[x])]
*Sin[2*ArcSec[x]]^2 + (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSec[x])]
*Sin[2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*ArcSec[
x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13*Log[Cos
[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*ArcSec[x]]
 - 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[5*ArcSe
c[x]] - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 13*Log[C
```

```
os[ArcSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Sin[5*ArcSec[x
]]])/(-1 + x^2)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
[Out] Could not integrate
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-1}\,x^6\,\text{arcsec}(x)}{x^6-3\,x^4+3\,x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\,\text{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

maple [A] time = 0.89, size = 240, normalized size = 1.37

method	result
default	$\frac{\sqrt{x^2-1}\,x\left(3x^4\text{arcsec}(x)-3\sqrt{\frac{x^2-1}{x^2}}\,x^3-20\,\text{arcsec}(x)x^2+2\sqrt{\frac{x^2-1}{x^2}}\,x+15\,\text{arcsec}(x)\right)}{6x^4-12x^2+6}+\frac{i\sqrt{\frac{x^2-1}{x^2}}\,x\left(15i\text{arcsec}(x)\ln\left(1+i\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^2}}\right)\right)-15\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(3*x^4*arcsec(x)-3*((x^2-1)/x^2)^(1/2)*x^
3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15*arcsec(x))+1/6*I/(x^2-1)^(1/2
)*((x^2-1)/x^2)^(1/2)*x*(15*I*arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))-15*
I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))+13*I*ln(1/x+I*(1-1/x^2)^(1/2)+1
)-13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1)+15*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))-15
*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\,\text{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6 \operatorname{acos}\left(\frac{1}{x}\right)}{\left(x^2-1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*acos(1/x))/(x^2 - 1)^(5/2),x)
```

```
[Out] int((x^6*acos(1/x))/(x^2 - 1)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\,\sec^{-1}(x)}{x}$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {264, 5238, 30}

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\,\sec^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{-1+x^2}\,\sec^{-1}(x)}{x}-\frac{x\int \frac{1}{x^2}\,dx}{\sqrt{x^2}}\\ &= \frac{1}{\sqrt{x^2}}+\frac{\sqrt{-1+x^2}\,\sec^{-1}(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 1.52

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x+\left(x^2-1\right)\sec^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.17, size = 16, normalized size = 0.70

$$\frac{\sqrt{x^2-1}\,\operatorname{arcsec}(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

giac [B] time = 1.04, size = 50, normalized size = 2.17

$$\frac{2\arccos\left(\frac{1}{x}\right)}{\left(x-\sqrt{x^2-1}\right)^2+1}-\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\operatorname{sgn}(x)}+\frac{1}{x\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^2 - 1))/sg
n(x) + 1/(x*sgn(x))
```

maple [C] time = 0.51, size = 178, normalized size = 7.74

method	result
default	$-\frac{\sqrt{\frac{x^2-1}{x^2}}\,x^3-3ix^2-4\sqrt{\frac{x^2-1}{x^2}}\,x+4i}{4\sqrt{x^2-1}\left(i\sqrt{\frac{x^2-1}{x^2}}\,x+1\right)x}+\frac{\left(x^2-2-2i\sqrt{\frac{x^2-1}{x^2}}\,x\right)\operatorname{arcsec}(x)}{4\sqrt{x^2-1}\,x}-\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x-1\right)(\operatorname{arcsec}(x)+i)}{4\sqrt{x^2-1}\,x}+\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)(3\operatorname{arcs}}{4\sqrt{x^2-1}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2)*x+4*I)/(x^2-1)^
(1/2)/(I*((x^2-1)/x^2)^(1/2)*x+1)/x+1/4/(x^2-1)^(1/2)/x*(x^2-2-2*I*((x^2-1)
/x^2)^(1/2)*x)*arcsec(x)-1/4/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x-1)*(arc
sec(x)+I)/x+1/4/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(3*arcsec(x)-
I)/x
```

maxima [A] time = 1.03, size = 17, normalized size = 0.74

$$\frac{\sqrt{x^2-1}\,\operatorname{arcsec}(x)}{x}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acos}\left(\frac{1}{x}\right)}{x^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)),x)
```

```
[Out] int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(x)}{x^2\sqrt{(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(asec(x)/(x**2*sqrt((x - 1)*(x + 1))), x)
```

3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2}}{6\left(x^2-1\right)}-\frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right)+\frac{\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{3x\left(x^2-1\right)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {271, 192, 191, 5239, 12, 1259, 453, 206}

$$-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}-\frac{11x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] -(1/Sqrt[x^2]) - Sqrt[x^2]/(6*(1 - x^2)) + ArcCsc[x]/(x*(-1 + x^2)^(3/2)) -
 (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8*x*ArcCsc[x])/(3*Sqrt[-1 + x^2])
- (11*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx &= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{3x^2\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{x^2\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= -\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int\frac{-6+17x^2}{x^2\left(1-x^2\right)}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{(11x)\int\frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{11x\tanh^{-1}(x)}{6\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 79, normalized size = 1.13

$$
\frac{\sqrt{1-\frac{1}{x^2}}\,x\left(-10x^2+11\left(x^2-1\right)x\log(1-x)-11\left(x^2-1\right)x\log(x+1)+12\right)+4\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{12x\left(x^2-1\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] (4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x^2 + 11*x*
(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(12*x*(-1 + x^2)^(3/2)
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.09, size = 81, normalized size = 1.16

$$-\frac{10\,x^4-4\left(8\,x^4-12\,x^2+3\right)\sqrt{x^2-1}\,\operatorname{arccsc}(x)-22\,x^2+11\left(x^5-2\,x^3+x\right)\log\left(x+1\right)-11\left(x^5-2\,x^3+x\right)\log}{12\left(x^5-2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) - 22*x^2 + 1
1*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*log(x - 1) + 12)/(x^5
 - 2*x^3 + x)
```

giac [A] time = 1.19, size = 105, normalized size = 1.50

$$\frac{1}{3}\left(\frac{\left(5\,x^2-6\right)x}{\left(x^2-1\right)^{\frac{3}{2}}}+\frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right)+\frac{2\,\arctan\left(-x+\sqrt{x^2-1}\right)}{\operatorname{sgn}(x)}-\frac{11\,\log\left(|x+1|\right)}{12\,\operatorname{sgn}(x)}+\frac{11\,\log\left(|x-1|\right.}{12\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1))*arcsin(
1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sgn(x) - 11/12*log(abs(x + 1))/sgn(x) +
 11/12*log(abs(x - 1))/sgn(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*sgn(x))
```

maple [C] time = 0.83, size = 702, normalized size = 10.03

method	result
default	$-\frac{3ix^2-4i-4\sqrt{\frac{x^2-1}{x^2}}\,x+\sqrt{\frac{x^2-1}{x^2}}\,x^3}{4\left(i\sqrt{\frac{x^2-1}{x^2}}\,x-1\right)x\sqrt{x^2-1}}+\frac{\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}\,x\right)\operatorname{arccsc}(x)}{4\sqrt{x^2-1}\,x}+\frac{x\operatorname{arccsc}(x)}{2\sqrt{x^2-1}}+\frac{\left(x^2-2-2i\sqrt{\frac{x^2-1}{x^2}}\,x\right)\operatorname{arccsc}(x)}{4\sqrt{x^2-1}\,x}+\frac{}{4\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x)/x^2/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(I*((x^2-1)/x^2)^(1/2)*x-1)/x/(x^2-1)^(1/2)*(3*I*x^2-4*I-4*((x^2-1)/x^
2)^(1/2)*x+((x^2-1)/x^2)^(1/2)*x^3)+1/4/(x^2-1)^(1/2)/x*(x^2-2+2*I*((x^2-1)
/x^2)^(1/2)*x)*arccsc(x)+1/2*x*arccsc(x)/(x^2-1)^(1/2)+1/4/(x^2-1)^(1/2)*(x
^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/x+1/4*x^3/(x^2-1)^(1/2)/(I*x^2-2*
((x^2-1)/x^2)^(1/2)*x-2*I)-1/24*x^5*(((x^2-1)/x^2)^(1/2)*x+I)/(x^2-1)^(1/2)
/(I*((x^2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/
```

```
x^2)^(1/2)*x+7*x^2-4)+1/24*x*(5*I*x^4-20*I*x^2-12*((x^2-1)/x^2)^(1/2)*x^3+(
(x^2-1)/x^2)^(1/2)*x^5+16*I+16*((x^2-1)/x^2)^(1/2)*x)/(x^2-1)^(1/2)/(I*((x^
2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/x^2)^(1/
2)*x+7*x^2-4)+2/3*(x^2-1)^(1/2)*x^3/(x^4-2*x^2+1)*arccsc(x)+1/2*(x^2-1)^(1/
2)*x*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/(x^4-2*x^2+1)+1/2*(x^2-1)^
(1/2)*x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/(x^4-2*x^2+1)+11/12/(x^
2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*ln(I/x+(1-1/x^2)^(1/2)-I)+11/12/(x^2-1
)^(1/2)*(((x^2-1)/x^2)^(1/2)*x-I)*ln(I/x+(1-1/x^2)^(1/2)-I)-11/12/(x^2-1)^(
1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*ln(I/x+(1-1/x^2)^(1/2)+I)-11/12/(x^2-1)^(1/2
)*(((x^2-1)/x^2)^(1/2)*x-I)*ln(I/x+(1-1/x^2)^(1/2)+I)
```

maxima [B] time = 2.21, size = 123, normalized size = 1.76

$$\frac{32\,x^4\arctan\left(1,\sqrt{x+1}\,\sqrt{x-1}\right)-\left(x^3-x\right)\sqrt{x+1}\,\sqrt{x-1}\left(\frac{2\left(5\,x^2-6\right)}{x^3-x}+11\,\log\left(x+1\right)-11\,\log\left(x-1\right)\right)-48\,x^2\,\mathrm{arct}}{12\left(x^3-x\right)\sqrt{x+1}\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(32*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - (x^3 - x)*sqrt(x + 1)*sq
rt(x - 1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*log(x + 1) - 11*log(x - 1)) - 48*x^
2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 12*arctan2(1, sqrt(x + 1)*sqrt(x -
1)))/((x^3 - x)*sqrt(x + 1)*sqrt(x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asin}\left(\frac{1}{x}\right)}{x^2\,{\left(x^2-1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)),x)
```

```
[Out] int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{24\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2-1}\,\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{x^2-1}\,\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

Rubi [A] time = 0.18, antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5243, 4677, 4619, 261}

$$\frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\,\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\,\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*Sqrt[1 - x^(-2)]*Sqrt[x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[1 -
 x^(-2)]*Sqrt[x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[1 - x
^(-2)]*Sqrt[x^2]*ArcCsc[x]^4)/x
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 5243

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcSin[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{x\sin^{-1}(x)^4}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(4\sqrt{x^2}\right)\text{Subst}\left(\int \sin^{-1}(x)^3\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(12\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x\sin^{-1}(x)^2}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(24\sqrt{x^2}\right)\text{Subst}\left(\int \text{si}\right.}{x}$$

$$= \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(24\sqrt{x^2}\right.}{}$$

$$= \frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x} + \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\,\text{cs}}{x}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 1.03

$$\frac{24\left(x^2-1\right)+\left(x^2-1\right)\csc^{-1}(x)^4-4\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x)^3-12\left(x^2-1\right)\csc^{-1}(x)^2+24\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]^
2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1 +
 x^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.19, size = 37, normalized size = 0.50

$$-\frac{4\,\text{arccsc}(x)^3-\left(\text{arccsc}(x)^4-12\,\text{arccsc}(x)^2+24\right)\sqrt{x^2-1}\,-24\,\text{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 - 1) - 24*ar
ccsc(x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccsc}(x)^4}{\sqrt{x^2-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

maple [C] time = 0.66, size = 330, normalized size = 4.46

method	result
default	$\frac{\left(i x^2-2\sqrt{\frac{x^2-1}{x^2}}\, x-2i\right)\operatorname{arccsc}(x)^3}{\sqrt{x^2-1}\, x}+\frac{\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}\, x\right)\operatorname{arccsc}(x)^4}{4\sqrt{x^2-1}\, x}-\frac{6\left(i x^2-2\sqrt{\frac{x^2-1}{x^2}}\, x-2i\right)\operatorname{arccsc}(x)}{\sqrt{x^2-1}\, x}-\frac{3\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}\, x\right)\mathrm{arc}}{\sqrt{x^2-1}\, x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(x^2-1)^(1/2)/x*(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)^3+1/4/(x^2-
1)^(1/2)/x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^4-6/(x^2-1)^(1/2)/x*
(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)-3/(x^2-1)^(1/2)/x*(x^2-2+2*I*
((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^2+6*(I*((x^2-1)/x^2)^(1/2)*x^3-4*I*((x^2-1
)/x^2)^(1/2)*x-3*x^2+4)/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x-1)/x+1/4/(x^
2-1)^(1/2)*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(3*arccsc(x)^4-4*I*arccsc(x)^3-
36*arccsc(x)^2+24*I*arccsc(x)+72)/x+1/4/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2
)*x+1)*(arccsc(x)^4+4*I*arccsc(x)^3-12*arccsc(x)^2-24*I*arccsc(x)+24)/x
```

maxima [A] time = 1.03, size = 58, normalized size = 0.78

$$\frac{\sqrt{x^2-1}\,\operatorname{arccsc}(x)^4}{x}-12\sqrt{-\frac{1}{x^2}+1}\,\operatorname{arccsc}(x)^2-\frac{4\,\operatorname{arccsc}(x)^3}{x}+24\sqrt{-\frac{1}{x^2}+1}+\frac{24\,\operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x)
^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}\left(\frac{1}{x}\right)^4}{x^2\,\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)),x)
```

```
[Out] int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsc}^4(x)}{x^2\sqrt{(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(acsc(x)**4/(x**2*sqrt((x - 1)*(x + 1))), x)
```

3.693 $\int \frac{\left(-1+x^2\right)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx$

Optimal. Leaf size=133

$$\frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}-\frac{3\sqrt{x^2-1}\sec^{-1}(x)^2}{8x^2}+\frac{9x\sec^{-1}(x)}{64\sqrt{x^2}}-\frac{3\sec^{-1}(x)}{8x\sqrt{x^2}}+\frac{\sqrt{x^2-1}\left(17x^2-2\right)}{64x^4}-\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)^2}{4x^4}+\frac{\left(x^2-1\right)^2}{8x^3}$$

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {5242, 4650, 4648, 4642, 4628, 321, 216, 4678, 195}

$$\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}}+\frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}}-\frac{9\sqrt{x^2}\csc^{-1}(x)}{64x}+\frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (15*Sqrt[1 - x^(-2)])/(64*Sqrt[x^2]) + (1 - x^(-2))^(3/2)/(32*Sqrt[x^2]) -
(9*Sqrt[x^2]*ArcCsc[x])/(64*x) - (3*Sqrt[x^2]*ArcSec[x])/(8*x^3) + ((1 - x^
(-2))^2*Sqrt[x^2]*ArcSec[x])/(8*x) - (3*Sqrt[1 - x^(-2)]*ArcSec[x]^2)/(8*Sq
rt[x^2]) - ((1 - x^(-2))^(3/2)*ArcSec[x]^2)/(4*Sqrt[x^2]) + (Sqrt[x^2]*ArcS
ec[x]^3)/(8*x)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
```

```
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = -\frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int (1-x^2)^{3/2}\cos^{-1}(x)^2\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} - \frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int x(1-x^2)\cos^{-1}(x)\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(3\sqrt{x^2}\right)\mathrm{Su}}{}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\,\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\,\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\,\mathrm{Sub}}{}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\,\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\,\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\,\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{(}{}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\,\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\,\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}}{8\sqrt{}}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{9\sqrt{x^2}\,\csc^{-1}(x)}{64x} - \frac{3\sqrt{x^2}\,\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\,\mathrm{sec}}{8x}$$

Mathematica [A] time = 0.31, size = 84, normalized size = 0.63

$$\frac{\sqrt{x^2-1}\left(32\sec^{-1}(x)^3 + 4\sec^{-1}(x)\left(\cos\left(4\sec^{-1}(x)\right) - 16\cos\left(2\sec^{-1}(x)\right)\right) + 8\sec^{-1}(x)^2\left(\sin\left(4\sec^{-1}(x)\right) - 8\sin\right.\right.}{256\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[4
*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8*S
in[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 59, normalized size = 0.44

$$\frac{8\,x^4\,\mathrm{arcsec}(x)^3 + \left(17\,x^4 - 40\,x^2 + 8\right)\mathrm{arcsec}(x) - \left(8\left(5\,x^2-2\right)\mathrm{arcsec}(x)^2 - 17\,x^2 + 2\right)\sqrt{x^2-1}}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5*x^2 - 2)*
arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

maple [C] time = 0.70, size = 386, normalized size = 2.90

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}}\, x \operatorname{arcsec}(x)^3}{8\sqrt{x^2-1}} + \frac{\left(-5i\sqrt{\frac{x^2-1}{x^2}}\, x^5+x^6+20i\sqrt{\frac{x^2-1}{x^2}}\, x^3-13x^4-16i\sqrt{\frac{x^2-1}{x^2}}\, x+28x^2-16\right)\left(4i\operatorname{arcsec}(x)+8\operatorname{arcsec}(x)^2-1\right)}{1024\sqrt{x^2-1}\, x^4} - \left(-i\sqrt{\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)^3+1/1024/(x^2-1)^(1/2)/x^
4*(-5*I*((x^2-1)/x^2)^(1/2)*x^5+x^6+20*I*((x^2-1)/x^2)^(1/2)*x^3-13*x^4-16*
I*((x^2-1)/x^2)^(1/2)*x+28*x^2-16)*(4*I*arcsec(x)+8*arcsec(x)^2-1)-1/32/(x^
2-1)^(1/2)*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(2*arcsec(x)^2-1+2*I*arcsec(x))
+1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*
x-2*x^2+2)*(2*arcsec(x)^2-1-2*I*arcsec(x))-1/1024/(x^2-1)^(1/2)*(-5*x^2+4+3
*I*((x^2-1)/x^2)^(1/2)*x^3+x^4-4*I*((x^2-1)/x^2)^(1/2)*x)*(-4*I*arcsec(x)+8
*arcsec(x)^2-1)/x^2+1/128/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(7*
I*arcsec(x)+8*arcsec(x)^2-4)*cos(4*arcsec(x))+1/512/(x^2-1)^(1/2)*(I*x^2-((
x^2-1)/x^2)^(1/2)*x-I)*(32*I*arcsec(x)+24*arcsec(x)^2-15)*sin(4*arcsec(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{x}\right)^2 (x^2-1)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5,x)
```

```
[Out] int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

```
[Out] Timed out
```

3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

Optimal. Leaf size=110

$$\frac{2(1-21x^2)}{27(x^2)^{3/2}}+\frac{(x^2-1)\sec^{-1}(x)^2}{3(x^2)^{3/2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{4\sqrt{x^2-1}\sec^{-1}(x)}{3x}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)^3}{3x^3}-\frac{2(x^2-1)^{3/2}\sec^{-1}(x)}{9x^3}$$

Rubi [A] time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5242, 4678, 4650, 4620, 8}

$$\frac{2\sqrt{x^2}}{27x^4}-\frac{14}{9\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}+\frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}-\frac{4\sqrt{1-\frac{1}{x^2}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] -14/(9*Sqrt[x^2]) + (2*Sqrt[x^2])/(27*x^4) - (4*Sqrt[1 - x^(-2)]*Sqrt[x^2]*
ArcSec[x])/(3*x) - (2*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x])/(9*x) + (2*Ar
cSec[x]^2)/(3*Sqrt[x^2]) + ((1 - x^(-2))*ArcSec[x]^2)/(3*Sqrt[x^2]) + ((1 -
 x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x]^3)/(3*x)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
```

```
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int x\sqrt{1-x^2}\cos^{-1}(x)^3\,dx,x,\frac{1}{x}\right)}{x}$$
$$=\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}+\frac{\sqrt{x^2}\,\text{Subst}\left(\int\left(1-x^2\right)\cos^{-1}(x)^2\,dx,x,\frac{1}{x}\right)}{x}$$
$$=\frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}+\frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int x\sqrt{1-x^2}\,\text{co}\right.}{3x}$$
$$=-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x}}{3x}$$
$$=-\frac{2}{9\sqrt{x^2}}+\frac{2\sqrt{x^2}}{27x^4}-\frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x}-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}+\frac{2\sec}{3\sqrt{}}$$
$$=-\frac{14}{9\sqrt{x^2}}+\frac{2\sqrt{x^2}}{27x^4}-\frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x}-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}+\frac{2\sec}{3\sqrt{}}$$

Mathematica [A] time = 0.11, size = 92, normalized size = 0.84

$$\frac{2\sqrt{1-\frac{1}{x^2}}\,x\left(1-21x^2\right)+9\left(x^2-1\right)^2\sec^{-1}(x)^3+9\sqrt{1-\frac{1}{x^2}}\,x\left(3x^2-1\right)\sec^{-1}(x)^2-6\left(7x^4-8x^2+1\right)\sec^{-1}(x)}{27x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*Sq
rt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(27
*x^3*Sqrt[-1 + x^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.06, size = 57, normalized size = 0.52

$$\frac{9\left(3x^2-1\right)\text{arcsec}(x)^2-42x^2+3\left(3\left(x^2-1\right)\text{arcsec}(x)^3-2\left(7x^2-1\right)\text{arcsec}(x)\right)\sqrt{x^2-1}+2}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x)^3 - 2*(
7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}\,\text{arcsec}(x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

maple [C] time = 0.84, size = 536, normalized size = 4.87

method	result
default	$\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x^5-8i\sqrt{\frac{x^2-1}{x^2}}\,x^3+4x^4+8i\sqrt{\frac{x^2-1}{x^2}}\,x-12x^2+8\right)\text{arcsec}(x)^3}{48\sqrt{x^2-1}\,x^3}+\frac{\sqrt{x^2-1}\left(\sqrt{\frac{x^2-1}{x^2}}\,x^5-5ix^4-12\sqrt{\frac{x^2-1}{x^2}}\,x^3+20ix^2+16\sqrt{\frac{x^2-1}{x^2}}\,x-16\right.}{216\left(-i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/(x^2-1)^(1/2)/x^3*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x
^3+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)^3+1/216*(x^2-1)^(1/2
)*(((x^2-1)/x^2)^(1/2)*x^5-5*I*x^4-12*((x^2-1)/x^2)^(1/2)*x^3+20*I*x^2+16*(
(x^2-1)/x^2)^(1/2)*x-16*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x^3-1/72/(x^2-1
)^(1/2)/x^3*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*
I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)-1/48/(x^2-1)^(1/2)/x^3*(((x^2-1
)/x^2)^(1/2)*x^5-4*I*x^4-8*((x^2-1)/x^2)^(1/2)*x^3+12*I*x^2+8*((x^2-1)/x^2)
^(1/2)*x-8*I)*arcsec(x)^2+1/216/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-
1)*(9*arcsec(x)^3-117*I*arcsec(x)^2-78*arcsec(x)+242*I)/x-1/216*(45*I*arcse
c(x)^2+9*arcsec(x)^3-82*I-78*arcsec(x))*(I*((x^2-1)/x^2)^(1/2)*x-1)*(x^2-1)
^(1/2)/x-1/432/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(63*I*arcsec(x
)^2+27*arcsec(x)^3-158*I-162*arcsec(x))*cos(3*arcsec(x))-1/144/(x^2-1)^(1/2
)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*(27*I*arcsec(x)^2+3*arcsec(x)^3-54*I-50*a
rcsec(x))*sin(3*arcsec(x))
```

maxima [A] time = 2.19, size = 93, normalized size = 0.85

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}\text{arcsec}(x)^3}{3\,x^3}+\frac{\left(3\,x^2-1\right)\text{arcsec}(x)^2}{3\,x^3}-\frac{2\left(\left(21\,x^2-1\right)\sqrt{x+1}\,\sqrt{x-1}+3\left(7\,x^4-8\,x^2+1\right)\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\right)\right)}{27\,\sqrt{x+1}\,\sqrt{x-1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/2
7*((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sqrt
(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{acos}\left(\frac{1}{x}\right)^3\sqrt{x^2-1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4,x)
```

```
[Out] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)} \operatorname{asec}^3(x)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)**3/x**4, x)
```

3.695 $\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal. Leaf size=55

$$(a+x)\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)-\frac{\sqrt{2}\,a\sqrt{\frac{x-a}{a+x}}}{\sqrt{\frac{a}{a+x}}}$$

Rubi [B] time = 0.84, antiderivative size = 118, normalized size of antiderivative = 2.15, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4840, 12, 6677, 6720, 385, 217, 206}

$$-\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x)+x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{a\sqrt{\frac{a}{a+x}}\tanh^{-1}\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] -(Sqrt[2]*Sqrt[a/(a + x)]*Sqrt[-((a - x)/(a + x))]*(a + x)) + x*ArcSin[Sqrt
[-((a - x)/(a + x))]] - (a*Sqrt[a/(a + x)]*ArcTanh[Sqrt[-((a - x)/(a + x))]
/(Sqrt[2]*Sqrt[-(a/(a + x))])])/Sqrt[-(a/(a + x))]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 6677

```
Int[(u_)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(p_), x_Symbol] :> Dist[(c^IntPa
rt[p]*(c*(a + b*x)^n)^FracPart[p])/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*
```

```
x)^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[
u, x^(n1_.)*(v_.) /; EqQ[n, n1 + 1]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{2}\,a\sqrt{\frac{-a+x}{a+x}}}\,dx$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{\frac{-a+x}{a+x}}}\,dx}{\sqrt{2}\,a}$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(\sqrt{\frac{a}{a+x}}\sqrt{a+x}\right)\int \frac{x}{\sqrt{\frac{-a+x}{a+x}}\,(a+x)^{3/2}}\,dx}{\sqrt{2}}$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \left(a\sqrt{\frac{a}{a+x}}\sqrt{a+x}\right)\text{Subst}\left(\int \frac{1+x^2}{\sqrt{-\frac{a}{-1+x^2}}\left(-1+x^2\right)^2}\,dx, x, \sqrt{\right.$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1+x^2}{\left(-1+x^2\right)^{3/2}}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}\,(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\right.}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}\,(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}\,(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{a\sqrt{\frac{a}{a+x}}\tanh^{-1}\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

Mathematica [A] time = 0.17, size = 99, normalized size = 1.80

$$x\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right) + \frac{\sqrt{\frac{a}{a+x}}\left(\sqrt{2}\sqrt{a}\sqrt{x-a}\tan^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right) + 2a - 2x\right)}{\sqrt{2}\sqrt{\frac{x-a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*Sq
rt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[2]*Sqrt[(
-a + x)/(a + x)])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.14, size = 51, normalized size = 0.93

$$-\sqrt{2}\,(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(sq
rt(-(a - x)/(a + x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arcsin(sqrt(-(a - x)/(a + x))), x)
```

maple [A] time = 0.06, size = 87, normalized size = 1.58

method	result	size
default	$x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)-\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\left(-\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)+2\sqrt{-a+x}\right)}{2\sqrt{-\frac{a-x}{a+x}}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsin(((-a+x)/(a+x))^(1/2))-1/2/(-(a-x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/2
)*(a/(a+x))^(1/2)*(-a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2)*2^(1/2)/a^(1/2)
)+2*(-a+x)^(1/2))
```

maxima [B] time = 1.01, size = 103, normalized size = 1.87

$$a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(a
 + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt(-
(a - x)/(a + x)) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}\left(\sqrt{-\frac{a-x}{a+x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin((-(a - x)/(a + x))^(1/2)),x)
[Out] int(asin((-(a - x)/(a + x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(((-a+x)/(a+x))**(1/2)),x)
[Out] Integral(asin(sqrt((-a + x)/(a + x))), x)
```

3.696 $\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal. Leaf size=40

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5203, 12, 1961, 208}

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right)dx = x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \int \frac{a}{2\sqrt{\frac{-a+x}{a+x}}(a+x)}dx$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2}a\int \frac{1}{\sqrt{\frac{-a+x}{a+x}}(a+x)}dx$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - (2a^2)\,\text{Subst}\left(\int \frac{1}{2a-2ax^2}dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Mathematica [A] time = 0.06, size = 71, normalized size = 1.78

$$x\tan^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right) - \frac{a\sqrt{x-a}\tanh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{a+x}}\right)}{\sqrt{\frac{x-a}{a+x}}\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
[Out] x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[-a + x]/Sqr
t[a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right)dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
[Out] Could not integrate
```

fricas [A] time = 1.12, size = 58, normalized size = 1.45

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2}a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \frac{1}{2}a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
[Out] x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x)) + 1) +
1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

giac [A] time = 1.19, size = 49, normalized size = 1.22

$$\frac{1}{2}a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\text{sgn}(a+x) + x\arctan\left(\frac{\sqrt{-a^2+x^2}\,\text{sgn}(a+x)}{a+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^2
)*sgn(a + x)/(a + x))
```

maple [A] time = 0.06, size = 66, normalized size = 1.65

method	result	size
default	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x)a\ln\left(x+\sqrt{-a^2+x^2}\right)}{2\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(((-a+x)/(a+x))^(1/2))+1/2*(a-x)*a*ln(x+(-a^2+x^2)^(1/2))/(-(a-x)/(
a+x))^(1/2)/(-(a-x)*(a+x))^(1/2)
```

maxima [B] time = 0.98, size = 89, normalized size = 2.22

$$\frac{1}{2}a\left(\frac{4\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}-2\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2*arctan(sq
rt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1) + log(sqrt(-(a - x)
/(a + x)) - 1))
```

mupad [B] time = 0.36, size = 36, normalized size = 0.90

$$x\,\mathrm{atan}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((-(a - x)/(a + x))^(1/2)),x)
```

```
[Out] x*atan((-(a - x)/(a + x))^(1/2)) - a*atanh((-(a - x)/(a + x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] Integral(atan(sqrt((-a + x)/(a + x))), x)
```

3.697 $\int \frac{\tan^{-1}(x)}{(1+x)^3} dx$

Optimal. Leaf size=39

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4862, 710, 801, 260}

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] -1/(4*(1 + x)) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 + x^2]/8
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)}{(1+x)^3}\,dx = -\frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{2}\int \frac{1}{(1+x)^2\left(1+x^2\right)}\,dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\int \frac{1-x}{(1+x)\left(1+x^2\right)}\,dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\int \left(\frac{1}{1+x} - \frac{x}{1+x^2}\right)dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{4}\int \frac{x}{1+x^2}\,dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{8}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.90

$$\frac{1}{8}\left(-\log\left(x^2+1\right) - \frac{2}{x+1} + 2\log(x+1) - \frac{4\tan^{-1}(x)}{(x+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] (-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(x)}{(1+x)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] Could not integrate
```

fricas [A] time = 1.06, size = 50, normalized size = 1.28

$$-\frac{\left(x^2+2x+1\right)\log\left(x^2+1\right) - 2\left(x^2+2x+1\right)\log\left(x+1\right) + 2x + 4\arctan(x) + 2}{8\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1) + 2*x + 4
*arctan(x) + 2)/(x^2 + 2*x + 1)
```

giac [A] time = 0.93, size = 32, normalized size = 0.82

$$-\frac{1}{4(x+1)} - \frac{\arctan(x)}{2(x+1)^2} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{4}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x +
 1))
```

maple [A] time = 0.36, size = 32, normalized size = 0.82

method	result	size
default	$-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{8}$	32
risch	$\frac{i\ln(ix+1)}{4(1+x)^2}-\frac{i(2i\ln(1+x)x^2-i\ln(x^2+1)x^2+4i\ln(1+x)x-2i\ln(x^2+1)x+2i\ln(1+x)-i\ln(x^2+1)-2ix-2i+2\ln(-ix+1))}{8(1+x)^2}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/(1+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

maxima [A] time = 0.96, size = 31, normalized size = 0.79

$$-\frac{1}{4(x+1)}-\frac{\arctan(x)}{2(x+1)^2}-\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(x + 1)
```

mupad [B] time = 0.36, size = 31, normalized size = 0.79

$$\frac{\ln(x+1)}{4}-\frac{\ln\left(x^2+1\right)}{8}-\frac{\frac{x}{4}+\frac{\operatorname{atan}(x)}{2}+\frac{1}{4}}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x + 1)^3,x)
```

```
[Out] log(x + 1)/4 - log(x^2 + 1)/8 - (x/4 + atan(x)/2 + 1/4)/(x + 1)^2
```

sympy [B] time = 0.64, size = 153, normalized size = 3.92

$$\frac{2x^2\log(x+1)}{8x^2+16x+8}-\frac{x^2\log\left(x^2+1\right)}{8x^2+16x+8}+\frac{4x\log(x+1)}{8x^2+16x+8}-\frac{2x\log\left(x^2+1\right)}{8x^2+16x+8}-\frac{2x}{8x^2+16x+8}+\frac{2\log(x+1)}{8x^2+16x+8}-\frac{\log\left(x^2+1\right)}{8x^2+16x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/(1+x)**3,x)
```

```
[Out] 2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**2 + 16*x +
 8) + 4*x*log(x + 1)/(8*x**2 + 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 + 16*x
 + 8) - 2*x/(8*x**2 + 16*x + 8) + 2*log(x + 1)/(8*x**2 + 16*x + 8) - log(x*
*2 + 1)/(8*x**2 + 16*x + 8) - 4*atan(x)/(8*x**2 + 16*x + 8) - 2/(8*x**2 + 1
6*x + 8)
```

3.698 $\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx$

Optimal. Leaf size=122

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\text{lo}$$

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I -
 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)*P
olyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx = \text{Subst}\left(\int \frac{\tan^{-1}(x)}{2a-x}\,dx, x, a-x\right)$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \text{Subst}\left(\int\right.$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) + \frac{1}{2}i\text{Li}_2\left(1-\right.$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\text{Li}_2\left(1-\right.$$

Mathematica [A] time = 0.03, size = 105, normalized size = 0.86

$$-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{a-x+i}{2a+i}\right) - \text{PolyLog}\left(2,\frac{-a+x+i}{-2a+i}\right) - \log(1+i(a-x))\log\left(\frac{a+x}{2a-i}\right) + \log(-ia+ix+1)\,\text{lo}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] (-1/2*I)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*x
]*Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2, (
I - a + x)/(I - 2*a)])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] Could not integrate
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(-a+x)}{a+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")
```

```
[Out] integral(arctan(-a + x)/(a + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.37, size = 102, normalized size = 0.84

method	result
derivativedivides	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i-a+x}{i-2a}\right)}{2}$
default	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i-a+x}{i-2a}\right)}{2}$
risch	$-\frac{i\,\mathrm{dilog}\left(\frac{ia+ix}{2ia-1}\right)}{2}-\frac{i\ln(-ia+ix+1)\ln\left(\frac{ia+ix}{2ia-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-ia-ix}{-2ia-1}\right)}{2}+\frac{i\ln(ia-ix+1)\ln\left(\frac{-ia-ix}{-2ia-1}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(a-x)/(a+x),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((I+a-x)/(2*a+I))-1/2*I*ln(a+x)*ln((I-
a+x)/(I-2*a))+1/2*I*dilog((I+a-x)/(2*a+I))-1/2*I*dilog((I-a+x)/(I-2*a))
```

maxima [A] time = 1.20, size = 118, normalized size = 0.97

$$-\frac{1}{2}\arctan\left(\frac{a+x}{4a^2+1},\frac{2\left(a^2+ax\right)}{4a^2+1}\right)\log\left(a^2-2ax+x^2+1\right)+\frac{1}{2}\arctan(-a+x)\log\left(\frac{a^2+2ax+x^2}{4a^2+1}\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(a^2 - 2*a*
x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^2)/(4*a^2 + 1)) - 1/
2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/2*I*dilog(-(-I*a + I*x - 1)/(2
*I*a + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atan(a - x)/(a + x),x)
```

```
[Out] -int(atan(a - x)/(a + x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(a-x)/(a+x),x)
```

```
[Out] -Integral(atan(a - x)/(a + x), x)
```

3.699 $\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4834, 4641}

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

```
[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4834

```
Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[-(b*x^2)]/(b*x), Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx, x, \sqrt{1-x^2}\right)}{x}\\ &= -\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.00

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
[Out] Could not integrate
```

fricas [A] time = 1.10, size = 14, normalized size = 0.50

$$-\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*arcsin(sqrt(-x^2 + 1))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
[Out] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{asin}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2),x)
[Out] int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2), x)
```

sympy [**A**] time = 2.56, size = 22, normalized size = 0.79

$$-\frac{\sqrt{x^2}\,\mathrm{asin}^2\left(\sqrt{1-x^2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(x**2)*asin(sqrt(1 - x**2))**2/(2*x)
```

3.700 $\int \frac{x\tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=31

$$\sqrt{x^2+1}\,\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {261, 5207, 260}

$$\sqrt{x^2+1}\,\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \sqrt{1+x^2}\,\tan^{-1}\left(\sqrt{1+x^2}\right)-\int \frac{x}{2+x^2}\,dx\\ &= \sqrt{1+x^2}\,\tan^{-1}\left(\sqrt{1+x^2}\right)-\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\sqrt{x^2+1}\,\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Could not integrate
```

fricas [A] time = 1.16, size = 25, normalized size = 0.81

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

giac [A] time = 1.00, size = 25, normalized size = 0.81

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

maple [A] time = 0.31, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26
default	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

maxima [A] time = 0.47, size = 25, normalized size = 0.81

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

mupad [B] time = 0.57, size = 25, normalized size = 0.81

$$\mathrm{atan}\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}-\frac{\ln\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
[Out] atan((x^2 + 1)^(1/2))*(x^2 + 1)^(1/2) - log(x^2 + 2)/2
```

sympy [A] time = 2.42, size = 26, normalized size = 0.84

$$\sqrt{x^2+1}\,\operatorname{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

3.701 $\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4743, 627, 51, 63, 206}

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] -Sqrt[1 + x]/(3*(1 - x)) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcTanh[Sqrt[1
 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}}\,dx &= \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1-x^2}}\,dx\\
&= \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{1}{6}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x}\right)\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 61, normalized size = 1.07

$$\frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\sin^{-1}(x)\right)}{(1-x)^{3/2}}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] ((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTanh[Sqrt[1
 - x^2]/Sqrt[2 - 2*x]])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.14, size = 90, normalized size = 1.58

$$\frac{\sqrt{2}\left(x^2-2x+1\right)\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)-4\sqrt{-x+1}\left(\sqrt{-x^2+1}-2\arcsin(x)\right)}{12\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
 + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arcsi
n(x)))/(x^2 - 2*x + 1)
```

giac [A] time = 0.97, size = 58, normalized size = 1.02

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+\frac{\sqrt{x+1}}{3(x-1)}-\frac{2\arcsin(x)}{3(x-1)\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sqr
t(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

maple [A] time = 0.32, size = 70, normalized size = 1.23

method	result	size
derivativedivides	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$	70
default	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*arctanh(2^(1
/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\frac{1}{8}\left(7\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+16\sqrt{x+1}-\frac{4\sqrt{x+1}}{x-1}\right)(x-1)\sqrt{-x+1}+\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)\right)}{3(x-1)\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(x - 1)*sqrt(-x + 1)*integrate(1/3*sqrt(x + 1)*x^2/(x^5 - x^4 - x^3
 + x^2 + (x^3 - x^2 - x + 1)*e^(log(x + 1) + log(-x + 1))), x) + arctan2(x,
 sqrt(x + 1)*sqrt(-x + 1)))/((x - 1)*sqrt(-x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(x)}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(1 - x)^(5/2),x)
```

```
[Out] int(asin(x)/(1 - x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}(x)}{(1-x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(1-x)**(5/2),x)
```

```
[Out] Integral(asin(x)/(1 - x)**(5/2), x)
```

3.702 $\int (-1+x)^{5/2} \csc^{-1}(x) \, dx$

Optimal. Leaf size=82

$$\frac{4x\sqrt{x^2-1}\left(3x^2-19x+83\right)}{105\sqrt{x^2}\sqrt{x-1}}+\frac{4x\tanh^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{x-1}}\right)}{7\sqrt{x^2}}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5227, 1574, 892, 88, 63, 207}

$$\frac{4(x+1)^3\sqrt{x-1}}{35\sqrt{1-\frac{1}{x^2}}\,x}-\frac{20(x+1)^2\sqrt{x-1}}{21\sqrt{1-\frac{1}{x^2}}\,x}+\frac{4(x+1)\sqrt{x-1}}{\sqrt{1-\frac{1}{x^2}}\,x}+\frac{4\sqrt{x+1}\sqrt{x-1}\tanh^{-1}\left(\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}\,x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[-1 + x]*(1 + x))/(Sqrt[1 - x^(-2)]*x) - (20*Sqrt[-1 + x]*(1 + x)^2)
/(21*Sqrt[1 - x^(-2)]*x) + (4*Sqrt[-1 + x]*(1 + x)^3)/(35*Sqrt[1 - x^(-2)]*
x) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[S
qrt[1 + x]])/(7*Sqrt[1 - x^(-2)]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
```

```
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{2}{7}\int \frac{(-1+x)^{7/2}}{\sqrt{1-\frac{1}{x^2}}\,x^2}\,dx$$

$$= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x^2}\right)\int \frac{(-1+x)^{7/2}}{x\sqrt{-1+x^2}}\,dx}{7\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\,\sqrt{1+x}\right)\int \frac{(-1+x)^3}{x\sqrt{1+x}}\,dx}{7\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\,\sqrt{1+x}\right)\int \left(\frac{7}{\sqrt{1+x}}-\frac{1}{x\sqrt{1+x}}-5\sqrt{1+x}+(1+x\right.}{7\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= \frac{4\sqrt{-1+x}\,(1+x)}{\sqrt{1-\frac{1}{x^2}}\,x}-\frac{20\sqrt{-1+x}\,(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}\,x}+\frac{4\sqrt{-1+x}\,(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}\,x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}$$

$$= \frac{4\sqrt{-1+x}\,(1+x)}{\sqrt{1-\frac{1}{x^2}}\,x}-\frac{20\sqrt{-1+x}\,(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}\,x}+\frac{4\sqrt{-1+x}\,(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}\,x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}$$

$$= \frac{4\sqrt{-1+x}\,(1+x)}{\sqrt{1-\frac{1}{x^2}}\,x}-\frac{20\sqrt{-1+x}\,(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}\,x}+\frac{4\sqrt{-1+x}\,(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}\,x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 0.88

$$\frac{4\sqrt{1-\frac{1}{x^2}}\,x\left(3x^2-19x+83\right)}{105\sqrt{x-1}}+\frac{4}{7}\tanh^{-1}\left(\frac{\sqrt{1-\frac{1}{x^2}}\,x}{\sqrt{x-1}}\right)+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x)
^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.16, size = 125, normalized size = 1.52

$$\frac{2\left(15\left(x^4-4x^3+6x^2-4x+1\right)\sqrt{x-1}\,\operatorname{arccsc}(x)+2\left(3x^2-19x+83\right)\sqrt{x^2-1}\,\sqrt{x-1}\,+15\,(x-1)\log\left(\frac{x^2+\sqrt{x^2-}}{x^2}\right.\right.}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")
```

```
[Out] 2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2
 - 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 - 1
)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqrt(x
 - 1) - 1)/(x^2 - 1)))/(x - 1)
```

giac [B] time = 1.75, size = 228, normalized size = 2.78

$$\frac{2}{35}\left(5\,(x-1)^{\frac{7}{2}}+21\,(x-1)^{\frac{5}{2}}+35\,(x-1)^{\frac{3}{2}}+35\,\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right)-\frac{2}{5}\left(3\,(x-1)^{\frac{5}{2}}+10\,(x-1)^{\frac{3}{2}}+15\,\sqrt{x-1}\right)\operatorname{arcs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 35*(x - 1)^(3/2) + 35*sqrt(x - 1
))*arcsin(1/x) - 2/5*(3*(x - 1)^(5/2) + 10*(x - 1)^(3/2) + 15*sqrt(x - 1))*
arcsin(1/x) + 2*((x - 1)^(3/2) + 3*sqrt(x - 1))*arcsin(1/x) - 2*sqrt(x - 1)
*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 4*(x + 1)^(3/2) + 21*sqrt(x + 1))/s
gn((x - 1)^(3/2) + sqrt(x - 1)) - 4/5*((x + 1)^(3/2) + sqrt(x + 1))/sgn((x
 - 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2) + sq
rt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1)) + 4*
sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))
```

maple [A] time = 0.33, size = 76, normalized size = 0.93

method	result	size
derivativedivides	$\frac{2(-1+x)^{\frac{7}{2}}\operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\,\operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$	76
default	$\frac{2(-1+x)^{\frac{7}{2}}\operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\,\operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(5/2)*arccsc(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x)
^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1+x
)*(1+x)/x^2)^(1/2)/x
```

maxima [A] time = 3.44, size = 116, normalized size = 1.41

$$\frac{4}{35}\,(x+1)^{\frac{5}{2}}-\frac{20}{21}\,(x+1)^{\frac{3}{2}}+\frac{2}{7}\left(x^3\arctan\left(1,\sqrt{x+1}\,\sqrt{x-1}\right)-3\,x^2\arctan\left(1,\sqrt{x+1}\,\sqrt{x-1}\right)+3\,x\arctan\left(1,\sqrt{x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")
```

```
[Out] 4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)*
sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1, s
qrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x - 1)
+ 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{asin}\left(\frac{1}{x}\right)\,(x-1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(1/x)*(x - 1)^(5/2),x)
```

```
[Out] int(asin(1/x)*(x - 1)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(5/2)*acsc(x),x)
```

```
[Out] Timed out
```

3.703 $\int \sin^{-1}(\sinh(x))\mathbf{sech}^4(x)\,dx$

Optimal. Leaf size=49

$$-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\sqrt{1-\sinh^2(x)}\,\text{sech}(x)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

Rubi [A] time = 0.14, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3767, 4844, 12, 4357, 451, 216}

$$\frac{1}{6}\sqrt{2-\cosh^2(x)}\,\text{sech}(x)-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sqrt[2 - Cosh[x]^2]*Sech[x])/6 + ArcSin[S
inh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4844

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/Sq
rt[1 - u^2], x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
```

```
eQ[{c, d, m}, x]]
```

Rubi steps

$$\int \sin^{-1}(\sinh(x))\text{sech}^4(x)\,dx = \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \int \frac{(2+\cosh(2x))\text{sech}(}{3\sqrt{1-\sinh^2}}$$

$$= \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \frac{1}{3}\int \frac{(2+\cosh(2x))\text{sec}}{\sqrt{1-\text{sinh}}}$$

$$= \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \frac{1}{3}\text{Subst}\left(\int \frac{1+2x^2}{x^2\sqrt{2-x^2}}\right.$$

$$= \frac{1}{6}\sqrt{2-\cosh^2(x)}\,\text{sech}(x) + \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3($$

$$= -\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right) + \frac{1}{6}\sqrt{2-\cosh^2(x)}\,\text{sech}(x) + \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}$$

Mathematica [C] time = 0.27, size = 66, normalized size = 1.35

$$\frac{1}{12}\left(8i\log\left(\sqrt{3-\cosh(2x)} + i\sqrt{2}\cosh(x)\right) + \sqrt{6-2\cosh(2x)}\,\text{sech}(x) + 4(\cosh(2x)+2)\tanh(x)\text{sech}^2(x)\,\text{sin}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] ((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*Cosh[2*x]]
*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-1}(\sinh(x))\text{sech}^4(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 1.15, size = 519, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-(
cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4
*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)
^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 +
 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + co
sh(x))*sinh(x) + 1)*arctan(sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sin
h(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)) + 8
```

```
*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 1)*arctan(sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)
^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 -
 3*cosh(x))*sinh(x) + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3
*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(co
sh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [C] time = 1.34, size = 218, normalized size = 4.45

$$-\frac{16\left(-8i\sqrt{2}\arctan(-i) - 3\sqrt{2} + 32\arctan(-i) - 3i\right)}{96i\sqrt{2} - 384} + \frac{\sqrt{2} + \frac{2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}}{e^{(2x)} - 3}}{6\left(\frac{\sqrt{2}\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)}{e^{(2x)} - 3} + \frac{\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)^2}{\left(e^{(2x)} - 3\right)^2} + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

```
[Out] -16*(-8*I*sqrt(2)*arctan(-I) - 3*sqrt(2) + 32*arctan(-I) - 3*I)/(96*I*sqrt(
2) - 384) + 1/6*(sqrt(2) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^
(2*x) - 3))/(sqrt(2)*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*x)
- 3) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))^2/(e^(2*x) - 3)^2 + 1)
- 4/3*(3*e^(2*x) + 1)*arcsin(1/2*(e^(2*x) - 1)*e^(-x))/(e^(2*x) + 1)^3 - 4/
3*arctan(-2*sqrt(2) - 3*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*
x) - 3))
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \arcsin(\sinh(x))\,\mathrm{sech}(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(sinh(x))*sech(x)^4,x)
```

```
[Out] int(arcsin(sinh(x))*sech(x)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(3e^{(2x)} + 1\right)\arctan\left(e^{(2x)} - 1, \sqrt{e^{(2x)} + 2e^{x} - 1}\sqrt{-e^{(2x)} + 2e^{x} + 1}\right) + 16\left(e^{(6x)} + 3e^{(4x)} + 3e^{(2x)} + 1\right)\int -\frac{}{\left(e^{(8x)} - \right.}}{3\left(e^{(6x)} + 3e^{(4x)} + 3e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqrt
(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integrate
(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e^(2
*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e^(lo
g(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(10*x)
 - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*e^(4*x
) + 3*e^(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asin}(\sinh(x))}{\cosh(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(sinh(x))/cosh(x)^4,x)
```

```
[Out] int(asin(sinh(x))/cosh(x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}(\sinh(x)) \operatorname{sech}^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(sinh(x))*sech(x)**4,x)
```

```
[Out] Integral(asin(sinh(x))*sech(x)**4, x)
```

3.704 $\int \cot^{-1}(\cosh(x)) \coth(x)\mathbf{csch}^3(x)\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2606, 30, 5208, 12, 453, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x]]*Csch[x]
^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 5208

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
```

```
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx &= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)+\int \frac{2\text{csch}^2(x)}{3(-3-\cosh(2x))}\,dx \\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)+\frac{2}{3}\int \frac{\text{csch}^2(x)}{-3-\cosh(2x)}\,dx \\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)-\frac{2}{3}\text{Subst}\left(\int \frac{1-x^2}{2x^2\left(2-x^2\right)}\,dx,x,\tanh(x)\right) \\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1-x^2}{x^2\left(2-x^2\right)}\,dx,x,\tanh(x)\right) \\
&= \frac{\coth(x)}{6}-\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\tanh
\end{aligned}$$

$$= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)$$

Mathematica [A] time = 0.23, size = 40, normalized size = 1.11

$$\frac{1}{24}\left(2\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)+\text{csch}^3(x)\left(-\cosh(x)+\cosh(3x)-8\cot^{-1}(\cosh(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + (-8*ArcCot[Cosh[x]] - Cosh[x] + Cosh[
3*x])*Csch[x]^3)/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] Could not integrate
```

fricas [B] time = 0.70, size = 423, normalized size = 11.75

$$\frac{8\cosh(x)^4+32\cosh(x)\sinh(x)^3+8\sinh(x)^4+16\left(3\cosh(x)^2-1\right)\sinh(x)^2-64\left(\cosh(x)^3+3\cosh(x)^2 s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(8*cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + 16*(3*cosh(x)^2 -
1)*sinh(x)^2 - 64*(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
```

```
nh(x)^2 + 1)) - 16*cosh(x)^2 + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(
x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*
sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 3*sqrt
(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x
))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)
*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2
+ sinh(x)^2 + 3)) + 32*(cosh(x)^3 - cosh(x))*sinh(x) + 8)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

giac [B] time = 1.13, size = 70, normalized size = 1.94

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{1}{3\left(e^{(2x)}-1\right)}+\frac{8\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) - e^x)^3
```

maple [C] time = 0.64, size = 854, normalized size = 23.72

method	result
risch	$\frac{4ie^{3x}\ln\left(e^{2x}+1+2ie^{x}\right)}{3\left(-1+e^{2x}\right)^{3}}-\frac{-8+16\,e^{2x}-8\,e^{4x}+16\pi\,\mathrm{csgn}(ie^{-x})\mathrm{csgn}\left(i\left(-e^{2x}-1+2ie^{x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(-e^{2x}-1+2ie^{x}\right)\right)e^{3x}-16\pi\,\mathrm{csgn}(ie^{-x})\mathrm{csgn}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I*exp(3*x)/(-1+exp(2*x))^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8+16*exp(2*
x)-8*exp(4*x)-16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)+16*Pi*
csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(-exp(
2*x)-1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*(exp(2*x)+1+2*I*exp(x)))*csgn(I
*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(-exp(2*x
)-1+2*I*exp(x)))*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^2*exp(3*x)-16*Pi*cs
gn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))
*exp(3*x)-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)+16*Pi*cs
gn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)
))*exp(3*x)-16*Pi*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^3*exp(3*x)+16*Pi
*csgn(I*(-exp(2*x)-1+2*I*exp(x)))*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^
2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(
2*x)+1+2*I*exp(x)))^2*exp(3*x)-2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)+2^(1/2)*l
n(exp(2*x)+(2^(1/2)-1)^2)-3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(4*x)-3*2
^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)*exp(2*x)+3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2)
)^2)*exp(2*x)+32*I*exp(3*x)*ln(exp(2*x)+1-2*I*exp(x))-2^(1/2)*ln(exp(2*x)+(
2^(1/2)-1)^2)*exp(6*x)+2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(6*x)+3*2^(1/2
)*ln(exp(2*x)+(2^(1/2)-1)^2)*exp(4*x)+16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*ex
p(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x))*csgn(I*(-exp(2*x)-1+2*I*exp(x)))*cs
gn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))*exp(3*x)-16*Pi*csgn(I*exp(-x))*csgn(
I*(exp(2*x)+1+2*I*exp(x)))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*exp(3*x)
-16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^2*exp(3*x)+
16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x))/(
-1+exp(2*x))^3
```

maxima [A] time = 1.01, size = 54, normalized size = 1.50

$$-\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-\frac{1}{3\left(e^{(-2x)}-1\right)}-\frac{\operatorname{arccot}(\cosh(x))}{3\sinh(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)^3
```

mupad [B] time = 0.54, size = 103, normalized size = 2.86

$$\frac{\sqrt{2}\ \ln\left(-\frac{2\,e^{2x}}{3}-\frac{\sqrt{2}\,\left(12\,e^{2x}+4\right)}{24}\right)}{24}-\frac{\sqrt{2}\ \ln\left(\frac{\sqrt{2}\,\left(12\,e^{2x}+4\right)}{24}-\frac{2\,e^{2x}}{3}\right)}{24}+\frac{1}{3\,\left(e^{2x}-1\right)}-\frac{8\,e^{3x}\,\mathrm{acot}\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}{3\,\left(3\,e^{2x}-3\,e^{4x}+e^{6x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acot(cosh(x))*cosh(x))/sinh(x)^4,x)
```

```
[Out] (2^(1/2)*log(- (2*exp(2*x))/3 - (2^(1/2)*(12*exp(2*x) + 4))/24))/24 - (2^(1
/2)*log((2^(1/2)*(12*exp(2*x) + 4))/24 - (2*exp(2*x))/3))/24 + 1/(3*(exp(2*
x) - 1)) - (8*exp(3*x)*acot(exp(-x)/2 + exp(x)/2))/(3*(3*exp(2*x) - 3*exp(4
*x) + exp(6*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

```
[Out] Timed out
```

3.705 $\int e^x \sin^{-1}(\tanh(x))\,dx$

Optimal. Leaf size=28

$$e^x \sin^{-1}(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

Rubi [A] time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2194, 4844, 6720, 2282, 12, 260}

$$e^x \sin^{-1}(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4844

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/Sq
rt[1 - u^2], x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int e^x \sin^{-1}(\tanh(x))\,dx &= e^x \sin^{-1}(\tanh(x)) - \int e^x\sqrt{\operatorname{sech}^2(x)}\,dx\\ &= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\int e^x\operatorname{sech}(x)\,dx\\ &= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^x\right)\\ &= e^x \sin^{-1}(\tanh(x)) - \left(2\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^x\right)\\ &= e^x \sin^{-1}(\tanh(x)) - \cosh(x)\log\left(1+e^{2x}\right)\sqrt{\operatorname{sech}^2(x)}\end{aligned}$$

Mathematica [B] time = 0.97, size = 64, normalized size = 2.29

$$e^x \sin^{-1}\left(\frac{e^{2x}-1}{e^{2x}+1}\right) - e^{-x}\sqrt{\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}+1\right)\log\left(e^{2x}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*ArcSin[Tanh[x]],x]
[Out] E^x*ArcSin[(-1 + E^(2*x))/(1 + E^(2*x))] - (Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(
1 + E^(2*x))*Log[1 + E^(2*x)])/E^x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \sin^{-1}(\tanh(x))\,dx$$

Verification is Not applicable to the result.

```
[In] IntegrateAlgebraic[E^x*ArcSin[Tanh[x]],x]
[Out] Could not integrate
```

fricas [A] time = 1.04, size = 26, normalized size = 0.93

$$(\cosh(x)+\sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")
[Out] (cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 1.02, size = 29, normalized size = 1.04

$$\arcsin\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)e^x - \log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="giac")
[Out] arcsin((e^(2*x) - 1)/(e^(2*x) + 1))*e^x - log(e^(2*x) + 1)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^x \arcsin(\tanh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*arcsin(tanh(x)),x)
```

```
[Out] int(exp(x)*arcsin(tanh(x)),x)
```

maxima [A] time = 1.00, size = 16, normalized size = 0.57

$$\arcsin\left(\tanh(x)\right)e^{x} - \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")
```

```
[Out] arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{asin}\left(\tanh(x)\right)\ \mathrm{e}^{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(tanh(x))*exp(x),x)
```

```
[Out] int(asin(tanh(x))*exp(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{asin}\left(\tanh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*asin(tanh(x)),x)
```

```
[Out] Integral(exp(x)*asin(tanh(x)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
```

```
HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
```

```
            print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                    print("both result and optimal complex");
                fi;
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                if debug then
                    print("result contains complex but optimal is not");
                fi;
                return "C";
            end if
        else # result do not contain complex
             # this assumes optimal do not as well
             if debug then
                 print("result do not contain complex, this assumes optimal do
    not as well");
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B";
             end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C";
    end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
```

```
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
```

```
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
```

```
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
```

```
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
```

```
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
```

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    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
   :
            return expn in expn.parent().base_ring() or expn in expn.parent().
   gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
   Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
   )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
   Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
   Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
   expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
   [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
   isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
   .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
```

```
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```