Computer algebra independent integration tests

0-Independent-test-suites/Stewart-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [376]. This is test number [9].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (376)	% 0.00 (0)
Mathematica	% 100.00 (376)	% 0.00 (0)
Maple	% 100.00 (376)	% 0.00 (0)
Maxima	% 99.47 (374)	% 0.53 (2)
Fricas	% 100.00 (376)	% 0.00 (0)
Sympy	% 92.55 (348)	% 7.45 (28)
Giac	% 99.73 (375)	% 0.27 (1)
Mupad	% 98.94 (372)	% 1.06 (4)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

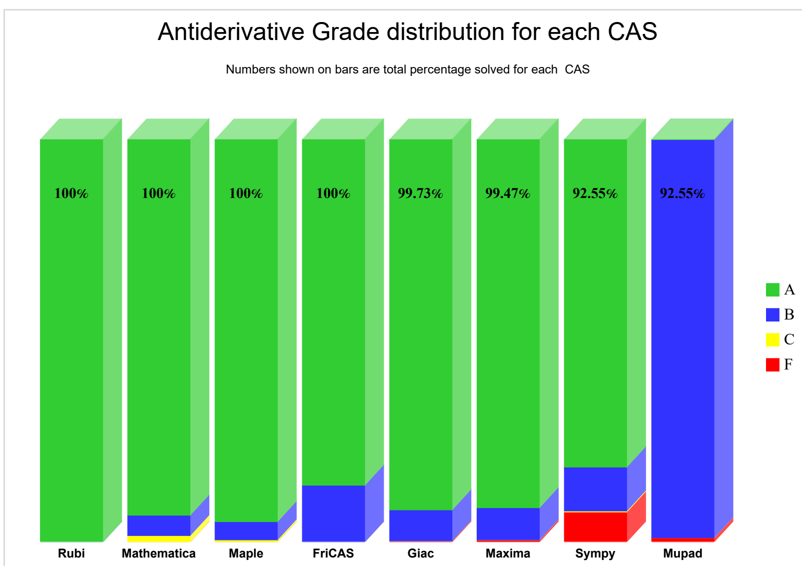
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

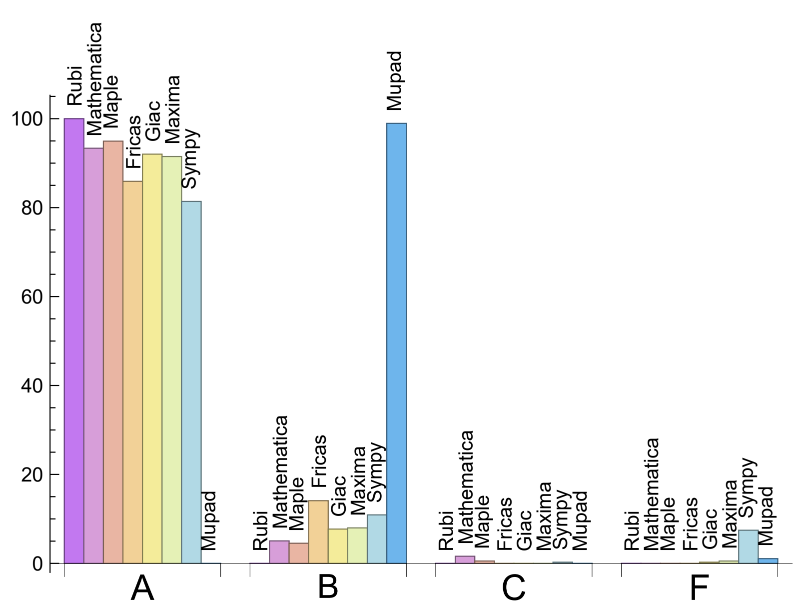
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.35	5.05	1.60	0.00
Maple	94.95	4.52	0.53	0.00
Maxima	91.49	7.98	0.00	0.53
Fricas	85.90	14.10	0.00	0.00
Sympy	81.38	10.90	0.27	7.45
Giac	92.02	7.71	0.00	0.27
Mupad	0.00	98.94	0.00	1.06

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	2	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Sympy	28	85.71 %	14.29 %	0.00 %
Giac	1	0.00 %	100.00 %	0.00 %
Mupad	4	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

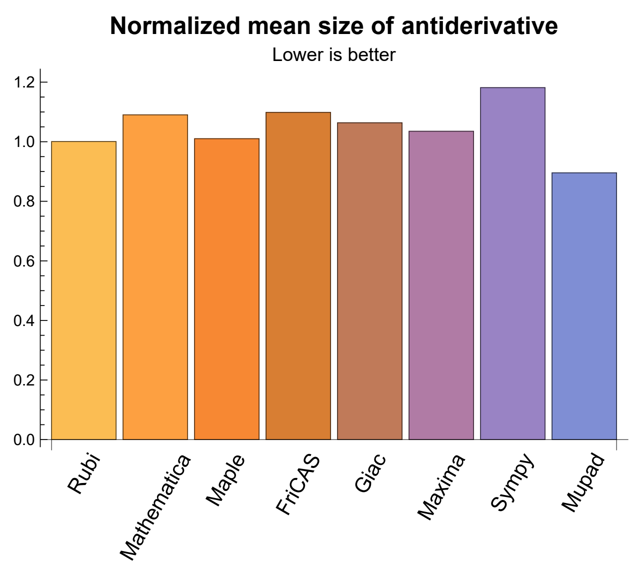
1.3 Performance

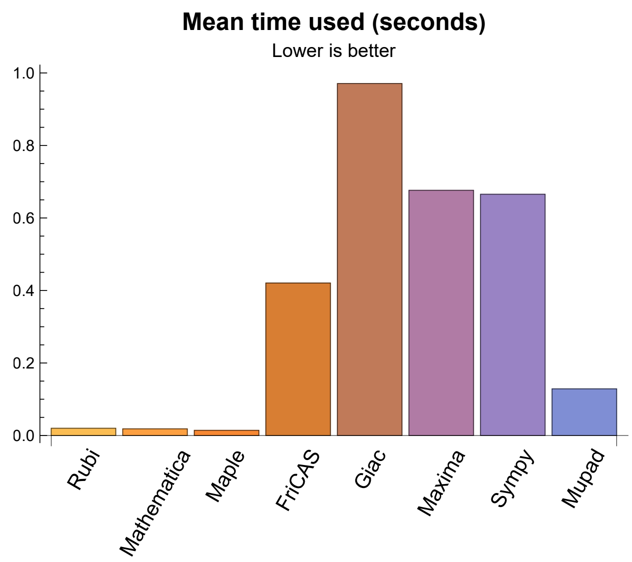
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	22.98	1.00	19.00	1.00
Mathematica	0.02	22.10	1.09	20.00	1.00
Maple	0.01	21.60	1.01	18.00	0.88
Maxima	0.68	21.78	1.04	16.00	0.80
Fricas	0.42	24.64	1.10	18.00	0.86
Sympy	0.67	26.13	1.18	19.00	0.89
Giac	0.97	20.80	1.06	17.00	0.82
Mupad	0.13	19.37	0.90	16.00	0.80

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {316, 360}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

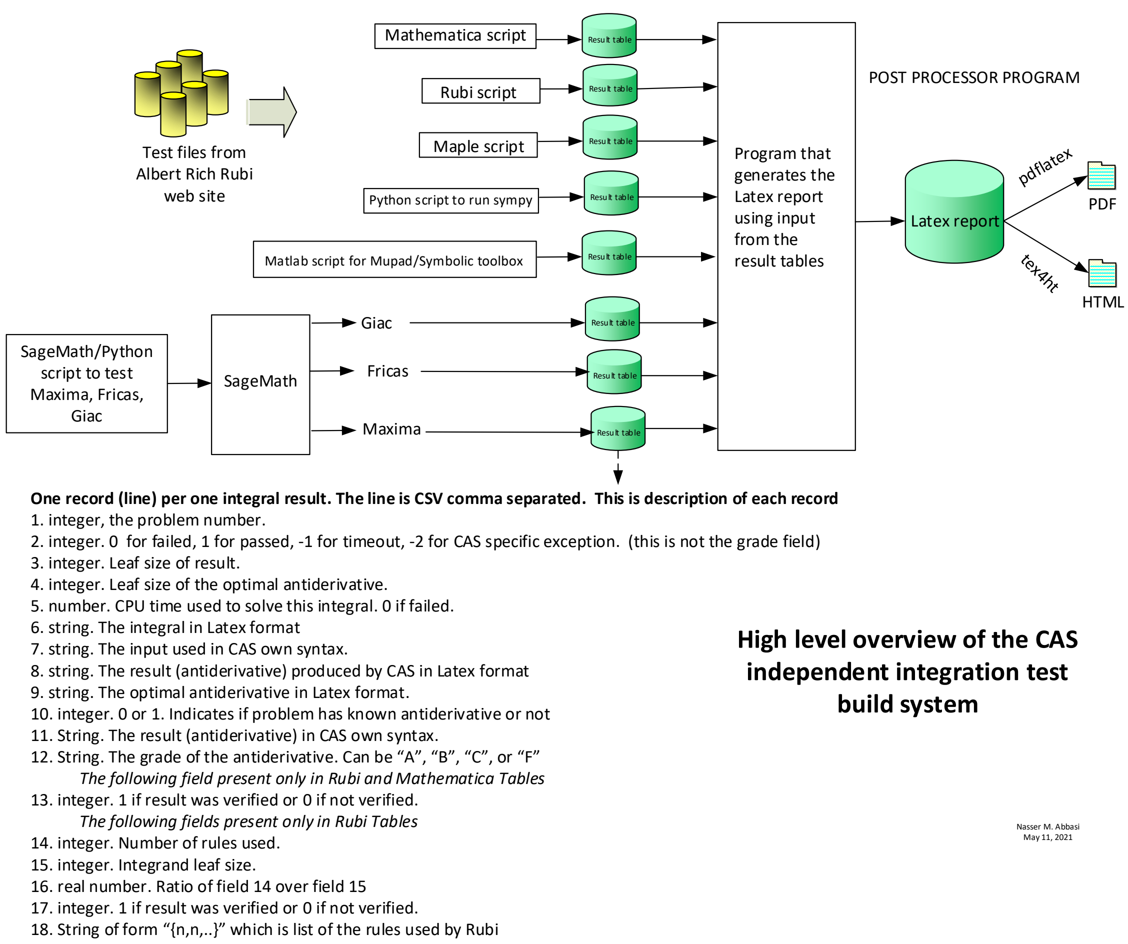
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 80, 81, 100, 102, 103, 104, 113, 121, 152, 195, 211, 212, 221, 245, 246, 297, 328, 355, 370 }

C grade: { 220, 235, 236, 244, 316, 334 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 63, 74, 89, 92, 93, 95, 117, 226, 227, 256, 270, 296, 308, 314, 329, 341, 370 }

C grade: { 323, 363 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 247, 249, 250, 251, 252, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade: { 29, 34, 35, 41, 95, 97, 103, 104, 112, 113, 220, 221, 225, 235, 241, 244, 245, 246, 248, 253, 255, 271, 276, 291, 298, 313, 328, 329, 355, 363 }

C grade: { }

F grade: { 330, 337 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 245, 247, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 338, 339, 340, 342, 343, 345, 346, 347, 348, 349, 350, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 7, 13, 14, 29, 41, 67, 79, 86, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 113, 115, 130, 139, 143, 145, 195, 220, 221, 225, 235, 241, 244, 246, 248, 249, 250, 251, 255, 270, 291, 295, 298, 311, 312, 313, 314, 328, 334, 337, 341, 344, 351, 355, 370 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 77, 78, 79, 81, 83, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 103, 105, 107, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 143, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 239, 240, 242, 243, 244, 245, 246, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 289, 290, 292, 293, 294, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 323, 326, 327, 329, 331, 332, 333, 335, 336, 338, 339, 342, 343, 345, 346, 347, 348, 349, 350, 352, 354, 355, 356, 357, 358, 361, 362, 363, 364, 366, 367, 368, 369, 371, 373, 374, 375, 376 }

B grade: { 7, 8, 42, 67, 75, 76, 80, 82, 84, 86, 87, 100, 102, 104, 106, 108, 111, 113, 127, 139, 142, 181, 195, 226, 230, 241, 266, 270, 283, 291, 296, 297, 298, 306, 320, 330, 334, 341, 344, 370, 372 }

C grade: { 324 }

F grade: { 29, 41, 74, 123, 144, 145, 146, 147, 149, 220, 238, 247, 248, 249, 251, 288, 295, 301, 322, 325, 328, 337, 340, 351, 353, 359, 360, 365 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade: { 11, 12, 29, 41, 97, 98, 103, 104, 113, 124, 130, 133, 138, 145, 195, 205, 225, 241, 244, 255, 263, 270, 291, 293, 298, 306, 328, 329, 344 }

C grade: { }

F grade: { 269 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

C grade: { }

F grade: { 147, 323, 359, 363 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	10	12	11	20
normalized size	1	1.00	1.00	1.09	1.00	0.91	1.09	1.00	1.82
time (sec)	N/A	0.002	0.001	0.003	0.414	0.420	0.058	0.929	0.321

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.002	0.422	0.413	0.037	0.907	0.006

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.000	0.417	0.407	0.055	0.928	0.002

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.001	0.423	0.422	0.086	0.828	0.185

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.002	0.001	0.000	0.419	0.429	0.059	0.824	0.020

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.000	0.434	0.439	0.058	0.981	0.003

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	2
normalized size	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	1.00
time (sec)	N/A	0.006	0.004	0.011	0.424	0.427	0.064	1.108	0.023

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	8	7	6	4
normalized size	1	1.00	1.00	1.25	1.50	2.00	1.75	1.50	1.00
time (sec)	N/A	0.005	0.002	0.014	0.426	0.408	0.066	1.044	0.009

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	4	4	3	4	12
normalized size	1	1.00	1.00	1.50	2.00	2.00	1.50	2.00	6.00
time (sec)	N/A	0.007	0.001	0.013	0.423	0.418	0.067	0.892	0.265

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	6	6
normalized size	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50
time (sec)	N/A	0.008	0.002	0.015	0.427	0.445	0.068	1.036	0.261

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.003	0.001	0.002	0.419	0.406	0.125	1.046	0.019

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.002	0.001	0.000	0.437	0.412	0.125	0.979	0.016

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	5
normalized size	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.00
time (sec)	N/A	0.002	0.002	0.001	0.436	0.433	0.063	1.022	0.027

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3
normalized size	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.002	0.000	0.421	0.431	0.065	0.853	0.002

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.431	0.425	0.177	0.891	0.021

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75
time (sec)	N/A	0.001	0.001	0.002	0.425	0.412	0.079	0.939	0.018

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	11	11	10	11	11
normalized size	1	1.00	0.63	0.63	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.020	0.005	0.000	0.440	0.413	0.086	1.080	0.023

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
normalized size	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.007	0.005	0.000	0.428	0.422	0.301	0.941	0.002

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.004	0.002	0.002	0.425	0.419	0.197	0.917	0.157

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	12	11	11	10	11	11
normalized size	1	1.00	0.75	0.60	0.55	0.55	0.50	0.55	0.55
time (sec)	N/A	0.008	0.004	0.003	0.428	0.426	0.084	0.928	0.019

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.419	0.430	0.178	0.784	0.002

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.010	0.011	0.016	0.431	0.426	0.186	0.874	0.028

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.001	0.001	0.414	0.407	0.086	0.758	0.002

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	21	21	27	21	23
normalized size	1	1.00	0.86	0.83	0.72	0.72	0.93	0.72	0.79
time (sec)	N/A	0.025	0.028	0.017	0.429	0.433	0.333	0.829	0.065

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	21	21	24	21	24
normalized size	1	1.00	0.86	0.83	0.72	0.72	0.83	0.72	0.83
time (sec)	N/A	0.023	0.026	0.022	0.430	0.452	0.325	0.949	0.030

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
normalized size	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.003	0.001	0.000	0.424	0.418	0.091	1.021	0.032

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	0.88
time (sec)	N/A	0.004	0.003	0.001	1.015	0.453	0.132	1.051	0.002

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	14	17	24	14	18
normalized size	1	1.00	0.78	0.78	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.013	0.003	0.004	0.424	0.426	0.335	0.955	0.046

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	0	103	8
normalized size	1	1.00	1.00	1.12	9.25	2.25	0.00	12.88	1.00
time (sec)	N/A	0.018	0.006	0.016	0.966	0.439	0.000	0.927	0.022

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.007	0.002	0.003	0.417	0.402	0.088	0.823	0.028

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	17	17	16	16	15	16	16
normalized size	1	1.00	0.63	0.63	0.59	0.59	0.56	0.59	0.59
time (sec)	N/A	0.031	0.007	0.003	0.443	0.384	0.085	1.072	0.018

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.011	0.036	0.014	0.427	0.421	0.306	0.987	0.027

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	17	21	20	17	17
normalized size	1	1.00	0.74	0.81	0.63	0.78	0.74	0.63	0.63
time (sec)	N/A	0.011	0.029	0.017	0.420	0.430	0.454	1.067	0.026

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
normalized size	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.012	0.003	0.002	0.432	0.397	0.188	0.809	0.019

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	57	18	20	30	18
normalized size	1	1.00	1.00	1.00	3.00	0.95	1.05	1.58	0.95
time (sec)	N/A	0.016	0.014	0.018	0.437	0.395	0.230	0.775	0.060

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
normalized size	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.009	0.005	0.000	0.447	0.408	0.079	0.951	0.021

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	66	13	9
normalized size	1	1.00	0.71	0.67	0.62	0.67	3.14	0.62	0.43
time (sec)	N/A	0.008	0.003	0.008	0.433	0.401	1.917	0.889	0.031

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.012	0.009	0.016	0.475	0.435	0.182	0.852	0.020

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	14	14	12	14	14
normalized size	1	1.00	0.62	0.58	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.018	0.006	0.003	0.437	0.375	0.086	1.035	0.029

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	12	16	16
normalized size	1	1.00	1.00	0.94	0.89	0.89	0.67	0.89	0.89
time (sec)	N/A	0.004	0.003	0.001	0.984	0.417	0.132	0.822	0.169

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	0	52	9
normalized size	1	1.00	1.00	1.11	11.56	2.22	0.00	5.78	1.00
time (sec)	N/A	0.016	0.017	0.013	0.446	0.422	0.000	0.957	0.147

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	25	26	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.47	1.53	0.76	0.76
time (sec)	N/A	0.009	0.008	0.142	0.438	0.422	0.519	0.800	0.040

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	13	14	22	13	13
normalized size	1	1.00	1.00	0.53	0.76	0.82	1.29	0.76	0.76
time (sec)	N/A	0.008	0.007	0.027	0.428	0.443	0.574	0.767	0.173

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8
normalized size	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.010	0.003	0.015	0.433	0.432	0.895	1.004	0.201

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	11	11	10	11	11
normalized size	1	1.00	0.64	0.55	0.50	0.50	0.45	0.50	0.50
time (sec)	N/A	0.019	0.002	0.003	0.439	0.405	0.087	1.043	0.029

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	9	9	12	8	7	8	8
normalized size	1	1.00	0.60	0.60	0.80	0.53	0.47	0.53	0.53
time (sec)	N/A	0.008	0.012	0.001	0.437	0.397	0.081	0.716	0.033

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	14	14	14	14	14
normalized size	1	1.00	0.74	0.79	0.74	0.74	0.74	0.74	0.74
time (sec)	N/A	0.009	0.003	0.007	0.966	0.422	0.096	0.860	0.023

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	13	13
normalized size	1	1.00	1.00	0.82	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.003	0.001	0.428	0.432	0.415	0.777	0.167

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	11	11	20	11	11
normalized size	1	1.00	0.67	0.71	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.008	0.006	0.000	0.444	0.410	0.255	0.850	0.021

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	9	9	8	9	7
normalized size	1	1.00	0.86	0.71	0.64	0.64	0.57	0.64	0.50
time (sec)	N/A	0.001	0.001	0.003	0.423	0.408	0.109	0.935	0.024

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	12	13	15	13	13
normalized size	1	1.00	1.00	0.82	0.71	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.003	0.002	0.429	0.428	0.535	0.871	0.022

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.013	0.002	0.439	0.425	0.309	0.962	0.244

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.016	0.010	0.021	0.443	0.437	1.905	0.964	0.194

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	17	16	16	15	16	16
normalized size	1	1.00	0.68	0.61	0.57	0.57	0.54	0.57	0.57
time (sec)	N/A	0.034	0.002	0.003	0.451	0.403	0.091	0.792	0.028

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14
normalized size	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.009	0.003	0.000	0.954	0.408	0.244	0.917	0.019

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	18	16
normalized size	1	1.00	1.00	0.94	0.89	0.89	0.83	1.00	0.89
time (sec)	N/A	0.013	0.016	0.019	0.435	0.418	0.194	0.993	0.023

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	66	13	9
normalized size	1	1.00	0.71	0.67	0.62	0.67	3.14	0.62	0.43
time (sec)	N/A	0.006	0.002	0.002	0.417	0.417	1.939	1.088	0.017

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	14	15	10	14	14	10	10
normalized size	1	1.00	0.78	0.83	0.56	0.78	0.78	0.56	0.56
time (sec)	N/A	0.006	0.007	0.020	0.425	0.425	0.066	0.837	0.050

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.000	0.433	0.433	0.061	0.990	0.002

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
normalized size	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.013	0.002	0.000	0.437	0.436	0.063	0.871	0.031

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
normalized size	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.005	0.002	0.002	0.427	0.415	0.066	0.932	0.034

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	18	13	13	12	13	14
normalized size	1	1.00	1.82	1.06	0.76	0.76	0.71	0.76	0.82
time (sec)	N/A	0.023	0.011	0.008	0.435	0.415	0.067	0.957	0.039

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	30	13	22	12	13	14
normalized size	1	1.00	1.82	1.76	0.76	1.29	0.71	0.76	0.82
time (sec)	N/A	0.022	0.010	0.009	0.431	0.404	0.066	0.956	0.151

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	29	18	25	31	22	24
normalized size	1	1.00	0.83	0.81	0.50	0.69	0.86	0.61	0.67
time (sec)	N/A	0.045	0.008	0.010	0.444	0.425	0.063	0.918	0.044

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	10	18
normalized size	1	1.00	0.58	0.79	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.025	0.004	0.000	0.431	0.409	0.070	1.075	0.041

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	14	18	37	14	14
normalized size	1	1.00	0.82	0.86	0.64	0.82	1.68	0.64	0.64
time (sec)	N/A	0.009	0.008	0.052	0.432	0.411	0.202	1.011	0.261

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
normalized size	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.014	0.011	0.075	0.443	0.434	0.565	0.893	0.146

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	19	19	19	19	19
normalized size	1	1.00	1.00	1.12	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.030	0.010	0.013	0.452	0.412	0.070	0.918	0.050

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	22	22
normalized size	1	1.00	0.88	0.71	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.017	0.002	0.000	0.431	0.432	0.064	0.815	0.036

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	22	22
normalized size	1	1.00	0.88	0.71	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.017	0.002	0.103	0.453	0.420	0.063	0.941	0.035

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	36	18	33	41	22	37
normalized size	1	1.00	0.65	0.78	0.39	0.72	0.89	0.48	0.80
time (sec)	N/A	0.036	0.013	0.017	0.432	0.429	0.067	0.833	0.071

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
normalized size	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.007	0.002	0.000	0.438	0.421	0.070	0.913	0.038

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	16	31	31	16	32
normalized size	1	1.00	0.48	0.78	0.35	0.67	0.67	0.35	0.70
time (sec)	N/A	0.052	0.006	0.011	0.436	0.435	0.072	0.958	0.042

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	39	13	17	0	13	13
normalized size	1	1.00	1.62	1.86	0.62	0.81	0.00	0.62	0.62
time (sec)	N/A	0.022	0.062	0.057	0.438	0.433	0.000	0.901	0.086

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	14	167	13	25
normalized size	1	1.00	0.86	0.67	0.62	0.67	7.95	0.62	1.19
time (sec)	N/A	0.022	0.012	0.039	0.433	0.451	27.121	0.958	0.207

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	12	13	39	12	12
normalized size	1	1.00	0.95	0.74	0.63	0.68	2.05	0.63	0.63
time (sec)	N/A	0.020	0.028	0.038	0.444	0.444	0.361	0.990	0.273

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	15	15	22	15	14
normalized size	1	1.00	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.014	0.016	0.021	0.438	0.441	0.620	0.918	0.177

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	13	16
normalized size	1	1.00	1.00	0.93	1.14	1.00	0.71	0.93	1.14
time (sec)	N/A	0.013	0.006	0.024	0.448	0.456	0.090	1.077	0.199

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
normalized size	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.029	0.023	0.033	0.444	0.441	0.103	0.943	0.226

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	5	5	19	5	5
normalized size	1	1.00	7.20	1.20	1.00	1.00	3.80	1.00	1.00
time (sec)	N/A	0.013	0.007	0.042	0.445	0.437	0.335	0.907	0.166

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	10
normalized size	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	0.91
time (sec)	N/A	0.008	0.012	0.036	0.434	0.407	0.385	0.868	0.029

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	7	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.004	0.003	0.000	0.968	0.412	0.070	0.841	0.026

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	12	12	19	12	12
normalized size	1	1.00	1.29	0.93	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.008	0.004	0.000	0.956	0.407	0.074	0.865	0.025

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	13	9	16	19	9	17
normalized size	1	1.00	1.55	1.18	0.82	1.45	1.73	0.82	1.55
time (sec)	N/A	0.007	0.003	0.101	0.443	0.397	0.075	0.950	0.029

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	19	15	22	31	15	27
normalized size	1	1.00	1.42	1.00	0.79	1.16	1.63	0.79	1.42
time (sec)	N/A	0.009	0.003	0.114	0.444	0.404	0.070	0.971	0.038

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	6	20	29	6	6
normalized size	1	1.00	1.00	1.38	0.75	2.50	3.62	0.75	0.75
time (sec)	N/A	0.020	0.002	0.029	0.438	0.422	0.073	0.882	0.029

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	13	20	29	13	13
normalized size	1	1.00	1.59	1.29	0.76	1.18	1.71	0.76	0.76
time (sec)	N/A	0.022	0.020	0.026	0.439	0.430	0.070	0.770	0.172

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.012	0.005	0.020	0.428	0.412	0.072	0.780	0.290

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	14	14	14	14	13
normalized size	1	1.00	1.00	2.47	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.023	0.024	0.027	0.427	0.420	0.105	0.916	0.379

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	23	34	24	20	22	18
normalized size	1	1.00	0.91	1.05	1.55	1.09	0.91	1.00	0.82
time (sec)	N/A	0.010	0.003	0.002	0.442	0.433	0.114	0.990	0.033

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	19	18	18	31	18	18
normalized size	1	1.00	1.36	0.86	0.82	0.82	1.41	0.82	0.82
time (sec)	N/A	0.013	0.004	0.003	0.978	0.426	0.077	0.919	0.034

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	48	20	20	22	20	17
normalized size	1	1.00	1.00	2.53	1.05	1.05	1.16	1.05	0.89
time (sec)	N/A	0.016	0.011	0.024	0.428	0.440	0.115	1.034	0.269

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	20	20	22	20	19
normalized size	1	1.00	1.00	2.32	0.80	0.80	0.88	0.80	0.76
time (sec)	N/A	0.029	0.013	0.029	0.439	0.420	0.124	1.003	0.529

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	10	6	7	6	18
normalized size	1	1.00	1.00	0.88	1.25	0.75	0.88	0.75	2.25
time (sec)	N/A	0.013	0.005	0.019	0.432	0.424	0.074	0.940	0.173

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	32	36	14	14	14	20
normalized size	1	1.00	1.00	1.88	2.12	0.82	0.82	0.82	1.18
time (sec)	N/A	0.028	0.013	0.031	0.445	0.420	0.116	0.943	0.167

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	10	6	7	6	6
normalized size	1	1.00	1.00	0.88	1.25	0.75	0.88	0.75	0.75
time (sec)	N/A	0.012	0.004	0.024	0.435	0.430	0.076	1.022	0.032

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	27	34	27	29	30
normalized size	1	1.00	1.00	1.50	1.69	2.12	1.69	1.81	1.88
time (sec)	N/A	0.015	0.007	0.021	0.451	0.430	0.127	1.002	0.258

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	10	20	8	18	8
normalized size	1	1.00	1.00	1.50	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.005	0.002	0.000	0.983	0.410	0.073	1.134	0.018

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	14	28	14	22	18
normalized size	1	1.00	1.00	1.21	1.00	2.00	1.00	1.57	1.29
time (sec)	N/A	0.007	0.003	0.006	0.442	0.448	0.101	0.938	0.026

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	22	14	39	41	14	14
normalized size	1	1.00	2.18	1.29	0.82	2.29	2.41	0.82	0.82
time (sec)	N/A	0.026	0.028	0.030	0.434	0.426	0.076	0.921	0.185

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	14
normalized size	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.82
time (sec)	N/A	0.026	0.008	0.028	0.445	0.423	0.112	0.906	0.180

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	8	19	15	6	5
normalized size	1	1.00	3.40	1.80	1.60	3.80	3.00	1.20	1.00
time (sec)	N/A	0.002	0.003	0.003	0.433	0.440	0.111	0.998	0.043

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	47	18	27	44	27	54	16
normalized size	1	1.00	2.94	1.12	1.69	2.75	1.69	3.38	1.00
time (sec)	N/A	0.007	0.005	0.108	0.451	0.416	0.134	1.117	0.155

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	17	21	19	19	8
normalized size	1	1.00	2.38	1.50	2.12	2.62	2.38	2.38	1.00
time (sec)	N/A	0.011	0.004	0.020	0.441	0.442	0.108	0.985	0.148

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	12	14	25	20	14	17
normalized size	1	1.00	1.31	0.92	1.08	1.92	1.54	1.08	1.31
time (sec)	N/A	0.007	0.002	0.100	0.436	0.410	0.064	0.984	0.028

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	24	26	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.41	1.53	0.76	0.76
time (sec)	N/A	0.008	0.007	0.099	0.423	0.433	0.566	0.878	0.062

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.008	0.006	0.057	0.424	0.432	0.540	0.918	0.026

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	24	26	11	11
normalized size	1	1.00	1.00	0.80	0.73	1.60	1.73	0.73	0.73
time (sec)	N/A	0.008	0.006	0.102	0.431	0.434	0.558	0.973	0.174

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	13	14	24	13	13
normalized size	1	1.00	1.00	0.53	0.76	0.82	1.41	0.76	0.76
time (sec)	N/A	0.008	0.007	0.027	0.442	0.426	0.606	0.925	0.058

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	19
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	2.38
time (sec)	N/A	0.013	0.001	0.005	0.426	0.419	0.067	0.803	0.031

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	112	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.73	0.73	0.73
time (sec)	N/A	0.031	0.008	0.090	0.429	0.432	13.533	0.797	0.293

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	11	5	7	9	6
normalized size	1	1.00	1.60	1.20	2.20	1.00	1.40	1.80	1.20
time (sec)	N/A	0.022	0.002	0.033	0.425	0.433	0.435	1.029	0.170

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	69	35	32	29	24
normalized size	1	1.00	4.07	1.33	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.053	0.009	0.097	0.979	0.444	1.571	1.112	0.396

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
normalized size	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.015	0.005	0.028	0.440	0.441	0.087	0.922	0.141

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
normalized size	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.052	0.010	0.031	0.422	0.447	0.103	0.846	0.149

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.015	0.004	0.001	0.456	0.423	0.073	0.911	0.002

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	14	14	14	14	13
normalized size	1	1.00	1.00	2.47	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.026	0.014	0.000	0.422	0.431	0.105	0.819	0.002

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	21	35	15	39	21
normalized size	1	1.00	1.00	1.36	0.84	1.40	0.60	1.56	0.84
time (sec)	N/A	0.004	0.007	0.006	0.979	0.403	0.258	0.984	0.039

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	14	12	19	12
normalized size	1	1.00	1.00	0.81	0.75	0.88	0.75	1.19	0.75
time (sec)	N/A	0.003	0.003	0.003	0.986	0.390	0.779	1.067	0.028

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.002	0.001	0.003	0.431	0.406	0.152	0.889	0.028

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	18	18	19	19	14
normalized size	1	1.00	2.88	0.94	1.12	1.12	1.19	1.19	0.88
time (sec)	N/A	0.002	0.003	0.005	0.483	0.420	1.084	0.985	0.080

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	18	27	23	24
normalized size	1	1.00	0.71	0.61	0.84	0.58	0.87	0.74	0.77
time (sec)	N/A	0.015	0.008	0.004	0.962	0.408	0.576	0.977	0.047

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	42	0	23	38
normalized size	1	1.00	1.00	0.89	0.78	1.56	0.00	0.85	1.41
time (sec)	N/A	0.010	0.007	0.006	0.959	0.421	0.000	1.050	0.221

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	14	14	27	33	14
normalized size	1	1.00	1.00	1.25	0.88	0.88	1.69	2.06	0.88
time (sec)	N/A	0.003	0.003	0.007	0.971	0.408	0.796	0.954	0.230

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	26	23	39	30	23
normalized size	1	1.00	0.71	0.81	0.84	0.74	1.26	0.97	0.74
time (sec)	N/A	0.014	0.010	0.004	0.969	0.414	0.641	0.882	0.036

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	11	11	8	11	11
normalized size	1	1.00	1.00	1.31	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.002	0.001	0.003	0.441	0.402	0.145	1.044	0.032

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	11	16	24	11	11
normalized size	1	1.00	1.00	1.20	0.73	1.07	1.60	0.73	0.73
time (sec)	N/A	0.002	0.002	0.003	0.417	0.405	0.206	1.023	0.026

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	19	19	18
normalized size	1	1.00	1.00	0.80	0.76	1.28	0.76	0.76	0.72
time (sec)	N/A	0.003	0.008	0.004	0.980	0.395	0.216	0.941	0.033

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	18	15	22	14	24	19	14
normalized size	1	1.00	0.72	0.60	0.88	0.56	0.96	0.76	0.56
time (sec)	N/A	0.011	0.005	0.005	0.967	0.410	0.370	0.917	0.023

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	14	3	14	4
normalized size	1	1.00	1.00	0.83	0.67	2.33	0.50	2.33	0.67
time (sec)	N/A	0.001	0.004	0.003	0.958	0.414	0.149	1.096	0.030

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	15	25	15	25	15
normalized size	1	1.00	0.86	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.003	0.005	0.004	0.951	0.411	0.212	0.931	0.029

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	26	22	33	66	25	25
normalized size	1	1.00	1.31	0.74	0.63	0.94	1.89	0.71	0.71
time (sec)	N/A	0.013	0.022	0.009	0.960	0.423	2.187	0.950	0.346

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	19	23	76	48	19
normalized size	1	1.00	1.00	1.22	0.83	1.00	3.30	2.09	0.83
time (sec)	N/A	0.004	0.005	0.006	0.970	0.396	0.757	0.975	0.341

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	28	92	24	24
normalized size	1	1.00	1.00	0.83	0.63	0.93	3.07	0.80	0.80
time (sec)	N/A	0.016	0.005	0.009	0.961	0.414	1.403	0.925	0.310

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	18	37	23	14
normalized size	1	1.00	1.00	1.39	0.78	1.00	2.06	1.28	0.78
time (sec)	N/A	0.003	0.004	0.005	0.978	0.400	0.815	0.990	0.254

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	31	22	58	49	24	34
normalized size	1	1.00	1.15	0.91	0.65	1.71	1.44	0.71	1.00
time (sec)	N/A	0.006	0.031	0.013	0.983	0.413	1.681	1.062	0.222

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	22	29	24	22	22
normalized size	1	1.00	1.00	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.006	0.010	0.009	0.976	0.423	0.226	0.890	0.036

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	14	24	15	37	18
normalized size	1	1.00	1.00	0.78	0.61	1.04	0.65	1.61	0.78
time (sec)	N/A	0.011	0.005	0.006	0.963	0.399	1.062	0.906	0.056

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	26	9	9
normalized size	1	1.00	1.00	0.77	0.69	1.62	2.00	0.69	0.69
time (sec)	N/A	0.002	0.002	0.003	0.430	0.402	2.237	0.979	0.161

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	26	23	44	32	23
normalized size	1	1.00	0.71	0.94	0.84	0.74	1.42	1.03	0.74
time (sec)	N/A	0.014	0.011	0.005	0.957	0.399	0.657	1.103	0.167

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	32	31	39	112	26	27
normalized size	1	1.00	0.73	0.71	0.69	0.87	2.49	0.58	0.60
time (sec)	N/A	0.010	0.017	0.007	0.979	0.407	2.816	1.016	0.036

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	26	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69
time (sec)	N/A	0.002	0.002	0.003	0.426	0.403	0.203	0.991	0.029

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	23	12	30	36	12	12
normalized size	1	1.00	1.00	1.44	0.75	1.88	2.25	0.75	0.75
time (sec)	N/A	0.001	0.004	0.003	0.428	0.401	0.783	1.041	0.251

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	26	36	35	0	23	24
normalized size	1	1.00	0.97	0.79	1.09	1.06	0.00	0.70	0.73
time (sec)	N/A	0.008	0.042	0.006	0.966	0.408	0.000	0.888	0.165

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	18	0	18	14
normalized size	1	1.00	1.00	0.88	0.75	2.25	0.00	2.25	1.75
time (sec)	N/A	0.006	0.006	0.003	0.962	0.395	0.000	1.182	0.201

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	22	20	0	21	20
normalized size	1	1.00	0.96	1.20	0.88	0.80	0.00	0.84	0.80
time (sec)	N/A	0.006	0.007	0.004	0.973	0.408	0.000	1.105	0.260

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	37	36	35	0	25	-1
normalized size	1	1.00	1.07	0.84	0.82	0.80	0.00	0.57	-0.02
time (sec)	N/A	0.017	0.053	0.004	0.993	0.408	0.000	1.004	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	25	22	28	19	22	23
normalized size	1	1.00	0.88	0.96	0.85	1.08	0.73	0.85	0.88
time (sec)	N/A	0.006	0.008	0.004	1.095	0.395	0.129	0.972	0.189

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	59	49	0	36	29
normalized size	1	1.00	0.72	0.84	1.37	1.14	0.00	0.84	0.67
time (sec)	N/A	0.008	0.010	0.005	0.419	0.397	0.000	1.067	0.050

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	23	22	35	29	22	22
normalized size	1	1.00	0.97	0.70	0.67	1.06	0.88	0.67	0.67
time (sec)	N/A	0.025	0.013	0.016	1.093	0.411	1.545	1.042	0.210

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	22	22	34
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.73	0.73	1.13
time (sec)	N/A	0.015	0.007	0.009	1.000	0.423	1.493	0.920	0.227

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	6	16	3	16	12
normalized size	1	1.00	3.00	0.93	0.43	1.14	0.21	1.14	0.86
time (sec)	N/A	0.002	0.003	0.003	0.437	0.397	1.029	0.848	0.188

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	15	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	1.00	0.87
time (sec)	N/A	0.004	0.004	0.007	0.432	0.378	0.103	0.777	0.177

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
normalized size	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.016	0.005	0.001	0.423	0.382	0.078	0.868	0.033

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	22	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.042	0.007	0.010	0.434	0.409	0.147	0.755	0.185

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
normalized size	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.030	0.017	0.008	0.431	0.398	0.102	1.050	0.050

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.038	0.005	0.006	0.976	0.405	0.143	0.937	0.052

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	32	31	31	34	31	30
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.89	0.82	0.79
time (sec)	N/A	0.038	0.011	0.005	0.987	0.405	0.128	0.858	0.167

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	77	136	88	74	96
normalized size	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.537	0.049	0.012	0.969	0.409	0.542	1.025	0.262

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
normalized size	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.047	0.022	0.011	1.250	0.387	0.157	0.907	0.161

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16
normalized size	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84
time (sec)	N/A	0.003	0.005	0.002	1.164	0.374	0.112	1.079	0.024

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	12
normalized size	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.63
time (sec)	N/A	0.003	0.003	0.005	0.589	0.384	0.104	0.971	0.098

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	15	15	17	17	8
normalized size	1	1.00	1.11	0.84	0.79	0.79	0.89	0.89	0.42
time (sec)	N/A	0.007	0.003	0.006	0.446	0.388	0.108	1.081	0.122

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	37	26	43	22
normalized size	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.026	0.017	0.009	0.522	0.386	0.143	0.893	0.100

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	34	53	32	31	29
normalized size	1	1.00	0.77	0.79	0.79	1.23	0.74	0.72	0.67
time (sec)	N/A	0.042	0.024	0.009	0.482	0.380	0.166	0.963	0.230

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	19	26	17	21	16
normalized size	1	1.00	1.00	0.86	0.90	1.24	0.81	1.00	0.76
time (sec)	N/A	0.011	0.002	0.008	0.632	0.378	0.111	0.921	0.040

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	19	17	26	19
normalized size	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.050	0.006	0.009	0.625	0.388	0.111	1.171	0.168

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
normalized size	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.012	0.004	0.006	0.479	0.394	0.102	1.022	0.175

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
normalized size	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.118	0.017	0.005	1.280	0.401	0.214	0.900	0.108

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
normalized size	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.113	0.017	0.011	1.321	0.399	0.183	0.874	0.168

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
normalized size	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.254	0.054	0.011	1.058	0.406	0.267	0.901	0.124

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	25	30	39	27	25	30
normalized size	1	1.00	0.76	0.68	0.81	1.05	0.73	0.68	0.81
time (sec)	N/A	0.008	0.013	0.010	1.142	0.390	0.136	0.985	0.156

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	75	134	88	71	84
normalized size	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.077	0.044	0.014	1.182	0.411	0.237	0.931	0.241

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
normalized size	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.059	0.026	0.010	1.113	0.410	0.176	0.985	0.237

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	37	46	48	38	51
normalized size	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.024	0.012	0.008	1.268	0.405	0.162	0.969	0.109

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	13	13	10	14	13
normalized size	1	1.00	1.27	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.006	0.003	0.002	0.461	0.396	0.076	0.941	0.025

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	8	9	8	8	7	9	8
normalized size	1	1.00	0.80	0.90	0.80	0.80	0.70	0.90	0.80
time (sec)	N/A	0.004	0.002	0.004	0.539	0.409	0.075	1.184	0.028

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	13	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	1.00	0.85
time (sec)	N/A	0.006	0.004	0.006	0.585	0.408	0.108	1.101	0.168

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	10
normalized size	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	0.91
time (sec)	N/A	0.001	0.003	0.005	0.530	0.416	0.104	1.008	0.068

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	13	12	12	10	13	10
normalized size	1	1.00	1.25	1.08	1.00	1.00	0.83	1.08	0.83
time (sec)	N/A	0.006	0.003	0.003	0.484	0.398	0.081	1.003	0.158

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	27	26	20	80	28	18
normalized size	1	1.00	0.73	1.04	1.00	0.77	3.08	1.08	0.69
time (sec)	N/A	0.006	0.007	0.007	0.589	0.397	0.227	0.906	0.219

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.018	0.004	0.006	0.502	0.404	0.104	1.000	0.044

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	17	20	14
normalized size	1	1.00	1.00	0.73	0.69	0.69	0.65	0.77	0.54
time (sec)	N/A	0.016	0.006	0.008	0.511	0.397	0.107	0.834	0.041

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	15	14
normalized size	1	1.00	1.00	1.07	1.00	1.21	0.71	1.07	1.00
time (sec)	N/A	0.005	0.005	0.006	0.491	0.387	0.086	1.089	0.030

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	22	17
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.96	0.74
time (sec)	N/A	0.008	0.005	0.006	0.512	0.411	0.144	0.864	0.083

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.040	0.006	0.010	0.505	0.416	0.142	0.927	0.066

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	16	8	13	12
normalized size	1	1.00	1.00	1.08	1.00	1.33	0.67	1.08	1.00
time (sec)	N/A	0.005	0.005	0.006	0.454	0.413	0.082	0.857	0.034

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	21	20	26	19	21	22
normalized size	1	1.00	0.73	0.70	0.67	0.87	0.63	0.70	0.73
time (sec)	N/A	0.010	0.009	0.007	0.475	0.408	0.131	0.966	0.061

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	20	27	22	26	22
normalized size	1	1.00	0.93	0.75	0.71	0.96	0.79	0.93	0.79
time (sec)	N/A	0.010	0.017	0.009	0.563	0.400	0.127	0.972	0.212

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
normalized size	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.023	0.004	0.009	0.514	0.388	0.123	0.884	0.042

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
normalized size	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.027	0.007	0.010	0.502	0.403	0.146	0.899	0.181

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.009	0.005	0.001	0.546	0.380	0.110	0.886	0.051

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	27	40	20	30	27
normalized size	1	1.00	1.00	0.96	1.08	1.60	0.80	1.20	1.08
time (sec)	N/A	0.008	0.012	0.012	0.437	0.385	0.120	0.947	0.050

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	22	31	19	18	21
normalized size	1	1.00	1.00	0.95	1.05	1.48	0.90	0.86	1.00
time (sec)	N/A	0.007	0.010	0.006	0.520	0.400	0.095	0.927	0.026

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	22	17	16	20	15	18	8
normalized size	1	1.00	2.75	2.12	2.00	2.50	1.88	2.25	1.00
time (sec)	N/A	0.005	0.003	0.009	0.477	0.413	0.111	0.895	0.167

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.009	0.004	0.001	0.609	0.395	0.099	0.864	0.040

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78
time (sec)	N/A	0.008	0.003	0.003	0.630	0.385	0.081	1.028	0.025

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	17	18	18
normalized size	1	1.00	1.00	0.95	0.90	0.90	0.85	0.90	0.90
time (sec)	N/A	0.009	0.004	0.003	1.340	0.400	0.112	0.915	0.160

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	26	34	26	28
normalized size	1	1.00	1.00	0.87	0.84	0.84	1.10	0.84	0.90
time (sec)	N/A	0.014	0.007	0.003	1.174	0.389	0.122	0.932	0.160

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
normalized size	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.025	0.005	0.005	1.187	0.397	0.122	0.939	0.042

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
normalized size	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.035	0.008	0.007	1.370	0.417	0.145	0.846	0.054

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	23	23	29	24	55
normalized size	1	1.00	1.00	0.86	0.82	0.82	1.04	0.86	1.96
time (sec)	N/A	0.022	0.010	0.007	1.394	0.414	0.145	0.955	0.300

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	32	41	33	46
normalized size	1	1.00	1.00	0.80	0.78	0.78	1.00	0.80	1.12
time (sec)	N/A	0.020	0.006	0.002	1.263	0.403	0.143	1.016	0.063

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	36	35	35	42	36	47
normalized size	1	1.00	1.02	0.88	0.85	0.85	1.02	0.88	1.15
time (sec)	N/A	0.024	0.008	0.006	1.456	0.407	0.144	0.875	0.229

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
normalized size	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.034	0.016	0.008	1.192	0.397	0.149	0.947	0.044

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	26	19	18	18	19	20	10
normalized size	1	1.00	1.86	1.36	1.29	1.29	1.36	1.43	0.71
time (sec)	N/A	0.006	0.005	0.005	1.208	0.411	0.134	1.054	0.056

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
normalized size	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.114	0.015	0.005	1.248	0.399	0.207	1.033	0.075

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.031	0.010	0.006	1.186	0.406	0.195	0.943	0.186

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	35	32	39	41	32	36
normalized size	1	1.00	1.00	0.90	0.82	1.00	1.05	0.82	0.92
time (sec)	N/A	0.014	0.026	0.006	1.102	0.399	0.132	0.932	0.041

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	8	14	10
normalized size	1	1.00	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.017	0.007	0.000	0.439	0.408	0.103	0.914	0.155

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	11	15	12	15	11
normalized size	1	1.00	2.36	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.048	0.095	0.050	0.561	0.448	0.239	1.039	0.087

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
normalized size	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.052	0.226	0.031	1.501	0.447	0.539	0.803	0.060

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	8
normalized size	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.42
time (sec)	N/A	0.004	0.003	0.006	0.574	0.398	0.110	0.842	0.076

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	13	6
normalized size	1	1.00	1.00	0.71	0.65	0.65	0.59	0.76	0.35
time (sec)	N/A	0.002	0.002	0.006	0.517	0.399	0.103	0.898	0.097

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.005	0.005	0.006	0.587	0.407	0.116	0.976	0.205

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	27	37	44	46	40	36
normalized size	1	1.00	0.90	0.55	0.76	0.90	0.94	0.82	0.73
time (sec)	N/A	0.012	0.023	0.003	1.321	0.388	0.115	0.963	0.177

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
normalized size	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.087	0.031	0.011	1.273	0.419	0.370	1.032	0.280

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	54	59	103	65	59	71
normalized size	1	1.00	0.78	0.63	0.69	1.20	0.76	0.69	0.83
time (sec)	N/A	0.098	0.064	0.017	1.200	0.409	0.232	0.900	0.128

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	28	28	44	29	18
normalized size	1	1.00	1.00	1.21	1.17	1.17	1.83	1.21	0.75
time (sec)	N/A	0.005	0.006	0.008	0.506	0.414	0.923	0.846	0.036

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	22	175	272	638	0	139	223
normalized size	1	1.00	0.11	0.88	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.378	0.014	0.053	1.358	1.353	0.000	1.633	0.240

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	22	30	27	20	23	11
normalized size	1	1.00	2.28	1.22	1.67	1.50	1.11	1.28	0.61
time (sec)	N/A	0.012	0.018	0.064	0.549	0.443	0.288	0.965	0.488

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	15	14	15	14	14
normalized size	1	1.00	1.00	1.50	0.83	0.78	0.83	0.78	0.78
time (sec)	N/A	0.006	0.008	0.005	0.606	0.405	0.134	1.054	0.056

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	21	28	20	26	20	20
normalized size	1	1.00	0.88	0.66	0.88	0.62	0.81	0.62	0.62
time (sec)	N/A	0.013	0.012	0.004	0.506	0.410	0.137	0.925	0.027

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.003	0.004	0.006	1.235	0.398	0.173	0.910	0.031

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	19	19	26	20	8
normalized size	1	1.00	1.00	0.90	1.90	1.90	2.60	2.00	0.80
time (sec)	N/A	0.003	0.002	0.003	0.512	0.418	0.599	0.949	0.159

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	50	17	8	22	18	8
normalized size	1	1.00	1.00	3.57	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.004	0.002	0.020	0.510	0.411	0.205	1.236	0.150

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	21	21	36	22	25
normalized size	1	1.00	1.00	1.74	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.023	0.011	0.020	0.605	0.417	2.762	0.958	0.192

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	18	22	17	76	22	19
normalized size	1	1.00	0.66	0.56	0.69	0.53	2.38	0.69	0.59
time (sec)	N/A	0.005	0.006	0.003	0.695	0.411	1.329	1.015	0.033

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	76	24	24
normalized size	1	1.00	1.00	0.81	0.77	0.77	2.45	0.77	0.77
time (sec)	N/A	0.008	0.008	0.008	1.281	0.399	1.330	1.058	0.159

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	20	19	14	117	19	12
normalized size	1	1.00	0.76	0.69	0.66	0.48	4.03	0.66	0.41
time (sec)	N/A	0.007	0.007	0.006	0.558	0.402	0.914	0.968	0.236

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.005	0.002	0.007	1.219	0.402	0.311	0.919	0.243

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	15	15	17	16	15
normalized size	1	1.00	0.95	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.011	0.008	0.003	0.526	0.395	0.159	0.846	0.172

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	26	23	22
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.017	0.013	0.004	0.608	0.423	0.181	0.995	0.039

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	19	16	26	19	16
normalized size	1	1.00	0.74	0.63	0.70	0.59	0.96	0.70	0.59
time (sec)	N/A	0.010	0.007	0.004	0.479	0.406	0.940	1.099	0.262

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	29	242	293	547	311	140	208
normalized size	1	1.00	0.14	1.20	1.46	2.72	1.55	0.70	1.03
time (sec)	N/A	0.216	0.008	0.026	1.219	1.329	25.121	1.604	0.062

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	24	46	45	47	68	45	73
normalized size	1	1.00	0.39	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.037	0.008	0.009	1.281	0.424	0.645	0.959	0.171

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	82	76	121	82	82
normalized size	1	1.00	1.00	0.64	0.63	0.58	0.93	0.63	0.63
time (sec)	N/A	0.046	0.040	0.005	0.483	0.421	3.168	1.034	0.143

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
normalized size	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.009	0.008	0.007	1.201	0.402	0.000	1.135	0.157

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
normalized size	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.021	0.009	0.056	0.655	0.423	0.217	0.828	0.143

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	15	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	0.88	0.88
time (sec)	N/A	0.032	0.014	0.011	0.469	0.404	0.134	0.883	0.213

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	21	21	26	21	9
normalized size	1	1.00	1.00	0.83	1.75	1.75	2.17	1.75	0.75
time (sec)	N/A	0.008	0.004	0.005	0.644	0.428	1.414	0.973	0.030

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	36	35	35	32	37	40
normalized size	1	1.00	1.00	1.29	1.25	1.25	1.14	1.32	1.43
time (sec)	N/A	0.013	0.009	0.006	0.499	0.473	1.724	1.020	0.190

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	30	27	20	23	11
normalized size	1	1.00	1.00	0.51	0.70	0.63	0.47	0.53	0.26
time (sec)	N/A	0.019	0.014	0.026	0.474	0.464	0.237	0.984	0.392

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
normalized size	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.010	0.022	0.000	1.154	0.418	0.524	1.024	0.373

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	24	16	25	17	14	17	11
normalized size	1	1.00	2.18	1.45	2.27	1.55	1.27	1.55	1.00
time (sec)	N/A	0.011	0.017	0.059	0.593	0.467	0.251	0.938	0.068

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	43	22	30	27	20	23	11
normalized size	1	1.00	2.39	1.22	1.67	1.50	1.11	1.28	0.61
time (sec)	N/A	0.012	0.017	0.057	0.633	0.461	0.290	0.967	0.480

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
normalized size	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.052	0.021	0.064	0.605	0.456	0.000	1.111	0.300

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
normalized size	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.027	0.037	0.106	0.539	0.444	0.000	1.147	0.338

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	33	0	25	22
normalized size	1	1.00	1.00	1.33	1.28	1.83	0.00	1.39	1.22
time (sec)	N/A	0.030	0.019	0.045	0.434	0.431	0.000	1.212	0.128

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	61	98	139	61	31
normalized size	1	1.00	1.06	0.97	1.69	2.72	3.86	1.69	0.86
time (sec)	N/A	0.022	0.046	0.070	1.311	0.422	8.115	1.120	0.728

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	0	26	15
normalized size	1	1.00	1.00	1.07	1.00	2.87	0.00	1.73	1.00
time (sec)	N/A	0.025	0.047	0.000	1.127	0.445	0.000	0.821	0.499

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	8	8	7	9	8
normalized size	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.002	0.002	0.000	0.473	0.401	0.080	0.863	0.042

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
normalized size	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.003	0.003	0.003	0.615	0.396	0.142	0.929	0.028

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
normalized size	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.009	0.010	0.000	0.451	0.394	0.356	0.969	0.196

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	27	34	27	29	30
normalized size	1	1.00	1.00	1.50	1.69	2.12	1.69	1.81	1.88
time (sec)	N/A	0.014	0.008	0.002	0.583	0.432	0.122	1.125	0.002

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	14	14	14	14	13
normalized size	1	1.00	1.00	2.47	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.025	0.018	0.002	0.559	0.414	0.105	0.965	0.002

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	11	11	20	11	11
normalized size	1	1.00	0.67	0.71	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.009	0.006	0.000	0.460	0.411	0.207	0.991	0.002

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
normalized size	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.043	0.009	0.010	0.519	0.422	0.161	0.934	0.184

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.014	0.002	0.006	0.484	0.413	0.424	0.925	0.069

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.006	0.003	0.004	0.478	0.437	0.077	1.040	0.031

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	3	3	3
normalized size	1	1.00	1.00	0.80	0.60	0.60	0.60	0.60	0.60
time (sec)	N/A	0.006	0.005	0.009	0.456	0.425	0.701	0.944	0.031

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	10	18
normalized size	1	1.00	0.58	0.79	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.028	0.005	0.002	0.474	0.431	0.071	0.969	0.002

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	18	32
normalized size	1	1.00	1.00	1.12	1.00	1.38	1.00	2.25	4.00
time (sec)	N/A	0.023	0.025	0.049	0.505	0.433	0.147	0.985	0.420

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	11	11	8	11	11
normalized size	1	1.00	1.00	1.31	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.002	0.002	0.001	0.481	0.395	0.188	0.983	0.002

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.007	0.001	0.008	0.589	0.432	0.096	0.932	0.188

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	18	18	107	18	18
normalized size	1	1.00	1.00	0.79	0.75	0.75	4.46	0.75	0.75
time (sec)	N/A	0.006	0.005	0.009	1.280	0.399	1.279	1.124	0.039

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	16	8	13	12
normalized size	1	1.00	1.00	1.08	1.00	1.33	0.67	1.08	1.00
time (sec)	N/A	0.005	0.003	0.007	0.505	0.402	0.085	0.997	0.034

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16
normalized size	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.005	0.002	0.008	1.173	0.404	0.129	1.013	0.180

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	0	18
normalized size	1	1.00	1.00	1.36	1.32	1.32	1.45	0.00	0.82
time (sec)	N/A	0.056	0.023	0.003	0.521	0.393	2.326	0.000	0.254

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	43	19	46	54	42	42
normalized size	1	1.00	0.81	1.59	0.70	1.70	2.00	1.56	1.56
time (sec)	N/A	0.006	0.019	0.002	0.473	0.399	0.965	1.275	0.032

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	30	14	14	14	14
normalized size	1	1.00	1.00	1.29	1.76	0.82	0.82	0.82	0.82
time (sec)	N/A	0.025	0.011	0.030	0.472	0.420	0.113	1.398	0.187

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.008	0.004	0.004	1.299	0.396	0.110	1.253	0.162

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	24	24	26	24
normalized size	1	1.00	0.88	0.78	0.75	0.75	0.75	0.81	0.75
time (sec)	N/A	0.010	0.010	0.004	1.203	0.438	0.211	1.273	0.028

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	73	40	30
normalized size	1	1.00	1.00	0.83	1.17	0.93	2.43	1.33	1.00
time (sec)	N/A	0.015	0.006	0.006	1.148	0.395	1.356	1.397	0.063

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	13
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.00
time (sec)	N/A	0.003	0.003	0.001	0.490	0.412	0.100	1.269	0.050

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	44	14	17	27	16
normalized size	1	1.00	0.88	1.06	2.75	0.88	1.06	1.69	1.00
time (sec)	N/A	0.027	0.013	0.016	0.475	0.406	0.356	1.329	0.180

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
normalized size	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.009	0.006	0.002	0.481	0.387	0.105	1.298	0.070

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	3	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.018	0.005	0.025	1.119	0.420	0.232	0.939	0.073

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.014	0.002	0.498	0.405	0.326	1.019	0.225

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.004	0.004	0.006	0.505	0.409	0.065	0.929	0.018

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.020	0.008	0.000	0.672	0.409	0.099	1.003	0.046

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.010	0.031	0.015	0.474	0.417	0.325	1.008	0.028

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	22	26	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.29	1.53	0.76	0.76
time (sec)	N/A	0.008	0.007	0.075	0.539	0.433	0.577	1.038	0.058

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.014	0.006	0.006	1.268	0.395	0.131	0.922	0.176

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	28	46	29	25	29	49	25
normalized size	1	1.00	0.72	1.18	0.74	0.64	0.74	1.26	0.64
time (sec)	N/A	0.015	0.009	0.004	0.473	0.396	0.115	1.030	0.041

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	13
normalized size	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.027	0.003	0.003	0.482	0.407	0.096	0.873	0.050

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	10	10	12	10	10
normalized size	1	1.00	1.50	0.92	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.005	0.009	0.006	1.111	0.416	0.072	1.295	0.178

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	22	20	0	21	20
normalized size	1	1.00	0.96	1.20	0.88	0.80	0.00	0.84	0.80
time (sec)	N/A	0.005	0.006	0.005	1.094	0.404	0.000	1.138	0.273

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	21	21	20	21	21
normalized size	1	1.00	0.85	0.81	0.78	0.78	0.74	0.78	0.78
time (sec)	N/A	0.015	0.006	0.004	0.486	0.426	0.346	0.900	0.168

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
normalized size	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.003	0.004	0.002	0.577	0.390	1.674	1.013	0.026

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	19	15	15	16	15
normalized size	1	1.00	1.00	1.00	3.17	2.50	2.50	2.67	2.50
time (sec)	N/A	0.010	0.004	0.005	0.584	0.415	0.111	0.794	0.099

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.013	0.006	0.005	1.248	0.390	0.109	1.651	0.186

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.003	0.002	0.001	1.485	0.398	0.236	0.897	0.219

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	18	25	31	22	26
normalized size	1	1.00	0.88	0.76	0.53	0.74	0.91	0.65	0.76
time (sec)	N/A	0.033	0.009	0.001	0.504	0.428	0.066	0.910	0.039

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	8	29	0	6	6
normalized size	1	1.00	1.00	0.58	0.67	2.42	0.00	0.50	0.50
time (sec)	N/A	0.007	0.006	0.005	1.217	0.401	0.000	1.018	0.163

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	59	14	21	44	14	21
normalized size	1	1.00	1.00	3.69	0.88	1.31	2.75	0.88	1.31
time (sec)	N/A	0.046	0.028	0.018	0.489	0.407	5.471	0.819	0.134

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	5	7	12	7	5
normalized size	1	1.00	2.86	0.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.013	0.006	0.029	0.532	0.428	2.257	0.834	0.049

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	16	15
normalized size	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.50
time (sec)	N/A	0.017	0.002	0.009	0.479	0.419	0.107	0.910	0.027

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	32	32	41	33	46
normalized size	1	1.00	1.00	0.81	0.74	0.74	0.95	0.77	1.07
time (sec)	N/A	0.020	0.009	0.005	1.113	0.406	0.139	0.816	0.088

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	74	30	42	57	37
normalized size	1	1.00	0.78	1.03	2.00	0.81	1.14	1.54	1.00
time (sec)	N/A	0.076	0.017	0.017	0.482	0.405	2.150	1.030	0.210

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
normalized size	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.023	0.008	0.043	0.503	0.431	0.000	1.078	2.593

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	12	16	15
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.60	0.80	0.75
time (sec)	N/A	0.002	0.000	0.002	0.598	0.352	0.057	0.967	0.027

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	9	9	8	10	9
normalized size	1	1.00	1.00	1.00	0.75	0.75	0.67	0.83	0.75
time (sec)	N/A	0.021	0.011	0.010	0.647	0.418	0.087	1.283	0.181

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.011	0.005	0.010	0.443	0.407	0.108	0.974	0.053

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	30	27	20	23	11
normalized size	1	1.00	1.00	0.51	0.70	0.63	0.47	0.53	0.26
time (sec)	N/A	0.018	0.013	0.029	0.466	0.436	0.246	1.130	0.396

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	18	15	16	22	144	43	14
normalized size	1	1.00	0.75	0.62	0.67	0.92	6.00	1.79	0.58
time (sec)	N/A	0.004	0.007	0.003	0.628	0.389	1.104	0.936	0.161

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	26	16	16	34	16	21
normalized size	1	1.00	0.63	0.68	0.42	0.42	0.89	0.42	0.55
time (sec)	N/A	0.019	0.008	0.005	0.575	0.396	0.341	1.074	0.032

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	30	32	39	30	32
normalized size	1	1.00	1.00	2.51	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.036	0.017	0.015	1.346	0.400	2.558	0.952	0.196

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
normalized size	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.029	0.004	0.009	0.545	0.402	0.112	0.982	0.043

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	38	26	39	26	34
normalized size	1	1.00	0.72	0.88	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.067	0.061	0.022	0.562	0.410	0.354	0.899	0.200

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.009	2.753	0.011	0.465	0.426	0.522	0.906	0.219

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	18	3	4	4
normalized size	1	1.00	1.00	0.83	0.67	3.00	0.50	0.67	0.67
time (sec)	N/A	0.001	0.005	0.004	1.125	0.403	0.154	0.980	0.011

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	30	62	62	61	22	29
normalized size	1	1.00	0.65	0.81	1.68	1.68	1.65	0.59	0.78
time (sec)	N/A	0.011	0.007	0.007	0.480	0.389	0.147	0.875	0.124

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	58	18	36	19	18	18
normalized size	1	1.00	1.00	2.76	0.86	1.71	0.90	0.86	0.86
time (sec)	N/A	0.029	0.024	0.062	0.524	0.439	0.124	1.025	0.377

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.034	0.058	0.023	0.578	0.433	0.220	0.935	0.308

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	27	4	3	3	3	3	3
normalized size	1	1.00	6.75	1.00	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.021	0.005	0.005	0.499	0.422	0.816	0.953	0.249

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	11	10	15	11
normalized size	1	1.00	1.00	0.92	1.15	0.85	0.77	1.15	0.85
time (sec)	N/A	0.005	0.003	0.006	0.473	0.401	0.107	0.903	0.069

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	21	21	22	21	21
normalized size	1	1.00	0.55	0.55	0.48	0.48	0.50	0.48	0.48
time (sec)	N/A	0.039	0.009	0.003	0.465	0.413	0.096	0.943	0.042

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	27	25	37	27	27
normalized size	1	1.00	1.00	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.012	0.009	0.003	1.134	0.440	3.708	1.028	0.043

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	14
normalized size	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	0.56
time (sec)	N/A	0.030	0.010	0.078	0.607	0.453	13.251	0.989	0.223

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	7	10	8
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.58	0.83	0.67
time (sec)	N/A	0.001	0.001	0.002	0.535	0.442	0.090	1.011	0.175

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	62	41	43	32	0	30	43
normalized size	1	1.00	1.51	1.00	1.05	0.78	0.00	0.73	1.05
time (sec)	N/A	0.013	0.024	0.008	1.201	0.425	0.000	1.132	0.050

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	27	27	29	28	-1
normalized size	1	1.00	0.79	3.50	0.79	0.79	0.85	0.82	-0.03
time (sec)	N/A	0.037	0.021	0.000	1.312	0.444	2.855	0.942	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	42	17	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	2.21	0.89	0.89
time (sec)	N/A	0.006	0.004	0.004	1.274	0.402	0.111	1.028	0.043

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	30	39	37	0	31	28
normalized size	1	1.00	1.03	0.79	1.03	0.97	0.00	0.82	0.74
time (sec)	N/A	0.012	0.015	0.003	1.324	0.422	0.000	0.936	0.046

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67
time (sec)	N/A	0.005	0.004	0.003	1.343	0.398	0.119	0.791	0.182

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	26	36	26	28
normalized size	1	1.00	1.00	0.87	0.84	0.84	1.16	0.84	0.90
time (sec)	N/A	0.015	0.010	0.002	1.283	0.409	0.114	1.016	0.038

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	37	16	121	29	0	150	19
normalized size	1	1.00	3.70	1.60	12.10	2.90	0.00	15.00	1.90
time (sec)	N/A	0.010	0.010	0.013	1.280	0.449	0.000	1.025	0.111

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	29	26	22	30	13
normalized size	1	1.00	1.00	2.00	1.93	1.73	1.47	2.00	0.87
time (sec)	N/A	0.007	0.004	0.005	0.522	0.417	0.157	1.066	0.059

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
normalized size	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.006	0.016	0.001	0.000	0.410	0.360	1.098	0.199

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	19	17	17	18	17
normalized size	1	1.00	0.70	0.78	0.83	0.74	0.74	0.78	0.74
time (sec)	N/A	0.017	0.011	0.007	0.444	0.404	0.123	0.970	0.249

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.008	0.008	0.005	0.545	0.444	0.431	0.906	0.251

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	19	14	20	18
normalized size	1	1.00	1.00	0.89	1.00	1.06	0.78	1.11	1.00
time (sec)	N/A	0.008	0.003	0.010	0.502	0.419	0.136	0.860	0.168

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	19	20	19	25	20	20	19
normalized size	1	1.00	1.58	1.67	1.58	2.08	1.67	1.67	1.58
time (sec)	N/A	0.012	0.005	0.012	0.610	0.426	0.124	0.883	0.232

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	15	12	16	8
normalized size	1	1.00	1.00	1.38	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.037	0.008	0.049	1.319	0.436	1.149	0.906	0.213

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	44	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.67	2.44	0.67	0.67
time (sec)	N/A	0.003	0.002	0.006	1.698	0.410	1.011	1.118	0.145

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	24	0	9	18
normalized size	1	1.00	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.050	0.017	0.049	0.000	0.521	0.000	1.038	0.430

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	30	27	22	22
normalized size	1	1.00	1.00	0.77	0.73	1.00	0.90	0.73	0.73
time (sec)	N/A	0.006	0.008	0.005	1.303	0.413	0.264	0.996	0.173

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	21	21	26	21	23
normalized size	1	1.00	0.83	1.04	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.037	0.016	0.012	0.684	0.441	0.683	0.957	0.031

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	42	49	39	0	40	39
normalized size	1	1.00	0.76	0.84	0.98	0.78	0.00	0.80	0.78
time (sec)	N/A	0.015	0.020	0.007	1.424	0.405	0.000	0.952	0.212

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	47	9	46	51	9	9
normalized size	1	1.00	1.00	4.27	0.82	4.18	4.64	0.82	0.82
time (sec)	N/A	0.001	0.002	0.001	0.584	0.344	0.064	0.983	0.206

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	28	19	19	20	19	19
normalized size	1	1.00	1.24	1.12	0.76	0.76	0.80	0.76	0.76
time (sec)	N/A	0.027	0.010	0.009	0.476	0.422	0.070	0.869	0.172

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.010	0.029	0.016	0.493	0.420	0.494	0.923	0.029

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	41	24	34	56	36	70	18
normalized size	1	1.00	1.71	1.00	1.42	2.33	1.50	2.92	0.75
time (sec)	N/A	0.011	0.009	0.018	0.513	0.446	0.136	1.043	0.069

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	47	33	35	17	44	32
normalized size	1	1.00	1.03	1.38	0.97	1.03	0.50	1.29	0.94
time (sec)	N/A	0.008	0.016	0.006	1.241	0.407	0.255	1.028	0.516

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	75	38	37
normalized size	1	1.00	1.00	0.83	1.17	0.93	2.50	1.27	1.23
time (sec)	N/A	0.017	0.006	0.006	1.194	0.406	1.558	0.952	0.107

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	17	16	16	15	16	16
normalized size	1	1.00	0.59	0.53	0.50	0.50	0.47	0.50	0.50
time (sec)	N/A	0.019	0.006	0.003	0.518	0.395	0.091	0.885	0.033

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	17	12
normalized size	1	1.00	1.35	0.78	0.74	0.52	1.13	0.74	0.52
time (sec)	N/A	0.037	0.023	0.016	0.433	0.426	0.357	0.887	0.103

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	21	21	19	21	21
normalized size	1	1.00	0.89	0.81	0.78	0.78	0.70	0.78	0.78
time (sec)	N/A	0.015	0.005	0.004	1.049	0.452	0.203	0.888	0.278

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	53	28	29	32	28
normalized size	1	1.00	0.84	0.82	1.39	0.74	0.76	0.84	0.74
time (sec)	N/A	0.023	0.011	0.003	1.082	0.434	0.614	0.953	0.220

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	5	4	16	0	6	6
normalized size	1	1.00	1.00	1.00	0.80	3.20	0.00	1.20	1.20
time (sec)	N/A	0.010	0.005	0.005	0.476	0.421	0.000	0.941	0.047

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	21	21	27	21	23
normalized size	1	1.00	0.86	0.83	0.72	0.72	0.93	0.72	0.79
time (sec)	N/A	0.028	0.024	0.003	0.458	0.418	0.333	0.920	0.193

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	36	47	0	26	27
normalized size	1	1.00	0.92	0.81	1.00	1.31	0.00	0.72	0.75
time (sec)	N/A	0.010	0.017	0.003	1.120	0.413	0.000	1.093	0.176

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	23	22	22	24	22	22
normalized size	1	1.00	1.11	0.82	0.79	0.79	0.86	0.79	0.79
time (sec)	N/A	0.019	0.010	0.013	1.162	0.408	0.131	0.962	0.056

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	58	25	42	43	46	38	29
normalized size	1	1.00	2.23	0.96	1.62	1.65	1.77	1.46	1.12
time (sec)	N/A	0.014	0.122	0.066	0.648	0.459	0.157	0.902	0.058

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	32	24	33	46	22	22
normalized size	1	1.00	0.65	0.70	0.52	0.72	1.00	0.48	0.48
time (sec)	N/A	0.020	0.007	0.017	0.526	0.435	0.069	1.019	0.207

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	11
normalized size	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.55
time (sec)	N/A	0.035	0.015	0.009	0.484	0.430	5.392	1.043	0.248

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	16	24	17	19	19
normalized size	1	1.00	1.00	1.05	0.76	1.14	0.81	0.90	0.90
time (sec)	N/A	0.026	0.017	0.008	0.544	0.422	0.106	0.858	0.069

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	31	47	58	0	50	-1
normalized size	1	1.00	0.89	0.84	1.27	1.57	0.00	1.35	-0.03
time (sec)	N/A	0.041	0.020	0.027	1.369	0.557	0.000	1.049	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	40	26	29	27	0	28	23
normalized size	1	1.00	1.43	0.93	1.04	0.96	0.00	1.00	0.82
time (sec)	N/A	0.008	0.028	0.004	0.425	0.413	0.000	1.131	0.152

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	15	18	17	15	21
normalized size	1	1.00	1.21	0.89	0.79	0.95	0.89	0.79	1.11
time (sec)	N/A	0.009	0.002	0.062	0.419	0.436	0.071	1.048	0.035

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	26	25	24	24	22	24	24
normalized size	1	1.00	0.57	0.54	0.52	0.52	0.48	0.52	0.52
time (sec)	N/A	0.040	0.007	0.003	0.437	0.407	0.094	0.877	0.027

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	34	33	16	34	17	-1
normalized size	1	1.00	1.00	1.89	1.83	0.89	1.89	0.94	-0.06
time (sec)	N/A	0.007	0.003	0.079	0.424	0.415	1.089	1.014	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	24	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.89	0.70	0.70
time (sec)	N/A	0.009	0.044	0.014	0.422	0.423	0.317	0.956	0.200

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	18	17	16	15	0	16	18
normalized size	1	1.22	1.00	0.94	0.89	0.83	0.00	0.89	1.00
time (sec)	N/A	0.030	0.006	0.003	0.422	0.431	0.000	1.039	0.235

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	21	21	20	21	21
normalized size	1	1.00	0.85	0.81	0.78	0.78	0.74	0.78	0.78
time (sec)	N/A	0.016	0.005	0.001	0.417	0.427	0.354	0.958	0.002

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	19	20	31
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.73	0.77	1.19
time (sec)	N/A	0.012	0.006	0.006	0.961	0.424	1.388	0.868	0.257

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	14	9	9	15	9	9
normalized size	1	1.00	0.73	0.93	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.017	0.016	0.020	0.444	0.437	1.991	0.948	0.274

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	35	34	37	122	29	30
normalized size	1	1.00	0.74	0.74	0.72	0.79	2.60	0.62	0.64
time (sec)	N/A	0.010	0.019	0.005	0.958	0.420	2.793	0.997	0.037

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	36	27	9	26	27	9	26
normalized size	1	1.00	3.27	2.45	0.82	2.36	2.45	0.82	2.36
time (sec)	N/A	0.002	0.002	0.002	0.418	0.338	0.058	0.873	0.024

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	13	13	12	13	14
normalized size	1	1.00	1.00	1.06	0.76	0.76	0.71	0.76	0.82
time (sec)	N/A	0.022	0.006	0.008	0.435	0.452	0.068	0.975	0.050

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	13	20	29	13	13
normalized size	1	1.00	1.59	1.29	0.76	1.18	1.71	0.76	0.76
time (sec)	N/A	0.023	0.016	0.001	0.422	0.404	0.076	0.857	0.190

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	17	36	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.63	1.33	0.70	0.52
time (sec)	N/A	0.005	0.006	0.002	0.419	0.403	1.023	0.937	0.035

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
normalized size	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.009	0.002	0.001	0.424	0.422	0.063	0.902	0.032

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16
normalized size	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33
time (sec)	N/A	0.006	0.003	0.003	0.445	0.452	0.093	1.027	0.019

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	22	34	26	53	28	25
normalized size	1	1.00	0.62	0.55	0.85	0.65	1.32	0.70	0.62
time (sec)	N/A	0.014	0.008	0.004	0.969	0.405	2.025	0.990	0.025

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [19] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	3	0.333
2	A	1	1	1.00	3	0.333
3	A	1	1	1.00	3	0.333
4	A	1	1	1.00	3	0.333
5	A	1	1	1.00	2	0.500
6	A	1	1	1.00	2	0.500
7	A	2	2	1.00	4	0.500
8	A	2	2	1.00	4	0.500
9	A	2	2	1.00	5	0.400
10	A	2	2	1.00	5	0.400
11	A	1	1	1.00	2	0.500
12	A	1	1	1.00	2	0.500
13	A	1	1	1.00	2	0.500
14	A	1	1	1.00	2	0.500
15	A	2	2	1.00	4	0.500
16	A	1	1	1.00	2	0.500
17	A	3	2	1.00	7	0.286
18	A	1	1	1.00	6	0.167
19	A	2	2	1.00	2	1.000
20	A	2	2	1.00	7	0.286
21	A	2	2	1.00	4	0.500
22	A	2	2	1.00	6	0.333
23	A	1	1	1.00	4	0.250
24	A	3	2	1.00	8	0.250
25	A	3	2	1.00	8	0.250
26	A	2	2	1.00	4	0.500
27	A	2	2	1.00	2	1.000
28	A	3	3	1.00	6	0.500
29	A	2	2	1.00	6	0.333
30	A	1	1	1.00	6	0.167
31	A	4	2	1.00	7	0.286
32	A	1	1	1.00	10	0.100
33	A	1	1	1.00	10	0.100
34	A	2	2	1.00	4	0.500
35	A	2	2	1.00	6	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	2	2	1.00	7	0.286
37	A	1	1	1.00	8	0.125
38	A	2	2	1.00	6	0.333
39	A	3	2	1.00	9	0.222
40	A	2	2	1.00	2	1.000
41	A	2	2	1.00	6	0.333
42	A	1	1	1.00	9	0.111
43	A	1	1	1.00	9	0.111
44	A	2	2	1.00	6	0.333
45	A	2	2	1.00	9	0.222
46	A	2	2	1.00	9	0.222
47	A	2	2	1.00	5	0.400
48	A	1	1	1.00	3	0.333
49	A	3	3	1.00	7	0.429
50	A	1	1	1.00	6	0.167
51	A	1	1	1.00	3	0.333
52	A	3	3	1.00	6	0.500
53	A	3	3	1.00	8	0.375
54	A	3	2	1.00	9	0.222
55	A	3	3	1.00	4	0.750
56	A	2	2	1.00	6	0.333
57	A	1	1	1.00	8	0.125
58	A	2	2	1.00	6	0.333
59	A	2	2	1.00	4	0.500
60	A	3	2	1.00	4	0.500
61	A	2	1	1.00	4	0.250
62	A	3	2	1.00	9	0.222
63	A	3	2	1.00	9	0.222
64	A	4	3	1.00	9	0.333
65	A	3	3	1.00	9	0.333
66	A	1	1	1.00	10	0.100
67	A	3	2	1.00	11	0.182
68	A	4	3	1.00	9	0.333
69	A	4	2	1.00	4	0.500
70	A	4	2	1.00	4	0.500
71	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	2	1	1.00	4	0.250
73	A	5	3	1.00	9	0.333
74	A	3	2	1.00	11	0.182
75	A	3	2	1.00	11	0.182
76	A	3	3	1.00	14	0.214
77	A	3	2	1.00	8	0.250
78	A	3	2	1.00	7	0.286
79	A	4	3	1.00	9	0.333
80	A	2	2	1.00	9	0.222
81	A	1	1	1.00	8	0.125
82	A	2	2	1.00	4	0.500
83	A	3	2	1.00	4	0.500
84	A	2	1	1.00	4	0.250
85	A	2	1	1.00	4	0.250
86	A	2	2	1.00	9	0.222
87	A	3	2	1.00	9	0.222
88	A	2	2	1.00	7	0.286
89	A	3	2	1.00	9	0.222
90	A	3	2	1.00	4	0.500
91	A	4	2	1.00	4	0.500
92	A	3	2	1.00	7	0.286
93	A	3	2	1.00	9	0.222
94	A	2	2	1.00	7	0.286
95	A	3	2	1.00	9	0.222
96	A	2	2	1.00	7	0.286
97	A	2	2	1.00	7	0.286
98	A	2	2	1.00	4	0.500
99	A	2	2	1.00	4	0.500
100	A	3	2	1.00	9	0.222
101	A	3	2	1.00	9	0.222
102	A	1	1	1.00	2	0.500
103	A	2	2	1.00	4	0.500
104	A	3	3	1.00	5	0.600
105	A	2	1	1.00	4	0.250
106	A	1	1	1.00	9	0.111
107	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	1	1	1.00	9	0.111
109	A	1	1	1.00	9	0.111
110	A	2	2	1.00	7	0.286
111	A	5	2	1.00	11	0.182
112	A	2	2	1.00	13	0.154
113	A	6	4	1.00	10	0.400
114	A	3	2	1.00	7	0.286
115	A	4	3	1.00	9	0.333
116	A	2	2	1.00	7	0.286
117	A	3	2	1.00	9	0.222
118	A	2	2	1.00	15	0.133
119	A	1	1	1.00	13	0.077
120	A	1	1	1.00	11	0.091
121	A	2	2	1.00	13	0.154
122	A	3	2	1.00	15	0.133
123	A	3	3	1.00	16	0.188
124	A	1	1	1.00	15	0.067
125	A	3	2	1.00	15	0.133
126	A	1	1	1.00	13	0.077
127	A	1	1	1.00	13	0.077
128	A	2	2	1.00	11	0.182
129	A	3	2	1.00	13	0.154
130	A	1	1	1.00	9	0.111
131	A	2	2	1.00	9	0.222
132	A	4	4	1.00	13	0.308
133	A	1	1	1.00	17	0.059
134	A	4	4	1.00	15	0.267
135	A	1	1	1.00	15	0.067
136	A	3	3	1.00	17	0.176
137	A	2	2	1.00	15	0.133
138	A	3	3	1.00	13	0.231
139	A	1	1	1.00	11	0.091
140	A	3	2	1.00	15	0.133
141	A	3	3	1.00	15	0.200
142	A	2	2	1.00	12	0.167
143	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	3	3	1.00	13	0.231
145	A	2	2	1.00	12	0.167
146	A	2	2	1.00	14	0.143
147	A	4	4	1.00	17	0.235
148	A	3	3	1.00	10	0.300
149	A	2	2	1.00	14	0.143
150	A	3	3	1.00	17	0.176
151	A	4	4	1.00	11	0.364
152	A	2	2	1.00	11	0.182
153	A	3	2	1.00	12	0.167
154	A	3	2	1.00	11	0.182
155	A	3	2	1.00	25	0.080
156	A	2	1	1.00	29	0.034
157	A	6	5	1.00	20	0.250
158	A	6	5	1.00	23	0.217
159	A	14	10	1.00	32	0.312
160	A	6	5	1.00	26	0.192
161	A	2	2	1.00	7	0.286
162	A	3	2	1.00	11	0.182
163	A	4	3	1.00	14	0.214
164	A	2	1	1.00	23	0.043
165	A	3	2	1.00	22	0.091
166	A	3	2	1.00	11	0.182
167	A	4	3	1.00	26	0.115
168	A	3	2	1.00	16	0.125
169	A	8	4	1.00	25	0.160
170	A	6	3	1.00	23	0.130
171	A	6	5	1.00	26	0.192
172	A	3	2	1.00	11	0.182
173	A	8	7	1.00	20	0.350
174	A	7	6	1.00	23	0.261
175	A	8	8	1.00	11	0.727
176	A	2	1	1.00	9	0.111
177	A	2	1	1.00	7	0.143
178	A	2	1	1.00	16	0.062
179	A	3	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	2	1	1.00	13	0.077
181	A	3	2	1.00	11	0.182
182	A	3	2	1.00	15	0.133
183	A	5	3	1.00	20	0.150
184	A	2	1	1.00	11	0.091
185	A	2	1	1.00	16	0.062
186	A	3	2	1.00	25	0.080
187	A	3	2	1.00	12	0.167
188	A	2	1	1.00	11	0.091
189	A	2	1	1.00	14	0.071
190	A	3	2	1.00	22	0.091
191	A	2	1	1.00	24	0.042
192	A	1	1	1.00	20	0.050
193	A	2	1	1.00	9	0.111
194	A	2	1	1.00	9	0.111
195	A	3	3	1.00	11	0.273
196	A	1	1	1.00	22	0.045
197	A	3	2	1.00	11	0.182
198	A	4	4	1.00	14	0.286
199	A	4	4	1.00	10	0.400
200	A	6	5	1.00	23	0.217
201	A	5	4	1.00	23	0.174
202	A	6	5	1.00	15	0.333
203	A	6	6	1.00	7	0.857
204	A	7	7	1.00	11	0.636
205	A	5	4	1.00	21	0.190
206	A	4	4	1.00	11	0.364
207	A	6	4	1.00	30	0.133
208	A	7	6	1.00	24	0.250
209	A	3	3	1.00	14	0.214
210	A	3	2	1.00	16	0.125
211	A	2	2	1.00	21	0.095
212	A	3	3	1.00	15	0.200
213	A	3	2	1.00	10	0.200
214	A	1	1	1.00	9	0.111
215	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	3	2	1.00	10	0.200
217	A	6	5	1.00	43	0.116
218	A	7	5	1.00	50	0.100
219	A	3	3	1.00	11	0.273
220	A	9	9	1.00	15	0.600
221	A	2	2	1.00	11	0.182
222	A	3	2	1.00	9	0.222
223	A	3	2	1.00	9	0.222
224	A	3	3	1.00	11	0.273
225	A	2	2	1.00	11	0.182
226	A	2	2	1.00	11	0.182
227	A	4	2	1.00	13	0.154
228	A	2	1	1.00	11	0.091
229	A	3	3	1.00	13	0.231
230	A	3	2	1.00	11	0.182
231	A	3	3	1.00	13	0.231
232	A	3	2	1.00	17	0.118
233	A	4	3	1.00	17	0.176
234	A	3	2	1.00	13	0.154
235	A	10	9	1.00	21	0.429
236	A	9	9	1.00	13	0.692
237	A	4	3	1.00	13	0.231
238	A	5	5	1.00	13	0.385
239	A	2	2	1.00	12	0.167
240	A	4	3	1.00	20	0.150
241	A	3	3	1.00	9	0.333
242	A	4	4	1.00	11	0.364
243	A	4	3	1.00	8	0.375
244	A	2	2	1.00	7	0.286
245	A	2	2	1.00	10	0.200
246	A	2	2	1.00	11	0.182
247	A	6	6	1.00	7	0.857
248	A	4	2	1.00	11	0.182
249	A	4	3	1.00	9	0.333
250	A	2	2	1.00	11	0.182
251	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	1	1	1.00	9	0.111
253	A	2	1	1.00	13	0.077
254	A	1	1	1.00	8	0.125
255	A	2	2	1.00	7	0.286
256	A	3	2	1.00	9	0.222
257	A	3	3	1.00	7	0.429
258	A	3	2	1.00	20	0.100
259	A	2	2	1.00	10	0.200
260	A	2	1	1.00	11	0.091
261	A	2	2	1.00	7	0.286
262	A	3	3	1.00	9	0.333
263	A	1	1	1.00	15	0.067
264	A	1	1	1.00	13	0.077
265	A	1	1	1.00	6	0.167
266	A	3	3	1.00	13	0.231
267	A	2	1	1.00	7	0.143
268	A	3	3	1.00	6	0.500
269	A	4	4	1.00	16	0.250
270	A	3	2	1.00	9	0.222
271	A	3	2	1.00	9	0.222
272	A	4	4	1.00	12	0.333
273	A	3	3	1.00	4	0.750
274	A	4	4	1.00	15	0.267
275	A	3	2	1.00	12	0.167
276	A	3	2	1.00	6	0.333
277	A	1	1	1.00	21	0.048
278	A	2	2	1.00	11	0.182
279	A	3	3	1.00	6	0.500
280	A	1	1	1.00	4	0.250
281	A	3	2	1.00	13	0.154
282	A	1	1	1.00	10	0.100
283	A	1	1	1.00	9	0.111
284	A	5	4	1.00	11	0.364
285	A	3	2	1.00	8	0.250
286	A	2	2	1.00	11	0.182
287	A	2	2	1.00	6	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	2	2	1.00	14	0.143
289	A	4	3	1.00	6	0.500
290	A	2	1	1.00	15	0.067
291	A	2	2	1.00	13	0.154
292	A	3	3	1.00	14	0.214
293	A	2	2	1.00	9	0.222
294	A	4	3	1.00	9	0.333
295	A	2	2	1.00	14	0.143
296	A	3	2	1.00	22	0.091
297	A	2	2	1.00	7	0.286
298	A	2	2	1.00	13	0.154
299	A	6	6	1.00	7	0.857
300	A	6	2	1.00	6	0.333
301	A	1	3	1.00	8	0.375
302	A	1	0	1.00	10	0.000
303	A	3	2	1.00	15	0.133
304	A	4	3	1.00	16	0.188
305	A	4	3	1.00	8	0.375
306	A	2	1	1.00	9	0.111
307	A	4	3	1.00	7	0.429
308	A	5	4	1.00	12	0.333
309	A	3	2	1.00	17	0.118
310	A	8	4	1.00	13	0.308
311	A	2	2	1.00	6	0.333
312	A	1	1	1.00	11	0.091
313	A	2	1	1.00	9	0.111
314	A	3	2	1.00	13	0.154
315	A	6	5	1.00	6	0.833
316	A	1	1	1.00	12	0.083
317	A	4	4	1.00	11	0.364
318	A	4	2	1.00	9	0.222
319	A	7	4	1.00	15	0.267
320	A	5	2	1.00	11	0.182
321	A	1	1	1.00	6	0.167
322	A	3	3	1.00	15	0.200
323	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	3	3	1.00	13	0.231
325	A	3	3	1.00	12	0.250
326	A	2	2	1.00	11	0.182
327	A	4	4	1.00	12	0.333
328	A	2	2	1.00	6	0.333
329	A	2	2	1.00	13	0.154
330	A	3	3	1.00	15	0.200
331	A	4	3	1.00	16	0.188
332	A	2	2	1.00	12	0.167
333	A	4	4	1.00	8	0.500
334	A	3	3	1.00	13	0.231
335	A	4	4	1.00	17	0.235
336	A	2	2	1.00	13	0.154
337	A	5	4	1.00	17	0.235
338	A	2	2	1.00	15	0.133
339	A	4	2	1.00	6	0.333
340	A	4	4	1.00	14	0.286
341	A	1	1	1.00	9	0.111
342	A	3	2	1.00	9	0.222
343	A	1	1	1.00	10	0.100
344	A	2	2	1.00	8	0.250
345	A	3	3	1.00	15	0.200
346	A	4	4	1.00	15	0.267
347	A	3	2	1.00	9	0.222
348	A	3	2	1.00	13	0.154
349	A	3	3	1.00	6	0.500
350	A	5	5	1.00	8	0.625
351	A	2	2	1.00	8	0.250
352	A	3	2	1.00	8	0.250
353	A	3	3	1.00	14	0.214
354	A	3	2	1.00	15	0.133
355	A	3	2	1.00	4	0.500
356	A	4	2	1.00	6	0.333
357	A	4	4	1.00	10	0.400
358	A	3	2	1.00	15	0.133
359	A	4	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	3	3	1.00	13	0.231
361	A	2	1	1.00	4	0.250
362	A	5	2	1.00	9	0.222
363	A	3	3	1.00	13	0.231
364	A	1	1	1.00	10	0.100
365	A	4	4	1.22	10	0.400
366	A	4	3	1.00	6	0.500
367	A	4	4	1.00	11	0.364
368	A	4	3	1.00	9	0.333
369	A	3	3	1.00	15	0.200
370	A	1	1	1.00	11	0.091
371	A	3	2	1.00	9	0.222
372	A	3	2	1.00	9	0.222
373	A	2	1	1.00	11	0.091
374	A	3	2	1.00	4	0.500
375	A	2	2	1.00	4	0.500
376	A	3	2	1.00	13	0.154

Chapter 3

Listing of integrals

3.1 $\int x^n\,dx$

Optimal. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

```
[Out] x^(1+n)/(1+n)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n\,dx = \frac{x^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

fricas [A] time = 0.42, size = 10, normalized size = 0.91

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
```

```
[Out] x*x^n/(n + 1)
```

giac [A] time = 0.93, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] x^(n+1)/(n+1)
```

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
```

```
[Out] x^(n + 1)/(n + 1)
```

mupad [B] time = 0.32, size = 20, normalized size = 1.82

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

sympy [A] time = 0.06, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

3.2 $\int e^x\,dx$

Optimal. Leaf size=3

$$e^x$$

[Out] exp(x)

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$e^x$$

Antiderivative was successfully verified.

[In] Int[E^x,x]

[Out] E^x

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x\,dx = e^x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Antiderivative was successfully verified.

[In] Integrate[E^x,x]

[Out] E^x

fricas [A] time = 0.41, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x),x, algorithm="fricas")

[Out] e^x

giac [A] time = 0.91, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x),x, algorithm="giac")

[Out] e^x

maple [A] time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
```

```
[Out] e^x
```

mupad [B] time = 0.01, size = 2, normalized size = 0.67

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

sympy [A] time = 0.04, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

3.3 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
[Out] Log[x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

giac [A] time = 0.93, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
[Out] log(abs(x))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
[Out] ln(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
[Out] log(x)
```

mupad [B] time = 0.00, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
[Out] log(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
[Out] log(x)
```

3.4 $\int a^x \, dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

[Out] a^x/ln(a)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

[In] Int[a^x,x]

[Out] a^x/Log[a]

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[a^x,x]

[Out] a^x/Log[a]

fricas [A] time = 0.42, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x,x, algorithm="fricas")

[Out] a^x/log(a)

giac [A] time = 0.83, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x,x, algorithm="giac")

```
[Out] a^x/log(a)
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
[Out] a^x/ln(a)
```

maxima [A] time = 0.42, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
[Out] a^x/log(a)
```

mupad [B] time = 0.18, size = 8, normalized size = 1.00

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
[Out] a^x/log(a)
```

sympy [A] time = 0.09, size = 8, normalized size = 1.00

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x,x)
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

3.5 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2638}

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x],x]
[Out] -Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x)\,dx = -\cos(x)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x],x]
[Out] -Cos[x]
```

fricas [A] time = 0.43, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="fricas")
[Out] -cos(x)
```

giac [A] time = 0.82, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="giac")
[Out] -cos(x)
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
[Out] -cos(x)
```

maxima [A] time = 0.42, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="maxima")
[Out] -cos(x)
```

mupad [B] time = 0.02, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
[Out] -cos(x)
```

sympy [A] time = 0.06, size = 3, normalized size = 0.75

$$-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
[Out] -cos(x)
```

3.6 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

[Out] sin(x)

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x],x]

[Out] Sin[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x],x]

[Out] Sin[x]

fricas [A] time = 0.44, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x),x, algorithm="fricas")

[Out] sin(x)

giac [A] time = 0.98, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x),x, algorithm="giac")

[Out] sin(x)

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
[Out] sin(x)
```

maxima [A] time = 0.43, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

mupad [B] time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
[Out] sin(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

3.7 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

```
[Out] tan(x)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8}

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2,x]
[Out] Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2,x]
[Out] Tan[x]
```

fricas [B] time = 0.43, size = 7, normalized size = 3.50

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="fricas")
[Out] sin(x)/cos(x)
```

giac [A] time = 1.11, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

maple [A] time = 0.01, size = 3, normalized size = 1.50

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2,x)
```

```
[Out] tan(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

sympy [B] time = 0.06, size = 5, normalized size = 2.50

$$\frac{\sin{\left(x \right)}}{\cos{\left(x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

3.8 $\int \csc^2(x)\,dx$

Optimal. Leaf size=4

$$-\cot(x)$$

```
[Out] -cot(x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8}

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)\\ &= -\cot(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

fricas [A] time = 0.41, size = 8, normalized size = 2.00

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/sin(x)
```

giac [A] time = 1.04, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x)
```

maple [A] time = 0.01, size = 5, normalized size = 1.25

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2,x)
```

```
[Out] -cot(x)
```

maxima [A] time = 0.43, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x)
```

mupad [B] time = 0.01, size = 4, normalized size = 1.00

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^2,x)
```

```
[Out] -cot(x)
```

sympy [B] time = 0.07, size = 7, normalized size = 1.75

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2,x)
```

```
[Out] -cos(x)/sin(x)
```

3.9 $\int \sec(x)\tan(x)\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

[Out] sec(x)

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Tan[x],x]

[Out] Sec[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan(x)\,dx &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right)\\ &= \sec(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Tan[x],x]

[Out] Sec[x]

fricas [A] time = 0.42, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x),x, algorithm="fricas")

[Out] 1/cos(x)

giac [A] time = 0.89, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

maple [A] time = 0.01, size = 3, normalized size = 1.50

$$\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x),x)
```

```
[Out] sec(x)
```

maxima [A] time = 0.42, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

mupad [B] time = 0.26, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x),x)
```

```
[Out] -2/(tan(x/2)^2 - 1)
```

sympy [A] time = 0.07, size = 3, normalized size = 1.50

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

3.10 $\int \cot(x)\csc(x)\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\ &= -\csc(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

fricas [A] time = 0.44, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

giac [A] time = 1.04, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

maple [A] time = 0.02, size = 5, normalized size = 1.25

$$-\csc(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x),x)
```

```
[Out] -csc(x)
```

maxima [A] time = 0.43, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

mupad [B] time = 0.26, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/sin(x),x)
```

```
[Out] -1/sin(x)
```

sympy [A] time = 0.07, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x)
```

```
[Out] -1/sin(x)
```

3.11 $\int \sinh(x)\,dx$

Optimal. Leaf size=2

$$\cosh(x)$$

```
[Out] cosh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2638}

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x],x]
[Out] Cosh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(x)\,dx = \cosh(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x],x]
[Out] Cosh[x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="fricas")
[Out] cosh(x)
```

giac [B] time = 1.05, size = 11, normalized size = 5.50

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x)
[Out] cosh(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="maxima")
[Out] cosh(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x)
[Out] cosh(x)
```

sympy [A] time = 0.12, size = 2, normalized size = 1.00

$$\cosh{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x)
[Out] cosh(x)
```

3.12 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

```
[Out] sinh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2637}

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(x)\,dx = \sinh(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x],x]
```

```
[Out] Sinh[x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

giac [B] time = 0.98, size = 11, normalized size = 5.50

$$-\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x)
```

```
[Out] sinh(x)
```

maxima [A] time = 0.44, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="maxima")
```

```
[Out] sinh(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x)
```

```
[Out] sinh(x)
```

sympy [A] time = 0.13, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x)
```

```
[Out] sinh(x)
```

3.13 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(x)\,dx = -\log(\cos(x))$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

fricas [B] time = 0.43, size = 11, normalized size = 2.20

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 1.02, size = 6, normalized size = 1.20

$$-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="giac")
```

```
[Out] -log(abs(cos(x)))
```

maple [A] time = 0.00, size = 6, normalized size = 1.20

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
[Out] -ln(cos(x))
```

maxima [A] time = 0.44, size = 3, normalized size = 0.60

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="maxima")
[Out] log(sec(x))
```

mupad [B] time = 0.03, size = 5, normalized size = 1.00

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
[Out] -log(cos(x))
```

sympy [A] time = 0.06, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
[Out] -log(cos(x))
```

3.14 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
[Out] Log[Sin[x]]
```

fricas [B] time = 0.43, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

giac [A] time = 0.85, size = 4, normalized size = 1.33

$$\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
[Out] log(abs(sin(x)))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] ln(sin(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$\log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

mupad [B] time = 0.00, size = 3, normalized size = 1.00

$$\ln\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] log(sin(x))
```

sympy [A] time = 0.06, size = 3, normalized size = 1.00

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

3.15 $\int x \sin(x) \, dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

[Out] -x*cos(x)+sin(x)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \sin(x) \, dx &= -x\cos(x) + \int \cos(x) \, dx \\ &= -x\cos(x) + \sin(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]

fricas [A] time = 0.42, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x),x, algorithm="fricas")

[Out] -x*cos(x) + sin(x)

giac [A] time = 0.89, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

maxima [A] time = 0.43, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\sin(x) - x\ \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] sin(x) - x*cos(x)
```

sympy [A] time = 0.18, size = 7, normalized size = 0.88

$$-x\cos\left(x\right) + \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

3.16 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x)-x$$

```
[Out] -x+x*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2295}

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log(x)\,dx=-x+x\log(x)$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

fricas [A] time = 0.41, size = 8, normalized size = 1.00

$$x\log(x)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

giac [A] time = 0.94, size = 8, normalized size = 1.00

$$x\log(x)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
```

```
[Out] x*log(x) - x
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$x\ln(x)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x)
```

```
[Out] x*ln(x)-x
```

maxima [A] time = 0.42, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$x\,(\ln(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x),x)
```

```
[Out] x*(log(x) - 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.62

$$x \log{(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

3.17 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

[In] Int[E^x*x^2,x]

[Out] 2*E^x - 2*E^x*x + E^x*x^2

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x^2 \, dx &= e^x x^2 - 2 \int e^x x \, dx \\ &= -2e^x x + e^x x^2 + 2 \int e^x \, dx \\ &= 2e^x - 2e^x x + e^x x^2 \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.63

$$e^x \left(x^2 - 2x + 2\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x*x^2,x]

[Out] E^x*(2 - 2*x + x^2)

fricas [A] time = 0.41, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x^2,x, algorithm="fricas")

[Out] (x^2 - 2*x + 2)*e^x

giac [**A**] time = 1.08, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
[Out] (x^2 - 2*x + 2)*e^x
```

maple [**A**] time = 0.00, size = 12, normalized size = 0.63

$$\left(x^2 - 2x + 2\right)\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x)
[Out] (x^2-2*x+2)*exp(x)
```

maxima [**A**] time = 0.44, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
[Out] (x^2 - 2*x + 2)*e^x
```

mupad [**B**] time = 0.02, size = 11, normalized size = 0.58

$$\mathrm{e}^x\,\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x),x)
[Out] exp(x)*(x^2 - 2*x + 2)
```

sympy [**A**] time = 0.09, size = 10, normalized size = 0.53

$$\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2,x)
[Out] (x**2 - 2*x + 2)*exp(x)
```

3.18 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x)-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

fricas [A] time = 0.42, size = 13, normalized size = 0.68

$$-\frac{1}{2}\cos(x)e^x+\frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

giac [A] time = 0.94, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x)-\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{\cos(x)e^x}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -1/2*cos(x)*exp(x)+1/2*exp(x)*sin(x)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

mupad [B] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{e^x\,(\cos(x) - \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

sympy [A] time = 0.30, size = 15, normalized size = 0.79

$$\frac{e^x \sin{(x)}}{2} - \frac{e^x \cos{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.19 $\int \tan^{-1}(x)\,dx$

Optimal. Leaf size=15

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4846, 260}

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(x)\,dx &= x\tan^{-1}(x)-\int \frac{x}{1+x^2}\,dx\\ &= x\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.42, size = 13, normalized size = 0.87

$$x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

giac [A] time = 0.92, size = 13, normalized size = 0.87

$$x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$x\arctan(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x),x)
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.43, size = 13, normalized size = 0.87

$$x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="maxima")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.16, size = 13, normalized size = 0.87

$$x\,\mathrm{atan}(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x),x)
[Out] x*atan(x) - log(x^2 + 1)/2
```

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$x\,\mathrm{atan}\,(x) - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x),x)
[Out] x*atan(x) - log(x**2 + 1)/2
```

3.20 $\int e^{2x} x \, dx$

Optimal. Leaf size=20

$$\frac{1}{2}e^{2x}x - \frac{e^{2x}}{4}$$

```
[Out] -1/4*exp(2*x)+1/2*exp(2*x)*x
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$\frac{1}{2}e^{2x}x - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*x,x]
```

```
[Out] -E^(2*x)/4 + (E^(2*x)*x)/2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{2x} x \, dx &= \frac{1}{2}e^{2x}x - \frac{1}{2}\int e^{2x} \, dx \\ &= -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x}x \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.75

$$e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*x,x]
```

```
[Out] E^(2*x)*(-1/4 + x/2)
```

fricas [A] time = 0.43, size = 11, normalized size = 0.55

$$\frac{1}{4}(2x - 1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="fricas")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

giac [A] time = 0.93, size = 11, normalized size = 0.55

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="giac")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.60

$$\frac{(2x-1)\,\mathrm{e}^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(2*x),x)
[Out] 1/4*(2*x-1)*exp(2*x)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.55

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="maxima")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.55

$$\frac{\mathrm{e}^{2x}\,(2x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(2*x),x)
[Out] (exp(2*x)*(2*x - 1))/4
```

sympy [A] time = 0.08, size = 10, normalized size = 0.50

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x)
[Out] (2*x - 1)*exp(2*x)/4
```

3.21 $\int x \cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x)+\cos(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2638}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
[Out] Cos[x] + x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int \sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
[Out] Cos[x] + x*Sin[x]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

giac [A] time = 0.78, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] x*sin(x)+cos(x)
```

maxima [A] time = 0.42, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

mupad [B] time = 0.00, size = 7, normalized size = 1.00

$$\cos(x) + x\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x) + x*sin(x)
```

sympy [A] time = 0.18, size = 7, normalized size = 1.00

$$x\sin\left(x\right) + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

3.22 $\int x\sin(4x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

```
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[4*x],x]
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(4x)\,dx&=-\frac{1}{4}x\cos(4x)+\frac{1}{4}\int\cos(4x)\,dx\\&=-\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[4*x],x]
[Out] -1/4*(x*Cos[4*x]) + Sin[4*x]/16
```

fricas [A] time = 0.43, size = 14, normalized size = 0.78

$$-\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="fricas")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

giac [A] time = 0.87, size = 14, normalized size = 0.78

$$-\frac{1}{4}\,x\cos\left(4\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="giac")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

maple [A] time = 0.02, size = 15, normalized size = 0.83

$$-\frac{x\cos\left(4x\right)}{4} + \frac{\sin\left(4x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(4*x),x)
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.78

$$-\frac{1}{4}\,x\cos\left(4\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="maxima")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.78

$$\frac{\sin\left(4\,x\right)}{16} - \frac{x\,\cos\left(4\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(4*x),x)
[Out] sin(4*x)/16 - (x*cos(4*x))/4
```

sympy [A] time = 0.19, size = 14, normalized size = 0.78

$$-\frac{x\cos\left(4x\right)}{4} + \frac{\sin\left(4x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x)
[Out] -x*cos(4*x)/4 + sin(4*x)/16
```

3.23 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2304}

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

giac [A] time = 0.76, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x^2 \ln(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
[Out] 1/2*x^2*ln(x)-1/4*x^2
```

maxima [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

mupad [B] time = 0.00, size = 9, normalized size = 0.53

$$\frac{x^2\left(\ln(x) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

3.24 $\int x^2 \cos(3x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2\sin(3x)-\frac{2}{27}\sin(3x)+\frac{2}{9}x\cos(3x)$$

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3296, 2637}

$$\frac{1}{3}x^2\sin(3x)-\frac{2}{27}\sin(3x)+\frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\cos(3x)\,dx &= \frac{1}{3}x^2\sin(3x)-\frac{2}{3}\int x\sin(3x)\,dx\\ &= \frac{2}{9}x\cos(3x)+\frac{1}{3}x^2\sin(3x)-\frac{2}{9}\int\cos(3x)\,dx\\ &= \frac{2}{9}x\cos(3x)-\frac{2}{27}\sin(3x)+\frac{1}{3}x^2\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2-2\right)\sin(3x)+\frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

fricas [A] time = 0.43, size = 21, normalized size = 0.72

$$\frac{2}{9}\,x\cos\left(3\,x\right)+\frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

giac [A] time = 0.83, size = 21, normalized size = 0.72

$$\frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\left(9\,x^2 - 2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

maple [A] time = 0.02, size = 24, normalized size = 0.83

$$\frac{x^2\sin\left(3x\right)}{3} + \frac{2x\cos\left(3x\right)}{9} - \frac{2\sin\left(3x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

maxima [A] time = 0.43, size = 21, normalized size = 0.72

$$\frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\left(9\,x^2 - 2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

mupad [B] time = 0.07, size = 23, normalized size = 0.79

$$\frac{2\,x\,\cos\left(3\,x\right)}{9} - \frac{2\,\sin\left(3\,x\right)}{27} + \frac{x^2\,\sin\left(3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
[Out] (2*x*cos(3*x))/9 - (2*sin(3*x))/27 + (x^2*sin(3*x))/3
```

sympy [A] time = 0.33, size = 27, normalized size = 0.93

$$\frac{x^2\sin\left(3x\right)}{3} + \frac{2x\cos\left(3x\right)}{9} - \frac{2\sin\left(3x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x),x)
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

3.25 $\int x^2 \sin(2x)\, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[Out] 1/4*cos(2*x)-1/2*x^2*cos(2*x)+1/2*x*sin(2*x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3296, 2638}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[2*x],x]
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sin(2x)\,dx&=-\frac{1}{2}x^2\cos(2x)+\int x\cos(2x)\,dx\\&=-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\&=\frac{1}{4}\cos(2x)-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.86

$$\frac{1}{2}x\sin(2x)-\frac{1}{4}\left(2x^2-1\right)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[2*x],x]
[Out] -1/4*((-1 + 2*x^2)*Cos[2*x]) + (x*Sin[2*x])/2
```

fricas [A] time = 0.45, size = 21, normalized size = 0.72

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

giac [A] time = 0.95, size = 21, normalized size = 0.72

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

maple [A] time = 0.02, size = 24, normalized size = 0.83

$$-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x),x)
[Out] 1/4*cos(2*x)-1/2*x^2*cos(2*x)+1/2*x*sin(2*x)
```

maxima [A] time = 0.43, size = 21, normalized size = 0.72

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.83

$$\frac{x\,\sin\left(2\,x\right)}{2}+\left(2\,\sin(x)^2-1\right)\,\left(\frac{x^2}{2}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x),x)
[Out] (x*sin(2*x))/2 + (2*sin(x)^2 - 1)*(x^2/2 - 1/4)
```

sympy [A] time = 0.33, size = 24, normalized size = 0.83

$$-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(2*x),x)
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 + cos(2*x)/4
```

3.26 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

[Out] 2*x-2*x*ln(x)+x*ln(x)^2

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

fricas [A] time = 0.42, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2,x, algorithm="fricas")

[Out] x*log(x)^2 - 2*x*log(x) + 2*x

giac [A] time = 1.02, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$x\ln(x)^2 - 2x\ln(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] x*ln(x)^2-2*x*ln(x)+2*x
```

maxima [A] time = 0.42, size = 12, normalized size = 0.80

$$\left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

mupad [B] time = 0.03, size = 12, normalized size = 0.80

$$x\,\left(\ln(x)^2 - 2\,\ln(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2,x)
```

```
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

sympy [A] time = 0.09, size = 15, normalized size = 1.00

$$x\log{(x)}^2 - 2x\log{(x)} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

3.27 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4619, 261}

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)\,dx &= x\sin^{-1}(x)-\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2}+x\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

fricas [A] time = 0.45, size = 14, normalized size = 0.88

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="fricas")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

giac [A] time = 1.05, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x),x)
```

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

maxima [A] time = 1.01, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="maxima")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

mupad [B] time = 0.00, size = 14, normalized size = 0.88

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x),x)
```

```
[Out] x*asin(x) + (1 - x^2)^(1/2)
```

sympy [A] time = 0.13, size = 12, normalized size = 0.75

$$x \operatorname{asin}{\left(x \right)} + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(1 - x**2)
```

3.28 $\int t \cos(t) \sin(t) \, dt$

Optimal. Leaf size=23

$$-\frac{t}{4}+\frac{1}{2}t\sin^2(t)+\frac{1}{4}\sin(t)\cos(t)$$

```
[Out] -1/4*t+1/4*cos(t)*sin(t)+1/2*t*sin(t)^2
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3443, 2635, 8}

$$-\frac{t}{4}+\frac{1}{2}t\sin^2(t)+\frac{1}{4}\sin(t)\cos(t)$$

Antiderivative was successfully verified.

```
[In] Int[t*Cos[t]*Sin[t],t]
```

```
[Out] -t/4 + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int t\cos(t)\sin(t)\,dt &= \frac{1}{2}t\sin^2(t)-\frac{1}{2}\int\sin^2(t)\,dt\\ &= \frac{1}{4}\cos(t)\sin(t)+\frac{1}{2}t\sin^2(t)-\frac{\int 1\,dt}{4}\\ &= -\frac{t}{4}+\frac{1}{4}\cos(t)\sin(t)+\frac{1}{2}t\sin^2(t)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2t)-\frac{1}{4}t\cos(2t)$$

Antiderivative was successfully verified.

```
[In] Integrate[t*Cos[t]*Sin[t],t]
```

```
[Out] -1/4*(t*Cos[2*t]) + Sin[2*t]/8
```

fricas [A] time = 0.43, size = 17, normalized size = 0.74

$$-\frac{1}{2}\,t\cos(t)^2+\frac{1}{4}\,\cos(t)\sin(t)+\frac{1}{4}\,t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="fricas")
[Out] -1/2*t*cos(t)^2 + 1/4*cos(t)*sin(t) + 1/4*t
```

giac [A] time = 0.95, size = 14, normalized size = 0.61

$$-\frac{1}{4}\,t\cos\left(2\,t\right)+\frac{1}{8}\,\sin\left(2\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="giac")
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{t\left(\cos^2(t)\right)}{2}+\frac{\cos(t)\sin(t)}{4}+\frac{t}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*cos(t)*sin(t),t)
[Out] -1/2*t*cos(t)^2+1/4*cos(t)*sin(t)+1/4*t
```

maxima [A] time = 0.42, size = 14, normalized size = 0.61

$$-\frac{1}{4}\,t\cos\left(2\,t\right)+\frac{1}{8}\,\sin\left(2\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="maxima")
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.78

$$\frac{\sin\left(2\,t\right)}{8}+\frac{t\,\left(2\,\sin(t)^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*cos(t)*sin(t),t)
[Out] sin(2*t)/8 + (t*(2*sin(t)^2 - 1))/4
```

sympy [A] time = 0.33, size = 24, normalized size = 1.04

$$\frac{t\sin^2\left(t\right)}{4}-\frac{t\cos^2\left(t\right)}{4}+\frac{\sin\left(t\right)\cos\left(t\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t)
[Out] t*sin(t)**2/4 - t*cos(t)**2/4 + sin(t)*cos(t)/4
```

3.29 $\int t \sec^2(t)\,dt$

Optimal. Leaf size=8

$$t\tan(t) + \log(\cos(t))$$

```
[Out] ln(cos(t))+t*tan(t)
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$t\tan(t) + \log(\cos(t))$$

Antiderivative was successfully verified.

```
[In] Int[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int t\sec^2(t)\,dt &= t\tan(t) - \int \tan(t)\,dt\\ &= \log(\cos(t)) + t\tan(t)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$t\tan(t) + \log(\cos(t))$$

Antiderivative was successfully verified.

```
[In] Integrate[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

fricas [B] time = 0.44, size = 18, normalized size = 2.25

$$\frac{\cos(t)\log(-\cos(t)) + t\sin(t)}{\cos(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="fricas")
```

```
[Out] (cos(t)*log(-cos(t)) + t*sin(t))/cos(t)
```

giac [B] time = 0.93, size = 103, normalized size = 12.88

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^{4}-2\tan\left(\frac{1}{2}t\right)^{2}+1\right)}{\tan\left(\frac{1}{2}t\right)^{4}+2\tan\left(\frac{1}{2}t\right)^{2}+1}\right)\tan\left(\frac{1}{2}t\right)^{2}-4t\tan\left(\frac{1}{2}t\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^{4}-2\tan\left(\frac{1}{2}t\right)^{2}+1\right)}{\tan\left(\frac{1}{2}t\right)^{4}+2\tan\left(\frac{1}{2}t\right)^{2}+1}\right)}{2\left(\tan\left(\frac{1}{2}t\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)
^2 + 1))*tan(1/2*t)^2 - 4*t*tan(1/2*t) - log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)
^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)^2 + 1)))/(tan(1/2*t)^2 - 1)
```

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$t\tan(t)+\ln(\cos(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*sec(t)^2,t)
```

```
[Out] ln(cos(t))+t*tan(t)
```

maxima [B] time = 0.97, size = 74, normalized size = 9.25

$$\frac{\left(\cos(2t)^{2}+\sin(2t)^{2}+2\cos(2t)+1\right)\log\left(\cos(2t)^{2}+\sin(2t)^{2}+2\cos(2t)+1\right)+4t\sin(2t)}{2\left(\cos(2t)^{2}+\sin(2t)^{2}+2\cos(2t)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2
 + 2*cos(2*t) + 1) + 4*t*sin(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) +
1)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\ln(\cos(t))+t\tan(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/cos(t)^2,t)
```

```
[Out] log(cos(t)) + t*tan(t)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int t\sec^{2}(t)\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)**2,t)
```

```
[Out] Integral(t*sec(t)**2, t)
```

3.30 $\int t^2 \log(t)\,dt$

Optimal. Leaf size=17

$$\frac{1}{3}t^3\log(t)-\frac{t^3}{9}$$

```
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{3}t^3\log(t)-\frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[t^2*Log[t],t]
[Out] -t^3/9 + (t^3*Log[t])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int t^2\log(t)\,dt=-\frac{t^3}{9}+\frac{1}{3}t^3\log(t)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{3}t^3\log(t)-\frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[t^2*Log[t],t]
[Out] -1/9*t^3 + (t^3*Log[t])/3
```

fricas [A] time = 0.40, size = 13, normalized size = 0.76

$$\frac{1}{3}\,t^3\log(t)-\frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="fricas")
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

giac [A] time = 0.82, size = 13, normalized size = 0.76

$$\frac{1}{3}\,t^3\log(t)-\frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="giac")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{t^3 \ln(t)}{3} - \frac{t^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2*ln(t),t)
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{3} t^3 \log(t) - \frac{1}{9} t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="maxima")
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

mupad [B] time = 0.03, size = 9, normalized size = 0.53

$$\frac{t^3 \left(\ln(t) - \frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2*log(t),t)
[Out] (t^3*(log(t) - 1/3))/3
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{t^3 \log(t)}{3} - \frac{t^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**2*ln(t),t)
[Out] t**3*log(t)/3 - t**3/9
```

3.31 $\int e^t t^3 \, dt$

Optimal. Leaf size=27

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

```
[Out] -6*exp(t)+6*exp(t)*t-3*exp(t)*t^2+exp(t)*t^3
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

Antiderivative was successfully verified.

```
[In] Int[E^t*t^3,t]
```

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^t t^3 \, dt &= e^t t^3 - 3 \int e^t t^2 \, dt \\
&= -3e^t t^2 + e^t t^3 + 6 \int e^t t \, dt \\
&= 6e^t t - 3e^t t^2 + e^t t^3 - 6 \int e^t \, dt \\
&= -6e^t + 6e^t t - 3e^t t^2 + e^t t^3
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.63

$$e^t \left(t^3 - 3t^2 + 6t - 6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*t^3,t]
```

```
[Out] E^t*(-6 + 6*t - 3*t^2 + t^3)
```

fricas [A] time = 0.38, size = 16, normalized size = 0.59

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="fricas")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

giac [A] time = 1.07, size = 16, normalized size = 0.59

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="giac")
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

maple [A] time = 0.00, size = 17, normalized size = 0.63

$$\left(t^3 - 3t^2 + 6t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*t^3,t)
[Out] (t^3-3*t^2+6*t-6)*exp(t)
```

maxima [A] time = 0.44, size = 16, normalized size = 0.59

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="maxima")
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

mupad [B] time = 0.02, size = 16, normalized size = 0.59

$$e^t \left(t^3 - 3\,t^2 + 6\,t - 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3*exp(t),t)
[Out] exp(t)*(6*t - 3*t^2 + t^3 - 6)
```

sympy [A] time = 0.09, size = 15, normalized size = 0.56

$$\left(t^3 - 3t^2 + 6t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t**3,t)
[Out] (t**3 - 3*t**2 + 6*t - 6)*exp(t)
```

3.32 $\int e^{2t}\sin(3t)\,dt$

Optimal. Leaf size=27

$$\frac{2}{13}e^{2t}\sin(3t)-\frac{3}{13}e^{2t}\cos(3t)$$

```
[Out] -3/13*exp(2*t)*cos(3*t)+2/13*exp(2*t)*sin(3*t)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4432}

$$\frac{2}{13}e^{2t}\sin(3t)-\frac{3}{13}e^{2t}\cos(3t)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*t)*Sin[3*t],t]
```

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{2t}\sin(3t)\,dt=-\frac{3}{13}e^{2t}\cos(3t)+\frac{2}{13}e^{2t}\sin(3t)$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{2t}(2\sin(3t)-3\cos(3t))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*t)*Sin[3*t],t]
```

```
[Out] (E^(2*t)*(-3*Cos[3*t] + 2*Sin[3*t]))/13
```

fricas [A] time = 0.42, size = 21, normalized size = 0.78

$$-\frac{3}{13}\cos(3\,t)\,e^{(2\,t)}+\frac{2}{13}\,e^{(2\,t)}\sin(3\,t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="fricas")
```

```
[Out] -3/13*cos(3*t)*e^(2*t) + 2/13*e^(2*t)*sin(3*t)
```

giac [A] time = 0.99, size = 19, normalized size = 0.70

$$-\frac{1}{13}(3\cos(3\,t)-2\sin(3\,t))e^{(2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="giac")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$-\frac{3\cos\left(3t\right)\mathrm{e}^{2t}}{13}+\frac{2\,\mathrm{e}^{2t}\sin\left(3t\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)*sin(3*t),t)
```

```
[Out] -3/13*exp(2*t)*cos(3*t)+2/13*exp(2*t)*sin(3*t)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.70

$$-\frac{1}{13}\left(3\,\cos\left(3\,t\right)-2\,\sin\left(3\,t\right)\right)e^{\left(2\,t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{2\,t}\,\left(3\,\cos\left(3\,t\right)-2\,\sin\left(3\,t\right)\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*t)*exp(2*t),t)
```

```
[Out] -(exp(2*t)*(3*cos(3*t) - 2*sin(3*t)))/13
```

sympy [A] time = 0.31, size = 26, normalized size = 0.96

$$\frac{2e^{2t}\sin\left(3t\right)}{13}-\frac{3e^{2t}\cos\left(3t\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t)
```

```
[Out] 2*exp(2*t)*sin(3*t)/13 - 3*exp(2*t)*cos(3*t)/13
```

3.33 $\int e^{-t}\cos(3t)\,dt$

Optimal. Leaf size=27

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

```
[Out] -1/10*cos(3*t)/exp(t)+3/10*sin(3*t)/exp(t)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*t]/E^t,t]
```

```
[Out] -Cos[3*t]/(10*E^t) + (3*Sin[3*t])/(10*E^t)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-t}\cos(3t)\,dt=-\frac{1}{10}e^{-t}\cos(3t)+\frac{3}{10}e^{-t}\sin(3t)$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.74

$$-\frac{1}{10}e^{-t}(\cos(3t)-3\sin(3t))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*t]/E^t,t]
```

```
[Out] -1/10*(Cos[3*t] - 3*Sin[3*t])/E^t
```

fricas [A] time = 0.43, size = 21, normalized size = 0.78

$$-\frac{1}{10}\cos\left(3\,t\right)e^{(-t)}+\frac{3}{10}\,e^{(-t)}\sin\left(3\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="fricas")
```

```
[Out] -1/10*cos(3*t)*e^(-t) + 3/10*e^(-t)*sin(3*t)
```

giac [A] time = 1.07, size = 17, normalized size = 0.63

$$-\frac{1}{10}\left(\cos\left(3\,t\right)-3\,\sin\left(3\,t\right)\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="giac")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

maple [A] time = 0.02, size = 22, normalized size = 0.81

$$-\frac{\cos(3t)\,\mathrm{e}^{-t}}{10}+\frac{3\,\mathrm{e}^{-t}\sin(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*t)/exp(t),t)
```

```
[Out] -1/10*exp(-t)*cos(3*t)+3/10*exp(-t)*sin(3*t)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.63

$$-\frac{1}{10}\,(\cos(3\,t)-3\,\sin(3\,t))e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="maxima")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.63

$$-\frac{\mathrm{e}^{-t}\,\left(\cos(3\,t)-3\,\sin(3\,t)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*t)*exp(-t),t)
```

```
[Out] -(exp(-t)*(cos(3*t) - 3*sin(3*t)))/10
```

sympy [A] time = 0.45, size = 20, normalized size = 0.74

$$\frac{3e^{-t}\sin(3t)}{10}-\frac{e^{-t}\cos(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t)
```

```
[Out] 3*exp(-t)*sin(3*t)/10 - exp(-t)*cos(3*t)/10
```

3.34 $\int y \sinh(y)\, dy$

Optimal. Leaf size=9

$$y \cosh(y) - \sinh(y)$$

```
[Out] y*cosh(y)-sinh(y)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2637}

$$y \cosh(y) - \sinh(y)$$

Antiderivative was successfully verified.

```
[In] Int[y*Sinh[y],y]
[Out] y*Cosh[y] - Sinh[y]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int y \sinh(y)\, dy &= y \cosh(y) - \int \cosh(y)\, dy\\ &= y \cosh(y) - \sinh(y)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$y \cosh(y) - \sinh(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[y*Sinh[y],y]
[Out] y*Cosh[y] - Sinh[y]
```

fricas [A] time = 0.40, size = 9, normalized size = 1.00

$$y \cosh(y) - \sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="fricas")
[Out] y*cosh(y) - sinh(y)
```

giac [A] time = 0.81, size = 17, normalized size = 1.89

$$\frac{1}{2} \left(y + 1\right) e^{\left(-y\right)} + \frac{1}{2} \left(y - 1\right) e^{y}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="giac")
```

```
[Out] 1/2*(y + 1)*e^(-y) + 1/2*(y - 1)*e^y
```

maple [A] time = 0.00, size = 10, normalized size = 1.11

$$y\cosh(y)-\sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*sinh(y),y)
```

```
[Out] y*cosh(y)-sinh(y)
```

maxima [B] time = 0.43, size = 34, normalized size = 3.78

$$\frac{1}{2}y^2\sinh(y)+\frac{1}{4}\left(y^2+2y+2\right)e^{(-y)}-\frac{1}{4}\left(y^2-2y+2\right)e^y$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="maxima")
```

```
[Out] 1/2*y^2*sinh(y) + 1/4*(y^2 + 2*y + 2)*e^(-y) - 1/4*(y^2 - 2*y + 2)*e^y
```

mupad [B] time = 0.02, size = 9, normalized size = 1.00

$$y\cosh(y)-\sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*sinh(y),y)
```

```
[Out] y*cosh(y) - sinh(y)
```

sympy [A] time = 0.19, size = 7, normalized size = 0.78

$$y\cosh{\left(y \right)}-\sinh{\left(y \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y)
```

```
[Out] y*cosh(y) - sinh(y)
```

3.35 $\int y\cosh(ay)\,dy$

Optimal. Leaf size=19

$$\frac{y\sinh(ay)}{a}-\frac{\cosh(ay)}{a^2}$$

```
[Out] -cosh(a*y)/a^2+y*sinh(a*y)/a
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{y\sinh(ay)}{a}-\frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[y*Cosh[a*y],y]
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int y\cosh(ay)\,dy&=\frac{y\sinh(ay)}{a}-\frac{\int\sinh(ay)\,dy}{a}\\&=-\frac{\cosh(ay)}{a^2}+\frac{y\sinh(ay)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{y\sinh(ay)}{a}-\frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[y*Cosh[a*y],y]
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

fricas [A] time = 0.40, size = 18, normalized size = 0.95

$$\frac{ay\sinh\left(ay\right)-\cosh\left(ay\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="fricas")
[Out] (a*y*sinh(a*y) - cosh(a*y))/a^2
```

giac [A] time = 0.77, size = 30, normalized size = 1.58

$$\frac{\left(ay-1\right)e^{\left(ay\right)}}{2\,a^2}-\frac{\left(ay+1\right)e^{\left(-ay\right)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="giac")
[Out] 1/2*(a*y - 1)*e^(a*y)/a^2 - 1/2*(a*y + 1)*e^(-a*y)/a^2
```

maple [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{ay\sinh\left(ay\right)-\cosh\left(ay\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*cosh(a*y),y)
[Out] 1/a^2*(y*a*sinh(a*y)-cosh(a*y))
```

maxima [B] time = 0.44, size = 57, normalized size = 3.00

$$\frac{1}{2}\,y^2\cosh\left(ay\right)-\frac{1}{4}\,a\left(\frac{\left(a^2y^2-2\,ay+2\right)e^{\left(ay\right)}}{a^3}+\frac{\left(a^2y^2+2\,ay+2\right)e^{\left(-ay\right)}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="maxima")
[Out] 1/2*y^2*cosh(a*y) - 1/4*a*((a^2*y^2 - 2*a*y + 2)*e^(a*y)/a^3 + (a^2*y^2 + 2
*a*y + 2)*e^(-a*y)/a^3)
```

mupad [B] time = 0.06, size = 18, normalized size = 0.95

$$-\frac{\cosh\left(a\,y\right)-a\,y\sinh\left(a\,y\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*cosh(a*y),y)
[Out] -(cosh(a*y) - a*y*sinh(a*y))/a^2
```

sympy [A] time = 0.23, size = 20, normalized size = 1.05

$$\begin{cases} \frac{y\sinh\left(ay\right)}{a}-\frac{\cosh\left(ay\right)}{a^2} & \text{for } a\neq 0 \\ \frac{y^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y)
[Out] Piecewise((y*sinh(a*y)/a - cosh(a*y)/a**2, Ne(a, 0)), (y**2/2, True))
```

3.36 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t}t - e^{-t}$$

```
[Out] -1/exp(t)-t/exp(t)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-t}t - e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{-t} t \, dt &= -e^{-t}t + \int e^{-t} \, dt \\ &= -e^{-t} - e^{-t}t \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 0.69

$$e^{-t}(-t-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]
```

```
[Out] (-1 - t)/E^t
```

fricas [A] time = 0.41, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

giac [A] time = 0.95, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

maple [A] time = 0.00, size = 10, normalized size = 0.62

$$-(t+1)\,\mathrm{e}^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t)
```

```
[Out] -(t+1)/exp(t)
```

maxima [A] time = 0.45, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="maxima")
```

```
[Out] -(t + 1)*e^(-t)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.56

$$-\mathrm{e}^{-t}\,(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*exp(-t),t)
```

```
[Out] -exp(-t)*(t + 1)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

3.37 $\int \sqrt{t} \log(t) \, dt$

Optimal. Leaf size=21

$$\frac{2}{3}t^{3/2}\log(t)-\frac{4t^{3/2}}{9}$$

```
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{2}{3}t^{3/2}\log(t)-\frac{4t^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[t]*Log[t],t]
```

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{t}\log(t)\,dt=-\frac{4t^{3/2}}{9}+\frac{2}{3}t^{3/2}\log(t)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.71

$$\frac{2}{9}t^{3/2}(3\log(t)-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[t]*Log[t],t]
```

```
[Out] (2*t^(3/2)*(-2 + 3*Log[t]))/9
```

fricas [A] time = 0.40, size = 14, normalized size = 0.67

$$\frac{2}{9}\left(3\,t\log(t)-2\,t\right)\sqrt{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="fricas")
```

```
[Out] 2/9*(3*t*log(t) - 2*t)*sqrt(t)
```

giac [A] time = 0.89, size = 13, normalized size = 0.62

$$\frac{2}{3}\,t^{\frac{3}{2}}\log(t)-\frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="giac")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

maple [A] time = 0.01, size = 14, normalized size = 0.67

$$\frac{2t^{\frac{3}{2}}\ln(t)}{3}-\frac{4t^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)*t^(1/2),t)
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

maxima [A] time = 0.43, size = 13, normalized size = 0.62

$$\frac{2}{3}\,t^{\frac{3}{2}}\log(t)-\frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="maxima")
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.43

$$\frac{2\,t^{3/2}\,\left(\ln(t)-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^(1/2)*log(t),t)
[Out] (2*t^(3/2)*(log(t) - 2/3))/3
```

sympy [A] time = 1.92, size = 66, normalized size = 3.14

$$\begin{cases}\frac{2t^{\frac{3}{2}}\log(t)}{3}-\frac{4t^{\frac{3}{2}}}{9} & \text{for}\ |t|<1\\ -\frac{2t^{\frac{3}{2}}\log\left(\frac{1}{t}\right)}{3}-\frac{4t^{\frac{3}{2}}}{9} & \text{for}\ \frac{1}{|t|}<1\\ -G_{3,3}^{2,1}\left(\left.\begin{matrix}1 & \frac{5}{2},\frac{5}{2}\\ \frac{3}{2},\frac{3}{2} & 0\end{matrix}\right|t\right)+G_{3,3}^{0,3}\left(\left.\begin{matrix}\frac{5}{2},\frac{5}{2},1 & \\ & \frac{3}{2},\frac{3}{2},0\end{matrix}\right|t\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)*t**(1/2),t)
[Out] Piecewise((2*t**(3/2)*log(t)/3 - 4*t**(3/2)/9, Abs(t) < 1), (-2*t**(3/2)*lo
g(1/t)/3 - 4*t**(3/2)/9, 1/Abs(t) < 1), (-meijerg(((1,), (5/2, 5/2)), ((3/2
, 3/2), (0,)), t) + meijerg(((5/2, 5/2, 1), ()), ((), (3/2, 3/2, 0)), t), T
rue))
```

3.38 $\int x\cos(2x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x],x]
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\,dx&=\frac{1}{2}x\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\&=\frac{1}{4}\cos(2x)+\frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x],x]
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

fricas [A] time = 0.43, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="fricas")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

giac [A] time = 0.85, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="giac")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

maple [A] time = 0.02, size = 15, normalized size = 0.83

$$\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x),x)
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

maxima [A] time = 0.48, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="maxima")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$\frac{\cos\left(2\,x\right)}{4}+\frac{x\,\sin\left(2\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x),x)
[Out] cos(2*x)/4 + (x*sin(2*x))/2
```

sympy [A] time = 0.18, size = 14, normalized size = 0.78

$$\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x)
[Out] x*sin(2*x)/2 + cos(2*x)/4
```

3.39 $\int e^{-x} x^2 \, dx$

Optimal. Leaf size=26

$$-e^{-x}x^2 - 2e^{-x}x - 2e^{-x}$$

```
[Out] -2/exp(x)-2*x/exp(x)-x^2/exp(x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-x}x^2 - 2e^{-x}x - 2e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^x,x]
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}x^2\,dx &= -e^{-x}x^2 + 2\int e^{-x}x\,dx\\ &= -2e^{-x}x - e^{-x}x^2 + 2\int e^{-x}\,dx\\ &= -2e^{-x} - 2e^{-x}x - e^{-x}x^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.62

$$e^{-x}\left(-x^2 - 2x - 2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^x,x]
[Out] (-2 - 2*x - x^2)/E^x
```

fricas [A] time = 0.37, size = 14, normalized size = 0.54

$$-\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="fricas")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

giac [**A**] time = 1.03, size = 14, normalized size = 0.54

$$-\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="giac")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.58

$$-\left(x^2 + 2x + 2\right)\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/exp(x),x)
[Out] -(x^2+2*x+2)/exp(x)
```

maxima [**A**] time = 0.44, size = 14, normalized size = 0.54

$$-\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="maxima")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

mupad [**B**] time = 0.03, size = 14, normalized size = 0.54

$$-\mathrm{e}^{-x}\,\left(x^2 + 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-x),x)
[Out] -exp(-x)*(2*x + x^2 + 2)
```

sympy [**A**] time = 0.09, size = 12, normalized size = 0.46

$$\left(-x^2 - 2x - 2\right)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/exp(x),x)
[Out] (-x**2 - 2*x - 2)*exp(-x)
```

3.40 $\int \cos^{-1}(x)\,dx$

Optimal. Leaf size=18

$$x\cos^{-1}(x)-\sqrt{1-x^2}$$

```
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4620, 261}

$$x\cos^{-1}(x)-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x],x]
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(x)\,dx &= x\cos^{-1}(x)+\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x^2}+x\cos^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$x\cos^{-1}(x)-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x],x]
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

fricas [A] time = 0.42, size = 16, normalized size = 0.89

$$x\arccos(x)-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="fricas")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

giac [A] time = 0.82, size = 16, normalized size = 0.89

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="giac")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x),x)
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

maxima [A] time = 0.98, size = 16, normalized size = 0.89

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="maxima")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

mupad [B] time = 0.17, size = 16, normalized size = 0.89

$$x \operatorname{acos}(x) - \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x),x)
[Out] x*acos(x) - (1 - x^2)^(1/2)
```

sympy [A] time = 0.13, size = 12, normalized size = 0.67

$$x \operatorname{acos}{\left(x \right)} - \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x),x)
[Out] x*acos(x) - sqrt(1 - x**2)
```

3.41 $\int x \csc^2(x)\, dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x\cot(x)$$

```
[Out] -x*cot(x)+ln(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]^2,x]
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \csc^2(x)\, dx &= -x\cot(x) + \int \cot(x)\, dx \\ &= -x\cot(x) + \log(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]^2,x]
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

fricas [B] time = 0.42, size = 20, normalized size = 2.22

$$-\frac{x\cos(x) - \log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="fricas")
[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

giac [B] time = 0.96, size = 52, normalized size = 5.78

$$\frac{x\tan\left(\frac{1}{2}\,x\right)^{2}+\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)-x}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
1))*tan(1/2*x) - x)/tan(1/2*x)
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$-x\cot(x)+\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2,x)
```

```
[Out] -x*cot(x)+ln(sin(x))
```

maxima [B] time = 0.45, size = 104, normalized size = 11.56

$$\frac{\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1\right)\log\left(\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1\right)+\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\cdots\right.}{2\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)
```

mupad [B] time = 0.15, size = 9, normalized size = 1.00

$$\ln(\sin(x))-x\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(x)^2,x)
```

```
[Out] log(sin(x)) - x*cot(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^{2}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

3.42 $\int \cos(5x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

```
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4284}

$$\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Sin[3*x],x]
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(5x)\sin(3x)\,dx=\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\cos^2(x)}{2}-\frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Sin[3*x],x]
[Out] Cos[x]^2/2 - Cos[8*x]/16
```

fricas [A] time = 0.42, size = 25, normalized size = 1.47

$$-8\,\cos(x)^8+16\,\cos(x)^6-10\,\cos(x)^4+\frac{5}{2}\,\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="fricas")
[Out] -8*cos(x)^8 + 16*cos(x)^6 - 10*cos(x)^4 + 5/2*cos(x)^2
```

giac [A] time = 0.80, size = 13, normalized size = 0.76

$$-\frac{1}{16}\,\cos\left(8\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="giac")
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

maple [A] time = 0.14, size = 14, normalized size = 0.82

$$\frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)*sin(3*x),x)
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$-\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="maxima")
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.76

$$\frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)*sin(3*x),x)
[Out] cos(2*x)/4 - cos(8*x)/16
```

sympy [B] time = 0.52, size = 26, normalized size = 1.53

$$\frac{5\sin(3x)\sin(5x)}{16} + \frac{3\cos(3x)\cos(5x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x)
[Out] 5*sin(3*x)*sin(5*x)/16 + 3*cos(3*x)*cos(5*x)/16
```

3.43 $\int \sin(2x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)-\frac{1}{12}\sin(6x)$$

```
[Out] 1/4*sin(2*x)-1/12*sin(6*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{4}\sin(2x)-\frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(2x)\sin(4x)\,dx=\frac{1}{4}\sin(2x)-\frac{1}{12}\sin(6x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x)-\frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

fricas [A] time = 0.44, size = 14, normalized size = 0.82

$$-\frac{1}{3}\left(\cos\left(2\,x\right)^{2}-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(2*x)^2 - 1)*sin(2*x)
```

giac [A] time = 0.77, size = 13, normalized size = 0.76

$$-\frac{1}{12}\sin\left(6\,x\right)+\frac{1}{4}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="giac")
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

maple [A] time = 0.03, size = 9, normalized size = 0.53

$$\frac{\left(\sin^3(2x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(4*x),x)
[Out] 1/3*sin(2*x)^3
```

maxima [A] time = 0.43, size = 13, normalized size = 0.76

$$-\frac{1}{12}\sin(6x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="maxima")
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.17, size = 13, normalized size = 0.76

$$\frac{\sin(2x)}{4} - \frac{\sin(6x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(4*x),x)
[Out] sin(2*x)/4 - sin(6*x)/12
```

sympy [A] time = 0.57, size = 22, normalized size = 1.29

$$-\frac{\sin(2x)\cos(4x)}{3} + \frac{\sin(4x)\cos(2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x)
[Out] -sin(2*x)*cos(4*x)/3 + sin(4*x)*cos(2*x)/6
```

3.44 $\int \cos(x)\log(\sin(x))\,dx$

Optimal. Leaf size=11

$$\sin(x)\log(\sin(x)) - \sin(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2637, 2554}

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x) - \int \cos(x)\,dx\\ &= -\sin(x) + \log(\sin(x))\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

fricas [A] time = 0.43, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
[Out] log(sin(x))*sin(x) - sin(x)
```

giac [A] time = 1.00, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
[Out] log(sin(x))*sin(x) - sin(x)
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$\ln(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x)),x)
[Out] -sin(x)+ln(sin(x))*sin(x)
```

maxima [A] time = 0.43, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
[Out] log(sin(x))*sin(x) - sin(x)
```

mupad [B] time = 0.20, size = 8, normalized size = 0.73

$$\sin(x)\,(\ln(\sin(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x),x)
[Out] sin(x)*(log(sin(x)) - 1)
```

sympy [A] time = 0.90, size = 10, normalized size = 0.91

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x)),x)
[Out] log(sin(x))*sin(x) - sin(x)
```

3.45 $\int e^{x^2} x^3 \, dx$

Optimal. Leaf size=22

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

```
[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2212, 2209}

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x^3,x]
```

```
[Out] -E^x^2/2 + (E^x^2*x^2)/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned} \int e^{x^2}x^3 \, dx &= \frac{1}{2}e^{x^2}x^2 - \int e^{x^2}x \, dx \\ &= -\frac{e^{x^2}}{2} + \frac{1}{2}e^{x^2}x^2 \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.64

$$\frac{1}{2}e^{x^2}\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^3,x]
```

```
[Out] (E^x^2*(-1 + x^2))/2
```

fricas [A] time = 0.40, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

giac [A] time = 1.04, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.55

$$\frac{\left(x^2-1\right)\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x^3,x)
[Out] 1/2*(x^2-1)*exp(x^2)
```

maxima [A] time = 0.44, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.50

$$\frac{\mathrm{e}^{x^2}\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(x^2),x)
[Out] (exp(x^2)*(x^2 - 1))/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.45

$$\frac{\left(x^2-1\right)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x**3,x)
[Out] (x**2 - 1)*exp(x**2)/2
```

3.46 $\int e^x(3+2x)\,dx$

Optimal. Leaf size=15

$$e^x(2x+3)-2e^x$$

```
[Out] -2*exp(x)+exp(x)*(3+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$e^x(2x+3)-2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*(3 + 2*x),x]
```

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x(3+2x)\,dx &= e^x(3+2x)-2\int e^x\,dx\\ &= -2e^x+e^x(3+2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.60

$$e^x(2x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(3 + 2*x),x]
```

```
[Out] E^x*(1 + 2*x)
```

fricas [A] time = 0.40, size = 8, normalized size = 0.53

$$(2x+1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="fricas")
```

```
[Out] (2*x + 1)*e^x
```

giac [A] time = 0.72, size = 8, normalized size = 0.53

$$(2x+1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="giac")
```

```
[Out] (2*x + 1)*e^x
```

maple [A]   time = 0.00, size = 9, normalized size = 0.60

$$(2x+1)\,\mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(2*x+3),x)
```

```
[Out] (2*x+1)*exp(x)
```

maxima [A]   time = 0.44, size = 12, normalized size = 0.80

$$2\,(x-1)e^{x}+3\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)*e^x + 3*e^x
```

mupad [B]   time = 0.03, size = 8, normalized size = 0.53

$$\mathrm{e}^{x}\,\,(2\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(2*x + 3),x)
```

```
[Out] exp(x)*(2*x + 1)
```

sympy [A]   time = 0.08, size = 7, normalized size = 0.47

$$(2x+1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x)
```

```
[Out] (2*x + 1)*exp(x)
```

3.47 $\int 5^x x\,dx$

Optimal. Leaf size=19

$$\frac{5^x x}{\log(5)}-\frac{5^x}{\log^2(5)}$$

```
[Out] -5^x/ln(5)^2+5^x*x/ln(5)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2176, 2194}

$$\frac{5^x x}{\log(5)}-\frac{5^x}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In] Int[5^x*x,x]
```

```
[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int 5^x x\,dx &= \frac{5^x x}{\log(5)}-\frac{\int 5^x\,dx}{\log(5)}\\ &= -\frac{5^x}{\log^2(5)}+\frac{5^x x}{\log(5)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{5^x(x\log(5)-1)}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[5^x*x,x]
```

```
[Out] (5^x*(-1 + x*Log[5]))/Log[5]^2
```

fricas [A] time = 0.42, size = 14, normalized size = 0.74

$$\frac{\left(x\log(5)-1\right)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="fricas")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

giac [A] time = 0.86, size = 14, normalized size = 0.74

$$\frac{\left(x\log(5)-1\right)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="giac")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

maple [A] time = 0.01, size = 15, normalized size = 0.79

$$\frac{(\ln(5)x-1)\,5^x}{\ln(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^x*x,x)
[Out] (ln(5)*x-1)*5^x/ln(5)^2
```

maxima [A] time = 0.97, size = 14, normalized size = 0.74

$$\frac{\left(x\log(5)-1\right)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="maxima")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

mupad [B] time = 0.02, size = 14, normalized size = 0.74

$$\frac{5^x\ (x\ \ln(5)-1)}{\ln(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^x*x,x)
[Out] (5^x*(x*log(5) - 1))/log(5)^2
```

sympy [A] time = 0.10, size = 14, normalized size = 0.74

$$\frac{5^x\left(x\log{(5)}-1\right)}{\log{(5)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5**x*x,x)
[Out] 5**x*(x*log(5) - 1)/log(5)**2
```

3.48 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx=\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

giac [A] time = 0.78, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

maxima [A] time = 0.43, size = 10, normalized size = 0.59

$$\frac{1}{2}x\left(\cos\left(\log(x)\right)+\sin\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

mupad [B] time = 0.17, size = 13, normalized size = 0.76

$$\frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(x)),x)
```

```
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

sympy [A] time = 0.41, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

3.49 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt{x}}\,dx&=2\,\text{Subst}\left(\int e^x x\,dx,x,\sqrt{x}\right)\\&=2e^{\sqrt{x}}\sqrt{x}-2\,\text{Subst}\left(\int e^x\,dx,x,\sqrt{x}\right)\\&=-2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

fricas [A] time = 0.41, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

giac [A] time = 0.85, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.71

$$2\sqrt{x}\,\mathrm{e}^{\sqrt{x}}-2\,\mathrm{e}^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*x^(1/2)*exp(x^(1/2))-2*exp(x^(1/2))
```

maxima [A] time = 0.44, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.46

$$2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

sympy [A] time = 0.25, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

3.50 $\int \log\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=14

$$x\log\left(\sqrt{x}\right)-\frac{x}{2}$$

```
[Out] -1/2*x+1/2*x*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2295}

$$x\log\left(\sqrt{x}\right)-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x]],x]
[Out] -x/2 + x*Log[Sqrt[x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log\left(\sqrt{x}\right)\,dx=-\frac{x}{2}+x\log\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{1}{2}(x\log(x)-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x]],x]
[Out] (-x + x*Log[x])/2
```

fricas [A] time = 0.41, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="fricas")
[Out] 1/2*x*log(x) - 1/2*x
```

giac [A] time = 0.94, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="giac")
[Out] 1/2*x*log(x) - 1/2*x
```

maple [A] time = 0.00, size = 10, normalized size = 0.71

$$\frac{x\ln(x)}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(x),x)
[Out] -1/2*x+1/2*x*ln(x)
```

maxima [A] time = 0.42, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log(x) - \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="maxima")
[Out] 1/2*x*log(x) - 1/2*x
```

mupad [B] time = 0.02, size = 7, normalized size = 0.50

$$\frac{x\,(\ln(x)-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/2,x)
[Out] (x*(log(x) - 1))/2
```

sympy [A] time = 0.11, size = 8, normalized size = 0.57

$$\frac{x\log{(x)}}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(x),x)
[Out] x*log(x)/2 - x/2
```

3.51 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))-\frac{1}{2}x\cos(\log(x))$$

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4475}

$$\frac{1}{2}x\sin(\log(x))-\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]],x]
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4475

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Sin[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] - Simp[(b*d*n*x*Cos[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(\log(x))\,dx=-\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\sin(\log(x))-\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]],x]
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$-\frac{1}{2}x\cos\left(\log(x)\right)+\frac{1}{2}x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

giac [A] time = 0.87, size = 13, normalized size = 0.76

$$-\frac{1}{2}x\cos\left(\log(x)\right)+\frac{1}{2}x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="giac")
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x)
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

maxima [A] time = 0.43, size = 12, normalized size = 0.71

$$-\frac{1}{2}\,x\left(\cos\left(\log(x)\right)-\sin\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2}\,x\,\cos\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x)),x)
[Out] -(2^(1/2)*x*cos(pi/4 + log(x)))/2
```

sympy [A] time = 0.53, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}-\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x)),x)
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

3.52 $\int \sin\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3361, 3296, 2637}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\sqrt{x}\right) dx &= 2\,\text{Subst}\left(\int x\sin(x)\,dx,x,\sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\,\text{Subst}\left(\int \cos(x)\,dx,x,\sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

fricas [A] time = 0.43, size = 16, normalized size = 0.73

$$-2\,\sqrt{x}\cos\left(\sqrt{x}\right)+2\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

giac [A] time = 0.96, size = 16, normalized size = 0.73

$$-2\,\sqrt{x}\,\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$-2\sqrt{x}\,\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] -2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

maxima [A] time = 0.44, size = 16, normalized size = 0.73

$$-2\,\sqrt{x}\,\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

mupad [B] time = 0.24, size = 16, normalized size = 0.73

$$2\,\sin\left(\sqrt{x}\right) - 2\,\sqrt{x}\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

sympy [A] time = 0.31, size = 20, normalized size = 0.91

$$-2\sqrt{x}\,\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.53 $\int x^5 \cos\left(x^3\right) dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^5\cos\left(x^3\right)dx&=\frac{1}{3}\text{Subst}\left(\int x\cos(x)\,dx,x,x^3\right)\\&=\frac{1}{3}x^3\sin\left(x^3\right)-\frac{1}{3}\text{Subst}\left(\int\sin(x)\,dx,x,x^3\right)\\&=\frac{\cos\left(x^3\right)}{3}+\frac{1}{3}x^3\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

fricas [A] time = 0.44, size = 16, normalized size = 0.80

$$\frac{1}{3} x^3 \sin\left(x^3\right) + \frac{1}{3} \cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

giac [A] time = 0.96, size = 16, normalized size = 0.80

$$\frac{1}{3} x^3 \sin\left(x^3\right) + \frac{1}{3} \cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$\frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

maxima [A] time = 0.44, size = 16, normalized size = 0.80

$$\frac{1}{3} x^3 \sin\left(x^3\right) + \frac{1}{3} \cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

mupad [B] time = 0.19, size = 16, normalized size = 0.80

$$\frac{\cos\left(x^3\right)}{3} + \frac{x^3 \sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
[Out] cos(x^3)/3 + (x^3*sin(x^3))/3
```

sympy [A] time = 1.91, size = 15, normalized size = 0.75

$$\frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(x**3),x)
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

3.54 $\int e^{x^2} x^5 \, dx$

Optimal. Leaf size=28

$$-e^{x^2}x^2+e^{x^2}+\frac{1}{2}e^{x^2}x^4$$

```
[Out] exp(x^2)-exp(x^2)*x^2+1/2*exp(x^2)*x^4
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2212, 2209}

$$\frac{1}{2}e^{x^2}x^4-e^{x^2}x^2+e^{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x^5,x]
```

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{x^2}x^5\,dx&=\frac{1}{2}e^{x^2}x^4-2\int e^{x^2}x^3\,dx\\&=-e^{x^2}x^2+\frac{1}{2}e^{x^2}x^4+2\int e^{x^2}x\,dx\\&=e^{x^2}-e^{x^2}x^2+\frac{1}{2}e^{x^2}x^4\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.68

$$\frac{1}{2}e^{x^2}\left(x^4-2x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^5,x]
```

```
[Out] (E^x^2*(2 - 2*x^2 + x^4))/2
```

fricas [A] time = 0.40, size = 16, normalized size = 0.57

$$\frac{1}{2}\left(x^4-2\,x^2+2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

giac [A] time = 0.79, size = 16, normalized size = 0.57

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="giac")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.61

$$\frac{\left(x^4 - 2x^2 + 2\right)\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x^5,x)
```

```
[Out] 1/2*(x^4-2*x^2+2)*exp(x^2)
```

maxima [A] time = 0.45, size = 16, normalized size = 0.57

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.57

$$\frac{\mathrm{e}^{x^2}\,\left(x^4 - 2\,x^2 + 2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*exp(x^2),x)
```

```
[Out] (exp(x^2)*(x^4 - 2*x^2 + 2))/2
```

sympy [A] time = 0.09, size = 15, normalized size = 0.54

$$\frac{\left(x^4 - 2x^2 + 2\right)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x**5,x)
```

```
[Out] (x**4 - 2*x**2 + 2)*exp(x**2)/2
```

3.55 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \\
&= -\frac{x}{2} + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

giac [A] time = 0.92, size = 15, normalized size = 0.71

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}x+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{x^2\arctan(x)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
```

```
[Out] 1/2*x^2*arctan(x)-1/2*x+1/2*arctan(x)
```

maxima [A] time = 0.95, size = 15, normalized size = 0.71

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}x+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.67

$$\mathrm{atan}(x)\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
```

```
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

sympy [A] time = 0.24, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}(x)}{2}-\frac{x}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

3.56 $\int x\cos(\pi x)\,dx$

Optimal. Leaf size=18

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

```
[Out] cos(Pi*x)/Pi^2+x*sin(Pi*x)/Pi
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[Pi*x],x]
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(\pi x)\,dx &= \frac{x\sin(\pi x)}{\pi}-\frac{\int \sin(\pi x)\,dx}{\pi}\\ &= \frac{\cos(\pi x)}{\pi^2}+\frac{x\sin(\pi x)}{\pi}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[Pi*x],x]
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

fricas [A] time = 0.42, size = 16, normalized size = 0.89

$$\frac{\pi x\sin(\pi x)+\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="fricas")
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

giac [A] time = 0.99, size = 18, normalized size = 1.00

$$\frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="giac")
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi^2
```

maple [A] time = 0.02, size = 17, normalized size = 0.94

$$\frac{\pi x\sin(\pi x) + \cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(Pi*x),x)
```

```
[Out] 1/Pi^2*(cos(Pi*x)+x*Pi*sin(Pi*x))
```

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$\frac{\pi x\sin(\pi x) + \cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="maxima")
```

```
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

mupad [B] time = 0.02, size = 16, normalized size = 0.89

$$\frac{\cos(\Pi\,x) + \Pi\,x\,\sin(\Pi\,x)}{\Pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(Pi*x),x)
```

```
[Out] (cos(Pi*x) + Pi*x*sin(Pi*x))/Pi^2
```

sympy [A] time = 0.19, size = 15, normalized size = 0.83

$$\frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x)
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi**2
```

3.57 $\int \sqrt{x} \log(x) \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}x^{3/2}\log(x) - \frac{4x^{3/2}}{9}$$

```
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{2}{3}x^{3/2}\log(x) - \frac{4x^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[x],x]
```

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x} \log(x) \, dx = -\frac{4x^{3/2}}{9} + \frac{2}{3}x^{3/2}\log(x)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.71

$$\frac{2}{9}x^{3/2}(3\log(x) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[x],x]
```

```
[Out] (2*x^(3/2)*(-2 + 3*Log[x]))/9
```

fricas [A] time = 0.42, size = 14, normalized size = 0.67

$$\frac{2}{9}\left(3\,x\log(x) - 2\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x*log(x) - 2*x)*sqrt(x)
```

giac [A] time = 1.09, size = 13, normalized size = 0.62

$$\frac{2}{3}\,x^{\frac{3}{2}}\log(x) - \frac{4}{9}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}\ln(x)}{3}-\frac{4x^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*x^(1/2),x)
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.62

$$\frac{2}{3}\,x^{\frac{3}{2}}\log(x)-\frac{4}{9}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.43

$$\frac{2\,x^{3/2}\left(\ln(x)-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*log(x),x)
[Out] (2*x^(3/2)*(log(x) - 2/3))/3
```

sympy [A] time = 1.94, size = 66, normalized size = 3.14

$$\begin{cases} \frac{2x^{\frac{3}{2}}\log{(x)}}{3}-\frac{4x^{\frac{3}{2}}}{9} & \text{for } |x|<1 \\ -\frac{2x^{\frac{3}{2}}\log{\left(\frac{1}{x}\right)}}{3}-\frac{4x^{\frac{3}{2}}}{9} & \text{for } \frac{1}{|x|}<1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{5}{2},\frac{5}{2} \\ \frac{3}{2},\frac{3}{2} & 0 \end{matrix}\middle| x\right)+G_{3,3}^{0,3}\left(\begin{matrix} \frac{5}{2},\frac{5}{2},1 & \\ & \frac{3}{2},\frac{3}{2},0 \end{matrix}\middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*x**(1/2),x)
[Out] Piecewise((2*x**(3/2)*log(x)/3 - 4*x**(3/2)/9, Abs(x) < 1), (-2*x**(3/2)*lo
g(1/x)/3 - 4*x**(3/2)/9, 1/Abs(x) < 1), (-meijerg(((1,), (5/2, 5/2)), ((3/2
, 3/2), (0,)), x) + meijerg(((5/2, 5/2, 1), ()), ((), (3/2, 3/2, 0)), x), T
rue))
```

3.58 $\int \sin^2(3x)\,dx$

Optimal. Leaf size=18

$$\frac{x}{2}-\frac{1}{6}\sin(3x)\cos(3x)$$

```
[Out] 1/2*x-1/6*cos(3*x)*sin(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{6}\sin(3x)\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[3*x]^2,x]
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(3x)\,dx &= -\frac{1}{6}\cos(3x)\sin(3x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{6}\cos(3x)\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.78

$$\frac{x}{2}-\frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]^2,x]
[Out] x/2 - Sin[6*x]/12
```

fricas [A] time = 0.42, size = 14, normalized size = 0.78

$$-\frac{1}{6}\cos(3x)\sin(3x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="fricas")
[Out] -1/6*cos(3*x)*sin(3*x) + 1/2*x
```

giac [**A**] time = 0.84, size = 10, normalized size = 0.56

$$\frac{1}{2}\,x - \frac{1}{12}\,\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/12*sin(6*x)
```

maple [**A**] time = 0.02, size = 15, normalized size = 0.83

$$-\frac{\cos\left(3x\right)\sin\left(3x\right)}{6} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)^2,x)
[Out] 1/2*x-1/6*cos(3*x)*sin(3*x)
```

maxima [**A**] time = 0.43, size = 10, normalized size = 0.56

$$\frac{1}{2}\,x - \frac{1}{12}\,\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/12*sin(6*x)
```

mupad [**B**] time = 0.05, size = 10, normalized size = 0.56

$$\frac{x}{2} - \frac{\sin\left(6\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)^2,x)
[Out] x/2 - sin(6*x)/12
```

sympy [**A**] time = 0.07, size = 14, normalized size = 0.78

$$\frac{x}{2} - \frac{\sin\left(3x\right)\cos\left(3x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)**2,x)
[Out] x/2 - sin(3*x)*cos(3*x)/6
```

3.59 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

fricas [A] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [A]   time = 0.99, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

maple [A]   time = 0.00, size = 11, normalized size = 0.79

$$\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*cos(x)*sin(x)+1/2*x
```

maxima [A]   time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B]   time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

sympy [A]   time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.60 $\int \cos^4(x)\, dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] 3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\, dx &= \frac{1}{4}\cos^3(x)\sin(x)+\frac{3}{4}\int \cos^2(x)\, dx\\ &= \frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+\frac{3\int 1\, dx}{8}\\ &= \frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

fricas [A] time = 0.44, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

giac [A] time = 0.87, size = 16, normalized size = 0.67

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.75

$$\frac{3x}{8} + \frac{\left(\cos^3(x) + \frac{3\cos(x)}{2}\right)\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
```

```
[Out] 3/8*x+1/4*(cos(x)^3+3/2*cos(x))*sin(x)
```

maxima [A] time = 0.44, size = 16, normalized size = 0.67

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.67

$$\frac{3x}{8} + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
```

```
[Out] (3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

sympy [A] time = 0.06, size = 24, normalized size = 1.00

$$\frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

3.61 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

fricas [A] time = 0.42, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.93, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{\left(\sin^2(x)+2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] -1/3*(sin(x)^2+2)*cos(x)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.77

$$\frac{\cos(x)\left(\cos(x)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

3.62 $\int \cos^4(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}$$

```
[Out] -1/5*cos(x)^5+1/7*cos(x)^7
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 14}

$$\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] -Cos[x]^5/5 + Cos[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\sin^3(x)\,dx &= -\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(x)\right)\\
&= -\frac{1}{5}\cos^5(x)+\frac{\cos^7(x)}{7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.82

$$-\frac{3\cos(x)}{64}-\frac{1}{64}\cos(3x)+\frac{1}{320}\cos(5x)+\frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/64 - Cos[3*x]/64 + Cos[5*x]/320 + Cos[7*x]/448
```

fricas [A] time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{7}\cos(x)^7-\frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="fricas")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

giac [A] time = 0.96, size = 13, normalized size = 0.76

$$\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="giac")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$-\frac{\left(\cos^5(x)\right)\left(\sin^2(x)\right)}{7} - \frac{2\left(\cos^5(x)\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^3,x)
[Out] -1/7*cos(x)^5*sin(x)^2-2/35*cos(x)^5
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="maxima")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

mupad [B] time = 0.04, size = 14, normalized size = 0.82

$$\frac{\cos(x)^5\,\left(5\cos(x)^2-7\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^3,x)
[Out] (cos(x)^5*(5*cos(x)^2 - 7))/35
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**3,x)
[Out] cos(x)**7/7 - cos(x)**5/5
```

3.63 $\int \cos^3(x)\sin^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

```
[Out] 1/5*sin(x)^5-1/7*sin(x)^7
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2564, 14}

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^4(x)\,dx &= \text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\sin(x)\right)\\ &= \frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.82

$$\frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] (3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448
```

fricas [A] time = 0.40, size = 22, normalized size = 1.29

$$\frac{1}{35}\left(5\cos(x)^6-8\cos(x)^4+\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="fricas")
[Out] 1/35*(5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

giac [**A**] time = 0.96, size = 13, normalized size = 0.76

$$-\frac{1}{7}\,\sin(x)^7+\frac{1}{5}\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="giac")
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

maple [**B**] time = 0.01, size = 30, normalized size = 1.76

$$-\frac{\left(\cos^4(x)\right)\left(\sin^3(x)\right)}{7}-\frac{3\left(\cos^4(x)\right)\sin(x)}{35}+\frac{\left(\cos^2(x)+2\right)\sin(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^4,x)
[Out] -1/7*cos(x)^4*sin(x)^3-3/35*sin(x)*cos(x)^4+1/35*(cos(x)^2+2)*sin(x)
```

maxima [**A**] time = 0.43, size = 13, normalized size = 0.76

$$-\frac{1}{7}\,\sin(x)^7+\frac{1}{5}\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="maxima")
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

mupad [**B**] time = 0.15, size = 14, normalized size = 0.82

$$-\frac{\sin(x)^5\,\left(5\,\sin(x)^2-7\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^4,x)
[Out] -(sin(x)^5*(5*sin(x)^2 - 7))/35
```

sympy [**A**] time = 0.07, size = 12, normalized size = 0.71

$$-\frac{\sin^7(x)}{7}+\frac{\sin^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**4,x)
[Out] -sin(x)**7/7 + sin(x)**5/5
```

3.64 $\int \cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=36

$$\frac{x}{16}-\frac{1}{6}\sin^3(x)\cos^3(x)-\frac{1}{8}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] 1/16*x+1/16*cos(x)*sin(x)-1/8*cos(x)^3*sin(x)-1/6*cos(x)^3*sin(x)^3
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin^3(x)\cos^3(x)-\frac{1}{8}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x]^3)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x)\sin^4(x)\,dx &= -\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{1}{2}\int \cos^2(x)\sin^2(x)\,dx\\
&= -\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{1}{8}\int \cos^2(x)\,dx\\
&= \frac{1}{16}\cos(x)\sin(x)-\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)-\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192
```

fricas [A] time = 0.43, size = 25, normalized size = 0.69

$$\frac{1}{48}\left(8\cos(x)^5 - 14\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

giac [A] time = 0.92, size = 22, normalized size = 0.61

$$\frac{1}{16}x + \frac{1}{192}\sin(6x) - \frac{1}{64}\sin(4x) - \frac{1}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*x) - 1/64*sin(4*x) - 1/64*sin(2*x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.81

$$-\frac{\left(\cos^3(x)\right)\left(\sin^3(x)\right)}{6} - \frac{\left(\cos^3(x)\right)\sin(x)}{8} + \frac{\cos(x)\sin(x)}{16} + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^4,x)
```

```
[Out] 1/16*x+1/16*cos(x)*sin(x)-1/8*cos(x)^3*sin(x)-1/6*cos(x)^3*sin(x)^3
```

maxima [A] time = 0.44, size = 18, normalized size = 0.50

$$-\frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{\cos(x)\sin(x)^5}{6} + \frac{x}{16} - \frac{\sin(2x)}{24} + \frac{\sin(4x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^4,x)
```

```
[Out] x/16 - sin(2*x)/24 + sin(4*x)/192 + (cos(x)*sin(x)^5)/6
```

sympy [A] time = 0.06, size = 31, normalized size = 0.86

$$\frac{x}{16} + \frac{\sin^5(x)\cos(x)}{6} - \frac{\sin^3(x)\cos(x)}{24} - \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**4,x)
```

```
[Out] x/16 + sin(x)**5*cos(x)/6 - sin(x)**3*cos(x)/24 - sin(x)*cos(x)/16
```

3.65 $\int \cos^2(x) \sin^2(x) \, dx$

Optimal. Leaf size=24

$$\frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x) \sin^2(x) \, dx &= -\frac{1}{4} \cos^3(x) \sin(x) + \frac{1}{4} \int \cos^2(x) \, dx \\
&= \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x) + \frac{\int 1 \, dx}{8} \\
&= \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32} \sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

fricas [A] time = 0.41, size = 19, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

giac [A] time = 1.07, size = 10, normalized size = 0.42

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.79

$$-\frac{\left(\cos^3(x)\right)\sin(x)}{4} + \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] -1/4*cos(x)^3*sin(x)+1/8*cos(x)*sin(x)+1/8*x
```

maxima [A] time = 0.43, size = 10, normalized size = 0.42

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos(x)\sin(x)^3}{4} - \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

sympy [A] time = 0.07, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

3.66 $\int (1-\sin(2x))^2\,dx$

Optimal. Leaf size=22

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

```
[Out] 3/2*x+cos(2*x)-1/4*cos(2*x)*sin(2*x)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2644}

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (1-\sin(2x))^2\,dx=\frac{3x}{2}+\cos(2x)-\frac{1}{4}\cos(2x)\sin(2x)$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$\frac{3x}{2}-\frac{1}{8}\sin(4x)+\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - Sin[4*x]/8
```

fricas [A] time = 0.41, size = 18, normalized size = 0.82

$$-\frac{1}{4}\cos(2x)\sin(2x)+\frac{3}{2}x+\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x)*sin(2*x) + 3/2*x + cos(2*x)
```

giac [A] time = 1.01, size = 14, normalized size = 0.64

$$\frac{3}{2}x+\cos(2x)-\frac{1}{8}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

maple [A] time = 0.05, size = 19, normalized size = 0.86

$$-\frac{\cos(2x)\sin(2x)}{4}+\frac{3x}{2}+\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2*x))^2,x)
```

```
[Out] 3/2*x+cos(2*x)-1/4*cos(2*x)*sin(2*x)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.64

$$\frac{3}{2}\,x+\cos\left(2\,x\right)-\frac{1}{8}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

mupad [B] time = 0.26, size = 14, normalized size = 0.64

$$\frac{3\,x}{2}+\cos\left(2\,x\right)-\frac{\sin\left(4\,x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(2*x) - 1)^2,x)
```

```
[Out] (3*x)/2 + cos(2*x) - sin(4*x)/8
```

sympy [A] time = 0.20, size = 37, normalized size = 1.68

$$\frac{x\sin^2(2x)}{2}+\frac{x\cos^2(2x)}{2}+x-\frac{\sin(2x)\cos(2x)}{4}+\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))**2,x)
```

```
[Out] x*sin(2*x)**2/2 + x*cos(2*x)**2/2 + x - sin(2*x)*cos(2*x)/4 + cos(2*x)
```

3.67 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal. Leaf size=20

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx &= \int \left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{2}\int \sin\left(\frac{\pi}{6}+2x\right)\,dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

fricas [B] time = 0.43, size = 31, normalized size = 1.55

$$-\frac{1}{4}\sqrt{3}\cos\left(\frac{1}{6}\pi+x\right)^2-\frac{1}{4}\cos\left(\frac{1}{6}\pi+x\right)\sin\left(\frac{1}{6}\pi+x\right)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

giac [A] time = 0.89, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

maple [A] time = 0.08, size = 15, normalized size = 0.75

$$\frac{x}{4} - \frac{\cos\left(2x + \frac{\pi}{6}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(1/6*Pi+x),x)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

maxima [A] time = 0.44, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

mupad [B] time = 0.15, size = 18, normalized size = 0.90

$$\frac{x\sin\left(\frac{\Pi}{6}\right)}{2} - \frac{\cos\left(\frac{\Pi}{6} + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(Pi/6 + x),x)
```

```
[Out] (x*sin(Pi/6))/2 - cos(Pi/6 + 2*x)/4
```

sympy [B] time = 0.56, size = 37, normalized size = 1.85

$$-\frac{x\sin(x)\cos\left(x + \frac{\pi}{6}\right)}{2} + \frac{x\sin\left(x + \frac{\pi}{6}\right)\cos(x)}{2} - \frac{\cos(x)\cos\left(x + \frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 - cos(x)*cos(x + pi/6)
/2
```

3.68 $\int \cos^5(x)\sin^5(x)\,dx$

Optimal. Leaf size=25

$$\frac{\sin^{10}(x)}{10}-\frac{\sin^8(x)}{4}+\frac{\sin^6(x)}{6}$$

```
[Out] 1/6*sin(x)^6-1/4*sin(x)^8+1/10*sin(x)^10
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2564, 266, 43}

$$\frac{\sin^{10}(x)}{10}-\frac{\sin^8(x)}{4}+\frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^5(x)\sin^5(x)\,dx &= \text{Subst}\left(\int x^5\left(1-x^2\right)^2\,dx,x,\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int (1-x)^2x^2\,dx,x,\sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)\,dx,x,\sin^2(x)\right)\\
&= \frac{\sin^6(x)}{6}-\frac{\sin^8(x)}{4}+\frac{\sin^{10}(x)}{10}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{5}{512}\cos(2x)+\frac{5\cos(6x)}{3072}-\frac{\cos(10x)}{5120}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] (-5*Cos[2*x])/512 + (5*Cos[6*x])/3072 - Cos[10*x]/5120
```

fricas [A] time = 0.41, size = 19, normalized size = 0.76

$$-\frac{1}{10}\cos(x)^{10}+\frac{1}{4}\cos(x)^{8}-\frac{1}{6}\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

giac [A] time = 0.92, size = 19, normalized size = 0.76

$$-\frac{1}{10}\cos(x)^{10}+\frac{1}{4}\cos(x)^{8}-\frac{1}{6}\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$-\frac{\left(\cos^{6}(x)\right)\left(\sin^{4}(x)\right)}{10}-\frac{\left(\cos^{6}(x)\right)\left(\sin^{2}(x)\right)}{20}-\frac{\left(\cos^{6}(x)\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x)^5,x)
```

```
[Out] -1/10*cos(x)^6*sin(x)^4-1/20*sin(x)^2*cos(x)^6-1/60*cos(x)^6
```

maxima [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{1}{10}\sin(x)^{10}-\frac{1}{4}\sin(x)^{8}+\frac{1}{6}\sin(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/10*sin(x)^10 - 1/4*sin(x)^8 + 1/6*sin(x)^6
```

mupad [B] time = 0.05, size = 19, normalized size = 0.76

$$\frac{\sin(x)^{10}}{10}-\frac{\sin(x)^{8}}{4}+\frac{\sin(x)^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x)^5,x)
```

```
[Out] sin(x)^6/6 - sin(x)^8/4 + sin(x)^10/10
```

sympy [A] time = 0.07, size = 19, normalized size = 0.76

$$\frac{\sin^{10}(x)}{10}-\frac{\sin^{8}(x)}{4}+\frac{\sin^{6}(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5*sin(x)**5,x)
```

```
[Out] sin(x)**10/10 - sin(x)**8/4 + sin(x)**6/6
```

3.69 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] 5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^6(x)\,dx &= -\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{6}\int \sin^4(x)\,dx\\ &= -\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{8}\int \sin^2(x)\,dx\\ &= -\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16}-\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

fricas [A] time = 0.43, size = 25, normalized size = 0.74

$$-\frac{1}{48}\left(8\cos(x)^5-26\cos(x)^3+33\cos(x)\right)\sin(x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

giac [A] time = 0.81, size = 22, normalized size = 0.65

$$\frac{5}{16}x - \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) - \frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.71

$$\frac{5x}{16} - \frac{\left(\sin^5(x) + \frac{5\left(\sin^3(x)\right)}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] 5/16*x-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)
```

maxima [A] time = 0.43, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin(2x)^3 + \frac{5}{16}x + \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{5x}{16} - \frac{15\sin(2x)}{64} + \frac{3\sin(4x)}{64} - \frac{\sin(6x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] (5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

sympy [A] time = 0.06, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

3.70 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] 5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^6(x)\,dx &= \frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{6}\int \cos^4(x)\,dx\\ &= \frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{8}\int \cos^2(x)\,dx\\ &= \frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

fricas [A] time = 0.42, size = 25, normalized size = 0.74

$$\frac{1}{48}\left(8\,\cos(x)^5+10\,\cos(x)^3+15\,\cos(x)\right)\sin(x)+\frac{5}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

giac [A] time = 0.94, size = 22, normalized size = 0.65

$$\frac{5}{16}\,x + \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

maple [A] time = 0.10, size = 24, normalized size = 0.71

$$\frac{5x}{16} + \frac{\left(\cos^{5}(x) + \frac{5\left(\cos^{3}(x)\right)}{4} + \frac{15\cos(x)}{8}\right)\sin(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

maxima [A] time = 0.45, size = 24, normalized size = 0.71

$$-\frac{1}{48}\,\sin\left(2\,x\right)^{3} + \frac{5}{16}\,x + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.65

$$\frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] (5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

sympy [A] time = 0.06, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin\left(x\right)\cos^{5}\left(x\right)}{6} + \frac{5\sin\left(x\right)\cos^{3}\left(x\right)}{24} + \frac{5\sin\left(x\right)\cos\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

3.71 $\int \cos^4(2x)\sin^2(2x)\,dx$

Optimal. Leaf size=46

$$\frac{x}{16}-\frac{1}{12}\sin(2x)\cos^5(2x)+\frac{1}{48}\sin(2x)\cos^3(2x)+\frac{1}{32}\sin(2x)\cos(2x)$$

```
[Out] 1/16*x+1/32*cos(2*x)*sin(2*x)+1/48*cos(2*x)^3*sin(2*x)-1/12*cos(2*x)^5*sin(
2*x)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{12}\sin(2x)\cos^5(2x)+\frac{1}{48}\sin(2x)\cos^3(2x)+\frac{1}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2*x]^5*Sin[
2*x])/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(2x)\sin^2(2x)\,dx &= -\frac{1}{12}\cos^5(2x)\sin(2x)+\frac{1}{6}\int \cos^4(2x)\,dx\\
&= \frac{1}{48}\cos^3(2x)\sin(2x)-\frac{1}{12}\cos^5(2x)\sin(2x)+\frac{1}{8}\int \cos^2(2x)\,dx\\
&= \frac{1}{32}\cos(2x)\sin(2x)+\frac{1}{48}\cos^3(2x)\sin(2x)-\frac{1}{12}\cos^5(2x)\sin(2x)+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{1}{32}\cos(2x)\sin(2x)+\frac{1}{48}\cos^3(2x)\sin(2x)-\frac{1}{12}\cos^5(2x)\sin(2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.65

$$\frac{x}{16}+\frac{1}{128}\sin(4x)-\frac{1}{128}\sin(8x)-\frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + Sin[4*x]/128 - Sin[8*x]/128 - Sin[12*x]/384
```

fricas [A] time = 0.43, size = 33, normalized size = 0.72

$$-\frac{1}{96}\left(8\cos\left(2x\right)^{5}-2\cos\left(2x\right)^{3}-3\cos\left(2x\right)\right)\sin\left(2x\right)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 2*cos(2*x)^3 - 3*cos(2*x))*sin(2*x) + 1/16*x
```

giac [A] time = 0.83, size = 22, normalized size = 0.48

$$\frac{1}{16}x-\frac{1}{384}\sin\left(12x\right)-\frac{1}{128}\sin\left(8x\right)+\frac{1}{128}\sin\left(4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/384*sin(12*x) - 1/128*sin(8*x) + 1/128*sin(4*x)
```

maple [A] time = 0.02, size = 36, normalized size = 0.78

$$-\frac{\left(\cos^{5}\left(2x\right)\right)\sin\left(2x\right)}{12}+\frac{x}{16}+\frac{\left(\cos^{3}\left(2x\right)+\frac{3\cos\left(2x\right)}{2}\right)\sin\left(2x\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*sin(2*x)^2,x)
```

```
[Out] -1/12*cos(2*x)^5*sin(2*x)+1/48*(cos(2*x)^3+3/2*cos(2*x))*sin(2*x)+1/16*x
```

maxima [A] time = 0.43, size = 18, normalized size = 0.39

$$\frac{1}{96}\sin\left(4x\right)^{3}+\frac{1}{16}x-\frac{1}{128}\sin\left(8x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 1/16*x - 1/128*sin(8*x)
```

mupad [B] time = 0.07, size = 37, normalized size = 0.80

$$\frac{x}{16}-\frac{\cos\left(2x\right)\sin\left(2x\right)}{32}+\frac{\sin\left(2x\right)^{3}\left(\frac{\cos\left(2x\right)^{3}}{6}+\frac{\cos\left(2x\right)}{8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*sin(2*x)^2,x)
```

```
[Out] x/16 - (cos(2*x)*sin(2*x))/32 + (sin(2*x)^3*(cos(2*x)/8 + cos(2*x)^3/6))/2
```

sympy [A] time = 0.07, size = 41, normalized size = 0.89

$$\frac{x}{16}-\frac{\sin\left(2x\right)\cos^{5}\left(2x\right)}{12}+\frac{\sin\left(2x\right)\cos^{3}\left(2x\right)}{48}+\frac{\sin\left(2x\right)\cos\left(2x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**4*sin(2*x)**2,x)
```

```
[Out] x/16 - sin(2*x)*cos(2*x)**5/12 + sin(2*x)*cos(2*x)**3/48 + sin(2*x)*cos(2*x
)/32
```

3.72 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

fricas [A] time = 0.42, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.91, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="giac")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.81

$$-\frac{\left(\sin^4(x)+\frac{4\left(\sin^2(x)\right)}{3}+\frac{8}{3}\right)\cos(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
[Out] -1/5*(sin(x)^4+4/3*sin(x)^2+8/3)*cos(x)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$-\frac{\cos(x)^5}{5}+\frac{2\cos(x)^3}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
[Out] (2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

sympy [A] time = 0.07, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5,x)
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

3.73 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

```
[Out] 3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*c
os(x)^5*sin(x)^3
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\sin^4(x)\,dx &= -\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{8}\int \cos^4(x)\sin^2(x)\,dx\\
&= -\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{1}{16}\int \cos^4(x)\,dx\\
&= \frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{64}\int \cos^2(x)\,dx\\
&= \frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{128}\\
&= \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.48

$$\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

fricas [A] time = 0.43, size = 31, normalized size = 0.67

$$\frac{1}{128}\left(16\cos(x)^7 - 24\cos(x)^5 + 2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

giac [A] time = 0.96, size = 16, normalized size = 0.35

$$\frac{3}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$-\frac{\left(\cos^5(x)\right)\left(\sin^3(x)\right)}{8} - \frac{\left(\cos^5(x)\right)\sin(x)}{16} + \frac{3x}{128} + \frac{\left(\cos^3(x) + \frac{3\cos(x)}{2}\right)\sin(x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x)
[Out] -1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*cos(x))*sin(
x)+3/128*x
```

maxima [A] time = 0.44, size = 16, normalized size = 0.35

$$\frac{3}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.70

$$\left(\frac{\cos(x)^3}{8} + \frac{\cos(x)}{16}\right)\sin(x)^5 + \frac{3x}{128} - \frac{\sin(2x)}{64} + \frac{\sin(4x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x)
[Out] (3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

sympy [A] time = 0.07, size = 31, normalized size = 0.67

$$\frac{3x}{128} - \frac{\sin^3(2x)\cos(2x)}{128} - \frac{3\sin(2x)\cos(2x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

3.74 $\int \sqrt{\cos(x)}\,\sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[Out] -2/3*cos(x)^(3/2)+2/7*cos(x)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2565, 14}

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)}\,\sin^3(x)\,dx &= -\text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)dx,x,\cos(x)\right)\\ &= -\frac{2}{3}\cos^{\frac{3}{2}}(x)+\frac{2}{7}\cos^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.62

$$\frac{(3\cos(2x)-11)\cos^2(x)+8\sqrt[4]{\cos^2(x)}}{21\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] (8*(Cos[x]^2)^(1/4) + Cos[x]^2*(-11 + 3*Cos[2*x]))/(21*Sqrt[Cos[x]])
```

fricas [A] time = 0.43, size = 17, normalized size = 0.81

$$\frac{2}{21}\left(3\,\cos(x)^3-7\,\cos(x)\right)\sqrt{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^3 - 7*cos(x))*sqrt(cos(x))
```

giac [A] time = 0.90, size = 13, normalized size = 0.62

$$\frac{2}{7}\cos(x)^{\frac{7}{2}} - \frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

maple [B] time = 0.06, size = 39, normalized size = 1.86

$$-\frac{8\sqrt{-2\left(\sin^2\left(\frac{x}{2}\right)\right)+1}\,\left(6\left(\sin^6\left(\frac{x}{2}\right)\right)-9\left(\sin^4\left(\frac{x}{2}\right)\right)+\sin^2\left(\frac{x}{2}\right)+1\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*cos(x)^(1/2),x)
```

```
[Out] -8/21*(-2*sin(1/2*x)^2+1)^(1/2)*(6*sin(1/2*x)^6-9*sin(1/2*x)^4+sin(1/2*x)^2
+1)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.62

$$\frac{2}{7}\cos(x)^{\frac{7}{2}} - \frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

mupad [B] time = 0.09, size = 13, normalized size = 0.62

$$\cos(x)^{3/2}\left(\frac{2\cos(x)^2}{7} - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(1/2)*sin(x)^3,x)
```

```
[Out] cos(x)^(3/2)*((2*cos(x)^2)/7 - 2/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3*cos(x)**(1/2),x)
```

```
[Out] Timed out
```

3.75 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sqrt{\sin(x)}\,dx &= \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)dx,x,\sin(x)\right)\\ &= \frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{1}{21}\sin^{\frac{3}{2}}(x)(3\cos(2x)+11)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

fricas [A] time = 0.45, size = 14, normalized size = 0.67

$$\frac{2}{21}\left(3\cos(x)^2+4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

giac [A] time = 0.96, size = 13, normalized size = 0.62

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

maple [A] time = 0.04, size = 14, normalized size = 0.67

$$-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7} + \frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

maxima [A] time = 0.43, size = 13, normalized size = 0.62

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

mupad [B] time = 0.21, size = 25, normalized size = 1.19

$$-\frac{\cos(x)^4\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
[Out] -(cos(x)^4*sin(x)^(3/2)*hypergeom([1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(3/
4))
```

sympy [B] time = 27.12, size = 167, normalized size = 7.95

$$\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{11}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{8\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{7}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{3}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
[Out] 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(11/2)/(21*tan(x/2)**6 + 63*
tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*ta
n(x/2)**(7/2)/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 28*
sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(3/2)/(21*tan(x/2)**6 + 63*tan(
x/2)**4 + 63*tan(x/2)**2 + 21)
```

3.76 $\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$\sqrt{x} + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3380, 2635, 8}

$$\sqrt{x} + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \cos^2(x)\,dx, x, \sqrt{x}\right)\\
&= \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)\\
&= \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.95

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

fricas [A] time = 0.44, size = 13, normalized size = 0.68

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

giac [A] time = 0.99, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

maple [A] time = 0.04, size = 14, normalized size = 0.74

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

maxima [A] time = 0.44, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

mupad [B] time = 0.27, size = 12, normalized size = 0.63

$$\frac{\sin\left(2\sqrt{x}\right)}{2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
[Out] sin(2*x^(1/2))/2 + x^(1/2)
```

sympy [B] time = 0.36, size = 39, normalized size = 2.05

$$\sqrt{x}\sin^{2}\left(\sqrt{x}\right) + \sqrt{x}\cos^{2}\left(\sqrt{x}\right) + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))**2/x**(1/2),x)
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x
))
```

3.77 $\int x \sin^3\left(x^2\right) dx$

Optimal. Leaf size=19

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

```
[Out] -1/2*cos(x^2)+1/6*cos(x^2)^3
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3379, 2633}

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\sin^3\left(x^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int \sin^3(x)\,dx,x,x^2\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(1-x^2\right)dx,x,\cos\left(x^2\right)\right)\right)\\
&= -\frac{1}{2}\cos\left(x^2\right)+\frac{1}{6}\cos^3\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{1}{24}\cos\left(3x^2\right)-\frac{3\cos\left(x^2\right)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2]^3,x]
```

```
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

fricas [A] time = 0.44, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

giac [A] time = 0.92, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

maple [A] time = 0.02, size = 15, normalized size = 0.79

$$-\frac{\left(\sin^2\left(x^2\right)+2\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.79

$$\frac{1}{24}\cos\left(3x^2\right) - \frac{3}{8}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

mupad [B] time = 0.18, size = 14, normalized size = 0.74

$$\frac{\cos\left(x^2\right)\left(\cos\left(x^2\right)^2-3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] (cos(x^2)*(cos(x^2)^2 - 3))/6
```

sympy [A] time = 0.62, size = 22, normalized size = 1.16

$$-\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2} - \frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

3.78 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

[Out] 1/2*cos(x)^2-ln(cos(x))

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2*Tan[x],x]

[Out] Cos[x]^2/2 - Log[Cos[x]]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cos(x)\right)\\
&= \frac{\cos^2(x)}{2} - \log(\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2*Tan[x],x]

[Out] Cos[x]^2/2 - Log[Cos[x]]

fricas [A] time = 0.46, size = 14, normalized size = 1.00

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="fricas")
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

giac [A] time = 1.08, size = 13, normalized size = 0.93

$$\frac{1}{2}\cos(x)^2 - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="giac")
[Out] 1/2*cos(x)^2 - log(abs(cos(x)))
```

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$-\frac{\left(\sin^2(x)\right)}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*tan(x)^3,x)
[Out] -1/2*sin(x)^2-ln(cos(x))
```

maxima [A] time = 0.45, size = 16, normalized size = 1.14

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="maxima")
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.20, size = 16, normalized size = 1.14

$$\frac{\cos(x)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*tan(x)^3,x)
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*tan(x)**3,x)
[Out] -log(cos(x)) + cos(x)**2/2
```

3.79 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\cot^3(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx,x,-\sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx,x,\sin^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx,x,\sin^2(x)\right)\\ &= -\frac{1}{2}\csc^2(x)-2\log(\sin(x))+\frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x)-\csc^2(x)-4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

fricas [B] time = 0.44, size = 37, normalized size = 1.68

$$-\frac{2 \cos(x)^4 - 3 \cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

giac [A] time = 0.94, size = 28, normalized size = 1.27

$$-\frac{1}{2}\cos(x)^2 + \frac{1}{2\left(\cos(x)^2 - 1\right)} - \log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

maple [A] time = 0.03, size = 29, normalized size = 1.32

$$-\frac{\cos^6(x)}{2\sin(x)^2} - \frac{\left(\cos^4(x)\right)}{2} - \left(\cos^2(x)\right) - 2\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^5*sin(x)^2,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

maxima [A] time = 0.44, size = 20, normalized size = 0.91

$$\frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

mupad [B] time = 0.23, size = 32, normalized size = 1.45

$$\ln\left(\tan(x)^2 + 1\right) - 2\ln(\tan(x)) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan(x)^4 + \tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^5*sin(x)^2,x)
```

```
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

sympy [**A**] time = 0.10, size = 20, normalized size = 0.91

$$-2\log\left(\sin\left(x\right)\right)+\frac{\sin^{2}\left(x\right)}{2}-\frac{1}{2\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**5*sin(x)**2,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

3.80 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] ln(1+sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(x)(1-\sin(x))\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,-\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 36, normalized size = 7.20

$$\log(\cos(x))-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 0.44, size = 5, normalized size = 1.00

$$\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

giac [A] time = 0.91, size = 5, normalized size = 1.00

$$\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="giac")
[Out] log(sin(x) + 1)
```

maple [A] time = 0.04, size = 6, normalized size = 1.20

$$\ln(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(x)+1)/cos(x),x)
[Out] ln(sin(x)+1)
```

maxima [A] time = 0.44, size = 5, normalized size = 1.00

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="maxima")
[Out] log(sin(x) + 1)
```

mupad [B] time = 0.17, size = 5, normalized size = 1.00

$$\ln(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 1)/cos(x),x)
[Out] log(sin(x) + 1)
```

sympy [B] time = 0.33, size = 19, normalized size = 3.80

$$2\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x)
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)**2 + 1)
```

3.81 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x])^(-1),x]
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x])^(-1),x]
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

fricas [A] time = 0.41, size = 17, normalized size = 1.55

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

giac [A] time = 0.87, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) - 1)
```

maple [A] time = 0.04, size = 11, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-sin(x)+1),x)
[Out] -2/(tan(1/2*x)-1)
```

maxima [A] time = 0.43, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1),x)
[Out] -2/(tan(x/2) - 1)
```

sympy [A] time = 0.39, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
[Out] -2/(tan(x/2) - 1)
```

3.82 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x+tan(x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\,dx &= \tan(x) - \int 1\,dx \\ &= -x + \tan(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

giac [A] time = 0.84, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
```

```
[Out] -x+tan(x)
```

maxima [A] time = 0.97, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
```

```
[Out] -x + tan(x)
```

mupad [B] time = 0.03, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
```

```
[Out] tan(x) - x
```

sympy [B] time = 0.07, size = 7, normalized size = 1.17

$$-x + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

3.83 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

[Out] x-tan(x)+1/3*tan(x)^3

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^4,x]

[Out] x - Tan[x] + Tan[x]^3/3

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\,dx &= \frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\ &= -\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\ &= x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]^4,x]

[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3

fricas [A] time = 0.41, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3+x-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^4,x, algorithm="fricas")

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

giac [A] time = 0.87, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{\left(\tan^3(x)\right)}{3} + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
[Out] 1/3*tan(x)^3+x-tan(x)
```

maxima [A] time = 0.96, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.86

$$\frac{\tan(x)^3}{3} - \tan(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
[Out] x - tan(x) + tan(x)^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

3.84 $\int \sec^4(x)\,dx$

Optimal. Leaf size=11

$$\frac{\tan^3(x)}{3}+\tan(x)$$

[Out] tan(x)+1/3*tan(x)^3

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767}

$$\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^4,x]

[Out] Tan[x] + Tan[x]^3/3

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\,dx &= -\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{2\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^4,x]

[Out] (2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3

fricas [A] time = 0.40, size = 16, normalized size = 1.45

$$\frac{\left(2\cos(x)^2+1\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4,x, algorithm="fricas")

[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/cos(x)^3

giac [A] time = 0.95, size = 9, normalized size = 0.82

$$\frac{1}{3}\tan(x)^3+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + tan(x)
```

maple [A] time = 0.10, size = 13, normalized size = 1.18

$$-\left(-\frac{\left(\sec^2(x)\right)}{3}-\frac{2}{3}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4,x)
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)
```

maxima [A] time = 0.44, size = 9, normalized size = 0.82

$$\frac{1}{3}\tan(x)^3+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + tan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 1.55

$$\frac{2\sin(x)\cos(x)^2+\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^4,x)
[Out] (sin(x) + 2*cos(x)^2*sin(x))/(3*cos(x)^3)
```

sympy [B] time = 0.07, size = 19, normalized size = 1.73

$$\frac{2\sin{(x)}}{3\cos{(x)}}+\frac{\sin{(x)}}{3\cos^3{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4,x)
[Out] 2*sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

3.85 $\int \sec^6(x)\,dx$

Optimal. Leaf size=19

$$\frac{\tan^5(x)}{5}+\frac{2\tan^3(x)}{3}+\tan(x)$$

```
[Out] tan(x)+2/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767}

$$\frac{\tan^5(x)}{5}+\frac{2\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6,x]
```

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\,dx &= -\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{2\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.42

$$\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6,x]
```

```
[Out] (8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

fricas [A] time = 0.40, size = 22, normalized size = 1.16

$$\frac{\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/cos(x)^5
```

giac [A] time = 0.97, size = 15, normalized size = 0.79

$$\frac{1}{5}\,\tan(x)^5+\frac{2}{3}\,\tan(x)^3+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

maple [A] time = 0.11, size = 19, normalized size = 1.00

$$-\left(-\frac{\left(\sec^{4}(x)\right)}{5}-\frac{4\left(\sec^{2}(x)\right)}{15}-\frac{8}{15}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6,x)
[Out] -(-8/15-1/5*sec(x)^4-4/15*sec(x)^2)*tan(x)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.79

$$\frac{1}{5}\tan(x)^{5}+\frac{2}{3}\tan(x)^{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

mupad [B] time = 0.04, size = 27, normalized size = 1.42

$$\frac{8\sin(x)\cos(x)^{4}+4\sin(x)\cos(x)^{2}+3\sin(x)}{15\cos(x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^6,x)
[Out] (3*sin(x) + 4*cos(x)^2*sin(x) + 8*cos(x)^4*sin(x))/(15*cos(x)^5)
```

sympy [A] time = 0.07, size = 31, normalized size = 1.63

$$\frac{8\sin(x)}{15\cos(x)}+\frac{4\sin(x)}{15\cos^{3}(x)}+\frac{\sin(x)}{5\cos^{5}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6,x)
[Out] 8*sin(x)/(15*cos(x)) + 4*sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

3.86 $\int \sec^2(x)\tan^4(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^5(x)}{5}$$

```
[Out] 1/5*tan(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 30}

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan^4(x)\,dx &= \text{Subst}\left(\int x^4\,dx, x, \tan(x)\right)\\ &= \frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

fricas [B] time = 0.42, size = 20, normalized size = 2.50

$$\frac{\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\sin(x)}{5\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)^5
```

giac [A] time = 0.88, size = 6, normalized size = 0.75

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="giac")
[Out] 1/5*tan(x)^5
```

maple [A] time = 0.03, size = 11, normalized size = 1.38

$$\frac{\sin^5(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^4,x)
[Out] 1/5*sin(x)^5/cos(x)^5
```

maxima [A] time = 0.44, size = 6, normalized size = 0.75

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="maxima")
[Out] 1/5*tan(x)^5
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{\tan(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/cos(x)^2,x)
[Out] tan(x)^5/5
```

sympy [B] time = 0.07, size = 29, normalized size = 3.62

$$\frac{\sin(x)}{5\cos(x)} - \frac{2\sin(x)}{5\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**4,x)
[Out] sin(x)/(5*cos(x)) - 2*sin(x)/(5*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

3.87 $\int \sec^4(x) \tan^2(x)\, dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5} + \frac{\tan^3(x)}{3}$$

```
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\tan^5(x)}{5} + \frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \sec^4(x) \tan^2(x)\, dx &= \text{Subst}\left(\int x^2 \left(1+x^2\right) \, dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(x^2+x^4\right) \, dx, x, \tan(x)\right)\\
&= \frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.59

$$-\frac{2 \tan(x)}{15} + \frac{1}{5} \tan(x) \sec^4(x) - \frac{1}{15} \tan(x) \sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

fricas [A] time = 0.43, size = 20, normalized size = 1.18

$$-\frac{\left(2 \cos(x)^4 + \cos(x)^2 - 3\right) \sin(x)}{15 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

giac [A] time = 0.77, size = 13, normalized size = 0.76

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

maple [A] time = 0.03, size = 22, normalized size = 1.29

$$\frac{2\left(\sin^3(x)\right)}{15\cos(x)^3} + \frac{\sin^3(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^2,x)
[Out] 1/5*sin(x)^3/cos(x)^5+2/15*sin(x)^3/cos(x)^3
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

mupad [B] time = 0.17, size = 13, normalized size = 0.76

$$\frac{\tan(x)^5}{5} + \frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x)^4,x)
[Out] tan(x)^3/3 + tan(x)^5/5
```

sympy [B] time = 0.07, size = 29, normalized size = 1.71

$$-\frac{2\sin(x)}{15\cos(x)} - \frac{\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**2,x)
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

3.88 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

[Out] 1/3*sec(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^3*Tan[x],x]

[Out] Sec[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^3*Tan[x],x]

[Out] Sec[x]^3/3

fricas [A] time = 0.41, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")

[Out] 1/3/cos(x)^3

giac [A] time = 0.78, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\frac{\left(\sec^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x)
[Out] 1/3*sec(x)^3
```

maxima [A] time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

mupad [B] time = 0.29, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
[Out] 1/(3*cos(x)**3)
```

3.89 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

fricas [A] time = 0.42, size = 14, normalized size = 0.82

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

giac [A] time = 0.92, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

maple [B] time = 0.03, size = 42, normalized size = 2.47

$$-\frac{\sin^4(x)}{15\cos(x)}-\frac{\left(\sin^2(x)+2\right)\cos(x)}{15}+\frac{\sin^4(x)}{15\cos(x)^3}+\frac{\sin^4(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)-1/15*(sin
(x)^2+2)*cos(x)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

mupad [B] time = 0.38, size = 13, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^3,x)
```

```
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{3-5\cos^2(x)}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

3.90 $\int \tan^5(x)\,dx$

Optimal. Leaf size=22

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

```
[Out] -ln(cos(x))-1/2*tan(x)^2+1/4*tan(x)^4
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^5(x)\,dx&=\frac{\tan^4(x)}{4}-\int \tan^3(x)\,dx\\&=-\frac{1}{2}\tan^2(x)+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\&=-\log(\cos(x))-\frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.91

$$\frac{\sec^4(x)}{4}-\sec^2(x)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Sec[x]^2 + Sec[x]^4/4
```

fricas [A] time = 0.43, size = 24, normalized size = 1.09

$$\frac{1}{4}\tan(x)^4-\frac{1}{2}\tan(x)^2-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 0.99, size = 22, normalized size = 1.00

$$\frac{1}{4}\tan(x)^4 - \frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*log(tan(x)^2 + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(\tan^4(x)\right)}{4} - \frac{\left(\tan^2(x)\right)}{2} + \frac{\ln\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5,x)
```

```
[Out] 1/4*tan(x)^4-1/2*tan(x)^2+1/2*ln(tan(x)^2+1)
```

maxima [A] time = 0.44, size = 34, normalized size = 1.55

$$\frac{4\sin(x)^2 - 3}{4\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$\frac{\tan(x)^4}{4} - \frac{\tan(x)^2}{2} - \ln\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5,x)
```

```
[Out] tan(x)^4/4 - tan(x)^2/2 - log(cos(x))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.91

$$-\frac{4\cos^2(x) - 1}{4\cos^4(x)} - \log\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

3.91 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \tan^6(x)\,dx &= \frac{\tan^5(x)}{5}-\int \tan^4(x)\,dx\\
&= -\frac{1}{3}\tan^3(x)+\frac{\tan^5(x)}{5}+\int \tan^2(x)\,dx\\
&= \tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}-\int 1\,dx\\
&= -x+\tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.36

$$-x+\frac{23\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

fricas [A] time = 0.43, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^5-\frac{1}{3}\tan(x)^3-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

giac [A] time = 0.92, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\left(\tan^5(x)\right)}{5} - \frac{\left(\tan^3(x)\right)}{3} - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

maxima [A] time = 0.98, size = 18, normalized size = 0.82

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$\frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
[Out] tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

sympy [A] time = 0.08, size = 31, normalized size = 1.41

$$-x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**6,x)
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

3.92 $\int \sec(x)\tan^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

```
[Out] sec(x)-2/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 194}

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^5(x)\,dx&=\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\sec(x)\right)\\&=\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\sec(x)\right)\\&=\sec(x)-\frac{2\sec^3(x)}{3}+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

fricas [A] time = 0.44, size = 20, normalized size = 1.05

$$\frac{15\cos(x)^4-10\cos(x)^2+3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

giac [A] time = 1.03, size = 20, normalized size = 1.05

$$\frac{15\cos(x)^4 - 10\cos(x)^2 + 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

maple [B] time = 0.02, size = 48, normalized size = 2.53

$$\frac{\sin^6(x)}{5\cos(x)} - \frac{\sin^6(x)}{15\cos(x)^3} + \frac{\left(\sin^4(x) + \frac{4\left(\sin^2(x)\right)}{3} + \frac{8}{3}\right)\cos(x)}{5} + \frac{\sin^6(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^5,x)
```

```
[Out] 1/5*sin(x)^6/cos(x)^5-1/15*sin(x)^6/cos(x)^3+1/5*sin(x)^6/cos(x)+1/5*(sin(x
)^4+4/3*sin(x)^2+8/3)*cos(x)
```

maxima [A] time = 0.43, size = 20, normalized size = 1.05

$$\frac{15\cos(x)^4 - 10\cos(x)^2 + 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

mupad [B] time = 0.27, size = 17, normalized size = 0.89

$$\frac{\cos(x)^4 - \frac{2\cos(x)^2}{3} + \frac{1}{5}}{\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5/cos(x),x)
```

```
[Out] (cos(x)^4 - (2*cos(x)^2)/3 + 1/5)/cos(x)^5
```

sympy [A] time = 0.11, size = 22, normalized size = 1.16

$$-\frac{-15\cos^4(x) + 10\cos^2(x) - 3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**5,x)
```

```
[Out] -(-15*cos(x)**4 + 10*cos(x)**2 - 3)/(15*cos(x)**5)
```

3.93 $\int \sec^3(x)\tan^5(x)\,dx$

Optimal. Leaf size=25

$$\frac{\sec^7(x)}{7}-\frac{2\sec^5(x)}{5}+\frac{\sec^3(x)}{3}$$

```
[Out] 1/3*sec(x)^3-2/5*sec(x)^5+1/7*sec(x)^7
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$\frac{\sec^7(x)}{7}-\frac{2\sec^5(x)}{5}+\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^5,x]
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan^5(x)\,dx&=\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(x)\right)\\&=\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sec(x)\right)\\&=\frac{\sec^3(x)}{3}-\frac{2\sec^5(x)}{5}+\frac{\sec^7(x)}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\sec^7(x)}{7}-\frac{2\sec^5(x)}{5}+\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^5,x]
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

fricas [A] time = 0.42, size = 20, normalized size = 0.80

$$\frac{35\cos(x)^4-42\cos(x)^2+15}{105\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

giac [A] time = 1.00, size = 20, normalized size = 0.80

$$\frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

maple [B] time = 0.03, size = 58, normalized size = 2.32

$$\frac{\sin^6(x)}{35\cos(x)} - \frac{\sin^6(x)}{105\cos(x)^3} + \frac{\left(\sin^4(x) + \frac{4\left(\sin^2(x)\right)}{3} + \frac{8}{3}\right)\cos(x)}{35} + \frac{\sin^6(x)}{35\cos(x)^5} + \frac{\sin^6(x)}{7\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^5,x)
```

```
[Out] 1/7*sin(x)^6/cos(x)^7+1/35/cos(x)^5*sin(x)^6-1/105/cos(x)^3*sin(x)^6+1/35/c
os(x)*sin(x)^6+1/35*(sin(x)^4+4/3*sin(x)^2+8/3)*cos(x)
```

maxima [A] time = 0.44, size = 20, normalized size = 0.80

$$\frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

mupad [B] time = 0.53, size = 19, normalized size = 0.76

$$\frac{\frac{\cos(x)^4}{3} - \frac{2\cos(x)^2}{5} + \frac{1}{7}}{\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5/cos(x)^3,x)
```

```
[Out] (cos(x)^4/3 - (2*cos(x)^2)/5 + 1/7)/cos(x)^7
```

sympy [A] time = 0.12, size = 22, normalized size = 0.88

$$-\frac{-35\cos^4(x) + 42\cos^2(x) - 15}{105\cos^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**5,x)
```

```
[Out] -(-35*cos(x)**4 + 42*cos(x)**2 - 15)/(105*cos(x)**7)
```

3.94 $\int \sec^6(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^6(x)}{6}$$

```
[Out] 1/6*sec(x)^6
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\tan(x)\,dx &= \text{Subst}\left(\int x^5\,dx, x, \sec(x)\right)\\ &= \frac{\sec^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

fricas [A] time = 0.42, size = 6, normalized size = 0.75

$$\frac{1}{6\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="fricas")
```

```
[Out] 1/6/cos(x)^6
```

giac [A] time = 0.94, size = 6, normalized size = 0.75

$$\frac{1}{6\,\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="giac")
[Out] 1/6/cos(x)^6
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\frac{\left(\sec^6(x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6*tan(x),x)
[Out] 1/6*sec(x)^6
```

maxima [A] time = 0.43, size = 10, normalized size = 1.25

$$-\frac{1}{6\left(\sin(x)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="maxima")
[Out] -1/6/(sin(x)^2 - 1)^3
```

mupad [B] time = 0.17, size = 18, normalized size = 2.25

$$\frac{\tan(x)^2\,\left(\tan(x)^4+3\tan(x)^2+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^6,x)
[Out] (tan(x)^2*(3*tan(x)^2 + tan(x)^4 + 3))/6
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{6\cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6*tan(x),x)
[Out] 1/(6*cos(x)**6)
```

3.95 $\int \sec^6(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

```
[Out] -1/6*sec(x)^6+1/8*sec(x)^8
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^5\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^5+x^7\right)\,dx,x,\sec(x)\right)\\ &= -\frac{1}{6}\sec^6(x)+\frac{\sec^8(x)}{8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -1/6*Sec[x]^6 + Sec[x]^8/8
```

fricas [A] time = 0.42, size = 14, normalized size = 0.82

$$-\frac{4\cos(x)^2-3}{24\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="fricas")
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

giac [A] time = 0.94, size = 14, normalized size = 0.82

$$-\frac{4\cos(x)^2-3}{24\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="giac")
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

maple [B] time = 0.03, size = 32, normalized size = 1.88

$$\frac{\sin^4(x)}{24\cos(x)^4}+\frac{\sin^4(x)}{12\cos(x)^6}+\frac{\sin^4(x)}{8\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6*tan(x)^3,x)
[Out] 1/8*sin(x)^4/cos(x)^8+1/12*sin(x)^4/cos(x)^6+1/24*sin(x)^4/cos(x)^4
```

maxima [B] time = 0.45, size = 36, normalized size = 2.12

$$\frac{4\sin(x)^2-1}{24\left(\sin(x)^8-4\sin(x)^6+6\sin(x)^4-4\sin(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="maxima")
[Out] 1/24*(4*sin(x)^2 - 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*sin(x)^2 + 1)
```

mupad [B] time = 0.17, size = 20, normalized size = 1.18

$$\frac{\tan(x)^4\left(3\tan(x)^4+8\tan(x)^2+6\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^6,x)
[Out] (tan(x)^4*(8*tan(x)^2 + 3*tan(x)^4 + 6))/24
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{3-4\cos^2(x)}{24\cos^8(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6*tan(x)**3,x)
[Out] (3 - 4*cos(x)**2)/(24*cos(x)**8)
```

3.96 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

fricas [A] time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

giac [A] time = 1.02, size = 6, normalized size = 0.75

$$\frac{1}{2\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\frac{\left(\sec^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/cot(x),x)
```

```
[Out] 1/2*sec(x)^2
```

maxima [A] time = 0.44, size = 10, normalized size = 1.25

$$-\frac{1}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1)
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{\tan(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*cot(x)),x)
```

```
[Out] tan(x)^2/2
```

sympy [A] time = 0.08, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/cot(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

3.97 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -1/2*arctanh(sin(x))+1/2*sec(x)*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2611, 3770}

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^2(x)\,dx&=\frac{1}{2}\sec(x)\tan(x)-\frac{1}{2}\int \sec(x)\,dx\\&=-\frac{1}{2}\tanh^{-1}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

fricas [B] time = 0.43, size = 34, normalized size = 2.12

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

giac [B] time = 1.00, size = 29, normalized size = 1.81

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log\left(\sin(x)+1\right)+\frac{1}{4}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

maple [A] time = 0.02, size = 24, normalized size = 1.50

$$\frac{\sin^3(x)}{2\cos(x)^2}-\frac{\ln\left(\sec(x)+\tan(x)\right)}{2}+\frac{\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

maxima [B] time = 0.45, size = 27, normalized size = 1.69

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log\left(\sin(x)+1\right)+\frac{1}{4}\log\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

mupad [B] time = 0.26, size = 30, normalized size = 1.88

$$\frac{\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2}-\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x),x)
```

```
[Out] (tan(x/2) + tan(x/2)^3)/(tan(x/2)^2 - 1)^2 - atanh(tan(x/2))
```

sympy [A] time = 0.13, size = 27, normalized size = 1.69

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}-\frac{\sin\left(x\right)}{2\sin^2\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

3.98 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x-cot(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]
[Out] -x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\,dx &= -\cot(x)-\int 1\,dx\\ &= -x-\cot(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]
[Out] -x - Cot[x]
```

fricas [B] time = 0.41, size = 20, normalized size = 2.50

$$-\frac{x\sin(2x)+\cos(2x)+1}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

giac [B] time = 1.13, size = 18, normalized size = 2.25

$$-x-\frac{1}{2\tan\left(\frac{1}{2}x\right)}+\frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="giac")
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$-x - \cot(x) + \frac{\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
[Out] -x-cot(x)+1/2*Pi
```

maxima [A] time = 0.98, size = 10, normalized size = 1.25

$$-x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
[Out] -x - 1/tan(x)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$-x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
[Out] - x - cot(x)
```

sympy [A] time = 0.07, size = 8, normalized size = 1.00

$$-x - \frac{\cos{(x)}}{\sin{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2,x)
[Out] -x - cos(x)/sin(x)
```

3.99 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

```
[Out] -1/2*cot(x)^2-ln(sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3,x]
```

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\,dx &= -\frac{1}{2}\cot^2(x)-\int \cot(x)\,dx\\ &= -\frac{1}{2}\cot^2(x)-\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3,x]
```

```
[Out] -1/2*Csc[x]^2 - Log[Sin[x]]
```

fricas [B] time = 0.45, size = 28, normalized size = 2.00

$$-\frac{(\cos(2x)-1)\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)-2}{2(\cos(2x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2) - 2)/(cos(2*x) - 1)
```

giac [A] time = 0.94, size = 22, normalized size = 1.57

$$\frac{1}{2\left(\cos(x)^2 - 1\right)} - \frac{1}{2}\log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="giac")
[Out] 1/2/(cos(x)^2 - 1) - 1/2*log(-cos(x)^2 + 1)
```

maple [A] time = 0.01, size = 17, normalized size = 1.21

$$-\frac{\left(\cot^2(x)\right)}{2} + \frac{\ln\left(\cot^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3,x)
[Out] -1/2*cot(x)^2+1/2*ln(cot(x)^2+1)
```

maxima [A] time = 0.44, size = 14, normalized size = 1.00

$$-\frac{1}{2\sin(x)^2} - \frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="maxima")
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

mupad [B] time = 0.03, size = 18, normalized size = 1.29

$$\frac{\sin(x)^2 - 1}{2\sin(x)^2} - \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3,x)
[Out] (sin(x)^2 - 1)/(2*sin(x)^2) - log(sin(x))
```

sympy [A] time = 0.10, size = 14, normalized size = 1.00

$$-\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3,x)
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

3.100 $\int \cot^4(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

```
[Out] -1/5*cot(x)^5-1/7*cot(x)^7
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\csc^4(x)\,dx &= \text{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,-\cot(x)\right)\\ &= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,-\cot(x)\right)\\ &= -\frac{1}{5}\cot^5(x)-\frac{\cot^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.03, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35}-\frac{1}{7}\cot(x)\csc^6(x)+\frac{8}{35}\cot(x)\csc^4(x)-\frac{1}{35}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

fricas [B] time = 0.43, size = 39, normalized size = 2.29

$$-\frac{2\cos(x)^7-7\cos(x)^5}{35\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

giac [A] time = 0.92, size = 14, normalized size = 0.82

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="giac")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

maple [A] time = 0.03, size = 22, normalized size = 1.29

$$-\frac{2\left(\cos^5(x)\right)}{35\sin(x)^5}-\frac{\cos^5(x)}{7\sin(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*csc(x)^4,x)
```

```
[Out] -1/7/sin(x)^7*cos(x)^5-2/35/sin(x)^5*cos(x)^5
```

maxima [A] time = 0.43, size = 14, normalized size = 0.82

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

mupad [B] time = 0.18, size = 14, normalized size = 0.82

$$-\frac{\cot(x)^5\,\left(5\cot(x)^2+7\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/sin(x)^4,x)
```

```
[Out] -(cot(x)^5*(5*cot(x)^2 + 7))/35
```

sympy [B] time = 0.08, size = 41, normalized size = 2.41

$$-\frac{2\cos(x)}{35\sin(x)}-\frac{\cos(x)}{35\sin^3(x)}+\frac{8\cos(x)}{35\sin^5(x)}-\frac{\cos(x)}{7\sin^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*csc(x)**4,x)
```

```
[Out] -2*cos(x)/(35*sin(x)) - cos(x)/(35*sin(x)**3) + 8*cos(x)/(35*sin(x)**5) - c
os(x)/(7*sin(x)**7)
```

3.101 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] 1/4*csc(x)^4-1/6*csc(x)^6
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\csc^4(x)\,dx &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\ &= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\ &= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

fricas [B] time = 0.42, size = 30, normalized size = 1.76

$$\frac{3\,\cos(x)^2-1}{12\left(\cos(x)^6-3\,\cos(x)^4+3\,\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

giac [A] time = 0.91, size = 14, normalized size = 0.82

$$\frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

maple [A] time = 0.03, size = 22, normalized size = 1.29

$$-\frac{\cos^4(x)}{12\sin(x)^4} - \frac{\cos^4(x)}{6\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x)^4,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

maxima [A] time = 0.45, size = 14, normalized size = 0.82

$$\frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

mupad [B] time = 0.18, size = 14, normalized size = 0.82

$$-\frac{\cot(x)^4\,\left(2\cot(x)^2 + 3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/sin(x)^4,x)
```

```
[Out] -(cot(x)^4*(2*cot(x)^2 + 3))/12
```

sympy [A] time = 0.11, size = 15, normalized size = 0.88

$$-\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x)**4,x)
```

```
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

3.102 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))
```

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3770}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(x)\,dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.00, size = 17, normalized size = 3.40

$$\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.44, size = 19, normalized size = 3.80

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 1.00, size = 6, normalized size = 1.20

$$\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))
```

maple [A] time = 0.00, size = 9, normalized size = 1.80

$$-\ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x),x)
[Out] -ln(csc(x)+cot(x))
```

maxima [A] time = 0.43, size = 8, normalized size = 1.60

$$-\log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="maxima")
[Out] -log(cot(x) + csc(x))
```

mupad [B] time = 0.04, size = 5, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x)
[Out] log(tan(x/2))
```

sympy [B] time = 0.11, size = 15, normalized size = 3.00

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

3.103 $\int \csc^3(x)\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -1/2*arctanh(cos(x))-1/2*cot(x)*csc(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3,x]
```

```
[Out] -ArcTanh[Cos[x]]/2 - (Cot[x]*Csc[x])/2
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(x)\,dx &= -\frac{1}{2}\cot(x)\csc(x)+\frac{1}{2}\int \csc(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 47, normalized size = 2.94

$$-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3,x]
```

```
[Out] -1/8*Csc[x/2]^2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8
```

fricas [B] time = 0.42, size = 44, normalized size = 2.75

$$-\frac{\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

giac [B] time = 1.12, size = 54, normalized size = 3.38

$$-\frac{\left(\frac{2(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{8(\cos(x)-1)}-\frac{\cos(x)-1}{8(\cos(x)+1)}+\frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.11, size = 18, normalized size = 1.12

$$-\frac{\cot(x)\csc(x)}{2}+\frac{\ln(-\cot(x)+\csc(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3,x)
```

```
[Out] -1/2*cot(x)*csc(x)+1/2*ln(csc(x)-cot(x))
```

maxima [B] time = 0.45, size = 27, normalized size = 1.69

$$\frac{\cos(x)}{2\left(\cos(x)^2-1\right)}-\frac{1}{4}\log(\cos(x)+1)+\frac{1}{4}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*cos(x)/(cos(x)^2 - 1) - 1/4*log(cos(x) + 1) + 1/4*log(cos(x) - 1)
```

mupad [B] time = 0.15, size = 16, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{\cos(x)}{2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^3,x)
```

```
[Out] log(tan(x/2))/2 - cos(x)/(2*sin(x)^2)
```

sympy [A] time = 0.13, size = 27, normalized size = 1.69

$$\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}+\frac{\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3,x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)/(2*cos(x)**2 - 2)
```

3.104 $\int \cos(x)\cot(x)\,dx$

Optimal. Leaf size=8

$$\cos(x)-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))+cos(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2592, 321, 206}

$$\cos(x)-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(x)\,dx &= -\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(x)\right)\\ &= \cos(x)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(x)\right)\\ &= -\tanh^{-1}(\cos(x))+\cos(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 2.38

$$\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.44, size = 21, normalized size = 2.62

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="fricas")
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 0.98, size = 19, normalized size = 2.38

$$\cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="giac")
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

maple [A] time = 0.02, size = 12, normalized size = 1.50

$$\cos(x) + \ln(-\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x),x)
[Out] cos(x)+ln(-cot(x)+csc(x))
```

maxima [B] time = 0.44, size = 17, normalized size = 2.12

$$\cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="maxima")
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

mupad [B] time = 0.15, size = 8, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x),x)
[Out] log(tan(x/2)) + cos(x)
```

sympy [B] time = 0.11, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/sin(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

3.105 $\int \csc^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

```
[Out] -cot(x)-1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767}

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4,x]
[Out] -Cot[x] - Cot[x]^3/3
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(x)\,dx &= -\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(x)\right)\\ &= -\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.31

$$-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4,x]
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

fricas [B] time = 0.41, size = 25, normalized size = 1.92

$$-\frac{2\,\cos(x)^3-3\,\cos(x)}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="fricas")
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.98, size = 14, normalized size = 1.08

$$-\frac{3\,\tan(x)^2+1}{3\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

maple [A] time = 0.10, size = 12, normalized size = 0.92

$$\left(-\frac{\left(\csc^2(x)\right)}{3}-\frac{2}{3}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^4,x)
```

```
[Out] (-2/3-1/3*csc(x)^2)*cot(x)
```

maxima [A] time = 0.44, size = 14, normalized size = 1.08

$$-\frac{3\tan(x)^2+1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

mupad [B] time = 0.03, size = 17, normalized size = 1.31

$$-\frac{2\cos(x)\sin(x)^2+\cos(x)}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^4,x)
```

```
[Out] -(cos(x) + 2*cos(x)*sin(x)^2)/(3*sin(x)^3)
```

sympy [A] time = 0.06, size = 20, normalized size = 1.54

$$-\frac{2\cos(x)}{3\sin(x)}-\frac{\cos(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**4,x)
```

```
[Out] -2*cos(x)/(3*sin(x)) - cos(x)/(3*sin(x)**3)
```

3.106 $\int \sin(2x)\sin(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

```
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]*Sin[5*x],x]
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(2x)\sin(5x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]*Sin[5*x],x]
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

fricas [A] time = 0.43, size = 24, normalized size = 1.41

$$-\frac{2}{21}\left(48\cos(x)^6 - 60\cos(x)^4 + 11\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="fricas")
[Out] -2/21*(48*cos(x)^6 - 60*cos(x)^4 + 11*cos(x)^2 + 1)*sin(x)
```

giac [A] time = 0.88, size = 13, normalized size = 0.76

$$-\frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="giac")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

maple [A]   time = 0.10, size = 14, normalized size = 0.82

$$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(5*x),x)
```

```
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

maxima [A]   time = 0.42, size = 13, normalized size = 0.76

$$-\frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="maxima")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

mupad [B]   time = 0.06, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(5*x),x)
```

```
[Out] sin(3*x)/6 - sin(7*x)/14
```

sympy [B]   time = 0.57, size = 26, normalized size = 1.53

$$-\frac{5\sin(2x)\cos(5x)}{21} + \frac{2\sin(5x)\cos(2x)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x)
```

```
[Out] -5*sin(2*x)*cos(5*x)/21 + 2*sin(5*x)*cos(2*x)/21
```

3.107 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(3x)\,dx=-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[3*x],x]
```

```
[Out] -1/2*Cos[x]^2 - Cos[4*x]/8
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$-\cos(x)^4+\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

giac [A] time = 0.92, size = 13, normalized size = 0.76

$$-\cos(x)^4+\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

maple [A] time = 0.06, size = 14, normalized size = 0.82

$$-\left(\cos^{4}(x)\right)+\frac{\left(\cos^{2}(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(3*x),x)
```

```
[Out] -cos(x)^4+1/2*cos(x)^2
```

maxima [A] time = 0.42, size = 13, normalized size = 0.76

$$-\frac{1}{8}\cos\left(4\,x\right)-\frac{1}{4}\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\cos(x)^2}{2}-\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*cos(x),x)
```

```
[Out] cos(x)^2/2 - cos(x)^4
```

sympy [A] time = 0.54, size = 22, normalized size = 1.29

$$-\frac{\sin\left(x\right)\sin\left(3x\right)}{8}-\frac{3\cos\left(x\right)\cos\left(3x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

3.108 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(4x)\,dx=\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

fricas [B] time = 0.43, size = 24, normalized size = 1.60

$$\frac{1}{7}\left(32\cos(x)^6-40\cos(x)^4+12\cos(x)^2+3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

giac [A] time = 0.97, size = 11, normalized size = 0.73

$$\frac{1}{14}\sin(7x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

maple [A] time = 0.10, size = 12, normalized size = 0.80

$$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{1}{14} \sin(7x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

mupad [B] time = 0.17, size = 11, normalized size = 0.73

$$\frac{\sin(7x)}{14} + \frac{\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x)
```

```
[Out] sin(7*x)/14 + sin(x)/2
```

sympy [B] time = 0.56, size = 26, normalized size = 1.73

$$-\frac{3\sin(3x)\cos(4x)}{7} + \frac{4\sin(4x)\cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

3.109 $\int \sin(3x)\sin(6x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

```
[Out] 1/6*sin(3*x)-1/18*sin(9*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[3*x]*Sin[6*x],x]
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(3x)\sin(6x)\,dx=\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]*Sin[6*x],x]
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

fricas [A] time = 0.43, size = 14, normalized size = 0.82

$$-\frac{2}{9}\left(\cos\left(3\,x\right)^{2}-1\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="fricas")
[Out] -2/9*(cos(3*x)^2 - 1)*sin(3*x)
```

giac [A] time = 0.93, size = 13, normalized size = 0.76

$$-\frac{1}{18}\sin\left(9\,x\right)+\frac{1}{6}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="giac")
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

maple [A] time = 0.03, size = 9, normalized size = 0.53

$$\frac{2\left(\sin^3(3x)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(6*x),x)
[Out] 2/9*sin(3*x)^3
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$-\frac{1}{18}\sin(9x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="maxima")
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} - \frac{\sin(9x)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(6*x),x)
[Out] sin(3*x)/6 - sin(9*x)/18
```

sympy [A] time = 0.61, size = 24, normalized size = 1.41

$$-\frac{2\sin(3x)\cos(6x)}{9} + \frac{\sin(6x)\cos(3x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x)
[Out] -2*sin(3*x)*cos(6*x)/9 + sin(6*x)*cos(3*x)/9
```

3.110 $\int \cos^5(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{6}\cos^6(x)$$

```
[Out] -1/6*cos(x)^6
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 30}

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5*Sin[x],x]
```

```
[Out] -Cos[x]^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\sin(x)\,dx &= -\text{Subst}\left(\int x^5\,dx, x, \cos(x)\right)\\ &= -\frac{1}{6}\cos^6(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5*Sin[x],x]
```

```
[Out] -1/6*Cos[x]^6
```

fricas [A] time = 0.42, size = 6, normalized size = 0.75

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="fricas")
```

```
[Out] -1/6*cos(x)^6
```

giac [**A**] time = 0.80, size = 6, normalized size = 0.75

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*cos(x)^6
```

maple [**A**] time = 0.00, size = 7, normalized size = 0.88

$$-\frac{\left(\cos^6(x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x),x)
```

```
[Out] -1/6*cos(x)^6
```

maxima [**A**] time = 0.43, size = 6, normalized size = 0.75

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(x)^6
```

mupad [**B**] time = 0.03, size = 19, normalized size = 2.38

$$\frac{\sin(x)^6}{6} - \frac{\sin(x)^4}{2} + \frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x),x)
```

```
[Out] sin(x)^2/2 - sin(x)^4/2 + sin(x)^6/6
```

sympy [**A**] time = 0.07, size = 7, normalized size = 0.88

$$-\frac{\cos^6(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5*sin(x),x)
```

```
[Out] -cos(x)**6/6
```

3.111 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

fricas [A] time = 0.43, size = 25, normalized size = 0.83

$$\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

giac [A] time = 0.80, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

maple [A] time = 0.09, size = 23, normalized size = 0.77

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

maxima [A] time = 0.43, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

mupad [B] time = 0.29, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

sympy [B] time = 13.53, size = 112, normalized size = 3.73

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
24 - sin(2*x)*cos(x)*cos(3*x)/8 + sin(3*x)*cos(x)*cos(2*x)/3
```

3.112 $\int \cos^2(x) \left(1-\tan^2(x)\right) \, dx$

Optimal. Leaf size=5

$$\sin(x)\cos(x)$$

```
[Out] cos(x)*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 383}

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Cos[x]*Sin[x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \cos^2(x)\left(1-\tan^2(x)\right) dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx, x, \tan(x)\right) \\ &= \cos(x)\sin(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.60

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Sin[2*x]/2
```

fricas [A] time = 0.43, size = 5, normalized size = 1.00

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

giac [**A**] time = 1.03, size = 9, normalized size = 1.80

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="giac")
[Out] 1/(1/tan(x) + tan(x))
```

maple [**A**] time = 0.03, size = 6, normalized size = 1.20

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tan(x)^2)/sec(x)^2,x)
[Out] cos(x)*sin(x)
```

maxima [**B**] time = 0.42, size = 11, normalized size = 2.20

$$\frac{\tan(x)}{\tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="maxima")
[Out] tan(x)/(tan(x)^2 + 1)
```

mupad [**B**] time = 0.17, size = 6, normalized size = 1.20

$$\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2*(tan(x)^2 - 1),x)
[Out] sin(2*x)/2
```

sympy [**A**] time = 0.44, size = 7, normalized size = 1.40

$$\frac{\tan(x)}{\sec^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)**2)/sec(x)**2,x)
[Out] tan(x)/sec(x)**2
```

3.113 $\int \csc(2x)(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -1/2*arctanh(cos(x))+1/2*arctanh(sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4401, 4287, 3770, 4288}

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx &= \int (\cos(x)\csc(2x)+\csc(2x)\sin(x))\,dx\\ &= \int \cos(x)\csc(2x)\,dx+\int \csc(2x)\sin(x)\,dx\\ &= \frac{1}{2}\int \csc(x)\,dx+\frac{1}{2}\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 61, normalized size = 4.07

$$\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -1/2*Log[Cos[x/2]] - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 + Log[Cos
[x/2] + Sin[x/2]]/2
```

fricas [B] time = 0.44, size = 35, normalized size = 2.33

$$-\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

giac [B] time = 1.11, size = 29, normalized size = 1.93

$$\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))
```

maple [A] time = 0.10, size = 20, normalized size = 1.33

$$\frac{\ln(-\cot(x)+\csc(x))}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(-cot(x)+csc(x))
```

maxima [B] time = 0.98, size = 69, normalized size = 4.60

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 0.40, size = 24, normalized size = 1.60

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/sin(2*x),x)
```

```
[Out] log(tan(x/2) + tan(x/2)^2)/2 - log(tan(x/2) - 1)/2
```

sympy [**B**] time = 1.57, size = 32, normalized size = 2.13

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{4}+\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4
```

3.114 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

```
[Out] 1/2*cos(x)^2-ln(cos(x))
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx,x,\cos(x)\right)\\&= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx,x,\cos(x)\right)\\&= \frac{\cos^2(x)}{2}-\log(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

fricas [A] time = 0.44, size = 14, normalized size = 1.00

$$\frac{1}{2}\cos(x)^2-\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

giac [A] time = 0.92, size = 18, normalized size = 1.29

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(-\sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

maple [A] time = 0.03, size = 13, normalized size = 0.93

$$-\frac{\left(\sin^2(x)\right)}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
[Out] -1/2*sin(x)^2-ln(cos(x))
```

maxima [A] time = 0.44, size = 16, normalized size = 1.14

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.14, size = 16, normalized size = 1.14

$$\frac{\cos(x)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*tan(x),x)
[Out] -log(cos(x)) + cos(x)**2/2
```

3.115 $\int \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x)\cot^3(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx, x, -\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(x)\right)\\
&= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x) - \csc^2(x) - 4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

fricas [B] time = 0.45, size = 37, normalized size = 1.68

$$-\frac{2\cos(x)^4-3\cos(x)^2+8\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\sin(x)\right)-1}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

giac [A] time = 0.85, size = 28, normalized size = 1.27

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2\left(\cos(x)^2-1\right)}-\log\left(-\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

maple [A] time = 0.03, size = 29, normalized size = 1.32

$$-\frac{\cos^6(x)}{2\sin(x)^2}-\frac{\left(\cos^4(x)\right)}{2}-\left(\cos^2(x)\right)-2\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] -1/2*cos(x)^6/sin(x)^2-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

maxima [A] time = 0.42, size = 20, normalized size = 0.91

$$\frac{1}{2}\sin(x)^2-\frac{1}{2\sin(x)^2}-\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

mupad [B] time = 0.15, size = 32, normalized size = 1.45

$$\ln\left(\tan(x)^2+1\right)-2\ln(\tan(x))-\frac{\tan(x)^2+\frac{1}{2}}{\tan(x)^4+\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

sympy [**A**] time = 0.10, size = 20, normalized size = 0.91

$$-2\log\left(\sin\left(x\right)\right)+\frac{\sin^{2}\left(x\right)}{2}-\frac{1}{2\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*cot(x)**3,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

3.116 $\int \sec^3(x) \tan(x) \, dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] 1/3*sec(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \sec^3(x) \tan(x) \, dx &= \text{Subst}\left(\int x^2 \, dx, x, \sec(x)\right) \\ &= \frac{\sec^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

fricas [A] time = 0.42, size = 6, normalized size = 0.75

$$\frac{1}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
[Out] 1/3/cos(x)^3
```

giac [A] time = 0.91, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sec^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x)
[Out] 1/3*sec(x)^3
```

maxima [A] time = 0.46, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

mupad [B] time = 0.00, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
[Out] 1/(3*cos(x)**3)
```

3.117 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

fricas [A] time = 0.43, size = 14, normalized size = 0.82

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

giac [A] time = 0.82, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

maple [B] time = 0.00, size = 42, normalized size = 2.47

$$-\frac{\sin^4(x)}{15\cos(x)}-\frac{\left(\sin^2(x)+2\right)\cos(x)}{15}+\frac{\sin^4(x)}{15\cos(x)^3}+\frac{\sin^4(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
[Out] -1/15/cos(x)*sin(x)^4-1/15*(sin(x)^2+2)*cos(x)+1/15/cos(x)^3*sin(x)^4+1/5/c
os(x)^5*sin(x)^4
```

maxima [A] time = 0.42, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

mupad [B] time = 0.00, size = 13, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^3,x)
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{3-5\cos^2(x)}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

3.118 $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -arcsin(1/3*x)-(-x^2+9)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-x^2}}{x^2}\,dx &= -\frac{\sqrt{9-x^2}}{x}-\int \frac{1}{\sqrt{9-x^2}}\,dx\\ &= -\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

fricas [A] time = 0.40, size = 35, normalized size = 1.40

$$\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)-\sqrt{-x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan((sqrt(-x^2 + 9) - 3)/x) - sqrt(-x^2 + 9))/x
```

giac [A] time = 0.98, size = 39, normalized size = 1.56

$$\frac{x}{2\left(\sqrt{-x^2+9}-3\right)} - \frac{\sqrt{-x^2+9}-3}{2\,x} - \arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 9) - 3) - 1/2*(sqrt(-x^2 + 9) - 3)/x - arcsin(1/3*x)
```

maple [A] time = 0.01, size = 34, normalized size = 1.36

$$-\frac{\sqrt{-x^2+9}\,x}{9} - \arcsin\left(\frac{x}{3}\right) - \frac{\left(-x^2+9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(-x^2+9)^(3/2)-1/9*x*(-x^2+9)^(1/2)-arcsin(1/3*x)
```

maxima [A] time = 0.98, size = 21, normalized size = 0.84

$$-\frac{\sqrt{-x^2+9}}{x} - \arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 9)/x - arcsin(1/3*x)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$-\mathrm{asin}\left(\frac{x}{3}\right) - \frac{\sqrt{9-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - x^2)^(1/2)/x^2,x)
```

```
[Out] - asin(x/3) - (9 - x^2)^(1/2)/x
```

sympy [A] time = 0.26, size = 15, normalized size = 0.60

$$-\operatorname{asin}\left(\frac{x}{3}\right) - \frac{\sqrt{9-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+9)**(1/2)/x**2,x)
```

```
[Out] -asin(x/3) - sqrt(9 - x**2)/x
```

3.119 $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^2+4}}{4x}$$

[Out] -1/4*(x^2+4)^(1/2)/x

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[4 + x^2]),x]

[Out] -Sqrt[4 + x^2]/(4*x)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{4+x^2}}{4x}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[4 + x^2]),x]

[Out] -1/4*Sqrt[4 + x^2]/x

fricas [A] time = 0.39, size = 14, normalized size = 0.88

$$-\frac{x+\sqrt{x^2+4}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="fricas")

[Out] -1/4*(x + sqrt(x^2 + 4))/x

giac [A] time = 1.07, size = 19, normalized size = 1.19

$$\frac{2}{\left(x-\sqrt{x^2+4}\right)^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="giac")
[Out] 2/((x - sqrt(x^2 + 4))^2 - 4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{x^2+4}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+4)^(1/2),x)
[Out] -1/4*(x^2+4)^(1/2)/x
```

maxima [A] time = 0.99, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^2+4}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(x^2 + 4)/x
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^2+4}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^2 + 4)^(1/2)),x)
[Out] -(x^2 + 4)^(1/2)/(4*x)
```

sympy [A] time = 0.78, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{4}{x^2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+4)**(1/2),x)
[Out] -sqrt(1 + 4/x**2)/4
```

3.120 $\int \frac{x}{\sqrt{4+x^2}} \, dx$

Optimal. Leaf size=9

$$\sqrt{x^2+4}$$

```
[Out] (x^2+4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{4+x^2}} \, dx = \sqrt{4+x^2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

fricas [A] time = 0.41, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 4)
```

giac [A] time = 0.89, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 4)
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4)^(1/2),x)
[Out] (x^2+4)^(1/2)
```

maxima [A] time = 0.43, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 4)
```

mupad [B] time = 0.03, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + 4)^(1/2),x)
[Out] (x^2 + 4)^(1/2)
```

sympy [A] time = 0.15, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4)**(1/2),x)
[Out] sqrt(x**2 + 4)
```

3.121 $\int \frac{1}{\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

```
[Out] arctanh(x/(-a^2+x^2)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a^2+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-a^2+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-a^2+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] -1/2*Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1 + x/Sqrt[-a^2 + x^2]]/2
```

fricas [A] time = 0.42, size = 18, normalized size = 1.12

$$-\log\left(-x+\sqrt{-a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(-a^2 + x^2))
```

giac [A] time = 0.99, size = 19, normalized size = 1.19

$$-\log\left(\left|-x + \sqrt{-a^2 + x^2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(-a^2 + x^2)))
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\ln\left(x + \sqrt{-a^2 + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2+x^2)^(1/2),x)
[Out] ln(x+(-a^2+x^2)^(1/2))
```

maxima [A] time = 0.48, size = 18, normalized size = 1.12

$$\log\left(2\,x + 2\,\sqrt{-a^2 + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(-a^2 + x^2))
```

mupad [B] time = 0.08, size = 14, normalized size = 0.88

$$\ln\left(x + \sqrt{x^2 - a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - a^2)^(1/2),x)
[Out] log(x + (x^2 - a^2)^(1/2))
```

sympy [A] time = 1.08, size = 19, normalized size = 1.19

$$\begin{cases} \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\: \left|\frac{x^2}{a^2}\right| > 1 \\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2+x**2)**(1/2),x)
[Out] Piecewise((acosh(x/a), Abs(x**2/a**2) > 1), (-I*asin(x/a), True))
```

3.122 $\int \frac{x^3}{(9+4x^2)^{3/2}} dx$

Optimal. Leaf size=31

$$\frac{1}{16}\sqrt{4x^2+9} + \frac{9}{16\sqrt{4x^2+9}}$$

```
[Out] 9/16/(4*x^2+9)^(1/2)+1/16*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{16}\sqrt{4x^2+9} + \frac{9}{16\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(9+4x^2)^{3/2}} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(9+4x)^{3/2}} dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4(9+4x)^{3/2}} + \frac{1}{4\sqrt{9+4x}}\right) dx, x, x^2\right) \\
&= \frac{9}{16\sqrt{9+4x^2}} + \frac{1}{16}\sqrt{9+4x^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{2x^2+9}{8\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] (9 + 2*x^2)/(8*Sqrt[9 + 4*x^2])
```

fricas [A] time = 0.41, size = 18, normalized size = 0.58

$$\frac{2\,x^2+9}{8\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="fricas")
[Out] 1/8*(2*x^2 + 9)/sqrt(4*x^2 + 9)
```

giac [A] time = 0.98, size = 23, normalized size = 0.74

$$\frac{1}{16}\,\sqrt{4\,x^2+9}+\frac{9}{16\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="giac")
[Out] 1/16*sqrt(4*x^2 + 9) + 9/16/sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{2x^2+9}{8\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2+9)^(3/2),x)
[Out] 1/8*(2*x^2+9)/(4*x^2+9)^(1/2)
```

maxima [A] time = 0.96, size = 26, normalized size = 0.84

$$\frac{x^2}{4\,\sqrt{4\,x^2+9}}+\frac{9}{8\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="maxima")
[Out] 1/4*x^2/sqrt(4*x^2 + 9) + 9/8/sqrt(4*x^2 + 9)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.77

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(2\,x^2+9\right)}{4\,\left(4\,x^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 + 9)^(3/2),x)
[Out] ((x^2 + 9/4)^(1/2)*(2*x^2 + 9))/(4*(4*x^2 + 9))
```

sympy [A] time = 0.58, size = 27, normalized size = 0.87

$$\frac{x^2}{4\sqrt{4x^2+9}}+\frac{9}{8\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2+9)**(3/2),x)
[Out] x**2/(4*sqrt(4*x**2 + 9)) + 9/(8*sqrt(4*x**2 + 9))
```

3.123 $\int \frac{x}{\sqrt{3-2x-x^2}} \, dx$

Optimal. Leaf size=27

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

```
[Out] -arcsin(1/2+1/2*x)-(-x^2-2*x+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 216}

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{3-2x-x^2}} \, dx &= -\sqrt{3-2x-x^2}-\int \frac{1}{\sqrt{3-2x-x^2}} \, dx\\ &= -\sqrt{3-2x-x^2}+\frac{1}{4} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} \, dx,x,-2-2x\right)\\ &= -\sqrt{3-2x-x^2}+\sin^{-1}\left(\frac{1}{2}(-1-x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sin^{-1}\left(\frac{1}{4}(-2x-2)\right)-\sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-2 - 2*x)/4]
```

fricas [A] time = 0.42, size = 42, normalized size = 1.56

$$-\sqrt{-x^2-2\,x+3}+\arctan\left(\frac{\sqrt{-x^2-2\,x+3}\,(x+1)}{x^2+2\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) + arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)
)
```

giac [A] time = 1.05, size = 23, normalized size = 0.85

$$-\sqrt{-x^2-2\,x+3}-\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) - arcsin(1/2*x + 1/2)
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\arcsin\left(\frac{x}{2}+\frac{1}{2}\right)-\sqrt{-x^2-2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2-2*x+3)^(1/2),x)
```

```
[Out] -arcsin(1/2*x+1/2)-(-x^2-2*x+3)^(1/2)
```

maxima [A] time = 0.96, size = 21, normalized size = 0.78

$$-\sqrt{-x^2-2\,x+3}+\arcsin\left(-\frac{1}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) + arcsin(-1/2*x - 1/2)
```

mupad [B] time = 0.22, size = 38, normalized size = 1.41

$$-\sqrt{-x^2-2\,x+3}+\ln\left(x\,1\mathrm{i}+\sqrt{-x^2-2\,x+3}+1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3 - x^2 - 2*x)^(1/2),x)
```

```
[Out] log(x*1i + (3 - x^2 - 2*x)^(1/2) + 1i)*1i - (3 - x^2 - 2*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)\,(x+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2-2*x+3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(x - 1)*(x + 3)), x)
```

3.124 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{1-x^2}}{x}$$

```
[Out] -(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

fricas [A] time = 0.41, size = 14, normalized size = 0.88

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)/x
```

giac [B] time = 0.95, size = 33, normalized size = 2.06

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x
```

maple [A] time = 0.01, size = 20, normalized size = 1.25

$$\frac{(x-1)(x+1)}{\sqrt{-x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/2),x)
[Out] 1/x*(x-1)*(x+1)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 14, normalized size = 0.88

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/x
```

mupad [B] time = 0.23, size = 14, normalized size = 0.88

$$-\frac{\sqrt{1-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^2)^(1/2)),x)
[Out] -(1 - x^2)^(1/2)/x
```

sympy [A] time = 0.80, size = 27, normalized size = 1.69

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/2),x)
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
```

3.125 $\int x^3\sqrt{4-x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] -4/3*(-x^2+4)^(3/2)+1/5*(-x^2+4)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[4 - x^2],x]
```

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{4-x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{4-x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(4\sqrt{4-x}-(4-x)^{3/2}\right)dx,x,x^2\right)\\&=-\frac{4}{3}\left(4-x^2\right)^{3/2}+\frac{1}{5}\left(4-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{1}{15}\left(4-x^2\right)^{3/2}\left(3x^2+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[4 - x^2],x]
```

```
[Out] -1/15*((4 - x^2)^(3/2)*(8 + 3*x^2))
```

fricas [A] time = 0.41, size = 23, normalized size = 0.74

$$\frac{1}{15}\left(3\,x^4-4\,x^2-32\right)\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^4 - 4*x^2 - 32)*sqrt(-x^2 + 4)
```

giac [A] time = 0.88, size = 30, normalized size = 0.97

$$\frac{1}{5}\left(x^2-4\right)^2\sqrt{-x^2+4}-\frac{4}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 - 4)^2*sqrt(-x^2 + 4) - 4/3*(-x^2 + 4)^(3/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.81

$$\frac{(x-2)(x+2)\left(3x^2+8\right)\sqrt{-x^2+4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+4)^(1/2),x)
[Out] 1/15*(x-2)*(x+2)*(3*x^2+8)*(-x^2+4)^(1/2)
```

maxima [A] time = 0.97, size = 26, normalized size = 0.84

$$-\frac{1}{5}\left(-x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{15}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/5*(-x^2 + 4)^(3/2)*x^2 - 8/15*(-x^2 + 4)^(3/2)
```

mupad [B] time = 0.04, size = 23, normalized size = 0.74

$$-\sqrt{4-x^2}\,\left(-\frac{x^4}{5}+\frac{4\,x^2}{15}+\frac{32}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4 - x^2)^(1/2),x)
[Out] -(4 - x^2)^(1/2)*((4*x^2)/15 - x^4/5 + 32/15)
```

sympy [A] time = 0.64, size = 39, normalized size = 1.26

$$\frac{x^4\sqrt{4-x^2}}{5}-\frac{4x^2\sqrt{4-x^2}}{15}-\frac{32\sqrt{4-x^2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+4)**(1/2),x)
[Out] x**4*sqrt(4 - x**2)/5 - 4*x**2*sqrt(4 - x**2)/15 - 32*sqrt(4 - x**2)/15
```

3.126 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

fricas [A] time = 0.40, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

giac [A] time = 1.04, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.31

$$\frac{(x-1)(x+1)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)*x,x)
[Out] (x-1)*(x+1)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.44, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.85

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^2)^(1/2),x)
[Out] -(1 - x^2)^(1/2)
```

sympy [A] time = 0.15, size = 8, normalized size = 0.62

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2)
```

3.127 $\int x\sqrt{4-x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] -1/3*(-x^2+4)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[4 - x^2],x]
```

```
[Out] -(4 - x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{4-x^2}\,dx=-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[4 - x^2],x]
```

```
[Out] -1/3*(4 - x^2)^(3/2)
```

fricas [A] time = 0.41, size = 16, normalized size = 1.07

$$\frac{1}{3}\left(x^2-4\right)\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 - 4)*sqrt(-x^2 + 4)
```

giac [A] time = 1.02, size = 11, normalized size = 0.73

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="giac")
[Out] -1/3*(-x^2 + 4)^(3/2)
```

maple [**A**] time = 0.00, size = 18, normalized size = 1.20

$$\frac{(x-2)(x+2)\sqrt{-x^2+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+4)^(1/2),x)
[Out] 1/3*(x-2)*(x+2)*(-x^2+4)^(1/2)
```

maxima [**A**] time = 0.42, size = 11, normalized size = 0.73

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/3*(-x^2 + 4)^(3/2)
```

mupad [**B**] time = 0.03, size = 11, normalized size = 0.73

$$-\frac{\left(4-x^2\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4 - x^2)^(1/2),x)
[Out] -(4 - x^2)^(3/2)/3
```

sympy [**B**] time = 0.21, size = 24, normalized size = 1.60

$$\frac{x^2\sqrt{4-x^2}}{3}-\frac{4\sqrt{4-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+4)**(1/2),x)
[Out] x**2*sqrt(4 - x**2)/3 - 4*sqrt(4 - x**2)/3
```

3.128 $\int \sqrt{1-4x^2}\, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sqrt{1-4x^2}\,x+\frac{1}{4}\sin^{-1}(2x)$$

```
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-4x^2}\,x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-4x^2}\, dx &= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-4x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\sqrt{1-4x^2}\,x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

fricas [A] time = 0.39, size = 32, normalized size = 1.28

$$\frac{1}{2}\sqrt{-4\,x^2+1}\,x-\frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2+1}-1}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-4*x^2 + 1)*x - 1/2*arctan(1/2*(sqrt(-4*x^2 + 1) - 1)/x)
```

giac [A] time = 0.94, size = 19, normalized size = 0.76

$$\frac{1}{2}\sqrt{-4x^2+1}\,x+\frac{1}{4}\arcsin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\sqrt{-4x^2+1}\,x}{2}+\frac{\arcsin\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2),x)
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

maxima [A] time = 0.98, size = 19, normalized size = 0.76

$$\frac{1}{2}\sqrt{-4x^2+1}\,x+\frac{1}{4}\arcsin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.72

$$\frac{\mathrm{asin}\left(2x\right)}{4}+x\sqrt{\frac{1}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 4*x^2)^(1/2),x)
[Out] asin(2*x)/4 + x*(1/4 - x^2)^(1/2)
```

sympy [A] time = 0.22, size = 19, normalized size = 0.76

$$\frac{x\sqrt{1-4x^2}}{2}+\frac{\mathrm{asin}\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2),x)
[Out] x*sqrt(1 - 4*x**2)/2 + asin(2*x)/4
```

3.129 $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

[Out] 1/3*(x^2+4)^(3/2)-4*(x^2+4)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[4 + x^2],x]

[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{4+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{4+x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{4}{\sqrt{4+x}}+\sqrt{4+x}\right)dx,x,x^2\right)\\ &= -4\sqrt{4+x^2}+\frac{1}{3}\left(4+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.72

$$\frac{1}{3}\left(x^2-8\right)\sqrt{x^2+4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[4 + x^2],x]

[Out] ((-8 + x^2)*Sqrt[4 + x^2])/3

fricas [A] time = 0.41, size = 14, normalized size = 0.56

$$\frac{1}{3}\sqrt{x^2+4}\left(x^2-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(x^2 + 4)*(x^2 - 8)
```

giac [A] time = 0.92, size = 19, normalized size = 0.76

$$\frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}} - 4\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 4)^(3/2) - 4*sqrt(x^2 + 4)
```

maple [A] time = 0.00, size = 15, normalized size = 0.60

$$\frac{\sqrt{x^2+4}\,\left(x^2-8\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+4)^(1/2),x)
[Out] 1/3*(x^2+4)^(1/2)*(x^2-8)
```

maxima [A] time = 0.97, size = 22, normalized size = 0.88

$$\frac{1}{3}\sqrt{x^2+4}\,x^2 - \frac{8}{3}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^2 + 4)*x^2 - 8/3*sqrt(x^2 + 4)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.56

$$\frac{\sqrt{x^2+4}\,\left(x^2-8\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 + 4)^(1/2),x)
[Out] ((x^2 + 4)^(1/2)*(x^2 - 8))/3
```

sympy [A] time = 0.37, size = 24, normalized size = 0.96

$$\frac{x^2\sqrt{x^2+4}}{3} - \frac{8\sqrt{x^2+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+4)**(1/2),x)
[Out] x**2*sqrt(x**2 + 4)/3 - 8*sqrt(x**2 + 4)/3
```

3.130 $\int \frac{1}{\sqrt{9+x^2}}\,dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] arcsinh(1/3*x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{3}\right)$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

fricas [B] time = 0.41, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 9))
```

giac [B] time = 1.10, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 9))
```

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\operatorname{arcsinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+9)^(1/2),x)
[Out] arcsinh(1/3*x)
```

maxima [A] time = 0.96, size = 4, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*x)
```

mupad [B] time = 0.03, size = 4, normalized size = 0.67

$$\operatorname{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 9)^(1/2),x)
[Out] asinh(x/3)
```

sympy [A] time = 0.15, size = 3, normalized size = 0.50

$$\operatorname{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+9)**(1/2),x)
[Out] asinh(x/3)
```

3.131 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\,dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

fricas [A] time = 0.41, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.93, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{\sqrt{x^2+1}\;x}{2}+\frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x)
[Out] 1/2*arcsinh(x)+1/2*(x^2+1)^(1/2)*x
```

maxima [A] time = 0.95, size = 15, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{\operatorname{asinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2),x)
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

sympy [A] time = 0.21, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

3.132 $\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

```
[Out] 1/128*arctan(1/4*(x^2-16)^(1/2))+1/32*(x^2-16)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-16+x}\,x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{64}\,\text{Subst}\left(\int \frac{1}{\sqrt{-16+x}\,x}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{32}\,\text{Subst}\left(\int \frac{1}{16+x^2}\,dx,x,\sqrt{-16+x^2}\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{1}{4}\sqrt{-16+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.31

$$\frac{1}{256}\sqrt{x^2-16}\left(\frac{8}{x^2}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{x^2}{16}}\right)}{\sqrt{16-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] (Sqrt[-16 + x^2]*(8/x^2 + (2*ArcTanh[Sqrt[1 - x^2/16]])/Sqrt[16 - x^2]))/25
6
```

fricas [A] time = 0.42, size = 33, normalized size = 0.94

$$\frac{x^2\arctan\left(-\frac{1}{4}\,x+\frac{1}{4}\,\sqrt{x^2-16}\right)+2\,\sqrt{x^2-16}}{64\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(x^2*arctan(-1/4*x + 1/4*sqrt(x^2 - 16)) + 2*sqrt(x^2 - 16))/x^2
```

giac [A] time = 0.95, size = 25, normalized size = 0.71

$$\frac{\sqrt{x^2-16}}{32\,x^2}+\frac{1}{128}\,\arctan\left(\frac{1}{4}\,\sqrt{x^2-16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 + 1/128*arctan(1/4*sqrt(x^2 - 16))
```

maple [A] time = 0.01, size = 26, normalized size = 0.74

$$-\frac{\arctan\left(\frac{4}{\sqrt{x^2-16}}\right)}{128}+\frac{\sqrt{x^2-16}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2-16)^(1/2),x)
```

```
[Out] 1/32*(x^2-16)^(1/2)/x^2-1/128*arctan(4/(x^2-16)^(1/2))
```

maxima [A] time = 0.96, size = 22, normalized size = 0.63

$$\frac{\sqrt{x^2-16}}{32\,x^2}-\frac{1}{128}\,\arcsin\left(\frac{4}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 - 1/128*arcsin(4/abs(x))
```

mupad [B] time = 0.35, size = 25, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x^2-16}}{4}\right)}{128}+\frac{\sqrt{x^2-16}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^2 - 16)^(1/2)),x)
```

```
[Out] atan((x^2 - 16)^(1/2)/4)/128 + (x^2 - 16)^(1/2)/(32*x^2)
```

sympy [A] time = 2.19, size = 66, normalized size = 1.89

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{4}{x}\right)}{128}+\frac{i\sqrt{-1+\frac{16}{x^2}}}{32x} & \text{for}\ \frac{16}{|x^2|}>1 \\ -\frac{\operatorname{asin}\left(\frac{4}{x}\right)}{128}+\frac{1}{32x\sqrt{1-\frac{16}{x^2}}}-\frac{1}{2x^3\sqrt{1-\frac{16}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2-16)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(4/x)/128 + I*sqrt(-1 + 16/x**2)/(32*x), 16/Abs(x**2) > 1
), (-asin(4/x)/128 + 1/(32*x*sqrt(1 - 16/x**2)) - 1/(2*x**3*sqrt(1 - 16/x**
2)), True))
```

3.133 $\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

```
[Out] 1/3*(-a^2+x^2)^(3/2)/a^2/x^3
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {264}

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx=\frac{\left(-a^2+x^2\right)^{3/2}}{3a^2x^3}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

fricas [A] time = 0.40, size = 23, normalized size = 1.00

$$\frac{x^3+\left(-a^2+x^2\right)^{\frac{3}{2}}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + (-a^2 + x^2)^(3/2))/(a^2*x^3)
```

giac [B] time = 0.98, size = 48, normalized size = 2.09

$$\frac{2\left(a^4 + 3\left(x - \sqrt{-a^2 + x^2}\right)^4\right)}{3\left(a^2 + \left(x - \sqrt{-a^2 + x^2}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(a^4 + 3*(x - sqrt(-a^2 + x^2))^4)/(a^2 + (x - sqrt(-a^2 + x^2))^2)^3
```

maple [A] time = 0.01, size = 28, normalized size = 1.22

$$-\frac{(a+x)(a-x)\sqrt{-a^2+x^2}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2+x^2)^(1/2)/x^4,x)
```

```
[Out] -1/3/x^3*(a+x)*(a-x)/a^2*(-a^2+x^2)^(1/2)
```

maxima [A] time = 0.97, size = 19, normalized size = 0.83

$$\frac{\left(-a^2+x^2\right)^{\frac{3}{2}}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(-a^2 + x^2)^(3/2)/(a^2*x^3)
```

mupad [B] time = 0.34, size = 19, normalized size = 0.83

$$\frac{\left(x^2-a^2\right)^{3/2}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - a^2)^(1/2)/x^4,x)
```

```
[Out] (x^2 - a^2)^(3/2)/(3*a^2*x^3)
```

sympy [A] time = 0.76, size = 76, normalized size = 3.30

$$\begin{cases} -\frac{i\sqrt{\frac{a^2}{x^2}-1}}{3x^2} + \frac{i\sqrt{\frac{a^2}{x^2}-1}}{3a^2} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\sqrt{-\frac{a^2}{x^2}+1}}{3x^2} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{3a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2+x**2)**(1/2)/x**4,x)
```

```
[Out] Piecewise((-I*sqrt(a**2/x**2 - 1)/(3*x**2) + I*sqrt(a**2/x**2 - 1)/(3*a**2)
, Abs(a**2/x**2) > 1), (-sqrt(-a**2/x**2 + 1)/(3*x**2) + sqrt(-a**2/x**2 +
1)/(3*a**2), True))
```

3.134 $\int \frac{\sqrt{-4+9x^2}}{x} dx$

Optimal. Leaf size=30

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

[Out] -2*arctan(1/2*(9*x^2-4)^(1/2))+(9*x^2-4)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 203}

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-4 + 9*x^2]/x,x]

[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-4+9x^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-4+9x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{-4+9x^2} - 2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-4+9x}}\,dx, x, x^2\right)\\
&= \sqrt{-4+9x^2} - \frac{4}{9}\operatorname{Subst}\left(\int \frac{1}{\frac{4}{9}+\frac{x^2}{9}}\,dx, x, \sqrt{-4+9x^2}\right)\\
&= \sqrt{-4+9x^2} - 2\tan^{-1}\left(\frac{1}{2}\sqrt{-4+9x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\sqrt{9x^2-4} - 2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 + 9*x^2]/x,x]
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

fricas [A] time = 0.41, size = 28, normalized size = 0.93

$$\sqrt{9\,x^2-4} - 4\,\arctan\left(-\frac{3}{2}\,x + \frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(9*x^2 - 4) - 4*arctan(-3/2*x + 1/2*sqrt(9*x^2 - 4))
```

giac [A] time = 0.93, size = 24, normalized size = 0.80

$$\sqrt{9\,x^2-4} - 2\,\arctan\left(\frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(9*x^2 - 4) - 2*arctan(1/2*sqrt(9*x^2 - 4))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$2\arctan\left(\frac{2}{\sqrt{9x^2-4}}\right) + \sqrt{9x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-4)^(1/2)/x,x)
[Out] (9*x^2-4)^(1/2)+2*arctan(2/(9*x^2-4)^(1/2))
```

maxima [A] time = 0.96, size = 19, normalized size = 0.63

$$\sqrt{9\,x^2-4} + 2\,\arcsin\left(\frac{2}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(9*x^2 - 4) + 2*arcsin(2/3/abs(x))
```

mupad [B] time = 0.31, size = 24, normalized size = 0.80

$$\sqrt{9\,x^2-4}-2\,\mathrm{atan}\left(\frac{\sqrt{9\,x^2-4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2 - 4)^(1/2)/x,x)
```

```
[Out] (9*x^2 - 4)^(1/2) - 2*atan((9*x^2 - 4)^(1/2)/2)
```

sympy [A] time = 1.40, size = 92, normalized size = 3.07

$$\begin{cases} -\frac{3ix}{\sqrt{-1+\frac{4}{9x^2}}}-2i\,\mathrm{acosh}\left(\frac{2}{3x}\right)+\frac{4i}{3x\sqrt{-1+\frac{4}{9x^2}}} & \text{for}\ \frac{4}{9|x^2|}>1 \\ \frac{3x}{\sqrt{1-\frac{4}{9x^2}}}+2\,\mathrm{asin}\left(\frac{2}{3x}\right)-\frac{4}{3x\sqrt{1-\frac{4}{9x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-4)**(1/2)/x,x)
```

```
[Out] Piecewise((-3*I*x/sqrt(-1 + 4/(9*x**2)) - 2*I*acosh(2/(3*x)) + 4*I/(3*x*sqr
t(-1 + 4/(9*x**2))), 4/(9*Abs(x**2)) > 1), (3*x/sqrt(1 - 4/(9*x**2)) + 2*as
in(2/(3*x)) - 4/(3*x*sqrt(1 - 4/(9*x**2))), True))
```

3.135 $\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{16x^2-9}}{9x}$$

```
[Out] 1/9*(16*x^2-9)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{-9+16x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

fricas [A] time = 0.40, size = 18, normalized size = 1.00

$$\frac{4\,x+\sqrt{16\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(4*x + sqrt(16*x^2 - 9))/x
```

giac [A] time = 0.99, size = 23, normalized size = 1.28

$$\frac{8}{\left(4\,x-\sqrt{16\,x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 8/((4*x - sqrt(16*x^2 - 9))^2 + 9)
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(4x-3)(4x+3)}{9\sqrt{16x^2-9}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(16*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(4*x-3)*(4*x+3)/(16*x^2-9)^(1/2)
```

maxima [A] time = 0.98, size = 14, normalized size = 0.78

$$\frac{\sqrt{16\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(16*x^2 - 9)/x
```

mupad [B] time = 0.25, size = 14, normalized size = 0.78

$$\frac{\sqrt{16\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(16*x^2 - 9)^(1/2)),x)
```

```
[Out] (16*x^2 - 9)^(1/2)/(9*x)
```

sympy [A] time = 0.82, size = 37, normalized size = 2.06

$$\begin{cases} \frac{4i\sqrt{-1+\frac{9}{16x^2}}}{9} & \text{for}\ \frac{9}{16|x^2|} > 1 \\ \frac{4\sqrt{1-\frac{9}{16x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(16*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 9/(16*x**2))/9, 9/(16*Abs(x**2)) > 1), (4*sqrt(1 -
 9/(16*x**2))/9, True))
```

3.136 $\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{a^2-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] -arctan(x/(a^2-x^2)^(1/2))+x/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {288, 217, 203}

$$\frac{x}{\sqrt{a^2-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx &= \frac{x}{\sqrt{a^2-x^2}}-\int \frac{1}{\sqrt{a^2-x^2}}\,dx\\
&= \frac{x}{\sqrt{a^2-x^2}}-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{a^2-x^2}}\right)\\
&= \frac{x}{\sqrt{a^2-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.15

$$\frac{x-a\sqrt{1-\frac{x^2}{a^2}}\,\sin^{-1}\left(\frac{x}{a}\right)}{\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 - x^2)^(3/2),x]
[Out] (x - a*Sqrt[1 - x^2/a^2]*ArcSin[x/a])/Sqrt[a^2 - x^2]
```

fricas [A] time = 0.41, size = 58, normalized size = 1.71

$$\frac{2\left(a^2-x^2\right)\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\sqrt{a^2-x^2}\,x}{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="fricas")
[Out] (2*(a^2 - x^2)*arctan(-(a - sqrt(a^2 - x^2))/x) + sqrt(a^2 - x^2)*x)/(a^2 -
 x^2)
```

giac [A] time = 1.06, size = 24, normalized size = 0.71

$$-\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)+\frac{x}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="giac")
[Out] -arcsin(x/a)*sgn(a) + x/sqrt(a^2 - x^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{x}{\sqrt{a^2-x^2}}-\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2-x^2)^(3/2),x)
[Out] -arctan(1/(a^2-x^2)^(1/2)*x)+1/(a^2-x^2)^(1/2)*x
```

maxima [A] time = 0.98, size = 22, normalized size = 0.65

$$\frac{x}{\sqrt{a^2-x^2}}-\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(a^2 - x^2) - arcsin(x/a)
```

mupad [B] time = 0.22, size = 34, normalized size = 1.00

$$\frac{x}{\sqrt{a^2-x^2}}+\ln\left(\sqrt{a^2-x^2}+x\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 - x^2)^(3/2),x)
[Out] log(x*1i + (a^2 - x^2)^(1/2))*1i + x/(a^2 - x^2)^(1/2)
```

sympy [A] time = 1.68, size = 49, normalized size = 1.44

$$\begin{cases} i\operatorname{acosh}\left(\frac{x}{a}\right) - \frac{ix}{a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ -\operatorname{asin}\left(\frac{x}{a}\right) + \frac{x}{a\sqrt{1-\frac{x^2}{a^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2-x**2)**(3/2),x)
```

```
[Out] Piecewise((I*acosh(x/a) - I*x/(a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1)
, (-asin(x/a) + x/(a*sqrt(1 - x**2/a**2)), True))
```

3.137 $\int \frac{x^2}{\sqrt{5-x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

```
[Out] 5/2*arcsin(1/5*x*5^(1/2))-1/2*x*(-x^2+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[5 - x^2],x]
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{5-x^2}}\,dx &= -\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\int \frac{1}{\sqrt{5-x^2}}\,dx\\ &= -\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[5 - x^2],x]
[Out] -1/2*(x*Sqrt[5 - x^2]) + (5*ArcSin[x/Sqrt[5]])/2
```

fricas [A] time = 0.42, size = 29, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-x^2+5}\,x-\frac{5}{2}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x - 5/2*arctan(sqrt(-x^2 + 5)/x)
```

giac [A] time = 0.89, size = 22, normalized size = 0.76

$$-\frac{1}{2}\sqrt{-x^2+5}\,x+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

maple [A] time = 0.01, size = 23, normalized size = 0.79

$$-\frac{\sqrt{-x^2+5}\,x}{2}+\frac{5\arcsin\left(\frac{\sqrt{5}\,x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+5)^(1/2),x)
```

```
[Out] 5/2*arcsin(1/5*5^(1/2)*x)-1/2*x*(-x^2+5)^(1/2)
```

maxima [A] time = 0.98, size = 22, normalized size = 0.76

$$-\frac{1}{2}\sqrt{-x^2+5}\,x+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.76

$$\frac{5\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}-\frac{x\,\sqrt{5-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(5 - x^2)^(1/2),x)
```

```
[Out] (5*asin((5^(1/2)*x)/5))/2 - (x*(5 - x^2)^(1/2))/2
```

sympy [A] time = 0.23, size = 24, normalized size = 0.83

$$-\frac{x\sqrt{5-x^2}}{2}+\frac{5\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(5 - x**2)/2 + 5*asin(sqrt(5)*x/5)/2
```

3.138 $\int \frac{1}{x\sqrt{3+x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctanh(1/3*(x^2+3)^(1/2)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{3+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+x}}\,dx, x, x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \sqrt{3+x^2}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[3 + x^2]),x]
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

fricas [A] time = 0.40, size = 24, normalized size = 1.04

$$\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 3))/x)
```

giac [B] time = 0.91, size = 37, normalized size = 1.61

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}+\sqrt{x^2+3}\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(sqrt(3) + sqrt(x^2 + 3)) + 1/6*sqrt(3)*log(-sqrt(3) + sqrt
(x^2 + 3))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{x^2+3}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3)^(1/2),x)
[Out] -1/3*3^(1/2)*arctanh(3^(1/2)/(x^2+3)^(1/2))
```

maxima [A] time = 0.96, size = 14, normalized size = 0.61

$$-\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{\sqrt{3}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsinh(sqrt(3)/abs(x))
```

mupad [B] time = 0.06, size = 18, normalized size = 0.78

$$-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,\sqrt{x^2+3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 3)^(1/2)),x)
```

```
[Out] -(3^(1/2)*atanh((3^(1/2)*(x^2 + 3)^(1/2))/3))/3
```

sympy [**A**] time = 1.06, size = 15, normalized size = 0.65

$$-\frac{\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{3}}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*asinh(sqrt(3)/x)/3
```

3.139 $\int \frac{x}{(4+x^2)^{5/2}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

[Out] -1/3/(x^2+4)^(3/2)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(4 + x^2)^(5/2),x]

[Out] -1/(3*(4 + x^2)^(3/2))

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(4+x^2\right)^{5/2}}\,dx = -\frac{1}{3\left(4+x^2\right)^{3/2}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(4 + x^2)^(5/2),x]

[Out] -1/3*1/(4 + x^2)^(3/2)

fricas [B] time = 0.40, size = 21, normalized size = 1.62

$$-\frac{\sqrt{x^2+4}}{3\left(x^4+8\,x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+4)^(5/2),x, algorithm="fricas")

[Out] -1/3*sqrt(x^2 + 4)/(x^4 + 8*x^2 + 16)

giac [A] time = 0.98, size = 9, normalized size = 0.69

$$-\frac{1}{3\left(x^{2}+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{3\left(x^{2}+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4)^(5/2),x)
```

```
[Out] -1/3/(x^2+4)^(3/2)
```

maxima [A] time = 0.43, size = 9, normalized size = 0.69

$$-\frac{1}{3\left(x^{2}+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

mupad [B] time = 0.16, size = 9, normalized size = 0.69

$$-\frac{1}{3\left(x^{2}+4\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + 4)^(5/2),x)
```

```
[Out] -1/(3*(x^2 + 4)^(3/2))
```

sympy [B] time = 2.24, size = 26, normalized size = 2.00

$$-\frac{1}{3x^{2}\sqrt{x^{2}+4}+12\sqrt{x^{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4)**(5/2),x)
```

```
[Out] -1/(3*x**2*sqrt(x**2 + 4) + 12*sqrt(x**2 + 4))
```

3.140 $\int x^3\sqrt{4-9x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

```
[Out] -4/243*(-9*x^2+4)^(3/2)+1/405*(-9*x^2+4)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{4-9x^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int\sqrt{4-9x}\,x\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{4}{9}\sqrt{4-9x}-\frac{1}{9}(4-9x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{4}{243}\left(4-9x^2\right)^{3/2}+\frac{1}{405}\left(4-9x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{\left(4-9x^2\right)^{3/2}\left(27x^2+8\right)}{1215}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] -1/1215*((4 - 9*x^2)^(3/2)*(8 + 27*x^2))
```

fricas [A] time = 0.40, size = 23, normalized size = 0.74

$$\frac{1}{1215}\left(243\,x^4-36\,x^2-32\right)\sqrt{-9\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1215*(243*x^4 - 36*x^2 - 32)*sqrt(-9*x^2 + 4)
```

giac [A] time = 1.10, size = 32, normalized size = 1.03

$$\frac{1}{405}\left(9\,x^2-4\right)^2\sqrt{-9\,x^2+4}-\frac{4}{243}\left(-9\,x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(9*x^2 - 4)^2*sqrt(-9*x^2 + 4) - 4/243*(-9*x^2 + 4)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(3x-2)\,(3x+2)\left(27x^2+8\right)\sqrt{-9x^2+4}}{1215}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/1215*(3*x-2)*(3*x+2)*(27*x^2+8)*(-9*x^2+4)^(1/2)
```

maxima [A] time = 0.96, size = 26, normalized size = 0.84

$$-\frac{1}{45}\left(-9\,x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{1215}\left(-9\,x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/45*(-9*x^2 + 4)^(3/2)*x^2 - 8/1215*(-9*x^2 + 4)^(3/2)
```

mupad [B] time = 0.17, size = 23, normalized size = 0.74

$$-\frac{\sqrt{\frac{4}{9}-x^2}\,\left(-\frac{9\,x^4}{5}+\frac{4\,x^2}{15}+\frac{32}{135}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4 - 9*x^2)^(1/2),x)
```

```
[Out] -((4/9 - x^2)^(1/2)*((4*x^2)/15 - (9*x^4)/5 + 32/135))/3
```

sympy [A] time = 0.66, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4-9x^2}}{5}-\frac{4x^2\sqrt{4-9x^2}}{135}-\frac{32\sqrt{4-9x^2}}{1215}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-9*x**2+4)**(1/2),x)
```

```
[Out] x**4*sqrt(4 - 9*x**2)/5 - 4*x**2*sqrt(4 - 9*x**2)/135 - 32*sqrt(4 - 9*x**2)
/1215
```

3.141 $\int x^2\sqrt{9-x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{8}\sqrt{9-x^2}\,x+\frac{1}{4}\sqrt{9-x^2}\,x^3+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] 81/8*arcsin(1/3*x)-9/8*x*(-x^2+9)^(1/2)+1/4*x^3*(-x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{9-x^2}\,x^3-\frac{9}{8}\sqrt{9-x^2}\,x+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 - x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9-x^2}\,dx &= \frac{1}{4}x^3\sqrt{9-x^2}+\frac{9}{4}\int\frac{x^2}{\sqrt{9-x^2}}\,dx\\ &= -\frac{9}{8}x\sqrt{9-x^2}+\frac{1}{4}x^3\sqrt{9-x^2}+\frac{81}{8}\int\frac{1}{\sqrt{9-x^2}}\,dx\\ &= -\frac{9}{8}x\sqrt{9-x^2}+\frac{1}{4}x^3\sqrt{9-x^2}+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.73

$$\frac{1}{8}\left(x\sqrt{9-x^2}\left(2x^2-9\right)+81\sin^{-1}\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - x^2],x]
[Out] (x*Sqrt[9 - x^2]*(-9 + 2*x^2) + 81*ArcSin[x/3])/8
```

fricas [A] time = 0.41, size = 39, normalized size = 0.87

$$\frac{1}{8}\left(2\,x^3-9\,x\right)\sqrt{-x^2+9}-\frac{81}{4}\,\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*x^3 - 9*x)*sqrt(-x^2 + 9) - 81/4*arctan((sqrt(-x^2 + 9) - 3)/x)
```

giac [A] time = 1.02, size = 26, normalized size = 0.58

$$\frac{1}{8}\left(2\,x^2-9\right)\sqrt{-x^2+9}\,x+\frac{81}{8}\,\arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/8*(2*x^2 - 9)*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

maple [A] time = 0.01, size = 32, normalized size = 0.71

$$-\frac{\left(-x^2+9\right)^{\frac{3}{2}}x}{4}+\frac{9\sqrt{-x^2+9}\,x}{8}+\frac{81\arcsin\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+9)^(1/2),x)
[Out] -1/4*x*(-x^2+9)^(3/2)+9/8*(-x^2+9)^(1/2)*x+81/8*arcsin(1/3*x)
```

maxima [A] time = 0.98, size = 31, normalized size = 0.69

$$-\frac{1}{4}\left(-x^2+9\right)^{\frac{3}{2}}x+\frac{9}{8}\sqrt{-x^2+9}\,x+\frac{81}{8}\,\arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/4*(-x^2 + 9)^(3/2)*x + 9/8*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

mupad [B] time = 0.04, size = 27, normalized size = 0.60

$$\frac{81\,\mathrm{asin}\left(\frac{x}{3}\right)}{8}-\sqrt{9-x^2}\,\left(\frac{9\,x}{8}-\frac{x^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(9 - x^2)^(1/2),x)
[Out] (81*asin(x/3))/8 - (9 - x^2)^(1/2)*((9*x)/8 - x^3/4)
```

sympy [A] time = 2.82, size = 112, normalized size = 2.49

$$\begin{cases} \frac{ix^5}{4\sqrt{x^2-9}}-\frac{27ix^3}{8\sqrt{x^2-9}}+\frac{81ix}{8\sqrt{x^2-9}}-\frac{81i\,\mathrm{acosh}\left(\frac{x}{3}\right)}{8} & \text{for}\ \frac{\left|x^2\right|}{9}>1 \\ -\frac{x^5}{4\sqrt{9-x^2}}+\frac{27x^3}{8\sqrt{9-x^2}}-\frac{81x}{8\sqrt{9-x^2}}+\frac{81\,\mathrm{asin}\left(\frac{x}{3}\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 9)) - 27*I*x**3/(8*sqrt(x**2 - 9)) + 81*I*
x/(8*sqrt(x**2 - 9)) - 81*I*acosh(x/3)/8, Abs(x**2)/9 > 1), (-x**5/(4*sqrt(
9 - x**2)) + 27*x**3/(8*sqrt(9 - x**2)) - 81*x/(8*sqrt(9 - x**2)) + 81*asin
(x/3)/8, True))
```

3.142 $\int 5x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] 5/3*(x^2+1)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 261}

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int 5x\sqrt{1+x^2}\,dx &= 5\int x\sqrt{1+x^2}\,dx\\ &= \frac{5}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

fricas [A] time = 0.40, size = 9, normalized size = 0.69

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

giac [**A**] time = 0.99, size = 9, normalized size = 0.69

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 5/3*(x^2 + 1)^(3/2)
```

maple [**A**] time = 0.00, size = 10, normalized size = 0.77

$$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*(x^2+1)^(1/2)*x,x)
[Out] 5/3*(x^2+1)^(3/2)
```

maxima [**A**] time = 0.43, size = 9, normalized size = 0.69

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 5/3*(x^2 + 1)^(3/2)
```

mupad [**B**] time = 0.03, size = 9, normalized size = 0.69

$$\frac{5\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x*(x^2 + 1)^(1/2),x)
[Out] (5*(x^2 + 1)^(3/2))/3
```

sympy [**B**] time = 0.20, size = 26, normalized size = 2.00

$$\frac{5x^2\sqrt{x^2+1}}{3}+\frac{5\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x**2+1)**(1/2),x)
[Out] 5*x**2*sqrt(x**2 + 1)/3 + 5*sqrt(x**2 + 1)/3
```

3.143 $\int \frac{1}{(-25+4x^2)^{3/2}} dx$

Optimal. Leaf size=16

$$-\frac{x}{25\sqrt{4x^2-25}}$$

[Out] -1/25*x/(4*x^2-25)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

[In] Int[(-25 + 4*x^2)^(-3/2),x]

[Out] -x/(25*Sqrt[-25 + 4*x^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(-25+4x^2)^{3/2}} dx = -\frac{x}{25\sqrt{-25+4x^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

[In] Integrate[(-25 + 4*x^2)^(-3/2),x]

[Out] -1/25*x/Sqrt[-25 + 4*x^2]

fricas [B] time = 0.40, size = 30, normalized size = 1.88

$$-\frac{4x^2+2\sqrt{4x^2-25}\,x-25}{50\left(4x^2-25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="fricas")

[Out] -1/50*(4*x^2 + 2*sqrt(4*x^2 - 25)*x - 25)/(4*x^2 - 25)

giac [A] time = 1.04, size = 12, normalized size = 0.75

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="giac")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

maple [A] time = 0.00, size = 23, normalized size = 1.44

$$-\frac{(2x-5)(2x+5)x}{25\left(4x^2-25\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-25)^(3/2),x)
```

```
[Out] -1/25*(2*x-5)*(2*x+5)*x/(4*x^2-25)^(3/2)
```

maxima [A] time = 0.43, size = 12, normalized size = 0.75

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

mupad [B] time = 0.25, size = 12, normalized size = 0.75

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 25)^(3/2),x)
```

```
[Out] -x/(25*(4*x^2 - 25)^(1/2))
```

sympy [A] time = 0.78, size = 36, normalized size = 2.25

$$\begin{cases} -\frac{x}{25\sqrt{4x^2-25}} & \text{for}\ \frac{4|x^2|}{25} > 1 \\ \frac{ix}{25\sqrt{25-4x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-25)**(3/2),x)
```

```
[Out] Piecewise((-x/(25*sqrt(4*x**2 - 25)), 4*Abs(x**2)/25 > 1), (I*x/(25*sqrt(25
 - 4*x**2)), True))
```

3.144 $\int \sqrt{2x-x^2}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{2x-x^2}\,(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

```
[Out] 1/2*arcsin(-1+x)-1/2*(1-x)*(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{2}\sqrt{2x-x^2}\,(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x - x^2],x]
```

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - ArcSin[1 - x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2x-x^2}\,dx &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2x-x^2}}\,dx\\ &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,2-2x\right)\\ &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{2}\sin^{-1}(1-x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-1)\sqrt{-((x-2)x)}-\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x - x^2],x]
[Out] ((-1 + x)*Sqrt[-((-2 + x)*x)])/2 - ArcSin[Sqrt[1 - x/2]]
```

fricas [A] time = 0.41, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+2x}\,(x-1)-\arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) - arctan(sqrt(-x^2 + 2*x)/x)
```

giac [A] time = 0.89, size = 23, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+2x}\,(x-1)+\frac{1}{2}\arcsin(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) + 1/2*arcsin(x - 1)
```

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{\arcsin(x-1)}{2}-\frac{(-2x+2)\sqrt{-x^2+2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x)^(1/2),x)
[Out] -1/4*(-2*x+2)*(-x^2+2*x)^(1/2)+1/2*arcsin(x-1)
```

maxima [A] time = 0.97, size = 36, normalized size = 1.09

$$\frac{1}{2}\sqrt{-x^2+2x}\,x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\arcsin(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*arcsin(-x + 1)
```

mupad [B] time = 0.17, size = 24, normalized size = 0.73

$$\frac{\mathrm{asin}(x-1)}{2}+\left(\frac{x}{2}-\frac{1}{2}\right)\sqrt{2x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2)^(1/2),x)
[Out] asin(x - 1)/2 + (x/2 - 1/2)*(2*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x)**(1/2),x)
[Out] Integral(sqrt(-x**2 + 2*x), x)
```

3.145 $\int \frac{1}{\sqrt{8+4x+x^2}}\,dx$

Optimal. Leaf size=8

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

```
[Out] arcsinh(1+1/2*x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 215}

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{8+4x+x^2}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 4+2x\right)\\ &= \sinh^{-1}\left(\frac{2+x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

fricas [B] time = 0.40, size = 18, normalized size = 2.25

$$-\log\left(-x+\sqrt{x^2+4x+8}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

giac [B] time = 1.18, size = 18, normalized size = 2.25

$$-\log\left(-x+\sqrt{x^2+4\,x+8}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="giac")
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\operatorname{arcsinh}\left(\frac{x}{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x+8)^(1/2),x)
[Out] arcsinh(1/2*x+1)
```

maxima [A] time = 0.96, size = 6, normalized size = 0.75

$$\operatorname{arsinh}\left(\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/2*x + 1)
```

mupad [B] time = 0.20, size = 14, normalized size = 1.75

$$\ln\left(x+\sqrt{x^2+4\,x+8}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^2 + 8)^(1/2),x)
[Out] log(x + (4*x + x^2 + 8)^(1/2) + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+4x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x+8)**(1/2),x)
[Out] Integral(1/sqrt(x**2 + 4*x + 8), x)
```

3.146 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

[Out] 1/3*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]

[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]

[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3

fricas [A] time = 0.41, size = 20, normalized size = 0.80

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

giac [A] time = 1.10, size = 21, normalized size = 0.84

$$-\frac{1}{3}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

maple [A] time = 0.00, size = 30, normalized size = 1.20

$$\frac{\sqrt{9}\ \ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x)
[Out] 1/9*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

maxima [A] time = 0.97, size = 22, normalized size = 0.88

$$\frac{1}{3}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

mupad [B] time = 0.26, size = 20, normalized size = 0.80

$$\frac{\ln\left(3x+\sqrt{9x^2+6x-8}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x + 9*x^2 - 8)^(1/2),x)
[Out] log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{9x^2+6x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

3.147 $\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{4x-x^2}\,x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] 6*arcsin(-1+1/2*x)-3*(-x^2+4*x)^(1/2)-1/2*x*(-x^2+4*x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}\,x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[4*x - x^2],x]
```

```
[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = -\frac{1}{2}x\sqrt{4x-x^2} + 3\int \frac{x}{\sqrt{4x-x^2}}\,dx$$
$$= -3\sqrt{4x-x^2} - \frac{1}{2}x\sqrt{4x-x^2} + 6\int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -3\sqrt{4x-x^2} - \frac{1}{2}x\sqrt{4x-x^2} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)$$
$$= -3\sqrt{4x-x^2} - \frac{1}{2}x\sqrt{4x-x^2} - 6\sin^{-1}\left(1-\frac{x}{2}\right)$$

Mathematica [A] time = 0.05, size = 47, normalized size = 1.07

$$\frac{1}{2}\left(-\sqrt{4-x}\,x^{3/2} - 6\sqrt{-((x-4)x)} - 24\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[4*x - x^2],x]
[Out] (-(Sqrt[4 - x]*x^(3/2)) - 6*Sqrt[-((-4 + x)*x)] - 24*ArcSin[Sqrt[1 - x/4]])
/2
```

fricas [A] time = 0.41, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2+4x}\,(x+6) - 12\arctan\left(\frac{\sqrt{-x^2+4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) - 12*arctan(sqrt(-x^2 + 4*x)/x)
```

giac [A] time = 1.00, size = 25, normalized size = 0.57

$$-\frac{1}{2}\sqrt{-x^2+4x}\,(x+6) + 6\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) + 6*arcsin(1/2*x - 1)
```

maple [A] time = 0.00, size = 37, normalized size = 0.84

$$-\frac{\sqrt{-x^2+4x}\,x}{2} + 6\arcsin\left(\frac{x}{2}-1\right) - 3\sqrt{-x^2+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+4*x)^(1/2),x)
[Out] 6*arcsin(-1+1/2*x)-3*(-x^2+4*x)^(1/2)-1/2*x*(-x^2+4*x)^(1/2)
```

maxima [A] time = 0.99, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2+4x}\,x - 3\sqrt{-x^2+4x} - 6\arcsin\left(-\frac{1}{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*x - 3*sqrt(-x^2 + 4*x) - 6*arcsin(-1/2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{4\,x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x - x^2)^(1/2),x)
```

```
[Out] int(x^2/(4*x - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x - 4)), x)
```

3.148 $\int \frac{1}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=26

$$\frac{x+1}{2(x^2+2x+2)}+\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] 1/2*(1+x)/(x^2+2*x+2)+1/2*arctan(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {614, 617, 204}

$$\frac{x+1}{2(x^2+2x+2)}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+2x+x^2)^2}\,dx &= \frac{1+x}{2(2+2x+x^2)}+\frac{1}{2}\int \frac{1}{2+2x+x^2}\,dx\\ &= \frac{1+x}{2(2+2x+x^2)}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2(2+2x+x^2)}+\frac{1}{2}\tan^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{1}{2}\left(\frac{x+1}{x^2+2x+2}+\tan^{-1}(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^2)^(-2),x]
[Out] ((1 + x)/(2 + 2*x + x^2) + ArcTan[1 + x])/2
```

fricas [A] time = 0.40, size = 28, normalized size = 1.08

$$\frac{(x^2 + 2x + 2)\arctan(x + 1) + x + 1}{2(x^2 + 2x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 1)/(x^2 + 2*x + 2)
```

giac [A] time = 0.97, size = 22, normalized size = 0.85

$$\frac{x + 1}{2(x^2 + 2x + 2)} + \frac{1}{2}\arctan(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{\arctan(x + 1)}{2} + \frac{2x + 2}{4x^2 + 8x + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+2)^2,x)
[Out] 1/4*(2*x+2)/(x^2+2*x+2)+1/2*arctan(x+1)
```

maxima [A] time = 1.10, size = 22, normalized size = 0.85

$$\frac{x + 1}{2(x^2 + 2x + 2)} + \frac{1}{2}\arctan(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

mupad [B] time = 0.19, size = 23, normalized size = 0.88

$$\frac{\operatorname{atan}(x + 1)}{2} + \frac{\frac{x}{2} + \frac{1}{2}}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + x^2 + 2)^2,x)
[Out] atan(x + 1)/2 + (x/2 + 1/2)/(2*x + x^2 + 2)
```

sympy [A] time = 0.13, size = 19, normalized size = 0.73

$$\frac{x + 1}{2x^2 + 4x + 4} + \frac{\operatorname{atan}(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+2)**2,x)
```

```
[Out] (x + 1)/(2*x**2 + 4*x + 4) + atan(x + 1)/2
```

3.149 $\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

```
[Out] 1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2}{27}\int \frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx\\ &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
[Out] -1/243*((2 + x)*(-19 + 8*x + 2*x^2))/(5 - 4*x - x^2)^(3/2)
```

fricas [A] time = 0.40, size = 49, normalized size = 1.14

$$-\frac{\left(2\,x^{3}+12\,x^{2}-3\,x-38\right)\sqrt{-x^{2}-4\,x+5}}{243\,\left(x^{4}+8\,x^{3}+6\,x^{2}-40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

giac [A] time = 1.07, size = 36, normalized size = 0.84

$$-\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^{2}-4\,x+5}}{243\,\left(x^{2}+4\,x-5\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{(x+5)\,(x-1)\left(2x^{3}+12x^{2}-3x-38\right)}{243\left(-x^{2}-4x+5\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(x+5)*(x-1)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

maxima [A] time = 0.42, size = 59, normalized size = 1.37

$$\frac{2\,x}{243\,\sqrt{-x^{2}-4\,x+5}}+\frac{4}{243\,\sqrt{-x^{2}-4\,x+5}}+\frac{x}{27\,\left(-x^{2}-4\,x+5\right)^{\frac{3}{2}}}+\frac{2}{27\,\left(-x^{2}-4\,x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

mupad [B] time = 0.05, size = 29, normalized size = 0.67

$$-\frac{(4\,x+8)\,\left(8\,x^{2}+32\,x-76\right)}{3888\,\left(-x^{2}-4\,x+5\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5 - x^2 - 4*x)^(5/2),x)
```

```
[Out] -((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-x^{2}-4x+5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

3.150 $\int e^t \sqrt{9-e^{2t}}\, dt$

Optimal. Leaf size=33

$$\frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)$$

```
[Out] 9/2*arcsin(1/3*exp(t))+1/2*exp(t)*(9-exp(2*t))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 195, 216}

$$\frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int e^t\sqrt{9-e^{2t}}\, dt &= \text{Subst}\left(\int \sqrt{9-t^2}\, dt, t, e^t\right)\\
&= \frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-t^2}}\, dt, t, e^t\right)\\
&= \frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{1}{2}\left(e^t\sqrt{9-e^{2t}} + 9\sin^{-1}\left(\frac{e^t}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*Sqrt[9 - E^(2*t)],t]
[Out] (E^t*Sqrt[9 - E^(2*t)] + 9*ArcSin[E^t/3])/2
```

fricas [A] time = 0.41, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}\,e^{t}-9\arctan\left(\left(\sqrt{-e^{(2t)}+9}-3\right)e^{(-t)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="fricas")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t - 9*arctan((sqrt(-e^(2*t) + 9) - 3)*e^(-t))
```

giac [A] time = 1.04, size = 22, normalized size = 0.67

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}\,e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="giac")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

maple [A] time = 0.02, size = 23, normalized size = 0.70

$$\frac{9\arcsin\left(\frac{e^{t}}{3}\right)}{2}+\frac{\sqrt{-e^{2t}+9}\,e^{t}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*(9-exp(2*t))^(1/2),t)
[Out] 1/2*exp(t)*(9-exp(t)^2)^(1/2)+9/2*arcsin(1/3*exp(t))
```

maxima [A] time = 1.09, size = 22, normalized size = 0.67

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}\,e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="maxima")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

mupad [B] time = 0.21, size = 22, normalized size = 0.67

$$\frac{9\,\mathrm{asin}\left(\frac{e^{t}}{3}\right)}{2}+\frac{e^{t}\,\sqrt{9-e^{2t}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*(9 - exp(2*t))^(1/2),t)
[Out] (9*asin(exp(t)/3))/2 + (exp(t)*(9 - exp(2*t))^(1/2))/2
```

sympy [A] time = 1.54, size = 29, normalized size = 0.88

$$\begin{cases} \frac{\sqrt{9-e^{2t}}\,e^{t}}{2}+\frac{9\,\mathrm{asin}\left(\frac{e^{t}}{3}\right)}{2} & \text{for } e^{t} < \log(3) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(9 - exp(2*t))*exp(t)/2 + 9*asin(exp(t)/3)/2, exp(t) < log(3
)))
```

3.151 $\int \sqrt{-9+e^{2t}}\,dt$

Optimal. Leaf size=30

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

```
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 203}

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-9+e^{2t}}\,dt &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+t}}{t}\,dt, t, e^{2t}\right)\\
&= \sqrt{-9+e^{2t}} - \frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9+t}\,t}\,dt, t, e^{2t}\right)\\
&= \sqrt{-9+e^{2t}} - 9\operatorname{Subst}\left(\int \frac{1}{9+t^2}\,dt, t, \sqrt{-9+e^{2t}}\right)\\
&= \sqrt{-9+e^{2t}} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+e^{2t}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\sqrt{e^{2t}-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + E^(2*t)],t]
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

fricas [A] time = 0.42, size = 22, normalized size = 0.73

$$\sqrt{e^{(2t)}-9} - 3\arctan\left(\frac{1}{3}\sqrt{e^{(2t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="fricas")
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

giac [A] time = 0.92, size = 22, normalized size = 0.73

$$\sqrt{e^{(2t)}-9} - 3\arctan\left(\frac{1}{3}\sqrt{e^{(2t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="giac")
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

maple [A] time = 0.01, size = 23, normalized size = 0.77

$$-3\arctan\left(\frac{\sqrt{e^{2t}-9}}{3}\right) + \sqrt{e^{2t}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9+exp(2*t))^(1/2),t)
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

maxima [A] time = 1.00, size = 22, normalized size = 0.73

$$\sqrt{e^{(2t)}-9} - 3\arctan\left(\frac{1}{3}\sqrt{e^{(2t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="maxima")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

mupad [B] time = 0.23, size = 34, normalized size = 1.13

$$\left(\frac{3\,\mathrm{e}^{-t}\,\mathrm{asin}\left(3\,\mathrm{e}^{-t}\right)}{\sqrt{1-9\,\mathrm{e}^{-2\,t}}}+1\right)\,\sqrt{\mathrm{e}^{2\,t}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(2*t) - 9)^(1/2),t)
```

```
[Out] ((3*exp(-t)*asin(3*exp(-t)))/(1 - 9*exp(-2*t))^(1/2) + 1)*(exp(2*t) - 9)^(1
/2)
```

sympy [A] time = 1.49, size = 22, normalized size = 0.73

$$\begin{cases} \sqrt{e^{2t}-9} - 3\,\mathrm{acos}\left(3e^{-t}\right) & \text{for}\ e^{t} < \log\left(3\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(exp(2*t) - 9) - 3*acos(3*exp(-t)), exp(t) < log(3)))
```

3.152 $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[Out] arctanh(x/(a^2+x^2)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{a^2+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 42, normalized size = 3.00

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + x^2],x]
```

```
[Out] -1/2*Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 + x/Sqrt[a^2 + x^2]]/2
```

fricas [A] time = 0.40, size = 16, normalized size = 1.14

$$-\log\left(-x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(a^2 + x^2))
```

giac [A] time = 0.85, size = 16, normalized size = 1.14

$$-\log\left(-x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] -log(-x + sqrt(a^2 + x^2))
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\ln\left(x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2)^(1/2),x)
[Out] ln(x+(a^2+x^2)^(1/2))
```

maxima [A] time = 0.44, size = 6, normalized size = 0.43

$$\operatorname{arsinh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x/a)
```

mupad [B] time = 0.19, size = 12, normalized size = 0.86

$$\ln\left(x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + x^2)^(1/2),x)
[Out] log(x + (a^2 + x^2)^(1/2))
```

sympy [A] time = 1.03, size = 3, normalized size = 0.21

$$\operatorname{asinh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2)**(1/2),x)
[Out] asinh(x/a)
```

3.153 $\int \frac{5+x}{-2+x+x^2} \, dx$

Optimal. Leaf size=15

$$2\log(1-x)-\log(x+2)$$

[Out] 2*ln(1-x)-ln(2+x)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {632, 31}

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(5 + x)/(-2 + x + x^2),x]

[Out] 2*Log[1 - x] - Log[2 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x}{-2+x+x^2} \, dx &= 2\int \frac{1}{-1+x} \, dx-\int \frac{1}{2+x} \, dx\\ &= 2\log(1-x)-\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x)/(-2 + x + x^2),x]

[Out] 2*Log[1 - x] - Log[2 + x]

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$-\log\left(x+2\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x)/(x^2+x-2),x, algorithm="fricas")

[Out] -log(x + 2) + 2*log(x - 1)

giac [A] time = 0.78, size = 15, normalized size = 1.00

$$-\log\left(|x+2|\right)+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="giac")
[Out] -log(abs(x + 2)) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$2\ln(x-1)-\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+5)/(x^2+x-2),x)
[Out] -ln(x+2)+2*ln(x-1)
```

maxima [A] time = 0.43, size = 13, normalized size = 0.87

$$-\log(x+2)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="maxima")
[Out] -log(x + 2) + 2*log(x - 1)
```

mupad [B] time = 0.18, size = 13, normalized size = 0.87

$$2\ln(x-1)-\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 5)/(x + x^2 - 2),x)
[Out] 2*log(x - 1) - log(x + 2)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.67

$$2\log(x-1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x**2+x-2),x)
[Out] 2*log(x - 1) - log(x + 2)
```

3.154 $\int \frac{x+x^3}{-1+x} dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

```
[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 772}

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^3}{-1+x} dx &= \int \frac{x\left(1+x^2\right)}{-1+x} dx \\
&= \int \left(2+\frac{2}{-1+x}+x+x^2\right) dx \\
&= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

fricas [A] time = 0.38, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3+\frac{1}{2}x^2+2x+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

giac [A] time = 0.87, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(x-1),x)
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

maxima [A] time = 0.42, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$2x + 2\ln(x - 1) + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)/(x - 1),x)
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

sympy [A] time = 0.08, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

3.155 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} \, dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} \, dx &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)} \, dx \\
&= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right) dx \\
&= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

fricas [A] time = 0.41, size = 19, normalized size = 0.76

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

giac [A] time = 0.76, size = 22, normalized size = 0.88

$$\frac{1}{10}\log\left(|2x-1|\right)-\frac{1}{10}\log\left(|x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
[Out] -1/10*ln(x+2)+1/10*ln(2*x-1)+1/2*ln(x)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

mupad [B] time = 0.19, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

sympy [A] time = 0.15, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

3.156 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2074}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right)dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

fricas [A] time = 0.40, size = 36, normalized size = 1.20

$$\frac{x^3+x^2-2\,(x-1)\log\left(x+1\right)+2\,(x-1)\log\left(x-1\right)-2\,x-4}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
 - 1)
```

giac [A] time = 1.05, size = 26, normalized size = 0.87

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(|x+1|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + x + \ln(x-1) - \ln(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)
[Out] 1/2*x^2+x-ln(x+1)+ln(x-1)-2/(x-1)
```

maxima [A] time = 0.43, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

mupad [B] time = 0.05, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \operatorname{atan}(x\,1\mathrm{i})\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

3.157 $\int \frac{4-x+2x^2}{4x+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx\\ &= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx\\ &= \log(x)+\int \frac{-1+x}{4+x^2}\,dx\\ &= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx\\ &= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

fricas [A] time = 0.41, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

giac [A] time = 0.94, size = 18, normalized size = 0.78

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x)
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

maxima [A] time = 0.98, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.91

$$\ln(x) + \ln(x - 2i)\left(\frac{1}{2} + \frac{1}{4}i\right) + \ln(x + 2i)\left(\frac{1}{2} - \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
```

```
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\log(x) + \frac{\log\left(x^2 + 4\right)}{2} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

3.158 $\int \frac{2-3x+4x^2}{3-4x+4x^2} \, dx$

Optimal. Leaf size=38

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] x+1/8*ln(4*x^2-4*x+3)+1/8*arctan(1/2*(1-2*x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx &= \int \left(1-\frac{1-x}{3-4x+4x^2}\right)dx\\ &= x-\int \frac{1-x}{3-4x+4x^2}\,dx\\ &= x+\frac{1}{8}\int \frac{-4+8x}{3-4x+4x^2}\,dx-\frac{1}{2}\int \frac{1}{3-4x+4x^2}\,dx\\ &= x+\frac{1}{8}\log\left(3-4x+4x^2\right)+\text{Subst}\left(\int \frac{1}{-32-x^2}\,dx,x,-4+8x\right)\\ &= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{8}\log\left(3-4x+4x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

fricas [A] time = 0.40, size = 31, normalized size = 0.82

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(2\,x-1)\right)+x+\frac{1}{8}\log\left(4\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

giac [A] time = 0.86, size = 31, normalized size = 0.82

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(2\,x-1)\right)+x+\frac{1}{8}\log\left(4\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="giac")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

maple [A] time = 0.00, size = 32, normalized size = 0.84

$$x-\frac{\sqrt{2}\,\arctan\left(\frac{(8x-4)\sqrt{2}}{8}\right)}{8}+\frac{\ln\left(4x^2-4x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-3*x+2)/(4*x^2-4*x+3),x)
[Out] x+1/8*ln(4*x^2-4*x+3)-1/8*2^(1/2)*arctan(1/8*(8*x-4)*2^(1/2))
```

maxima [A] time = 0.99, size = 31, normalized size = 0.82

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)+x+\frac{1}{8}\log\left(4x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

mupad [B] time = 0.17, size = 30, normalized size = 0.79

$$x+\frac{\ln\left(x^2-x+\frac{3}{4}\right)}{8}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 3*x + 2)/(4*x^2 - 4*x + 3),x)
[Out] x + log(x^2 - x + 3/4)/8 - (2^(1/2)*atan(2^(1/2)*x - 2^(1/2)/2))/8
```

sympy [A] time = 0.13, size = 34, normalized size = 0.89

$$x+\frac{\log\left(x^2-x+\frac{3}{4}\right)}{8}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-3*x+2)/(4*x**2-4*x+3),x)
[Out] x + log(x**2 - x + 3/4)/8 - sqrt(2)*atan(sqrt(2)*x - sqrt(2)/2)/8
```

3.159 $\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan}{}$$

```
[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(
1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2\left(1+x^2\right)^3} + \frac{3(1+x)}{4\left(1+x^2\right)^2} + \frac{-1+15x}{8\left(1+x^2\right)} + \frac{-1-}{1+x}\right.$$

$$= \frac{1}{8}\log(1-x) - \log(x) + \frac{1}{8}\int \frac{-1+15x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)^3}\,dx + \frac{3}{4}\int$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{1}{8}\log(1-x) - \log(x) - \frac{1}{8}\int \frac{1}{1+x^2}\,dx +$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{8}\log(1-x) -$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.05, size = 93, normalized size = 0.90

$$\frac{1}{48}\left(-14\log\left(1-x^3\right) + \frac{6(x+1)}{\left(x^2+1\right)^2} + \frac{9(3x-2)}{x^2+1} + 45\log\left(x^2+1\right) - 10\log\left(x^2+x+1\right) + 20\log(1-x) - 48\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

fricas [A] time = 0.41, size = 136, normalized size = 1.32

$$\frac{27\,x^3-16\,\sqrt{3}\left(x^4+2\,x^2+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-18\,x^2+21\left(x^4+2\,x^2+1\right)\arctan(x)-24\left(x^4+2\,x^2+1\right)\mathrm{l}}{48\left(x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 1.02, size = 74, normalized size = 0.72

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{9\,x^3-6\,x^2+11\,x-4}{16\left(x^2+1\right)^2}+\frac{7}{16}\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+x+1\right)+\frac{15}{16}\,\log\left(x^2+1\right)+\frac{1}{8}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 73, normalized size = 0.71

$$\frac{7\arctan(x)}{16}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\ln(x)+\frac{\ln(x-1)}{8}+\frac{15\ln\left(x^2+1\right)}{16}-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)+1/8*l
n(x-1)-ln(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 0.97, size = 77, normalized size = 0.75

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{9\,x^3-6\,x^2+11\,x-4}{16\left(x^4+2\,x^2+1\right)}+\frac{7}{16}\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+x+1\right)+\frac{15}{16}\,\log\left(x^2+1\right)+\frac{1}{8}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

mupad [B] time = 0.26, size = 96, normalized size = 0.93

$$\frac{\ln(x-1)}{8}-\ln(x)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\frac{9\,x^3}{16}-\frac{3\,x^2}{8}+\frac{11\,x}{16}-\frac{1}{4}}{x^4+2\,x^2+1}+\mathrm{ln}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.54, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log\left(x^2+1\right)}{16}-\frac{\log\left(x^2+x+1\right)}{2}+\frac{7\,\mathrm{atan}(x)}{16}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x}{16x^4+32x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

3.160 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x) - 2\tan^{-1}(x)$$

```
[Out] 1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x) - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.39, size = 44, normalized size = 1.33

$$-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2\,x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

giac [A] time = 0.91, size = 30, normalized size = 0.91

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-2\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}-\frac{x+\frac{1}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

maxima [A] time = 1.25, size = 29, normalized size = 0.88

$$-\frac{2\,x+1}{2\,\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 0.16, size = 33, normalized size = 1.00

$$\ln(x)-\frac{x+\frac{1}{2}}{x^2+1}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+1\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
 1)
```

sympy [A] time = 0.16, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

3.161 $\int \frac{1}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 203}

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-2),x]
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)^2} \, dx &= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-2),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

fricas [A] time = 0.37, size = 19, normalized size = 1.00

$$\frac{\left(x^2+1\right)\arctan(x)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

giac [A] time = 1.08, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2,x)
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 1.16, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.02, size = 16, normalized size = 0.84

$$\frac{\operatorname{atan}(x)}{2}+\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^2,x)
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2,x)
[Out] x/(2*x**2 + 2) + atan(x)/2
```

3.162 $\int \frac{1}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

```
[Out] 1/3*ln(1-x)-1/3*ln(2+x)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)(2+x)}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx-\frac{1}{3}\int \frac{1}{2+x}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{3}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

fricas [A] time = 0.38, size = 13, normalized size = 0.68

$$-\frac{1}{3}\,\log\left(x+2\right)+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="fricas")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

giac [A] time = 0.97, size = 15, normalized size = 0.79

$$-\frac{1}{3}\log\left(|x+2|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(x + 2)) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{\ln\left(x-1\right)}{3}-\frac{\ln\left(x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)/(x+2),x)
```

```
[Out] -1/3*ln(x+2)+1/3*ln(x-1)
```

maxima [A] time = 0.59, size = 13, normalized size = 0.68

$$-\frac{1}{3}\log\left(x+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="maxima")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

mupad [B] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\ln\left(\frac{x-1}{x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x + 2)),x)
```

```
[Out] log((x - 1)/(x + 2))/3
```

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x)
```

```
[Out] log(x - 1)/3 - log(x + 2)/3
```

3.163 $\int \frac{7}{-12+5x+2x^2} \, dx$

Optimal. Leaf size=19

$$\frac{7}{11}\log(3-2x)-\frac{7}{11}\log(x+4)$$

```
[Out] 7/11*ln(3-2*x)-7/11*ln(4+x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 616, 31}

$$\frac{7}{11}\log(3-2x)-\frac{7}{11}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{7}{-12+5x+2x^2} \, dx &= 7\int \frac{1}{-12+5x+2x^2} \, dx\\
&= \frac{14}{11}\int \frac{1}{-3+2x} \, dx-\frac{14}{11}\int \frac{1}{8+2x} \, dx\\
&= \frac{7}{11}\log(3-2x)-\frac{7}{11}\log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.11

$$7\left(\frac{1}{11}\log(3-2x)-\frac{1}{11}\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] 7*(Log[3 - 2*x]/11 - Log[4 + x]/11)
```

fricas [A] time = 0.39, size = 15, normalized size = 0.79

$$\frac{7}{11} \log(2x - 3) - \frac{7}{11} \log(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="fricas")
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

giac [A] time = 1.08, size = 17, normalized size = 0.89

$$\frac{7}{11} \log(|2x - 3|) - \frac{7}{11} \log(|x + 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="giac")
[Out] 7/11*log(abs(2*x - 3)) - 7/11*log(abs(x + 4))
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{7 \ln(2x - 3)}{11} - \frac{7 \ln(x + 4)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7/(2*x^2+5*x-12),x)
[Out] -7/11*ln(4+x)+7/11*ln(-3+2*x)
```

maxima [A] time = 0.45, size = 15, normalized size = 0.79

$$\frac{7}{11} \log(2x - 3) - \frac{7}{11} \log(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="maxima")
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

mupad [B] time = 0.12, size = 8, normalized size = 0.42

$$-\frac{14 \operatorname{atanh}\left(\frac{4x}{11} + \frac{5}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7/(5*x + 2*x^2 - 12),x)
[Out] -(14*atanh((4*x)/11 + 5/11))/11
```

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$\frac{7 \log\left(x - \frac{3}{2}\right)}{11} - \frac{7 \log(x + 4)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x**2+5*x-12),x)
[Out] 7*log(x - 3/2)/11 - 7*log(x + 4)/11
```

3.164 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {893}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

fricas [A] time = 0.39, size = 37, normalized size = 1.16

$$-\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x - 1)
```

giac [A] time = 0.89, size = 43, normalized size = 1.34

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^{2}}\right)-\frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/(2*x - 1) - 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$-\frac{25\ln(2x+3)}{128}+\frac{41\ln(2x-1)}{128}+\frac{9}{32\,(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(2*x+3),x)
```

```
[Out] -25/128*ln(2*x+3)+9/32/(2*x-1)+41/128*ln(2*x-1)
```

maxima [A] time = 0.52, size = 26, normalized size = 0.81

$$\frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log(2\,x+3)+\frac{41}{128}\,\log(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

mupad [B] time = 0.10, size = 22, normalized size = 0.69

$$\frac{41\,\ln\left(x-\frac{1}{2}\right)}{128}-\frac{25\,\ln\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
```

```
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

sympy [A] time = 0.14, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

3.165 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*x)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 148}

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3}\,dx\\ &= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right)dx\\ &= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2}+2500\log(x-6)+1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

fricas [A] time = 0.38, size = 53, normalized size = 1.23

$$\frac{1493\left(25\,x^2+30\,x+9\right)\log\left(5\,x+3\right)+2500\left(25\,x^2+30\,x+9\right)\log\left(x-6\right)+33165\,x+15543}{499125\left(25\,x^2+30\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

giac [A] time = 0.96, size = 31, normalized size = 0.72

$$\frac{3\left(335\,x+157\right)}{15125\left(5\,x+3\right)^2}+\frac{1493}{499125}\,\log\left(|5\,x+3|\right)+\frac{20}{3993}\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$\frac{20\ln\left(x-6\right)}{3993}+\frac{1493\ln\left(5x+3\right)}{499125}-\frac{12}{1375\left(5x+3\right)^2}+\frac{201}{15125\left(5x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(-6+x)/(3+5*x)^3,x)
```

```
[Out] 20/3993*ln(-6+x)-12/1375/(3+5*x)^2+201/15125/(3+5*x)+1493/499125*ln(3+5*x)
```

maxima [A] time = 0.48, size = 34, normalized size = 0.79

$$\frac{3\left(335\,x+157\right)}{15125\left(25\,x^2+30\,x+9\right)}+\frac{1493}{499125}\,\log\left(5\,x+3\right)+\frac{20}{3993}\,\log\left(x-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

mupad [B] time = 0.23, size = 29, normalized size = 0.67

$$\frac{20\,\ln\left(x-6\right)}{3993}+\frac{1493\,\ln\left(x+\frac{3}{5}\right)}{499125}+\frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

sympy [A] time = 0.17, size = 32, normalized size = 0.74

$$\frac{1005x+471}{378125x^2+453750x+136125}+\frac{20\log\left(x-6\right)}{3993}+\frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

3.166 $\int \frac{1}{-x^3+x^4}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

```
[Out] 1/2/x^2+1/x+ln(1-x)-ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 44}

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^3+x^4}\,dx &= \int \frac{1}{(-1+x)x^3}\,dx\\ &= \int \left(\frac{1}{-1+x}-\frac{1}{x^3}-\frac{1}{x^2}-\frac{1}{x}\right)dx\\ &= \frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

fricas [A] time = 0.38, size = 26, normalized size = 1.24

$$\frac{2\,x^2\log\left(x-1\right)-2\,x^2\log\left(x\right)+2\,x+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x - 1) - 2*x^2*log(x) + 2*x + 1)/x^2
```

giac [**A**] time = 0.92, size = 21, normalized size = 1.00

$$\frac{2\,x+1}{2\,x^2}+\log\left(|x-1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="giac")
[Out] 1/2*(2*x + 1)/x^2 + log(abs(x - 1)) - log(abs(x))
```

maple [**A**] time = 0.01, size = 18, normalized size = 0.86

$$-\ln(x)+\ln\left(x-1\right)+\frac{1}{x}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^3),x)
[Out] ln(x-1)+1/2/x^2+1/x-ln(x)
```

maxima [**A**] time = 0.63, size = 19, normalized size = 0.90

$$\frac{2\,x+1}{2\,x^2}+\log\left(x-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="maxima")
[Out] 1/2*(2*x + 1)/x^2 + log(x - 1) - log(x)
```

mupad [**B**] time = 0.04, size = 16, normalized size = 0.76

$$\frac{x+\frac{1}{2}}{x^2}-2\,\mathrm{atanh}\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3 - x^4),x)
[Out] (x + 1/2)/x^2 - 2*atanh(2*x - 1)
```

sympy [**A**] time = 0.11, size = 17, normalized size = 0.81

$$-\log\left(x\right)+\log\left(x-1\right)+\frac{2x+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**3),x)
[Out] -log(x) + log(x - 1) + (2*x + 1)/(2*x**2)
```

3.167 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx &= \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)} \, dx\\ &= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right) dx\\ &= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2} \, dx\\ &= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

fricas [A] time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2 - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

giac [A] time = 1.17, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x + 1|) + \frac{1}{2}\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{x^2}{2} + x - \ln(x) + \frac{\ln(x - 1)}{2} + \frac{\ln(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)
[Out] 1/2*x^2+x+1/2*ln(x+1)+1/2*ln(x-1)-ln(x)
```

maxima [A] time = 0.63, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x + 1) + \frac{1}{2}\log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

mupad [B] time = 0.17, size = 19, normalized size = 0.76

$$x + \frac{\ln\left(x^2 - 1\right)}{2} - \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

sympy [A] time = 0.11, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

3.168 $\int \frac{-2+x^2}{x(2+x^2)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

```
[Out] -ln(x)+ln(x^2+2)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {446, 72}

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)dx, x, x^2\right)\\
&= -\log(x)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

fricas [A] time = 0.39, size = 11, normalized size = 1.00

$$\log\left(x^2+2\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

giac [A] time = 1.02, size = 13, normalized size = 1.18

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

maple [A] time = 0.01, size = 12, normalized size = 1.09

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)
[Out] -ln(x)+ln(x^2+2)
```

maxima [A] time = 0.48, size = 13, normalized size = 1.18

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

mupad [B] time = 0.18, size = 11, normalized size = 1.00

$$\ln\left(x^2+2\right)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
[Out] log(x^2 + 2) - log(x)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.73

$$-\log(x)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
[Out] -log(x) + log(x**2 + 2)
```

3.169 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-4x^2+x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx\\
&= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\
&= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= 6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

fricas [A] time = 0.40, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

giac [A] time = 0.90, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$6\arctan(x)-5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.28, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

mupad [B] time = 0.11, size = 56, normalized size = 1.56

$$\ln(x-\mathrm{i})\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+3\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

sympy [A] time = 0.21, size = 36, normalized size = 1.00

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}\,(x)-5\sqrt{2}\,\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

3.170 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx &= \int \left(\frac{1}{9(1+x^2)}-\frac{13}{3(4+x^2)^2}+\frac{8}{9(4+x^2)}\right) dx \\
&= \frac{1}{9}\int \frac{1}{1+x^2} \, dx+\frac{8}{9}\int \frac{1}{4+x^2} \, dx-\frac{13}{3}\int \frac{1}{(4+x^2)^2} \, dx \\
&= -\frac{13x}{24(4+x^2)}+\frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)-\frac{13}{24}\int \frac{1}{4+x^2} \, dx \\
&= -\frac{13x}{24(4+x^2)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

fricas [A] time = 0.40, size = 33, normalized size = 1.14

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+16\left(x^2+4\right)\arctan(x)-78x}{144\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

giac [A] time = 0.87, size = 21, normalized size = 0.72

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\arctan\left(\frac{1}{2}x\right)+\frac{1}{9}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{\arctan(x)}{9}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

maxima [A] time = 1.32, size = 21, normalized size = 0.72

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\arctan\left(\frac{1}{2}x\right)+\frac{1}{9}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

mupad [B] time = 0.17, size = 23, normalized size = 0.79

$$\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}(x)}{9}-\frac{13x}{24\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
```

```
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

sympy [A] time = 0.18, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\operatorname{atan}\left(\frac{x}{2}\right)}{144}+\frac{\operatorname{atan}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

3.171 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+45
1/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \int\left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right.$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int\frac{-15-481x}{1+x+x^2}\,dx}{2793}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int\frac{1}{1+x+x^2}\,dx}{5586}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+\right.}{5586}$$

$$= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

fricas [A] time = 0.41, size = 60, normalized size = 1.00

$$\frac{152438\sqrt{3}\,(x+5)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-243867\,(x+5)\log\left(x^2+x+1\right)+176400\,(x+5)\log\left(2x-3\right)+3}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

giac [A] time = 0.90, size = 60, normalized size = 1.00

$$\frac{451}{8379}\sqrt{3}\,\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\,\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\,\log\left(\left|-\frac{13}{x+5}+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{451\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{8379}+\frac{200\ln\left(2x-3\right)}{3211}+\frac{2731\ln\left(x+5\right)}{24843}-\frac{481\ln\left(x^2+x+1\right)}{5586}-\frac{79}{273\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(x+5)^2/(2*x-3)/(x^2+x+1),x)
```

```
[Out] 200/3211*ln(2*x-3)-79/273/(x+5)+2731/24843*ln(x+5)-481/5586*ln(x^2+x+1)+451
/8379*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.06, size = 47, normalized size = 0.78

$$\frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{79}{273(x+5)}-\frac{481}{5586}\log\left(x^2+x+1\right)+\frac{200}{3211}\log(2x-3)+\frac{2731}{24843}\log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

mupad [B] time = 0.12, size = 61, normalized size = 1.02

$$\frac{200\ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\ln(x+5)}{24843}-\frac{79}{273(x+5)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{4}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

sympy [A] time = 0.27, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log(x+5)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

3.172 $\int \frac{x^4}{\left(9+x^2\right)^3} dx$

Optimal. Leaf size=37

$$-\frac{3x}{8\left(x^2+9\right)}-\frac{x^3}{4\left(x^2+9\right)^2}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -1/4*x^3/(x^2+9)^2-3/8*x/(x^2+9)+1/8*arctan(1/3*x)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$-\frac{x^3}{4\left(x^2+9\right)^2}-\frac{3x}{8\left(x^2+9\right)}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(9 + x^2)^3,x]
```

```
[Out] -x^3/(4*(9 + x^2)^2) - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(9+x^2\right)^3} dx &= -\frac{x^3}{4\left(9+x^2\right)^2}+\frac{3}{4}\int \frac{x^2}{\left(9+x^2\right)^2} dx \\
&= -\frac{x^3}{4\left(9+x^2\right)^2}-\frac{3x}{8\left(9+x^2\right)}+\frac{3}{8}\int \frac{1}{9+x^2} dx \\
&= -\frac{x^3}{4\left(9+x^2\right)^2}-\frac{3x}{8\left(9+x^2\right)}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.76

$$\frac{1}{8}\left(\tan^{-1}\left(\frac{x}{3}\right)-\frac{x\left(5x^2+27\right)}{\left(x^2+9\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(9 + x^2)^3,x]
[Out] (-((x*(27 + 5*x^2))/(9 + x^2)^2) + ArcTan[x/3])/8
```

fricas [A] time = 0.39, size = 39, normalized size = 1.05

$$-\frac{5\,x^{3} - \left(x^{4} + 18\,x^{2} + 81\right)\arctan\left(\frac{1}{3}\,x\right) + 27\,x}{8\,\left(x^{4} + 18\,x^{2} + 81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="fricas")
[Out] -1/8*(5*x^3 - (x^4 + 18*x^2 + 81)*arctan(1/3*x) + 27*x)/(x^4 + 18*x^2 + 81)
```

giac [A] time = 0.98, size = 25, normalized size = 0.68

$$-\frac{5\,x^{3} + 27\,x}{8\,\left(x^{2} + 9\right)^{2}} + \frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="giac")
[Out] -1/8*(5*x^3 + 27*x)/(x^2 + 9)^2 + 1/8*arctan(1/3*x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.68

$$\frac{\arctan\left(\frac{x}{3}\right)}{8} + \frac{-\frac{5}{8}x^{3} - \frac{27}{8}x}{\left(x^{2} + 9\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^2+9)^3,x)
[Out] (-5/8*x^3-27/8*x)/(x^2+9)^2+1/8*arctan(1/3*x)
```

maxima [A] time = 1.14, size = 30, normalized size = 0.81

$$-\frac{5\,x^{3} + 27\,x}{8\,\left(x^{4} + 18\,x^{2} + 81\right)} + \frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="maxima")
[Out] -1/8*(5*x^3 + 27*x)/(x^4 + 18*x^2 + 81) + 1/8*arctan(1/3*x)
```

mupad [B] time = 0.16, size = 30, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{x}{3}\right)}{8} - \frac{\frac{5\,x^{3}}{8} + \frac{27\,x}{8}}{x^{4} + 18\,x^{2} + 81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^2 + 9)^3,x)
[Out] atan(x/3)/8 - ((27*x)/8 + (5*x^3)/8)/(18*x^2 + x^4 + 81)
```

sympy [A] time = 0.14, size = 27, normalized size = 0.73

$$\frac{-5x^{3} - 27x}{8x^{4} + 144x^{2} + 648} + \frac{\mathrm{atan}\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**2+9)**3,x)
```

```
[Out] (-5*x**3 - 27*x)/(8*x**4 + 144*x**2 + 648) + atan(x/3)/8
```

3.173 $\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{19(44x+39)}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{209\log\left(4x^2+5x+3\right)}{4608}-\frac{1843}{4416(1-x)}-\frac{399}{736(1-x)^2}+\frac{209\log(1-x)}{2304}+\frac{114437\tan^{-1}\left(\frac{8}{\cdot}\right.}{52992\sqrt{23}}$$

```
[Out] -399/736/(1-x)^2-1843/4416/(1-x)+19/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+209
/2304*ln(1-x)-209/4608*ln(4*x^2+5*x+3)+114437/1218816*arctan(1/23*(5+8*x)*2
3^(1/2))*23^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {12, 822, 800, 634, 618, 204, 628}

$$\frac{19(44x+39)}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{209\log\left(4x^2+5x+3\right)}{4608}-\frac{1843}{4416(1-x)}-\frac{399}{736(1-x)^2}+\frac{209\log(1-x)}{2304}+\frac{114437\tan^{-1}\left(\frac{8}{\cdot}\right.}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(276*(1 - x)^
2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23])
 + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x + 4*x^2])/4608
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{19x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = 19\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$$

$$= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{19}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}\,dx$$

$$= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{19}{276}\int \left(\frac{63}{4(-1+x)^3} - \frac{97}{16(-1+x)^2} + \frac{253}{192(-1}\right.$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{114437\tan^{-1}(}{52992\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.80

$$\frac{19\left(\frac{184(2204x+975)}{4x^2+5x+3} - 17457\log\left(4x^2+5x+3\right) + \frac{59248}{x-1} - \frac{25392}{(x-1)^2} + 34914\log(1-x) + 36138\sqrt{23}\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (19*(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4
*x^2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 1745
7*Log[3 + 5*x + 4*x^2]))/7312896
```

fricas [A] time = 0.41, size = 134, normalized size = 1.38

$$\frac{19\left(214176\,x^3+12046\,\sqrt{23}\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(8\,x+5)\right)-224664\,x^2-5819\left(4\,x^4-3\,x^3\right.\right.}{2437632\left(4\,x^4-3\,x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 19/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arc
tan(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
 + 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x -
1) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

giac [A] time = 0.93, size = 71, normalized size = 0.73

$$\frac{114437}{1218816}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(8\,x+5)\right)+\frac{19\left(388\,x^3-407\,x^2-120\,x-45\right)}{4416\left(4\,x^2+5\,x+3\right)(x-1)^2}-\frac{209}{4608}\,\log\left(4\,x^2+5\,x+3\right)+\frac{209}{2304}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 209/4608*log(4*x^2
+ 5*x + 3) + 209/2304*log(abs(x - 1))
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$\frac{114437\sqrt{23}\ \arctan\left(\frac{(8x+5)\sqrt{23}}{23}\right)}{1218816}+\frac{209\ln(x-1)}{2304}-\frac{209\ln\left(4x^2+5x+3\right)}{4608}-\frac{19\left(-\frac{2204x}{23}-\frac{975}{23}\right)}{6912\left(x^2+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{19}{288\,(x-1)^2}+\frac{1}{864\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(19*x/(x-1)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -19/6912*(-2204/23*x-975/23)/(x^2+5/4*x+3/4)-209/4608*ln(4*x^2+5*x+3)+11443
7/1218816*arctan(1/23*(5+8*x)*23^(1/2))*23^(1/2)-19/288/(x-1)^2+133/864/(x-
1)+209/2304*ln(x-1)
```

maxima [A] time = 1.18, size = 75, normalized size = 0.77

$$\frac{114437}{1218816}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(8\,x+5)\right)+\frac{19\left(388\,x^3-407\,x^2-120\,x-45\right)}{4416\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)}-\frac{209}{4608}\,\log\left(4\,x^2+5\,x+3\right)+\frac{209}{2304}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 209/4608*log(4*x^
2 + 5*x + 3) + 209/2304*log(x - 1)
```

mupad [B] time = 0.24, size = 84, normalized size = 0.87

$$\frac{209\ \ln(x-1)}{2304}+\frac{-\frac{1843\,x^3}{4416}+\frac{7733\,x^2}{17664}+\frac{95\,x}{736}+\frac{285}{5888}}{-x^4+\frac{3\,x^3}{4}+\frac{3\,x^2}{4}+\frac{x}{4}-\frac{3}{4}}-\ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\ 1\mathrm{i}}{8}\right)\left(\frac{209}{4608}+\frac{\sqrt{23}\ 114437\mathrm{i}}{2437632}\right)+\ln\left(x+\frac{5}{8}+\frac{\sqrt{23}}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x)/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

```
[Out] (209*log(x - 1))/2304 + ((95*x)/736 + (7733*x^2)/17664 - (1843*x^3)/4416 +
285/5888)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)
/8 + 5/8)*((23^(1/2)*114437i)/2437632 + 209/4608) + log(x + (23^(1/2)*1i)/8
 + 5/8)*((23^(1/2)*114437i)/2437632 - 209/4608)
```

sympy [A] time = 0.24, size = 88, normalized size = 0.91

$$\frac{19\left(388x^3-407x^2-120x-45\right)}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{209\log\left(x-1\right)}{2304}-\frac{209\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{114437\sqrt{23}\ \mathrm{atan}\left(\right.}{12188}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] 19*(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2
 - 4416*x + 13248) + 209*log(x - 1)/2304 - 209*log(x**2 + 5*x/4 + 3/4)/4608
 + 114437*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

3.174 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right) \\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

fricas [A] time = 0.41, size = 39, normalized size = 0.85

$$\frac{2\sqrt{7}\,x\arctan\left(\frac{1}{7}\sqrt{7}\,(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log(x)-28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

giac [A] time = 0.98, size = 36, normalized size = 0.78

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{\sqrt{7}\ \arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{28}-\frac{\ln(x)}{4}+\frac{5\ln\left(x^{2}+x+2\right)}{8}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(2*x+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 1.11, size = 35, normalized size = 0.76

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^{2}+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

mupad [B] time = 0.24, size = 49, normalized size = 1.07

$$-\frac{\ln(x)}{4}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

sympy [A] time = 0.18, size = 46, normalized size = 1.00

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^{2}+x+2\right)}{8}+\frac{\sqrt{7}\ \mathrm{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

3.175 $\int \frac{1}{-x^3+x^6} \, dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx\\
&= \frac{1}{2x^2}+\int \frac{1}{-1+x^3}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

fricas [A] time = 0.40, size = 46, normalized size = 0.96

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x^2\log\left(x^2+x+1\right)-2\,x^2\log\left(x-1\right)-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

giac [A] time = 0.97, size = 38, normalized size = 0.79

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x)
```

```
[Out] 1/2/x^2+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.27, size = 37, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

mupad [B] time = 0.11, size = 51, normalized size = 1.06

$$\frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3 - x^6),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

sympy [A] time = 0.16, size = 48, normalized size = 1.00

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

3.176 $\int \frac{x^2}{1+x} dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - x + \log(x+1)$$

```
[Out] -x+1/2*x^2+ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^2}{2} - x + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x),x]
[Out] -x + x^2/2 + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1+x} dx &= \int \left(-1 + x + \frac{1}{1+x}\right) dx \\ &= -x + \frac{x^2}{2} + \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.27

$$\frac{1}{2}(x+1)^2 - 2(x+1) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x),x]
[Out] -2*(1 + x) + (1 + x)^2/2 + Log[1 + x]
```

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{2} x^2 - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + log(x + 1)
```

giac [A] time = 0.94, size = 14, normalized size = 0.93

$$\frac{1}{2} x^2 - x + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + log(abs(x + 1))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2}{2} - x + \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1),x)
```

```
[Out] -x+1/2*x^2+ln(x+1)
```

maxima [A] time = 0.46, size = 13, normalized size = 0.87

$$\frac{1}{2}x^2 - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 1)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(x+1) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + 1),x)
```

```
[Out] log(x + 1) - x + x^2/2
```

sympy [A] time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^2}{2} - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x),x)
```

```
[Out] x**2/2 - x + log(x + 1)
```

3.177 $\int \frac{x}{-5+x} dx$

Optimal. Leaf size=10

$$x + 5\log(5 - x)$$

[Out] x+5*ln(5-x)

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$x + 5\log(5 - x)$$

Antiderivative was successfully verified.

[In] Int[x/(-5 + x),x]

[Out] x + 5*Log[5 - x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{-5+x}\,dx &= \int \left(1+\frac{5}{-5+x}\right)dx\\ &= x+5\log(5-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 0.80

$$x + 5\log(x - 5)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-5 + x),x]

[Out] x + 5*Log[-5 + x]

fricas [A] time = 0.41, size = 8, normalized size = 0.80

$$x + 5\log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-5+x),x, algorithm="fricas")

[Out] x + 5*log(x - 5)

giac [A] time = 1.18, size = 9, normalized size = 0.90

$$x + 5\log(|x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-5+x),x, algorithm="giac")

[Out] x + 5*log(abs(x - 5))

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$x + 5\ln(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-5+x),x)
[Out] x+5*ln(-5+x)
```

maxima [A] time = 0.54, size = 8, normalized size = 0.80

$$x + 5\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x, algorithm="maxima")
[Out] x + 5*log(x - 5)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$x + 5\ln(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - 5),x)
[Out] x + 5*log(x - 5)
```

sympy [A] time = 0.07, size = 7, normalized size = 0.70

$$x + 5\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x)
[Out] x + 5*log(x - 5)
```

3.178 $\int \frac{-1+4x}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=13

$$\log(1-x)+3\log(x+2)$$

[Out] ln(1-x)+3*ln(2+x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x)/((-1 + x)*(2 + x)),x]

[Out] Log[1 - x] + 3*Log[2 + x]

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+4x}{(-1+x)(2+x)}\,dx &= \int \left(\frac{1}{-1+x}+\frac{3}{2+x}\right)dx\\ &= \log(1-x)+3\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x)/((-1 + x)*(2 + x)),x]

[Out] Log[1 - x] + 3*Log[2 + x]

fricas [A] time = 0.41, size = 11, normalized size = 0.85

$$3\,\log\left(x+2\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="fricas")

[Out] 3*log(x + 2) + log(x - 1)

giac [A] time = 1.10, size = 13, normalized size = 1.00

$$3\,\log\left(|x+2|\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="giac")

```
[Out] 3*log(abs(x + 2)) + log(abs(x - 1))
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\ln(x-1)+3\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+4*x)/(x-1)/(x+2),x)
[Out] 3*ln(x+2)+ln(x-1)
```

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$3\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="maxima")
[Out] 3*log(x + 2) + log(x - 1)
```

mupad [B] time = 0.17, size = 11, normalized size = 0.85

$$\ln(x-1)+3\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 1)/((x - 1)*(x + 2)),x)
[Out] log(x - 1) + 3*log(x + 2)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\log(x-1)+3\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x)
[Out] log(x - 1) + 3*log(x + 2)
```

3.179 $\int \frac{1}{(1+x)(2+x)}\,dx$

Optimal. Leaf size=11

$$\log(x+1)-\log(x+2)$$

```
[Out] ln(1+x)-ln(2+x)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)}\,dx &= \int \frac{1}{1+x}\,dx-\int \frac{1}{2+x}\,dx\\ &= \log(1+x)-\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

fricas [A] time = 0.42, size = 11, normalized size = 1.00

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + log(x + 1)
```

giac [A] time = 1.01, size = 13, normalized size = 1.18

$$-\log(|x+2|)+\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="giac")
[Out] -log(abs(x + 2)) + log(abs(x + 1))
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\ln(x+1) - \ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x+2),x)
[Out] ln(x+1)-ln(x+2)
```

maxima [A] time = 0.53, size = 11, normalized size = 1.00

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="maxima")
[Out] -log(x + 2) + log(x + 1)
```

mupad [B] time = 0.07, size = 10, normalized size = 0.91

$$\ln\left(1 - \frac{1}{x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)),x)
[Out] log(1 - 1/(x + 2))
```

sympy [A] time = 0.10, size = 8, normalized size = 0.73

$$\log(x+1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x)
[Out] log(x + 1) - log(x + 2)
```

3.180 $\int \frac{-5+6x}{3+2x} \, dx$

Optimal. Leaf size=12

$$3x - 7\log(2x+3)$$

```
[Out] 3*x-7*ln(3+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$3x - 7\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-5+6x}{3+2x} \, dx &= \int \left(3 - \frac{14}{3+2x}\right) dx \\ &= 3x - 7\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.25

$$3x - 7\log(2x+3) + \frac{9}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 9/2 + 3*x - 7*Log[3 + 2*x]
```

fricas [A] time = 0.40, size = 12, normalized size = 1.00

$$3x - 7\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="fricas")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

giac [A] time = 1.00, size = 13, normalized size = 1.08

$$3x - 7\log(|2x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="giac")
```

```
[Out] 3*x - 7*log(abs(2*x + 3))
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$3x - 7\ln(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+6*x)/(2*x+3),x)
```

```
[Out] 3*x-7*ln(2*x+3)
```

maxima [A] time = 0.48, size = 12, normalized size = 1.00

$$3\,x - 7\,\log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="maxima")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

mupad [B] time = 0.16, size = 10, normalized size = 0.83

$$3\,x - 7\,\ln\left(x + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - 5)/(2*x + 3),x)
```

```
[Out] 3*x - 7*log(x + 3/2)
```

sympy [A] time = 0.08, size = 10, normalized size = 0.83

$$3x - 7\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x - 7*log(2*x + 3)
```

3.181 $\int \frac{1}{(a+x)(b+x)} dx$

Optimal. Leaf size=26

$$\frac{\log(b+x)}{a-b} - \frac{\log(a+x)}{a-b}$$

```
[Out] -ln(a+x)/(a-b)+ln(b+x)/(a-b)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {36, 31}

$$\frac{\log(b+x)}{a-b} - \frac{\log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + x)*(b + x)),x]
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+x)(b+x)} dx &= \frac{\int \frac{1}{a+x} dx}{-a+b} - \frac{\int \frac{1}{b+x} dx}{-a+b} \\
&= -\frac{\log(a+x)}{a-b} + \frac{\log(b+x)}{a-b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{\log(b+x) - \log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + x)*(b + x)),x]
[Out] (-Log[a + x] + Log[b + x])/(a - b)
```

fricas [A] time = 0.40, size = 20, normalized size = 0.77

$$-\frac{\log(a+x) - \log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="fricas")
```

```
[Out] -(log(a + x) - log(b + x))/(a - b)
```

giac [A] time = 0.91, size = 28, normalized size = 1.08

$$-\frac{\log\left(|a+x|\right)}{a-b}+\frac{\log\left(|b+x|\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="giac")
[Out] -log(abs(a + x))/(a - b) + log(abs(b + x))/(a - b)
```

maple [A] time = 0.01, size = 27, normalized size = 1.04

$$-\frac{\ln\left(a+x\right)}{a-b}+\frac{\ln\left(b+x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x)/(b+x),x)
[Out] -ln(a+x)/(a-b)+ln(b+x)/(a-b)
```

maxima [A] time = 0.59, size = 26, normalized size = 1.00

$$-\frac{\log\left(a+x\right)}{a-b}+\frac{\log\left(b+x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="maxima")
[Out] -log(a + x)/(a - b) + log(b + x)/(a - b)
```

mupad [B] time = 0.22, size = 18, normalized size = 0.69

$$\frac{\ln\left(\frac{b+x}{a+x}\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + x)*(b + x)),x)
[Out] log((b + x)/(a + x))/(a - b)
```

sympy [B] time = 0.23, size = 80, normalized size = 3.08

$$\frac{\log\left(-\frac{a^2}{2(a-b)}+\frac{ab}{a-b}+\frac{a}{2}-\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}-\frac{\log\left(\frac{a^2}{2(a-b)}-\frac{ab}{a-b}+\frac{a}{2}+\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x)
[Out] log(-a**2/(2*(a - b)) + a*b/(a - b) + a/2 - b**2/(2*(a - b)) + b/2 + x)/(a
- b) - log(a**2/(2*(a - b)) - a*b/(a - b) + a/2 + b**2/(2*(a - b)) + b/2 +
x)/(a - b)
```

3.182 $\int \frac{1+x^2}{-x+x^2}\,dx$

Optimal. Leaf size=14

$$x + 2\log(1 - x) - \log(x)$$

[Out] x+2*ln(1-x)-ln(x)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 894}

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(-x + x^2),x]

[Out] x + 2*Log[1 - x] - Log[x]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{-x+x^2}\,dx &= \int \frac{1+x^2}{(-1+x)x}\,dx\\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right)dx\\
&= x + 2\log(1-x) - \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(-x + x^2),x]

[Out] x + 2*Log[1 - x] - Log[x]

fricas [A] time = 0.40, size = 12, normalized size = 0.86

$$x + 2\,\log\left(x - 1\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")

```
[Out] x + 2*log(x - 1) - log(x)
```

giac [**A**] time = 1.00, size = 14, normalized size = 1.00

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [**A**] time = 0.01, size = 13, normalized size = 0.93

$$x - \ln(x) + 2\ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x)
[Out] x+2*ln(x-1)-ln(x)
```

maxima [**A**] time = 0.50, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
[Out] x + 2*log(x - 1) - log(x)
```

mupad [**B**] time = 0.04, size = 12, normalized size = 0.86

$$x + 2 \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

sympy [**A**] time = 0.10, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

3.183 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

```
[Out] 1/2*x^2+1/7*ln(3-x)-1/7*ln(4+x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1657, 616, 31}

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx &= \int \left(x+\frac{1}{-12+x+x^2}\right) dx \\
&= \frac{x^2}{2} + \int \frac{1}{-12+x+x^2} \, dx \\
&= \frac{x^2}{2} + \frac{1}{7}\int \frac{1}{-3+x} \, dx - \frac{1}{7}\int \frac{1}{4+x} \, dx \\
&= \frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

fricas [A] time = 0.40, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

giac [A] time = 0.83, size = 20, normalized size = 0.77

$$\frac{1}{2}x^2 - \frac{1}{7}\log(|x+4|) + \frac{1}{7}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

maple [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^2}{2} + \frac{\ln(x-3)}{7} - \frac{\ln(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
[Out] 1/2*x^2+1/7*ln(x-3)-1/7*ln(x+4)
```

maxima [A] time = 0.51, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.54

$$\frac{x^2}{2} - \frac{2\,\mathrm{atanh}\left(\frac{2x}{7} + \frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

sympy [A] time = 0.11, size = 17, normalized size = 0.65

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

3.184 $\int \frac{3+2x}{(1+x)^2} \, dx$

Optimal. Leaf size=14

$$2\log(x+1)-\frac{1}{x+1}$$

```
[Out] -1/(1+x)+2*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(1+x)^2} \, dx &= \int \left(\frac{1}{(1+x)^2}+\frac{2}{1+x}\right) dx\\ &= -\frac{1}{1+x}+2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

fricas [A] time = 0.39, size = 17, normalized size = 1.21

$$\frac{2\,(x+1)\log\left(x+1\right)-1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (2*(x + 1)*log(x + 1) - 1)/(x + 1)
```

giac [A] time = 1.09, size = 15, normalized size = 1.07

$$-\frac{1}{x+1}+2\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] -1/(x + 1) + 2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2\ln\left(x+1\right)-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(x+1)^2,x)
```

```
[Out] -1/(x+1)+2*ln(x+1)
```

maxima [A] time = 0.49, size = 14, normalized size = 1.00

$$-\frac{1}{x+1}+2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/(x + 1) + 2*log(x + 1)
```

mupad [B] time = 0.03, size = 14, normalized size = 1.00

$$2\,\ln\left(x+1\right)-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(x + 1)^2,x)
```

```
[Out] 2*log(x + 1) - 1/(x + 1)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.71

$$2\log\left(x+1\right)-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)**2,x)
```

```
[Out] 2*log(x + 1) - 1/(x + 1)
```

3.185 $\int \frac{1}{x(1+x)(3+2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

```
[Out] 1/3*ln(x)-ln(1+x)+2/3*ln(3+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {72}

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x)(3+2x)}\,dx &= \int \left(\frac{1}{-1-x} + \frac{1}{3x} + \frac{4}{3(3+2x)}\right) dx\\ &= \frac{\log(x)}{3} - \log(1+x) + \frac{2}{3}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

fricas [A] time = 0.41, size = 19, normalized size = 0.83

$$\frac{2}{3}\log(2x+3) - \log(x+1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="fricas")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

giac [A] time = 0.86, size = 22, normalized size = 0.96

$$\frac{2}{3}\log(|2x+3|) - \log(|x+1|) + \frac{1}{3}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(2*x + 3)) - log(abs(x + 1)) + 1/3*log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\ln(x)}{3} - \ln(x+1) + \frac{2\ln(2x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)/(2*x+3),x)
```

```
[Out] 1/3*ln(x)-ln(x+1)+2/3*ln(2*x+3)
```

maxima [A] time = 0.51, size = 19, normalized size = 0.83

$$\frac{2}{3}\log(2x+3) - \log(x+1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="maxima")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

mupad [B] time = 0.08, size = 17, normalized size = 0.74

$$\frac{2\ln\left(x+\frac{3}{2}\right)}{3} - \ln(x+1) + \frac{\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x + 3)*(x + 1)),x)
```

```
[Out] (2*log(x + 3/2))/3 - log(x + 1) + log(x)/3
```

sympy [A] time = 0.14, size = 19, normalized size = 0.83

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2\log\left(x+\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x)
```

```
[Out] log(x)/3 - log(x + 1) + 2*log(x + 3/2)/3
```

3.186 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\ &= 2\log(1-x)+\log(x)+3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

fricas [A] time = 0.42, size = 15, normalized size = 0.88

$$3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

giac [A] time = 0.93, size = 18, normalized size = 1.06

$$3 \log(|x+3|) + 2 \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) + 2\ln(x-1) + 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
[Out] 3*ln(x+3)+2*ln(x-1)+ln(x)
```

maxima [A] time = 0.51, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x-1) + 3 \ln(x+3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x-1) + 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

3.187 $\int \frac{x}{4+4x+x^2} \, dx$

Optimal. Leaf size=12

$$\frac{2}{x+2} + \log(x+2)$$

[Out] 2/(2+x)+ln(2+x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {27, 43}

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[x/(4 + 4*x + x^2),x]

[Out] 2/(2 + x) + Log[2 + x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{4+4x+x^2} \, dx &= \int \frac{x}{(2+x)^2} \, dx \\ &= \int \left(-\frac{2}{(2+x)^2} + \frac{1}{2+x}\right) dx \\ &= \frac{2}{2+x} + \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(4 + 4*x + x^2),x]

[Out] 2/(2 + x) + Log[2 + x]

fricas [A] time = 0.41, size = 16, normalized size = 1.33

$$\frac{(x+2)\log(x+2)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

giac [A] time = 0.86, size = 13, normalized size = 1.08

$$\frac{2}{x+2} + \log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="giac")
```

```
[Out] 2/(x + 2) + log(abs(x + 2))
```

maple [A] time = 0.01, size = 13, normalized size = 1.08

$$\ln\left(x+2\right) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4*x+4),x)
```

```
[Out] 2/(x+2)+ln(x+2)
```

maxima [A] time = 0.45, size = 12, normalized size = 1.00

$$\frac{2}{x+2} + \log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$\ln\left(x+2\right) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x + x^2 + 4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.67

$$\log\left(x+2\right) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4*x+4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

3.188 $\int \frac{1}{(-1+x)^2(4+x)}\,dx$

Optimal. Leaf size=30

$$\frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(x+4)$$

```
[Out] 1/5/(1-x)-1/25*ln(1-x)+1/25*ln(4+x)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^2(4+x)}\,dx &= \int \left(\frac{1}{5(-1+x)^2}-\frac{1}{25(-1+x)}+\frac{1}{25(4+x)}\right)dx\\ &= \frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.73

$$\frac{1}{25}\left(-\frac{5}{x-1}-\log(x-1)+\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] (-5/(-1 + x) - Log[-1 + x] + Log[4 + x])/25
```

fricas [A] time = 0.41, size = 26, normalized size = 0.87

$$\frac{(x-1)\log(x+4)-(x-1)\log(x-1)-5}{25\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="fricas")
```

```
[Out] 1/25*((x - 1)*log(x + 4) - (x - 1)*log(x - 1) - 5)/(x - 1)
```

giac [**A**] time = 0.97, size = 21, normalized size = 0.70

$$-\frac{1}{5\,(x-1)} + \frac{1}{25}\,\log\left(\left|-\frac{5}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="giac")
[Out] -1/5/(x - 1) + 1/25*log(abs(-5/(x - 1) - 1))
```

maple [**A**] time = 0.01, size = 21, normalized size = 0.70

$$-\frac{\ln\left(x-1\right)}{25} + \frac{\ln\left(x+4\right)}{25} - \frac{1}{5\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^2/(x+4),x)
[Out] 1/25*ln(x+4)-1/5/(x-1)-1/25*ln(x-1)
```

maxima [**A**] time = 0.48, size = 20, normalized size = 0.67

$$-\frac{1}{5\,(x-1)} + \frac{1}{25}\,\log\left(x+4\right) - \frac{1}{25}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="maxima")
[Out] -1/5/(x - 1) + 1/25*log(x + 4) - 1/25*log(x - 1)
```

mupad [**B**] time = 0.06, size = 22, normalized size = 0.73

$$-\frac{\ln\left(\frac{x-1}{x+4}\right)}{25} - \frac{1}{5\;(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^2*(x + 4)),x)
[Out] - log((x - 1)/(x + 4))/25 - 1/(5*(x - 1))
```

sympy [**A**] time = 0.13, size = 19, normalized size = 0.63

$$-\frac{\log\left(x-1\right)}{25} + \frac{\log\left(x+4\right)}{25} - \frac{1}{5x-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(4+x),x)
[Out] -log(x - 1)/25 + log(x + 4)/25 - 1/(5*x - 5)
```

3.189 $\int \frac{x^2}{(-3+x)(2+x)^2} \, dx$

Optimal. Leaf size=28

$$\frac{4}{5(x+2)}+\frac{9}{25}\log(3-x)+\frac{16}{25}\log(x+2)$$

```
[Out] 4/5/(2+x)+9/25*ln(3-x)+16/25*ln(2+x)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {88}

$$\frac{4}{5(x+2)}+\frac{9}{25}\log(3-x)+\frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(-3+x)(2+x)^2} \, dx &= \int \left(\frac{9}{25(-3+x)}-\frac{4}{5(2+x)^2}+\frac{16}{25(2+x)}\right) dx\\ &= \frac{4}{5(2+x)}+\frac{9}{25}\log(3-x)+\frac{16}{25}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.93

$$\frac{4}{5(x+2)}+\frac{9}{25}\log(x-3)+\frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[-3 + x])/25 + (16*Log[2 + x])/25
```

fricas [A] time = 0.40, size = 27, normalized size = 0.96

$$\frac{16\,(x+2)\log\left(x+2\right)+9\,(x+2)\log\left(x-3\right)+20}{25\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="fricas")
```

```
[Out] 1/25*(16*(x + 2)*log(x + 2) + 9*(x + 2)*log(x - 3) + 20)/(x + 2)
```

giac [A] time = 0.97, size = 26, normalized size = 0.93

$$\frac{4}{5\,(x+2)} + \log\left(|x+2|\right) + \frac{9}{25}\,\log\left(\left|-\frac{5}{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="giac")
[Out] 4/5/(x + 2) + log(abs(x + 2)) + 9/25*log(abs(-5/(x + 2) + 1))
```

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{9\ln\left(x-3\right)}{25} + \frac{16\ln\left(x+2\right)}{25} + \frac{4}{5\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-3)/(x+2)^2,x)
[Out] 9/25*ln(x-3)+4/5/(x+2)+16/25*ln(x+2)
```

maxima [A] time = 0.56, size = 20, normalized size = 0.71

$$\frac{4}{5\,(x+2)} + \frac{16}{25}\,\log\left(x+2\right) + \frac{9}{25}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="maxima")
[Out] 4/5/(x + 2) + 16/25*log(x + 2) + 9/25*log(x - 3)
```

mupad [B] time = 0.21, size = 22, normalized size = 0.79

$$\frac{16\,\ln\left(x+2\right)}{25} + \frac{9\,\ln\left(x-3\right)}{25} + \frac{4}{5\,\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 2)^2*(x - 3)),x)
[Out] (16*log(x + 2))/25 + (9*log(x - 3))/25 + 4/(5*(x + 2))
```

sympy [A] time = 0.13, size = 22, normalized size = 0.79

$$\frac{9\log\left(x-3\right)}{25} + \frac{16\log\left(x+2\right)}{25} + \frac{4}{5x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3+x)/(2+x)**2,x)
[Out] 9*log(x - 3)/25 + 16*log(x + 2)/25 + 4/(5*x + 10)
```

3.190 $\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx$

Optimal. Leaf size=14

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 893}

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx &= \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx\\ &= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx\\ &= \frac{1}{x}+2\log(x)+3\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

fricas [A] time = 0.39, size = 18, normalized size = 1.29

$$\frac{3\,x\log\left(x+2\right)+2\,x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

giac [A] time = 0.88, size = 16, normalized size = 1.14

$$\frac{1}{x} + 3 \log(|x + 2|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2 \ln(x) + 3 \ln(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(x+2)
```

maxima [A] time = 0.51, size = 14, normalized size = 1.00

$$\frac{1}{x} + 3 \log(x + 2) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$3 \ln(x + 2) + 2 \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
```

```
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$2 \log(x) + 3 \log(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

3.191 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right)dx\\ &= \log(1-x)-2\log(2+x)-3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

fricas [A] time = 0.40, size = 17, normalized size = 0.89

$$-3\,\log\left(x+3\right)-2\,\log\left(x+2\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

giac [A] time = 0.90, size = 20, normalized size = 1.05

$$-3\,\log\left(|x+3|\right)-2\,\log\left(|x+2|\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 0.95

$$\ln(x-1) - 2\ln(x+2) - 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
[Out] -3*ln(x+3)-2*ln(x+2)+ln(x-1)
```

maxima [A] time = 0.50, size = 17, normalized size = 0.89

$$-3\log(x+3) - 2\log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

mupad [B] time = 0.18, size = 17, normalized size = 0.89

$$\ln(x-1) - 2\ln(x+2) - 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

sympy [A] time = 0.15, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.192 $\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(x^3+3\,x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

giac [A] time = 0.89, size = 14, normalized size = 0.93

$$\frac{1}{3}\log\left(\left|x^3+3\,x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x)
```

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

maxima [A] time = 0.55, size = 13, normalized size = 0.87

$$\frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
```

```
[Out] log(3*x^2 + x^3 + 4)/3
```

sympy [A] time = 0.11, size = 12, normalized size = 0.80

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

3.193 $\int \frac{1}{(-1+x)^2x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

[Out] 1/(1-x)-1/x-2*ln(1-x)+2*ln(x)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {44}

$$\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x)^2*x^2),x]

[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^2x^2}\,dx &= \int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx\\ &= \frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{x-1}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x)^2*x^2),x]

[Out] -(-1 + x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]

fricas [A] time = 0.39, size = 40, normalized size = 1.60

$$-\frac{2\left(x^2-x\right)\log\left(x-1\right)-2\left(x^2-x\right)\log(x)+2\,x-1}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^2/x^2,x, algorithm="fricas")

[Out] -(2*(x^2 - x)*log(x - 1) - 2*(x^2 - x)*log(x) + 2*x - 1)/(x^2 - x)

giac [A] time = 0.95, size = 30, normalized size = 1.20

$$-\frac{1}{x-1}+\frac{1}{\frac{1}{x-1}+1}+2\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/(1/(x - 1) + 1) + 2*log(abs(-1/(x - 1) - 1))
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$2\ln(x)-2\ln(x-1)-\frac{1}{x}-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^2/x^2,x)
```

```
[Out] -1/(x-1)-2*ln(x-1)-1/x+2*ln(x)
```

maxima [A] time = 0.44, size = 27, normalized size = 1.08

$$-\frac{2x-1}{x^2-x}-2\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - x) - 2*log(x - 1) + 2*log(x)
```

mupad [B] time = 0.05, size = 27, normalized size = 1.08

$$\frac{1}{x(x-1)}-\frac{2}{x-1}-2\ln\left(\frac{x-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x - 1)^2),x)
```

```
[Out] 1/(x*(x - 1)) - 2/(x - 1) - 2*log((x - 1)/x)
```

sympy [A] time = 0.12, size = 20, normalized size = 0.80

$$\frac{1-2x}{x^2-x}+2\log(x)-2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/x**2,x)
```

```
[Out] (1 - 2*x)/(x**2 - x) + 2*log(x) - 2*log(x - 1)
```

3.194 $\int \frac{x^2}{(1+x)^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{x+1}-\frac{1}{2(x+1)^2}+\log(x+1)$$

```
[Out] -1/2/(1+x)^2+2/(1+x)+ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{x+1}-\frac{1}{2(x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x)^3}\,dx &= \int \left(\frac{1}{(1+x)^3}-\frac{2}{(1+x)^2}+\frac{1}{1+x}\right)dx\\ &= -\frac{1}{2(1+x)^2}+\frac{2}{1+x}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{x+1}-\frac{1}{2(x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x)^3,x]
```

```
[Out] -1/2*1/(1 + x)^2 + 2/(1 + x) + Log[1 + x]
```

fricas [A] time = 0.40, size = 31, normalized size = 1.48

$$\frac{2\left(x^2+2x+1\right)\log(x+1)+4x+3}{2\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 2*x + 1)*log(x + 1) + 4*x + 3)/(x^2 + 2*x + 1)
```

giac [A] time = 0.93, size = 18, normalized size = 0.86

$$\frac{4\,x+3}{2\,(x+1)^2}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="giac")
[Out] 1/2*(4*x + 3)/(x + 1)^2 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.95

$$\ln\left(x+1\right)-\frac{1}{2\,(x+1)^2}+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^3,x)
[Out] -1/2/(x+1)^2+2/(x+1)+ln(x+1)
```

maxima [A] time = 0.52, size = 22, normalized size = 1.05

$$\frac{4\,x+3}{2\left(x^2+2\,x+1\right)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="maxima")
[Out] 1/2*(4*x + 3)/(x^2 + 2*x + 1) + log(x + 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 1.00

$$\ln\left(x+1\right)+\frac{2\,x+\frac{3}{2}}{x^2+2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + 1)^3,x)
[Out] log(x + 1) + (2*x + 3/2)/(2*x + x^2 + 1)
```

sympy [A] time = 0.10, size = 19, normalized size = 0.90

$$\frac{4x+3}{2x^2+4x+2}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**3,x)
[Out] (4*x + 3)/(2*x**2 + 4*x + 2) + log(x + 1)
```

3.195 $\int \frac{1}{-x^2+x^4} \, dx$

Optimal. Leaf size=8

$$\frac{1}{x}-\tanh^{-1}(x)$$

```
[Out] 1/x-arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 325, 207}

$$\frac{1}{x}-\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) - ArcTanh[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^2+x^4} \, dx &= \int \frac{1}{x^2\left(-1+x^2\right)} \, dx\\
&= \frac{1}{x}+\int \frac{1}{-1+x^2} \, dx\\
&= \frac{1}{x}-\tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 22, normalized size = 2.75

$$\frac{1}{x}+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) + Log[1 - x]/2 - Log[1 + x]/2
```

fricas [B] time = 0.41, size = 20, normalized size = 2.50

$$-\frac{x\log\left(x+1\right)-x\log\left(x-1\right)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="fricas")
[Out] -1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

giac [B] time = 0.89, size = 18, normalized size = 2.25

$$\frac{1}{x}-\frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="giac")
[Out] 1/x - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 17, normalized size = 2.12

$$\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2),x)
[Out] -1/2*ln(x+1)+1/x+1/2*ln(x-1)
```

maxima [A] time = 0.48, size = 16, normalized size = 2.00

$$\frac{1}{x}-\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="maxima")
[Out] 1/x - 1/2*log(x + 1) + 1/2*log(x - 1)
```

mupad [B] time = 0.17, size = 8, normalized size = 1.00

$$\frac{1}{x}-\mathrm{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2 - x^4),x)
[Out] 1/x - atanh(x)
```

sympy [B] time = 0.11, size = 15, normalized size = 1.88

$$\frac{\log\left(x-1\right)}{2}-\frac{\log\left(x+1\right)}{2}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2),x)
[Out] log(x - 1)/2 - log(x + 1)/2 + 1/x
```

3.196 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2} \log \left(x^4 - x^2 + 1\right)$$

```
[Out] 1/2*ln(x^4-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1587}

$$\frac{1}{2} \log \left(x^4 - x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x + 2x^3}{1 - x^2 + x^4} \, dx = \frac{1}{2} \log \left(1 - x^2 + x^4\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2} \log \left(x^4 - x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

fricas [A] time = 0.39, size = 13, normalized size = 0.87

$$\frac{1}{2} \log \left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

giac [A] time = 0.86, size = 13, normalized size = 0.87

$$\frac{1}{2} \log \left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
[Out] 1/2*ln(x^4-x^2+1)
```

maxima [A] time = 0.61, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
[Out] log(x^4 - x^2 + 1)/2
```

sympy [A] time = 0.10, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
[Out] log(x**4 - x**2 + 1)/2
```

3.197 $\int \frac{x^3}{1+x^2} dx$

Optimal. Leaf size=18

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 43}

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

fricas [A] time = 0.39, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

giac [A] time = 1.03, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - \frac{\ln\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1),x)
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

maxima [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$\frac{x^2}{2} - \frac{\ln\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 + 1),x)
[Out] x^2/2 - log(x^2 + 1)/2
```

sympy [A] time = 0.08, size = 12, normalized size = 0.67

$$\frac{x^2}{2} - \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1),x)
[Out] x**2/2 - log(x**2 + 1)/2
```

3.198 $\int \frac{-1+x}{2+2x+x^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

[Out] -2*arctan(1+x)+1/2*ln(x^2+2*x+2)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {634, 617, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(2 + 2*x + x^2),x]

[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{2+2x+x^2}\,dx &= \frac{1}{2}\int \frac{2+2x}{2+2x+x^2}\,dx-2\int \frac{1}{2+2x+x^2}\,dx\\
&= \frac{1}{2}\log\left(2+2x+x^2\right)+2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\
&= -2\tan^{-1}(1+x)+\frac{1}{2}\log\left(2+2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(2 + 2*x + x^2),x]
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

fricas [A] time = 0.40, size = 18, normalized size = 0.90

$$-2\arctan\left(x+1\right)+\frac{1}{2}\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="fricas")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

giac [A] time = 0.91, size = 18, normalized size = 0.90

$$-2\arctan\left(x+1\right)+\frac{1}{2}\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="giac")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-2\arctan\left(x+1\right)+\frac{\ln\left(x^2+2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x^2+2*x+2),x)
[Out] -2*arctan(x+1)+1/2*ln(x^2+2*x+2)
```

maxima [A] time = 1.34, size = 18, normalized size = 0.90

$$-2\arctan\left(x+1\right)+\frac{1}{2}\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="maxima")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

mupad [B] time = 0.16, size = 18, normalized size = 0.90

$$\frac{\ln\left(x^2+2\,x+2\right)}{2}-2\,\mathrm{atan}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(2*x + x^2 + 2),x)
[Out] log(2*x + x^2 + 2)/2 - 2*atan(x + 1)
```

sympy [A] time = 0.11, size = 17, normalized size = 0.85

$$\frac{\log\left(x^2+2x+2\right)}{2}-2\,\mathrm{atan}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2+2*x+2),x)
```

```
[Out] log(x**2 + 2*x + 2)/2 - 2*atan(x + 1)
```

3.199 $\int \frac{x}{1+x+x^2} \, dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

fricas [A] time = 0.39, size = 26, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

giac [A] time = 0.93, size = 26, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1),x)
```

```
[Out] 1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.17, size = 26, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

mupad [B] time = 0.16, size = 28, normalized size = 0.90

$$\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^2 + 1),x)
```

```
[Out] log(x + x^2 + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

sympy [**A**] time = 0.12, size = 34, normalized size = 1.10

$$\frac{\log\left(x^{2}+x+1\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1),x)
```

```
[Out] log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

3.200 $\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx$

Optimal. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right)dx\\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx\\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx\\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right)\\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

fricas [A] time = 0.40, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

giac [A] time = 0.94, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

maxima [A] time = 1.19, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.63

$$x + \frac{\ln\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
```

```
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

sympy [A] time = 0.12, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

3.201 $\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2} \log \left(x^2+1\right)+2 \log(1-x)-3 \tan^{-1}(x)$$

[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2} \log \left(x^2+1\right)+2 \log(1-x)-3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]

[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)} \, dx &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right) \, dx\\
&= 2 \log(1-x)+\int \frac{-3+x}{1+x^2} \, dx\\
&= 2 \log(1-x)-3 \int \frac{1}{1+x^2} \, dx+\int \frac{x}{1+x^2} \, dx\\
&= -3 \tan^{-1}(x)+2 \log(1-x)+\frac{1}{2} \log \left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

fricas [A] time = 0.42, size = 19, normalized size = 0.83

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

giac [A] time = 0.85, size = 20, normalized size = 0.87

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-3\arctan(x)+2\ln(x-1)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)
[Out] 2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)
```

maxima [A] time = 1.37, size = 19, normalized size = 0.83

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 1.09

$$2\ln(x-1)+\ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

sympy [A] time = 0.15, size = 19, normalized size = 0.83

$$2\log(x-1)+\frac{\log(x^2+1)}{2}-3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

3.202 $\int \frac{3+2x}{3x+x^3}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+2x}{3x+x^3}\,dx &= \int \frac{3+2x}{x\left(3+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{2-x}{3+x^2}\right)dx \\
&= \log(x)+\int \frac{2-x}{3+x^2}\,dx \\
&= \log(x)+2\int \frac{1}{3+x^2}\,dx-\int \frac{x}{3+x^2}\,dx \\
&= \frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(3+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$
-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(3*x + x^3),x]
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

fricas [A] time = 0.41, size = 23, normalized size = 0.82

$$
\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)-\frac{1}{2}\log\left(x^2+3\right)+\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

giac [A] time = 0.96, size = 24, normalized size = 0.86

$$
\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)-\frac{1}{2}\log\left(x^2+3\right)+\log\left(|x|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.86

$$
\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}+\ln(x)-\frac{\ln\left(x^2+3\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(x^3+3*x),x)
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*3^(1/2)*x)*3^(1/2)
```

maxima [A] time = 1.39, size = 23, normalized size = 0.82

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{2}\log\left(x^2+3\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

mupad [B] time = 0.30, size = 55, normalized size = 1.96

$$\ln(x)-\frac{\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)1\mathrm{i}}{3}+\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(3*x + x^3),x)
```

```
[Out] log(x) - log(x + 3^(1/2)*1i)/2 - log(x - 3^(1/2)*1i)/2 - (3^(1/2)*log(x - 3
^(1/2)*1i)*1i)/3 + (3^(1/2)*log(x + 3^(1/2)*1i)*1i)/3
```

sympy [A] time = 0.14, size = 29, normalized size = 1.04

$$\log(x)-\frac{\log\left(x^2+3\right)}{2}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**3+3*x),x)
```

```
[Out] log(x) - log(x**2 + 3)/2 + 2*sqrt(3)*atan(sqrt(3)*x/3)/3
```

3.203 $\int \frac{1}{-1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(-1),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

fricas [A] time = 0.40, size = 32, normalized size = 0.78

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

giac [A] time = 1.02, size = 33, normalized size = 0.80

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(abs(x - 1))
```

maple [A] time = 0.00, size = 33, normalized size = 0.80

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1),x)
[Out] 1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.26, size = 32, normalized size = 0.78

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

mupad [B] time = 0.06, size = 46, normalized size = 1.12

$$\frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 1),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [A] time = 0.14, size = 41, normalized size = 1.00

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

3.204 $\int \frac{x^3}{1+x^3} \, dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x-1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {321, 200, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+x^3}\,dx &= x-\int \frac{1}{1+x^3}\,dx\\ &= x-\frac{1}{3}\int \frac{1}{1+x}\,dx-\frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx\\ &= x-\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= x-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.02

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^3),x]
```

```
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

fricas [A] time = 0.41, size = 35, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

giac [A] time = 0.87, size = 36, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(abs(x + 1))
```

maple [A] time = 0.01, size = 36, normalized size = 0.88

$$x-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3+1),x)
```

```
[Out] x-1/3*ln(x+1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.46, size = 35, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

mupad [B] time = 0.23, size = 47, normalized size = 1.15

$$x-\frac{\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 + 1),x)
```

```
[Out] x - log(x + 1)/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.14, size = 42, normalized size = 1.02

$$x-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**3+1),x)
```

```
[Out] x - log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(
3)/3)/3
```

3.205 $\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

fricas [A] time = 0.40, size = 36, normalized size = 1.50

$$\frac{2\,(x-1)\arctan(x)-(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)+2}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

giac [B] time = 0.95, size = 47, normalized size = 1.96

$$\frac{1}{4}\pi-\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x)+\ln\left(x-1\right)-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)
```

```
[Out] ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)
```

maxima [A] time = 1.19, size = 20, normalized size = 0.83

$$\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.17

$$\ln\left(x-1\right)+\frac{1}{x-1}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

sympy [**A**] time = 0.15, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

3.206 $\int \frac{x^4}{-1+x^4} \, dx$

Optimal. Leaf size=14

$$x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

[Out] x-1/2*arctan(x)-1/2*arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {321, 212, 206, 203}

$$x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^4/(-1 + x^4),x]

[Out] x - ArcTan[x]/2 - ArcTanh[x]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{-1+x^4} \, dx &= x+\int \frac{1}{-1+x^4} \, dx\\ &= x-\frac{1}{2}\int \frac{1}{1-x^2} \, dx-\frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.86

$$x+\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-1 + x^4),x]
[Out] x - ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

fricas [A] time = 0.41, size = 18, normalized size = 1.29

$$x-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="fricas")
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

giac [A] time = 1.05, size = 20, normalized size = 1.43

$$x-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="giac")
[Out] x - 1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 1.36

$$x-\frac{\arctan(x)}{2}+\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1),x)
[Out] x+1/4*ln(x-1)-1/4*ln(x+1)-1/2*arctan(x)
```

maxima [A] time = 1.21, size = 18, normalized size = 1.29

$$x-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="maxima")
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.71

$$x-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 - 1),x)
[Out] x - atan(x)/2 - atanh(x)/2
```

sympy [A] time = 0.13, size = 19, normalized size = 1.36

$$x+\frac{\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{4}-\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1),x)
```

```
[Out] x + log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

3.207 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+6x-x^2+3x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\
&= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\
&= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\
&= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

fricas [A] time = 0.40, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

giac [A] time = 1.03, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.86

$$-3\arctan(x)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{3\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
[Out] -3*arctan(x)+3/2*ln(x^2+1)+2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.25, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

mupad [B] time = 0.07, size = 51, normalized size = 1.76

$$-\sqrt{2}\,\mathrm{atan}\left(\frac{24\sqrt{2}}{24\,x-64}+\frac{32\sqrt{2}\,x}{24\,x-64}\right)+\ln\left(x-\mathrm{i}\right)\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

sympy [**A**] time = 0.21, size = 29, normalized size = 1.00

$$\frac{3\log\left(x^2+1\right)}{2} - 3\,\mathrm{atan}\,(x) + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

3.208 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{4+5x^2+x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

fricas [A] time = 0.41, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

giac [A] time = 0.94, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\arctan(x)-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

maxima [**A**] time = 1.19, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

mupad [**B**] time = 0.19, size = 33, normalized size = 1.43

$$-\mathrm{atan}\left(\frac{1305}{4\,(144\,x-162)}+\frac{9}{8}\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
```

```
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

sympy [**A**] time = 0.20, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

3.209 $\int \frac{-3+x}{\left(4+2x+x^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{-4x-7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/6*(-7-4*x)/(x^2+2*x+4)-2/9*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {638, 618, 204}

$$-\frac{4x+7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] -(7 + 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x}{\left(4+2x+x^2\right)^2}\,dx &= -\frac{7+4x}{6\left(4+2x+x^2\right)}-\frac{2}{3}\int \frac{1}{4+2x+x^2}\,dx\\
&= -\frac{7+4x}{6\left(4+2x+x^2\right)}+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,2+2x\right)\\
&= -\frac{7+4x}{6\left(4+2x+x^2\right)}-\frac{2\tan^{-1}\left(\frac{1+x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{-4x-7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(4 + 2*x + x^2)^2,x]
[Out] (-7 - 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.40, size = 39, normalized size = 1.00

$$-\frac{4\sqrt{3}\left(x^2+2x+4\right)\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)+12x+21}{18\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="fricas")
[Out] -1/18*(4*sqrt(3)*(x^2 + 2*x + 4)*arctan(1/3*sqrt(3)*(x + 1)) + 12*x + 21)/(
x^2 + 2*x + 4)
```

giac [A] time = 0.93, size = 32, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{4x+7}{6\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="giac")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$-\frac{2\sqrt{3}\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\right)}{9}+\frac{-8x-14}{12x^2+24x+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)/(x^2+2*x+4)^2,x)
[Out] 1/12*(-8*x-14)/(x^2+2*x+4)-2/9*3^(1/2)*arctan(1/6*(2*x+2)*3^(1/2))
```

maxima [A] time = 1.10, size = 32, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{4x+7}{6\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.92

$$-\frac{\frac{2x}{3}+\frac{7}{6}}{x^2+2x+4}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + x^2 + 4)^2,x)
```

```
[Out] - ((2*x)/3 + 7/6)/(2*x + x^2 + 4) - (2*3^(1/2)*atan((3^(1/2)*x)/3 + 3^(1/2)
/3))/9
```

sympy [A] time = 0.13, size = 41, normalized size = 1.05

$$\frac{-4x-7}{6x^2+12x+24}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**2+2*x+4)**2,x)
```

```
[Out] (-4*x - 7)/(6*x**2 + 12*x + 24) - 2*sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.210 $\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1}+\log(x)$$

[Out] 1/(x^2+1)+ln(x)

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1252, 894}

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right) \, dx,x,x^2\right)\\
&= \frac{1}{1+x^2}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]

fricas [A] time = 0.41, size = 18, normalized size = 1.80

$$\frac{\left(x^2+1\right)\log(x)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

giac [A] time = 0.91, size = 14, normalized size = 1.40

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln(x)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x/(x^2+1)^2,x)
[Out] ln(x)+1/(x^2+1)
```

maxima [A] time = 0.44, size = 14, normalized size = 1.40

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 0.16, size = 10, normalized size = 1.00

$$\ln(x)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) + 1/(x^2 + 1)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.80

$$\log\left(x\right)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
[Out] log(x) + 1/(x**2 + 1)
```

3.211 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4334, 628}

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2}\,dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [B] time = 0.10, size = 26, normalized size = 2.36

$$\log(2-\sin(x))+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

fricas [A] time = 0.45, size = 15, normalized size = 1.36

$$\log\left(-\frac{1}{2}\sin(x)+1\right)+\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

giac [A] time = 1.04, size = 15, normalized size = 1.36

$$\log\left(-\sin(x)+2\right)+\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

maple [A] time = 0.05, size = 12, normalized size = 1.09

$$\ln\left(\sin^2(x)-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)
[Out] ln(2-3*sin(x)+sin(x)^2)
```

maxima [A] time = 0.56, size = 11, normalized size = 1.00

$$\log\left(\sin(x)^2-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

mupad [B] time = 0.09, size = 11, normalized size = 1.00

$$\ln\left(\sin(x)^2-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

sympy [A] time = 0.24, size = 12, normalized size = 1.09

$$\log\left(\sin\left(x\right)-2\right)+\log\left(\sin\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

3.212 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4335, 321, 203}

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^2}{5+x^2}\,dx,x,\cos(x)\right)\\ &= -\cos(x)+5\,\text{Subst}\left(\int \frac{1}{5+x^2}\,dx,x,\cos(x)\right)\\ &= \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)\end{aligned}$$

Mathematica [B] time = 0.23, size = 82, normalized size = 4.10

$$\frac{1}{20}\left(-20\cos(x)+21\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right)+21\sqrt{5}\tan^{-1}\left(\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)+\frac{1}{\sqrt{5}}\right)-\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

fricas [A] time = 0.45, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

giac [A] time = 0.80, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

maple [A] time = 0.03, size = 18, normalized size = 0.90

$$\sqrt{5}\, \arctan\left(\frac{\sqrt{5}\, \cos(x)}{5}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x)
```

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.50, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.85

$$\sqrt{5}\, \mathrm{atan}\left(\frac{\sqrt{5}\, \cos(x)}{5}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)
```

```
[Out] 5^(1/2)*atan((5^(1/2)*cos(x))/5) - cos(x)
```

<u>**sympy**</u> **[A]**   time = 0.54, size = 19, normalized size = 0.95

$$-\cos(x) + \sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

```
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

3.213 $\int \frac{1}{-3+2x+x^2} dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+3)$$

[Out] 1/4*ln(1-x)-1/4*ln(3+x)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {616, 31}

$$\frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x + x^2)^(-1),x]

[Out] Log[1 - x]/4 - Log[3 + x]/4

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-3+2x+x^2} dx &= \frac{1}{4}\int \frac{1}{-1+x} dx - \frac{1}{4}\int \frac{1}{3+x} dx\\ &= \frac{1}{4}\log(1-x) - \frac{1}{4}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2*x + x^2)^(-1),x]

[Out] Log[1 - x]/4 - Log[3 + x]/4

fricas [A] time = 0.40, size = 13, normalized size = 0.68

$$-\frac{1}{4}\log(x+3) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x-3),x, algorithm="fricas")

```
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

giac [A] time = 0.84, size = 15, normalized size = 0.79

$$-\frac{1}{4}\log\left(|x+3|\right)+\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="giac")
[Out] -1/4*log(abs(x + 3)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{\ln\left(x-1\right)}{4}-\frac{\ln\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x-3),x)
[Out] -1/4*ln(x+3)+1/4*ln(x-1)
```

maxima [A] time = 0.57, size = 13, normalized size = 0.68

$$-\frac{1}{4}\log\left(x+3\right)+\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="maxima")
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + x^2 - 3),x)
[Out] -atanh(x/2 + 1/2)/2
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$\frac{\log\left(x-1\right)}{4}-\frac{\log\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x-3),x)
[Out] log(x - 1)/4 - log(x + 3)/4
```

3.214 $\int \frac{1}{-2x+x^2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

[Out] 1/2*ln(2-x)-1/2*ln(x)

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {615}

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \frac{1}{-2x+x^2}\,dx=\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

fricas [A] time = 0.40, size = 11, normalized size = 0.65

$$\frac{1}{2}\log\left(x-2\right)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

giac [A] time = 0.90, size = 13, normalized size = 0.76

$$\frac{1}{2}\log\left(|x-2|\right)-\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x - 2)) - 1/2*log(abs(x))
```

maple [**A**] time = 0.01, size = 12, normalized size = 0.71

$$-\frac{\ln(x)}{2}+\frac{\ln\left(x-2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x),x)
[Out] 1/2*ln(x-2)-1/2*ln(x)
```

maxima [**A**] time = 0.52, size = 11, normalized size = 0.65

$$\frac{1}{2}\log\left(x-2\right)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="maxima")
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

mupad [**B**] time = 0.10, size = 6, normalized size = 0.35

$$-\operatorname{atanh}\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(2*x - x^2),x)
[Out] -atanh(x - 1)
```

sympy [**A**] time = 0.10, size = 10, normalized size = 0.59

$$-\frac{\log\left(x\right)}{2}+\frac{\log\left(x-2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x),x)
[Out] -log(x)/2 + log(x - 2)/2
```

3.215 $\int \frac{1+2x}{-7+12x+4x^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

```
[Out] 1/8*ln(1-2*x)+3/8*ln(7+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {632, 31}

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{-7+12x+4x^2}\,dx &= \frac{1}{2}\int \frac{1}{-2+4x}\,dx+\frac{3}{2}\int \frac{1}{14+4x}\,dx\\ &= \frac{1}{8}\log(1-2x)+\frac{3}{8}\log(7+2x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

fricas [A] time = 0.41, size = 17, normalized size = 0.81

$$\frac{3}{8}\,\log\left(2\,x+7\right)+\frac{1}{8}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="fricas")
```

```
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

giac [A] time = 0.98, size = 19, normalized size = 0.90

$$\frac{3}{8}\log\left(|2x+7|\right)+\frac{1}{8}\log\left(|2x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="giac")
[Out] 3/8*log(abs(2*x + 7)) + 1/8*log(abs(2*x - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{3\ln\left(2x+7\right)}{8}+\frac{\ln\left(2x-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(4*x^2+12*x-7),x)
[Out] 3/8*ln(7+2*x)+1/8*ln(2*x-1)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$\frac{3}{8}\log\left(2x+7\right)+\frac{1}{8}\log\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="maxima")
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

mupad [B] time = 0.20, size = 13, normalized size = 0.62

$$\frac{\ln\left(x-\frac{1}{2}\right)}{8}+\frac{3\,\ln\left(x+\frac{7}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(12*x + 4*x^2 - 7),x)
[Out] log(x - 1/2)/8 + (3*log(x + 7/2))/8
```

sympy [A] time = 0.12, size = 17, normalized size = 0.81

$$\frac{\log\left(x-\frac{1}{2}\right)}{8}+\frac{3\log\left(x+\frac{7}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x**2+12*x-7),x)
[Out] log(x - 1/2)/8 + 3*log(x + 7/2)/8
```

3.216 $\int \frac{x}{-1+x+x^2} dx$

Optimal. Leaf size=49

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] 1/10*ln(1+2*x-5^(1/2))*(5-5^(1/2))+1/10*ln(1+2*x+5^(1/2))*(5+5^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {632, 31}

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x + x^2),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{-1+x+x^2}\,dx &= \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx\\ &= \frac{1}{10}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{1}{10}\left(\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x + x^2),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) + (5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

fricas [A] time = 0.39, size = 44, normalized size = 0.90

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
*log(x^2 + x - 1)
```

giac [A] time = 0.96, size = 40, normalized size = 0.82

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)+\frac{1}{2}\log\left(\left|x^2+x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) + 1/2*log(
abs(x^2 + x - 1))
```

maple [A] time = 0.00, size = 27, normalized size = 0.55

$$\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x-1),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))
```

maxima [A] time = 1.32, size = 37, normalized size = 0.76

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) + 1/2*log(x^2 +
x - 1)
```

mupad [B] time = 0.18, size = 36, normalized size = 0.73

$$\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)-\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^2 - 1),x)
```

```
[Out] log(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 1/2) - log(x - 5^(1/2)/2 + 1/2)*(5^(
1/2)/10 - 1/2)
```

sympy [A] time = 0.11, size = 46, normalized size = 0.94

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x-1),x)
```

```
[Out] (sqrt(5)/10 + 1/2)*log(x + 1/2 + sqrt(5)/2) + (1/2 - sqrt(5)/10)*log(x - sq
rt(5)/2 + 1/2)
```

3.217 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} \, dx$

Optimal. Leaf size=63

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

```
[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln
(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int \left(-\frac{668}{323(1+2x)} - \frac{9438}{80155(-7+3x)} + \frac{24110}{4879(2+5x)} + \frac{48}{2600}\right.$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} +$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} +$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} +$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) +$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{453009\log\left(x^2+x+5\right) - 418418\log(7-3x) - 11023670\log(2x+1) + 10536070\log(5x+2) + 163508\sqrt{19}\tan}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

fricas [A] time = 0.42, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right) + \frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log(5x+2) - \frac{3146}{80155}\log(3x-7) - \frac{334}{323}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

giac [A] time = 1.03, size = 53, normalized size = 0.84

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right) + \frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log(|5x+2|) - \frac{3146}{80155}\log(|3x-7|) - \frac{3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{3988\sqrt{19}\arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)}{260015} - \frac{334\ln(2x+1)}{323} + \frac{4822\ln(5x+2)}{4879} - \frac{3146\ln(3x-7)}{80155} + \frac{11049\ln\left(x^2+x+5\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)+11049/260015*ln(x^2+x+5)+3988/260015*arctan(1/19*(2*x+1
)*19^(1/2))*19^(1/2)-334/323*ln(2*x+1)-3146/80155*ln(3*x-7)
```

maxima [A] time = 1.27, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

mupad [B] time = 0.28, size = 58, normalized size = 0.92

$$\frac{4822\ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\ln\left(x-\frac{7}{3}\right)}{80155}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)+\ln\left(x+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

sympy [A] time = 0.37, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

3.218 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}\,x\right)}{726\sqrt{2}}$$

```
[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/
7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}\,x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+31\ldots}{363\left(1+2x\ldots\right.}\right.$$

$$=\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int\frac{816+2843x}{1+2x^2}dx}{3993}$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{59096\log(2-5x)}{99825}\ldots$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}+\ldots$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.78

$$\frac{142150\log\left(2x^2+1\right)-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}-236384\log(2-5x)+12575\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

fricas [A] time = 0.41, size = 103, normalized size = 1.20

$$\frac{12575\sqrt{2}\left(10x^3-4x^2+5x-2\right)\arctan\left(\sqrt{2}x\right)-1203114x^2+142150\left(10x^3-4x^2+5x-2\right)\log\left(2x^2+1\right)\ldots}{399300\left(10x^3-4x^2+5x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

giac [A] time = 0.90, size = 59, normalized size = 0.69

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458x^2+4675x+2554}{12100\left(2x^2+1\right)(5x-2)}+\frac{2843}{7986}\log\left(2x^2+1\right)-\frac{59096}{99825}\log\left(|5x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

maple [A] time = 0.02, size = 54, normalized size = 0.63

$$\frac{503\sqrt{2}\arctan\left(\sqrt{2}x\right)}{15972}-\frac{59096\ln(5x-2)}{99825}+\frac{2843\ln\left(2x^2+1\right)}{7986}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}-\frac{5828}{9075(5x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
)
```

```
[Out] 1/3993*(-2761/4*x-3443/8)/(x^2+1/2)+2843/7986*ln(2*x^2+1)+503/15972*arctan(
2^(1/2)*x)*2^(1/2)-5828/9075/(5*x-2)-59096/99825*ln(5*x-2)
```

maxima [A] time = 1.20, size = 59, normalized size = 0.69

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458x^2+4675x+2554}{12100\left(10x^3-4x^2+5x-2\right)}+\frac{2843}{7986}\log\left(2x^2+1\right)-\frac{59096}{99825}\log\left(5x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

mupad [B] time = 0.13, size = 71, normalized size = 0.83

$$-\frac{59096\ln\left(x-\frac{2}{5}\right)}{99825}-\frac{\frac{18229x^2}{60500}+\frac{17x}{440}+\frac{1277}{60500}}{x^3-\frac{2x^2}{5}+\frac{x}{2}-\frac{1}{5}}-\ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,5}{319}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

sympy [A] time = 0.23, size = 65, normalized size = 0.76

$$\frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200}-\frac{59096\log\left(x-\frac{2}{5}\right)}{99825}+\frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986}+\frac{503\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

3.219 $\int \frac{\sqrt{4+x}}{x} dx$

Optimal. Leaf size=24

$$2\sqrt{x+4} - 4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

```
[Out] -4*arctanh(1/2*(4+x)^(1/2))+2*(4+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {50, 63, 207}

$$2\sqrt{x+4} - 4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4+x}}{x} dx &= 2\sqrt{4+x} + 4\int \frac{1}{x\sqrt{4+x}} dx \\
&= 2\sqrt{4+x} + 8\,\text{Subst}\left(\int \frac{1}{-4+x^2} dx, x, \sqrt{4+x}\right) \\
&= 2\sqrt{4+x} - 4\tanh^{-1}\left(\frac{\sqrt{4+x}}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$2\sqrt{x+4} - 4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x]/x,x]
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

fricas [A] time = 0.41, size = 28, normalized size = 1.17

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="fricas")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

giac [A] time = 0.85, size = 29, normalized size = 1.21

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\left|\sqrt{x+4}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="giac")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(abs(sqrt(x + 4) - 2))
```

maple [A] time = 0.01, size = 29, normalized size = 1.21

$$2\ln\left(\sqrt{x+4}-2\right) - 2\ln\left(\sqrt{x+4}+2\right) + 2\sqrt{x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)^(1/2)/x,x)
[Out] 2*(x+4)^(1/2)+2*ln((x+4)^(1/2)-2)-2*ln((x+4)^(1/2)+2)
```

maxima [A] time = 0.51, size = 28, normalized size = 1.17

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="maxima")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.75

$$2\sqrt{x+4} - 4\,\mathrm{atanh}\left(\frac{\sqrt{x+4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)^(1/2)/x,x)
[Out] 2*(x + 4)^(1/2) - 4*atanh((x + 4)^(1/2)/2)
```

<u>**sympy**</u> **[A]** time = 0.92, size = 44, normalized size = 1.83

$$\begin{cases} 2\sqrt{x+4} - 4\operatorname{acoth}\left(\frac{\sqrt{x+4}}{2}\right) & \text{for } \frac{|x+4|}{4} > 1 \\ 2\sqrt{x+4} - 4\operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)**(1/2)/x,x)
```

```
[Out] Piecewise((2*sqrt(x + 4) - 4*acoth(sqrt(x + 4)/2), Abs(x + 4)/4 > 1), (2*sq
rt(x + 4) - 4*atanh(sqrt(x + 4)/2), True))
```

3.220 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1)
-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*arc
tan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*ar
ctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
egQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + 6\,\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} - \frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{}\right.$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) + \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x}\right.}{\sqrt{5}}$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\,\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log$$

$$= 2\sqrt{x} + 6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\,\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}}\left(5+\sqrt{5}\right.\right.$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.11

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{5},1;\frac{8}{5};x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

fricas [B] time = 1.35, size = 638, normalized size = 3.19

$$\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

giac [A] time = 1.63, size = 139, normalized size = 0.70

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

maple [A] time = 0.05, size = 175, normalized size = 0.88

$$\frac{12\sqrt{5}\ \arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{12\sqrt{5}\ \arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\sqrt{5}\ \ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\ x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^
(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+12/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1
/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1
/6)*5^(1/2))+3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-12/5/(10-
2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)
+6/5*ln(x^(1/6)-1)
```

maxima [B] time = 1.36, size = 272, normalized size = 1.36

$$-\frac{6}{5}\left(-1\right)^{\frac{3}{5}}\log\left(\left(-1\right)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}\left(-1\right)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+\left(-1\right)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+\left(-1\right)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}\left(-1\right)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)}{\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)}\right)}{5\sqrt{-2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

mupad [B] time = 0.24, size = 223, normalized size = 1.12

$$\frac{6\ \ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\ \sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\ \sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

3.221 $\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

```
[Out] -1/5*arctanh(3/5*cos(x)+4/5*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] + 4*Sin[x])/5]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x)+4\sin(x)\right)\\ &= -\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)+4\sin(x))\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] Log[Cos[x/2] - 2*Sin[x/2]]/5 - Log[2*Cos[x/2] + Sin[x/2]]/5
```

fricas [B] time = 0.44, size = 27, normalized size = 1.50

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) + 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) - 2*sin(x) +
5/2)
```

giac [A] time = 0.96, size = 23, normalized size = 1.28

$$\frac{1}{5}\log\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)-\frac{1}{5}\log\left(\left|\tan\left(\frac{1}{2}x\right)+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*log(abs(2*tan(1/2*x) - 1)) - 1/5*log(abs(tan(1/2*x) + 2))
```

maple [A] time = 0.06, size = 22, normalized size = 1.22

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}+\frac{\ln\left(2\tan\left(\frac{x}{2}\right)-1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] -1/5*ln(tan(1/2*x)+2)+1/5*ln(2*tan(1/2*x)-1)
```

maxima [B] time = 0.55, size = 30, normalized size = 1.67

$$\frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}-1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) - 1) - 1/5*log(sin(x)/(cos(x) + 1) + 2)
```

mupad [B] time = 0.49, size = 11, normalized size = 0.61

$$-\frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{5}+\frac{3}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*cos(x) - 3*sin(x)),x)
```

```
[Out] -(2*atanh((4*tan(x/2))/5 + 3/5))/5
```

sympy [A] time = 0.29, size = 20, normalized size = 1.11

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{2}\right)}{5}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] log(tan(x/2) - 1/2)/5 - log(tan(x/2) + 2)/5
```

3.222 $\int \frac{1}{1+\sqrt{x}} dx$

Optimal. Leaf size=18

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

```
[Out] -2*ln(1+x^(1/2))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 43}

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt{x}} dx &= 2\,\text{Subst}\left(\int \frac{x}{1+x} dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

giac [A] time = 1.05, size = 14, normalized size = 0.78

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

maple [A] time = 0.00, size = 27, normalized size = 1.50

$$-\ln(x-1) + \ln\left(\sqrt{x} - 1\right) - \ln\left(\sqrt{x} + 1\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+1),x)
[Out] 2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)-ln(x-1)
```

maxima [A] time = 0.61, size = 15, normalized size = 0.83

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

mupad [B] time = 0.06, size = 14, normalized size = 0.78

$$2\sqrt{x} - 2\ln\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1),x)
[Out] 2*x^(1/2) - 2*log(x^(1/2) + 1)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.83

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2)),x)
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

3.223 $\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=32

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\frac{1}{\sqrt[3]{x}}+1\right)-\log(x)$$

```
[Out] 3*x^(1/3)-3/2*x^(2/3)+x-3*ln(1+1/x^(1/3))-ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 44}

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\frac{1}{\sqrt[3]{x}}+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(1+x)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{1}{x^3}+\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= 3\sqrt[3]{x}-\frac{3x^{2/3}}{2}+x-3\log\left(1+\frac{1}{\sqrt[3]{x}}\right)-\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(1/3)]
```

fricas [A] time = 0.41, size = 20, normalized size = 0.62

$$x-\frac{3}{2}\,x^{\frac{2}{3}}+3\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="fricas")
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

giac [A] time = 0.92, size = 20, normalized size = 0.62

$$x-\frac{3}{2}\,x^{\frac{2}{3}}+3\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="giac")
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.66

$$x-3\ln\left(x^{\frac{1}{3}}+1\right)-\frac{3x^{\frac{2}{3}}}{2}+3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/x^(1/3)),x)
[Out] x-3/2*x^(2/3)+3*x^(1/3)-3*ln(x^(1/3)+1)
```

maxima [A] time = 0.51, size = 28, normalized size = 0.88

$$-\frac{1}{2}\,x\left(\frac{3}{x^{\frac{1}{3}}}-\frac{6}{x^{\frac{2}{3}}}-2\right)-\log\left(x\right)-3\,\log\left(\frac{1}{x^{\frac{1}{3}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="maxima")
[Out] -1/2*x*(3/x^(1/3) - 6/x^(2/3) - 2) - log(x) - 3*log(1/x^(1/3) + 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.62

$$x-3\,\ln\left(x^{1/3}+1\right)+3\,x^{1/3}-\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3) + 1),x)
[Out] x - 3*log(x^(1/3) + 1) + 3*x^(1/3) - (3*x^(2/3))/2
```

sympy [A] time = 0.14, size = 26, normalized size = 0.81

$$-\frac{3x^{\frac{2}{3}}}{2}+3\sqrt[3]{x}+x-3\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x**(1/3)),x)
[Out] -3*x**(2/3)/2 + 3*x**(1/3) + x - 3*log(x**(1/3) + 1)
```

3.224 $\int \frac{\sqrt{x}}{1+x} dx$

Optimal. Leaf size=16

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {50, 63, 203}

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{1+x} \, dx &= 2\sqrt{x} - \int \frac{1}{\sqrt{x}(1+x)} \, dx \\ &= 2\sqrt{x} - 2\,\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{x}\right) \\ &= 2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

giac [A] time = 0.91, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$-2\arctan\left(\sqrt{x}\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+1),x)
```

```
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

maxima [A] time = 1.24, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1),x)
```

```
[Out] 2*x^(1/2) - 2*atan(x^(1/2))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.88

$$2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x),x)
```

```
[Out] 2*sqrt(x) - 2*atan(sqrt(x))
```

3.225 $\int \frac{1}{x\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] -2*arctanh((1+x)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {63, 207}

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]),x]
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= -2\tanh^{-1}\left(\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x]),x]
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

fricas [B] time = 0.42, size = 19, normalized size = 1.90

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

giac [B] time = 0.95, size = 20, normalized size = 2.00

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-2\,\mathrm{arctanh}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(1/2),x)
[Out] -2*arctanh((x+1)^(1/2))
```

maxima [B] time = 0.51, size = 19, normalized size = 1.90

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="maxima")
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

mupad [B] time = 0.16, size = 8, normalized size = 0.80

$$-2\,\mathrm{atanh}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(1/2)),x)
[Out] -2*atanh((x + 1)^(1/2))
```

sympy [A] time = 0.60, size = 26, normalized size = 2.60

$$\begin{cases} -2\,\mathrm{acoth}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2\,\mathrm{atanh}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2),x)
[Out] Piecewise((-2*acoth(sqrt(x + 1)), Abs(x + 1) > 1), (-2*atanh(sqrt(x + 1)),
True))
```

3.226 $\int \frac{1}{-\sqrt[3]{x}+x}\,dx$

Optimal. Leaf size=14

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

[Out] 3/2*ln(1-x^(2/3))

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

[In] Int[(-x^(1/3) + x)^(-1),x]

[Out] (3*Log[1 - x^(2/3)])/2

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt[3]{x}+x}\,dx &= \int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx\\ &= \frac{3}{2}\log\left(1-x^{2/3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-x^(1/3) + x)^(-1),x]

[Out] (3*Log[1 - x^(2/3)])/2

fricas [A] time = 0.41, size = 8, normalized size = 0.57

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x),x, algorithm="fricas")

```
[Out] 3/2*log(x^(2/3) - 1)
```

giac [A] time = 1.24, size = 18, normalized size = 1.29

$$\frac{3}{2}\,\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="giac")
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(abs(x^(1/3) - 1))
```

maple [B] time = 0.02, size = 50, normalized size = 3.57

$$\frac{\ln\left(x-1\right)}{2}+\frac{\ln\left(x+1\right)}{2}+\ln\left(x^{\frac{1}{3}}-1\right)+\ln\left(x^{\frac{1}{3}}+1\right)-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{2}-\frac{\ln\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/3)+x),x)
[Out] 1/2*ln(x-1)+1/2*ln(x+1)+ln(x^(1/3)-1)-1/2*ln(x^(2/3)+x^(1/3)+1)+ln(x^(1/3)+
1)-1/2*ln(x^(2/3)-x^(1/3)+1)
```

maxima [A] time = 0.51, size = 17, normalized size = 1.21

$$\frac{3}{2}\,\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="maxima")
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(x^(1/3) - 1)
```

mupad [B] time = 0.15, size = 8, normalized size = 0.57

$$\frac{3\,\ln\left(x^{2/3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(1/3)),x)
[Out] (3*log(x^(2/3) - 1))/2
```

sympy [B] time = 0.20, size = 22, normalized size = 1.57

$$\frac{3\log\left(\sqrt[3]{x}-1\right)}{2}+\frac{3\log\left(\sqrt[3]{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/3)+x),x)
[Out] 3*log(x**(1/3) - 1)/2 + 3*log(x**(1/3) + 1)/2
```

3.227 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] 4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt{2+x}\right)+\frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

fricas [A] time = 0.42, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

giac [A] time = 0.96, size = 22, normalized size = 0.71

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

maple [B] time = 0.02, size = 54, normalized size = 1.74

$$\frac{2\ln(x-2)}{3}+\frac{\ln(x+1)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}+\frac{2\ln\left(\sqrt{x+2}-2\right)}{3}-\frac{\ln\left(\sqrt{x+2}-1\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+2)^(1/2)),x)
```

```
[Out] 1/3*ln(x+1)+2/3*ln(x-2)+1/3*ln(1+(x+2)^(1/2))+2/3*ln((x+2)^(1/2)-2)-2/3*ln(
(x+2)^(1/2)+2)-1/3*ln((x+2)^(1/2)-1)
```

maxima [A] time = 0.60, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

mupad [B] time = 0.19, size = 25, normalized size = 0.81

$$\frac{2\,\ln\left(\frac{2\,\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\,\ln\left(\frac{4}{3}-\frac{2\,\sqrt{x+2}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2)),x)
```

```
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

sympy [A] time = 2.76, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

3.228 $\int \frac{x^2}{\sqrt{-1+x}} \, dx$

Optimal. Leaf size=32

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[Out] 4/3*(-1+x)^(3/2)+2/5*(-1+x)^(5/2)+2*(-1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 + x],x]
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-1+x}}\,dx &= \int \left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\\ &= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}+\frac{2}{5}(-1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 + x],x]
[Out] (2*Sqrt[-1 + x]*(8 + 4*x + 3*x^2))/15
```

fricas [A] time = 0.41, size = 17, normalized size = 0.53

$$\frac{2}{15}\left(3x^2+4x+8\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

giac [A] time = 1.01, size = 22, normalized size = 0.69

$$\frac{2}{5}\,(x-1)^{\frac{5}{2}}+\frac{4}{3}\,(x-1)^{\frac{3}{2}}+2\,\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="giac")
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.56

$$\frac{2\sqrt{x-1}\,\left(3x^{2}+4x+8\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-1)^(1/2),x)
[Out] 2/15*(x-1)^(1/2)*(3*x^2+4*x+8)
```

maxima [A] time = 0.69, size = 22, normalized size = 0.69

$$\frac{2}{5}\,(x-1)^{\frac{5}{2}}+\frac{4}{3}\,(x-1)^{\frac{3}{2}}+2\,\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.59

$$\frac{2\,\sqrt{x-1}\,\left(10\,x+3\,{(x-1)}^{2}+5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x - 1)^(1/2),x)
[Out] (2*(x - 1)^(1/2)*(10*x + 3*(x - 1)^2 + 5))/15
```

sympy [A] time = 1.33, size = 76, normalized size = 2.38

$$\begin{cases} \frac{2x^{2}\sqrt{x-1}}{5}+\frac{8x\sqrt{x-1}}{15}+\frac{16\sqrt{x-1}}{15} & \text{for}\ |x|>1 \\ \frac{2ix^{2}\sqrt{1-x}}{5}+\frac{8ix\sqrt{1-x}}{15}+\frac{16i\sqrt{1-x}}{15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)**(1/2),x)
[Out] Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15 + 16*sqrt(x - 1)/15, A
bs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 - x)/15 + 16*I*sqrt(1 -
x)/15, True))
```

3.229 $\int \frac{\sqrt{-1+x}}{1+x}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

```
[Out] -2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(-1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 203}

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{1+x}\,dx &= 2\sqrt{-1+x}-2\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\ &= 2\sqrt{-1+x}-4\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= 2\sqrt{-1+x}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x),x]
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

fricas [A] time = 0.40, size = 24, normalized size = 0.77

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="fricas")
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

giac [A] time = 1.06, size = 24, normalized size = 0.77

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="giac")
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

maple [A] time = 0.01, size = 25, normalized size = 0.81

$$-2\sqrt{2}\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/(x+1),x)
[Out] -2*arctan(1/2*(x-1)^(1/2)*2^(1/2))*2^(1/2)+2*(x-1)^(1/2)
```

maxima [A] time = 1.28, size = 24, normalized size = 0.77

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="maxima")
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

mupad [B] time = 0.16, size = 24, normalized size = 0.77

$$2\sqrt{x-1}-2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1),x)
```

```
[Out] 2*(x - 1)^(1/2) - 2*2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2)
```

sympy [**A**] time = 1.33, size = 76, normalized size = 2.45

$$\begin{cases} 2\sqrt{x-1} + 2\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for } \frac{|x+1|}{2} > 1 \\ 2i\sqrt{1-x} + \sqrt{2}\,i\log(x+1) - 2\sqrt{2}\,i\log\left(\sqrt{\frac{1}{2}-\frac{x}{2}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((2*sqrt(x - 1) + 2*sqrt(2)*asin(sqrt(2)/sqrt(x + 1)), Abs(x + 1)/
2 > 1), (2*I*sqrt(1 - x) + sqrt(2)*I*log(x + 1) - 2*sqrt(2)*I*log(sqrt(1/2
- x/2) + 1), True))
```

3.230 $\int \frac{1}{\sqrt{1+\sqrt{x}}} \, dx$

Optimal. Leaf size=29

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

```
[Out] 4/3*(1+x^(1/2))^(3/2)-4*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {190, 43}

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sqrt[x]],x]
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\sqrt{x}}} \, dx &= 2 \operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x}} \, dx, x, \sqrt{x}\right) \\
&= 2 \operatorname{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right) dx, x, \sqrt{x}\right) \\
&= -4\sqrt{1+\sqrt{x}}+\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{4}{3}\left(\sqrt{x}-2\right)\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[x]],x]
[Out] (4*(-2 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/3
```

fricas [A] time = 0.40, size = 14, normalized size = 0.48

$$\frac{4}{3}\sqrt{\sqrt{x}+1}\left(\sqrt{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(x) + 1)*(sqrt(x) - 2)
```

giac [A] time = 0.97, size = 19, normalized size = 0.66

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

maple [A] time = 0.01, size = 20, normalized size = 0.69

$$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 4/3*(x^(1/2)+1)^(3/2)-4*(x^(1/2)+1)^(1/2)
```

maxima [A] time = 0.56, size = 19, normalized size = 0.66

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

mupad [B] time = 0.24, size = 12, normalized size = 0.41

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1)^(1/2),x)
```

```
[Out] x*hypergeom([1/2, 2], 3, -x^(1/2))
```

sympy [B] time = 0.91, size = 117, normalized size = 4.03

$$-\frac{4x^{\frac{5}{2}}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}+\frac{8x^{\frac{5}{2}}}{3x^{\frac{5}{2}}+3x^2}+\frac{4x^3\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}-\frac{8x^2\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}+\frac{8x^2}{3x^{\frac{5}{2}}+3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2))**(1/2),x)
```

```
[Out] -4*x**(5/2)*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**(5/2)/(3*x**(5/2
) + 3*x**2) + 4*x**3*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) - 8*x**2*sqrt(
sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**2/(3*x**(5/2) + 3*x**2)
```

3.231 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

Optimal. Leaf size=8

$$2 \tan^{-1}\left(\sqrt{x}\right)$$

[Out] 2*arctan(x^(1/2))

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {647, 63, 203}

$$2 \tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{x+x^2} \, dx &= \int \frac{1}{\sqrt{x}(1+x)} \, dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{x}\right) \\ &= 2 \tan^{-1}\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2 \tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]

fricas [**A**] time = 0.40, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

giac [**A**] time = 0.92, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

maple [**A**] time = 0.01, size = 7, normalized size = 0.88

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x)
[Out] 2*arctan(x^(1/2))
```

maxima [**A**] time = 1.22, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

mupad [**B**] time = 0.24, size = 6, normalized size = 0.75

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^2),x)
[Out] 2*atan(x^(1/2))
```

sympy [**A**] time = 0.31, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
[Out] 2*atan(sqrt(x))
```

3.232 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

```
[Out] x+4*ln(1-x^(1/2))+4*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {376, 77}

$$x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx, x, \sqrt{x}\right)\\ &= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.95

$$x+4\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])
```

fricas [A] time = 0.39, size = 15, normalized size = 0.71

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

giac [A] time = 0.85, size = 16, normalized size = 0.76

$$x + 4\sqrt{x} + 4\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/2)-1),x)
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

maxima [A] time = 0.53, size = 15, normalized size = 0.71

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

mupad [B] time = 0.17, size = 15, normalized size = 0.71

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

sympy [A] time = 0.16, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

3.233 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal. Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {374, 376, 77}

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}} \, dx \\
&= 3\,\text{Subst}\left(\int \frac{x^2(1+x)}{1-x} \, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(-2 - \frac{2}{-1+x} - 2x - x^2\right) dx, x, \sqrt[3]{x}\right) \\
&= -6\sqrt[3]{x} - 3x^{2/3} - x - 6\log\left(1 - \sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

fricas [A] time = 0.42, size = 22, normalized size = 0.73

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

giac [A] time = 1.00, size = 23, normalized size = 0.77

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

maple [A] time = 0.00, size = 23, normalized size = 0.77

$$-x - 6\ln\left(x^{\frac{1}{3}} - 1\right) - 3x^{\frac{2}{3}} - 6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

maxima [A] time = 0.61, size = 22, normalized size = 0.73

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.73

$$-x - 6\,\ln\left(x^{1/3} - 1\right) - 6\,x^{1/3} - 3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

sympy [**A**] time = 0.18, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

3.234 $\int \frac{x^3}{\sqrt[3]{1+x^2}} \, dx$

Optimal. Leaf size=27

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

```
[Out] -3/4*(x^2+1)^(2/3)+3/10*(x^2+1)^(5/3)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2)^(1/3),x]
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1+x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1+x}}+(1+x)^{2/3}\right) dx, x, x^2\right)\\
&= -\frac{3}{4}\left(1+x^2\right)^{2/3}+\frac{3}{10}\left(1+x^2\right)^{5/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{3}{20}\left(x^2+1\right)^{2/3}\left(2x^2-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2)^(1/3),x]
[Out] (3*(1 + x^2)^(2/3)*(-3 + 2*x^2))/20
```

fricas [A] time = 0.41, size = 16, normalized size = 0.59

$$\frac{3}{20}\left(2x^2-3\right)\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="fricas")
[Out] 3/20*(2*x^2 - 3)*(x^2 + 1)^(2/3)
```

giac [A] time = 1.10, size = 19, normalized size = 0.70

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="giac")
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

maple [A] time = 0.00, size = 17, normalized size = 0.63

$$\frac{3\left(x^2+1\right)^{\frac{2}{3}}\left(2x^2-3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1)^(1/3),x)
[Out] 3/20*(x^2+1)^(2/3)*(2*x^2-3)
```

maxima [A] time = 0.48, size = 19, normalized size = 0.70

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

mupad [B] time = 0.26, size = 16, normalized size = 0.59

$$\frac{3\left(x^2+1\right)^{2/3}\left(2\,x^2-3\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 + 1)^(1/3),x)
[Out] (3*(x^2 + 1)^(2/3)*(2*x^2 - 3))/20
```

sympy [A] time = 0.94, size = 26, normalized size = 0.96

$$\frac{3x^2\left(x^2+1\right)^{\frac{2}{3}}}{10}-\frac{9\left(x^2+1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1)**(1/3),x)
[Out] 3*x**2*(x**2 + 1)**(2/3)/10 - 9*(x**2 + 1)**(2/3)/20
```

3.235 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}\right.$$

```
[Out] 6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*
(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5*a
rctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/
2)
```

Rubi [A] time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\,\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x-\frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log$$

$$= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right.}\right.$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x}\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};x^{5/6}\right)+x+6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]
```

fricas [B] time = 1.33, size = 547, normalized size = 2.72

$$-\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

giac [A] time = 1.60, size = 140, normalized size = 0.70

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}(\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

maple [A] time = 0.03, size = 242, normalized size = 1.20

$$x-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{6\sqrt{5}\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{6\sqrt{5}\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)-3/10*ln(2*x^(1/3)+x^(1/6)-5^(1/2)*x^(1/6)+2)+3/10*5^(1/2)*ln(2*
x^(1/3)+x^(1/6)-5^(1/2)*x^(1/6)+2)-6/(10+2*5^(1/2))^(1/2)*arctan((4*x^(1/6)
+1-5^(1/2))/(10+2*5^(1/2))^(1/2))-6/5/(10+2*5^(1/2))^(1/2)*5^(1/2)*arctan((
4*x^(1/6)+1-5^(1/2))/(10+2*5^(1/2))^(1/2))-3/10*5^(1/2)*ln(2*x^(1/3)+x^(1/6
)+5^(1/2)*x^(1/6)+2)-3/10*ln(2*x^(1/3)+x^(1/6)+5^(1/2)*x^(1/6)+2)-6/(10-2*5
^(1/2))^(1/2)*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*
5^(1/2))^(1/2)*5^(1/2)*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+6
/5*ln(x^(1/6)-1)
```

maxima [B] time = 1.22, size = 293, normalized size = 1.46

$$-\frac{3\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}\cdots}{\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}\cdots}\right)}{5\sqrt{-2\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

mupad [B] time = 0.06, size = 208, normalized size = 1.03

$$x+\frac{6\,\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}-270\,\sqrt{5}+1080\,x^{1/6}+270\right)\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\,\sqrt{5}}{10}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

sympy [A] time = 25.12, size = 311, normalized size = 1.55

$$6\sqrt[6]{x}+x+\frac{6\log\left(\sqrt[6]{x}-1\right)}{5}-\frac{3\sqrt{5}\log\left(8\sqrt[6]{x}+8\sqrt{5}\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}-\frac{3\log\left(8\sqrt[6]{x}+8\sqrt{5}\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}-\frac{3\log\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + x + 6*log(x**(1/6) - 1)/5 - 3*sqrt(5)*log(8*x**(1/6) + 8*sqrt(
5)*x**(1/6) + 16*x**(1/3) + 16)/10 - 3*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6)
+ 16*x**(1/3) + 16)/10 - 3*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/
3) + 16)/10 + 3*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3)
+ 16)/10 - 3*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqr
t(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/5 -
 3*sqrt(2)*sqrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sq
rt(10)/(2*sqrt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/5
```

3.236 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3^
(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx &= \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx\\
&= 4\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}-4\,\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right)-\frac{4}{3}\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)-2\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x,\right.\\
&= 2\sqrt{x}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)+4\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.39

$$-2\sqrt{x}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-x^{3/4}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])
```

fricas [A] time = 0.42, size = 47, normalized size = 0.76

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

giac [A] time = 0.96, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.74

$$-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(x^{\frac{1}{4}}+1\right)}{3}-\frac{2\ln\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)}{3}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/
3*(2*x^(1/4)-1)*3^(1/2))
```

maxima [A] time = 1.28, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

mupad [B] time = 0.17, size = 73, normalized size = 1.18

$$\frac{4\ln\left(16x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)-\ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
```

```
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

sympy [A] time = 0.65, size = 68, normalized size = 1.10

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

3.237 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12$$

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx$$

$$= 12\,\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx, x, \sqrt[12]{x}\right)$$

$$= 12\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}+x^{12}-x^{13}+\right.\right.$$

$$= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-$$

Mathematica [A] time = 0.04, size = 130, normalized size = 1.00

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

fricas [A] time = 0.42, size = 76, normalized size = 0.58

$$\frac{4}{5}(x+5)x^{\frac{1}{4}}-\frac{6}{7}(x+7)x^{\frac{1}{6}}+\frac{12}{13}(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}-12\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

giac [A] time = 1.03, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\Big(x^{\frac{1}{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

maple [A] time = 0.00, size = 83, normalized size = 0.64

$$\frac{4x^{\frac{5}{4}}}{5}-\frac{6x^{\frac{7}{6}}}{7}+\frac{12x^{\frac{13}{12}}}{13}-x-12\ln\left(x^{\frac{1}{12}}+1\right)+\frac{12x^{\frac{11}{12}}}{11}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}-\frac{3x^{\frac{2}{3}}}{2}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}+\frac{12x^{\frac{5}{12}}}{5}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

maxima [A] time = 0.48, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\Big(x^{\frac{1}{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

mupad [B] time = 0.14, size = 82, normalized size = 0.63

$$4\,x^{1/4}-12\,\ln\left(x^{1/12}+1\right)-2\,\sqrt{x}-3\,x^{1/3}-x-\frac{3\,x^{2/3}}{2}-6\,x^{1/6}+\frac{4\,x^{3/4}}{3}+\frac{4\,x^{5/4}}{5}-\frac{6\,x^{5/6}}{5}+12\,x^{1/12}-\frac{6\,x^{7/6}}{7}+\frac{12\,x^{5/12}}{5}+\frac{12\,x^{7/12}}{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

sympy [A] time = 3.17, size = 121, normalized size = 0.93

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

3.238 $\int \sqrt{\frac{1-x}{x}}\, dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] -arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$
\sqrt{\frac{1}{x}-1}\,x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

fricas [A] time = 0.40, size = 26, normalized size = 1.08

$$
x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

giac [A] time = 1.14, size = 28, normalized size = 1.17

$$
\frac{1}{4}\pi\text{sgn}(x) + \frac{1}{2}\arcsin(2x-1)\,\text{sgn}(x) + \sqrt{-x^2+x}\,\text{sgn}(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.67

$$
\frac{\sqrt{-\frac{x-1}{x}}\left(\arcsin(2x-1) + 2\sqrt{-x^2+x}\right)x}{2\sqrt{-(x-1)\,x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/x)^(1/2),x)
[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1/2)
```

maxima [A] time = 1.20, size = 37, normalized size = 1.54

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

mupad [B] time = 0.16, size = 20, normalized size = 0.83

$$x\sqrt{\frac{1}{x}-1}-\operatorname{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/x)^(1/2),x)
```

```
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/x), x)
```

3.239 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3258, 615}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x+x^2}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

fricas [A] time = 0.42, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2}\sin(x)\right) - \log\left(\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

giac [A] time = 0.83, size = 12, normalized size = 1.09

$$-\log(\sin(x)+1)+\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

maple [A] time = 0.06, size = 12, normalized size = 1.09

$$-\ln(\sin(x)+1)+\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x)
[Out] ln(sin(x))-ln(sin(x)+1)
```

maxima [A] time = 0.65, size = 11, normalized size = 1.00

$$-\log(\sin(x)+1)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] -log(sin(x) + 1) + log(sin(x))
```

mupad [B] time = 0.14, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}\left(2\,\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^2),x)
[Out] -2*atanh(2*sin(x) + 1)
```

sympy [A] time = 0.22, size = 10, normalized size = 0.91

$$-\log(\sin(x)+1)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
[Out] -log(sin(x) + 1) + log(sin(x))
```

3.240 $\int \frac{e^{2x}}{2+3e^x+e^{2x}} \, dx$

Optimal. Leaf size=17

$$2\log(e^x+2)-\log(e^x+1)$$

```
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2282, 632, 31}

$$2\log(e^x+2)-\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{2+3e^x+e^{2x}} \, dx &= \text{Subst}\left(\int \frac{x}{2+3x+x^2} \, dx, x, e^x\right) \\
&= 2\,\text{Subst}\left(\int \frac{1}{2+x} \, dx, x, e^x\right)-\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\
&= -\log\left(1+e^x\right)+2\log\left(2+e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2\log(e^x+2)-\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

fricas [A] time = 0.40, size = 15, normalized size = 0.88

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

giac [A] time = 0.88, size = 15, normalized size = 0.88

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$-\ln(\mathrm{e}^x + 1) + 2\ln(\mathrm{e}^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
[Out] -ln(exp(x)+1)+2*ln(2+exp(x))
```

maxima [A] time = 0.47, size = 15, normalized size = 0.88

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

mupad [B] time = 0.21, size = 15, normalized size = 0.88

$$2 \ln(\mathrm{e}^x + 2) - \ln(\mathrm{e}^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(2*x) + 3*exp(x) + 2),x)
[Out] 2*log(exp(x) + 2) - log(exp(x) + 1)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$-\log(e^x + 1) + 2\log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
[Out] -log(exp(x) + 1) + 2*log(exp(x) + 2)
```

3.241 $\int \frac{1}{\sqrt{1+e^x}} \, dx$

Optimal. Leaf size=12

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

```
[Out] -2*arctanh((1+exp(x))^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2282, 63, 207}

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+e^x}} \, dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} \, dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \sqrt{1+e^x}\right)\\
&= -2\tanh^{-1}\left(\sqrt{1+e^x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + E^x],x]
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

fricas [B] time = 0.43, size = 21, normalized size = 1.75

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

giac [B] time = 0.97, size = 21, normalized size = 1.75

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="giac")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.83

$$-2\,\mathrm{arctanh}\left(\sqrt{\mathrm{e}^x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x)+1)^(1/2),x)
[Out] -2*arctanh((exp(x)+1)^(1/2))
```

maxima [B] time = 0.64, size = 21, normalized size = 1.75

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="maxima")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.75

$$-2\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) + 1)^(1/2),x)
[Out] -2*atanh((exp(x) + 1)^(1/2))
```

sympy [B] time = 1.41, size = 26, normalized size = 2.17

$$\log\left(-1+\frac{1}{\sqrt{e^x+1}}\right)-\log\left(1+\frac{1}{\sqrt{e^x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))**(1/2),x)
[Out] log(-1 + 1/sqrt(exp(x) + 1)) - log(1 + 1/sqrt(exp(x) + 1))
```

3.242 $\int \sqrt{1-e^x}\,dx$

Optimal. Leaf size=28

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

```
[Out] -2*arctanh((1-exp(x))^(1/2))+2*(1-exp(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 206}

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \sqrt{1-e^x}\,dx = \text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, e^x\right)$$
$$= 2\sqrt{1-e^x} + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, e^x\right)$$
$$= 2\sqrt{1-e^x} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-e^x}\right)$$
$$= 2\sqrt{1-e^x} - 2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$2\sqrt{1-e^x} - 2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - E^x],x]
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

fricas [A] time = 0.47, size = 35, normalized size = 1.25

$$2\sqrt{-e^x+1} - \log\left(\sqrt{-e^x+1}+1\right) + \log\left(\sqrt{-e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

giac [A] time = 1.02, size = 37, normalized size = 1.32

$$2\sqrt{-e^x+1} - \log\left(\sqrt{-e^x+1}+1\right) + \log\left(-\sqrt{-e^x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(-sqrt(-e^x + 1) + 1)
```

maple [A] time = 0.01, size = 36, normalized size = 1.29

$$\ln\left(\sqrt{-e^x+1}-1\right) - \ln\left(\sqrt{-e^x+1}+1\right) + 2\sqrt{-e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-exp(x))^(1/2),x)
[Out] 2*(1-exp(x))^(1/2)+ln((1-exp(x))^(1/2)-1)-ln((1-exp(x))^(1/2)+1)
```

maxima [A] time = 0.50, size = 35, normalized size = 1.25

$$2\sqrt{-e^x+1} - \log\left(\sqrt{-e^x+1}+1\right) + \log\left(\sqrt{-e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

mupad [B] time = 0.19, size = 40, normalized size = 1.43

$$2\sqrt{1-\mathrm{e}^x}+\frac{2\,\mathrm{e}^{-\frac{x}{2}}\,\mathrm{asin}\left(\mathrm{e}^{-\frac{x}{2}}\right)\,\sqrt{1-\mathrm{e}^x}}{\sqrt{1-\mathrm{e}^{-x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - exp(x))^(1/2),x)
```

```
[Out] 2*(1 - exp(x))^(1/2) + (2*exp(-x/2)*asin(exp(-x/2))*(1 - exp(x))^(1/2))/(1
- exp(-x))^(1/2)
```

sympy [A] time = 1.72, size = 32, normalized size = 1.14

$$2\sqrt{1-e^x}+\log\left(\sqrt{1-e^x}-1\right)-\log\left(\sqrt{1-e^x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))**(1/2),x)
```

```
[Out] 2*sqrt(1 - exp(x)) + log(sqrt(1 - exp(x)) - 1) - log(sqrt(1 - exp(x)) + 1)
```

3.243 $\int \frac{1}{3-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -1/4*ln(cos(1/2*x)-3*sin(1/2*x))+1/4*ln(3*cos(1/2*x)-sin(1/2*x))
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 616, 31}

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3-5\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{3-10x+3x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= \frac{3}{4}\text{Subst}\left(\int \frac{1}{-9+3x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)-\frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+3x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{1}{4}\log\left(1-3\tan\left(\frac{x}{2}\right)\right)+\frac{1}{4}\log\left(3-\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -1/4*Log[Cos[x/2] - 3*Sin[x/2]] + Log[3*Cos[x/2] - Sin[x/2]]/4
```

fricas [A] time = 0.46, size = 27, normalized size = 0.63

$$\frac{1}{8}\log\left(4\cos(x) - 3\sin(x) + 5\right) - \frac{1}{8}\log\left(-4\cos(x) - 3\sin(x) + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="fricas")
[Out] 1/8*log(4*cos(x) - 3*sin(x) + 5) - 1/8*log(-4*cos(x) - 3*sin(x) + 5)
```

giac [A] time = 0.98, size = 23, normalized size = 0.53

$$-\frac{1}{4}\log\left(\left|3\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{4}\log\left(\left|\tan\left(\frac{1}{2}x\right) - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="giac")
[Out] -1/4*log(abs(3*tan(1/2*x) - 1)) + 1/4*log(abs(tan(1/2*x) - 3))
```

maple [A] time = 0.03, size = 22, normalized size = 0.51

$$-\frac{\ln\left(3\tan\left(\frac{x}{2}\right) - 1\right)}{4} + \frac{\ln\left(\tan\left(\frac{x}{2}\right) - 3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-5*sin(x)),x)
[Out] 1/4*ln(tan(1/2*x)-3)-1/4*ln(-1+3*tan(1/2*x))
```

maxima [A] time = 0.47, size = 30, normalized size = 0.70

$$-\frac{1}{4}\log\left(\frac{3\sin(x)}{\cos(x) + 1} - 1\right) + \frac{1}{4}\log\left(\frac{\sin(x)}{\cos(x) + 1} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="maxima")
[Out] -1/4*log(3*sin(x)/(cos(x) + 1) - 1) + 1/4*log(sin(x)/(cos(x) + 1) - 3)
```

mupad [B] time = 0.39, size = 11, normalized size = 0.26

$$-\frac{\operatorname{atanh}\left(\frac{3\tan\left(\frac{x}{2}\right)}{4} - \frac{5}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(5*sin(x) - 3),x)
[Out] -atanh((3*tan(x/2))/4 - 5/4)/2
```

sympy [A] time = 0.24, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right) - 3\right)}{4} - \frac{\log\left(\tan\left(\frac{x}{2}\right) - \frac{1}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x)
[Out] log(tan(x/2) - 3)/4 - log(tan(x/2) - 1/3)/4
```

3.244 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

fricas [B] time = 0.42, size = 38, normalized size = 1.81

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

giac [B] time = 1.02, size = 37, normalized size = 1.76

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

maple [A] time = 0.00, size = 19, normalized size = 0.90

$$\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

maxima [B] time = 1.15, size = 39, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

mupad [B] time = 0.37, size = 21, normalized size = 1.00

$$-\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + sin(x)),x)
```

```
[Out] -2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

sympy [A] time = 0.52, size = 39, normalized size = 1.86

$$\frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2}-\frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1)
/2
```

3.245 $\int \frac{1}{1-\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=11

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

```
[Out] -ln(1+cot(1/2*x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3121, 31}

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -Log[1 + Cot[x/2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cot\left(\frac{x}{2}\right)\right)\\ &= -\log\left(1+\cot\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 24, normalized size = 2.18

$$\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] Log[Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 0.47, size = 17, normalized size = 1.55

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\frac{1}{2}\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(x) + 1/2) - 1/2*log(sin(x) + 1)
```

giac [A] time = 0.94, size = 17, normalized size = 1.55

$$-\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="giac")
[Out] -log(abs(tan(1/2*x) + 1)) + log(abs(tan(1/2*x)))
```

maple [A] time = 0.06, size = 16, normalized size = 1.45

$$-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)+sin(x)),x)
[Out] -ln(tan(1/2*x)+1)+ln(tan(1/2*x))
```

maxima [B] time = 0.59, size = 25, normalized size = 2.27

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="maxima")
[Out] -log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.07, size = 11, normalized size = 1.00

$$-2\,\mathrm{atanh}\left(2\tan\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - cos(x) + 1),x)
[Out] -2*atanh(2*tan(x/2) + 1)
```

sympy [A] time = 0.25, size = 14, normalized size = 1.27

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x)
[Out] -log(tan(x/2) + 1) + log(tan(x/2))
```

3.246 $\int \frac{1}{4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

```
[Out] -1/5*arctanh(3/5*cos(x)-4/5*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3074, 206}

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] - 4*Sin[x])/5]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4\cos(x)+3\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x)-4\sin(x)\right)\\ &= -\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 43, normalized size = 2.39

$$\frac{1}{5}\log\left(2\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -1/5*Log[2*Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + 2*Sin[x/2]]/5
```

fricas [B] time = 0.46, size = 27, normalized size = 1.50

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) - 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) + 2*sin(x) +
5/2)
```

giac [A] time = 0.97, size = 23, normalized size = 1.28

$$\frac{1}{5}\log\left(\left|2\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{5}\log\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*log(abs(2*tan(1/2*x) + 1)) - 1/5*log(abs(tan(1/2*x) - 2))
```

maple [A] time = 0.06, size = 22, normalized size = 1.22

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-2\right)}{5}+\frac{\ln\left(2\tan\left(\frac{x}{2}\right)+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -1/5*ln(tan(1/2*x)-2)+1/5*ln(2*tan(1/2*x)+1)
```

maxima [B] time = 0.63, size = 30, normalized size = 1.67

$$\frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) + 1) - 1/5*log(sin(x)/(cos(x) + 1) - 2)
```

mupad [B] time = 0.48, size = 11, normalized size = 0.61

$$\frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{5}-\frac{3}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(x) + 3*sin(x)),x)
```

```
[Out] (2*atanh((4*tan(x/2))/5 - 3/5))/5
```

sympy [A] time = 0.29, size = 20, normalized size = 1.11

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{5}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{1}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -log(tan(x/2) - 2)/5 + log(tan(x/2) + 1/2)/5
```

3.247 $\int \frac{1}{\sin(x)+\tan(x)} dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sin(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\cos(x)}\,dx \\
&= -\int \cot^2(x)\csc(x)\,dx + \int \cot(x)\csc^2(x)\,dx \\
&= \frac{1}{2}\cot(x)\csc(x) + \frac{1}{2}\int \csc(x)\,dx - \text{Subst}(\int x\,dx, x, \csc(x)) \\
&= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x) - \frac{\csc^2(x)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$
-\frac{1}{4}\sec^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

fricas [A] time = 0.46, size = 35, normalized size = 1.46

$$
-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) - (\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right) + 2}{4(\cos(x)+1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

giac [A] time = 1.11, size = 28, normalized size = 1.17

$$
\frac{\cos(x)-1}{4(\cos(x)+1)} + \frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.06, size = 24, normalized size = 1.00

$$
-\frac{\ln(\cos(x)+1)}{4} + \frac{\ln(\cos(x)-1)}{4} - \frac{1}{2(\cos(x)+1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x)),x)
```

```
[Out] -1/2/(cos(x)+1)-1/4*ln(cos(x)+1)+1/4*ln(cos(x)-1)
```

maxima [A] time = 0.61, size = 25, normalized size = 1.04

$$
-\frac{\sin(x)^2}{4(\cos(x)+1)^2} + \frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.30, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x)),x)
[Out] log(tan(x/2))/2 - tan(x/2)^2/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x) + \tan(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x)
[Out] Integral(1/(sin(x) + tan(x)), x)
```

3.248 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 14}

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\sin(x)+\sin(2x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

fricas [B] time = 0.44, size = 35, normalized size = 1.46

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

giac [A] time = 1.15, size = 28, normalized size = 1.17

$$-\frac{\cos(x)-1}{8(\cos(x)+1)}+\frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.11, size = 24, normalized size = 1.00

$$-\frac{\ln(\cos(x)+1)}{8}+\frac{\ln(\cos(x)-1)}{8}+\frac{1}{4\cos(x)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] 1/4/(cos(x)+1)-1/8*ln(cos(x)+1)+1/8*ln(cos(x)-1)
```

maxima [B] time = 0.54, size = 220, normalized size = 9.17

$$\frac{4\cos(2x)\cos(x)+8\cos(x)^2-\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(2x)\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

mupad [B] time = 0.34, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x) + 2*sin(x)),x)
```

```
[Out] log(tan(x/2))/4 + tan(x/2)^2/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{2\sin(x)+\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

3.249 $\int \frac{\sec(x)}{1+\sin(x)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

[Out] 1/2*arctanh(sin(x))-1/2/(1+sin(x))

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2667, 44, 207}

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]/(1 + Sin[x]),x]

[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{1+\sin(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{2(1+x)^2} - \frac{1}{2\left(-1+x^2\right)}\right) dx, x, \sin(x)\right)\\
&= -\frac{1}{2(1+\sin(x))} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(1+\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(1 + Sin[x]),x]
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

fricas [B] time = 0.43, size = 33, normalized size = 1.83

$$\frac{(\sin(x)+1)\log(\sin(x)+1)-(\sin(x)+1)\log(-\sin(x)+1)-2}{4(\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="fricas")
[Out] 1/4*((sin(x) + 1)*log(sin(x) + 1) - (sin(x) + 1)*log(-sin(x) + 1) - 2)/(sin
(x) + 1)
```

giac [A] time = 1.21, size = 25, normalized size = 1.39

$$-\frac{1}{2(\sin(x)+1)}+\frac{1}{4}\log(\sin(x)+1)-\frac{1}{4}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="giac")
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

maple [A] time = 0.04, size = 24, normalized size = 1.33

$$-\frac{\ln(\sin(x)-1)}{4}+\frac{\ln(\sin(x)+1)}{4}-\frac{1}{2(\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sin(x)+1),x)
[Out] -1/2/(sin(x)+1)+1/4*ln(sin(x)+1)-1/4*ln(sin(x)-1)
```

maxima [A] time = 0.43, size = 23, normalized size = 1.28

$$-\frac{1}{2(\sin(x)+1)}+\frac{1}{4}\log(\sin(x)+1)-\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="maxima")
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(sin(x) - 1)
```

mupad [B] time = 0.13, size = 22, normalized size = 1.22

$$\frac{\ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)}{2}-\frac{1}{2\ (\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(sin(x) + 1)),x)
[Out] log(tan(x/2 + pi/4))/2 - 1/(2*(sin(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x)
```

```
[Out] Integral(sec(x)/(sin(x) + 1), x)
```

3.250 $\int \frac{1}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b\cos(x)+a\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, a\cos(x)-b\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

fricas [B] time = 0.42, size = 98, normalized size = 2.72

$$\frac{\log\left(-\frac{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2\,b^2+2\,\sqrt{a^2+b^2}\,(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^2 - 2*b^2 + 2*sqrt
(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x
)^2 + a^2))/sqrt(a^2 + b^2)
```

giac [A]   time = 1.12, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*a
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

maple [A]   time = 0.07, size = 35, normalized size = 0.97

$$\frac{2\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a)/(a^2+b^2)^(1/2))
```

maxima [A]   time = 1.31, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
1) - sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

mupad [B]   time = 0.73, size = 31, normalized size = 0.86

$$-\frac{2\,\mathrm{atanh}\left(\frac{a-b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x) + a*sin(x)),x)
```

```
[Out] -(2*atanh((a - b*tan(x/2))/(a^2 + b^2)^(1/2)))/(a^2 + b^2)^(1/2)
```

sympy [A] time = 8.12, size = 139, normalized size = 3.86

$$\begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{a} & \text{for } b=0 \\ \frac{1}{b\sin(x)+i\sqrt{b^2}\cos(x)} & \text{for } a=-\sqrt{-b^2} \\ \frac{1}{b\sin(x)-i\sqrt{b^2}\cos(x)} & \text{for } a=\sqrt{-b^2} \\ -\frac{\log\left(-\frac{a}{b}+\tan\left(\frac{x}{2}\right)-\frac{\sqrt{a^2+b^2}}{b}\right)}{\sqrt{a^2+b^2}}+\frac{\log\left(-\frac{a}{b}+\tan\left(\frac{x}{2}\right)+\frac{\sqrt{a^2+b^2}}{b}\right)}{\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b, 0
)), (log(tan(x/2))/a, Eq(b, 0)), (1/(b*sin(x) + I*sqrt(b**2)*cos(x)), Eq(a,
 -sqrt(-b**2))), (1/(b*sin(x) - I*sqrt(b**2)*cos(x)), Eq(a, sqrt(-b**2))),
(-log(-a/b + tan(x/2) - sqrt(a**2 + b**2)/b)/sqrt(a**2 + b**2) + log(-a/b +
 tan(x/2) + sqrt(a**2 + b**2)/b)/sqrt(a**2 + b**2), True))
```

3.251 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.05, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

fricas [B] time = 0.45, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{\left(a^2+b^2\right)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

<u>**giac**</u> **[A]** time = 0.82, size = 26, normalized size = 1.73

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

<u>**maple**</u> **[A]** time = 0.00, size = 16, normalized size = 1.07

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
[Out] 1/a/b*arctan(a/b*tan(x))
```

<u>**maxima**</u> **[A]** time = 1.13, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x)/b)/(a*b)
```

<u>**mupad**</u> **[B]** time = 0.50, size = 15, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
[Out] Timed out
```

3.252 $\int \frac{x}{-1+x^2} dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {260}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
[Out] Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
[Out] Log[-1 + x^2]/2
```

fricas [A] time = 0.40, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1)
```

giac [A] time = 0.86, size = 9, normalized size = 0.75

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x)
[Out] 1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 0.47, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1),x)
[Out] log(x^2 - 1)/2
```

sympy [A] time = 0.08, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
[Out] log(x**2 - 1)/2
```

3.253 $\int \left(1+\sqrt{x}\right) \sqrt{x}\,dx$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx &= \int \left(\sqrt{x}+x\right)dx\\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

fricas [A] time = 0.40, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

giac [A] time = 0.93, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.71

$$\frac{x^2}{2}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2)+1),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

maxima [B] time = 0.61, size = 26, normalized size = 1.53

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{x^2}{2}+\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
```

```
[Out] x^2/2 + (2*x^(3/2))/3
```

sympy [A] time = 0.14, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

3.254 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

fricas [A] time = 0.39, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 0.97, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
[Out] -1/tan(1/2*x)
```

maxima [A] time = 0.45, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

mupad [B] time = 0.20, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

sympy [A] time = 0.36, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

3.255 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -1/2*arctanh(sin(x))+1/2*sec(x)*tan(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2611, 3770}

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^2(x)\,dx&=\frac{1}{2}\sec(x)\tan(x)-\frac{1}{2}\int \sec(x)\,dx\\&=-\frac{1}{2}\tanh^{-1}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

fricas [B] time = 0.43, size = 34, normalized size = 2.12

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\,\sin(x)}{4\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

giac [B] time = 1.13, size = 29, normalized size = 1.81

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log\left(\sin(x)+1\right)+\frac{1}{4}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$\frac{\sin^3(x)}{2\cos(x)^2}-\frac{\ln\left(\sec(x)+\tan(x)\right)}{2}+\frac{\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

maxima [B] time = 0.58, size = 27, normalized size = 1.69

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log\left(\sin(x)+1\right)+\frac{1}{4}\log\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

mupad [B] time = 0.00, size = 30, normalized size = 1.88

$$\frac{\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2}-\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x),x)
```

```
[Out] (tan(x/2) + tan(x/2)^3)/(tan(x/2)^2 - 1)^2 - atanh(tan(x/2))
```

sympy [A] time = 0.12, size = 27, normalized size = 1.69

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}-\frac{\sin\left(x\right)}{2\sin^2\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

3.256 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int\left(-x^2+x^4\right)\,dx,x,\sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

fricas [A] time = 0.41, size = 14, normalized size = 0.82

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

giac [A] time = 0.96, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

maple [B] time = 0.00, size = 42, normalized size = 2.47

$$-\frac{\sin^4(x)}{15\cos(x)}-\frac{\left(\sin^2(x)+2\right)\cos(x)}{15}+\frac{\sin^4(x)}{15\cos(x)^3}+\frac{\sin^4(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
```

```
[Out] -1/15/cos(x)*sin(x)^4-1/15*(sin(x)^2+2)*cos(x)+1/15/cos(x)^3*sin(x)^4+1/5/c
os(x)^5*sin(x)^4
```

maxima [A] time = 0.56, size = 14, normalized size = 0.82

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

mupad [B] time = 0.00, size = 13, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^3,x)
```

```
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{3-5\cos^2(x)}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

3.257 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int e^x x\,dx,x,\sqrt{x}\right)\\
&= 2e^{\sqrt{x}}\sqrt{x}-2\,\text{Subst}\left(\int e^x\,dx,x,\sqrt{x}\right)\\
&= -2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

fricas [**A**] time = 0.41, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

giac [**A**] time = 0.99, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

maple [**A**] time = 0.00, size = 17, normalized size = 0.71

$$2\sqrt{x}\,\mathrm{e}^{\sqrt{x}}-2\,\mathrm{e}^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*x^(1/2)*exp(x^(1/2))-2*exp(x^(1/2))
```

maxima [**A**] time = 0.46, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

mupad [**B**] time = 0.00, size = 11, normalized size = 0.46

$$2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

sympy [**A**] time = 0.21, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

3.258 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1594, 1628}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx &= \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx\\
&= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx\\
&= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

fricas [A] time = 0.42, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

giac [A] time = 0.93, size = 33, normalized size = 0.79

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(|x+2|) + \frac{3126}{35}\log(|x-5|) - \frac{1}{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\ln(x)}{10} + \frac{3126\ln(x-5)}{35} - \frac{31\ln(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x+3126/35*ln(x-5)-31/14*ln(x+2)-1/10*ln(x)
```

maxima [A] time = 0.52, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

mupad [B] time = 0.18, size = 30, normalized size = 0.71

$$19x - \frac{31\ln(x+2)}{14} + \frac{3126\ln(x-5)}{35} - \frac{\ln(x)}{10} + \frac{3x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

```
[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3
```

sympy [A] time = 0.16, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

3.259 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

```
[Out] 2*ln(x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2302, 30}

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\log(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

fricas [A] time = 0.41, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x))
```

giac [A] time = 0.92, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(log(x))
```

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)^(1/2),x)
```

```
[Out] 2*ln(x)^(1/2)
```

maxima [A] time = 0.48, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x))
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)^(1/2)),x)
```

```
[Out] 2*log(x)^(1/2)
```

sympy [A] time = 0.42, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

3.260 $\int \frac{5+2x}{-3+x} \, dx$

Optimal. Leaf size=12

$$2x + 11\log(3 - x)$$

```
[Out] 2*x+11*ln(3-x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2x + 11\log(3 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*x + 11*Log[3 - x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{5 + 2x}{-3 + x} \, dx &= \int \left(2 + \frac{11}{-3 + x}\right) \, dx \\ &= 2x + 11\log(3 - x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$2(x - 3) + 11\log(x - 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*(-3 + x) + 11*Log[-3 + x]
```

fricas [A] time = 0.44, size = 10, normalized size = 0.83

$$2x + 11\log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="fricas")
```

```
[Out] 2*x + 11*log(x - 3)
```

giac [A] time = 1.04, size = 11, normalized size = 0.92

$$2x + 11\log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="giac")
```

```
[Out] 2*x + 11*log(abs(x - 3))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$2x + 11\ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+5)/(x-3),x)
[Out] 2*x+11*ln(x-3)
```

maxima [A] time = 0.48, size = 10, normalized size = 0.83

$$2\,x + 11\,\log\left(x - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="maxima")
[Out] 2*x + 11*log(x - 3)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.83

$$2\,x + 11\,\ln\left(x - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)/(x - 3),x)
[Out] 2*x + 11*log(x - 3)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.67

$$2x + 11\log\left(x - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x)
[Out] 2*x + 11*log(x - 3)
```

3.261 $\int e^{e^x+x}\,dx$

Optimal. Leaf size=5

$$e^{e^x}$$

```
[Out] exp(exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2282, 2194}

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int e^{e^x+x}\,dx &= \text{Subst}\left(\int e^x\,dx, x, e^x\right)\\ &= e^{e^x} \end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(E^x + x),x]
```

```
[Out] E^E^x
```

fricas [A] time = 0.42, size = 3, normalized size = 0.60

$$e^{\left(e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="fricas")
```

```
[Out] e^(e^x)
```

giac [A] time = 0.94, size = 3, normalized size = 0.60

$$e^{\left(e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="giac")
```

```
[Out] e^(e^x)
```

maple [A] time = 0.01, size = 4, normalized size = 0.80

$$\mathrm{e}^{\mathrm{e}^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x+exp(x)),x)
```

```
[Out] exp(exp(x))
```

maxima [A] time = 0.46, size = 3, normalized size = 0.60

$$e^{\left(e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="maxima")
```

```
[Out] e^(e^x)
```

mupad [B] time = 0.03, size = 3, normalized size = 0.60

$$\mathrm{e}^{\mathrm{e}^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x + exp(x)),x)
```

```
[Out] exp(exp(x))
```

sympy [A] time = 0.70, size = 3, normalized size = 0.60

$$e^{e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x)
```

```
[Out] exp(exp(x))
```

3.262 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x)+\frac{1}{4}\int \cos^2(x)\,dx\\
&= \frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)+\frac{\int 1\,dx}{8}\\
&= \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

fricas [A] time = 0.43, size = 19, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos(x)^3-\cos(x)\right)\sin(x)+\frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

giac [A] time = 0.97, size = 10, normalized size = 0.42

$$\frac{1}{8}x-\frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.79

$$-\frac{\left(\cos^3(x)\right)\sin(x)}{4}+\frac{\cos(x)\sin(x)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] -1/4*cos(x)^3*sin(x)+1/8*cos(x)*sin(x)+1/8*x
```

maxima [A] time = 0.47, size = 10, normalized size = 0.42

$$\frac{1}{8}x-\frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

mupad [B] time = 0.00, size = 18, normalized size = 0.75

$$\frac{\cos(x)\sin(x)^3}{4}-\frac{\cos(x)\,\sin(x)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

sympy [A] time = 0.07, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

3.263 $\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)} \, dx$

Optimal. Leaf size=8

$$-\log(\sin(x)+\cos(x))$$

```
[Out] -ln(cos(x)+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3133}

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)} \, dx = -\log(\cos(x)+\sin(x))$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

fricas [A] time = 0.43, size = 11, normalized size = 1.38

$$-\frac{1}{2} \log(2 \cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.98, size = 18, normalized size = 2.25

$$\frac{1}{2} \log\left(\tan(x)^2+1\right) - \log(|\tan(x)+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*log(tan(x)^2 + 1) - log(abs(tan(x) + 1))
```

maple [A] time = 0.05, size = 9, normalized size = 1.12

$$-\ln(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
[Out] -ln(cos(x)+sin(x))
```

maxima [A] time = 0.51, size = 8, normalized size = 1.00

$$-\log(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -log(cos(x) + sin(x))
```

mupad [B] time = 0.42, size = 32, normalized size = 4.00

$$-2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{x}{2}\right)+128}{16\tan\left(\frac{x}{2}\right)^2+32\tan\left(\frac{x}{2}\right)+48}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) - sin(x))/(cos(x) + sin(x)),x)
[Out] -2*atanh((128*tan(x/2) + 128)/(32*tan(x/2) + 16*tan(x/2)^2 + 48) - 3)
```

sympy [A] time = 0.15, size = 8, normalized size = 1.00

$$-\log(\sin(x) + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
[Out] -log(sin(x) + cos(x))
```

3.264 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

fricas [A] time = 0.40, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

giac [A] time = 0.98, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.31

$$\frac{(x-1)(x+1)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)*x,x)
[Out] (x-1)*(x+1)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.48, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)
```

mupad [B] time = 0.00, size = 11, normalized size = 0.85

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^2)^(1/2),x)
[Out] -(1 - x^2)^(1/2)
```

sympy [A] time = 0.19, size = 8, normalized size = 0.62

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2)
```

3.265 $\int x^3 \log(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}x^4\log(x)-\frac{x^4}{16}$$

```
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{4}x^4\log(x)-\frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[x],x]
[Out] -x^4/16 + (x^4*Log[x])/4
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\log(x)\,dx=-\frac{x^4}{16}+\frac{1}{4}x^4\log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4}x^4\log(x)-\frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[x],x]
[Out] -1/16*x^4 + (x^4*Log[x])/4
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$\frac{1}{4}\,x^4\log(x)-\frac{1}{16}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="fricas")
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

giac [A] time = 0.93, size = 13, normalized size = 0.76

$$\frac{1}{4}\,x^4\log(x)-\frac{1}{16}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{x^4 \ln(x)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x),x)
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

maxima [A] time = 0.59, size = 13, normalized size = 0.76

$$\frac{1}{4}\, x^4 \log(x) - \frac{1}{16}\, x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="maxima")
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

mupad [B] time = 0.19, size = 9, normalized size = 0.53

$$\frac{x^4 \left(\ln(x) - \frac{1}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x),x)
[Out] (x^4*(log(x) - 1/4))/4
```

sympy [A] time = 0.10, size = 12, normalized size = 0.71

$$\frac{x^4 \log{(x)}}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(x),x)
[Out] x**4*log(x)/4 - x**4/16
```

3.266 $\int \frac{\sqrt{-2+x}}{2+x} \, dx$

Optimal. Leaf size=24

$$2\sqrt{x-2} - 4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

[Out] -4*arctan(1/2*(-2+x)^(1/2))+2*(-2+x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{50, 63, 203\}$

$$2\sqrt{x-2} - 4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2 + x]/(2 + x),x]

[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-2+x}}{2+x} \, dx &= 2\sqrt{-2+x} - 4\int \frac{1}{\sqrt{-2+x}(2+x)} \, dx \\
&= 2\sqrt{-2+x} - 8\,\text{Subst}\left(\int \frac{1}{4+x^2} \, dx, x, \sqrt{-2+x}\right) \\
&= 2\sqrt{-2+x} - 4\tan^{-1}\left(\frac{\sqrt{-2+x}}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$2\sqrt{x-2} - 4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + x]/(2 + x),x]
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

fricas [A] time = 0.40, size = 18, normalized size = 0.75

$$2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="fricas")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

giac [A] time = 1.12, size = 18, normalized size = 0.75

$$2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="giac")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$-4\arctan\left(\frac{\sqrt{x-2}}{2}\right) + 2\sqrt{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-2)^(1/2)/(x+2),x)
[Out] -4*arctan(1/2*(x-2)^(1/2))+2*(x-2)^(1/2)
```

maxima [A] time = 1.28, size = 18, normalized size = 0.75

$$2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="maxima")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

mupad [B] time = 0.04, size = 18, normalized size = 0.75

$$2\sqrt{x-2} - 4\,\mathrm{atan}\left(\frac{\sqrt{x-2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(1/2)/(x + 2),x)
[Out] 2*(x - 2)^(1/2) - 4*atan((x - 2)^(1/2)/2)
```

sympy [**B**] time = 1.28, size = 107, normalized size = 4.46

$$\begin{cases} -4i\,\mathrm{acosh}\left(\frac{2}{\sqrt{x+2}}\right) - \frac{2i\sqrt{x+2}}{\sqrt{-1+\frac{4}{x+2}}} + \frac{8i}{\sqrt{-1+\frac{4}{x+2}}\,\sqrt{x+2}} & \text{for } \frac{4}{|x+2|} > 1 \\ 4\,\mathrm{asin}\left(\frac{2}{\sqrt{x+2}}\right) + \frac{2\sqrt{x+2}}{\sqrt{1-\frac{4}{x+2}}} - \frac{8}{\sqrt{1-\frac{4}{x+2}}\,\sqrt{x+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)**(1/2)/(2+x),x)
```

```
[Out] Piecewise((-4*I*acosh(2/sqrt(x + 2)) - 2*I*sqrt(x + 2)/sqrt(-1 + 4/(x + 2))
 + 8*I/(sqrt(-1 + 4/(x + 2))*sqrt(x + 2)), 4/Abs(x + 2) > 1), (4*asin(2/sqr
t(x + 2)) + 2*sqrt(x + 2)/sqrt(1 - 4/(x + 2)) - 8/(sqrt(1 - 4/(x + 2))*sqrt
(x + 2)), True))
```

3.267 $\int \frac{x}{(2+x)^2} \, dx$

Optimal. Leaf size=12

$$\frac{2}{x+2} + \log(x+2)$$

[Out] 2/(2+x)+ln(2+x)

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[x/(2 + x)^2,x]

[Out] 2/(2 + x) + Log[2 + x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(2+x)^2} \, dx &= \int \left(-\frac{2}{(2+x)^2} + \frac{1}{2+x}\right) \, dx \\ &= \frac{2}{2+x} + \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(2 + x)^2,x]

[Out] 2/(2 + x) + Log[2 + x]

fricas [A] time = 0.40, size = 16, normalized size = 1.33

$$\frac{(x+2)\log(x+2) + 2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x)^2,x, algorithm="fricas")

[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)

giac [A] time = 1.00, size = 13, normalized size = 1.08

$$\frac{2}{x+2} + \log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)^2,x, algorithm="giac")
[Out] 2/(x + 2) + log(abs(x + 2))
```

maple [A] time = 0.01, size = 13, normalized size = 1.08

$$\ln(x+2)+\frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+2)^2,x)
[Out] ln(x+2)+2/(x+2)
```

maxima [A] time = 0.51, size = 12, normalized size = 1.00

$$\frac{2}{x+2}+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)^2,x, algorithm="maxima")
[Out] 2/(x + 2) + log(x + 2)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$\ln(x+2)+\frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 2)^2,x)
[Out] log(x + 2) + 2/(x + 2)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\log(x+2)+\frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)**2,x)
[Out] log(x + 2) + 2/(x + 2)
```

3.268 $\int \log\left(1+x^2\right)\,dx$

Optimal. Leaf size=16

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2448, 321, 203}

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(1+x^2\right)\,dx &= x\log\left(1+x^2\right)-2\int \frac{x^2}{1+x^2}\,dx\\ &= -2x+x\log\left(1+x^2\right)+2\int \frac{1}{1+x^2}\,dx\\ &= -2x+2\tan^{-1}(x)+x\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

fricas [A] time = 0.40, size = 16, normalized size = 1.00

$$x\log\left(x^2+1\right)-2\,x+2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="fricas")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

giac [A] time = 1.01, size = 16, normalized size = 1.00

$$x\log\left(x^2+1\right)-2\,x+2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="giac")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$x\ln\left(x^2+1\right)-2x+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+1),x)
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

maxima [A] time = 1.17, size = 16, normalized size = 1.00

$$x\log\left(x^2+1\right)-2\,x+2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="maxima")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

mupad [B] time = 0.18, size = 16, normalized size = 1.00

$$2\,\mathrm{atan}(x)-2\,x+x\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2 + 1),x)
[Out] 2*atan(x) - 2*x + x*log(x^2 + 1)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.94

$$x\log\left(x^2+1\right)-2x+2\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+1),x)
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

3.269 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

```
[Out] -2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2365, 50, 63, 207}

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)$$
$$= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)$$
$$= 2\sqrt{1+\log(x)} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)$$
$$= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

fricas [A] time = 0.39, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.00, size = 30, normalized size = 1.36

$$-\ln\left(1+\sqrt{\ln(x)+1}\right) + \ln\left(\sqrt{\ln(x)+1}-1\right) + 2\sqrt{\ln(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)+1)^(1/2)/x/ln(x),x)
[Out] 2*(ln(x)+1)^(1/2)+ln((ln(x)+1)^(1/2)-1)-ln(1+(ln(x)+1)^(1/2))
```

maxima [A] time = 0.52, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

mupad [B] time = 0.25, size = 18, normalized size = 0.82

$$2\sqrt{\ln(x)+1}-2\,\mathrm{atanh}\left(\sqrt{\ln(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^(1/2)/(x*log(x)),x)
[Out] 2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

sympy [A] time = 2.33, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1}+\log\left(\sqrt{\log(x)+1}-1\right)-\log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

3.270 $\int \left(1+\sqrt{x}\right)^8 \, dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^9$$

```
[Out] -2/9*(1+x^(1/2))^9+1/5*(1+x^(1/2))^10
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 43}

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^9$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^8,x]
```

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(1+\sqrt{x}\right)^8 \, dx &= 2\,\text{Subst}\left(\int x(1+x)^8 \, dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-(1+x)^8+(1+x)^9\right) \, dx, x, \sqrt{x}\right)\\
&= -\frac{2}{9}\left(1+\sqrt{x}\right)^9+\frac{1}{5}\left(1+\sqrt{x}\right)^{10}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{45}\left(\sqrt{x}+1\right)^9\left(9\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^8,x]
```

```
[Out] ((1 + Sqrt[x])^9*(-1 + 9*Sqrt[x]))/45
```

fricas [B] time = 0.40, size = 46, normalized size = 1.70

$$\frac{1}{5}\,x^5+7\,x^4+\frac{70}{3}\,x^3+14\,x^2+\frac{16}{45}\left(5\,x^4+45\,x^3+63\,x^2+15\,x\right)\sqrt{x}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 + 7*x^4 + 70/3*x^3 + 14*x^2 + 16/45*(5*x^4 + 45*x^3 + 63*x^2 + 15*x
)*sqrt(x) + x
```

giac [B] time = 1.28, size = 42, normalized size = 1.56

$$\frac{1}{5}\,x^5 + \frac{16}{9}\,x^{\frac{9}{2}} + 7\,x^4 + 16\,x^{\frac{7}{2}} + \frac{70}{3}\,x^3 + \frac{112}{5}\,x^{\frac{5}{2}} + 14\,x^2 + \frac{16}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="giac")
```

```
[Out] 1/5*x^5 + 16/9*x^(9/2) + 7*x^4 + 16*x^(7/2) + 70/3*x^3 + 112/5*x^(5/2) + 14
*x^2 + 16/3*x^(3/2) + x
```

maple [B] time = 0.00, size = 43, normalized size = 1.59

$$\frac{x^5}{5} + \frac{16x^{\frac{9}{2}}}{9} + 7x^4 + 16x^{\frac{7}{2}} + \frac{70x^3}{3} + \frac{112x^{\frac{5}{2}}}{5} + 14x^2 + \frac{16x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)^8,x)
```

```
[Out] 1/5*x^5+16/9*x^(9/2)+7*x^4+16*x^(7/2)+70/3*x^3+112/5*x^(5/2)+14*x^2+16/3*x^
(3/2)+x
```

maxima [A] time = 0.47, size = 19, normalized size = 0.70

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10} - \frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(sqrt(x) + 1)^10 - 2/9*(sqrt(x) + 1)^9
```

mupad [B] time = 0.03, size = 42, normalized size = 1.56

$$x + 14\,x^2 + \frac{70\,x^3}{3} + 7\,x^4 + \frac{16\,x^{3/2}}{3} + \frac{x^5}{5} + \frac{112\,x^{5/2}}{5} + 16\,x^{7/2} + \frac{16\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^8,x)
```

```
[Out] x + 14*x^2 + (70*x^3)/3 + 7*x^4 + (16*x^(3/2))/3 + x^5/5 + (112*x^(5/2))/5
+ 16*x^(7/2) + (16*x^(9/2))/9
```

sympy [B] time = 0.97, size = 54, normalized size = 2.00

$$\frac{16x^{\frac{9}{2}}}{9} + 16x^{\frac{7}{2}} + \frac{112x^{\frac{5}{2}}}{5} + \frac{16x^{\frac{3}{2}}}{3} + \frac{x^5}{5} + 7x^4 + \frac{70x^3}{3} + 14x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**8,x)
```

```
[Out] 16*x**(9/2)/9 + 16*x**(7/2) + 112*x**(5/2)/5 + 16*x**(3/2)/3 + x**5/5 + 7*x
**4 + 70*x**3/3 + 14*x**2 + x
```

3.271 $\int \sec^4(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

```
[Out] -1/4*sec(x)^4+1/6*sec(x)^6
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^3(x)\,dx&=\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\&=\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\sec(x)\right)\\&=-\frac{1}{4}\sec^4(x)+\frac{\sec^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -1/4*Sec[x]^4 + Sec[x]^6/6
```

fricas [A] time = 0.42, size = 14, normalized size = 0.82

$$-\frac{3\cos(x)^2-2}{12\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="fricas")
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

giac [A] time = 1.40, size = 14, normalized size = 0.82

$$-\frac{3 \cos(x)^2 - 2}{12 \cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="giac")
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

maple [A] time = 0.03, size = 22, normalized size = 1.29

$$\frac{\sin^4(x)}{12 \cos(x)^4} + \frac{\sin^4(x)}{6 \cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^3,x)
[Out] 1/6/cos(x)^6*sin(x)^4+1/12*sin(x)^4/cos(x)^4
```

maxima [B] time = 0.47, size = 30, normalized size = 1.76

$$-\frac{3 \sin(x)^2 - 1}{12 \left(\sin(x)^6 - 3 \sin(x)^4 + 3 \sin(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="maxima")
[Out] -1/12*(3*sin(x)^2 - 1)/(sin(x)^6 - 3*sin(x)^4 + 3*sin(x)^2 - 1)
```

mupad [B] time = 0.19, size = 14, normalized size = 0.82

$$\frac{\tan(x)^4 \left(2 \tan(x)^2 + 3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^4,x)
[Out] (tan(x)^4*(2*tan(x)^2 + 3))/12
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{2 - 3 \cos^2(x)}{12 \cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**3,x)
[Out] (2 - 3*cos(x)**2)/(12*cos(x)**6)
```

3.272 $\int \frac{x}{2-2x+x^2} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

```
[Out] arctan(-1+x)+1/2*ln(x^2-2*x+2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 617, 204, 628}

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{2-2x+x^2} \, dx &= \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2} \, dx+\int \frac{1}{2-2x+x^2} \, dx \\
&= \frac{1}{2}\log\left(2-2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1-x\right) \\
&= -\tan^{-1}(1-x)+\frac{1}{2}\log\left(2-2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

fricas [A] time = 0.40, size = 16, normalized size = 0.73

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="fricas")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

giac [A] time = 1.25, size = 16, normalized size = 0.73

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="giac")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\arctan\left(x-1\right)+\frac{\ln\left(x^{2}-2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-2*x+2),x)
```

```
[Out] arctan(x-1)+1/2*ln(x^2-2*x+2)
```

maxima [A] time = 1.30, size = 16, normalized size = 0.73

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="maxima")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.73

$$\mathrm{atan}\left(x-1\right)+\frac{\ln\left(x^2-2\,x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 2*x + 2),x)
```

```
[Out] atan(x - 1) + log(x^2 - 2*x + 2)/2
```

sympy [A] time = 0.11, size = 15, normalized size = 0.68

$$\frac{\log\left(x^{2}-2x+2\right)}{2}+\mathrm{atan}\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-2*x+2),x)
```

```
[Out] log(x**2 - 2*x + 2)/2 + atan(x - 1)
```

3.273 $\int x \sin^{-1}(x)\,dx$

Optimal. Leaf size=32

$$\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4627, 321, 216}

$$\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sin^{-1}(x)\,dx &= \frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{4}x\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)+\frac{1}{2}x^2\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(\sqrt{1-x^2}\,x+\left(2x^2-1\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSin[x],x]
[Out] (x*Sqrt[1 - x^2] + (-1 + 2*x^2)*ArcSin[x])/4
```

fricas [A] time = 0.44, size = 24, normalized size = 0.75

$$\frac{1}{4}\left(2x^2-1\right)\arcsin(x)+\frac{1}{4}\sqrt{-x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="fricas")
[Out] 1/4*(2*x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x
```

giac [A] time = 1.27, size = 26, normalized size = 0.81

$$\frac{1}{2}\left(x^2-1\right)\arcsin(x)+\frac{1}{4}\sqrt{-x^2+1}\,x+\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{x^2\arcsin(x)}{2}+\frac{\sqrt{-x^2+1}\,x}{4}-\frac{\arcsin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x),x)
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*(-x^2+1)^(1/2)*x
```

maxima [A] time = 1.20, size = 24, normalized size = 0.75

$$\frac{1}{2}x^2\arcsin(x)+\frac{1}{4}\sqrt{-x^2+1}\,x-\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="maxima")
[Out] 1/2*x^2*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$\frac{x\sqrt{1-x^2}}{4}+\frac{\mathrm{asin}(x)\left(2x^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asin(x),x)
[Out] (x*(1 - x^2)^(1/2))/4 + (asin(x)*(2*x^2 - 1))/4
```

sympy [A] time = 0.21, size = 24, normalized size = 0.75

$$\frac{x^2\,\mathrm{asin}\,(x)}{2}+\frac{x\sqrt{1-x^2}}{4}-\frac{\mathrm{asin}\,(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x),x)
[Out] x**2*asin(x)/2 + x*sqrt(1 - x**2)/4 - asin(x)/4
```

3.274 $\int \frac{\sqrt{9-x^2}}{x} dx$

Optimal. Leaf size=30

$$\sqrt{9-x^2} - 3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[Out] -3*arctanh(1/3*(-x^2+9)^(1/2))+(-x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{9-x^2} - 3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-x^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9-x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{9-x^2} + \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-x}\,x}\,dx, x, x^2\right)\\ &= \sqrt{9-x^2} - 9\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{9-x^2}\right)\\ &= \sqrt{9-x^2} - 3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\sqrt{9-x^2} - 3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

fricas [A] time = 0.40, size = 28, normalized size = 0.93

$$\sqrt{-x^2+9} + 3\log\left(\frac{\sqrt{-x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 9) + 3*log((sqrt(-x^2 + 9) - 3)/x)
```

giac [A] time = 1.40, size = 40, normalized size = 1.33

$$\sqrt{-x^2+9} - \frac{3}{2}\log\left(\sqrt{-x^2+9}+3\right) + \frac{3}{2}\log\left(-\sqrt{-x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 9) - 3/2*log(sqrt(-x^2 + 9) + 3) + 3/2*log(-sqrt(-x^2 + 9) + 3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-3\,\text{arctanh}\left(\frac{3}{\sqrt{-x^2+9}}\right) + \sqrt{-x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+9)^(1/2)/x,x)
```

```
[Out] (-x^2+9)^(1/2)-3*arctanh(3/(-x^2+9)^(1/2))
```

maxima [A] time = 1.15, size = 35, normalized size = 1.17

$$\sqrt{-x^2+9} - 3\log\left(\frac{6\sqrt{-x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 9) - 3*log(6*sqrt(-x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 0.06, size = 30, normalized size = 1.00

$$3\,\ln\left(\sqrt{\frac{9}{x^2}-1}-3\,\sqrt{\frac{1}{x^2}}\right)+\sqrt{9-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - x^2)^(1/2)/x,x)
```

```
[Out] 3*log((9/x^2 - 1)^(1/2) - 3*(1/x^2)^(1/2)) + (9 - x^2)^(1/2)
```

sympy [A] time = 1.36, size = 73, normalized size = 2.43

$$\begin{cases} -\frac{x}{\sqrt{-1+\frac{9}{x^2}}} - 3\,\mathrm{acosh}\left(\frac{3}{x}\right) + \frac{9}{x\sqrt{-1+\frac{9}{x^2}}} & \text{for } \frac{9}{|x^2|} > 1 \\ \frac{ix}{\sqrt{1-\frac{9}{x^2}}} + 3i\,\mathrm{asin}\left(\frac{3}{x}\right) - \frac{9i}{x\sqrt{1-\frac{9}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((-x/sqrt(-1 + 9/x**2) - 3*acosh(3/x) + 9/(x*sqrt(-1 + 9/x**2)), 9
/Abs(x**2) > 1), (I*x/sqrt(1 - 9/x**2) + 3*I*asin(3/x) - 9*I/(x*sqrt(1 - 9/
x**2)), True))
```

3.275 $\int \frac{x}{2+3x+x^2}\,dx$

Optimal. Leaf size=13

$$2\log(x+2)-\log(x+1)$$

```
[Out] -ln(1+x)+2*ln(2+x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 31}

$$2\log(x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2+3x+x^2}\,dx &= 2\int \frac{1}{2+x}\,dx-\int \frac{1}{1+x}\,dx\\ &= -\log(1+x)+2\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$2\log(x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

fricas [A] time = 0.41, size = 13, normalized size = 1.00

$$2\,\log\left(x+2\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

giac [A] time = 1.27, size = 15, normalized size = 1.15

$$2\,\log\left(|x+2|\right)-\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 2*log(abs(x + 2)) - log(abs(x + 1))
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$-\ln(x+1)+2\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+3*x+2),x)
```

```
[Out] -ln(x+1)+2*ln(x+2)
```

maxima [A] time = 0.49, size = 13, normalized size = 1.00

$$2\,\log(x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

mupad [B] time = 0.05, size = 13, normalized size = 1.00

$$2\,\ln(x+2)-\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x + x^2 + 2),x)
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$-\log(x+1)+2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+3*x+2),x)
```

```
[Out] -log(x + 1) + 2*log(x + 2)
```

3.276 $\int x^2 \cosh(x)\,dx$

Optimal. Leaf size=16

$$x^2 \sinh(x) + 2\sinh(x) - 2x\cosh(x)$$

```
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^2 \sinh(x) + 2\sinh(x) - 2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \cosh(x)\,dx &= x^2 \sinh(x) - 2\int x\sinh(x)\,dx\\ &= -2x\cosh(x) + x^2\sinh(x) + 2\int \cosh(x)\,dx\\ &= -2x\cosh(x) + 2\sinh(x) + x^2\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$\left(x^2+2\right)\sinh(x) - 2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + (2 + x^2)*Sinh[x]
```

fricas [A] time = 0.41, size = 14, normalized size = 0.88

$$-2\,x\cosh(x) + \left(x^2+2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="fricas")
```

```
[Out] -2*x*cosh(x) + (x^2 + 2)*sinh(x)
```

giac [A] time = 1.33, size = 27, normalized size = 1.69

$$-\frac{1}{2}\left(x^2+2x+2\right)e^{(-x)}+\frac{1}{2}\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="giac")
[Out] -1/2*(x^2 + 2*x + 2)*e^(-x) + 1/2*(x^2 - 2*x + 2)*e^x
```

maple [A] time = 0.02, size = 17, normalized size = 1.06

$$x^2\sinh(x)-2x\cosh(x)+2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x),x)
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

maxima [B] time = 0.48, size = 44, normalized size = 2.75

$$\frac{1}{3}x^3\cosh(x)-\frac{1}{6}\left(x^3+3x^2+6x+6\right)e^{(-x)}-\frac{1}{6}\left(x^3-3x^2+6x-6\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="maxima")
[Out] 1/3*x^3*cosh(x) - 1/6*(x^3 + 3*x^2 + 6*x + 6)*e^(-x) - 1/6*(x^3 - 3*x^2 + 6
*x - 6)*e^x
```

mupad [B] time = 0.18, size = 16, normalized size = 1.00

$$2\sinh(x)+x^2\sinh(x)-2x\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x),x)
[Out] 2*sinh(x) + x^2*sinh(x) - 2*x*cosh(x)
```

sympy [A] time = 0.36, size = 17, normalized size = 1.06

$$x^2\sinh(x)-2x\cosh(x)+2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x),x)
[Out] x**2*sinh(x) - 2*x*cosh(x) + 2*sinh(x)
```

3.277 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] 1/4*ln(x^4+2*x^2+4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1587}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

fricas [A] time = 0.39, size = 15, normalized size = 0.88

$$\frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

giac [A] time = 1.30, size = 18, normalized size = 1.06

$$\frac{1}{4}\,\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

```
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln\left(\left(x^3+2x+4\right)x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

```
[Out] 1/4*ln(x*(x^3+2*x+4))
```

maxima [A] time = 0.48, size = 15, normalized size = 0.88

$$\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

mupad [B] time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\,\left(x^3+2\,x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
```

```
[Out] log(x*(2*x + x^3 + 4))/4
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

3.278 $\int \frac{\cos(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3190, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{1+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)\\ &= \tan^{-1}(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

fricas [A] time = 0.42, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

giac [A] time = 0.94, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="giac")
[Out] arctan(sin(x))
```

maple [A] time = 0.02, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+sin(x)^2),x)
[Out] arctan(sin(x))
```

maxima [A] time = 1.12, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="maxima")
[Out] arctan(sin(x))
```

mupad [B] time = 0.07, size = 3, normalized size = 1.00

$$\operatorname{atan}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)^2 + 1),x)
[Out] atan(sin(x))
```

sympy [A] time = 0.23, size = 3, normalized size = 1.00

$$\operatorname{atan}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)**2),x)
[Out] atan(sin(x))
```

3.279 $\int \cos\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right) dx &= 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right)-2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right)+2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

fricas [A] time = 0.41, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

giac [A] time = 1.02, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*x^(1/2)*sin(x^(1/2))+2*cos(x^(1/2))
```

maxima [A] time = 0.50, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

mupad [B] time = 0.23, size = 16, normalized size = 0.73

$$2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

sympy [A] time = 0.33, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.280 $\int \sin(\pi x)\,dx$

Optimal. Leaf size=9

$$-\frac{\cos(\pi x)}{\pi}$$

```
[Out] -cos(Pi*x)/Pi
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2638}

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Pi*x],x]
[Out] -(Cos[Pi*x]/Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(\pi x)\,dx = -\frac{\cos(\pi x)}{\pi}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Pi*x],x]
[Out] -(Cos[Pi*x]/Pi)
```

fricas [A] time = 0.41, size = 9, normalized size = 1.00

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="fricas")
[Out] -cos(pi*x)/pi
```

giac [A] time = 0.93, size = 9, normalized size = 1.00

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="giac")
```

```
[Out] -cos(pi*x)/pi
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*x),x)
[Out] -cos(Pi*x)/Pi
```

maxima [A] time = 0.50, size = 9, normalized size = 1.00

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="maxima")
[Out] -cos(pi*x)/pi
```

mupad [B] time = 0.02, size = 9, normalized size = 1.00

$$-\frac{\cos(\Pi x)}{\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*x),x)
[Out] -cos(Pi*x)/Pi
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x)
[Out] -cos(pi*x)/pi
```

3.281 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{1+e^x} dx &= \text{Subst}\left(\int \frac{x}{1+x} dx, x, e^x\right) \\
&= \text{Subst}\left(\int \left(1 + \frac{1}{-1-x}\right) dx, x, e^x\right) \\
&= e^x - \log(1+e^x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

giac [A] time = 1.00, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x)+1),x)
```

```
[Out] exp(x)-ln(exp(x)+1)
```

maxima [A] time = 0.67, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.83

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x) + 1),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

3.282 $\int e^{3x}\cos(5x)\,dx$

Optimal. Leaf size=27

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

```
[Out] 3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*Cos[5*x],x]
```

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{3x}\cos(5x)\,dx=\frac{3}{34}e^{3x}\cos(5x)+\frac{5}{34}e^{3x}\sin(5x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{1}{34}e^{3x}(5\sin(5x)+3\cos(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*Cos[5*x],x]
```

```
[Out] (E^(3*x)*(3*Cos[5*x] + 5*Sin[5*x]))/34
```

fricas [A] time = 0.42, size = 21, normalized size = 0.78

$$\frac{3}{34}\cos\left(5\,x\right)e^{(3\,x)}+\frac{5}{34}\,e^{(3\,x)}\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] 3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)
```

giac [A] time = 1.01, size = 19, normalized size = 0.70

$$\frac{1}{34}\left(3\,\cos\left(5\,x\right)+5\,\sin\left(5\,x\right)\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="giac")
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

maple [A] time = 0.02, size = 22, normalized size = 0.81

$$\frac{3\cos(5x)\,\mathrm{e}^{3x}}{34}+\frac{5\,\mathrm{e}^{3x}\sin(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*cos(5*x),x)
[Out] 3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

maxima [A] time = 0.47, size = 19, normalized size = 0.70

$$\frac{1}{34}\,(3\,\cos(5\,x)+5\,\sin(5\,x))e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="maxima")
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$\frac{\mathrm{e}^{3\,x}\,\left(3\,\cos(5\,x)+5\,\sin(5\,x)\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)*exp(3*x),x)
[Out] (exp(3*x)*(3*cos(5*x) + 5*sin(5*x)))/34
```

sympy [A] time = 0.32, size = 26, normalized size = 0.96

$$\frac{5e^{3x}\sin(5x)}{34}+\frac{3e^{3x}\cos(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x)
[Out] 5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

3.283 $\int \cos(3x)\cos(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

```
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(5x)\,dx=\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

fricas [A] time = 0.43, size = 22, normalized size = 1.29

$$\left(8\,\cos(x)^7-12\,\cos(x)^5+5\,\cos(x)^3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] (8*cos(x)^7 - 12*cos(x)^5 + 5*cos(x)^3)*sin(x)
```

giac [A] time = 1.04, size = 13, normalized size = 0.76

$$\frac{1}{16}\,\sin\left(8\,x\right)+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

maple [A] time = 0.08, size = 14, normalized size = 0.82

$$\frac{\sin(2x)}{4} + \frac{\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(5*x),x)
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

maxima [A] time = 0.54, size = 13, normalized size = 0.76

$$\frac{1}{16}\sin(8x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="maxima")
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.76

$$\frac{\sin(2x)}{4} + \frac{\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(5*x),x)
[Out] sin(2*x)/4 + sin(8*x)/16
```

sympy [B] time = 0.58, size = 26, normalized size = 1.53

$$-\frac{3\sin(3x)\cos(5x)}{16} + \frac{5\sin(5x)\cos(3x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x)
[Out] -3*sin(3*x)*cos(5*x)/16 + 5*sin(5*x)*cos(3*x)/16
```

3.284 $\int \frac{1}{1+x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{4} \log \left(x^2+1\right)+\frac{1}{2} \log (x+1)+\frac{1}{2} \tan ^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4} \log \left(x^2+1\right)+\frac{1}{2} \log (x+1)+\frac{1}{2} \tan ^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x+x^2+x^3} \, dx &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) \, dx\\
&= \frac{1}{2} \log (1+x)+\frac{1}{2} \int \frac{1-x}{1+x^2} \, dx\\
&= \frac{1}{2} \log (1+x)+\frac{1}{2} \int \frac{1}{1+x^2} \, dx-\frac{1}{2} \int \frac{x}{1+x^2} \, dx\\
&= \frac{1}{2} \tan ^{-1}(x)+\frac{1}{2} \log (1+x)-\frac{1}{4} \log \left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4} \log \left(x^2+1\right)+\frac{1}{2} \log (x+1)+\frac{1}{2} \tan ^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

fricas [A] time = 0.40, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

giac [A] time = 0.92, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln\left(x+1\right)}{2} - \frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x)
[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

maxima [A] time = 1.27, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

mupad [B] time = 0.18, size = 25, normalized size = 1.00

$$\frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + x^3 + 1),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log\left(x+1\right)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.285 $\int x^2 \log(1+x)\,dx$

Optimal. Leaf size=39

$$-\frac{x^3}{9}+\frac{1}{3}x^3\log(x+1)+\frac{x^2}{6}-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -1/3*x+1/6*x^2-1/9*x^3+1/3*ln(1+x)+1/3*x^3*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 43}

$$-\frac{x^3}{9}+\frac{x^2}{6}+\frac{1}{3}x^3\log(x+1)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 + x],x]
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log(1+x)\,dx &= \frac{1}{3}x^3\log(1+x)-\frac{1}{3}\int\frac{x^3}{1+x}\,dx\\ &= \frac{1}{3}x^3\log(1+x)-\frac{1}{3}\int\left(1+\frac{1}{-1-x}-x+x^2\right)dx\\ &= -\frac{x}{3}+\frac{x^2}{6}-\frac{x^3}{9}+\frac{1}{3}\log(1+x)+\frac{1}{3}x^3\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.72

$$\frac{1}{18}\left(6\left(x^3+1\right)\log(x+1)+x\left(-2x^2+3x-6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 + x],x]
[Out] (x*(-6 + 3*x - 2*x^2) + 6*(1 + x^3)*Log[1 + x])/18
```

fricas [A] time = 0.40, size = 25, normalized size = 0.64

$$-\frac{1}{9}x^3+\frac{1}{6}x^2+\frac{1}{3}\left(x^3+1\right)\log\left(x+1\right)-\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="fricas")
```

```
[Out] -1/9*x^3 + 1/6*x^2 + 1/3*(x^3 + 1)*log(x + 1) - 1/3*x
```

giac [A] time = 1.03, size = 49, normalized size = 1.26

$$\frac{1}{3}(x+1)^{3}\log(x+1)-\frac{1}{9}(x+1)^{3}-(x+1)^{2}\log(x+1)+\frac{1}{2}(x+1)^{2}+(x+1)\log(x+1)-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^3*log(x + 1) - 1/9*(x + 1)^3 - (x + 1)^2*log(x + 1) + 1/2*(x +
1)^2 + (x + 1)*log(x + 1) - x - 1
```

maple [A] time = 0.00, size = 46, normalized size = 1.18

$$-\frac{x^{3}}{9}+\frac{x^{2}}{6}-\frac{x}{3}+\frac{(x+1)^{3}\ln(x+1)}{3}-(x+1)^{2}\ln(x+1)+(x+1)\ln(x+1)-\frac{11}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x+1),x)
```

```
[Out] 1/3*(x+1)^3*ln(x+1)-1/9*x^3+1/6*x^2-1/3*x-11/18-(x+1)^2*ln(x+1)+(x+1)*ln(x+
1)
```

maxima [A] time = 0.47, size = 29, normalized size = 0.74

$$\frac{1}{3}x^{3}\log(x+1)-\frac{1}{9}x^{3}+\frac{1}{6}x^{2}-\frac{1}{3}x+\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x + 1) - 1/9*x^3 + 1/6*x^2 - 1/3*x + 1/3*log(x + 1)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.64

$$\frac{x^{2}}{6}-\frac{x}{3}-\frac{x^{3}}{9}+\frac{\ln(x+1)\left(x^{3}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x + 1),x)
```

```
[Out] x^2/6 - x/3 - x^3/9 + (log(x + 1)*(x^3 + 1))/3
```

sympy [A] time = 0.11, size = 29, normalized size = 0.74

$$\frac{x^{3}\log(x+1)}{3}-\frac{x^{3}}{9}+\frac{x^{2}}{6}-\frac{x}{3}+\frac{\log(x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(1+x),x)
```

```
[Out] x**3*log(x + 1)/3 - x**3/9 + x**2/6 - x/3 + log(x + 1)/3
```

3.286 $\int e^{-x^3} x^5 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{3} e^{-x^3} x^3 - \frac{e^{-x^3}}{3}$$

```
[Out] -1/3/exp(x^3)-1/3*x^3/exp(x^3)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2212, 2209}

$$-\frac{1}{3} e^{-x^3} x^3 - \frac{e^{-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/E^x^3,x]
```

```
[Out] -1/(3*E^x^3) - x^3/(3*E^x^3)
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned} \int e^{-x^3} x^5 \, dx &= -\frac{1}{3} e^{-x^3} x^3 + \int e^{-x^3} x^2 \, dx \\ &= -\frac{e^{-x^3}}{3} - \frac{1}{3} e^{-x^3} x^3 \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.62

$$-\frac{1}{3} e^{-x^3} \left(x^3 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/E^x^3,x]
```

```
[Out] -1/3*(1 + x^3)/E^x^3
```

fricas [A] time = 0.41, size = 13, normalized size = 0.50

$$-\frac{1}{3} \left(x^3 + 1\right) e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

giac [A] time = 0.87, size = 13, normalized size = 0.50

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="giac")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.54

$$-\frac{\left(x^3+1\right)\mathrm{e}^{-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/exp(x^3),x)
```

```
[Out] -1/3*(x^3+1)/exp(x^3)
```

maxima [A] time = 0.48, size = 13, normalized size = 0.50

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.50

$$-\frac{\mathrm{e}^{-x^3}\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*exp(-x^3),x)
```

```
[Out] -(exp(-x^3)*(x^3 + 1))/3
```

sympy [A] time = 0.10, size = 12, normalized size = 0.46

$$\frac{\left(-x^3-1\right)e^{-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/exp(x**3),x)
```

```
[Out] (-x**3 - 1)*exp(-x**3)/3
```

3.287 $\int \tan^2(4x)\,dx$

Optimal. Leaf size=12

$$\frac{1}{4}\tan(4x)-x$$

```
[Out] -x+1/4*tan(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3473, 8}

$$\frac{1}{4}\tan(4x)-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[4*x]^2,x]
```

```
[Out] -x + Tan[4*x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(4x)\,dx &= \frac{1}{4}\tan(4x)-\int 1\,dx\\ &= -x+\frac{1}{4}\tan(4x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.50

$$\frac{1}{4}\tan(4x)-\frac{1}{4}\tan^{-1}(\tan(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[4*x]^2,x]
```

```
[Out] -1/4*ArcTan[Tan[4*x]] + Tan[4*x]/4
```

fricas [A] time = 0.42, size = 10, normalized size = 0.83

$$-x+\frac{1}{4}\tan\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="fricas")
```

```
[Out] -x + 1/4*tan(4*x)
```

giac [A] time = 1.29, size = 10, normalized size = 0.83

$$-x+\frac{1}{4}\tan\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="giac")
```

```
[Out] -x + 1/4*tan(4*x)
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$-x+\frac{\tan\left(4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)^2,x)
```

```
[Out] -x+1/4*tan(4*x)
```

maxima [A] time = 1.11, size = 10, normalized size = 0.83

$$-x+\frac{1}{4}\tan\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="maxima")
```

```
[Out] -x + 1/4*tan(4*x)
```

mupad [B] time = 0.18, size = 10, normalized size = 0.83

$$\frac{\tan\left(4\,x\right)}{4}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)^2,x)
```

```
[Out] tan(4*x)/4 - x
```

sympy [A] time = 0.07, size = 12, normalized size = 1.00

$$-x+\frac{\sin\left(4x\right)}{4\cos\left(4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)**2,x)
```

```
[Out] -x + sin(4*x)/(4*cos(4*x))
```

3.288 $\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

```
[Out] 1/3*arctanh((2+3*x)/(9*x^2+12*x-5)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{12+18x}{\sqrt{-5+12x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{2+3x}{\sqrt{-5+12x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+12x-5}+3x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] Log[2 + 3*x + Sqrt[-5 + 12*x + 9*x^2]]/3
```

fricas [A] time = 0.40, size = 20, normalized size = 0.80

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+12x-5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 12*x - 5) - 2)
```

giac [A] time = 1.14, size = 21, normalized size = 0.84

$$-\frac{1}{3}\log\left(\left|-3\,x+\sqrt{9\,x^2+12\,x-5}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="giac")
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 12*x - 5) - 2))
```

maple [A] time = 0.00, size = 30, normalized size = 1.20

$$\frac{\sqrt{9}\,\ln\left(\frac{(9x+6)\sqrt{9}}{9}+\sqrt{9x^2+12x-5}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x-5)^(1/2),x)
[Out] 1/9*ln(1/9*(9*x+6)*9^(1/2)+(9*x^2+12*x-5)^(1/2))*9^(1/2)
```

maxima [A] time = 1.09, size = 22, normalized size = 0.88

$$\frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+12\,x-5}+12\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 12*x - 5) + 12)
```

mupad [B] time = 0.27, size = 20, normalized size = 0.80

$$\frac{\ln\left(3\,x+\sqrt{9\,x^2+12\,x-5}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 9*x^2 - 5)^(1/2),x)
[Out] log(3*x + (12*x + 9*x^2 - 5)^(1/2) + 2)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{9x^2+12x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x-5)**(1/2),x)
[Out] Integral(1/sqrt(9*x**2 + 12*x - 5), x)
```

3.289 $\int x^2 \tan^{-1}(x)\, dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3\tan^{-1}(x)-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)$$

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 266, 43}

$$-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{3}x^3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(x)\,dx&=\frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{3}\int\frac{x^3}{1+x^2}\,dx\\&=\frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int\frac{x}{1+x}\,dx,x,x^2\right)\\&=\frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)dx,x,x^2\right)\\&=-\frac{x^2}{6}+\frac{1}{3}x^3\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{1}{6}\left(2x^3\tan^{-1}(x)-x^2+\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x],x]
```

```
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

fricas [A] time = 0.43, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

giac [A] time = 0.90, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$\frac{x^3\arctan(x)}{3} - \frac{x^2}{6} + \frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x),x)
```

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

maxima [A] time = 0.49, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

mupad [B] time = 0.17, size = 21, normalized size = 0.78

$$\frac{\ln\left(x^2+1\right)}{6} + \frac{x^3\,\mathrm{atan}(x)}{3} - \frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(x),x)
```

```
[Out] log(x^2 + 1)/6 + (x^3*atan(x))/3 - x^2/6
```

sympy [A] time = 0.35, size = 20, normalized size = 0.74

$$\frac{x^3\,\mathrm{atan}\,(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

3.290 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x}}-\sqrt[6]{x}\right)dx\\ &= \frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

fricas [A] time = 0.39, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

giac [A] time = 1.01, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

maxima [A] time = 0.58, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.63

$$-\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
```

```
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

sympy [A] time = 1.67, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

3.291 $\int \frac{1}{-e^{-x}+e^x} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}\left(e^x\right)$$

```
[Out] -arctanh(exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 207}

$$-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-e^{-x}+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx,x,e^x\right)\\ &= -\tanh^{-1}\left(e^x\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

fricas [B] time = 0.42, size = 15, normalized size = 2.50

$$-\frac{1}{2} \, \log\left(e^x + 1\right) + \frac{1}{2} \, \log\left(e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

giac [B] time = 0.79, size = 16, normalized size = 2.67

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.00, size = 6, normalized size = 1.00

$$-\operatorname{arctanh}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/exp(x)+exp(x)),x)
[Out] -arctanh(exp(x))
```

maxima [B] time = 0.58, size = 19, normalized size = 3.17

$$-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="maxima")
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

mupad [B] time = 0.10, size = 15, normalized size = 2.50

$$\frac{\ln\left(e^{x}-1\right)}{2}-\frac{\ln\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(exp(-x) - exp(x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

sympy [B] time = 0.11, size = 15, normalized size = 2.50

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.292 $\int \frac{x}{10+2x^2+x^4} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = $\{1107, 618, 204\}$

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{10+2x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(1+x^2\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

fricas [**A**] time = 0.39, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

giac [**A**] time = 1.65, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

maple [**A**] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x)
[Out] 1/6*arctan(1/3*x^2+1/3)
```

maxima [**A**] time = 1.25, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

mupad [**B**] time = 0.19, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 10),x)
[Out] atan(x^2/3 + 1/3)/6
```

sympy [**A**] time = 0.11, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
[Out] atan(x**2/3 + 1/3)/6
```

3.293 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

```
[Out] 3/4*ln(1+x^(4/3))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 260}

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx\\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

fricas [A] time = 0.40, size = 8, normalized size = 0.67

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

giac [B] time = 0.90, size = 32, normalized size = 2.67

$$\frac{3}{4} \log\left(\sqrt{2}\,x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{3}{4} \log\left(-\sqrt{2}\,x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{3 \ln\left(x^{\frac{4}{3}} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x)
```

```
[Out] 3/4*ln(1+x^(4/3))
```

maxima [A] time = 1.49, size = 8, normalized size = 0.67

$$\frac{3}{4} \log\left(x^{\frac{4}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

mupad [B] time = 0.22, size = 8, normalized size = 0.67

$$\frac{3\ \ln\left(x^{4/3} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1/x^(1/3)),x)
```

```
[Out] (3*log(x^(4/3) + 1))/4
```

sympy [A] time = 0.24, size = 10, normalized size = 0.83

$$\frac{3 \log\left(x^{\frac{4}{3}} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

3.294 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] 1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\sin^2(x)\,dx&=-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{6}\int \cos^4(x)\,dx\\&=\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{8}\int \cos^2(x)\,dx\\&=\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{\int 1\,dx}{16}\\&=\frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

fricas [A] time = 0.43, size = 25, normalized size = 0.74

$$-\frac{1}{48}\left(8\cos(x)^{5}-2\cos(x)^{3}-3\cos(x)\right)\sin(x)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

giac [A] time = 0.91, size = 22, normalized size = 0.65

$$\frac{1}{16}x-\frac{1}{192}\sin(6x)-\frac{1}{64}\sin(4x)+\frac{1}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.76

$$-\frac{\left(\cos^{5}(x)\right)\sin(x)}{6}+\frac{x}{16}+\frac{\left(\cos^{3}(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x)
[Out] -1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x
```

maxima [A] time = 0.50, size = 18, normalized size = 0.53

$$\frac{1}{48}\sin(2x)^{3}+\frac{1}{16}x-\frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.76

$$\left(\frac{\cos(x)^{3}}{6}+\frac{\cos(x)}{8}\right)\sin(x)^{3}-\frac{\cos(x)\sin(x)}{16}+\frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x)
[Out] x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

sympy [A] time = 0.07, size = 31, normalized size = 0.91

$$\frac{x}{16}-\frac{\sin(x)\cos^{5}(x)}{6}+\frac{\sin(x)\cos^{3}(x)}{24}+\frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**2,x)
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

3.295 $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] arcsin(2/3+1/3*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-4x-x^2}}\,dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, -4-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{3}(-2-x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

fricas [B] time = 0.40, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-4x+5}\,(x+2)}{x^2+4x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/(x^2 + 4*x - 5))
```

giac [A] time = 1.02, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{3}x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="giac")
[Out] arcsin(1/3*x + 2/3)
```

maple [A] time = 0.00, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{3} + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(1/2),x)
[Out] arcsin(2/3+1/3*x)
```

maxima [A] time = 1.22, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{3}x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/3*x - 2/3)
```

mupad [B] time = 0.16, size = 6, normalized size = 0.50

$$\operatorname{asin}\left(\frac{x}{3} + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5 - x^2 - 4*x)^(1/2),x)
[Out] asin(x/3 + 2/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 4x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 - 4*x + 5), x)
```

3.296 $\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{1-x^2}+1\right)$$

```
[Out] -ln(1+(-x^2+1)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{1-x}-x}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\log\left(1+\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

fricas [A] time = 0.41, size = 21, normalized size = 1.31

$$-\log(x)+\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -log(x) + log((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.82, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(-x^2 + 1) + 1)
```

maple [B] time = 0.02, size = 59, normalized size = 3.69

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\ln(x)-\frac{\sqrt{2x-(x+1)^2+2}}{2}-\frac{\sqrt{-2x-(x-1)^2+2}}{2}+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] -ln(x)-1/2*(-(x+1)^2+2*x+2)^(1/2)-1/2*(-(x-1)^2-2*x+2)^(1/2)+(-x^2+1)^(1/2)
-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.49, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 1) + 1)
```

mupad [B] time = 0.13, size = 21, normalized size = 1.31

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/2) - x^2 + 1),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - log(x)
```

sympy [B] time = 5.47, size = 44, normalized size = 2.75

$$\frac{\log\left(2\sqrt{1-x^2}\right)}{2}-\frac{\log\left(2\sqrt{1-x^2}+2\right)}{2}-\frac{\log\left(x^2-\sqrt{1-x^2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] log(2*sqrt(1 - x**2))/2 - log(2*sqrt(1 - x**2) + 2)/2 - log(x**2 - sqrt(1 -
 x**2) - 1)/2
```

3.297 $\int (1+\cos(x))\csc(x)\,dx$

Optimal. Leaf size=7

$$\log(1-\cos(x))$$

```
[Out] ln(1-cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2667, 31}

$$\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.83, size = 7, normalized size = 1.00

$$\log\left(-\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
[Out] log(-cos(x) + 1)
```

maple [A] time = 0.03, size = 6, normalized size = 0.86

$$\ln\left(\cos(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+1)*csc(x),x)
[Out] ln(cos(x)-1)
```

maxima [A] time = 0.53, size = 5, normalized size = 0.71

$$\log\left(\cos(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
[Out] log(cos(x) - 1)
```

mupad [B] time = 0.05, size = 5, normalized size = 0.71

$$\ln\left(\cos(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/sin(x),x)
[Out] log(cos(x) - 1)
```

sympy [B] time = 2.26, size = 12, normalized size = 1.71

$$-\log\left(\cot\left(x\right) + \csc\left(x\right)\right) + \log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x)
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

3.298 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

```
[Out] -arctanh(exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 207}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{-1+e^{2x}} \, dx &= \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, e^x\right)\\ &= -\tanh^{-1}(e^x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

fricas [B] time = 0.42, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

giac [B] time = 0.91, size = 16, normalized size = 2.67

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.01, size = 6, normalized size = 1.00

$$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-1+exp(2*x)),x)
[Out] -arctanh(exp(x))
```

maxima [B] time = 0.48, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

mupad [B] time = 0.03, size = 15, normalized size = 2.50

$$\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) - 1),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

sympy [B] time = 0.11, size = 15, normalized size = 2.50

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.299 $\int \frac{1}{-8+x^3} \, dx$

Optimal. Leaf size=43

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] 1/12*ln(2-x)-1/24*ln(x^2+2*x+4)-1/12*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 618, 204, 628}

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + x^3)^(-1),x]
```

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-8+x^3}\,dx &= \frac{1}{12}\int \frac{1}{-2+x}\,dx + \frac{1}{12}\int \frac{-4-x}{4+2x+x^2}\,dx\\
&= \frac{1}{12}\log(2-x) - \frac{1}{24}\int \frac{2+2x}{4+2x+x^2}\,dx - \frac{1}{4}\int \frac{1}{4+2x+x^2}\,dx\\
&= \frac{1}{12}\log(2-x) - \frac{1}{24}\log\left(4+2x+x^2\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 2+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{12}\log(2-x) - \frac{1}{24}\log\left(4+2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(2-x) - \frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + x^3)^(-1),x]
[Out] -1/4*ArcTan[(1 + x)/Sqrt[3]]/Sqrt[3] + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

fricas [A] time = 0.41, size = 32, normalized size = 0.74

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

giac [A] time = 0.82, size = 33, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(abs(x - 2))
```

maple [A] time = 0.00, size = 35, normalized size = 0.81

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\right)}{12} + \frac{\ln(x-2)}{12} - \frac{\ln\left(x^2+2x+4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-8),x)
```

```
[Out] -1/24*ln(x^2+2*x+4)-1/12*3^(1/2)*arctan(1/6*(2*x+2)*3^(1/2))+1/12*ln(x-2)
```

maxima [A] time = 1.11, size = 32, normalized size = 0.74

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

mupad [B] time = 0.09, size = 46, normalized size = 1.07

$$\frac{\ln(x-2)}{12}+\ln\left(x+1-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)-\ln\left(x+1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 8),x)
```

```
[Out] log(x - 2)/12 + log(x - 3^(1/2)*1i + 1)*((3^(1/2)*1i)/24 - 1/24) - log(x +
3^(1/2)*1i + 1)*((3^(1/2)*1i)/24 + 1/24)
```

sympy [A] time = 0.14, size = 41, normalized size = 0.95

$$\frac{\log(x-2)}{12}-\frac{\log\left(x^2+2x+4\right)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-8),x)
```

```
[Out] log(x - 2)/12 - log(x**2 + 2*x + 4)/24 - sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)
/3)/12
```

3.300 $\int x^5 \cosh(x)\, dx$

Optimal. Leaf size=37

$$x^5 \sinh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

```
[Out] -120*cosh(x)-60*x^2*cosh(x)-5*x^4*cosh(x)+120*x*sinh(x)+20*x^3*sinh(x)+x^5*
sinh(x)
```

Rubi [A] time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^5 \sinh(x) + 20x^3 \sinh(x) - 5x^4 \cosh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cosh[x],x]
```

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 20*x^3*Sinh
[x] + x^5*Sinh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5 \cosh(x)\, dx &= x^5 \sinh(x) - 5 \int x^4 \sinh(x)\, dx \\
&= -5x^4 \cosh(x) + x^5 \sinh(x) + 20 \int x^3 \cosh(x)\, dx \\
&= -5x^4 \cosh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) - 60 \int x^2 \sinh(x)\, dx \\
&= -60x^2 \cosh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) + 120 \int x \cosh(x)\, dx \\
&= -60x^2 \cosh(x) - 5x^4 \cosh(x) + 120x \sinh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) - 120 \int \sinh(x) \\
&= -120 \cosh(x) - 60x^2 \cosh(x) - 5x^4 \cosh(x) + 120x \sinh(x) + 20x^3 \sinh(x) + x^5 \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.78

$$x\left(x^4 + 20x^2 + 120\right) \sinh(x) - 5\left(x^4 + 12x^2 + 24\right) \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cosh[x],x]
```

```
[Out] -5*(24 + 12*x^2 + x^4)*Cosh[x] + x*(120 + 20*x^2 + x^4)*Sinh[x]
```

fricas [A] time = 0.40, size = 30, normalized size = 0.81

$$-5\left(x^4 + 12\,x^2 + 24\right) \cosh(x) + \left(x^5 + 20\,x^3 + 120\,x\right) \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="fricas")
```

```
[Out] -5*(x^4 + 12*x^2 + 24)*cosh(x) + (x^5 + 20*x^3 + 120*x)*sinh(x)
```

giac [A] time = 1.03, size = 57, normalized size = 1.54

$$-\frac{1}{2}\left(x^5+5\,x^4+20\,x^3+60\,x^2+120\,x+120\right)e^{(-x)}+\frac{1}{2}\left(x^5-5\,x^4+20\,x^3-60\,x^2+120\,x-120\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x) + 1/2*(x^5 - 5*x^
4 + 20*x^3 - 60*x^2 + 120*x - 120)*e^x
```

maple [A] time = 0.02, size = 38, normalized size = 1.03

$$x^5\sinh(x)-5x^4\cosh(x)+20x^3\sinh(x)-60x^2\cosh(x)+120x\sinh(x)-120\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(x),x)
```

```
[Out] -120*cosh(x)-60*x^2*cosh(x)-5*x^4*cosh(x)+120*x*sinh(x)+20*x^3*sinh(x)+x^5*
sinh(x)
```

maxima [A] time = 0.48, size = 74, normalized size = 2.00

$$\frac{1}{6}\,x^6\cosh(x)-\frac{1}{12}\left(x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)e^{(-x)}-\frac{1}{12}\left(x^6-6\,x^5+30\,x^4-120\,x^3+360\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*cosh(x) - 1/12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x +
720)*e^(-x) - 1/12*(x^6 - 6*x^5 + 30*x^4 - 120*x^3 + 360*x^2 - 720*x + 720)
*e^x
```

mupad [B] time = 0.21, size = 37, normalized size = 1.00

$$20\,x^3\sinh(x)-60\,x^2\cosh(x)-5\,x^4\cosh(x)-120\cosh(x)+x^5\sinh(x)+120\,x\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(x),x)
```

```
[Out] 20*x^3*sinh(x) - 60*x^2*cosh(x) - 5*x^4*cosh(x) - 120*cosh(x) + x^5*sinh(x)
 + 120*x*sinh(x)
```

sympy [A] time = 2.15, size = 42, normalized size = 1.14

$$x^5\sinh(x)-5x^4\cosh(x)+20x^3\sinh(x)-60x^2\cosh(x)+120x\sinh(x)-120\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(x),x)
```

```
[Out] x**5*sinh(x) - 5*x**4*cosh(x) + 20*x**3*sinh(x) - 60*x**2*cosh(x) + 120*x*s
inh(x) - 120*cosh(x)
```

3.301 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

[Out] 1/2*ln(tan(x))^2

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2

fricas [A] time = 0.43, size = 12, normalized size = 1.33

$$\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="fricas")

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

giac [A] time = 1.08, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="giac")
[Out] 1/2*log(tan(x))^2
```

maple [A] time = 0.04, size = 8, normalized size = 0.89

$$\frac{\ln(\tan(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tan(x))/cos(x)/sin(x),x)
[Out] 1/2*ln(tan(x))^2
```

maxima [A] time = 0.50, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="maxima")
[Out] 1/2*log(tan(x))^2
```

mupad [B] time = 2.59, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{e^{x\,2\mathrm{i}}+1}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/(cos(x)*sin(x)),x)
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(\tan(x))}{\sin(x)\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tan(x))/cos(x)/sin(x),x)
[Out] Integral(log(tan(x))/(sin(x)*cos(x)), x)
```

3.302 $\int \left(-2x+x^2+x^3\right) dx$

Optimal. Leaf size=20

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int \left(-2x+x^2+x^3\right) dx=-x^2+\frac{x^3}{3}+\frac{x^4}{4}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

fricas [A] time = 0.35, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

giac [A] time = 0.97, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x)
```

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

maxima [A] time = 0.60, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

mupad [B] time = 0.03, size = 15, normalized size = 0.75

$$\frac{x^2\,\left(3\,x^2+4\,x-12\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2 - 2*x + x^3,x)
```

```
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

sympy [A] time = 0.06, size = 12, normalized size = 0.60

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

3.303 $\int \frac{1+e^x}{1-e^x} \, dx$

Optimal. Leaf size=12

$$x - 2\log(1 - e^x)$$

```
[Out] x-2*ln(1-exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2282, 72}

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+e^x}{1-e^x} \, dx &= \text{Subst}\left(\int \frac{1+x}{(1-x)x} \, dx, x, e^x\right) \\
&= \text{Subst}\left(\int \left(-\frac{2}{-1+x} + \frac{1}{x}\right) dx, x, e^x\right) \\
&= x - 2\log(1 - e^x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

fricas [A] time = 0.42, size = 9, normalized size = 0.75

$$x - 2\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="fricas")
```

```
[Out] x - 2*log(e^x - 1)
```

giac [A] time = 1.28, size = 10, normalized size = 0.83

$$x - 2 \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="giac")
[Out] x - 2*log(abs(e^x - 1))
```

maple [A] time = 0.01, size = 12, normalized size = 1.00

$$-2\ln\left(e^{x} - 1\right) + \ln\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+1)/(-exp(x)+1),x)
[Out] -2*ln(-1+exp(x))+ln(exp(x))
```

maxima [A] time = 0.65, size = 9, normalized size = 0.75

$$x - 2 \log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="maxima")
[Out] x - 2*log(e^x - 1)
```

mupad [B] time = 0.18, size = 9, normalized size = 0.75

$$x - 2 \ln\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x) + 1)/(exp(x) - 1),x)
[Out] x - 2*log(exp(x) - 1)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$x - 2\log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x)
[Out] x - 2*log(exp(x) - 1)
```

3.304 $\int \frac{x}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\
&= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

fricas [A] time = 0.41, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

giac [A] time = 0.97, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+4),x)
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{3\,x^2}{5\,x^2+8}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x^2 + 4)),x)
[Out] atanh((3*x^2)/(5*x^2 + 8))/3
```

sympy [A] time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

3.305 $\int \frac{1}{4-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -1/3*ln(cos(1/2*x)-2*sin(1/2*x))+1/3*ln(2*cos(1/2*x)-sin(1/2*x))
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 616, 31}

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4-10x+4x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-8+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)-\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-2+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{1}{3}\log\left(1-2\tan\left(\frac{x}{2}\right)\right)+\frac{1}{3}\log\left(2-\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -1/3*Log[Cos[x/2] - 2*Sin[x/2]] + Log[2*Cos[x/2] - Sin[x/2]]/3
```

fricas [A] time = 0.44, size = 27, normalized size = 0.63

$$\frac{1}{6}\log\left(\frac{3}{2}\cos(x) - 2\sin(x) + \frac{5}{2}\right) - \frac{1}{6}\log\left(-\frac{3}{2}\cos(x) - 2\sin(x) + \frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="fricas")
[Out] 1/6*log(3/2*cos(x) - 2*sin(x) + 5/2) - 1/6*log(-3/2*cos(x) - 2*sin(x) + 5/2
)
```

giac [A] time = 1.13, size = 23, normalized size = 0.53

$$-\frac{1}{3}\log\left(\left|2\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{3}\log\left(\left|\tan\left(\frac{1}{2}x\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="giac")
[Out] -1/3*log(abs(2*tan(1/2*x) - 1)) + 1/3*log(abs(tan(1/2*x) - 2))
```

maple [A] time = 0.03, size = 22, normalized size = 0.51

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right) - 2\right)}{3} - \frac{\ln\left(2\tan\left(\frac{x}{2}\right) - 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-5*sin(x)),x)
[Out] 1/3*ln(tan(1/2*x)-2)-1/3*ln(2*tan(1/2*x)-1)
```

maxima [A] time = 0.47, size = 30, normalized size = 0.70

$$-\frac{1}{3}\log\left(\frac{2\sin(x)}{\cos(x) + 1} - 1\right) + \frac{1}{3}\log\left(\frac{\sin(x)}{\cos(x) + 1} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="maxima")
[Out] -1/3*log(2*sin(x)/(cos(x) + 1) - 1) + 1/3*log(sin(x)/(cos(x) + 1) - 2)
```

mupad [B] time = 0.40, size = 11, normalized size = 0.26

$$-\frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{3} - \frac{5}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(5*sin(x) - 4),x)
[Out] -(2*atanh((4*tan(x/2))/3 - 5/3))/3
```

sympy [A] time = 0.25, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right) - 2\right)}{3} - \frac{\log\left(\tan\left(\frac{x}{2}\right) - \frac{1}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x)
```

```
[Out] log(tan(x/2) - 2)/3 - log(tan(x/2) - 1/2)/3
```

3.306 $\int x\sqrt[3]{c+x}\,dx$

Optimal. Leaf size=24

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

```
[Out] -3/4*c*(c+x)^(4/3)+3/7*(c+x)^(7/3)
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + x)^(1/3),x]
```

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{c+x}\,dx&=\int\left(-c\sqrt[3]{c+x}+(c+x)^{4/3}\right)dx\\&=-\frac{3}{4}c(c+x)^{4/3}+\frac{3}{7}(c+x)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.75

$$\frac{3}{28}(c+x)^{4/3}(4x-3c)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + x)^(1/3),x]
```

```
[Out] (3*(c + x)^(4/3)*(-3*c + 4*x))/28
```

fricas [A] time = 0.39, size = 22, normalized size = 0.92

$$-\frac{3}{28}\left(3c^2-cx-4x^2\right)(c+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/28*(3*c^2 - c*x - 4*x^2)*(c + x)^(1/3)
```

giac [B] time = 0.94, size = 43, normalized size = 1.79

$$\frac{3}{7}(c+x)^{\frac{7}{3}}-\frac{3}{2}(c+x)^{\frac{4}{3}}c+3(c+x)^{\frac{1}{3}}c^2+\frac{3}{4}\left((c+x)^{\frac{4}{3}}-4(c+x)^{\frac{1}{3}}c\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/2*(c + x)^(4/3)*c + 3*(c + x)^(1/3)*c^2 + 3/4*((c + x
)^(4/3) - 4*(c + x)^(1/3)*c)*c
```

maple [A] time = 0.00, size = 15, normalized size = 0.62

$$-\frac{3\left(c+x\right)^{\frac{4}{3}}\left(3c-4x\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c+x)^(1/3),x)
```

```
[Out] -3/28*(c+x)^(4/3)*(3*c-4*x)
```

maxima [A] time = 0.63, size = 16, normalized size = 0.67

$$\frac{3}{7}\left(c+x\right)^{\frac{7}{3}}-\frac{3}{4}\left(c+x\right)^{\frac{4}{3}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

mupad [B] time = 0.16, size = 14, normalized size = 0.58

$$-\frac{3\left(c+x\right)^{4/3}\left(3\,c-4\,x\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + x)^(1/3),x)
```

```
[Out] -(3*(c + x)^(4/3)*(3*c - 4*x))/28
```

sympy [B] time = 1.10, size = 144, normalized size = 6.00

$$-\frac{9c^{\frac{13}{3}}\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{9c^{\frac{13}{3}}}{28c^2+28cx}-\frac{6c^{\frac{10}{3}}x\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{9c^{\frac{10}{3}}x}{28c^2+28cx}+\frac{15c^{\frac{7}{3}}x^2\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{12c^{\frac{4}{3}}x^3\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)**(1/3),x)
```

```
[Out] -9*c**(13/3)*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(13/3)/(28*c**2 + 2
8*c*x) - 6*c**(10/3)*x*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(10/3)*x/
(28*c**2 + 28*c*x) + 15*c**(7/3)*x**2*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) +
 12*c**(4/3)*x**3*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x)
```

3.307 $\int e^{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=38

$$3e^{\sqrt[3]{x}}x^{2/3}-6e^{\sqrt[3]{x}}\sqrt[3]{x}+6e^{\sqrt[3]{x}}$$

```
[Out] 6*exp(x^(1/3))-6*exp(x^(1/3))*x^(1/3)+3*exp(x^(1/3))*x^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$3e^{\sqrt[3]{x}}x^{2/3}-6e^{\sqrt[3]{x}}\sqrt[3]{x}+6e^{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^(1/3),x]
```

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k = Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int e^x x^2\,dx, x, \sqrt[3]{x}\right)\\ &= 3e^{\sqrt[3]{x}}x^{2/3}-6\,\text{Subst}\left(\int e^x x\,dx, x, \sqrt[3]{x}\right)\\ &= -6e^{\sqrt[3]{x}}\sqrt[3]{x}+3e^{\sqrt[3]{x}}x^{2/3}+6\,\text{Subst}\left(\int e^x\,dx, x, \sqrt[3]{x}\right)\\ &= 6e^{\sqrt[3]{x}}-6e^{\sqrt[3]{x}}\sqrt[3]{x}+3e^{\sqrt[3]{x}}x^{2/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.63

$$e^{\sqrt[3]{x}}\left(3x^{2/3}-6\sqrt[3]{x}+6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^(1/3),x]
```

```
[Out] E^x^(1/3)*(6 - 6*x^(1/3) + 3*x^(2/3))
```

fricas [A] time = 0.40, size = 16, normalized size = 0.42

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="fricas")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

giac [A] time = 1.07, size = 16, normalized size = 0.42

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="giac")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

maple [A] time = 0.00, size = 26, normalized size = 0.68

$$3x^{\frac{2}{3}}\mathrm{e}^{x^{\frac{1}{3}}} - 6x^{\frac{1}{3}}\mathrm{e}^{x^{\frac{1}{3}}} + 6\,\mathrm{e}^{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/3)),x)
[Out] 6*exp(x^(1/3))-6*exp(x^(1/3))*x^(1/3)+3*exp(x^(1/3))*x^(2/3)
```

maxima [A] time = 0.57, size = 16, normalized size = 0.42

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="maxima")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

mupad [B] time = 0.03, size = 21, normalized size = 0.55

$$3\,x\,\mathrm{e}^{x^{1/3}}\left(\frac{2}{x} + \frac{1}{x^{1/3}} - \frac{2}{x^{2/3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/3)),x)
[Out] 3*x*exp(x^(1/3))*(2/x + 1/x^(1/3) - 2/x^(2/3))
```

sympy [A] time = 0.34, size = 34, normalized size = 0.89

$$3x^{\frac{2}{3}}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}\,e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/3)),x)
[Out] 3*x**(2/3)*exp(x**(1/3)) - 6*x**(1/3)*exp(x**(1/3)) + 6*exp(x**(1/3))
```

3.308 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)+\text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \log\left(4+x+\sqrt{1+x}\right)+2\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}}+\log\left(4+x+\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

fricas [A] time = 0.40, size = 32, normalized size = 0.86

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

giac [A] time = 0.95, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

maple [B] time = 0.02, size = 93, normalized size = 2.51

$$\frac{\sqrt{11}\arctan\left(\frac{(2x+7)\sqrt{11}}{11}\right)}{11}-\frac{\sqrt{11}\arctan\left(\frac{\left(2\sqrt{x+1}-1\right)\sqrt{11}}{11}\right)}{11}-\frac{\sqrt{11}\arctan\left(\frac{\left(2\sqrt{x+1}+1\right)\sqrt{11}}{11}\right)}{11}+\frac{\ln\left(x+4+\sqrt{x+1}\right)}{2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(x+1)^(1/2)),x)
```

```
[Out] -1/2*ln(x+4-(x+1)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(x+1)^(1/2)-1)*11^(1/
2))+1/2*ln(4+x+(x+1)^(1/2))-1/11*arctan(1/11*(2*(x+1)^(1/2)+1)*11^(1/2))*11
^(1/2)+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

maxima [A] time = 1.35, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

mupad [B] time = 0.20, size = 32, normalized size = 0.86

$$\ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
```

```
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

sympy [A] time = 2.56, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

3.309 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

```
[Out] 1/x+x+2*ln(1-x)-ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1620}

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{-x^2+x^3} \, dx &= \int \frac{1+x^3}{(-1+x)x^2} \, dx\\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right) dx\\
&= \frac{1}{x}+x+2\log(1-x)-\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

fricas [A] time = 0.40, size = 21, normalized size = 1.24

$$\frac{x^2+2\,x\log\left(x-1\right)-x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

giac [A] time = 0.98, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$x - \ln(x) + 2\ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x)
```

```
[Out] x+2*ln(x-1)+1/x-ln(x)
```

maxima [A] time = 0.54, size = 15, normalized size = 0.88

$$x + \frac{1}{x} + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.88

$$x + 2 \ln(x - 1) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
```

```
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

3.310 $\int \left(-3+4x+x^2\right) \sin(2x) \, dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(-3+4x+x^2\right)\sin(2x)\,dx &= \int \left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx\\
&= -(3\int \sin(2x)\,dx)+4\int x\sin(2x)\,dx+\int x^2\sin(2x)\,dx\\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+2\int \cos(2x)\,dx+\int x\cos(2x)\,dx\\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int \sin(2x)\,d\\
&= \frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.72

$$\frac{1}{4}\left(\left(-2x^2-8x+7\right)\cos(2x)+2(x+2)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

fricas [A] time = 0.41, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos\left(2x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

giac [A] time = 0.90, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos\left(2x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

maple [A] time = 0.02, size = 35, normalized size = 0.88

$$-\frac{x^2\cos\left(2x\right)}{2}-2x\cos\left(2x\right)+\frac{x\sin\left(2x\right)}{2}+\frac{7\cos\left(2x\right)}{4}+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-3)*sin(2*x),x)
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

maxima [A] time = 0.56, size = 38, normalized size = 0.95

$$-\frac{1}{4}\left(2x^2-1\right)\cos\left(2x\right)-2x\cos\left(2x\right)+\frac{1}{2}x\sin\left(2x\right)+\frac{3}{2}\cos\left(2x\right)+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

mupad [B] time = 0.20, size = 34, normalized size = 0.85

$$\frac{7\cos\left(2x\right)}{4}+\sin\left(2x\right)-2x\cos\left(2x\right)+\frac{x\sin\left(2x\right)}{2}-\frac{x^2\cos\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(4*x + x^2 - 3),x)
[Out] (7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))/
2
```

sympy [A] time = 0.35, size = 39, normalized size = 0.98

$$-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}-2x\cos\left(2x\right)+\sin\left(2x\right)+\frac{7\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

3.311 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -sin(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4335, 2637}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(\cos(x))\sin(x)\,dx &= -\text{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right)\\ &= -\sin(\cos(x))\end{aligned}$$

Mathematica [A] time = 2.75, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

fricas [B] time = 0.43, size = 20, normalized size = 4.00

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

giac [A] time = 0.91, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
[Out] -sin(cos(x))
```

maple [A] time = 0.01, size = 6, normalized size = 1.20

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

maxima [A] time = 0.46, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
[Out] -sin(cos(x))
```

mupad [B] time = 0.22, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

sympy [A] time = 0.52, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

3.312 $\int \frac{1}{\sqrt{16-x^2}}\,dx$

Optimal. Leaf size=6

$$\sin^{-1}\left(\frac{x}{4}\right)$$

```
[Out] arcsin(1/4*x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \sin^{-1}\left(\frac{x}{4}\right)$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

fricas [B] time = 0.40, size = 18, normalized size = 3.00

$$-2\arctan\left(\frac{\sqrt{-x^2+16}-4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 16) - 4)/x)
```

giac [A] time = 0.98, size = 4, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x)
```

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\arcsin\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+16)^(1/2),x)
```

```
[Out] arcsin(1/4*x)
```

maxima [A] time = 1.12, size = 4, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/4*x)
```

mupad [B] time = 0.01, size = 4, normalized size = 0.67

$$\operatorname{asin}\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16 - x^2)^(1/2),x)
```

```
[Out] asin(x/4)
```

sympy [A] time = 0.15, size = 3, normalized size = 0.50

$$\operatorname{asin}\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+16)**(1/2),x)
```

```
[Out] asin(x/4)
```

3.313 $\int \frac{x^3}{(1+x)^{10}} dx$

Optimal. Leaf size=37

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

```
[Out] 1/9/(1+x)^9-3/8/(1+x)^8+3/7/(1+x)^7-1/6/(1+x)^6
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x)^10,x]
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(1+x)^{10}} dx &= \int \left(-\frac{1}{(1+x)^{10}}+\frac{3}{(1+x)^9}-\frac{3}{(1+x)^8}+\frac{1}{(1+x)^7}\right) dx\\ &= \frac{1}{9(1+x)^9}-\frac{3}{8(1+x)^8}+\frac{3}{7(1+x)^7}-\frac{1}{6(1+x)^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$-\frac{84x^3+36x^2+9x+1}{504(x+1)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x)^10,x]
[Out] -1/504*(1 + 9*x + 36*x^2 + 84*x^3)/(1 + x)^9
```

fricas [B] time = 0.39, size = 62, normalized size = 1.68

$$-\frac{84\,x^3+36\,x^2+9\,x+1}{504\,\left(x^9+9\,x^8+36\,x^7+84\,x^6+126\,x^5+126\,x^4+84\,x^3+36\,x^2+9\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="fricas")
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

giac [A] time = 0.87, size = 22, normalized size = 0.59

$$-\frac{84\,x^{3}+36\,x^{2}+9\,x+1}{504\,(x+1)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="giac")
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x + 1)^9
```

maple [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{1}{9\,(x+1)^{9}}-\frac{3}{8\,(x+1)^{8}}+\frac{3}{7\,(x+1)^{7}}-\frac{1}{6\,(x+1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x+1)^10,x)
[Out] 1/9/(x+1)^9-3/8/(x+1)^8+3/7/(x+1)^7-1/6/(x+1)^6
```

maxima [B] time = 0.48, size = 62, normalized size = 1.68

$$-\frac{84\,x^{3}+36\,x^{2}+9\,x+1}{504\,\left(x^{9}+9\,x^{8}+36\,x^{7}+84\,x^{6}+126\,x^{5}+126\,x^{4}+84\,x^{3}+36\,x^{2}+9\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="maxima")
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

mupad [B] time = 0.12, size = 29, normalized size = 0.78

$$\frac{3}{7\,(x+1)^{7}}-\frac{1}{6\,(x+1)^{6}}-\frac{3}{8\,(x+1)^{8}}+\frac{1}{9\,(x+1)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x + 1)^10,x)
[Out] 3/(7*(x + 1)^7) - 1/(6*(x + 1)^6) - 3/(8*(x + 1)^8) + 1/(9*(x + 1)^9)
```

sympy [A] time = 0.15, size = 61, normalized size = 1.65

$$\frac{-84x^{3}-36x^{2}-9x-1}{504x^{9}+4536x^{8}+18144x^{7}+42336x^{6}+63504x^{5}+63504x^{4}+42336x^{3}+18144x^{2}+4536x+504}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**10,x)
[Out] (-84*x**3 - 36*x**2 - 9*x - 1)/(504*x**9 + 4536*x**8 + 18144*x**7 + 42336*x
**6 + 63504*x**5 + 63504*x**4 + 42336*x**3 + 18144*x**2 + 4536*x + 504)
```

3.314 $\int \cot^3(2x)\csc^3(2x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

```
[Out] 1/6*csc(2*x)^3-1/10*csc(2*x)^5
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 14}

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(2x)\csc^3(2x)\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(2x)\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(2x)\right)\right)\\ &= \frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

fricas [B] time = 0.44, size = 36, normalized size = 1.71

$$-\frac{5\,\cos\left(2\,x\right)^2-2}{30\left(\cos\left(2\,x\right)^4-2\,\cos\left(2\,x\right)^2+1\right)\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(5*cos(2*x)^2 - 2)/((cos(2*x)^4 - 2*cos(2*x)^2 + 1)*sin(2*x))
```

giac [A] time = 1.02, size = 18, normalized size = 0.86

$$\frac{5 \sin(2x)^2 - 3}{30 \sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

maple [B] time = 0.06, size = 58, normalized size = 2.76

$$\frac{\cos^4(2x)}{30\sin(2x)} - \frac{\cos^4(2x)}{30\sin(2x)^3} + \frac{\left(\cos^2(2x) + 2\right)\sin(2x)}{30} - \frac{\cos^4(2x)}{10\sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)^3*csc(2*x)^3,x)
```

```
[Out] -1/10/sin(2*x)^5*cos(2*x)^4-1/30/sin(2*x)^3*cos(2*x)^4+1/30/sin(2*x)*cos(2*
x)^4+1/30*(2+cos(2*x)^2)*sin(2*x)
```

maxima [A] time = 0.52, size = 18, normalized size = 0.86

$$\frac{5 \sin(2x)^2 - 3}{30 \sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

mupad [B] time = 0.38, size = 18, normalized size = 0.86

$$\frac{5\sin(2x)^2 - 3}{30\sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)^3/sin(2*x)^3,x)
```

```
[Out] (5*sin(2*x)^2 - 3)/(30*sin(2*x)^5)
```

sympy [A] time = 0.12, size = 19, normalized size = 0.90

$$-\frac{3 - 5\sin^2(2x)}{30\sin^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)**3*csc(2*x)**3,x)
```

```
[Out] -(3 - 5*sin(2*x)**2)/(30*sin(2*x)**5)
```

3.315 $\int (x+\sin(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int (x+\sin(x))^2 \, dx &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right) dx \\
&= \frac{x^3}{3}+2\int x\sin(x)\, dx+\int \sin^2(x)\, dx \\
&= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\, dx}{2}+2\int \cos(x)\, dx \\
&= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.00

$$\frac{1}{6}x\left(2x^2+3\right)+2\sin(x)-\frac{1}{4}\sin(2x)-2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^2,x]
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

fricas [A] time = 0.43, size = 22, normalized size = 0.73

$$\frac{1}{3}x^3-2x\cos(x)-\frac{1}{2}(\cos(x)-4)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

giac [A] time = 0.94, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3-2x\cos(x)+\frac{1}{2}x-\frac{1}{4}\sin(2x)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

maple [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{x^3}{3}-2x\cos(x)-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^2,x)
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

maxima [A] time = 0.58, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3-2x\cos(x)+\frac{1}{2}x-\frac{1}{4}\sin(2x)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

mupad [B] time = 0.31, size = 24, normalized size = 0.80

$$\frac{x}{2}+2\sin(x)-\frac{\cos(x)\sin(x)}{2}-2x\cos(x)+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))^2,x)
[Out] x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3
```

sympy [**A**] time = 0.22, size = 41, normalized size = 1.37

$$\frac{x^{3}}{3} + \frac{x\sin^{2}(x)}{2} + \frac{x\cos^{2}(x)}{2} - 2x\cos(x) - \frac{\sin(x)\cos(x)}{2} + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

3.316 $\int \frac{e^{\tan^{-1}(x)}}{1+x^2} dx$

Optimal. Leaf size=4

$$e^{\tan^{-1}(x)}$$

```
[Out] exp(arctan(x))
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5071}

$$e^{\tan^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[x]/(1 + x^2),x]
[Out] E^ArcTan[x]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}}{1+x^2} dx = e^{\tan^{-1}(x)}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 6.75

$$(1-ix)^{\frac{i}{2}}(1+ix)^{-\frac{i}{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[x]/(1 + x^2),x]
[Out] (1 - I*x)^(I/2)/(1 + I*x)^(I/2)
```

fricas [A] time = 0.42, size = 3, normalized size = 0.75

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="fricas")
[Out] e^arctan(x)
```

giac [A] time = 0.95, size = 3, normalized size = 0.75

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="giac")
[Out] e^arctan(x)
```

maple [A] time = 0.00, size = 4, normalized size = 1.00

$$\mathrm{e}^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))/(x^2+1),x)
[Out] exp(arctan(x))
```

maxima [A] time = 0.50, size = 3, normalized size = 0.75

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="maxima")
[Out] e^arctan(x)
```

mupad [B] time = 0.25, size = 3, normalized size = 0.75

$$\mathrm{e}^{\mathrm{atan}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(x))/(x^2 + 1),x)
[Out] exp(atan(x))
```

sympy [A] time = 0.82, size = 3, normalized size = 0.75

$$e^{\operatorname{atan}{\left(x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))/(x**2+1),x)
[Out] exp(atan(x))
```

3.317 $\int \frac{1}{x(1+x^4)} dx$

Optimal. Leaf size=13

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] ln(x)-1/4*ln(x^4+1)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {266, 36, 29, 31}

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4)),x]
```

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+x^4\right)} dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x} dx, x, x^4\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x} dx, x, x^4\right) \\
&= \log(x) - \frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4)),x]
[Out] Log[x] - Log[1 + x^4]/4
```

fricas [A] time = 0.40, size = 11, normalized size = 0.85

$$-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 1) + log(x)
```

giac [A] time = 0.90, size = 15, normalized size = 1.15

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="giac")
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1),x)
[Out] ln(x)-1/4*ln(x^4+1)
```

maxima [A] time = 0.47, size = 15, normalized size = 1.15

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.07, size = 11, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)),x)
[Out] log(x) - log(x^4 + 1)/4
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1),x)
[Out] log(x) - log(x**4 + 1)/4
```

3.318 $\int e^{-2t}t^3\,dt$

Optimal. Leaf size=44

$$-\frac{1}{2}e^{-2t}t^3-\frac{3}{4}e^{-2t}t^2-\frac{3}{4}e^{-2t}t-\frac{3e^{-2t}}{8}$$

```
[Out] -3/8/exp(2*t)-3/4*t/exp(2*t)-3/4*t^2/exp(2*t)-1/2*t^3/exp(2*t)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-\frac{1}{2}e^{-2t}t^3-\frac{3}{4}e^{-2t}t^2-\frac{3}{4}e^{-2t}t-\frac{3e^{-2t}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/E^(2*t),t]
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2*E^(2*t))
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2t}t^3\,dt&=-\frac{1}{2}e^{-2t}t^3+\frac{3}{2}\int e^{-2t}t^2\,dt\\&=-\frac{3}{4}e^{-2t}t^2-\frac{1}{2}e^{-2t}t^3+\frac{3}{2}\int e^{-2t}t\,dt\\&=-\frac{3}{4}e^{-2t}t-\frac{3}{4}e^{-2t}t^2-\frac{1}{2}e^{-2t}t^3+\frac{3}{4}\int e^{-2t}\,dt\\&=-\frac{3}{8}e^{-2t}-\frac{3}{4}e^{-2t}t-\frac{3}{4}e^{-2t}t^2-\frac{1}{2}e^{-2t}t^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.55

$$-\frac{1}{8}e^{-2t}\left(4t^3+6t^2+6t+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/E^(2*t),t]
[Out] -1/8*(3 + 6*t + 6*t^2 + 4*t^3)/E^(2*t)
```

fricas [A] time = 0.41, size = 21, normalized size = 0.48

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="fricas")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

giac [A] time = 0.94, size = 21, normalized size = 0.48

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="giac")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

maple [A] time = 0.00, size = 24, normalized size = 0.55

$$-\frac{\left(4t^3+6t^2+6t+3\right)\mathrm{e}^{-2t}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/exp(2*t),t)
[Out] -1/8*(4*t^3+6*t^2+6*t+3)/exp(2*t)
```

maxima [A] time = 0.47, size = 21, normalized size = 0.48

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="maxima")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.48

$$-\frac{\mathrm{e}^{-2\,t}\,\left(8\,t^3+12\,t^2+12\,t+6\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3*exp(-2*t),t)
[Out] -(exp(-2*t)*(12*t + 12*t^2 + 8*t^3 + 6))/16
```

sympy [A] time = 0.10, size = 22, normalized size = 0.50

$$\frac{\left(-4t^3-6t^2-6t-3\right)e^{-2t}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/exp(2*t),t)
[Out] (-4*t**3 - 6*t**2 - 6*t - 3)*exp(-2*t)/8
```

3.319 $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$

Optimal. Leaf size=41

$$\frac{6t^{7/6}}{7}-\frac{6t^{5/6}}{5}+2\sqrt{t}-6\sqrt[6]{t}+6\tan^{-1}\left(\sqrt[6]{t}\right)$$

```
[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {341, 50, 63, 203}

$$\frac{6t^{7/6}}{7}-\frac{6t^{5/6}}{5}+2\sqrt{t}-6\sqrt[6]{t}+6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt &= 3\,\mathrm{Subst}\left(\int \frac{t^{7/2}}{1+t}\,dt, t, \sqrt[3]{t}\right) \\
&= \frac{6t^{7/6}}{7} - 3\,\mathrm{Subst}\left(\int \frac{t^{5/2}}{1+t}\,dt, t, \sqrt[3]{t}\right) \\
&= -\frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\,\mathrm{Subst}\left(\int \frac{t^{3/2}}{1+t}\,dt, t, \sqrt[3]{t}\right) \\
&= 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} - 3\,\mathrm{Subst}\left(\int \frac{\sqrt{t}}{1+t}\,dt, t, \sqrt[3]{t}\right) \\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{t}\,(1+t)}\,dt, t, \sqrt[3]{t}\right) \\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\,\mathrm{Subst}\left(\int \frac{1}{1+t^2}\,dt, t, \sqrt[6]{t}\right) \\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{t}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{6t^{7/6}}{7} - \frac{6t^{5/6}}{5} + 2\sqrt{t} - 6\sqrt[6]{t} + 6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[t]/(1 + t^(1/3)),t]
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

fricas [A] time = 0.44, size = 25, normalized size = 0.61

$$\frac{6}{7}\,(t-7)t^{\frac{1}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="fricas")
[Out] 6/7*(t - 7)*t^(1/6) - 6/5*t^(5/6) + 2*sqrt(t) + 6*arctan(t^(1/6))
```

giac [A] time = 1.03, size = 27, normalized size = 0.66

$$\frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="giac")
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

maple [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6t^{\frac{7}{6}}}{7} + 6\arctan\left(t^{\frac{1}{6}}\right) - \frac{6t^{\frac{5}{6}}}{5} + 2\sqrt{t} - 6t^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^(1/2)/(1+t^(1/3)),t)
```

```
[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)
```

maxima [A] time = 1.13, size = 27, normalized size = 0.66

$$\frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="maxima")
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

mupad [B] time = 0.04, size = 27, normalized size = 0.66

$$6\,\mathrm{atan}\left(t^{1/6}\right) + 2\,\sqrt{t} - 6\,t^{1/6} - \frac{6\,t^{5/6}}{5} + \frac{6\,t^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^(1/2)/(t^(1/3) + 1),t)
[Out] 6*atan(t^(1/6)) + 2*t^(1/2) - 6*t^(1/6) - (6*t^(5/6))/5 + (6*t^(7/6))/7
```

sympy [A] time = 3.71, size = 37, normalized size = 0.90

$$\frac{6t^{\frac{7}{6}}}{7} - \frac{6t^{\frac{5}{6}}}{5} - 6\sqrt[6]{t} + 2\sqrt{t} + 6\,\mathrm{atan}\left(\sqrt[6]{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**(1/2)/(1+t**(1/3)),t)
[Out] 6*t**(7/6)/7 - 6*t**(5/6)/5 - 6*t**(1/6) + 2*sqrt(t) + 6*atan(t**(1/6))
```

3.320 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

fricas [A] time = 0.45, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

giac [A] time = 0.99, size = 13, normalized size = 0.52

$$-\frac{4}{3}\,\sin(x)^6+\frac{3}{2}\,\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

maple [A] time = 0.08, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

maxima [A] time = 0.61, size = 19, normalized size = 0.76

$$\frac{1}{24}\,\cos(6\,x)-\frac{1}{16}\,\cos(4\,x)-\frac{1}{8}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

mupad [B] time = 0.22, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\,\left(8\,\sin(x)^2-9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

sympy [B] time = 13.25, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

3.321 $\int \log\left(\frac{x}{2}\right) dx$

Optimal. Leaf size=12

$$x\log\left(\frac{x}{2}\right)-x$$

```
[Out] -x+x*ln(1/2*x)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2295}

$$x\log\left(\frac{x}{2}\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log\left(\frac{x}{2}\right) dx = -x+x\log\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x\log\left(\frac{x}{2}\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

fricas [A] time = 0.44, size = 10, normalized size = 0.83

$$x\log\left(\frac{1}{2}x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="fricas")
```

```
[Out] x*log(1/2*x) - x
```

giac [A] time = 1.01, size = 10, normalized size = 0.83

$$x\log\left(\frac{1}{2}x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="giac")
```

```
[Out] x*log(1/2*x) - x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$x \ln\left(\frac{x}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/2*x),x)
```

```
[Out] -x+x*ln(1/2*x)
```

maxima [A] time = 0.53, size = 10, normalized size = 0.83

$$x \log\left(\frac{1}{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="maxima")
```

```
[Out] x*log(1/2*x) - x
```

mupad [B] time = 0.18, size = 8, normalized size = 0.67

$$x\,\left(\ln\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x/2),x)
```

```
[Out] x*(log(x/2) - 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.58

$$x \log\left(\frac{x}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/2*x),x)
```

```
[Out] x*log(x/2) - x
```

3.322 $\int \sqrt{\frac{1+x}{1-x}}\,dx$

Optimal. Leaf size=41

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

```
[Out] 2*arctan(((1+x)/(1-x))^(1/2))-(1-x)*((1+x)/(1-x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 203}

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{1-x}}\,dx &= 4\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1+x}{1-x}}\right)\\ &= -(1-x)\sqrt{\frac{1+x}{1-x}}+2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1+x}{1-x}}\right)\\ &= -(1-x)\sqrt{\frac{1+x}{1-x}}+2\tan^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.51

$$\frac{\sqrt{\frac{x+1}{1-x}}\left((x-1)\sqrt{x+1}-2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] (Sqrt[(1 + x)/(1 - x)]*((-1 + x)*Sqrt[1 + x] - 2*Sqrt[1 - x]*ArcSin[Sqrt[1
- x]/Sqrt[2]]))/Sqrt[1 + x]
```

fricas [A] time = 0.43, size = 32, normalized size = 0.78

$$(x-1)\sqrt{-\frac{x+1}{x-1}}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x - 1)*sqrt(-(x + 1)/(x - 1)) + 2*arctan(sqrt(-(x + 1)/(x - 1)))
```

giac [A] time = 1.13, size = 30, normalized size = 0.73

$$\frac{1}{2}\pi\operatorname{sgn}(x-1)-\arcsin(x)\operatorname{sgn}(x-1)+\sqrt{-x^2+1}\operatorname{sgn}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x - 1) - arcsin(x)*sgn(x - 1) + sqrt(-x^2 + 1)*sgn(x - 1)
```

maple [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{\sqrt{-\frac{x+1}{x-1}}\,(x-1)\left(-\arcsin(x)+\sqrt{-x^2+1}\right)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/(-x+1))^(1/2),x)
```

```
[Out] (-(x+1)/(x-1))^(1/2)*(x-1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)-arcsin(x))
```

maxima [A] time = 1.20, size = 43, normalized size = 1.05

$$\frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-(x + 1)/(x - 1))/((x + 1)/(x - 1) - 1) + 2*arctan(sqrt(-(x + 1)/(x
- 1)))
```

mupad [B] time = 0.05, size = 43, normalized size = 1.05

$$2\operatorname{atan}\left(\sqrt{-\frac{x+1}{x-1}}\right)+\frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x + 1)/(x - 1))^(1/2),x)
```

```
[Out] 2*atan((-(x + 1)/(x - 1))^(1/2)) + (2*(-(x + 1)/(x - 1))^(1/2))/((x + 1)/(x
 - 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x+1}{1-x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(1 - x)), x)
```

3.323 $\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

[Out] arctan((x^2-1)^(1/2))-(x^2-1)^(1/2)+ln(x)*(x^2-1)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2338, 266, 50, 63, 203}

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[(x*Log[x])/Sqrt[-1 + x^2],x]

[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[(f^m*(d + e*x^r)^(q + 1)*(a + b*L
og[c*x^n])^p)/(e*r*(q + 1)), x] - Dist[(b*f^m*n*p)/(e*r*(q + 1)), Int[((d +
 e*x^r)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx &= \sqrt{-1+x^2}\log(x) - \int \frac{\sqrt{-1+x^2}}{x}\,dx\\ &= \sqrt{-1+x^2}\log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^2\right)\\ &= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^2\right)\\ &= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^2}\right)\\ &= -\sqrt{-1+x^2} + \tan^{-1}\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2}\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}\,(\log(x)-1) - \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]
[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])
```

fricas [A] time = 0.44, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}\left(\log(x)-1\right) + 2\arctan\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))
```

giac [A] time = 0.94, size = 28, normalized size = 0.82

$$\sqrt{x^2-1}\log(x) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

maple [C] time = 0.00, size = 119, normalized size = 3.50

$$\frac{\sqrt{-\text{signum}\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)\ln(x)}{2\sqrt{\text{signum}\left(x^2-1\right)}} - \frac{\sqrt{-\text{signum}\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)}{4\sqrt{\text{signum}\left(x^2-1\right)}} + \frac{\sqrt{-\text{signum}\left(x^2-1\right)}\left(-3\right.}{32\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)/(x^2-1)^(1/2),x)
[Out] 1/2*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))*ln(x)/signum(x^2-1)^(1/2)-1
/4*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))/signum(x^2-1)^(1/2)+1/32*(-s
```

```
ignum(x^2-1))^(1/2)*(-32*ln(1/2+1/2*(-x^2+1)^(1/2))-16+16*(-x^2+1)^(1/2))/s
ignum(x^2-1)^(1/2)
```

maxima [A] time = 1.31, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}\log(x) - \sqrt{x^2-1} - \arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\ln(x)}{\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x))/(x^2 - 1)^(1/2),x)
```

```
[Out] int((x*log(x))/(x^2 - 1)^(1/2), x)
```

sympy [A] time = 2.86, size = 29, normalized size = 0.85

$$\sqrt{x^2-1}\log(x) - \begin{cases} \sqrt{x^2-1} - \operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x > -1) & (
x < 1)))
```

3.324 $\int \frac{a+x}{a^2+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

```
[Out] arctan(x/a)+1/2*ln(a^2+x^2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {635, 203, 260}

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{a+x}{a^2+x^2}\,dx &= a\int \frac{1}{a^2+x^2}\,dx+\int \frac{x}{a^2+x^2}\,dx\\ &= \tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

fricas [A] time = 0.40, size = 17, normalized size = 0.89

$$\arctan\left(\frac{x}{a}\right)+\frac{1}{2}\log\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

giac [A] time = 1.03, size = 17, normalized size = 0.89

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\arctan\left(\frac{x}{a}\right) + \frac{\ln\left(a^2 + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(a^2+x^2),x)
```

```
[Out] arctan(1/a*x)+1/2*ln(a^2+x^2)
```

maxima [A] time = 1.27, size = 17, normalized size = 0.89

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.89

$$\frac{\ln\left(a^2 + x^2\right)}{2} + \operatorname{atan}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x)/(a^2 + x^2),x)
```

```
[Out] log(a^2 + x^2)/2 + atan(x/a)
```

sympy [C] time = 0.11, size = 42, normalized size = 2.21

$$\left(\frac{1}{2} - \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} - \frac{i}{2}\right) + x\right) + \left(\frac{1}{2} + \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} + \frac{i}{2}\right) + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a**2+x**2),x)
```

```
[Out] (1/2 - I/2)*log(-a + 2*a*(1/2 - I/2) + x) + (1/2 + I/2)*log(-a + 2*a*(1/2 +
 I/2) + x)
```

3.325 $\int \sqrt{1+x-x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4}\sqrt{-x^2+x+1}\,(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

```
[Out] -5/8*arcsin(1/5*(1-2*x)*5^(1/2))-1/4*(1-2*x)*(-x^2+x+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {612, 619, 216}

$$-\frac{1}{4}\sqrt{-x^2+x+1}\,(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x - x^2],x]
```

```
[Out] -((1 - 2*x)*Sqrt[1 + x - x^2])/4 - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+x-x^2}\,dx &= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}+\frac{5}{8}\int \frac{1}{\sqrt{1+x-x^2}}\,dx\\
&= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx,x,1-2x\right)\\
&= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1}-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x - x^2],x]
[Out] (-1/4 + x/2)*Sqrt[1 + x - x^2] - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

fricas [A] time = 0.42, size = 37, normalized size = 0.97

$$\frac{1}{4}\sqrt{-x^2+x+1}\,(2x-1)-\frac{5}{4}\arctan\left(\frac{\sqrt{-x^2+x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) - 5/4*arctan((sqrt(-x^2 + x + 1) - 1)/x)
```

giac [A] time = 0.94, size = 31, normalized size = 0.82

$$\frac{1}{4}\sqrt{-x^2+x+1}\,(2x-1)+\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}\,(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) + 5/8*arcsin(1/5*sqrt(5)*(2*x - 1))
```

maple [A] time = 0.00, size = 30, normalized size = 0.79

$$\frac{5\arcsin\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8}-\frac{(-2x+1)\sqrt{-x^2+x+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x+1)^(1/2),x)
[Out] -1/4*(1-2*x)*(-x^2+x+1)^(1/2)+5/8*arcsin(2/5*5^(1/2)*(x-1/2))
```

maxima [A] time = 1.32, size = 39, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+x+1}\,x-\frac{1}{4}\sqrt{-x^2+x+1}-\frac{5}{8}\arcsin\left(-\frac{1}{5}\sqrt{5}\,(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + x + 1)*x - 1/4*sqrt(-x^2 + x + 1) - 5/8*arcsin(-1/5*sqrt(5)
*(2*x - 1))
```

mupad [B] time = 0.05, size = 28, normalized size = 0.74

$$\frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8}+\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2 + 1)^(1/2),x)
[Out] (5*asin((2*5^(1/2)*(x - 1/2))/5))/8 + (x/2 - 1/4)*(x - x^2 + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x + 1), x)
```

3.326 $\int \frac{x^4}{16+x^{10}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

```
[Out] 1/20*arctan(1/4*x^5)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 203}

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{16+x^{10}} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{16+x^2} \, dx, x, x^5\right)\\ &= \frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

fricas [A] time = 0.40, size = 8, normalized size = 0.67

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="fricas")
[Out] 1/20*arctan(1/4*x^5)
```

giac [A] time = 0.79, size = 8, normalized size = 0.67

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="giac")
[Out] 1/20*arctan(1/4*x^5)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{\arctan\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+16),x)
[Out] 1/20*arctan(1/4*x^5)
```

maxima [A] time = 1.34, size = 8, normalized size = 0.67

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="maxima")
[Out] 1/20*arctan(1/4*x^5)
```

mupad [B] time = 0.18, size = 8, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10 + 16),x)
[Out] atan(x^5/4)/20
```

sympy [A] time = 0.12, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+16),x)
[Out] atan(x**5/4)/20
```

3.327 $\int \frac{2+x}{2+x+x^2} dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] 1/2*ln(x^2+2+x+2)+3/7*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{2+x+x^2}\,dx &= \frac{1}{2}\int \frac{1+2x}{2+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{2+x+x^2}\,dx\\
&= \frac{1}{2}\log\left(2+x+x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right)\\
&= \frac{3\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{1}{2}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(2 + x + x^2),x]
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

fricas [A] time = 0.41, size = 26, normalized size = 0.84

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="fricas")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

giac [A] time = 1.02, size = 26, normalized size = 0.84

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="giac")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$\frac{3\sqrt{7}\,\arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{7}+\frac{\ln\left(x^2+x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+x+2),x)
[Out] 1/2*ln(x^2+x+2)+3/7*7^(1/2)*arctan(1/7*(2*x+1)*7^(1/2))
```

maxima [A] time = 1.28, size = 26, normalized size = 0.84

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="maxima")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.90

$$\frac{\ln\left(x^2+x+2\right)}{2}+\frac{3\,\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x + x^2 + 2),x)
```

```
[Out] log(x + x^2 + 2)/2 + (3*7^(1/2)*atan((2*7^(1/2)*x)/7 + 7^(1/2)/7))/7
```

sympy [**A**] time = 0.11, size = 36, normalized size = 1.16

$$\frac{\log\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}\,x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+2),x)
```

```
[Out] log(x**2 + x + 2)/2 + 3*sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)/7
```

3.328 $\int x \sec(x) \tan(x)\, dx$

Optimal. Leaf size=10

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

```
[Out] -arctanh(sin(x))+x*sec(x)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3757, 3770}

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]*Tan[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sec(x) \tan(x)\, dx &= x \sec(x) - \int \sec(x)\, dx \\ &= -\tanh^{-1}(\sin(x)) + x \sec(x) \end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.70

$$x \sec(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*Tan[x],x]
```

```
[Out] Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + x*Sec[x]
```

fricas [B] time = 0.45, size = 29, normalized size = 2.90

$$-\frac{\cos(x) \log(\sin(x) + 1) - \cos(x) \log(-\sin(x) + 1) - 2\, x}{2\, \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*log(sin(x) + 1) - cos(x)*log(-sin(x) + 1) - 2*x)/cos(x)
```

giac [B] time = 1.03, size = 150, normalized size = 15.00

$$-\frac{2\,x\tan\left(\frac{1}{2}\,x\right)^2+\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2-\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+2\,x-\log}{2\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*(2*x*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 + 2*x - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1
)))/(tan(1/2*x)^2 - 1)
```

maple [A] time = 0.01, size = 16, normalized size = 1.60

$$-\ln\left(\sec(x)+\tan(x)\right)+\frac{x}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)*tan(x),x)
```

```
[Out] x/cos(x)-ln(sec(x)+tan(x))
```

maxima [B] time = 1.28, size = 121, normalized size = 12.10

$$\frac{4\,x\cos\left(2\,x\right)\cos(x)+4\,x\sin\left(2\,x\right)\sin(x)+4\,x\cos(x)-\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)\log\left(\cos(x)^2-\right.}{2\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*x*cos(2*x)*cos(x) + 4*x*sin(2*x)*sin(x) + 4*x*cos(x) - (cos(2*x)^2 +
 sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (co
s(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.11, size = 19, normalized size = 1.90

$$\frac{x}{\cos(x)}+\mathrm{atan}\left(\cos(x)+\sin(x)\,1\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x))/cos(x),x)
```

```
[Out] atan(cos(x) + sin(x)*1i)*2i + x/cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan\left(x\right)\sec\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x)
```

```
[Out] Integral(x*tan(x)*sec(x), x)
```

3.329 $\int \frac{x}{-a^4+x^4} \, dx$

Optimal. Leaf size=15

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] -1/2*arctanh(x^2/a^2)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 207}

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a^4 + x^4),x]
```

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{-a^4+x^4} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{-a^4+x^2} \, dx, x, x^2\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a^4 + x^4),x]
```

```
[Out] -1/2*ArcTanh[x^2/a^2]/a^2
```

fricas [A] time = 0.42, size = 26, normalized size = 1.73

$$-\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="fricas")
[Out] -1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

giac [B] time = 1.07, size = 30, normalized size = 2.00

$$-\frac{\log\left(a^2+x^2\right)}{4\,a^2}+\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="giac")
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(abs(-a^2 + x^2))/a^2
```

maple [B] time = 0.00, size = 30, normalized size = 2.00

$$\frac{\ln\left(-a^2+x^2\right)}{4a^2}-\frac{\ln\left(a^2+x^2\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^4+x^4),x)
[Out] -1/4/a^2*ln(a^2+x^2)+1/4/a^2*ln(-a^2+x^2)
```

maxima [B] time = 0.52, size = 29, normalized size = 1.93

$$-\frac{\log\left(a^2+x^2\right)}{4\,a^2}+\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="maxima")
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(-a^2 + x^2)/a^2
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$-\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(a^4 - x^4),x)
[Out] -atanh(x^2/a^2)/(2*a^2)
```

sympy [A] time = 0.16, size = 22, normalized size = 1.47

$$\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**4+x**4),x)
[Out] (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

3.330 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx &= -\int \sqrt{x}\,dx+\int \sqrt{1+x}\,dx\\ &= -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

fricas [A] time = 0.41, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

giac [A] time = 1.10, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

mupad [B] time = 0.20, size = 21, normalized size = 1.00

$$\frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

sympy [B] time = 0.36, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x+1}}+\frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

3.331 $\int \frac{1}{1-e^{-x}+2e^x} \, dx$

Optimal. Leaf size=23

$$\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(e^x+1\right)$$

[Out] 1/3*ln(1-2*exp(x))-1/3*ln(1+exp(x))

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 616, 31}

$$\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - E^(-x) + 2*E^x)^(-1),x]

[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-e^{-x}+2e^x}\,dx &= \text{Subst}\left(\int \frac{1}{-1+x+2x^2}\,dx,x,e^x\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+2x}\,dx,x,e^x\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,e^x\right)\\
&= \frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(1+e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(4e^x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - E^(-x) + 2*E^x)^(-1),x]
[Out] (-2*ArcTanh[(1 + 4*E^x)/3])/3
```

fricas [A] time = 0.40, size = 17, normalized size = 0.74

$$\frac{1}{3}\log\left(2e^{x}-1\right)-\frac{1}{3}\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="fricas")
[Out] 1/3*log(2*e^x - 1) - 1/3*log(e^x + 1)
```

giac [A] time = 0.97, size = 18, normalized size = 0.78

$$-\frac{1}{3}\log\left(e^{x}+1\right)+\frac{1}{3}\log\left(|2e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="giac")
[Out] -1/3*log(e^x + 1) + 1/3*log(abs(2*e^x - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{\ln\left(e^{x}+1\right)}{3}+\frac{\ln\left(2e^{x}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-1/exp(x)+2*exp(x)),x)
[Out] -1/3*ln(exp(x)+1)+1/3*ln(2*exp(x)-1)
```

maxima [A] time = 0.44, size = 19, normalized size = 0.83

$$-\frac{1}{3}\log\left(e^{(-x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="maxima")
[Out] -1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 2)
```

mupad [B] time = 0.25, size = 17, normalized size = 0.74

$$\frac{\ln\left(2e^{x}-1\right)}{3}-\frac{\ln\left(e^{x}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*exp(x) - exp(-x) + 1),x)
[Out] log(2*exp(x) - 1)/3 - log(exp(x) + 1)/3
```

sympy [A] time = 0.12, size = 17, normalized size = 0.74

$$\frac{\log\left(e^{x}-\frac{1}{2}\right)}{3}-\frac{\log\left(e^{x}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x)
[Out] log(exp(x) - 1/2)/3 - log(exp(x) + 1)/3
```

3.332 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

```
[Out] -ln(1+x)+2*arctan(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 31}

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

fricas [A] time = 0.44, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

giac [A] time = 0.91, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$2\sqrt{x}\,\arctan\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(1/2),x)
[Out] -ln(x+1)+2*arctan(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.54, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

mupad [B] time = 0.25, size = 16, normalized size = 0.80

$$2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

sympy [A] time = 0.43, size = 17, normalized size = 0.85

$$2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

3.333 $\int \frac{\log(1+x)}{x^2} \, dx$

Optimal. Leaf size=18

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

```
[Out] ln(x)-ln(1+x)-ln(1+x)/x
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2395, 36, 29, 31}

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(1+x)}{x^2} \, dx &= -\frac{\log(1+x)}{x} + \int \frac{1}{x(1+x)} \, dx \\
&= -\frac{\log(1+x)}{x} + \int \frac{1}{x} \, dx - \int \frac{1}{1+x} \, dx \\
&= \log(x) - \log(1+x) - \frac{\log(1+x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]/x^2,x]
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

fricas [A] time = 0.42, size = 19, normalized size = 1.06

$$-\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="fricas")
[Out] -((x + 1)*log(x + 1) - x*log(x))/x
```

giac [A] time = 0.86, size = 20, normalized size = 1.11

$$-\frac{\log(x+1)}{x} - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="giac")
[Out] -log(x + 1)/x - log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.89

$$\ln(x) - \frac{(x+1)\ln(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+1)/x^2,x)
[Out] ln(x)-ln(x+1)*(x+1)/x
```

maxima [A] time = 0.50, size = 18, normalized size = 1.00

$$-\frac{\log(x+1)}{x} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="maxima")
[Out] -log(x + 1)/x - log(x + 1) + log(x)
```

mupad [B] time = 0.17, size = 18, normalized size = 1.00

$$-\ln\left(\frac{1}{x}+1\right) - \frac{\ln(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1)/x^2,x)
[Out] - log(1/x + 1) - log(x + 1)/x
```

sympy [A] time = 0.14, size = 14, normalized size = 0.78

$$\log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)/x**2,x)
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

3.334 $\int \frac{1}{-e^x+e^{3x}}\,dx$

Optimal. Leaf size=12

$$e^{-x} - \tanh^{-1}(e^x)$$

[Out] exp(-x)-arctanh(exp(x))

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 325, 207}

$$e^{-x} - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(-E^x + E^(3*x))^(-1),x]

[Out] E^(-x) - ArcTanh[E^x]

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-e^x+e^{3x}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx, x, e^x\right)\\
&= e^{-x}+\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right)\\
&= e^{-x}-\tanh^{-1}(e^x)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 19, normalized size = 1.58

$$e^{-x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^x + E^(3*x))^(-1),x]
[Out] Hypergeometric2F1[-1/2, 1, 1/2, E^(2*x)]/E^x
```

fricas [B] time = 0.43, size = 25, normalized size = 2.08

$$-\frac{1}{2}\left(e^{x}\log\left(e^{x}+1\right)-e^{x}\log\left(e^{x}-1\right)-2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="fricas")
[Out] -1/2*(e^x*log(e^x + 1) - e^x*log(e^x - 1) - 2)*e^(-x)
```

giac [A] time = 0.88, size = 20, normalized size = 1.67

$$e^{(-x)}-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="giac")
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.01, size = 20, normalized size = 1.67

$$\mathrm{e}^{-x}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-exp(x)+exp(3*x)),x)
[Out] -1/2*ln(exp(x)+1)+1/exp(x)+1/2*ln(exp(x)-1)
```

maxima [A] time = 0.61, size = 19, normalized size = 1.58

$$e^{(-x)}-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="maxima")
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

mupad [B] time = 0.23, size = 19, normalized size = 1.58

$$\mathrm{e}^{-x}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(3*x) - exp(x)),x)
[Out] exp(-x) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

sympy [B] time = 0.12, size = 20, normalized size = 1.67

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}+e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)
```

3.335 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-x-2\cot(x)$$

```
[Out] -x-2*cot(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx &= -x+2\int \frac{1}{1-\cos^2(x)}\,dx \\
&= -x+2\int \csc^2(x)\,dx \\
&= -x-2\,\text{Subst}\left(\int 1\,dx, x, \cot(x)\right) \\
&= -x-2\cot(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
[Out] -x - 2*Cot[x]
```

fricas [A] time = 0.44, size = 15, normalized size = 1.88

$$-\frac{x\sin(x)+2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

giac [A] time = 0.91, size = 16, normalized size = 2.00

$$-x-\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

maple [A] time = 0.05, size = 11, normalized size = 1.38

$$-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2+1)/(-cos(x)^2+1),x)
[Out] -2/tan(x)-x
```

maxima [A] time = 1.32, size = 10, normalized size = 1.25

$$-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="maxima")
[Out] -x - 2/tan(x)
```

mupad [B] time = 0.21, size = 8, normalized size = 1.00

$$-x-2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x)
[Out] - x - 2*cot(x)
```

sympy [A] time = 1.15, size = 12, normalized size = 1.50

$$-x+\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(-cos(x)**2+1),x)
[Out] -x + tan(x/2) - 1/tan(x/2)
```

3.336 $\int \frac{1}{x\sqrt{-25+2x}}\,dx$

Optimal. Leaf size=18

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

```
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 203}

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-25+2x}}\,dx &= \text{Subst}\left(\int \frac{1}{\frac{25}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{-25+2x}\right)\\ &= \frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{-25+2x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

fricas [A] time = 0.41, size = 12, normalized size = 0.67

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

giac [A] time = 1.12, size = 12, normalized size = 0.67

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="giac")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$\frac{2\arctan\left(\frac{\sqrt{2x-25}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-25+2*x)^(1/2),x)
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

maxima [A] time = 1.70, size = 12, normalized size = 0.67

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="maxima")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

mupad [B] time = 0.15, size = 12, normalized size = 0.67

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{2\,x-25}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x - 25)^(1/2)),x)
[Out] (2*atan((2*x - 25)^(1/2)/5))/5
```

sympy [A] time = 1.01, size = 44, normalized size = 2.44

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{for}\ \frac{25}{2|x|} > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)**(1/2),x)
[Out] Piecewise((2*I*acosh(5*sqrt(2)/(2*sqrt(x)))/5, 25/(2*Abs(x)) > 1), (-2*asin
(5*sqrt(2)/(2*sqrt(x)))/5, True))
```

3.337 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -arcsin(1/3*cos(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = $\{12, 1107, 619, 216\}$

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right)\\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right)\\
&= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
[Out] -ArcSin[Cos[x]^2/3]
```

fricas [B] time = 0.52, size = 24, normalized size = 2.18

$$\arctan\left(\frac{\sqrt{-\cos(x)^4+9}\,\cos(x)^2}{\cos(x)^4-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

giac [A] time = 1.04, size = 9, normalized size = 0.82

$$-\arcsin\left(\frac{1}{3}\,\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
[Out] -arcsin(1/3*cos(x)^2)
```

maple [A] time = 0.05, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{\left(\cos^2(x)\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x)
[Out] -arcsin(1/3*cos(x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2\,x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

mupad [B] time = 0.43, size = 18, normalized size = 1.64

$$-\mathrm{atan}\left(\frac{{\cos(x)}^2}{\sqrt{9-{\cos(x)}^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
```

```
[Out] -atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.338 $\int \frac{x^2}{\sqrt{5-4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

```
[Out] 5/16*arcsin(2/5*x*5^(1/2))-1/8*x*(-4*x^2+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[5 - 4*x^2],x]
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{5-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{5-4x^2}+\frac{5}{8}\int \frac{1}{\sqrt{5-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{5-4x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[5 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[5 - 4*x^2]) + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

fricas [A] time = 0.41, size = 30, normalized size = 1.00

$$-\frac{1}{8}\sqrt{-4x^2+5}\,x-\frac{5}{16}\arctan\left(\frac{\sqrt{-4x^2+5}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="fricas")
[Out] -1/8*sqrt(-4*x^2 + 5)*x - 5/16*arctan(1/2*sqrt(-4*x^2 + 5)/x)
```

giac [A] time = 1.00, size = 22, normalized size = 0.73

$$-\frac{1}{8}\sqrt{-4\,x^2+5}\,x+\frac{5}{16}\,\arcsin\left(\frac{2}{5}\,\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="giac")
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.77

$$-\frac{\sqrt{-4x^2+5}\,x}{8}+\frac{5\arcsin\left(\frac{2\sqrt{5}\,x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+5)^(1/2),x)
[Out] 5/16*arcsin(2/5*5^(1/2)*x)-1/8*x*(-4*x^2+5)^(1/2)
```

maxima [A] time = 1.30, size = 22, normalized size = 0.73

$$-\frac{1}{8}\sqrt{-4\,x^2+5}\,x+\frac{5}{16}\,\arcsin\left(\frac{2}{5}\,\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

mupad [B] time = 0.17, size = 22, normalized size = 0.73

$$\frac{5\,\mathrm{asin}\left(\frac{2\,\sqrt{5}\,x}{5}\right)}{16}-\frac{x\,\sqrt{\frac{5}{4}-x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(5 - 4*x^2)^(1/2),x)
[Out] (5*asin((2*5^(1/2)*x)/5))/16 - (x*(5/4 - x^2)^(1/2))/4
```

sympy [A] time = 0.26, size = 27, normalized size = 0.90

$$-\frac{x\sqrt{5-4x^2}}{8}+\frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\,x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+5)**(1/2),x)
[Out] -x*sqrt(5 - 4*x**2)/8 + 5*asin(2*sqrt(5)*x/5)/16
```

3.339 $\int x^3 \sin(x)\,dx$

Optimal. Leaf size=24

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$3x^2\sin(x) + x^3(-\cos(x)) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\sin(x)\,dx &= -x^3\cos(x) + 3\int x^2\cos(x)\,dx\\ &= -x^3\cos(x) + 3x^2\sin(x) - 6\int x\sin(x)\,dx\\ &= 6x\cos(x) - x^3\cos(x) + 3x^2\sin(x) - 6\int \cos(x)\,dx\\ &= 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x) - x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x],x]
```

```
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

fricas [A] time = 0.44, size = 21, normalized size = 0.88

$$-\left(x^3-6\,x\right)\cos(x) + 3\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

giac [A] time = 0.96, size = 21, normalized size = 0.88

$$-\left(x^3 - 6\,x\right)\cos(x) + 3\left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

maple [A] time = 0.01, size = 25, normalized size = 1.04

$$-x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
[Out] -x^3*cos(x)+3*x^2*sin(x)+6*x*cos(x)-6*sin(x)
```

maxima [A] time = 0.68, size = 21, normalized size = 0.88

$$-\left(x^3 - 6\,x\right)\cos(x) + 3\left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.96

$$\cos(x)\left(6\,x - x^3\right) + \sin(x)\left(3\,x^2 - 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
[Out] cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

sympy [A] time = 0.68, size = 26, normalized size = 1.08

$$-x^3\cos\left(x\right) + 3x^2\sin\left(x\right) + 6x\cos\left(x\right) - 6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x),x)
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

3.340 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-3/2*arcsinh(1/3*(1+x)*3^(1/2))-1/2*(1+x)*(x^2+2*x+4)^
(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{640, 612, 619, 215\}$

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(1
+ x)/Sqrt[3]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{4+2x+x^2}\,dx &= \frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\int \sqrt{4+2x+x^2}\,dx\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\int \frac{1}{\sqrt{4+2x+x^2}}\,dx\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{1}{4}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.76

$$
\frac{1}{6}\left(\sqrt{x^2+2x+4}\left(2x^2+x+5\right)-9\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] (Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2) - 9*ArcSinh[(1 + x)/Sqrt[3]])/6
```

fricas [A] time = 0.40, size = 39, normalized size = 0.78

$$
\frac{1}{6}\left(2\,x^2+x+5\right)\sqrt{x^2+2\,x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4)
- 1)
```

giac [A] time = 0.95, size = 40, normalized size = 0.80

$$
\frac{1}{6}\left((2\,x+1)x+5\right)\sqrt{x^2+2\,x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4
) - 1)
```

maple [A] time = 0.01, size = 42, normalized size = 0.84

$$
-\frac{3\,\text{arcsinh}\left(\frac{(x+1)\sqrt{3}}{3}\right)}{2}+\frac{\left(x^2+2x+4\right)^{\frac{3}{2}}}{3}-\frac{(2x+2)\sqrt{x^2+2x+4}}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-1/4*(2*x+2)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1/3*(x+1)*3
^(1/2))
```

maxima [A] time = 1.42, size = 49, normalized size = 0.98

$$\frac{1}{3}\left(x^{2}+2x+4\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^{2}+2x+4}\,x-\frac{1}{2}\sqrt{x^{2}+2x+4}-\frac{3}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\,(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(x^2 + 2*x
+ 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

mupad [B] time = 0.21, size = 39, normalized size = 0.78

$$\frac{\sqrt{x^{2}+2x+4}\,\left(8x^{2}+4x+20\right)}{24}-\frac{3\ln\left(x+\sqrt{x^{2}+2x+4}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x + x^2 + 4)^(1/2),x)
```

```
[Out] ((2*x + x^2 + 4)^(1/2)*(4*x + 8*x^2 + 20))/24 - (3*log(x + (2*x + x^2 + 4)^
(1/2) + 1))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{2}+2x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2 + 2*x + 4), x)
```

3.341 $\int x\left(5+x^2\right)^8\,dx$

Optimal. Leaf size=11

$$\frac{1}{18}\left(x^2+5\right)^9$$

```
[Out] 1/18*(x^2+5)^9
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {261}

$$\frac{1}{18}\left(x^2+5\right)^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(5+x^2\right)^8\,dx=\frac{1}{18}\left(5+x^2\right)^9$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{18}\left(x^2+5\right)^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

fricas [B] time = 0.34, size = 46, normalized size = 4.18

$$\frac{1}{18}x^{18}+\frac{5}{2}x^{16}+50x^{14}+\frac{1750}{3}x^{12}+4375x^{10}+21875x^8+\frac{218750}{3}x^6+156250x^4+\frac{390625}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 + 5/2*x^16 + 50*x^14 + 1750/3*x^12 + 4375*x^10 + 21875*x^8 + 2187
50/3*x^6 + 156250*x^4 + 390625/2*x^2
```

giac [A] time = 0.98, size = 9, normalized size = 0.82

$$\frac{1}{18}\left(x^2+5\right)^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="giac")
```

```
[Out] 1/18*(x^2 + 5)^9
```

maple [B] time = 0.00, size = 47, normalized size = 4.27

$$\frac{1}{18}x^{18} + \frac{5}{2}x^{16} + 50x^{14} + \frac{1750}{3}x^{12} + 4375x^{10} + 21875x^{8} + \frac{218750}{3}x^{6} + 156250x^{4} + \frac{390625}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+5)^8,x)
```

```
[Out] 1/18*x^18+5/2*x^16+50*x^14+1750/3*x^12+4375*x^10+21875*x^8+218750/3*x^6+156
250*x^4+390625/2*x^2
```

maxima [A] time = 0.58, size = 9, normalized size = 0.82

$$\frac{1}{18}\left(x^{2} + 5\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(x^2 + 5)^9
```

mupad [B] time = 0.21, size = 9, normalized size = 0.82

$$\frac{\left(x^{2} + 5\right)^{9}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 5)^8,x)
```

```
[Out] (x^2 + 5)^9/18
```

sympy [B] time = 0.06, size = 51, normalized size = 4.64

$$\frac{x^{18}}{18} + \frac{5x^{16}}{2} + 50x^{14} + \frac{1750x^{12}}{3} + 4375x^{10} + 21875x^{8} + \frac{218750x^{6}}{3} + 156250x^{4} + \frac{390625x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+5)**8,x)
```

```
[Out] x**18/18 + 5*x**16/2 + 50*x**14 + 1750*x**12/3 + 4375*x**10 + 21875*x**8 +
218750*x**6/3 + 156250*x**4 + 390625*x**2/2
```

3.342 $\int \cos^2(x)\sin^5(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

```
[Out] -1/3*cos(x)^3+2/5*cos(x)^5-1/7*cos(x)^7
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 270}

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] -Cos[x]^3/3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^5(x)\,dx &= -\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,\cos(x)\right)\\ &= -\frac{1}{3}\cos^3(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.24

$$-\frac{5\cos(x)}{64}-\frac{1}{192}\cos(3x)+\frac{3}{320}\cos(5x)-\frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/64 - Cos[3*x]/192 + (3*Cos[5*x])/320 - Cos[7*x]/448
```

fricas [A] time = 0.42, size = 19, normalized size = 0.76

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="fricas")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

giac [A] time = 0.87, size = 19, normalized size = 0.76

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="giac")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$-\frac{\left(\cos^3(x)\right)\left(\sin^4(x)\right)}{7}-\frac{4\left(\cos^3(x)\right)\left(\sin^2(x)\right)}{35}-\frac{8\left(\cos^3(x)\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^5,x)
[Out] -1/7*cos(x)^3*sin(x)^4-4/35*sin(x)^2*cos(x)^3-8/105*cos(x)^3
```

maxima [A] time = 0.48, size = 19, normalized size = 0.76

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="maxima")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

mupad [B] time = 0.17, size = 19, normalized size = 0.76

$$-\frac{\cos(x)^7}{7}+\frac{2\cos(x)^5}{5}-\frac{\cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^5,x)
[Out] (2*cos(x)^5)/5 - cos(x)^3/3 - cos(x)^7/7
```

sympy [A] time = 0.07, size = 20, normalized size = 0.80

$$-\frac{\cos^7(x)}{7}+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**5,x)
[Out] -cos(x)**7/7 + 2*cos(x)**5/5 - cos(x)**3/3
```

3.343 $\int e^{-3x}\cos(4x)\,dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

```
[Out] -3/25*cos(4*x)/exp(3*x)+4/25*sin(4*x)/exp(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(4x)\,dx=-\frac{3}{25}e^{-3x}\cos(4x)+\frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4\sin(4x)-3\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

fricas [A] time = 0.42, size = 21, normalized size = 0.78

$$-\frac{3}{25}\cos\left(4\,x\right)e^{\left(-3\,x\right)}+\frac{4}{25}\,e^{\left(-3\,x\right)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

giac [A] time = 0.92, size = 19, normalized size = 0.70

$$-\frac{1}{25}\left(3\cos\left(4\,x\right)-4\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

maple [A] time = 0.02, size = 22, normalized size = 0.81

$$-\frac{3\cos(4x)\,\mathrm{e}^{-3x}}{25}+\frac{4\,\mathrm{e}^{-3x}\sin(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/exp(3*x),x)
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

maxima [A] time = 0.49, size = 19, normalized size = 0.70

$$-\frac{1}{25}\,(3\,\cos(4\,x)-4\,\sin(4\,x))e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{-3\,x}\,\left(3\,\cos(4\,x)-4\,\sin(4\,x)\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*exp(-3*x),x)
[Out] -(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25
```

sympy [A] time = 0.49, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25}-\frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x)
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

3.344 $\int \csc^3\left(\frac{x}{2}\right)\,dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -arctanh(cos(1/2*x))-cot(1/2*x)*csc(1/2*x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3770}

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x/2]^3,x]
```

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3\left(\frac{x}{2}\right)\,dx &= -\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\frac{1}{2}\int \csc\left(\frac{x}{2}\right)\,dx\\ &= -\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.71

$$-\frac{1}{4}\csc^2\left(\frac{x}{4}\right)+\frac{1}{4}\sec^2\left(\frac{x}{4}\right)+\log\left(\sin\left(\frac{x}{4}\right)\right)-\log\left(\cos\left(\frac{x}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x/2]^3,x]
```

```
[Out] -1/4*Csc[x/4]^2 - Log[Cos[x/4]] + Log[Sin[x/4]] + Sec[x/4]^2/4
```

fricas [B] time = 0.45, size = 56, normalized size = 2.33

$$-\frac{\left(\cos\left(\frac{1}{2}\,x\right)^2-1\right)\log\left(\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right)+\frac{1}{2}\right)-\left(\cos\left(\frac{1}{2}\,x\right)^2-1\right)\log\left(-\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right)+\frac{1}{2}\right)-2\,\cos\left(\frac{1}{2}\,x\right)}{2\left(\cos\left(\frac{1}{2}\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(1/2*x)^2 - 1)*log(1/2*cos(1/2*x) + 1/2) - (cos(1/2*x)^2 - 1)*log
(-1/2*cos(1/2*x) + 1/2) - 2*cos(1/2*x))/(cos(1/2*x)^2 - 1)
```

giac [B] time = 1.04, size = 70, normalized size = 2.92

$$-\frac{\left(\frac{2\left(\cos\left(\frac{1}{2}x\right)-1\right)}{\cos\left(\frac{1}{2}x\right)+1}-1\right)\left(\cos\left(\frac{1}{2}x\right)+1\right)}{4\left(\cos\left(\frac{1}{2}x\right)-1\right)}-\frac{\cos\left(\frac{1}{2}x\right)-1}{4\left(\cos\left(\frac{1}{2}x\right)+1\right)}+\frac{1}{2}\log\left(-\frac{\cos\left(\frac{1}{2}x\right)-1}{\cos\left(\frac{1}{2}x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) - 1)*(cos(1/2*x) + 1)/(cos(1/2*x)
 - 1) - 1/4*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) + 1/2*log(-(cos(1/2*x) - 1)/(
cos(1/2*x) + 1))
```

maple [A] time = 0.02, size = 24, normalized size = 1.00

$$-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\ln\left(-\cot\left(\frac{x}{2}\right)+\csc\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(1/2*x)^3,x)
```

```
[Out] -cot(1/2*x)*csc(1/2*x)+ln(csc(1/2*x)-cot(1/2*x))
```

maxima [A] time = 0.51, size = 34, normalized size = 1.42

$$\frac{\cos\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)^{2}-1}-\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="maxima")
```

```
[Out] cos(1/2*x)/(cos(1/2*x)^2 - 1) - 1/2*log(cos(1/2*x) + 1) + 1/2*log(cos(1/2*x
) - 1)
```

mupad [B] time = 0.07, size = 18, normalized size = 0.75

$$\ln\left(\tan\left(\frac{x}{4}\right)\right)-\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x/2)^3,x)
```

```
[Out] log(tan(x/4)) - cos(x/2)/sin(x/2)^2
```

sympy [B] time = 0.14, size = 36, normalized size = 1.50

$$\frac{\log\left(\cos\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\cos\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\cos\left(\frac{x}{2}\right)}{2\cos^{2}\left(\frac{x}{2}\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)**3,x)
```

```
[Out] log(cos(x/2) - 1)/2 - log(cos(x/2) + 1)/2 + 2*cos(x/2)/(2*cos(x/2)**2 - 2)
```

3.345 $\int \frac{\sqrt{-1+9x^2}}{x^2} \, dx$

Optimal. Leaf size=34

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

```
[Out] 3*arctanh(3*x/(9*x^2-1)^(1/2))-(9*x^2-1)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{277, 217, 206\}$

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+9x^2}}{x^2} \, dx &= -\frac{\sqrt{-1+9x^2}}{x}+9\int \frac{1}{\sqrt{-1+9x^2}} \, dx\\
&= -\frac{\sqrt{-1+9x^2}}{x}+9\,\text{Subst}\left(\int \frac{1}{1-9x^2} \, dx,x,\frac{x}{\sqrt{-1+9x^2}}\right)\\
&= -\frac{\sqrt{-1+9x^2}}{x}+3\tanh^{-1}\left(\frac{3x}{\sqrt{-1+9x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.03

$$\sqrt{9x^2-1}\left(-\frac{3\sin^{-1}(3x)}{\sqrt{1-9x^2}}-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 9*x^2]/x^2,x]
[Out] Sqrt[-1 + 9*x^2]*(-x^(-1) - (3*ArcSin[3*x])/Sqrt[1 - 9*x^2])
```

fricas [A] time = 0.41, size = 35, normalized size = 1.03

$$-\frac{3\,x\log\left(-3\,x+\sqrt{9\,x^2-1}\right)+3\,x+\sqrt{9\,x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="fricas")
[Out] -(3*x*log(-3*x + sqrt(9*x^2 - 1)) + 3*x + sqrt(9*x^2 - 1))/x
```

giac [A] time = 1.03, size = 44, normalized size = 1.29

$$-\frac{6}{\left(3\,x-\sqrt{9\,x^2-1}\right)^2+1}-\frac{3}{2}\log\left(\left(3\,x-\sqrt{9\,x^2-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="giac")
[Out] -6/((3*x - sqrt(9*x^2 - 1))^2 + 1) - 3/2*log((3*x - sqrt(9*x^2 - 1))^2)
```

maple [A] time = 0.01, size = 47, normalized size = 1.38

$$-9\sqrt{9x^2-1}\;x+\sqrt{9}\;\ln\left(\sqrt{9}\;x+\sqrt{9x^2-1}\right)+\frac{\left(9x^2-1\right)^{\frac{3}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-1)^(1/2)/x^2,x)
[Out] 1/x*(9*x^2-1)^(3/2)-9*x*(9*x^2-1)^(1/2)+ln(x*9^(1/2)+(9*x^2-1)^(1/2))*9^(1/
2)
```

maxima [A] time = 1.24, size = 33, normalized size = 0.97

$$-\frac{\sqrt{9\,x^2-1}}{x}+3\,\log\left(18\,x+6\,\sqrt{9\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(9*x^2 - 1)/x + 3*log(18*x + 6*sqrt(9*x^2 - 1))
```

mupad [B] time = 0.52, size = 32, normalized size = 0.94

$$-\frac{\left(\frac{3\,x\,\mathrm{asin}(3\,x)}{\sqrt{1-9\,x^2}}+1\right)\sqrt{9\,x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2 - 1)^(1/2)/x^2,x)
[Out] -(((3*x*asin(3*x))/(1 - 9*x^2)^(1/2) + 1)*(9*x^2 - 1)^(1/2))/x
```

sympy [A] time = 0.25, size = 17, normalized size = 0.50

$$3\,\mathrm{acosh}\,(3x)-\frac{\sqrt{9x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-1)**(1/2)/x**2,x)
```

```
[Out] 3*acosh(3*x) - sqrt(9*x**2 - 1)/x
```

3.346 $\int \frac{\sqrt{4-3x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4-3x^2} - 2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

[Out] -2*arctanh(1/2*(-3*x^2+4)^(1/2))+(-3*x^2+4)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{4-3x^2} - 2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4 - 3*x^2]/x,x]

[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{4-3x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{4-3x^2} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{4-3x}\,x}\,dx, x, x^2\right)$$
$$= \sqrt{4-3x^2} - \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{\frac{4}{3}-\frac{x^2}{3}}\,dx, x, \sqrt{4-3x^2}\right)$$
$$= \sqrt{4-3x^2} - 2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Mathematica [**A**] time = 0.01, size = 30, normalized size = 1.00

$$\sqrt{4-3x^2} - 2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - 3*x^2]/x,x]
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

fricas [**A**] time = 0.41, size = 28, normalized size = 0.93

$$\sqrt{-3\,x^2+4} + 2\,\log\left(\frac{\sqrt{-3\,x^2+4}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(-3*x^2 + 4) + 2*log((sqrt(-3*x^2 + 4) - 2)/x)
```

giac [**A**] time = 0.95, size = 38, normalized size = 1.27

$$\sqrt{-3\,x^2+4} - \log\left(\sqrt{-3\,x^2+4}+2\right) + \log\left(-\sqrt{-3\,x^2+4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(-3*x^2 + 4) - log(sqrt(-3*x^2 + 4) + 2) + log(-sqrt(-3*x^2 + 4) + 2)
```

maple [**A**] time = 0.01, size = 25, normalized size = 0.83

$$-2\,\text{arctanh}\left(\frac{2}{\sqrt{-3x^2+4}}\right) + \sqrt{-3x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4)^(1/2)/x,x)
[Out] (-3*x^2+4)^(1/2)-2*arctanh(2/(-3*x^2+4)^(1/2))
```

maxima [**A**] time = 1.19, size = 35, normalized size = 1.17

$$\sqrt{-3\,x^2+4} - 2\,\log\left(\frac{4\sqrt{-3\,x^2+4}}{|x|} + \frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-3*x^2 + 4) - 2*log(4*sqrt(-3*x^2 + 4)/abs(x) + 8/abs(x))
```

mupad [B] time = 0.11, size = 37, normalized size = 1.23

$$2\ln\left(\sqrt{\frac{4}{3\,x^2}-1}-\frac{2\,\sqrt{3}\,\sqrt{\frac{1}{x^2}}}{3}\right)+\sqrt{3}\,\sqrt{\frac{4}{3}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - 3*x^2)^(1/2)/x,x)
```

```
[Out] 2*log((4/(3*x^2) - 1)^(1/2) - (2*3^(1/2)*(1/x^2)^(1/2))/3) + 3^(1/2)*(4/3 -
 x^2)^(1/2)
```

sympy [A] time = 1.56, size = 75, normalized size = 2.50

$$\begin{cases} i\sqrt{3x^2-4}-2\log(x)+\log\left(x^2\right)+2i\,\mathrm{asin}\left(\frac{2\sqrt{3}}{3x}\right) & \text{for } \frac{3|x^2|}{4}>1 \\ \sqrt{4-3x^2}+\log\left(x^2\right)-2\log\left(\sqrt{1-\frac{3x^2}{4}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(3*x**2 - 4) - 2*log(x) + log(x**2) + 2*I*asin(2*sqrt(3)/(
3*x)), 3*Abs(x**2)/4 > 1), (sqrt(4 - 3*x**2) + log(x**2) - 2*log(sqrt(1 - 3
*x**2/4) + 1), True))
```

3.347 $\int e^{3x}x^2\,dx$

Optimal. Leaf size=32

$$\frac{1}{3}e^{3x}x^2-\frac{2}{9}e^{3x}x+\frac{2e^{3x}}{27}$$

```
[Out] 2/27*exp(3*x)-2/9*exp(3*x)*x+1/3*exp(3*x)*x^2
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$\frac{1}{3}e^{3x}x^2-\frac{2}{9}e^{3x}x+\frac{2e^{3x}}{27}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*x^2,x]
```

```
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{3x}x^2\,dx&=\frac{1}{3}e^{3x}x^2-\frac{2}{3}\int e^{3x}x\,dx\\&=-\frac{2}{9}e^{3x}x+\frac{1}{3}e^{3x}x^2+\frac{2}{9}\int e^{3x}\,dx\\&=\frac{2e^{3x}}{27}-\frac{2}{9}e^{3x}x+\frac{1}{3}e^{3x}x^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.59

$$\frac{1}{27}e^{3x}\left(9x^2-6x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*x^2,x]
```

```
[Out] (E^(3*x)*(2 - 6*x + 9*x^2))/27
```

fricas [A] time = 0.40, size = 16, normalized size = 0.50

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="fricas")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

giac [A] time = 0.89, size = 16, normalized size = 0.50

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="giac")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.53

$$\frac{\left(9x^2-6x+2\right)\mathrm{e}^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*x^2,x)
[Out] 1/27*(9*x^2-6*x+2)*exp(3*x)
```

maxima [A] time = 0.52, size = 16, normalized size = 0.50

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="maxima")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.50

$$\frac{\mathrm{e}^{3\,x}\,\left(9\,x^2-6\,x+2\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(3*x),x)
[Out] (exp(3*x)*(9*x^2 - 6*x + 2))/27
```

sympy [A] time = 0.09, size = 15, normalized size = 0.47

$$\frac{\left(9x^2-6x+2\right)e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x**2,x)
[Out] (9*x**2 - 6*x + 2)*exp(3*x)/27
```

3.348 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\, dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\, dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\, dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right) dx, x, \sin(x)\right)\\ &= -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.35

$$\frac{2(\sin(x)-2)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{3\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

fricas [A] time = 0.43, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\sin(x)+1}\,(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

giac [A] time = 0.89, size = 17, normalized size = 0.74

$$\frac{2}{3}\,(\sin(x)+1)^{\frac{3}{2}} - 2\,\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

maple [A] time = 0.02, size = 18, normalized size = 0.78

$$\frac{2\,(\sin(x)+1)^{\frac{3}{2}}}{3} - 2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(sin(x)+1)^(1/2),x)
[Out] 2/3*(sin(x)+1)^(3/2)-2*(sin(x)+1)^(1/2)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.74

$$\frac{2}{3}\,(\sin(x)+1)^{\frac{3}{2}} - 2\,\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

mupad [B] time = 0.10, size = 12, normalized size = 0.52

$$\frac{2\,\sqrt{\sin(x)+1}\,\,(\sin(x)-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)
[Out] (2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3
```

sympy [A] time = 0.36, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin{(x)}+1}\,\sin{(x)}}{3} - \frac{4\sqrt{\sin{(x)}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

3.349 $\int x \sin^{-1}\left(x^2\right) dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)$$

```
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6715, 4619, 261}

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSin[x^2],x]
```

```
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x\sin^{-1}\left(x^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int\sin^{-1}(x)\,dx,x,x^2\right)\\&=\frac{1}{2}x^2\sin^{-1}\left(x^2\right)-\frac{1}{2}\operatorname{Subst}\left(\int\frac{x}{\sqrt{1-x^2}}\,dx,x,x^2\right)\\&=\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(\sqrt{1-x^4}+x^2\sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSin[x^2],x]
```

```
[Out] (Sqrt[1 - x^4] + x^2*ArcSin[x^2])/2
```

fricas [A] time = 0.45, size = 21, normalized size = 0.78

$$\frac{1}{2}x^2 \arcsin\left(x^2\right) + \frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="fricas")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

giac [A] time = 0.89, size = 21, normalized size = 0.78

$$\frac{1}{2}x^2 \arcsin\left(x^2\right) + \frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="giac")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$\frac{x^2 \arcsin\left(x^2\right)}{2} + \frac{\sqrt{-x^4+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x^2),x)
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

maxima [A] time = 1.05, size = 21, normalized size = 0.78

$$\frac{1}{2}x^2 \arcsin\left(x^2\right) + \frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="maxima")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

mupad [B] time = 0.28, size = 21, normalized size = 0.78

$$\frac{x^2 \operatorname{asin}\left(x^2\right)}{2} + \frac{\sqrt{1-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asin(x^2),x)
[Out] (x^2*asin(x^2))/2 + (1 - x^4)^(1/2)/2
```

sympy [A] time = 0.20, size = 19, normalized size = 0.70

$$\frac{x^2 \operatorname{asin}\left(x^2\right)}{2} + \frac{\sqrt{1-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x**2),x)
[Out] x**2*asin(x**2)/2 + sqrt(1 - x**4)/2
```

3.350 $\int x^3 \sin^{-1}\left(x^2\right) dx$

Optimal. Leaf size=38

$$-\frac{1}{8}\sin^{-1}\left(x^2\right)+\frac{1}{8}\sqrt{1-x^4}\,x^2+\frac{1}{4}x^4\sin^{-1}\left(x^2\right)$$

[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)

Rubi **[A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4842, 12, 275, 321, 216}

$$\frac{1}{8}\sqrt{1-x^4}\,x^2+\frac{1}{4}x^4\sin^{-1}\left(x^2\right)-\frac{1}{8}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcSin[x^2],x]

[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4842

```
Int[((a_.) + ArcSin[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSin[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sin^{-1}\left(x^2\right) dx &= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{4}\int \frac{2x^5}{\sqrt{1-x^4}}\, dx \\
&= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{2}\int \frac{x^5}{\sqrt{1-x^4}}\, dx \\
&= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}}\, dx, x, x^2\right) \\
&= \frac{1}{8}x^2\sqrt{1-x^4} + \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, x^2\right) \\
&= \frac{1}{8}x^2\sqrt{1-x^4} - \frac{1}{8}\sin^{-1}\left(x^2\right) + \frac{1}{4}x^4 \sin^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.84

$$\frac{1}{8}\left(\sqrt{1-x^4}\,x^2 + \left(2x^4-1\right)\sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSin[x^2],x]
[Out] (x^2*Sqrt[1 - x^4] + (-1 + 2*x^4)*ArcSin[x^2])/8
```

fricas [A] time = 0.43, size = 28, normalized size = 0.74

$$\frac{1}{8}\sqrt{-x^4+1}\,x^2 + \frac{1}{8}\left(2\,x^4-1\right)\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="fricas")
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/8*(2*x^4 - 1)*arcsin(x^2)
```

giac [A] time = 0.95, size = 32, normalized size = 0.84

$$\frac{1}{8}\sqrt{-x^4+1}\,x^2 + \frac{1}{4}\left(x^4-1\right)\arcsin\left(x^2\right) + \frac{1}{8}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="giac")
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/4*(x^4 - 1)*arcsin(x^2) + 1/8*arcsin(x^2)
```

maple [A] time = 0.00, size = 31, normalized size = 0.82

$$\frac{x^4\arcsin\left(x^2\right)}{4} + \frac{\sqrt{-x^4+1}\;x^2}{8} - \frac{\arcsin\left(x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsin(x^2),x)
[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)
```

maxima [A] time = 1.08, size = 53, normalized size = 1.39

$$\frac{1}{4}\,x^4\arcsin\left(x^2\right) - \frac{\sqrt{-x^4+1}}{8\,x^2\left(\frac{x^4-1}{x^4}-1\right)} + \frac{1}{8}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsin(x^2) - 1/8*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)) + 1/8*ar
ctan(sqrt(-x^4 + 1)/x^2)
```

mupad [B] time = 0.22, size = 28, normalized size = 0.74

$$\frac{x^2\sqrt{1-x^4}}{8}+\frac{\operatorname{asin}\left(x^2\right)\left(2x^4-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asin(x^2),x)
```

```
[Out] (x^2*(1 - x^4)^(1/2))/8 + (asin(x^2)*(2*x^4 - 1))/8
```

sympy [A] time = 0.61, size = 29, normalized size = 0.76

$$\frac{x^4\operatorname{asin}\left(x^2\right)}{4}+\frac{x^2\sqrt{1-x^4}}{8}-\frac{\operatorname{asin}\left(x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asin(x**2),x)
```

```
[Out] x**4*asin(x**2)/4 + x**2*sqrt(1 - x**4)/8 - asin(x**2)/8
```

3.351 $\int e^x \text{sech}(e^x)\,dx$

Optimal. Leaf size=5

$$\tan^{-1}(\sinh(e^x))$$

```
[Out] arctan(sinh(exp(x)))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 3770}

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[E^x],x]
```

```
[Out] ArcTan[Sinh[E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \text{sech}(e^x)\,dx &= \text{Subst}\left(\int \text{sech}(x)\,dx, x, e^x\right)\\ &= \tan^{-1}(\sinh(e^x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[E^x],x]
```

```
[Out] ArcTan[Sinh[E^x]]
```

fricas [B] time = 0.42, size = 16, normalized size = 3.20

$$2\,\arctan(\cosh(\cosh(x)+\sinh(x))+\sinh(\cosh(x)+\sinh(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))
```

giac [A] time = 0.94, size = 6, normalized size = 1.20

$$2\,\arctan\left(e^{(e^x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(e^x))
```

maple [A] time = 0.00, size = 5, normalized size = 1.00

$$\arctan(\sinh(\mathrm{e}^{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(exp(x)),x)
```

```
[Out] arctan(sinh(exp(x)))
```

maxima [A] time = 0.48, size = 4, normalized size = 0.80

$$\arctan(\sinh(e^{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="maxima")
```

```
[Out] arctan(sinh(e^x))
```

mupad [B] time = 0.05, size = 6, normalized size = 1.20

$$2\,\mathrm{atan}\left(\mathrm{e}^{\mathrm{e}^{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(exp(x)),x)
```

```
[Out] 2*atan(exp(exp(x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(e^{x})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x)
```

```
[Out] Integral(exp(x)*sech(exp(x)), x)
```

3.352 $\int x^2 \cos(3x)\, dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3296, 2637}

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos(3x)\,dx &= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx \\
&= \frac{2}{9}x\cos(3x) + \frac{1}{3}x^2\sin(3x) - \frac{2}{9}\int \cos(3x)\,dx \\
&= \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2-2\right)\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

fricas [A] time = 0.42, size = 21, normalized size = 0.72

$$\frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

giac [A] time = 0.92, size = 21, normalized size = 0.72

$$\frac{2}{9}x\cos(3x) + \frac{1}{27}\left(9x^2 - 2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.83

$$\frac{x^2\sin(3x)}{3} + \frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
[Out] 1/3*x^2*sin(3*x)+2/9*x*cos(3*x)-2/27*sin(3*x)
```

maxima [A] time = 0.46, size = 21, normalized size = 0.72

$$\frac{2}{9}x\cos(3x) + \frac{1}{27}\left(9x^2 - 2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

mupad [B] time = 0.19, size = 23, normalized size = 0.79

$$\frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27} + \frac{x^2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
[Out] (2*x*cos(3*x))/9 - (2*sin(3*x))/27 + (x^2*sin(3*x))/3
```

sympy [A] time = 0.33, size = 27, normalized size = 0.93

$$\frac{x^2\sin(3x)}{3} + \frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x),x)
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

3.353 $\int \sqrt{5-4x-x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] 9/2*arcsin(2/3+1/3*x)+1/2*(2+x)*(-x^2-4*x+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 - 4*x - x^2],x]
```

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{5-4x-x^2}\,dx &= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}+\frac{9}{2}\int \frac{1}{\sqrt{5-4x-x^2}}\,dx\\
&= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,-4-2x\right)\\
&= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-2-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.92

$$\frac{1}{2}\left(\sqrt{-x^2-4x+5}\,(x+2)+9\sin^{-1}\left(\frac{x+2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 - 4*x - x^2],x]
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2] + 9*ArcSin[(2 + x)/3])/2
```

fricas [A] time = 0.41, size = 47, normalized size = 1.31

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)-\frac{9}{2}\arctan\left(\frac{\sqrt{-x^2-4x+5}(x+2)}{x^2+4x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) - 9/2*arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/
(x^2 + 4*x - 5))
```

giac [A] time = 1.09, size = 26, normalized size = 0.72

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)+\frac{9}{2}\arcsin\left(\frac{1}{3}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arcsin(1/3*x + 2/3)
```

maple [A] time = 0.00, size = 29, normalized size = 0.81

$$\frac{9\arcsin\left(\frac{x}{3}+\frac{2}{3}\right)}{2}-\frac{(-2x-4)\sqrt{-x^2-4x+5}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x+5)^(1/2),x)
[Out] -1/4*(-2*x-4)*(-x^2-4*x+5)^(1/2)+9/2*arcsin(1/3*x+2/3)
```

maxima [A] time = 1.12, size = 36, normalized size = 1.00

$$\frac{1}{2}\sqrt{-x^2-4x+5}\,x+\sqrt{-x^2-4x+5}-\frac{9}{2}\arcsin\left(-\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*x + sqrt(-x^2 - 4*x + 5) - 9/2*arcsin(-1/3*x - 2/3
)
```

mupad [B] time = 0.18, size = 27, normalized size = 0.75

$$\frac{9\,\mathrm{asin}\left(\frac{x}{3}+\frac{2}{3}\right)}{2}+\left(\frac{x}{2}+1\right)\sqrt{-x^2-4x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5 - x^2 - 4*x)^(1/2),x)
[Out] (9*asin(x/3 + 2/3))/2 + (x/2 + 1)*(5 - x^2 - 4*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2-4x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 - 4*x + 5), x)
```

3.354 $\int \frac{x^5}{\sqrt{2}+x^2}\,dx$

Optimal. Leaf size=28

$$\frac{x^4}{4}-\frac{x^2}{\sqrt{2}}+\log\left(x^2+\sqrt{2}\right)$$

```
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*x^2*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{x^4}{4}-\frac{x^2}{\sqrt{2}}+\log\left(x^2+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{2}+x^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{2}+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\sqrt{2}+x+\frac{2}{\sqrt{2}+x}\right)dx,x,x^2\right)\\ &= -\frac{x^2}{\sqrt{2}}+\frac{x^4}{4}+\log\left(\sqrt{2}+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.11

$$\frac{1}{4}\left(x^4-2\sqrt{2}x^2+4\log\left(x^2+\sqrt{2}\right)-6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] (-6 - 2*Sqrt[2]*x^2 + x^4 + 4*Log[Sqrt[2] + x^2])/4
```

fricas [A] time = 0.41, size = 22, normalized size = 0.79

$$\frac{1}{4}x^4-\frac{1}{2}\sqrt{2}x^2+\log\left(x^2+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

giac [A] time = 0.96, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x^4 - \frac{1}{2}\,\sqrt{2}\,x^2 + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="giac")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{x^4}{4} - \frac{\sqrt{2}\,x^2}{2} + \ln\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2+2^(1/2)),x)
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*2^(1/2)*x^2
```

maxima [A] time = 1.16, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x^4 - \frac{1}{2}\,\sqrt{2}\,x^2 + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="maxima")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

mupad [B] time = 0.06, size = 22, normalized size = 0.79

$$\ln\left(x^2 + \sqrt{2}\right) - \frac{\sqrt{2}\,x^2}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(2^(1/2) + x^2),x)
[Out] log(2^(1/2) + x^2) - (2^(1/2)*x^2)/2 + x^4/4
```

sympy [A] time = 0.13, size = 24, normalized size = 0.86

$$\frac{x^4}{4} - \frac{\sqrt{2}\,x^2}{2} + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2+2**(1/2)),x)
[Out] x**4/4 - sqrt(2)*x**2/2 + log(x**2 + sqrt(2))
```

3.355 $\int \sec^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)+\frac{3}{8}\tan(x)\sec(x)$$

```
[Out] 3/8*arctanh(sin(x))+3/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3768, 3770}

$$\frac{3}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)+\frac{3}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(x)\,dx &= \frac{1}{4}\sec^3(x)\tan(x)+\frac{3}{4}\int \sec^3(x)\,dx\\ &= \frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)+\frac{3}{8}\int \sec(x)\,dx\\ &= \frac{3}{8}\tanh^{-1}(\sin(x))+\frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)\end{aligned}$$

Mathematica [B] time = 0.12, size = 58, normalized size = 2.23

$$\frac{1}{16}\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/16
```

fricas [B] time = 0.46, size = 43, normalized size = 1.65

$$\frac{3\,\cos(x)^4\log(\sin(x)+1)-3\,\cos(x)^4\log(-\sin(x)+1)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/cos(x)^4
```

giac [A] time = 0.90, size = 38, normalized size = 1.46

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^2-1\right)^2}+\frac{3}{16}\,\log\left(\sin(x)+1\right)-\frac{3}{16}\,\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^2 - 1)^2 + 3/16*log(sin(x) + 1) - 3/16
*log(-sin(x) + 1)
```

maple [A] time = 0.07, size = 25, normalized size = 0.96

$$\frac{3\ln\left(\sec(x)+\tan(x)\right)}{8}-\left(-\frac{\left(\sec^3(x)\right)}{4}-\frac{3\sec(x)}{8}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x))
```

maxima [B] time = 0.65, size = 42, normalized size = 1.62

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^4-2\,\sin(x)^2+1\right)}+\frac{3}{16}\,\log\left(\sin(x)+1\right)-\frac{3}{16}\,\log\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3/16*log(sin(x)
+ 1) - 3/16*log(sin(x) - 1)
```

mupad [B] time = 0.06, size = 29, normalized size = 1.12

$$\frac{3\,\ln\left(\frac{\sin(x)+1}{\cos(x)}\right)}{8}+\sin(x)\,\left(\frac{3}{8\cos(x)^2}+\frac{1}{4\cos(x)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^5,x)
```

```
[Out] (3*log((sin(x) + 1)/cos(x)))/8 + sin(x)*(3/(8*cos(x)^2) + 1/(4*cos(x)^4))
```

sympy [A] time = 0.16, size = 46, normalized size = 1.77

$$-\frac{3\sin^3(x)-5\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}-\frac{3\log\left(\sin(x)-1\right)}{16}+\frac{3\log\left(\sin(x)+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5,x)
```

```
[Out] -(3*sin(x)**3 - 5*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 3*log(sin(x) -
 1)/16 + 3*log(sin(x) + 1)/16
```

3.356 $\int \sin^6(2x)\,dx$

Optimal. Leaf size=46

$$\frac{5x}{16}-\frac{1}{12}\sin^5(2x)\cos(2x)-\frac{5}{48}\sin^3(2x)\cos(2x)-\frac{5}{32}\sin(2x)\cos(2x)$$

```
[Out] 5/16*x-5/32*cos(2*x)*sin(2*x)-5/48*cos(2*x)*sin(2*x)^3-1/12*cos(2*x)*sin(2*
x)^5
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2635, 8}

$$\frac{5x}{16}-\frac{1}{12}\sin^5(2x)\cos(2x)-\frac{5}{48}\sin^3(2x)\cos(2x)-\frac{5}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48 - (Cos[2*x
]*Sin[2*x]^5)/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^6(2x)\,dx &= -\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5}{6}\int \sin^4(2x)\,dx\\ &= -\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5}{8}\int \sin^2(2x)\,dx\\ &= -\frac{5}{32}\cos(2x)\sin(2x)-\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16}-\frac{5}{32}\cos(2x)\sin(2x)-\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.65

$$\frac{5x}{16}-\frac{15}{128}\sin(4x)+\frac{3}{128}\sin(8x)-\frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[4*x])/128 + (3*Sin[8*x])/128 - Sin[12*x]/384
```

fricas [A] time = 0.44, size = 33, normalized size = 0.72

$$-\frac{1}{96}\left(8\cos(2x)^5-26\cos(2x)^3+33\cos(2x)\right)\sin(2x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 26*cos(2*x)^3 + 33*cos(2*x))*sin(2*x) + 5/16*x
```

giac [A] time = 1.02, size = 22, normalized size = 0.48

$$\frac{5}{16}\,x - \frac{1}{384}\,\sin\left(12\,x\right) + \frac{3}{128}\,\sin\left(8\,x\right) - \frac{15}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/384*sin(12*x) + 3/128*sin(8*x) - 15/128*sin(4*x)
```

maple [A] time = 0.02, size = 32, normalized size = 0.70

$$\frac{5x}{16} - \frac{\left(\sin^{5}\left(2x\right) + \frac{5\left(\sin^{3}\left(2x\right)\right)}{4} + \frac{15\sin\left(2x\right)}{8}\right)\cos\left(2x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^6,x)
```

```
[Out] -1/12*(sin(2*x)^5+5/4*sin(2*x)^3+15/8*sin(2*x))*cos(2*x)+5/16*x
```

maxima [A] time = 0.53, size = 24, normalized size = 0.52

$$\frac{1}{96}\,\sin\left(4\,x\right)^{3} + \frac{5}{16}\,x + \frac{3}{128}\,\sin\left(8\,x\right) - \frac{1}{8}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 5/16*x + 3/128*sin(8*x) - 1/8*sin(4*x)
```

mupad [B] time = 0.21, size = 22, normalized size = 0.48

$$\frac{5\,x}{16} - \frac{15\,\sin\left(4\,x\right)}{128} + \frac{3\,\sin\left(8\,x\right)}{128} - \frac{\sin\left(12\,x\right)}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^6,x)
```

```
[Out] (5*x)/16 - (15*sin(4*x))/128 + (3*sin(8*x))/128 - sin(12*x)/384
```

sympy [A] time = 0.07, size = 46, normalized size = 1.00

$$\frac{5x}{16} - \frac{\sin^{5}\left(2x\right)\cos\left(2x\right)}{12} - \frac{5\sin^{3}\left(2x\right)\cos\left(2x\right)}{48} - \frac{5\sin\left(2x\right)\cos\left(2x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**6,x)
```

```
[Out] 5*x/16 - sin(2*x)**5*cos(2*x)/12 - 5*sin(2*x)**3*cos(2*x)/48 - 5*sin(2*x)*c
os(2*x)/32
```

3.357 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2564, 30, 2554, 12}

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\sin^2(x)\,dx &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)\\ &= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.75

$$\frac{1}{9}\sin^3(x)(3\log(\sin(x)) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

fricas [A] time = 0.43, size = 24, normalized size = 1.20

$$-\frac{1}{3}\left(\cos(x)^2-1\right)\log\left(\sin(x)\right)\sin(x)+\frac{1}{9}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

giac [A] time = 1.04, size = 16, normalized size = 0.80

$$\frac{1}{3}\log\left(\sin(x)\right)\sin(x)^3-\frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

maple [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{\ln\left(\sin(x)\right)\left(\sin^3(x)\right)}{3}-\frac{\left(\sin^3(x)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x)
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

maxima [A] time = 0.48, size = 16, normalized size = 0.80

$$\frac{1}{3}\log\left(\sin(x)\right)\sin(x)^3-\frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

mupad [B] time = 0.25, size = 11, normalized size = 0.55

$$\frac{\sin(x)^3\left(\ln\left(\sin(x)\right)-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x)*sin(x)^2,x)
[Out] (sin(x)^3*(log(sin(x)) - 1/3))/3
```

sympy [A] time = 5.39, size = 17, normalized size = 0.85

$$\frac{\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3}-\frac{\sin^3\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

3.358 $\int \frac{e^{-x}}{1+2e^x}\,dx$

Optimal. Leaf size=21

$$-2x-e^{-x}+2\log\left(2e^x+1\right)$$

```
[Out] -1/exp(x)-2*x+2*ln(1+2*exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2248, 44}

$$-2x-e^{-x}+2\log\left(2e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-x}}{1+2e^x}\,dx &= \text{Subst}\left(\int \frac{1}{x^2(1+2x)}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{4}{1+2x}\right)dx,x,e^x\right)\\
&= -e^{-x}-2x+2\log\left(1+2e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$-2x-e^{-x}+2\log\left(2e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

fricas [A] time = 0.42, size = 24, normalized size = 1.14

$$-\left(2\,xe^x-2\,e^x\log\left(2\,e^x+1\right)+1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="fricas")
[Out] -(2*x*e^x - 2*e^x*log(2*e^x + 1) + 1)*e^(-x)
```

giac [A] time = 0.86, size = 19, normalized size = 0.90

$$-2\,x - e^{(-x)} + 2\,\log\left(2\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="giac")
[Out] -2*x - e^(-x) + 2*log(2*e^x + 1)
```

maple [A] time = 0.01, size = 22, normalized size = 1.05

$$-\mathrm{e}^{-x} + 2\ln\left(2\,\mathrm{e}^{x} + 1\right) - 2\ln\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(x)/(1+2*exp(x)),x)
[Out] 2*ln(1+2*exp(x))-1/exp(x)-2*ln(exp(x))
```

maxima [A] time = 0.54, size = 16, normalized size = 0.76

$$-e^{(-x)} + 2\,\log\left(e^{(-x)} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="maxima")
[Out] -e^(-x) + 2*log(e^(-x) + 2)
```

mupad [B] time = 0.07, size = 19, normalized size = 0.90

$$2\,\ln\left(2\,\mathrm{e}^{x} + 1\right) - 2\,x - \mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)/(2*exp(x) + 1),x)
[Out] 2*log(2*exp(x) + 1) - 2*x - exp(-x)
```

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-2x + 2\log\left(e^{x} + \frac{1}{2}\right) - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x)
[Out] -2*x + 2*log(exp(x) + 1/2) - exp(-x)
```

3.359 $\int \sqrt{2+3\cos(x)}\,\tan(x)\,dx$

Optimal. Leaf size=37

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

```
[Out] 2*arctanh(1/2*(2+3*cos(x))^(1/2)*2^(1/2))*2^(1/2)-2*(2+3*cos(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2721, 50, 63, 207}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \sqrt{2+3\cos(x)}\,\tan(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{2+x}}{x}\,dx, x, 3\cos(x)\right)$$
$$= -2\sqrt{2+3\cos(x)} - 2\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, 3\cos(x)\right)$$
$$= -2\sqrt{2+3\cos(x)} - 4\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+3\cos(x)}\right)$$
$$= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2+3\cos(x)}}{\sqrt{2}}\right) - 2\sqrt{2+3\cos(x)}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$2\sqrt{2}\tanh^{-1}\left(\sqrt{\frac{3\cos(x)}{2}+1}\right) - 2\sqrt{3\cos(x)+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + (3*Cos[x])/2]] - 2*Sqrt[2 + 3*Cos[x]]
```

fricas [A] time = 0.56, size = 58, normalized size = 1.57

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{9\cos(x)^2 + 4\left(3\sqrt{2}\cos(x) + 4\sqrt{2}\right)\sqrt{3\cos(x)+2} + 48\cos(x) + 32}{\cos(x)^2}\right) - 2\sqrt{3\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(9*cos(x)^2 + 4*(3*sqrt(2)*cos(x) + 4*sqrt(2))*sqrt(3*cos(
x) + 2) + 48*cos(x) + 32)/cos(x)^2) - 2*sqrt(3*cos(x) + 2)
```

giac [A] time = 1.05, size = 50, normalized size = 1.35

$$-\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\sqrt{3\cos(x)+2}\right|}{2\left(\sqrt{2} + \sqrt{3\cos(x)+2}\right)}\right) - 2\sqrt{3\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="giac")
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*c
os(x) + 2))) - 2*sqrt(3*cos(x) + 2)
```

maple [A] time = 0.03, size = 31, normalized size = 0.84

$$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{3\cos(x)+2}\,\sqrt{2}}{2}\right) - 2\sqrt{3\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(x))^(1/2)*tan(x),x)
[Out] 2*arctanh(1/2*(2+3*cos(x))^(1/2)*2^(1/2))*2^(1/2)-2*(2+3*cos(x))^(1/2)
```

maxima [A] time = 1.37, size = 47, normalized size = 1.27

$$-\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\sqrt{3\,\cos(x)+2}}{\sqrt{2}+\sqrt{3\,\cos(x)+2}}\right)-2\,\sqrt{3\,\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*cos(x) + 2))
) - 2*sqrt(3*cos(x) + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \tan(x)\,\sqrt{3\,\cos(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(3*cos(x) + 2)^(1/2),x)
```

```
[Out] int(tan(x)*(3*cos(x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\cos(x)+2}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(3*cos(x) + 2)*tan(x), x)
```

3.360 $\int \frac{x}{\sqrt{-4x+x^2}} \, dx$

Optimal. Leaf size=28

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

```
[Out] 4*arctanh(x/(x^2-4*x)^(1/2))+(x^2-4*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {640, 620, 206}

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4*x + x^2],x]
```

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-4x+x^2}}\,dx &= \sqrt{-4x+x^2}+2\int \frac{1}{\sqrt{-4x+x^2}}\,dx\\
&= \sqrt{-4x+x^2}+4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-4x+x^2}}\right)\\
&= \sqrt{-4x+x^2}+4\tanh^{-1}\left(\frac{x}{\sqrt{-4x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.43

$$\frac{(x-4)x-4\sqrt{-((x-4)x)}\,\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)}{\sqrt{(x-4)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-4*x + x^2],x]
[Out] ((-4 + x)*x - 4*Sqrt[-((-4 + x)*x)]*ArcSin[Sqrt[1 - x/4]])/Sqrt[(-4 + x)*x]
```

fricas [A] time = 0.41, size = 27, normalized size = 0.96

$$\sqrt{x^2-4x}-2\log\left(-x+\sqrt{x^2-4x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 4*x) - 2*log(-x + sqrt(x^2 - 4*x) + 2)
```

giac [A] time = 1.13, size = 28, normalized size = 1.00

$$\sqrt{x^2-4x}-2\log\left(\left|-x+\sqrt{x^2-4x}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 4*x) - 2*log(abs(-x + sqrt(x^2 - 4*x) + 2))
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$2\ln\left(x-2+\sqrt{x^2-4x}\right)+\sqrt{x^2-4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-4*x)^(1/2),x)
[Out] (x^2-4*x)^(1/2)+2*ln(x-2+(x^2-4*x)^(1/2))
```

maxima [A] time = 0.43, size = 29, normalized size = 1.04

$$\sqrt{x^2-4x}+2\log\left(2x+2\sqrt{x^2-4x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

mupad [B] time = 0.15, size = 23, normalized size = 0.82

$$2\ln\left(x+\sqrt{x\,(x-4)}-2\right)+\sqrt{x^2-4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 4*x)^(1/2),x)
[Out] 2*log(x + (x*(x - 4))^(1/2) - 2) + (x^2 - 4*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-4*x)**(1/2),x)
[Out] Integral(x/sqrt(x*(x - 4)), x)
```

3.361 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

[Out] sin(x)-2/3*sin(x)^3+1/5*sin(x)^5

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5,x]

[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(x)\right)\\ &= \sin(x)-\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^5,x]

[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80

fricas [A] time = 0.44, size = 18, normalized size = 0.95

$$\frac{1}{15}\left(3\,\cos(x)^4+4\,\cos(x)^2+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^5,x, algorithm="fricas")

[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)

giac [A] time = 1.05, size = 15, normalized size = 0.79

$$\frac{1}{5}\,\sin(x)^5-\frac{2}{3}\,\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.06, size = 17, normalized size = 0.89

$$\frac{\left(\cos^{4}(x)+\frac{4\left(\cos^{2}(x)\right)}{3}+\frac{8}{3}\right)\sin(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x)
```

```
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

maxima [A] time = 0.42, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^{5}-\frac{2}{3}\sin(x)^{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 1.11

$$\frac{\sin(x)\cos(x)^{4}}{5}+\frac{4\sin(x)\cos(x)^{2}}{15}+\frac{8\sin(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x)
```

```
[Out] (8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

sympy [A] time = 0.07, size = 17, normalized size = 0.89

$$\frac{\sin^{5}(x)}{5}-\frac{2\sin^{3}(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

3.362 $\int e^{-x}x^4\,dx$

Optimal. Leaf size=46

$$-e^{-x}x^4-4e^{-x}x^3-12e^{-x}x^2-24e^{-x}x-24e^{-x}$$

```
[Out] -24/exp(x)-24*x/exp(x)-12*x^2/exp(x)-4*x^3/exp(x)-x^4/exp(x)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-x}x^4-4e^{-x}x^3-12e^{-x}x^2-24e^{-x}x-24e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^x,x]
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}x^4\,dx &= -e^{-x}x^4+4\int e^{-x}x^3\,dx\\ &= -4e^{-x}x^3-e^{-x}x^4+12\int e^{-x}x^2\,dx\\ &= -12e^{-x}x^2-4e^{-x}x^3-e^{-x}x^4+24\int e^{-x}x\,dx\\ &= -24e^{-x}x-12e^{-x}x^2-4e^{-x}x^3-e^{-x}x^4+24\int e^{-x}\,dx\\ &= -24e^{-x}-24e^{-x}x-12e^{-x}x^2-4e^{-x}x^3-e^{-x}x^4\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.57

$$e^{-x}\left(-x^4-4x^3-12x^2-24x-24\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^x,x]
[Out] (-24 - 24*x - 12*x^2 - 4*x^3 - x^4)/E^x
```

fricas [A] time = 0.41, size = 24, normalized size = 0.52

$$-\left(x^4+4\,x^3+12\,x^2+24\,x+24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="fricas")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

giac [A] time = 0.88, size = 24, normalized size = 0.52

$$-\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="giac")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.54

$$-\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right)\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/exp(x),x)
[Out] -(x^4+4*x^3+12*x^2+24*x+24)/exp(x)
```

maxima [A] time = 0.44, size = 24, normalized size = 0.52

$$-\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="maxima")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.52

$$-\mathrm{e}^{-x}\,\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(-x),x)
[Out] -exp(-x)*(24*x + 12*x^2 + 4*x^3 + x^4 + 24)
```

sympy [A] time = 0.09, size = 22, normalized size = 0.48

$$\left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/exp(x),x)
[Out] (-x**4 - 4*x**3 - 12*x**2 - 24*x - 24)*exp(-x)
```

3.363 $\int \frac{x^4}{\sqrt{-2+x^{10}}} \, dx$

Optimal. Leaf size=18

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

```
[Out] 1/5*arctanh(x^5/(x^10-2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 217, 206}

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{-2+x^{10}}} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}} \, dx, x, x^5\right)\\ &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x^5}{\sqrt{-2+x^{10}}}\right)\\ &= \frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{-2+x^{10}}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

fricas [A] time = 0.41, size = 16, normalized size = 0.89

$$-\frac{1}{5}\log\left(-x^5+\sqrt{x^{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="fricas")
[Out] -1/5*log(-x^5 + sqrt(x^10 - 2))
```

giac [A] time = 1.01, size = 17, normalized size = 0.94

$$-\frac{1}{5}\log\left(\left|-x^5+\sqrt{x^{10}-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="giac")
[Out] -1/5*log(abs(-x^5 + sqrt(x^10 - 2)))
```

maple [C] time = 0.08, size = 34, normalized size = 1.89

$$\frac{\sqrt{-\mathrm{signum}\left(\frac{x^{10}}{2}-1\right)}\,\arcsin\left(\frac{\sqrt{2}\,x^5}{2}\right)}{5\sqrt{\mathrm{signum}\left(\frac{x^{10}}{2}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10-2)^(1/2),x)
[Out] 1/5/signum(-1+1/2*x^10)^(1/2)*(-signum(-1+1/2*x^10))^(1/2)*arcsin(1/2*x^5*2
^(1/2))
```

maxima [B] time = 0.42, size = 33, normalized size = 1.83

$$\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}+1\right)-\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="maxima")
[Out] 1/10*log(sqrt(x^10 - 2)/x^5 + 1) - 1/10*log(sqrt(x^10 - 2)/x^5 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\sqrt{x^{10}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10 - 2)^(1/2),x)
[Out] int(x^4/(x^10 - 2)^(1/2), x)
```

sympy [A] time = 1.09, size = 34, normalized size = 1.89

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{2}\,x^5}{2}\right)}{5} & \text{for } \frac{|x^{10}|}{2} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\,x^5}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10-2)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(2)*x**5/2)/5, Abs(x**10)/2 > 1), (-I*asin(sqrt(2)*x**
5/2)/5, True))
```

3.364 $\int e^x \cos(4+3x)\,dx$

Optimal. Leaf size=27

$$\frac{3}{10}e^x\sin(3x+4)+\frac{1}{10}e^x\cos(3x+4)$$

```
[Out] 1/10*exp(x)*cos(4+3*x)+3/10*exp(x)*sin(4+3*x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{3}{10}e^x\sin(3x+4)+\frac{1}{10}e^x\cos(3x+4)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[4 + 3*x],x]
[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(4+3x)\,dx=\frac{1}{10}e^x\cos(4+3x)+\frac{3}{10}e^x\sin(4+3x)$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.81

$$\frac{1}{10}e^x(3\sin(3x+4)+\cos(3x+4))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[4 + 3*x],x]
[Out] (E^x*(Cos[4 + 3*x] + 3*Sin[4 + 3*x]))/10
```

fricas [A] time = 0.42, size = 21, normalized size = 0.78

$$\frac{1}{10}\cos\left(3x+4\right)e^x+\frac{3}{10}e^x\sin\left(3x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="fricas")
[Out] 1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)
```

giac [A] time = 0.96, size = 19, normalized size = 0.70

$$\frac{1}{10}\left(\cos\left(3x+4\right)+3\sin\left(3x+4\right)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="giac")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\cos(3x+4)\,\mathrm{e}^{x}}{10}+\frac{3\,\mathrm{e}^{x}\sin(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(3*x+4),x)
```

```
[Out] 1/10*exp(x)*cos(3*x+4)+3/10*exp(x)*sin(3*x+4)
```

maxima [A] time = 0.42, size = 19, normalized size = 0.70

$$\frac{1}{10}\,(\cos(3\,x+4)+3\,\sin(3\,x+4))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="maxima")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

mupad [B] time = 0.20, size = 19, normalized size = 0.70

$$\frac{\mathrm{e}^{x}\,\left(\cos(3\,x+4)+3\,\sin(3\,x+4)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(3*x + 4),x)
```

```
[Out] (exp(x)*(cos(3*x + 4) + 3*sin(3*x + 4)))/10
```

sympy [A] time = 0.32, size = 24, normalized size = 0.89

$$\frac{3e^{x}\sin(3x+4)}{10}+\frac{e^{x}\cos(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x)
```

```
[Out] 3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

3.365 $\int e^x \log(1+e^x)\,dx$

Optimal. Leaf size=18

$$(e^x+1)\log(e^x+1)-e^x$$

```
[Out] -exp(x)+(1+exp(x))*ln(1+exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2194, 2554, 2248, 43}

$$-e^x+e^x\log(e^x+1)+\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + Log[1 + E^x] + E^x*Log[1 + E^x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x\log(1+e^x)\,dx &= e^x\log(1+e^x)-\int\frac{e^{2x}}{1+e^x}\,dx\\ &= e^x\log(1+e^x)-\text{Subst}\left(\int\frac{x}{1+x}\,dx,x,e^x\right)\\ &= e^x\log(1+e^x)-\text{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)dx,x,e^x\right)\\ &= -e^x+\log(1+e^x)+e^x\log(1+e^x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$(e^x+1)\log(e^x+1)-e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[1 + E^x],x]
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

fricas [A] time = 0.43, size = 15, normalized size = 0.83

$$(e^x + 1)\log(e^x + 1) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="fricas")
[Out] (e^x + 1)*log(e^x + 1) - e^x
```

giac [A] time = 1.04, size = 16, normalized size = 0.89

$$(e^x + 1)\log(e^x + 1) - e^x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="giac")
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-e^x + (e^x + 1)\ln(e^x + 1) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(exp(x)+1),x)
[Out] (exp(x)+1)*ln(exp(x)+1)-1-exp(x)
```

maxima [A] time = 0.42, size = 16, normalized size = 0.89

$$(e^x + 1)\log(e^x + 1) - e^x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="maxima")
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

mupad [B] time = 0.24, size = 18, normalized size = 1.00

$$\ln(e^x + 1) - e^x + e^x \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*log(exp(x) + 1),x)
[Out] log(exp(x) + 1) - exp(x) + exp(x)*log(exp(x) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(1+exp(x)),x)
[Out] Timed out
```

3.366 $\int x^2 \tan^{-1}(x)\, dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4852, 266, 43}

$$-\frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right) + \frac{1}{3}x^3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tan^{-1}(x)\, dx &= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}\, dx \\
&= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{x}{1+x}\, dx, x, x^2\right) \\
&= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{6}\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx, x, x^2\right) \\
&= -\frac{x^2}{6} + \frac{1}{3}x^3 \tan^{-1}(x) + \frac{1}{6}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.85

$$\frac{1}{6}\left(2x^3 \tan^{-1}(x) - x^2 + \log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x],x]
```

```
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

fricas [A] time = 0.43, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

giac [A] time = 0.96, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$\frac{x^3\arctan(x)}{3} - \frac{x^2}{6} + \frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x),x)
```

```
[Out] 1/3*x^3*arctan(x)-1/6*x^2+1/6*ln(x^2+1)
```

maxima [A] time = 0.42, size = 21, normalized size = 0.78

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

mupad [B] time = 0.00, size = 21, normalized size = 0.78

$$\frac{\ln\left(x^2+1\right)}{6} + \frac{x^3\,\mathrm{atan}(x)}{3} - \frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(x),x)
```

```
[Out] log(x^2 + 1)/6 + (x^3*atan(x))/3 - x^2/6
```

sympy [A] time = 0.35, size = 20, normalized size = 0.74

$$\frac{x^3\,\mathrm{atan}\,(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

3.367 $\int \sqrt{-1+e^{2x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

```
[Out] -arctan((-1+exp(2*x))^(1/2))+(-1+exp(2*x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 203}

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + E^(2*x)],x]
```

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+e^{2x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, e^{2x}\right)\\ &= \sqrt{-1+e^{2x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, e^{2x}\right)\\ &= \sqrt{-1+e^{2x}} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+e^{2x}}\right)\\ &= \sqrt{-1+e^{2x}} - \tan^{-1}\left(\sqrt{-1+e^{2x}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\sqrt{e^{2x}-1} - \tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + E^(2*x)],x]
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

fricas [A] time = 0.42, size = 20, normalized size = 0.77

$$\sqrt{e^{(2x)}-1} - \arctan\left(\sqrt{e^{(2x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

giac [A] time = 0.87, size = 20, normalized size = 0.77

$$\sqrt{e^{(2x)}-1} - \arctan\left(\sqrt{e^{(2x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="giac")
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$-\arctan\left(\sqrt{e^{2x}-1}\right) + \sqrt{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(2*x))^(1/2),x)
[Out] -arctan((-1+exp(2*x))^(1/2))+(-1+exp(2*x))^(1/2)
```

maxima [A] time = 0.96, size = 20, normalized size = 0.77

$$\sqrt{e^{(2x)}-1} - \arctan\left(\sqrt{e^{(2x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

mupad [B] time = 0.26, size = 31, normalized size = 1.19

$$\sqrt{e^{2x}-1}\left(\frac{e^{-x}\,\mathrm{asin}\left(e^{-x}\right)}{\sqrt{1-e^{-2x}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(2*x) - 1)^(1/2),x)
```

```
[Out] (exp(2*x) - 1)^(1/2)*((exp(-x)*asin(exp(-x)))/(1 - exp(-2*x))^(1/2) + 1)
```

sympy [A] time = 1.39, size = 19, normalized size = 0.73

$$\begin{cases} \sqrt{e^{2x}-1} - \mathrm{acos}\left(e^{-x}\right) & \text{for}\ e^{x} < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((sqrt(exp(2*x) - 1) - acos(exp(-x)), exp(x) < 0))
```

3.368 $\int e^{\sin(x)} \sin(2x)\, dx$

Optimal. Leaf size=15

$$2e^{\sin(x)}\sin(x) - 2e^{\sin(x)}$$

```
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 2176, 2194}

$$2e^{\sin(x)}\sin(x) - 2e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sin[x]*Sin[2*x],x]
```

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\sin(x)}\sin(2x)\,dx &= \text{Subst}\left(\int 2e^x x\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int e^x x\,dx, x, \sin(x)\right)\\
&= 2e^{\sin(x)}\sin(x) - 2\,\text{Subst}\left(\int e^x\,dx, x, \sin(x)\right)\\
&= -2e^{\sin(x)} + 2e^{\sin(x)}\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 0.73

$$e^{\sin(x)}(2\sin(x) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Sin[2*x],x]
```

```
[Out] E^Sin[x]*(-2 + 2*Sin[x])
```

fricas [A] time = 0.44, size = 9, normalized size = 0.60

$$2\,(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="fricas")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

giac [A] time = 0.95, size = 9, normalized size = 0.60

$$2\,(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="giac")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$2\,\mathrm{e}^{\sin(x)}\sin(x)-2\,\mathrm{e}^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*sin(2*x),x)
```

```
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

maxima [A] time = 0.44, size = 9, normalized size = 0.60

$$2\,(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="maxima")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

mupad [B] time = 0.27, size = 9, normalized size = 0.60

$$2\,\mathrm{e}^{\sin(x)}\,\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*exp(sin(x)),x)
```

```
[Out] 2*exp(sin(x))*(sin(x) - 1)
```

sympy [A] time = 1.99, size = 15, normalized size = 1.00

$$2e^{\sin{(x)}}\sin{(x)}-2e^{\sin{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x)
```

```
[Out] 2*exp(sin(x))*sin(x) - 2*exp(sin(x))
```

3.369 $\int x^2\sqrt{5-x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{5}{8}\sqrt{5-x^2}\,x+\frac{1}{4}\sqrt{5-x^2}\,x^3+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 25/8*arcsin(1/5*x*5^(1/2))-5/8*x*(-x^2+5)^(1/2)+1/4*x^3*(-x^2+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{5-x^2}\,x^3-\frac{5}{8}\sqrt{5-x^2}\,x+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[5 - x^2],x]
```

```
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqrt[5]])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{5-x^2}\,dx &= \frac{1}{4}x^3\sqrt{5-x^2}+\frac{5}{4}\int\frac{x^2}{\sqrt{5-x^2}}\,dx\\
&= -\frac{5}{8}x\sqrt{5-x^2}+\frac{1}{4}x^3\sqrt{5-x^2}+\frac{25}{8}\int\frac{1}{\sqrt{5-x^2}}\,dx\\
&= -\frac{5}{8}x\sqrt{5-x^2}+\frac{1}{4}x^3\sqrt{5-x^2}+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.74

$$\frac{1}{8}\left(x\sqrt{5-x^2}\,\left(2x^2-5\right)+25\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[5 - x^2],x]
[Out] (x*Sqrt[5 - x^2]*(-5 + 2*x^2) + 25*ArcSin[x/Sqrt[5]])/8
```

fricas [A] time = 0.42, size = 37, normalized size = 0.79

$$\frac{1}{8}\left(2x^3-5x\right)\sqrt{-x^2+5}-\frac{25}{8}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 5) - 25/8*arctan(sqrt(-x^2 + 5)/x)
```

giac [A] time = 1.00, size = 29, normalized size = 0.62

$$\frac{1}{8}\left(2x^2-5\right)\sqrt{-x^2+5}\,x+\frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/8*(2*x^2 - 5)*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

maple [A] time = 0.00, size = 35, normalized size = 0.74

$$-\frac{\left(-x^2+5\right)^{\frac{3}{2}}x}{4}+\frac{5\sqrt{-x^2+5}\,x}{8}+\frac{25\arcsin\left(\frac{\sqrt{5}\,x}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+5)^(1/2),x)
[Out] -1/4*x*(-x^2+5)^(3/2)+5/8*(-x^2+5)^(1/2)*x+25/8*arcsin(1/5*5^(1/2)*x)
```

maxima [A] time = 0.96, size = 34, normalized size = 0.72

$$-\frac{1}{4}\left(-x^2+5\right)^{\frac{3}{2}}x+\frac{5}{8}\sqrt{-x^2+5}\,x+\frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="maxima")
[Out] -1/4*(-x^2 + 5)^(3/2)*x + 5/8*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.64

$$\frac{25\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{8}-\sqrt{5-x^2}\,\left(\frac{5\,x}{8}-\frac{x^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5 - x^2)^(1/2),x)
[Out] (25*asin((5^(1/2)*x)/5))/8 - (5 - x^2)^(1/2)*((5*x)/8 - x^3/4)
```

sympy [A] time = 2.79, size = 122, normalized size = 2.60

$$\begin{cases} \frac{ix^5}{4\sqrt{x^2-5}} - \frac{15ix^3}{8\sqrt{x^2-5}} + \frac{25ix}{8\sqrt{x^2-5}} - \frac{25i\,\mathrm{acosh}\left(\frac{\sqrt{5}\,x}{5}\right)}{8} & \text{for } \frac{|x^2|}{5} > 1 \\ -\frac{x^5}{4\sqrt{5-x^2}} + \frac{15x^3}{8\sqrt{5-x^2}} - \frac{25x}{8\sqrt{5-x^2}} + \frac{25\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 5)) - 15*I*x**3/(8*sqrt(x**2 - 5)) + 25*I*
x/(8*sqrt(x**2 - 5)) - 25*I*acosh(sqrt(5)*x/5)/8, Abs(x**2)/5 > 1), (-x**5/
(4*sqrt(5 - x**2)) + 15*x**3/(8*sqrt(5 - x**2)) - 25*x/(8*sqrt(5 - x**2)) +
 25*asin(sqrt(5)*x/5)/8, True))
```

3.370 $\int x^2 \left(1+x^3\right)^4 \, dx$

Optimal. Leaf size=11

$$\frac{1}{15} \left(x^3+1\right)^5$$

```
[Out] 1/15*(x^3+1)^5
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{15} \left(x^3+1\right)^5$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^3)^4,x]
```

```
[Out] (1 + x^3)^5/15
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(1+x^3\right)^4 \, dx = \frac{1}{15} \left(1+x^3\right)^5$$

Mathematica [B] time = 0.00, size = 36, normalized size = 3.27

$$\frac{x^{15}}{15}+\frac{x^{12}}{3}+\frac{2x^9}{3}+\frac{2x^6}{3}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^4,x]
```

```
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

fricas [B] time = 0.34, size = 26, normalized size = 2.36

$$\frac{1}{15}x^{15}+\frac{1}{3}x^{12}+\frac{2}{3}x^9+\frac{2}{3}x^6+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15 + 1/3*x^12 + 2/3*x^9 + 2/3*x^6 + 1/3*x^3
```

giac [A] time = 0.87, size = 9, normalized size = 0.82

$$\frac{1}{15} \left(x^3+1\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="giac")
```

```
[Out] 1/15*(x^3 + 1)^5
```

maple [B] time = 0.00, size = 27, normalized size = 2.45

$$\frac{1}{15}x^{15} + \frac{1}{3}x^{12} + \frac{2}{3}x^{9} + \frac{2}{3}x^{6} + \frac{1}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^4,x)
```

```
[Out] 1/15*x^15+1/3*x^12+2/3*x^9+2/3*x^6+1/3*x^3
```

maxima [A] time = 0.42, size = 9, normalized size = 0.82

$$\frac{1}{15}\left(x^{3} + 1\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(x^3 + 1)^5
```

mupad [B] time = 0.02, size = 26, normalized size = 2.36

$$\frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2\,x^{9}}{3} + \frac{2\,x^{6}}{3} + \frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^4,x)
```

```
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

sympy [B] time = 0.06, size = 27, normalized size = 2.45

$$\frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2x^{9}}{3} + \frac{2x^{6}}{3} + \frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**4,x)
```

```
[Out] x**15/15 + x**12/3 + 2*x**9/3 + 2*x**6/3 + x**3/3
```

3.371 $\int \cos^3(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}$$

```
[Out] 1/4*sin(x)^4-1/6*sin(x)^6
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2564, 14}

$$\frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^3(x)\,dx &= \text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(x)\right)\\ &= \frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{192}\cos(6x)-\frac{3}{64}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] (-3*Cos[2*x])/64 + Cos[6*x]/192
```

fricas [A] time = 0.45, size = 13, normalized size = 0.76

$$\frac{1}{6}\cos(x)^6-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="fricas")
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

giac [A] time = 0.98, size = 13, normalized size = 0.76

$$\frac{1}{6}\cos(x)^6 - \frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="giac")
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$-\frac{\left(\cos^4(x)\right)\left(\sin^2(x)\right)}{6} - \frac{\left(\cos^4(x)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3,x)
[Out] -1/6*cos(x)^4*sin(x)^2-1/12*cos(x)^4
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$-\frac{1}{6}\sin(x)^6 + \frac{1}{4}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="maxima")
[Out] -1/6*sin(x)^6 + 1/4*sin(x)^4
```

mupad [B] time = 0.05, size = 14, normalized size = 0.82

$$-\frac{\sin(x)^4\left(2\sin(x)^2 - 3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3,x)
[Out] -(sin(x)^4*(2*sin(x)^2 - 3))/12
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$-\frac{\sin^6(x)}{6} + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3,x)
[Out] -sin(x)**6/6 + sin(x)**4/4
```

3.372 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^2(x)\,dx &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(x)\right)\\ &= \frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{1}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

fricas [A] time = 0.40, size = 20, normalized size = 1.18

$$-\frac{\left(2\cos(x)^4+\cos(x)^2-3\right)\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

giac [A] time = 0.86, size = 13, normalized size = 0.76

$$\frac{1}{5}\,\tan(x)^5 + \frac{1}{3}\,\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

maple [A] time = 0.00, size = 22, normalized size = 1.29

$$\frac{2\left(\sin^3(x)\right)}{15\cos(x)^3} + \frac{\sin^3(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 2/15/cos(x)^3*sin(x)^3+1/5/cos(x)^5*sin(x)^3
```

maxima [A] time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{5}\,\tan(x)^5 + \frac{1}{3}\,\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

mupad [B] time = 0.19, size = 13, normalized size = 0.76

$$\frac{\tan(x)^5}{5} + \frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x)^4,x)
```

```
[Out] tan(x)^3/3 + tan(x)^5/5
```

sympy [B] time = 0.08, size = 29, normalized size = 1.71

$$-\frac{2\sin(x)}{15\cos(x)} - \frac{\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

3.373 $\int x\sqrt{1+2x}\,dx$

Optimal. Leaf size=27

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

```
[Out] -1/6*(1+2*x)^(3/2)+1/10*(1+2*x)^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 2*x],x]
[Out] -(1 + 2*x)^(3/2)/6 + (1 + 2*x)^(5/2)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+2x}\,dx &= \int\left(-\frac{1}{2}\sqrt{1+2x}+\frac{1}{2}(1+2x)^{3/2}\right)dx\\ &= -\frac{1}{6}(1+2x)^{3/2}+\frac{1}{10}(1+2x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{1}{15}(2x+1)^{3/2}(3x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 2*x],x]
[Out] ((1 + 2*x)^(3/2)*(-1 + 3*x))/15
```

fricas [A] time = 0.40, size = 17, normalized size = 0.63

$$\frac{1}{15}\left(6x^2+x-1\right)\sqrt{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/15*(6*x^2 + x - 1)*sqrt(2*x + 1)
```

giac [A] time = 0.94, size = 19, normalized size = 0.70

$$\frac{1}{10}(2x+1)^{\frac{5}{2}}-\frac{1}{6}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$\frac{(2x+1)^{\frac{3}{2}}(3x-1)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x+1)^(1/2),x)
```

```
[Out] 1/15*(2*x+1)^(3/2)*(3*x-1)
```

maxima [A] time = 0.42, size = 19, normalized size = 0.70

$$\frac{1}{10}(2x+1)^{\frac{5}{2}}-\frac{1}{6}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.52

$$\frac{(2x+1)^{3/2}(6x-2)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x + 1)^(1/2),x)
```

```
[Out] ((2*x + 1)^(3/2)*(6*x - 2))/30
```

sympy [A] time = 1.02, size = 36, normalized size = 1.33

$$\frac{2x^2\sqrt{2x+1}}{5}+\frac{x\sqrt{2x+1}}{15}-\frac{\sqrt{2x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(2*x + 1)/5 + x*sqrt(2*x + 1)/15 - sqrt(2*x + 1)/15
```

3.374 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

fricas [A] time = 0.42, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\cos(x)^3-5\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

giac [A] time = 0.90, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.75

$$\frac{3x}{8} - \frac{\left(\sin^{3}(x) + \frac{3\sin(x)}{2}\right)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
[Out] 3/8*x-1/4*(sin(x)^3+3/2*sin(x))*cos(x)
```

maxima [A] time = 0.42, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.67

$$\frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

sympy [A] time = 0.06, size = 24, normalized size = 1.00

$$\frac{3x}{8} - \frac{\sin^{3}\left(x\right)\cos\left(x\right)}{4} - \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

3.375 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

```
[Out] ln(cos(x))+1/2*tan(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\,dx &= \frac{\tan^2(x)}{2}-\int \tan(x)\,dx\\ &= \log(\cos(x))+\frac{\tan^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{\sec^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

fricas [A] time = 0.45, size = 18, normalized size = 1.50

$$\frac{1}{2}\,\tan(x)^2+\frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 1.03, size = 16, normalized size = 1.33

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="giac")
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{\left(\tan^2(x)\right)}{2} - \frac{\ln\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
[Out] 1/2*tan(x)^2-1/2*ln(tan(x)^2+1)
```

maxima [A] time = 0.44, size = 20, normalized size = 1.67

$$-\frac{1}{2\left(\sin(x)^2 - 1\right)} + \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.02, size = 16, normalized size = 1.33

$$\ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
[Out] log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)
```

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3,x)
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

3.376 $\int x^5\sqrt{1+x^2}\,dx$

Optimal. Leaf size=40

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)-2/5*(x^2+1)^(5/2)+1/7*(x^2+1)^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1+x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{1}{3}\left(1+x^2\right)^{3/2}-\frac{2}{5}\left(1+x^2\right)^{5/2}+\frac{1}{7}\left(1+x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.62

$$\frac{1}{105}\left(x^2+1\right)^{3/2}\left(15x^4-12x^2+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(8 - 12*x^2 + 15*x^4))/105
```

fricas [A] time = 0.41, size = 26, normalized size = 0.65

$$\frac{1}{105}\left(15\,x^6+3\,x^4-4\,x^2+8\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*x^6 + 3*x^4 - 4*x^2 + 8)*sqrt(x^2 + 1)
```

giac [A] time = 0.99, size = 28, normalized size = 0.70

$$\frac{1}{7}\left(x^2+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^2+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/7*(x^2 + 1)^(7/2) - 2/5*(x^2 + 1)^(5/2) + 1/3*(x^2 + 1)^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.55

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}\left(15x^4-12x^2+8\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)^(1/2),x)
[Out] 1/105*(x^2+1)^(3/2)*(15*x^4-12*x^2+8)
```

maxima [A] time = 0.97, size = 34, normalized size = 0.85

$$\frac{1}{7}\left(x^2+1\right)^{\frac{3}{2}}x^4-\frac{4}{35}\left(x^2+1\right)^{\frac{3}{2}}x^2+\frac{8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/7*(x^2 + 1)^(3/2)*x^4 - 4/35*(x^2 + 1)^(3/2)*x^2 + 8/105*(x^2 + 1)^(3/2)
```

mupad [B] time = 0.02, size = 25, normalized size = 0.62

$$\sqrt{x^2+1}\,\left(\frac{x^6}{7}+\frac{x^4}{35}-\frac{4\,x^2}{105}+\frac{8}{105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(1/2)*(x^4/35 - (4*x^2)/105 + x^6/7 + 8/105)
```

sympy [A] time = 2.03, size = 53, normalized size = 1.32

$$\frac{x^6\sqrt{x^2+1}}{7}+\frac{x^4\sqrt{x^2+1}}{35}-\frac{4x^2\sqrt{x^2+1}}{105}+\frac{8\sqrt{x^2+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)**(1/2),x)
[Out] x**6*sqrt(x**2 + 1)/7 + x**4*sqrt(x**2 + 1)/35 - 4*x**2*sqrt(x**2 + 1)/105
+ 8*sqrt(x**2 + 1)/105
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```