Computer algebra independent integration tests

0-Independent-test-suites/Moses-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (113)	% 0.00 (0)
Mathematica	% 99.12 (112)	% 0.88 (1)
Maple	% 100.00 (113)	% 0.00 (0)
Maxima	% 98.23 (111)	% 1.77 (2)
Fricas	% 99.12 (112)	% 0.88 (1)
Sympy	% 92.04 (104)	% 7.96 (9)
Giac	% 96.46 (109)	% 3.54 (4)
Mupad	% 93.81 (106)	% 6.19 (7)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

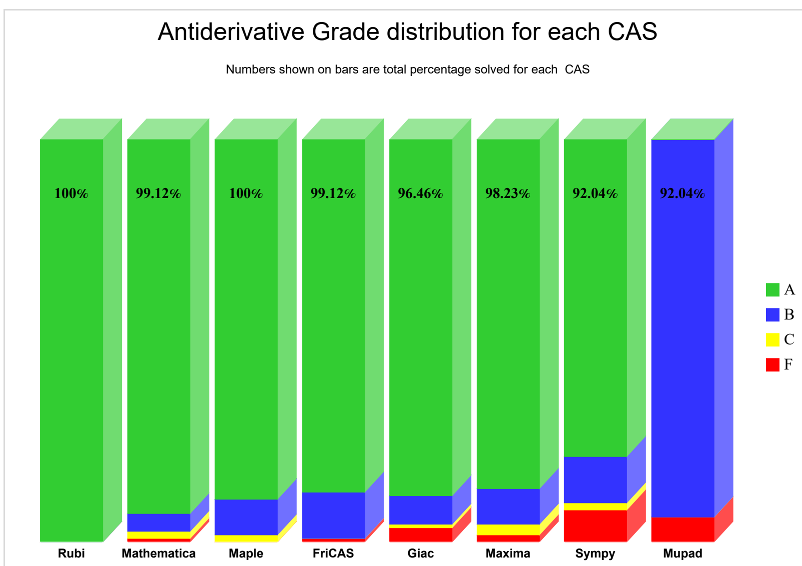
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

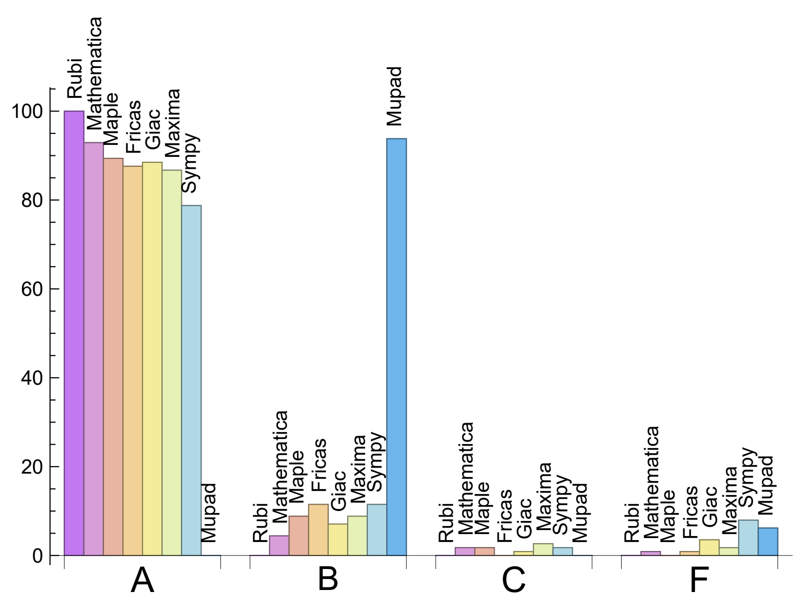
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.92	4.42	1.77	0.88
Maple	89.38	8.85	1.77	0.00
Maxima	86.73	8.85	2.65	1.77
Fricas	87.61	11.50	0.00	0.88
Sympy	78.76	11.50	1.77	7.96
Giac	88.50	7.08	0.88	3.54
Mupad	0.00	93.81	0.00	6.19

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	2	100.00 %	0.00 %	0.00 %
Fricas	1	100.00 %	0.00 %	0.00 %
Sympy	9	88.89 %	11.11 %	0.00 %
Giac	4	50.00 %	0.00 %	50.00 %
Mupad	7	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

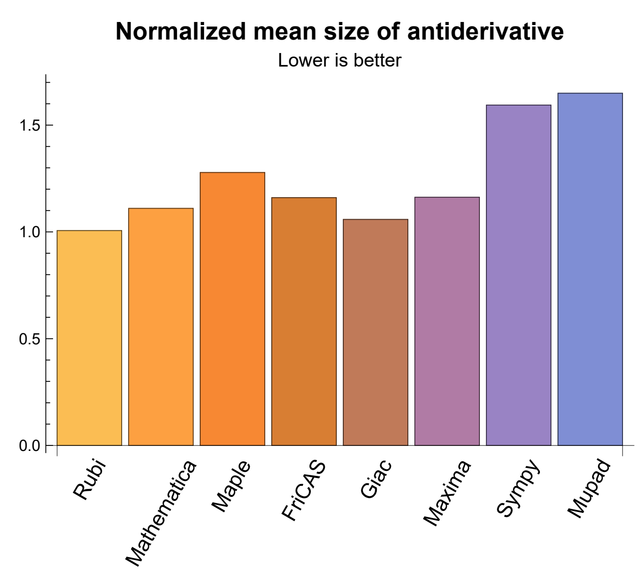
1.3 Performance

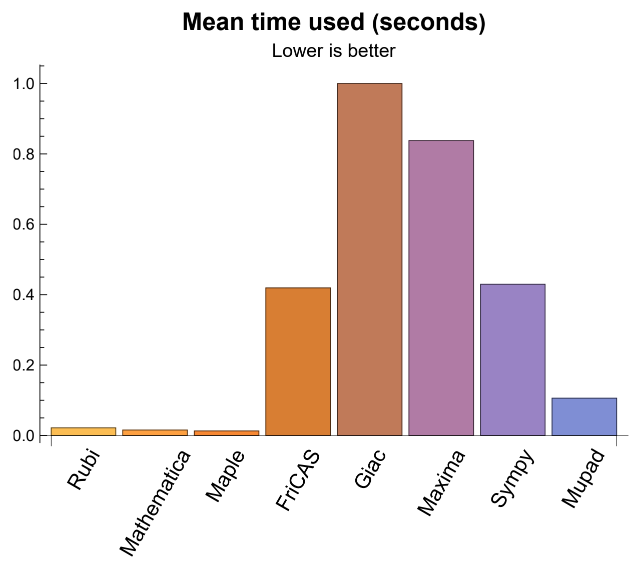
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	20.14	1.01	16.00	1.00
Mathematica	0.02	24.32	1.11	16.00	1.00
Maple	0.01	25.84	1.28	14.00	0.92
Maxima	0.84	23.75	1.16	13.00	0.88
Fricas	0.42	24.79	1.16	14.00	0.93
Sympy	0.43	29.46	1.59	15.00	0.83
Giac	1.00	18.50	1.06	13.00	0.83
Mupad	0.11	28.25	1.65	12.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

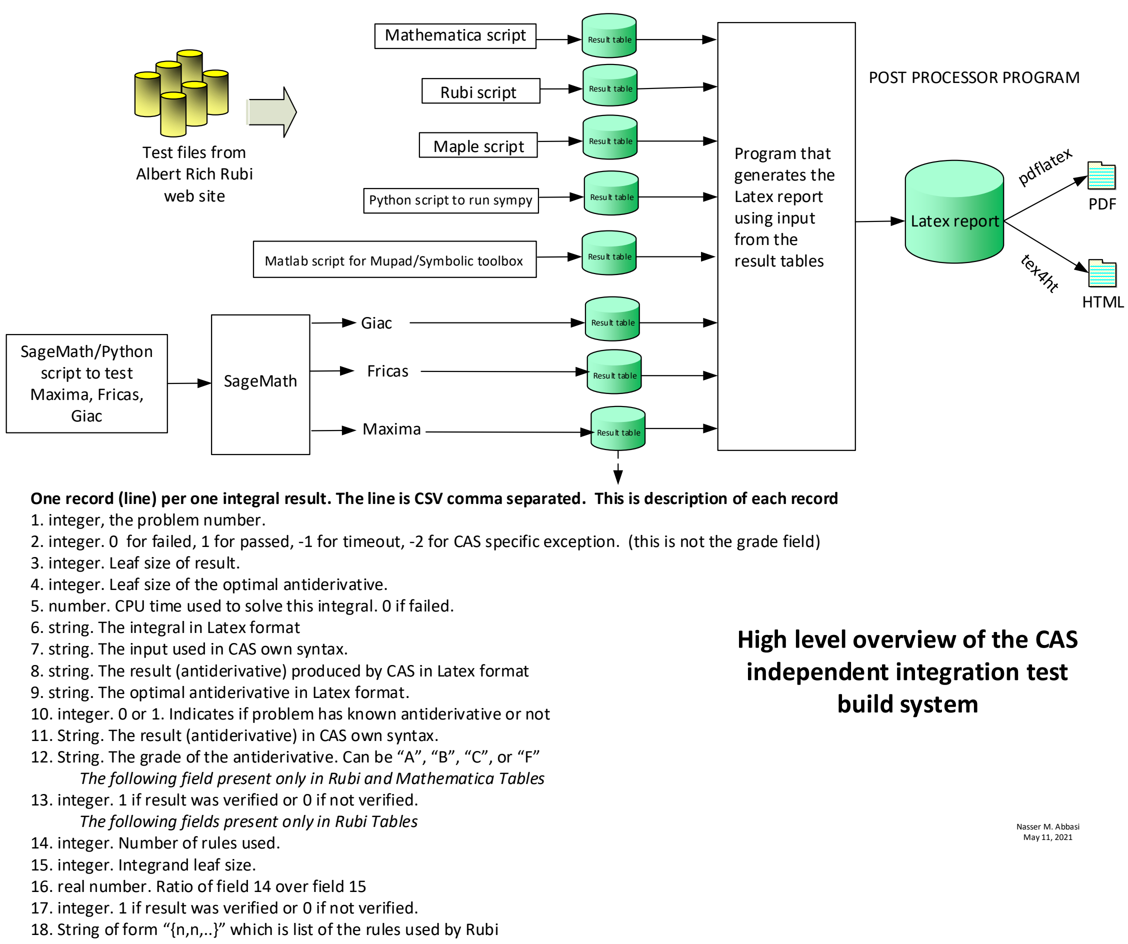
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 31, 42, 70, 71, 72 }

C grade: { 40, 69 }

F grade: { 57 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 35, 36, 40, 48, 51, 57, 69, 70, 71, 72 }

C grade: { 32, 42 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 36, 38, 40, 42, 68, 69, 70, 71, 72, 87 }

C grade: { 10, 11, 47 }

F grade: { 32, 67 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade: { }

F grade: { 32 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade: { 71, 72 }

F grade: { 35, 36, 40, 42, 69, 70, 87, 90, 98 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 25, 36, 43, 45, 70, 71, 72 }

C grade: { 47 }

F grade: { 32, 40, 42, 69 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

C grade: { }

F grade: { 10, 11, 32, 38, 40, 42, 69 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	16	48	19	34	10
normalized size	1	1.00	1.50	1.17	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.010	0.003	0.000	1.284	0.431	0.073	0.937	0.004

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	19	14	15	12
normalized size	1	1.00	1.00	0.92	1.15	1.46	1.08	1.15	0.92
time (sec)	N/A	0.005	0.004	0.007	1.287	0.419	0.108	0.805	0.158

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	11	14	15	11	10
normalized size	1	1.00	0.74	0.58	0.58	0.74	0.79	0.58	0.53
time (sec)	N/A	0.003	0.004	0.003	0.500	0.411	0.220	0.943	0.030

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.000	0.511	0.428	0.060	1.005	0.028

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.006	0.001	0.000	0.583	0.420	0.084	1.058	0.017

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.011	0.005	0.018	0.553	0.430	0.070	1.039	0.022

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.002	0.002	0.000	0.526	0.404	0.190	1.040	0.029

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
normalized size	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.007	0.015	0.000	0.572	0.420	0.300	1.052	0.019

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	8	6	6
normalized size	1	1.00	1.00	0.88	0.75	1.75	1.00	0.75	0.75
time (sec)	N/A	0.012	0.005	0.033	0.519	0.417	0.070	1.192	0.053

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	3	3	3	-1
normalized size	1	1.00	1.00	1.00	3.75	0.75	0.75	0.75	-0.25
time (sec)	N/A	0.012	0.006	0.014	0.829	0.429	0.628	0.996	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	-1
normalized size	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	-0.50
time (sec)	N/A	0.011	0.015	0.001	0.696	0.423	0.604	0.976	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	5	7	7
normalized size	1	1.00	1.00	1.00	0.88	0.88	0.62	0.88	0.88
time (sec)	N/A	0.003	0.003	0.003	0.484	0.434	0.078	1.082	0.042

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.020	0.006	0.004	0.466	0.408	0.086	0.882	0.153

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	19	19	20	19	21
normalized size	1	1.00	0.93	0.79	0.68	0.68	0.71	0.68	0.75
time (sec)	N/A	0.020	0.015	0.007	0.476	0.412	0.094	0.930	0.054

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.005	0.002	0.001	0.524	0.423	0.088	1.081	0.047

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.007	0.002	0.002	0.602	0.435	0.059	1.077	0.017

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.006	0.001	0.000	0.681	0.408	0.084	0.915	0.002

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.002	0.000	0.000	0.514	0.411	0.191	1.100	0.002

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
normalized size	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.015	0.005	0.002	0.562	0.409	0.077	0.905	0.029

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
normalized size	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.001	0.003	0.506	0.413	0.056	0.954	0.044

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.003	0.005	0.018	0.517	0.427	0.123	1.036	0.072

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	7
normalized size	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.003	0.001	0.003	0.479	0.411	0.056	1.098	0.016

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	7	7	8	7	7
normalized size	1	1.00	1.90	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.027	0.022	0.014	0.453	0.438	1.959	1.186	0.187

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	15	15	34	15	12
normalized size	1	1.00	0.70	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.004	0.005	0.003	0.549	0.408	0.934	0.961	0.031

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
normalized size	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.003	0.004	0.000	1.410	0.396	0.124	1.031	0.158

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	13	13	15	13	13
normalized size	1	1.00	1.00	0.78	0.72	0.72	0.83	0.72	0.72
time (sec)	N/A	0.020	0.008	0.007	1.326	0.418	0.114	1.108	0.107

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	19	76	22	19	19
normalized size	1	1.00	1.00	0.65	0.61	2.45	0.71	0.61	0.61
time (sec)	N/A	0.035	0.009	0.016	1.328	0.441	0.162	0.943	0.231

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.38	1.00	1.00	1.00
time (sec)	N/A	0.034	0.006	0.007	0.557	0.428	0.094	1.102	0.184

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	10	12	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.006	0.003	0.018	0.474	0.406	0.090	0.958	0.072

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.019	0.003	0.004	0.643	0.443	0.599	0.893	0.208

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	40	52
normalized size	1	1.00	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.036	0.118	0.004	1.156	0.417	0.169	0.894	0.138

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0	-1
normalized size	1	1.00	1.23	0.36	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.056	0.298	0.164	0.000	0.432	2.895	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.010	0.017	0.029	0.507	0.436	0.308	1.269	0.212

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	15	15	34	15	12
normalized size	1	1.00	0.70	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.004	0.005	0.000	0.473	0.409	0.951	0.928	0.002

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	24	24	0	24	24
normalized size	1	1.00	1.00	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.014	0.013	0.002	0.507	0.418	0.000	0.917	0.002

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	71	76	80	55	0	61	57
normalized size	1	1.00	1.61	1.73	1.82	1.25	0.00	1.39	1.30
time (sec)	N/A	0.016	0.037	0.012	1.314	0.428	0.000	1.055	0.206

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	44	63	105	29	91
normalized size	1	1.00	0.74	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.009	0.025	0.006	1.217	0.402	2.139	0.929	0.151

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	41	70	113	44	190	90	-1
normalized size	1	1.00	0.55	0.93	1.51	0.59	2.53	1.20	-0.01
time (sec)	N/A	0.013	0.020	0.003	0.532	0.414	8.746	1.156	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	44	63	105	29	91
normalized size	1	1.00	0.74	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.008	0.003	0.000	1.320	0.413	2.155	1.070	0.002

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	134	262	122	129	0	0	-1
normalized size	1	1.00	2.63	5.14	2.39	2.53	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.028	0.032	1.459	0.454	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.000	0.603	0.449	0.062	0.941	0.033

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	116	244	0	0	-1
normalized size	1	1.16	2.02	3.04	2.37	4.98	0.00	0.00	-0.02
time (sec)	N/A	0.085	0.112	0.193	1.357	0.550	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
normalized size	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.008	0.004	0.015	0.549	0.413	0.192	0.820	0.182

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
normalized size	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.007	0.001	0.000	0.570	0.406	0.078	1.092	0.017

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	5	30	8
normalized size	1	1.00	1.00	1.00	0.89	0.89	0.56	3.33	0.89
time (sec)	N/A	0.026	0.024	0.003	0.462	0.396	0.087	0.917	0.086

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.032	0.006	0.003	0.480	0.411	0.087	0.827	0.145

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
normalized size	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.002	0.002	0.001	0.545	0.420	0.197	1.048	0.019

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
normalized size	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.004	0.003	0.714	0.399	0.711	1.012	0.008

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	34	34	41	35	46
normalized size	1	1.00	0.98	0.85	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.022	0.009	0.001	1.088	0.407	0.133	0.889	0.106

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	65	65	83	67	88
normalized size	1	1.55	1.60	1.40	1.38	1.38	1.77	1.43	1.87
time (sec)	N/A	0.105	0.013	0.008	1.281	0.428	0.251	1.013	0.090

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	44	39	27	70	41	21
normalized size	1	1.00	1.00	2.10	1.86	1.29	3.33	1.95	1.00
time (sec)	N/A	0.009	0.004	0.009	0.510	0.400	0.304	1.143	0.240

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.004	0.001	0.005	0.609	0.414	0.089	1.067	0.035

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	38	24
normalized size	1	1.00	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.024	0.013	0.001	1.288	0.445	0.341	1.087	0.002

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.001	0.001	0.003	0.534	0.391	0.077	1.050	0.021

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.044	0.015	0.135	0.540	0.392	0.088	0.961	0.264

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.020	0.012	0.005	1.293	0.414	0.138	0.909	0.307

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	A	A	A	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	0	9	3	2	2	3	2
normalized size	1	1.00	0.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.002	0.002	0.000	0.646	0.412	0.448	0.892	0.011

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.021	0.005	0.026	0.576	0.408	0.181	1.108	0.059

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	16	11	14	8	11	15
normalized size	1	1.00	0.76	0.94	0.65	0.82	0.47	0.65	0.88
time (sec)	N/A	0.031	0.010	0.003	0.559	0.421	0.085	0.987	0.058

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	29	24	24	26	24	28
normalized size	1	1.00	0.76	0.76	0.63	0.63	0.68	0.63	0.74
time (sec)	N/A	0.033	0.028	0.004	0.619	0.402	0.095	0.952	0.157

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.506	0.432	0.175	0.921	0.018

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.012	0.013	0.001	0.616	0.422	0.308	0.916	0.002

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.660	0.426	0.175	1.008	0.002

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
normalized size	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.010	0.001	0.003	0.595	0.412	0.102	0.946	0.032

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	8	9	9
normalized size	1	1.00	1.00	0.91	0.82	1.36	0.73	0.82	0.82
time (sec)	N/A	0.012	0.004	0.031	0.532	0.449	0.570	0.962	0.040

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.019	0.010	0.001	1.364	0.436	0.141	0.915	0.002

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	3	3	43
normalized size	1	1.00	1.00	1.33	0.00	1.00	1.00	1.00	14.33
time (sec)	N/A	0.047	0.059	0.007	0.000	0.462	0.410	1.132	3.099

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	21	53	15	12	21	13
normalized size	1	1.00	1.00	1.31	3.31	0.94	0.75	1.31	0.81
time (sec)	N/A	0.041	0.022	0.050	1.279	0.420	0.150	1.082	0.029

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	127	262	123	128	0	0	-1
normalized size	1	1.00	2.40	4.94	2.32	2.42	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.057	0.008	1.495	0.436	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	127	69	67	0	83	360
normalized size	1	1.00	2.19	7.94	4.31	4.19	0.00	5.19	22.50
time (sec)	N/A	0.086	0.098	0.074	1.513	0.488	0.000	1.232	0.499

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	68	26	422	82	352
normalized size	1	1.00	2.19	7.56	4.25	1.62	26.38	5.12	22.00
time (sec)	N/A	0.124	0.019	0.010	1.414	0.431	1.927	1.158	0.236

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	65	26	422	79	352
normalized size	1	1.00	2.19	7.56	4.06	1.62	26.38	4.94	22.00
time (sec)	N/A	0.019	0.013	0.009	1.422	0.435	1.897	1.041	0.058

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	44	63	105	29	91
normalized size	1	1.00	0.74	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.008	0.021	0.001	1.269	0.412	2.070	1.051	0.002

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	12	26	19	12	12
normalized size	1	1.00	1.29	0.93	0.86	1.86	1.36	0.86	0.86
time (sec)	N/A	0.009	0.007	0.024	1.332	0.452	0.065	0.989	0.072

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.005	0.004	0.002	1.471	0.394	0.086	0.889	0.028

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.030	0.005	0.001	0.668	0.414	0.087	1.079	0.002

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	24	23	23	20	30	24
normalized size	1	1.00	0.83	1.00	0.96	0.96	0.83	1.25	1.00
time (sec)	N/A	0.357	0.073	0.005	1.800	0.434	0.120	1.058	0.223

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.003	0.003	0.001	0.469	0.406	0.075	1.147	0.032

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	24	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.96	0.76	0.76	0.76	0.76
time (sec)	N/A	0.045	0.002	0.004	0.567	0.428	0.095	0.958	0.227

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.003	0.003	0.001	1.416	0.400	0.092	1.057	0.056

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.008	0.004	0.004	1.470	0.412	0.095	1.029	0.022

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	16	20	36	16	10
normalized size	1	1.00	1.00	1.21	1.14	1.43	2.57	1.14	0.71
time (sec)	N/A	0.015	0.002	0.022	1.362	0.440	0.389	0.919	0.178

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	44	63	105	29	91
normalized size	1	1.00	0.74	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.011	0.003	0.000	1.396	0.394	2.086	1.096	0.002

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	32	16	15	45	34	21	37
normalized size	1	1.00	1.88	0.94	0.88	2.65	2.00	1.24	2.18
time (sec)	N/A	0.005	0.008	0.005	1.533	0.428	0.514	1.074	0.163

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
normalized size	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.008	0.015	0.001	0.815	0.407	0.297	0.948	0.002

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.002	0.513	0.385	0.055	0.951	0.006

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	256	71	0	33	32
normalized size	1	1.00	1.62	0.69	5.69	1.58	0.00	0.73	0.71
time (sec)	N/A	0.123	0.193	0.274	1.984	0.473	0.000	1.043	0.715

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.006	0.002	0.000	0.672	0.431	0.066	0.910	0.002

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	11	10	14	11	10
normalized size	1	1.00	0.82	0.65	0.65	0.59	0.82	0.65	0.59
time (sec)	N/A	0.003	0.004	0.006	0.520	0.409	0.215	0.946	0.024

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	20	19	27	0	27	27
normalized size	1	1.00	1.09	0.87	0.83	1.17	0.00	1.17	1.17
time (sec)	N/A	0.010	0.005	0.005	1.272	0.410	0.000	1.023	0.078

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	1.25	0.62	0.75	0.75
time (sec)	N/A	0.013	0.001	0.004	0.513	0.424	0.062	0.988	0.027

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
normalized size	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.016	0.005	0.001	0.554	0.410	0.078	0.970	0.002

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.026	0.008	0.004	0.687	0.411	0.088	0.981	0.053

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
normalized size	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.009	0.009	0.017	0.763	0.415	0.337	0.874	0.262

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.012	0.005	0.000	0.669	0.434	0.069	0.857	0.002

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.004	0.001	0.001	0.473	0.414	0.086	0.974	0.002

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.007	0.001	0.000	0.545	0.435	0.059	0.902	0.002

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	21	22	32	0	20	20
normalized size	1	1.00	1.12	0.88	0.92	1.33	0.00	0.83	0.83
time (sec)	N/A	0.009	0.032	0.007	1.202	0.407	0.000	1.027	0.268

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	13	19	15	13	13
normalized size	1	1.00	1.00	0.70	0.65	0.95	0.75	0.65	0.65
time (sec)	N/A	0.023	0.006	0.002	1.341	0.416	0.116	0.977	0.091

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	44	63	105	29	91
normalized size	1	1.00	0.74	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.009	0.003	0.000	1.335	0.417	2.142	0.905	0.002

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	24	14	14
normalized size	1	1.00	0.90	0.75	0.70	0.70	1.20	0.70	0.70
time (sec)	N/A	0.024	0.008	0.011	1.296	0.423	0.122	0.921	0.067

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	26	26
normalized size	1	1.00	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.011	0.013	0.000	1.272	0.419	0.137	1.122	0.002

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	31	15	19	19
normalized size	1	1.00	1.00	0.95	0.90	1.48	0.71	0.90	0.90
time (sec)	N/A	0.016	0.000	0.002	0.496	0.407	0.055	0.962	0.002

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	40	19	23	24
normalized size	1	1.00	1.00	0.96	0.92	1.54	0.73	0.88	0.92
time (sec)	N/A	0.019	0.000	0.003	0.684	0.392	0.057	0.919	0.003

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	43	22	25	25
normalized size	1	1.00	1.00	0.96	0.93	1.59	0.81	0.93	0.93
time (sec)	N/A	0.023	0.000	0.000	0.786	0.393	0.058	0.879	0.002

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	52	26	29	30
normalized size	1	1.00	1.00	0.97	0.94	1.62	0.81	0.91	0.94
time (sec)	N/A	0.026	0.000	0.001	0.543	0.399	0.060	0.889	0.002

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	23	41	20	23	23
normalized size	1	1.00	1.04	1.00	0.96	1.71	0.83	0.96	0.96
time (sec)	N/A	0.027	0.000	0.000	0.606	0.414	0.057	0.942	0.002

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	28	50	24	27	28
normalized size	1	1.00	1.03	1.00	0.97	1.72	0.83	0.93	0.97
time (sec)	N/A	0.032	0.000	0.000	0.576	0.409	0.059	0.965	0.002

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	17	17	18
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.89	0.89	0.95
time (sec)	N/A	0.009	0.000	0.002	0.595	0.386	0.055	0.984	0.002

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	22	20	21	23
normalized size	1	1.00	1.00	0.96	0.92	0.92	0.83	0.88	0.96
time (sec)	N/A	0.011	0.000	0.002	0.732	0.403	0.057	0.977	0.004

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	42	24	23	24
normalized size	1	1.00	1.00	0.96	0.92	1.68	0.96	0.92	0.96
time (sec)	N/A	0.013	0.000	0.001	0.475	0.400	0.058	0.918	0.002

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	28	50	27	27	29
normalized size	1	1.00	1.00	0.97	0.93	1.67	0.90	0.90	0.97
time (sec)	N/A	0.015	0.000	0.001	0.551	0.392	0.058	1.075	0.003

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	26	26	24	25	26
normalized size	1	1.00	1.04	1.00	0.96	0.96	0.89	0.93	0.96
time (sec)	N/A	0.006	0.000	0.001	0.649	0.404	0.059	1.284	0.003

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	4	0.500
2	A	3	2	1.00	11	0.182
3	A	2	1	1.00	11	0.091
4	A	1	1	1.00	2	0.500
5	A	1	1	1.00	7	0.143
6	A	2	2	1.00	7	0.286
7	A	1	1	1.00	11	0.091
8	A	1	1	1.00	6	0.167
9	A	2	2	1.00	7	0.286
10	A	2	2	1.00	4	0.500
11	A	1	1	1.00	6	0.167
12	A	3	2	1.00	6	0.333
13	A	4	2	1.00	16	0.125
14	A	5	3	1.00	7	0.429
15	A	3	1	1.00	14	0.071
16	A	2	2	1.00	5	0.400
17	A	1	1	1.00	7	0.143
18	A	1	1	1.00	11	0.091
19	A	2	2	1.00	11	0.182
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	6	0.167
22	A	2	2	1.00	9	0.222
23	A	3	3	1.00	14	0.214
24	A	2	1	1.00	9	0.111
25	A	3	3	1.00	7	0.429
26	A	2	2	1.00	15	0.133
27	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	3	1.00	13	0.231
29	A	1	1	1.00	5	0.200
30	A	3	3	1.00	8	0.375
31	A	7	7	1.00	11	0.636
32	A	3	2	1.00	12	0.167
33	A	3	3	1.00	6	0.500
34	A	2	1	1.00	9	0.111
35	A	4	3	1.00	13	0.231
36	A	4	4	1.00	15	0.267
37	A	3	2	1.00	15	0.133
38	A	6	3	1.00	13	0.231
39	A	3	2	1.00	15	0.133
40	A	5	5	1.00	29	0.172
41	A	2	2	1.00	4	0.500
42	A	6	6	1.16	19	0.316
43	A	1	1	1.00	6	0.167
44	A	2	2	1.00	5	0.400
45	A	1	1	1.00	10	0.100
46	A	5	3	1.00	13	0.231
47	A	1	1	1.00	5	0.200
48	A	1	1	1.00	7	0.143
49	A	6	6	1.00	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.00	27	0.037
52	A	1	1	1.00	4	0.250
53	A	4	3	1.00	6	0.500
54	A	2	2	1.00	10	0.200
55	A	7	4	1.00	9	0.444
56	A	2	1	1.00	12	0.083
57	A	1	1	1.00	4	0.250
58	A	6	4	1.00	7	0.571
59	A	4	3	1.00	11	0.273
60	A	6	3	1.00	9	0.333
61	A	2	2	1.00	4	0.500
62	A	3	3	1.00	6	0.500
63	A	2	2	1.00	4	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	2	2	1.00	6	0.333
65	A	2	1	1.00	9	0.111
66	A	2	1	1.00	12	0.083
67	A	2	2	1.00	20	0.100
68	A	2	2	1.00	10	0.200
69	A	6	6	1.00	30	0.200
70	A	5	5	1.00	39	0.128
71	A	6	5	1.00	45	0.111
72	A	4	4	1.00	31	0.129
73	A	3	2	1.00	15	0.133
74	A	3	2	1.00	4	0.500
75	A	3	2	1.00	11	0.182
76	A	5	3	1.00	13	0.231
77	A	10	6	1.00	33	0.182
78	A	1	1	1.00	7	0.143
79	A	5	5	1.00	8	0.625
80	A	2	2	1.00	9	0.222
81	A	3	3	1.00	11	0.273
82	A	3	3	1.00	8	0.375
83	A	3	2	1.00	15	0.133
84	A	2	2	1.00	16	0.125
85	A	1	1	1.00	6	0.167
86	A	1	1	1.00	3	0.333
87	A	4	2	1.00	17	0.118
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	11	0.091
90	A	3	3	1.00	14	0.214
91	A	2	2	1.00	7	0.286
92	A	2	2	1.00	11	0.182
93	A	3	2	1.00	13	0.154
94	A	1	1	1.00	8	0.125
95	A	2	2	1.00	7	0.286
96	A	1	1	1.00	4	0.250
97	A	2	2	1.00	5	0.400
98	A	3	3	1.00	17	0.176
99	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	3	2	1.00	15	0.133
101	A	3	3	1.00	15	0.200
102	A	3	3	1.00	8	0.375
103	A	1	1	1.00	20	0.050
104	A	1	1	1.00	25	0.040
105	A	1	1	1.00	26	0.038
106	A	1	1	1.00	31	0.032
107	A	1	1	1.00	24	0.042
108	A	1	1	1.00	29	0.034
109	A	1	1	1.00	18	0.056
110	A	1	1	1.00	23	0.043
111	A	1	1	1.00	24	0.042
112	A	1	1	1.00	29	0.034
113	A	1	1	1.00	27	0.037

Chapter 3

Listing of integrals

3.1 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

fricas [B] time = 0.43, size = 48, normalized size = 4.00

$$\frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

giac [B] time = 0.94, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$-\frac{\left(\cot^3(x)\right)}{3} + x + \cot(x) - \frac{\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+x+cot(x)-1/2*Pi
```

maxima [A] time = 1.28, size = 16, normalized size = 1.33

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

mupad [B] time = 0.00, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] x + cot(x) - cot(x)^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

```
[Out] -1/3/x^3+1/x+arctan(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {325, 203}

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1+x^2\right)}\,dx &= -\frac{1}{3x^3}-\int \frac{1}{x^2\left(1+x^2\right)}\,dx\\ &= -\frac{1}{3x^3}+\frac{1}{x}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/3*1/x^3 + x^(-1) + ArcTan[x]
```

fricas [A] time = 0.42, size = 19, normalized size = 1.46

$$\frac{3\,x^3\arctan(x)+3\,x^2-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="fricas")
[Out] 1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

giac [A] time = 0.80, size = 15, normalized size = 1.15

$$\frac{3\,x^2-1}{3\,x^3}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="giac")
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\arctan(x)+\frac{1}{x}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^2+1),x)
[Out] -1/3/x^3+1/x+arctan(x)
```

maxima [A] time = 1.29, size = 15, normalized size = 1.15

$$\frac{3\,x^2-1}{3\,x^3}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

mupad [B] time = 0.16, size = 12, normalized size = 0.92

$$\mathrm{atan}(x)+\frac{x^2-\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^2 + 1)),x)
[Out] atan(x) + (x^2 - 1/3)/x^3
```

sympy [A] time = 0.11, size = 14, normalized size = 1.08

$$\mathrm{atan}\,(x)+\frac{3x^2-1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**2+1),x)
[Out] atan(x) + (3*x**2 - 1)/(3*x**3)
```

3.3 $\int \frac{x+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

```
[Out] 2/3*x^(3/2)+2/5*x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{\sqrt{x}} \, dx &= \int \left(\sqrt{x}+x^{3/2}\right) \, dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2}{15}x^{3/2}(3x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(5 + 3*x))/15
```

fricas [A] time = 0.41, size = 14, normalized size = 0.74

$$\frac{2}{15}\left(3x^2+5x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 5*x)*sqrt(x)
```

giac [A] time = 0.94, size = 11, normalized size = 0.58

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$\frac{2 (3x + 5) x^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)*(3*x+5)
```

maxima [A] time = 0.50, size = 11, normalized size = 0.58

$$\frac{2}{5} x^{\frac{5}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.53

$$\frac{2\, x^{3/2}\, (3\, x + 5)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)/x^(1/2),x)
```

```
[Out] (2*x^(3/2)*(3*x + 5))/15
```

sympy [A] time = 0.22, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

3.4 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] sin(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

fricas [A] time = 0.43, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

giac [A] time = 1.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

maxima [A] time = 0.51, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

3.5 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

fricas [A] time = 0.42, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*x,x, algorithm="fricas")

[Out] 1/2*e^(x^2)

giac [A] time = 1.06, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
[Out] 1/2*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
[Out] 1/2*exp(x^2)
```

maxima [A] time = 0.58, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.67

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
[Out] exp(x^2)/2
```

sympy [A] time = 0.08, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
[Out] exp(x**2)/2
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right) \\ &= \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

fricas [A] time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

giac [A] time = 1.04, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
[Out] 1/2/cos(x)^2
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\frac{\left(\sec^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x)
[Out] 1/2*sec(x)^2
```

maxima [A] time = 0.55, size = 6, normalized size = 0.75

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
[Out] 1/2*tan(x)^2
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$\frac{\tan(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^2,x)
[Out] tan(x)^2/2
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
[Out] 1/(2*cos(x)**2)
```

3.7 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

[Out] 1/3*(x^2+1)^(3/2)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3

fricas [A] time = 0.40, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/3*(x^2 + 1)^(3/2)

giac [A] time = 1.04, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 1)^(3/2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*x,x)
[Out] 1/3*(x^2+1)^(3/2)
```

maxima [A] time = 0.53, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(3/2)/3
```

sympy [B] time = 0.19, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.8 $\int e^x \sin(x)\, dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x)\, dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

fricas [A] time = 0.42, size = 13, normalized size = 0.68

$$-\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

giac [A] time = 1.05, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{\cos(x)e^{x}}{2}+\frac{e^{x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -1/2*cos(x)*exp(x)+1/2*exp(x)*sin(x)
```

maxima [A] time = 0.57, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos(x)-\sin(x)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

mupad [B] time = 0.02, size = 11, normalized size = 0.58

$$-\frac{e^{x}\left(\cos(x)-\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

sympy [A] time = 0.30, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\csc^3(x)$$

[Out] -1/3*csc(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Csc[x]^3,x]

[Out] -Csc[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^3(x)\,dx &= -\text{Subst}\left(\int x^2\,dx, x, \csc(x)\right)\\ &= -\frac{1}{3}\csc^3(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]*Csc[x]^3,x]

[Out] -1/3*Csc[x]^3

fricas [B] time = 0.42, size = 14, normalized size = 1.75

$$\frac{1}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")

[Out] 1/3/((cos(x)^2 - 1)*sin(x))

giac [**A**] time = 1.19, size = 6, normalized size = 0.75

$$-\frac{1}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
[Out] -1/3/sin(x)^3
```

maple [**A**] time = 0.03, size = 7, normalized size = 0.88

$$-\frac{1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^2/sin(x)^2,x)
[Out] -1/3/sin(x)^3
```

maxima [**A**] time = 0.52, size = 6, normalized size = 0.75

$$-\frac{1}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/3/sin(x)^3
```

mupad [**B**] time = 0.05, size = 6, normalized size = 0.75

$$-\frac{1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^4,x)
[Out] -1/(3*sin(x)^3)
```

sympy [**A**] time = 0.07, size = 8, normalized size = 1.00

$$-\frac{1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)**2/sin(x)**2,x)
[Out] -1/(3*sin(x)**3)
```

3.10 $\int \sin(e^x)\,dx$

Optimal. Leaf size=4

$$\mathrm{Si}(e^x)$$

```
[Out] Si(exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 3299}

$$\mathrm{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(e^x)\,dx &= \mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, e^x\right)\\ &= \mathrm{Si}(e^x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\mathrm{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

fricas [A] time = 0.43, size = 3, normalized size = 0.75

$$\mathrm{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="fricas")
```

```
[Out] sin_integral(e^x)
```

giac [A] time = 1.00, size = 3, normalized size = 0.75

$$\mathrm{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="giac")
[Out] sin_integral(e^x)
```

maple [A] time = 0.01, size = 4, normalized size = 1.00

$$\mathrm{Si}\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x)
[Out] Si(exp(x))
```

maxima [C] time = 0.83, size = 15, normalized size = 3.75

$$-\frac{1}{2}i\,\mathrm{Ei}\left(i\,e^{x}\right) + \frac{1}{2}i\,\mathrm{Ei}\left(-i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="maxima")
[Out] -1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.25

$$\mathrm{sinint}\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x)
[Out] sinint(exp(x))
```

sympy [A] time = 0.63, size = 3, normalized size = 0.75

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x)
[Out] Si(exp(x))
```

3.11 $\int \frac{\sin(y)}{y} \, dy$

Optimal. Leaf size=2

$$\mathrm{Si}(y)$$

[Out] Si(y)

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3299}

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

[In] Int[Sin[y]/y,y]

[Out] SinIntegral[y]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(y)}{y} \, dy = \mathrm{Si}(y)$$

Mathematica [A] time = 0.02, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[y]/y,y]

[Out] SinIntegral[y]

fricas [A] time = 0.42, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y, algorithm="fricas")

[Out] sin_integral(y)

giac [A] time = 0.98, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y, algorithm="giac")

[Out] sin_integral(y)

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)/y,y)
```

```
[Out] Si(y)
```

maxima [C] time = 0.70, size = 13, normalized size = 6.50

$$-\frac{1}{2} i \operatorname{Ei}\left(i y\right) + \frac{1}{2} i \operatorname{Ei}\left(-i y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\operatorname{sinint}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)/y,y)
```

```
[Out] sinint(y)
```

sympy [A] time = 0.60, size = 2, normalized size = 1.00

$$\operatorname{Si}{\left(y \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y)
```

```
[Out] Si(y)
```

3.12 $\int (e^x + \sin(x)) \, dx$

Optimal. Leaf size=8

$$e^x - \cos(x)$$

[Out] exp(x)-cos(x)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194, 2638}

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

[In] Int[E^x + Sin[x],x]

[Out] E^x - Cos[x]

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (e^x + \sin(x)) \, dx &= \int e^x \, dx + \int \sin(x) \, dx \\ &= e^x - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x + Sin[x],x]

[Out] E^x - Cos[x]

fricas [A] time = 0.43, size = 7, normalized size = 0.88

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)+sin(x),x, algorithm="fricas")

[Out] -cos(x) + e^x

giac [A] time = 1.08, size = 7, normalized size = 0.88

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="giac")
[Out] -cos(x) + e^x
```

maple [A] time = 0.00, size = 8, normalized size = 1.00

$$-\cos(x) + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)+sin(x),x)
[Out] exp(x)-cos(x)
```

maxima [A] time = 0.48, size = 7, normalized size = 0.88

$$-\cos(x) + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="maxima")
[Out] -cos(x) + e^x
```

mupad [B] time = 0.04, size = 7, normalized size = 0.88

$$e^{x} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x) + sin(x),x)
[Out] exp(x) - cos(x)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.62

$$e^{x} - \cos{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x)
[Out] exp(x) - cos(x)
```

3.13 $\int \left(e^{x^2}+2 e^{x^2} x^2\right) \, dx$

Optimal. Leaf size=7

$$e^{x^2} x$$

```
[Out] exp(x^2)*x
```

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2204, 2212}

$$e^{x^2} x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int \left(e^{x^2}+2 e^{x^2} x^2\right) \, dx &= 2 \int e^{x^2} x^2 \, dx+\int e^{x^2} \, dx\\
&= e^{x^2} x+\frac{1}{2} \sqrt{\pi } \text{erfi}(x)-\int e^{x^2} \, dx\\
&= e^{x^2} x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$e^{x^2} x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

fricas [A] time = 0.41, size = 6, normalized size = 0.86

$$x e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

giac [A] time = 0.88, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
[Out] x*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 1.00

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)+2*exp(x^2)*x^2,x)
[Out] x*exp(x^2)
```

maxima [A] time = 0.47, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")
[Out] x*e^(x^2)
```

mupad [B] time = 0.15, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2) + 2*x^2*exp(x^2),x)
[Out] x*exp(x^2)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)+2*exp(x**2)*x**2,x)
[Out] x*exp(x**2)
```

3.14 $\int (e^x + x)^2 \, dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

```
[Out] -2*exp(x)+1/2*exp(2*x)+2*exp(x)*x+1/3*x^3
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 2194, 2176}

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)^2,x]
```

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int (e^x + x)^2 \, dx &= \int \left(e^{2x} + 2e^x x + x^2\right) dx \\
&= \frac{x^3}{3} + 2\int e^x x \, dx + \int e^{2x} \, dx \\
&= \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} - 2\int e^x \, dx \\
&= -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.93

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + e^x(2x - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)^2,x]
```

```
[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

fricas [A] time = 0.41, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

giac [A] time = 0.93, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{x^3}{3} + 2x\,\mathrm{e}^x - 2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+exp(x))^2,x)
[Out] 1/3*x^3+1/2*exp(x)^2+2*exp(x)*x-2*exp(x)
```

maxima [A] time = 0.48, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.75

$$\frac{\mathrm{e}^{2\,x}}{2} - 2\,\mathrm{e}^x + 2\,x\,\mathrm{e}^x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + exp(x))^2,x)
[Out] exp(2*x)/2 - 2*exp(x) + 2*x*exp(x) + x^3/3
```

sympy [A] time = 0.09, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)\,e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))**2,x)
[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

3.15 $\int \left(2e^x + e^{2x} + x^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

```
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2194}

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int \left(2e^x + e^{2x} + x^2\right) dx &= \frac{x^3}{3} + 2\int e^x\, dx + \int e^{2x}\, dx \\ &= 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

fricas [A] time = 0.42, size = 16, normalized size = 0.73

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

giac [A] time = 1.08, size = 16, normalized size = 0.73

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + 2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)+exp(2*x)+x^2,x)
```

```
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

maxima [A] time = 0.52, size = 16, normalized size = 0.73

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

mupad [B] time = 0.05, size = 16, normalized size = 0.73

$$\frac{\mathrm{e}^{2\,x}}{2} + 2\,\mathrm{e}^x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x) + 2*exp(x) + x^2,x)
```

```
[Out] exp(2*x)/2 + 2*exp(x) + x^3/3
```

sympy [A] time = 0.09, size = 15, normalized size = 0.68

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x**2,x)
```

```
[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

3.16 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] 1/2*sin(x)^2

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x],x]

[Out] -1/2*Cos[x]^2

fricas [A] time = 0.44, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)^2

giac [**A**] time = 1.08, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

maple [**A**] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sin^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] 1/2*sin(x)^2
```

maxima [**A**] time = 0.60, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

mupad [**B**] time = 0.02, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

sympy [**A**] time = 0.06, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

3.17 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

fricas [A] time = 0.41, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*x,x, algorithm="fricas")

[Out] 1/2*e^(x^2)

giac [A] time = 0.91, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
```

```
[Out] 1/2*exp(x^2)
```

maxima [A] time = 0.68, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

mupad [B] time = 0.00, size = 6, normalized size = 0.67

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
```

```
[Out] exp(x^2)/2
```

sympy [A] time = 0.08, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

3.18 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

[Out] 1/3*(x^2+1)^(3/2)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3

fricas [A] time = 0.41, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/3*(x^2 + 1)^(3/2)

giac [A] time = 1.10, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

maple [**A**] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*x,x)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

maxima [**A**] time = 0.51, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

mupad [**B**] time = 0.00, size = 9, normalized size = 0.69

$$\frac{\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
```

```
[Out] (x^2 + 1)^(3/2)/3
```

sympy [**B**] time = 0.19, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.19 $\int \frac{e^x}{1+e^x} dx$

Optimal. Leaf size=6

$$\log(e^x + 1)$$

```
[Out] ln(1+exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2246, 31}

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+e^x} dx &= \text{Subst}\left(\int \frac{1}{1+x} dx, x, e^x\right) \\ &= \log(1+e^x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

fricas [A] time = 0.41, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

giac [A] time = 0.91, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
[Out] log(e^x + 1)
```

maple [A] time = 0.00, size = 6, normalized size = 1.00

$$\ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x)+1),x)
[Out] ln(exp(x)+1)
```

maxima [A] time = 0.56, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
[Out] log(e^x + 1)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.83

$$\ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x) + 1),x)
[Out] log(exp(x) + 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
[Out] log(exp(x) + 1)
```

3.20 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5

fricas [A] time = 0.41, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2),x, algorithm="fricas")

[Out] 2/5*x^(5/2)

giac [A] time = 0.95, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2),x, algorithm="giac")

```
[Out] 2/5*x^(5/2)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] 2/5*x^(5/2)
```

maxima [A] time = 0.51, size = 5, normalized size = 0.56

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

mupad [B] time = 0.04, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] (2*x^(5/2))/5
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

3.21 $\int \cos(3+2x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin(2x+3)$$

```
[Out] 1/2*sin(3+2*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

fricas [A] time = 0.43, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sin(2*x + 3)
```

giac [A] time = 1.04, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*x + 3)
```

maple [A] time = 0.02, size = 9, normalized size = 0.90

$$\frac{\sin(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x+3),x)
[Out] 1/2*sin(2*x+3)
```

maxima [A] time = 0.52, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="maxima")
[Out] 1/2*sin(2*x + 3)
```

mupad [B] time = 0.07, size = 8, normalized size = 0.80

$$\frac{\sin(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x + 3),x)
[Out] sin(2*x + 3)/2
```

sympy [A] time = 0.12, size = 7, normalized size = 0.70

$$\frac{\sin(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x)
[Out] sin(2*x + 3)/2
```

3.22 $\int 2e^{2x}yz\,dx$

Optimal. Leaf size=8

$$e^{2x}yz$$

```
[Out] exp(2*x)*y*z
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 2194}

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In] Int[2*E^(2*x)*y*z,x]
```

```
[Out] E^(2*x)*y*z
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int 2e^{2x}yz\,dx &= (2yz)\int e^{2x}\,dx \\ &= e^{2x}yz \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In] Integrate[2*E^(2*x)*y*z,x]
```

```
[Out] E^(2*x)*y*z
```

fricas [A] time = 0.41, size = 7, normalized size = 0.88

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="fricas")
```

```
[Out] y*z*e^(2*x)
```

giac [A] time = 1.10, size = 7, normalized size = 0.88

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="giac")
```

```
[Out] y*z*e^(2*x)
```

maple [A] time = 0.00, size = 8, normalized size = 1.00

$$yz\,\mathrm{e}^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(2*x)*y*z,x)
```

```
[Out] exp(2*x)*y*z
```

maxima [A] time = 0.48, size = 7, normalized size = 0.88

$$yze^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="maxima")
```

```
[Out] y*z*e^(2*x)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.88

$$y\,z\,\mathrm{e}^{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*y*z*exp(2*x),x)
```

```
[Out] y*z*exp(2*x)
```

sympy [A] time = 0.06, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x)
```

```
[Out] y*z*exp(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{3} \cos^3(e^x)$$

```
[Out] -1/3*cos(exp(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 2565, 30}

$$-\frac{1}{3} \cos^3(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -Cos[E^x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int e^x \cos^2(e^x) \sin(e^x)\, dx &= \text{Subst}\left(\int \cos^2(x)\sin(x)\, dx, x, e^x\right)\\
&= -\text{Subst}\left(\int x^2\, dx, x, \cos(e^x)\right)\\
&= -\frac{1}{3}\cos^3(e^x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.90

$$-\frac{1}{4} \cos(e^x) - \frac{1}{12} \cos(3e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -1/4*Cos[E^x] - Cos[3*E^x]/12
```

fricas [A] time = 0.44, size = 7, normalized size = 0.70

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")
[Out] -1/3*cos(e^x)^3
```

giac [A] time = 1.19, size = 7, normalized size = 0.70

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")
[Out] -1/3*cos(e^x)^3
```

maple [A] time = 0.01, size = 8, normalized size = 0.80

$$-\frac{\left(\cos^{3}\left(\mathrm{e}^{x}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x)
[Out] -1/3*cos(exp(x))^3
```

maxima [A] time = 0.45, size = 7, normalized size = 0.70

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")
[Out] -1/3*cos(e^x)^3
```

mupad [B] time = 0.19, size = 7, normalized size = 0.70

$$-\frac{\cos\left(\mathrm{e}^{x}\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(x))^2*sin(exp(x))*exp(x),x)
[Out] -cos(exp(x))^3/3
```

sympy [A] time = 1.96, size = 8, normalized size = 0.80

$$-\frac{\cos^{3}\left(e^{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
[Out] -cos(exp(x))**3/3
```

3.24 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.70

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

fricas [A] time = 0.41, size = 15, normalized size = 0.65

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

giac [A] time = 0.96, size = 15, normalized size = 0.65

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$\frac{2\,(x+1)^{\frac{3}{2}}\,(3x-2)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(1/2),x)
[Out] 2/15*(x+1)^(3/2)*(3*x-2)
```

maxima [A] time = 0.55, size = 15, normalized size = 0.65

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.52

$$\frac{2\,\left(3\,x-2\right)\,{\left(x+1\right)}^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2),x)
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

sympy [A] time = 0.93, size = 34, normalized size = 1.48

$$\frac{2x^{2}\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

3.25 $\int \frac{1}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

fricas [A] time = 0.40, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

giac [B] time = 1.03, size = 19, normalized size = 1.46

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{\operatorname{arctanh}(x)}{2} - \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x)
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

maxima [A] time = 1.41, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 0.16, size = 9, normalized size = 0.69

$$-\frac{\operatorname{atan}(x)}{2} - \frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 1),x)
[Out] - atan(x)/2 - atanh(x)/2
```

sympy [A] time = 0.12, size = 17, normalized size = 1.31

$$\frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2249, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{2+3e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

fricas [**A**] time = 0.42, size = 13, normalized size = 0.72

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

giac [**A**] time = 1.11, size = 13, normalized size = 0.72

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

maple [**A**] time = 0.01, size = 14, normalized size = 0.78

$$\frac{\sqrt{6}\,\arctan\left(\frac{\sqrt{6}\,\mathrm{e}^{x}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(2+3*exp(2*x)),x)
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

maxima [**A**] time = 1.33, size = 13, normalized size = 0.72

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

mupad [**B**] time = 0.11, size = 13, normalized size = 0.72

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^{x}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(3*exp(2*x) + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/6
```

sympy [**A**] time = 0.11, size = 15, normalized size = 0.83

$$\mathrm{RootSum}\left(24z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x)
[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[Out] 1/2*arctan(exp(2*x)*B^(1/2)/A^(1/2))/A^(1/2)/B^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2x}}{A+Be^{4x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{A+Bx^2}\,dx, x, e^{2x}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

fricas [**A**] time = 0.44, size = 76, normalized size = 2.45

$$\left[-\frac{\sqrt{-AB}\log\left(\frac{Be^{(4x)}-2\sqrt{-AB}e^{(2x)}-A}{Be^{(4x)}+A}\right)}{4AB}, -\frac{\sqrt{AB}\arctan\left(\frac{\sqrt{AB}e^{(-2x)}}{B}\right)}{2AB}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A)
)/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]
```

giac [**A**] time = 0.94, size = 19, normalized size = 0.61

$$\frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

maple [**A**] time = 0.02, size = 20, normalized size = 0.65

$$\frac{\arctan\left(\frac{B\,e^{2x}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

maxima [**A**] time = 1.33, size = 19, normalized size = 0.61

$$\frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

mupad [**B**] time = 0.23, size = 19, normalized size = 0.61

$$\frac{\operatorname{atan}\left(\frac{B\,e^{2\,x}}{\sqrt{A\,B}}\right)}{2\,\sqrt{A\,B}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(A + B*exp(4*x)),x)
```

```
[Out] atan((B*exp(2*x))/(A*B)^(1/2))/(2*(A*B)^(1/2))
```

sympy [**A**] time = 0.16, size = 22, normalized size = 0.71

$$\operatorname{RootSum}\left(16z^2AB+1, \left(i \mapsto i\log\left(4iA+e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

3.28 $\int \frac{e^{1+x}}{1+e^x} \, dx$

Optimal. Leaf size=8

$$e \log(e^x + 1)$$

```
[Out] E*ln(1+exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2247, 2246, 31}

$$e \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 2247

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Dist[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x
], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log
[F], g*h*m*Log[G]]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{1+x}}{1+e^x} \, dx &= e \int \frac{e^x}{1+e^x} \, dx \\ &= e \operatorname{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\ &= e \log(1+e^x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$e \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

fricas [**A**] time = 0.43, size = 11, normalized size = 1.38

$$e \log \left(e + e^{(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")
[Out] e*log(e + e^(x + 1))
```

giac [**A**] time = 1.10, size = 8, normalized size = 1.00

$$e \log \left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
[Out] e*log(e^x + 1)
```

maple [**A**] time = 0.01, size = 9, normalized size = 1.12

$$\mathrm{e} \ln \left(\mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x+1)/(exp(x)+1),x)
[Out] exp(1)*ln(exp(x)+1)
```

maxima [**A**] time = 0.56, size = 8, normalized size = 1.00

$$e \log \left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")
[Out] e*log(e^x + 1)
```

mupad [**B**] time = 0.18, size = 8, normalized size = 1.00

$$\mathrm{e}\;\ln \left(\mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x + 1)/(exp(x) + 1),x)
[Out] exp(1)*log(exp(x) + 1)
```

sympy [**A**] time = 0.09, size = 8, normalized size = 1.00

$$e \log \left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x)
[Out] E*log(exp(x) + 1)
```

3.29 $\int (10e)^x \, dx$

Optimal. Leaf size=12

$$\frac{(10e)^x}{1+\log(10)}$$

```
[Out] (10*E)^x/(1+ln(10))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194}

$$\frac{(10e)^x}{1+\log(10)}$$

Antiderivative was successfully verified.

```
[In] Int[(10*E)^x,x]
```

```
[Out] (10*E)^x/(1 + Log[10])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int (10e)^x \, dx = \frac{(10e)^x}{1+\log(10)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{(10e)^x}{\log(10e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10*E)^x,x]
```

```
[Out] (10*E)^x/Log[10*E]
```

fricas [A] time = 0.41, size = 12, normalized size = 1.00

$$\frac{(10\,E)^x}{\log\,(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="fricas")
```

```
[Out] (10*E)^x/log(10*E)
```

giac [A] time = 0.96, size = 12, normalized size = 1.00

$$\frac{(10\,E)^x}{\log\,(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="giac")
```

```
[Out] (10*E)^x/log(10*E)
```

maple [**A**] time = 0.02, size = 13, normalized size = 1.08

$$\frac{(10E)^x}{\ln(10E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*E)^x,x)
```

```
[Out] 1/ln(10*E)*(10*E)^x
```

maxima [**A**] time = 0.47, size = 12, normalized size = 1.00

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="maxima")
```

```
[Out] (10*E)^x/log(10*E)
```

mupad [**B**] time = 0.07, size = 12, normalized size = 1.00

$$\frac{10^x\,\mathrm{e}^x}{\ln(10)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*exp(1))^x,x)
```

```
[Out] (10^x*exp(x))/(log(10) + 1)
```

sympy [**A**] time = 0.09, size = 10, normalized size = 0.83

$$\frac{(10e)^x}{1+\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)**x,x)
```

```
[Out] (10*E)**x/(1 + log(10))
```

3.30 $\int x^3 \sin \left(x^2\right) \, dx$

Optimal. Leaf size=20

$$\frac{\sin \left(x^2\right)}{2}-\frac{1}{2} x^2 \cos \left(x^2\right)$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin \left(x^2\right)}{2}-\frac{1}{2} x^2 \cos \left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x^3 \sin \left(x^2\right) \, dx &= \frac{1}{2} \text{Subst}\left(\int x \sin (x) \, dx,x,x^2\right)\\ &= -\frac{1}{2} x^2 \cos \left(x^2\right)+\frac{1}{2} \text{Subst}\left(\int \cos (x) \, dx,x,x^2\right)\\ &= -\frac{1}{2} x^2 \cos \left(x^2\right)+\frac{\sin \left(x^2\right)}{2}\\ \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\sin \left(x^2\right)}{2}-\frac{1}{2} x^2 \cos \left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

fricas [A] time = 0.44, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

giac [A] time = 0.89, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

maxima [A] time = 0.64, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

mupad [B] time = 0.21, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

sympy [A] time = 0.60, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

3.31 $\int \frac{x^7}{1+x^{12}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right) + \frac{1}{12}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\log\left(1-x^4+x^8\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [B] time = 0.12, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}\,x+1\right)-2\log\left(x^2+\sqrt{2}\,x+1\right)+\log\left(2x^2-\sqrt{6}\,x+\sqrt{2}\,x+2\right)+\log\left(2x^2+\sqrt{2}\,\left(\sqrt{3}\,-1\right)x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

fricas [A] time = 0.42, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\,\log\left(x^8-x^4+1\right)-\frac{1}{12}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

giac [A] time = 0.89, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\,\log\left(x^8-x^4+1\right)-\frac{1}{12}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

maple [A] time = 0.00, size = 41, normalized size = 0.84

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2
))
```

maxima [A] time = 1.16, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

mupad [B] time = 0.14, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^4+1\right)}{12}-\ln\left(x^4-\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\ 1\mathrm{i}}{24}\right)+\ln\left(x^4+\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\ 1\mathrm{i}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12 + 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

sympy [A] time = 0.17, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

3.32 $\int x^{3a}\sin\left(x^{2a}\right)dx$

Optimal. Leaf size=115

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

```
[Out] 1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,-I*x^(2*a))/a/((-I*x^(2*a))^(1/2*(1+3*a)/a)
)-1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,I*x^(2*a))/a/((I*x^(2*a))^(1/2*(1+3*a)/a)
)
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3423, 2218}

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^(
(1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/(a
*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x^{3a}\sin\left(x^{2a}\right)dx&=\frac{1}{2}i\int e^{-ix^{2a}}x^{3a}\,dx-\frac{1}{2}i\int e^{ix^{2a}}x^{3a}\,dx\\&=\frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a}-\frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.30, size = 142, normalized size = 1.23

$$-\frac{x^{a+1}\left(x^{4a}\right)^{-\frac{a+1}{2a}}\left((a+1)\left(-ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},ix^{2a}\right)+(a+1)\left(ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},-ix^{2a}\right)+4a\left(x^{4a}\right)^{\frac{a+1}{2a}}\cos\right.}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] -1/8*(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)*(I*x^
(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] + (1 + a)*((-I)*x
^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^(2*a)]))/(a^2*(x^(4*a))^((
1 + a)/(2*a)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{3\,a} \sin\left(x^{2\,a}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
[Out] integral(x^(3*a)*sin(x^(2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3\,a} \sin\left(x^{2\,a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

maple [C] time = 0.16, size = 41, normalized size = 0.36

$$\frac{x^{5a+1}\,\text{hypergeom}\left(\left[\frac{1}{4a}+\frac{5}{4}\right],\left[\frac{3}{2},\frac{1}{4a}+\frac{9}{4}\right],-\frac{x^{4a}}{4}\right)}{5a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*a)*sin(x^(2*a)),x)
[Out] 1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^(4*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x x^{a} \cos\left(x^{2\,a}\right) - \frac{(a+1) x x^{a} \Gamma\left(\frac{1}{4\,a}+\frac{1}{4}\right) {}_1F_2\left(\begin{matrix} \frac{1}{4\,a}+\frac{1}{4} \\ \frac{1}{2}, \frac{1}{4\,a}+\frac{5}{4} \end{matrix}; -\frac{1}{4} x^{4\,a}\right)}{4\,a \Gamma\left(\frac{1}{4\,a}+\frac{5}{4}\right)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
[Out] -1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3\,a}\,\sin\left(x^{2\,a}\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*a)*sin(x^(2*a)),x)
[Out] int(x^(3*a)*sin(x^(2*a)), x)
```

sympy [A] time = 2.89, size = 54, normalized size = 0.47

$$\frac{xx^{5a}\Gamma\left(\frac{5}{4}+\frac{1}{4a}\right){}_1F_2\left(\begin{matrix}\frac{5}{4}+\frac{1}{4a}\\ \frac{3}{2},\frac{9}{4}+\frac{1}{4a}\end{matrix}\middle|-\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4}+\frac{1}{4a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] x*x**(5*a)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a),), (3/2, 9/4 + 1/(4*a)
), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

3.33 $\int \cos\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right) dx &= 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right)-2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right)+2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

fricas [A] time = 0.44, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

giac [A] time = 1.27, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

maple [A] time = 0.03, size = 17, normalized size = 0.77

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.51, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

mupad [B] time = 0.21, size = 16, normalized size = 0.73

$$2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

sympy [A] time = 0.31, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.34 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x],x]

[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15

fricas [A] time = 0.41, size = 15, normalized size = 0.65

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")

[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)

giac [A] time = 0.93, size = 15, normalized size = 0.65

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$\frac{2\left(x+1\right)^{\frac{3}{2}}\left(3x-2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(1/2),x)
```

```
[Out] 2/15*(x+1)^(3/2)*(3*x-2)
```

maxima [A] time = 0.47, size = 15, normalized size = 0.65

$$\frac{2}{5}\left(x+1\right)^{\frac{5}{2}}-\frac{2}{3}\left(x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

mupad [B] time = 0.00, size = 12, normalized size = 0.52

$$\frac{2\,\left(3\,x-2\right)\,{\left(x+1\right)}^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2),x)
```

```
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

sympy [A] time = 0.95, size = 34, normalized size = 1.48

$$\frac{2x^{2}\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx,x,\sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx,x,\sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

fricas [A] time = 0.42, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

giac [A] time = 0.92, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

maple [B] time = 0.00, size = 92, normalized size = 2.88

$$-\ln(x-1) - 2\ln\left(x^{\frac{1}{6}}+1\right) + \ln\left(\sqrt{x}-1\right) - \ln\left(\sqrt{x}+1\right) - 2\ln\left(x^{\frac{1}{3}}-1\right) + 2\ln\left(x^{\frac{1}{6}}-1\right) + \ln\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - \ln\left(x^{\frac{1}{3}} + x^{\frac{1}{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] -ln(x-1)-2*ln(x^(1/6)+1)+ln(x^(1/2)-1)-ln(x^(1/2)+1)-2*ln(x^(1/3)-1)+2*ln(x
^(1/6)-1)+ln(x^(1/3)-x^(1/6)+1)-ln(x^(1/3)+x^(1/6)+1)+ln(x^(2/3)+x^(1/3)+1)
+2*x^(1/2)-3*x^(1/3)+6*x^(1/6)
```

maxima [A] time = 0.51, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

mupad [B] time = 0.00, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln\left(x^{1/6} + 1\right) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\, dx$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arcsinh(2^(1/2)*(1+x)^(1/2))*2^(1/2)+1/2*(1+x)^(1/2)*(3+2*x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1958, 50, 54, 215}

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{3+2x}}\,dx &= \int \frac{\sqrt{1+x}}{\sqrt{3+2x}}\,dx\\ &= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x}-\frac{1}{4}\int \frac{1}{\sqrt{1+x}\,\sqrt{3+2x}}\,dx\\ &= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx,x,\sqrt{1+x}\right)\\ &= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,\sqrt{1+x}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.61

$$\frac{2(x+1)\sqrt{2x+3}-\sqrt{2}\,\sqrt{x+1}\,\sinh^{-1}\left(\sqrt{2}\,\sqrt{x+1}\right)}{4\sqrt{\frac{x+1}{2x+3}}\,\sqrt{2x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (2*(1 + x)*Sqrt[3 + 2*x] - Sqrt[2]*Sqrt[1 + x]*ArcSinh[Sqrt[2]*Sqrt[1 + x]]
)/(4*Sqrt[(1 + x)/(3 + 2*x)]*Sqrt[3 + 2*x])
```

fricas [A] time = 0.43, size = 55, normalized size = 1.25

$$\frac{1}{2}\,(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}}+\frac{1}{8}\,\sqrt{2}\,\log\left(2\,\sqrt{2}\,(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}}-4\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3)
*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)
```

giac [B] time = 1.06, size = 61, normalized size = 1.39

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2+5\,x+3}\right)-5\right|\right)\operatorname{sgn}\left(2\,x+3\right)+\frac{1}{2}\,\sqrt{2\,x^2+5\,x+3}\operatorname{sgn}\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*sg
n(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)
```

maple [B] time = 0.01, size = 76, normalized size = 1.73

$$\frac{\sqrt{\frac{x+1}{2x+3}}\,\left(2x+3\right)\left(-\sqrt{2}\,\ln\left(\sqrt{2}\,x+\frac{5\sqrt{2}}{4}+\sqrt{2x^2+5x+3}\right)+4\sqrt{2x^2+5x+3}\right)}{8\sqrt{\left(2x+3\right)\left(x+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/(2*x+3))^(1/2),x)
```

```
[Out] 1/8*((x+1)/(2*x+3))^(1/2)*(2*x+3)*(-ln(5/4*2^(1/2)+2^(1/2)*x+(2*x^2+5*x+3)^
(1/2))*2^(1/2)+4*(2*x^2+5*x+3)^(1/2))/((2*x+3)*(x+1))^(1/2)
```

maxima [B] time = 1.31, size = 80, normalized size = 1.82

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+1}{2x+3}}}{\sqrt{2}+2\sqrt{\frac{x+1}{2x+3}}}\right)-\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2(x+1)}{2x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((x
 + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1)
```

mupad [B] time = 0.21, size = 57, normalized size = 1.30

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sqrt{\frac{x+1}{2x+3}}\right)}{4}-\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2x+2}{2x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(2*x + 3))^(1/2),x)
```

```
[Out] - (2^(1/2)*atanh(2^(1/2)*((x + 1)/(2*x + 3))^(1/2)))/4 - ((x + 1)/(2*x + 3)
)^(1/2)/(2*((2*x + 2)/(2*x + 3) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)
```

3.37 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

fricas [B] time = 0.40, size = 63, normalized size = 1.80

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

giac [A] time = 0.93, size = 29, normalized size = 0.83

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}\,x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.86

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

maxima [A] time = 1.22, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

mupad [B] time = 0.15, size = 91, normalized size = 2.60

$$\mathrm{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

sympy [**B**] time = 2.14, size = 105, normalized size = 3.00

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3\sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x\sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.38 $\int \sqrt{x}(1+x)^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 5/24*x^(3/2)*(1+x)^(3/2)+1/4*x^(3/2)*(1+x)^(5/2)-5/64*arcsinh(x^(1/2))+5/32
*x^(3/2)*(1+x)^(1/2)+5/64*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x
)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(1+x)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \sqrt{x}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{16}\int \sqrt{x}\sqrt{1+x}\,dx\\
&= \frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{64}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{128}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{1+}}\right.\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.55

$$\frac{1}{192}\left(\sqrt{x}\sqrt{x+1}\left(48x^3+136x^2+118x+15\right)-15\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3) - 15*ArcSinh[Sqrt[x]])
/192
```

fricas [A] time = 0.41, size = 44, normalized size = 0.59

$$\frac{1}{192}\left(48\,x^3+136\,x^2+118\,x+15\right)\sqrt{x+1}\,\sqrt{x}\,+\frac{5}{128}\,\log\left(2\,\sqrt{x+1}\,\sqrt{x}\,-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sqr
t(x + 1)*sqrt(x) - 2*x - 1)
```

giac [A] time = 1.16, size = 90, normalized size = 1.20

$$\frac{1}{192}\left(2\left(4\left(6\,x-19\right)(x+1)+163\right)(x+1)-279\right)\sqrt{x+1}\,\sqrt{x}+\frac{1}{8}\left(2\left(4\,x-9\right)(x+1)+33\right)\sqrt{x+1}\,\sqrt{x}+\frac{3}{4}\left(2\,x-3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 19)*(x + 1) + 163)*(x + 1) - 279)*sqrt(x + 1)*sqrt(x) +
1/8*(2*(4*x - 9)*(x + 1) + 33)*sqrt(x + 1)*sqrt(x) + 3/4*(2*x - 3)*sqrt(x +
 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 5/64*log(sqrt(x + 1) - sqrt(x))
```

maple [A] time = 0.00, size = 70, normalized size = 0.93

$$\frac{(x+1)^{\frac{7}{2}}\sqrt{x}}{4}-\frac{(x+1)^{\frac{5}{2}}\sqrt{x}}{24}-\frac{5\,(x+1)^{\frac{3}{2}}\sqrt{x}}{96}-\frac{5\sqrt{x+1}\,\sqrt{x}}{64}-\frac{5\sqrt{(x+1)\,x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{128\sqrt{x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+1)^(5/2),x)
```

```
[Out] 1/4*x^(1/2)*(x+1)^(7/2)-1/24*x^(1/2)*(x+1)^(5/2)-5/96*x^(1/2)*(x+1)^(3/2)-5
/64*(x+1)^(1/2)*x^(1/2)-5/128*((x+1)*x)^(1/2)/(x+1)^(1/2)/x^(1/2)*ln(x+1/2+
(x^2+x)^(1/2))
```

maxima [B] time = 0.53, size = 113, normalized size = 1.51

$$\frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}}+\frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}}-\frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{15\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^4}{x^4}-\frac{4\,(x+1)^3}{x^3}+\frac{6\,(x+1)^2}{x^2}-\frac{4\,(x+1)}{x}+1\right)}-\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3/
2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6*(
```

```
x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5/12
8*log(sqrt(x + 1)/sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int(x^(1/2)*(x + 1)^(5/2), x)
```

sympy [A] time = 8.75, size = 190, normalized size = 2.53

$$\begin{cases} -\frac{5\operatorname{acosh}\left(\sqrt{x+1}\right)}{64} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}} - \frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}} - \frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}} - \frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}} + \frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{5i\operatorname{asin}\left(\sqrt{x+1}\right)}{64} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}} + \frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}} + \frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}} + \frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}} - \frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x + 1
)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(192
*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(x +
 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x))
+ I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*
sqrt(x + 1)/(64*sqrt(-x)), True))
```

3.39 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

fricas [B] time = 0.41, size = 63, normalized size = 1.80

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

giac [A] time = 1.07, size = 29, normalized size = 0.83

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}\,x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.86

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3/(-x^2+1)^(3/2)*x^3-1/(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A] time = 1.32, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

mupad [B] time = 0.00, size = 91, normalized size = 2.60

$$\mathrm{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

sympy [B] time = 2.15, size = 105, normalized size = 3.00

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$

Optimal. Leaf size=51

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))+A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1/
2))
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {402, 217, 203, 377, 206}

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
- B^2*y^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = A^2\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}\,dy + B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy$$

$$= A^2\,\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right) + B^2\,\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy, y, \frac{}{\sqrt{A^2}}\right.$$

$$= B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) + A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

Mathematica [C] time = 0.03, size = 134, normalized size = 2.63

$$iB\log\left(2\sqrt{A^2-B^2y^2+B^2}-2iBy\right)+\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right)-\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] -1/2*(A*Log[1 - y]) + (A*Log[1 + y])/2 + I*B*Log[(-2*I)*B*y + 2*Sqrt[A^2 +
B^2 - B^2*y^2]] + (A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/
2 - (A*Log[A^2 + B^2 + B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
```

fricas [B] time = 0.45, size = 129, normalized size = 2.53

$$-B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)+\frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y^2+2\sqrt{-B^2y^2+A^2+B^2}\,Ay+A^2+B^2}{y^2}\right)-\frac{1}{4}A\log\left(-\frac{(A}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2 +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 - B^
2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,
0]%%%},0,%%%{1,[0,4,4]%%%}] at parameters values [88,76,-66]Warning, choosi
ng root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,0]%%%},0,%
%%{1,[0,4,4]%%%}] at parameters values [66,5,-23]-B^2*(1/2*pi*sign(y)-atan(
B^2*y*((-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))/B^2/y)^2-
1)/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))))/abs(B)+1/2*A*B^2*
ln(abs(B*(-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))/B^2/y+2
*B^2*y/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B)))+2*A))/B/abs(B)
-1/2*A*B^2*ln(abs(B*(-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(
B))/B^2/y+2*B^2*y/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B)))-2*A
))/B/abs(B)
```

maple [B] time = 0.03, size = 262, normalized size = 5.14

$$-\frac{A^2\ln\left(\frac{2A^2+2(y+1)B^2+2\sqrt{A^2}\sqrt{A^2-(y+1)^2B^2+2(y+1)B^2}}{y+1}\right)}{2\sqrt{A^2}}+\frac{A^2\ln\left(\frac{2A^2-2(y-1)B^2+2\sqrt{A^2}\sqrt{A^2-(y-1)^2B^2-2(y-1)B^2}}{y-1}\right)}{2\sqrt{A^2}}+\frac{B^2\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y)
```

```
[Out] 1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(
1/2)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2+
2*B^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))-1/2*
(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)
*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2-2*B^
2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))
```

maxima [B] time = 1.46, size = 122, normalized size = 2.39

$$B \arcsin\left(\frac{B^2 y}{\sqrt{A^2 B^2 + B^4}}\right) - \frac{1}{2} A \log\left(B^2 + \frac{2 A^2}{|2 y + 2|} + \frac{2 \sqrt{-B^2 y^2 + A^2 + B^2} A}{|2 y + 2|}\right) + \frac{1}{2} A \log\left(-B^2 + \frac{2 A^2}{|2 y - 2|} + \frac{2 \sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 + 2*A^2/abs(2*y + 2) +
2*sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y + 2)) + 1/2*A*log(-B^2 + 2*A^2/abs(2
*y - 2) + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} -\int \frac{\sqrt{-B^2 y^2}}{y^2-1}\,dy & \text{if } A^2 + B^2 = 0 \\ -\ln\left(2 y \sqrt{-B^2} + 2 \sqrt{A^2 - B^2 y^2 + B^2}\right) \sqrt{-B^2} - \operatorname{atan}\left(\frac{y \sqrt{A^2}\, 1\mathrm{i}}{\sqrt{A^2 - B^2 y^2 + B^2}}\right) \sqrt{A^2}\, 1\mathrm{i} & \text{if } A^2 + B^2 \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2 + B^2 - B^2*y^2)^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, -int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
 0, - atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i -
 log(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{A^2 - B^2 y^2 + B^2}}{y^2 - 1}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

```
[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

3.41 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

[Out] 1/2*x-1/2*cos(x)*sin(x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4

fricas [A] time = 0.45, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="fricas")

[Out] -1/2*cos(x)*sin(x) + 1/2*x

giac [A] time = 0.94, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
[Out] -1/2*cos(x)*sin(x)+1/2*x
```

maxima [A] time = 0.60, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
[Out] x/2 - sin(x)*cos(x)/2
```

3.42 $\int \csc(x)\sqrt{A^2 + B^2 \sin^2(x)}\, dx$

Optimal. Leaf size=49

$$-B \tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2 + B^2 \sin^2(x)}}\right) - A \tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2 + B^2 \sin^2(x)}}\right)$$

```
[Out] -B*arctan(B*cos(x)/(A^2+B^2*sin(x)^2)^(1/2))-A*arctanh(A*cos(x)/(A^2+B^2*si
n(x)^2)^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3186, 402, 217, 203, 377, 206}

$$-B \tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2 - B^2 \cos^2(x) + B^2}}\right) - A \tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2 - B^2 \cos^2(x) + B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTanh[(A*Cos[x
])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt{A^2+B^2-B^2x^2}}{1-x^2}\,dx, x, \cos(x)\right)$$
$$= -\left(A^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{A^2+B^2-B^2x^2}}\,dx, x, \cos(x)\right)\right) - B^2\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$
$$= -\left(A^2\,\text{Subst}\left(\int \frac{1}{1-A^2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)\right) - B^2\,\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right) - A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)$$

Mathematica [B] time = 0.11, size = 99, normalized size = 2.02

$$\sqrt{-B^2}\log\left(\sqrt{2A^2-B^2\cos(2x)+B^2}+\sqrt{2}\sqrt{-B^2}\cos(x)\right)-\sqrt{A^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2-B^2\cos(2x)+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[2
*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^2*
Cos[2*x]]]
```

fricas [B] time = 0.55, size = 244, normalized size = 4.98

$$\frac{1}{2}B\arctan\left(-\frac{\left(A^4+2A^2B^2+B^4\right)\cos(x)\sin(x)-2\left(2B^3\cos(x)^3-\left(A^2B+B^3\right)\cos(x)\right)\sqrt{-B^2\cos(x)^2+A^2+}}{4B^4\cos(x)^4+A^4+2A^2B^2+B^4-\left(A^4+6A^2B^2+5B^4\right)\cos(x)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 -
(A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 + A^
4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arctan(s
in(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2 + s
qrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(
x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*c
os(x) - B*sin(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{B^2\sin(x)^2+A^2}}{\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

maple [C] time = 0.19, size = 149, normalized size = 3.04

$$-\frac{\sqrt{\left(B^2\left(\sin^2(x)\right)+A^2\right)\left(\cos^2(x)\right)}\left(A\,\mathrm{csgn}(A)\ln\left(-\frac{A^2\left(\sin^2(x)\right)-B^2\left(\sin^2(x)\right)-2A^2-2\sqrt{\left(B^2\left(\sin^2(x)\right)+A^2\right)\left(\cos^2(x)\right)}\,A\,\mathrm{csgn}(A)}{\sin(x)^2}\right)-B}{2\sqrt{B^2\left(\sin^2(x)\right)+A^2}\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x)
```

```
[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*s
in(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-B*
csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2)*c
os(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)
```

maxima [B] time = 1.36, size = 116, normalized size = 2.37

$$-B\arcsin\left(\frac{B^2\cos(x)}{\sqrt{A^2B^2+B^4}}\right)-\frac{1}{2}A\log\left(B^2-\frac{A^2}{\cos(x)-1}-\frac{\sqrt{-B^2\cos(x)^2+A^2+B^2}\,A}{\cos(x)-1}\right)+\frac{1}{2}A\log\left(-B^2+\frac{A^2}{\cos(x)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")
```

```
[Out] -B*arcsin(B^2*cos(x)/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 - A^2/(cos(x) - 1
) - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) - 1)) + 1/2*A*log(-B^2 + A^2/
(cos(x) + 1) + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{A^2+B^2\sin(x)^2}}{\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x),x)
```

```
[Out] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{A^2+B^2\sin^2(x)}}{\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

```
[Out] Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] sin(x)/(1+cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

fricas [A] time = 0.41, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

giac [B] time = 0.82, size = 30, normalized size = 3.33

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

maple [A] time = 0.02, size = 5, normalized size = 0.56

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+1),x)
[Out] tan(1/2*x)
```

maxima [A] time = 0.55, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
[Out] sin(x)/(cos(x) + 1)
```

mupad [B] time = 0.18, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1),x)
[Out] tan(x/2)
```

sympy [A] time = 0.19, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
[Out] tan(x/2)
```

3.44 $\int e^x x \, dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

[Out] -exp(x)+exp(x)*x

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2176, 2194}

$$e^x x - e^x$$

Antiderivative was successfully verified.

[In] Int[E^x*x,x]

[Out] -E^x + E^x*x

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x \, dx &= e^x x - \int e^x \, dx \\ &= -e^x + e^x x \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.64

$$e^x (x - 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^x*x,x]

[Out] E^x*(-1 + x)

fricas [A] time = 0.41, size = 6, normalized size = 0.55

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x,x, algorithm="fricas")

[Out] (x - 1)*e^x

giac [A] time = 1.09, size = 6, normalized size = 0.55

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

maple [A] time = 0.00, size = 7, normalized size = 0.64

$$(x-1)\,\mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x),x)
```

```
[Out] (x-1)*exp(x)
```

maxima [A] time = 0.57, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\mathrm{e}^{x}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x),x)
```

```
[Out] exp(x)*(x - 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

3.45 $\int \frac{e^x x}{(1+x)^2} dx$

Optimal. Leaf size=9

$$\frac{e^x}{x+1}$$

```
[Out] exp(x)/(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2197}

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*x)/(1 + x)^2,x]
[Out] E^x/(1 + x)
```

Rule 2197

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[(g*u^(m + 1)*F^(c*v))/(b*c*e*Lo
g[F]), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^x x}{(1+x)^2} dx = \frac{e^x}{1+x}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*x)/(1 + x)^2,x]
[Out] E^x/(1 + x)
```

fricas [A] time = 0.40, size = 8, normalized size = 0.89

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")
[Out] e^x/(x + 1)
```

giac [B] time = 0.92, size = 30, normalized size = 3.33

$$-\frac{e^{\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)}}{(x+1)\left(\frac{1}{x+1}-1\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")
```

```
[Out] -e^(-(x + 1)*(1/(x + 1) - 1))/((x + 1)*(1/(x + 1) - 1) - 1)
```

maple [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{\mathrm{e}^{x}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x/(x+1)^2,x)
```

```
[Out] exp(x)/(x+1)
```

maxima [A] time = 0.46, size = 8, normalized size = 0.89

$$\frac{e^{x}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] e^x/(x + 1)
```

mupad [B] time = 0.09, size = 8, normalized size = 0.89

$$\frac{\mathrm{e}^{x}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(x))/(x + 1)^2,x)
```

```
[Out] exp(x)/(x + 1)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.56

$$\frac{e^{x}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)**2,x)
```

```
[Out] exp(x)/(x + 1)
```

3.46 $\int e^{x^2}\left(1+2x^2\right)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2226, 2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2}\left(1+2x^2\right)\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx\\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)-\int e^{x^2}\,dx\\
&= e^{x^2}x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

fricas [A] time = 0.41, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

giac [A] time = 0.83, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 1.00

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x)
```

```
[Out] x*exp(x^2)
```

maxima [A] time = 0.48, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

mupad [B] time = 0.15, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
```

```
[Out] x*exp(x^2)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

3.47 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\ \text{Erfi}(x)$$

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2204}

$$\frac{1}{2}\sqrt{\pi}\,\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\sqrt{\pi}\ \text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

fricas [A] time = 0.42, size = 7, normalized size = 0.64

$$\frac{1}{2}\sqrt{\pi}\ \text{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

giac [C] time = 1.05, size = 9, normalized size = 0.82

$$\frac{1}{2}i\sqrt{\pi}\ \text{erf}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\ \mathrm{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

maxima [C] time = 0.54, size = 9, normalized size = 0.82

$$-\frac{1}{2} i \sqrt{\pi}\, \mathrm{erf}\left(i\, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.64

$$\frac{\sqrt{\pi}\ \mathrm{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] (pi^(1/2)*erfi(x))/2
```

sympy [A] time = 0.20, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\, \mathrm{erfi}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

3.48 $\int \frac{e^x}{x} \, dx$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(x)$$

```
[Out] Ei(x)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2178}

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^x}{x} \, dx = \text{Ei}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

fricas [A] time = 0.40, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="fricas")
```

```
[Out] Ei(x)
```

giac [A] time = 1.01, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="giac")
```

```
[Out] Ei(x)
```

maple [**B**]   time = 0.00, size = 8, normalized size = 4.00

$$-\operatorname{Ei}(1,-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/x,x)
```

```
[Out] -Ei(1,-x)
```

maxima [**A**]   time = 0.71, size = 2, normalized size = 1.00

$$\operatorname{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="maxima")
```

```
[Out] Ei(x)
```

mupad [**B**]   time = 0.01, size = 2, normalized size = 1.00

$$\operatorname{ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/x,x)
```

```
[Out] ei(x)
```

sympy [**A**]   time = 0.71, size = 2, normalized size = 1.00

$$\operatorname{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x)
```

```
[Out] Ei(x)
```

3.49 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{1+x^3}\,dx = -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right) + \frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx$$
$$= -\frac{1}{3}\log(1+x) + \frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx$$
$$= -\frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$
$$= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

Mathematica [**A**] time = 0.01, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1) + \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

fricas [**A**] time = 0.41, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

giac [**A**] time = 0.89, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

maple [**A**] time = 0.00, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\ln\left(x+1\right)}{3} + \frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x)
[Out] 1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*ln(x+1)+1/6*ln(x^2-x+1)
```

maxima [A] time = 1.09, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

mupad [B] time = 0.11, size = 46, normalized size = 1.12

$$-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

sympy [A] time = 0.13, size = 41, normalized size = 1.00

$$-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

3.50 $\int \frac{1}{-1+x^6}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}}-\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/3*arctanh(x)-1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1/
2)
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x}\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\right.\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.60

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x\ldots}{\sqrt{\ldots}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

fricas [A] time = 0.43, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

giac [A] time = 1.01, size = 67, normalized size = 1.43

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 1.40

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x-1\right)}{6}-\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x)
```

```
[Out] -1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))+1/6*ln(x-1)-1/6*ln(x+1)+1/12*ln(x^2-x+1)-1/12*ln(x^2+x+1)
```

maxima [A] time = 1.28, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

mupad [B] time = 0.09, size = 88, normalized size = 1.87

$$-\frac{\operatorname{atanh}(x)}{3}-\operatorname{atan}\left(\frac{x\,1i}{1+\sqrt{3}\,1i}+\frac{\sqrt{3}\,x}{1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}i\right)-\operatorname{atan}\left(\frac{x\,1i}{-1+\sqrt{3}\,1i}-\frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 1),x)
```

```
[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))
*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1
i - 1))*(3^(1/2)/6 - 1i/6)
```

sympy [B] time = 0.25, size = 83, normalized size = 1.77

$$\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

3.51 $\int \frac{1}{A^4-A^2B^2+\left(-A^2+B^2\right)x^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

```
[Out] arctanh(x/A)/A/(A^2-B^2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{A^4-A^2B^2+\left(-A^2+B^2\right)x^2}\,dx=\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

fricas [A] time = 0.40, size = 27, normalized size = 1.29

$$\frac{\log\left(A+x\right)-\log\left(-A+x\right)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

giac [A] time = 1.14, size = 41, normalized size = 1.95

$$\frac{\log\left(|A+x|\right)}{2\left(A^3-AB^2\right)}-\frac{\log\left(|-A+x|\right)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

maple [B] time = 0.01, size = 44, normalized size = 2.10

$$-\frac{\ln \left(A-x\right)}{2 \left(A^{2}-B^{2}\right) A}+\frac{\ln \left(A+x\right)}{2 \left(A^{2}-B^{2}\right) A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x)
```

```
[Out] 1/2/(A^2-B^2)/A*ln(A+x)-1/2/(A^2-B^2)/A*ln(A-x)
```

maxima [A] time = 0.51, size = 39, normalized size = 1.86

$$\frac{\log \left(A+x\right)}{2 \left(A^{3}-AB^{2}\right)}-\frac{\log \left(-A+x\right)}{2 \left(A^{3}-AB^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

mupad [B] time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\operatorname{atanh}\left(\frac{x}{A}\right)}{A\,B^{2}-A^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(A^2 - B^2) - A^4 + A^2*B^2),x)
```

```
[Out] -atanh(x/A)/(A*B^2 - A^3)
```

sympy [B] time = 0.30, size = 70, normalized size = 3.33

$$-\frac{\log \left(-\frac{A^{3}}{(A-B)(A+B)}+\frac{AB^{2}}{(A-B)(A+B)}+x\right)}{2A\left(A-B\right)\left(A+B\right)}+\frac{\log \left(\frac{A^{3}}{(A-B)(A+B)}-\frac{AB^{2}}{(A-B)(A+B)}+x\right)}{2A\left(A-B\right)\left(A+B\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

```
[Out] -log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A*(A - B)*(
A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*(A + B)) + x)/(2*A*(
A - B)*(A + B))
```

3.52 $\int x \log(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2304}

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

giac [A] time = 1.07, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x^2\ln(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
[Out] 1/2*x^2*ln(x)-1/4*x^2
```

maxima [A] time = 0.61, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

mupad [B] time = 0.04, size = 9, normalized size = 0.53

$$\frac{x^2\left(\ln(x) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^2\log{(x)}}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

3.53 $\int x^2 \sin^{-1}(x)\,dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

```
[Out] -1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4627, 266, 43}

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\sin^{-1}(x)\,dx &= \frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx,x,x^2\right)\\
&= \frac{\sqrt{1-x^2}}{3}-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(3x^3\sin^{-1}(x)+\sqrt{1-x^2}\left(x^2+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
```

```
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

fricas [A] time = 0.44, size = 24, normalized size = 0.60

$$\frac{1}{3}\,x^3\arcsin(x) + \frac{1}{9}\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

giac [A] time = 1.09, size = 38, normalized size = 0.95

$$\frac{1}{3}\left(x^2-1\right)x\arcsin(x) + \frac{1}{3}\,x\arcsin(x) - \frac{1}{9}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.85

$$\frac{x^3\arcsin(x)}{3} + \frac{\sqrt{-x^2+1}\,x^2}{9} + \frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x)
```

```
[Out] 1/3*x^3*arcsin(x)+1/9*(-x^2+1)^(1/2)*x^2+2/9*(-x^2+1)^(1/2)
```

maxima [A] time = 1.29, size = 33, normalized size = 0.82

$$\frac{1}{3}\,x^3\arcsin(x) + \frac{1}{9}\,\sqrt{-x^2+1}\,x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

mupad [B] time = 0.00, size = 24, normalized size = 0.60

$$\frac{x^3\,\mathrm{asin}(x)}{3} + \frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x),x)
```

```
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

sympy [A] time = 0.34, size = 32, normalized size = 0.80

$$\frac{x^3\,\mathrm{asin}\,(x)}{3} + \frac{x^2\sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

3.54 $\int \frac{1}{1+2x+x^2}\,dx$

Optimal. Leaf size=7

$$-\frac{1}{x+1}$$

[Out] -1/(1+x)

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {27, 32}

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2)^(-1),x]

[Out] -(1 + x)^(-1)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2x+x^2}\,dx &= \int \frac{1}{(1+x)^2}\,dx\\ &= -\frac{1}{1+x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + x^2)^(-1),x]

[Out] -(1 + x)^(-1)

fricas [A] time = 0.39, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+1),x, algorithm="fricas")

[Out] -1/(x + 1)

giac [A] time = 1.05, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/(x + 1)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+1),x)
```

```
[Out] -1/(x+1)
```

maxima [A] time = 0.53, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -1/(x + 1)
```

mupad [B] time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + x^2 + 1),x)
```

```
[Out] -1/(x + 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.71

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+1),x)
```

```
[Out] -1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2}\,dx$

Optimal. Leaf size=8

$$\frac{x}{\log(x)+1}$$

```
[Out] x/(1+ln(x))
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2360, 2297, 2299, 2178}

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(x)}{(1+\log(x))^2}\,dx &= \int \left(-\frac{1}{(1+\log(x))^2}+\frac{1}{1+\log(x)}\right)dx\\
&= -\int \frac{1}{(1+\log(x))^2}\,dx+\int \frac{1}{1+\log(x)}\,dx\\
&= \frac{x}{1+\log(x)}-\int \frac{1}{1+\log(x)}\,dx+\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right)\\
&= \frac{\text{Ei}(1+\log(x))}{e}+\frac{x}{1+\log(x)}-\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right)\\
&= \frac{x}{1+\log(x)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(1 + Log[x])^2,x]
[Out] x/(1 + Log[x])
```

fricas [A] time = 0.39, size = 8, normalized size = 1.00

$$\frac{x}{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
[Out] x/(log(x) + 1)
```

giac [A] time = 0.96, size = 8, normalized size = 1.00

$$\frac{x}{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
[Out] x/(log(x) + 1)
```

maple [A] time = 0.14, size = 9, normalized size = 1.12

$$\frac{x}{\ln(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(ln(x)+1)^2,x)
[Out] x/(ln(x)+1)
```

maxima [A] time = 0.54, size = 8, normalized size = 1.00

$$\frac{x}{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")
```

```
[Out] x/(log(x) + 1)
```

mupad [B] time = 0.26, size = 8, normalized size = 1.00

$$\frac{x}{\ln(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(log(x) + 1)^2,x)
```

```
[Out] x/(log(x) + 1)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.62

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(1+ln(x))**2,x)
```

```
[Out] x/(log(x) + 1)
```

3.56 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

fricas [A] time = 0.41, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

giac [A] time = 0.91, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
[Out] arctan(ln(x))
```

maxima [A] time = 1.29, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

mupad [B] time = 0.31, size = 3, normalized size = 1.00

$$\mathrm{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

sympy [B] time = 0.14, size = 15, normalized size = 5.00

$$\mathrm{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log(x)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.57 $\int \frac{1}{\log(x)} dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] Li(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2298}

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(x)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] Integrate[Log[x]^(-1), x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

giac [A] time = 0.89, size = 3, normalized size = 1.50

$$\text{Ei}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(log(x))
```

maple [**B**] time = 0.00, size = 9, normalized size = 4.50

$$-\operatorname{Ei}(1, -\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x)
[Out] -Ei(1,-ln(x))
```

maxima [**A**] time = 0.65, size = 3, normalized size = 1.50

$$\operatorname{Ei}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
[Out] Ei(log(x))
```

mupad [**B**] time = 0.01, size = 2, normalized size = 1.00

$$\operatorname{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x),x)
[Out] logint(x)
```

sympy [**A**] time = 0.45, size = 2, normalized size = 1.00

$$\operatorname{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
[Out] li(x)
```

3.58 $\int x(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=14

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

```
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {14, 3296, 2638, 2637}

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x] + Sin[x]),x]
```

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x(\cos(x)+\sin(x))\,dx &= \int (x\cos(x)+x\sin(x))\,dx\\ &= \int x\cos(x)\,dx+\int x\sin(x)\,dx\\ &= -x\cos(x)+x\sin(x)+\int \cos(x)\,dx-\int \sin(x)\,dx\\ &= \cos(x)-x\cos(x)+\sin(x)+x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x] + Sin[x]),x]
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

fricas [A] time = 0.41, size = 14, normalized size = 1.00

$$-(x-1)\cos(x) + (x+1)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")
[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)
```

giac [A] time = 1.11, size = 14, normalized size = 1.00

$$-x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="giac")
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

maple [A] time = 0.03, size = 15, normalized size = 1.07

$$-x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x)+sin(x)),x)
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

maxima [A] time = 0.58, size = 14, normalized size = 1.00

$$-x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

mupad [B] time = 0.06, size = 14, normalized size = 1.00

$$\cos(x) + \sin(x) - x\ \cos(x) + x\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x) + sin(x)),x)
[Out] cos(x) + sin(x) - x*cos(x) + x*sin(x)
```

sympy [A] time = 0.18, size = 15, normalized size = 1.07

$$x\sin\left(x\right) - x\cos\left(x\right) + \sin\left(x\right) + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x)
[Out] x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

3.59 $\int e^{-x} (e^x + x) \, dx$

Optimal. Leaf size=17

$$-e^{-x}x + x - e^{-x}$$

```
[Out] -1/exp(x)+x-x/exp(x)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 2176, 2194}

$$-e^{-x}x + x - e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)/E^x,x]
```

```
[Out] -E^(-x) + x - x/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \int e^{-x} (e^x + x) \, dx &= \int (1 + e^{-x}x) \, dx \\ &= x + \int e^{-x}x \, dx \\ &= x - e^{-x}x + \int e^{-x} \, dx \\ &= -e^{-x} + x - e^{-x}x \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.76

$$e^{-x}(-x - 1) + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)/E^x,x]
```

```
[Out] (-1 - x)/E^x + x
```

fricas [A] time = 0.42, size = 14, normalized size = 0.82

$$(xe^x - x - 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] (x*e^x - x - 1)*e^(-x)
```

giac [A] time = 0.99, size = 11, normalized size = 0.65

$$-(x+1)e^{(-x)}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + x
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-x\,\mathrm{e}^{-x}+x-\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+exp(x))/exp(x),x)
```

```
[Out] -1/exp(x)+x-x/exp(x)
```

maxima [A] time = 0.56, size = 11, normalized size = 0.65

$$-(x+1)e^{(-x)}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + x
```

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$x-\mathrm{e}^{-x}-x\,\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*(x + exp(x)),x)
```

```
[Out] x - exp(-x) - x*exp(-x)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.47

$$x+(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x)
```

```
[Out] x + (-x - 1)*exp(-x)
```

3.60 $\int (1+e^x)^2 x\,dx$

Optimal. Leaf size=38

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

```
[Out] -2*exp(x)-1/4*exp(2*x)+2*exp(x)*x+1/2*exp(2*x)*x+1/2*x^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2183, 2176, 2194}

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)^2*x,x]
```

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (1+e^x)^2 x\,dx &= \int \left(x+2e^x x+e^{2x}x\right)\,dx\\ &= \frac{x^2}{2}+2\int e^x x\,dx+\int e^{2x}x\,dx\\ &= 2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}-\frac{1}{2}\int e^{2x}\,dx-2\int e^x\,dx\\ &= -2e^x-\frac{e^{2x}}{4}+2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.76

$$\frac{1}{4}\left(2x^2+8e^x(x-1)+e^{2x}(2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^2*x,x]
```

```
[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

fricas [A] time = 0.40, size = 24, normalized size = 0.63

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

giac [A] time = 0.95, size = 24, normalized size = 0.63

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

maple [A] time = 0.00, size = 29, normalized size = 0.76

$$\frac{x^2}{2} + 2x\,\mathrm{e}^x + \frac{x\,\mathrm{e}^{2x}}{2} - 2\,\mathrm{e}^x - \frac{\mathrm{e}^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+1)^2*x,x)
```

```
[Out] 1/2*x^2+1/2*exp(x)^2*x-1/4*exp(x)^2+2*x*exp(x)-2*exp(x)
```

maxima [A] time = 0.62, size = 24, normalized size = 0.63

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

mupad [B] time = 0.16, size = 28, normalized size = 0.74

$$\frac{x\,\mathrm{e}^{2\,x}}{2} - 2\,\mathrm{e}^x - \frac{\mathrm{e}^{2\,x}}{4} + 2\,x\,\mathrm{e}^x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(exp(x) + 1)^2,x)
```

```
[Out] (x*exp(2*x))/2 - 2*exp(x) - exp(2*x)/4 + 2*x*exp(x) + x^2/2
```

sympy [A] time = 0.09, size = 26, normalized size = 0.68

$$\frac{x^2}{2} + \frac{(2x-1)\,e^{2x}}{4} + \frac{(8x-8)\,e^x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))**2*x,x)
```

```
[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

3.61 $\int x \cos(x) \, dx$

Optimal. Leaf size=7

$$x\sin(x) + \cos(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2638}

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x) - \int \sin(x)\,dx\\ &= \cos(x) + x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

giac [A] time = 0.92, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] x*sin(x)+cos(x)
```

maxima [A] time = 0.51, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

mupad [B] time = 0.02, size = 7, normalized size = 1.00

$$\cos(x) + x\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x) + x*sin(x)
```

sympy [A] time = 0.17, size = 7, normalized size = 1.00

$$x\sin\left(x\right) + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

3.62 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x\cos(x)\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}\,\sin\left(\sqrt{x}\right)-2\,\text{Subst}\left(\int \sin(x)\,dx,x,\sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right)+2\sqrt{x}\,\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

fricas [A] time = 0.42, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

giac [A] time = 0.92, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*x^(1/2)*sin(x^(1/2))+2*cos(x^(1/2))
```

maxima [A] time = 0.62, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

mupad [B] time = 0.00, size = 16, normalized size = 0.73

$$2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

sympy [A] time = 0.31, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.63 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x)+\cos(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2638}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int \sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

giac [A] time = 1.01, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] x*sin(x)+cos(x)
```

maxima [A] time = 0.66, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

mupad [B] time = 0.00, size = 7, normalized size = 1.00

$$\cos(x) + x\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x) + x*sin(x)
```

sympy [A] time = 0.17, size = 7, normalized size = 1.00

$$x\sin\left(x\right) + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

3.64 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\log^2(x)\,dx &= \frac{1}{2}x^2\log^2(x)-\int x\log(x)\,dx\\ &= \frac{x^2}{4}-\frac{1}{2}x^2\log(x)+\frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

fricas [A] time = 0.41, size = 22, normalized size = 0.79

$$\frac{1}{2}x^2\log(x)^2-\frac{1}{2}x^2\log(x)+\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

giac [A] time = 0.95, size = 22, normalized size = 0.79

$$\frac{1}{2}x^2\log(x)^2 - \frac{1}{2}x^2\log(x) + \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{x^2\ln(x)^2}{2} - \frac{x^2\ln(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

maxima [A] time = 0.59, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\log(x)^2 - 2\log(x) + 1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2\left(2\ln(x)^2 - 2\ln(x) + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

sympy [A] time = 0.10, size = 22, normalized size = 0.79

$$\frac{x^2\log(x)^2}{2} - \frac{x^2\log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.65 $\int \cos(x)\left(1+\sin^3(x)\right)\,dx$

Optimal. Leaf size=11

$$\frac{\sin^4(x)}{4}+\sin(x)$$

```
[Out] sin(x)+1/4*sin(x)^4
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3223}

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^3),x]
[Out] Sin[x] + Sin[x]^4/4
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\left(1+\sin^3(x)\right)\,dx &= \text{Subst}\left(\int \left(1+x^3\right)\,dx, x, \sin(x)\right)\\ &= \sin(x)+\frac{\sin^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^3),x]
[Out] Sin[x] + Sin[x]^4/4
```

fricas [A] time = 0.45, size = 15, normalized size = 1.36

$$\frac{1}{4}\cos(x)^4-\frac{1}{2}\cos(x)^2+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")
[Out] 1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

giac [A] time = 0.96, size = 9, normalized size = 0.82

$$\frac{1}{4}\sin(x)^4+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

maple [A] time = 0.03, size = 10, normalized size = 0.91

$$\frac{\left(\sin^4(x)\right)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^3),x)
```

```
[Out] sin(x)+1/4*sin(x)^4
```

maxima [A] time = 0.53, size = 9, normalized size = 0.82

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

mupad [B] time = 0.04, size = 9, normalized size = 0.82

$$\frac{\sin(x)^4}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sin(x)^3 + 1),x)
```

```
[Out] sin(x) + sin(x)^4/4
```

sympy [A] time = 0.57, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**3),x)
```

```
[Out] sin(x)**4/4 + sin(x)
```

3.66 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)$$
$$= \tan^{-1}(\log(x))$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

fricas [A] time = 0.44, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

giac [A] time = 0.91, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
[Out] arctan(ln(x))
```

maxima [A] time = 1.36, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

mupad [B] time = 0.00, size = 3, normalized size = 1.00

$$\mathrm{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

sympy [B] time = 0.14, size = 15, normalized size = 5.00

$$\mathrm{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log(x)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}\left(1+\sin^{-1}(x)^2\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

```
[Out] arctan(arcsin(x))
```

Rubi [A] time = 0.05, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6696, 203}

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6696

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\left(1+\sin^{-1}(x)^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin^{-1}(x)\right)\\ &= \tan^{-1}\left(\sin^{-1}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 3, normalized size = 1.00

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

fricas [A] time = 0.46, size = 3, normalized size = 1.00

$$\arctan\left(\arcsin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(arcsin(x))
```

giac [A] time = 1.13, size = 3, normalized size = 1.00

$$\arctan\left(\arcsin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] arctan(arcsin(x))
```

maple [A] time = 0.01, size = 4, normalized size = 1.33

$$\arctan\left(\arcsin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x)
[Out] arctan(arcsin(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1}\left(\arcsin(x)^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

mupad [B] time = 3.10, size = 43, normalized size = 14.33

$$\frac{\ln\left(\frac{-1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2}-\frac{\ln\left(\frac{1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(asin(x)^2 + 1)),x)
[Out] (log((asin(x)*1i - 1)/(1 - x^2)^(1/2))*1i)/2 - (log((asin(x)*1i + 1)/(1 - x
^2)^(1/2))*1i)/2
```

sympy [A] time = 0.41, size = 3, normalized size = 1.00

$$\operatorname{atan}\left(\operatorname{asin}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)
[Out] atan(asin(x))
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

```
[Out] 1/2*x-1/2*ln(cos(x)+sin(x))
```

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3097, 3133}

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx &= \frac{x}{2}-\frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx\\ &= \frac{x}{2}-\frac{1}{2}\log(\cos(x)+\sin(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

fricas [A] time = 0.42, size = 15, normalized size = 0.94

$$\frac{1}{2}\,x-\frac{1}{4}\,\log\left(2\,\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

giac [A] time = 1.08, size = 21, normalized size = 1.31

$$\frac{1}{2}\,x + \frac{1}{4}\,\log\left(\tan(x)^2 + 1\right) - \frac{1}{2}\,\log\left(|\tan(x) + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))
```

maple [A] time = 0.05, size = 21, normalized size = 1.31

$$\frac{x}{2} + \frac{\ln\left(\tan^2(x) + 1\right)}{4} - \frac{\ln\left(\tan(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+sin(x)),x)
```

```
[Out] -1/2*ln(tan(x)+1)+1/4*ln(tan(x)^2+1)+1/2*x
```

maxima [B] time = 1.28, size = 53, normalized size = 3.31

$$\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right) - \frac{1}{2}\,\log\left(-\frac{2\,\sin(x)}{\cos(x) + 1} + \frac{\sin(x)^2}{(\cos(x) + 1)^2} - 1\right) + \frac{1}{2}\,\log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(co
s(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.81

$$\frac{x}{2} - \frac{\ln\left(\cos\left(x - \frac{\pi}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x)),x)
```

```
[Out] x/2 - log(cos(x - pi/4))/2
```

sympy [A] time = 0.15, size = 12, normalized size = 0.75

$$\frac{x}{2} - \frac{\log\left(\sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/2
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

Optimal. Leaf size=53

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] -B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))-A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1
/2))
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1974, 402, 217, 203, 377, 206}

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 + B
^2 - B^2*y^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
```

```
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rubi steps

$$\int -\frac{\sqrt{A^2+B^2\left(1-y^2\right)}}{1-y^2}\,dy = -\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$$
$$= -\left(A^2\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}\,dy\right) - B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy$$
$$= -\left(A^2\,\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)\right) - B^2\,\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy, y$$
$$= -B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) - A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

Mathematica [C] time = 0.06, size = 127, normalized size = 2.40

$$\frac{1}{2}\left(-2iB\log\left(2\left(\sqrt{A^2-B^2y^2+B^2}-iBy\right)\right) - A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right) + A\log\left(A\sqrt{A^2-B^2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] (A*Log[1 - y] - A*Log[1 + y] - (2*I)*B*Log[2*((-I)*B*y + Sqrt[A^2 + B^2 - B
^2*y^2])] - A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]] + A*Log[
A^2 + B^2*(1 + y) + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
```

fricas [B] time = 0.44, size = 128, normalized size = 2.42

$$B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right) - \frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y^2+2\sqrt{-B^2y^2+A^2+B^2}\,Ay+A^2+B^2}{y^2}\right) + \frac{1}{4}A\log\left(-\frac{\left(A^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 +
2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B^2
)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,
0]%%%},0,%%%{1,[0,4,4]%%%}] at parameters values [88,76,-66]Warning, choosi
ng root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,0]%%%},0,%
%%{1,[0,4,4]%%%}] at parameters values [66,5,-23]B^2*(1/2*pi*sign(y)-atan(B
^2*y*((-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))/B^2/y)^2-1
)/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))))/abs(B)-1/2*A*B^2*l
n(abs(B*(-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B))/B^2/y+2*
B^2*y/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B)))+2*A))/B/abs(B)+
```

```
1/2*A*B^2*ln(abs(B*(-1/2*(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B
))/B^2/y+2*B^2*y/(-2*B*sqrt(A^2+B^2)-2*sqrt(-B^2*y^2+A^2+B^2)*abs(B)))-2*A)
)/B/abs(B)
```

maple [B] time = 0.01, size = 262, normalized size = 4.94

$$\frac{A^2\ln\left(\frac{2A^2+2(y+1)B^2+2\sqrt{A^2}\sqrt{A^2-(y+1)^2B^2+2(y+1)B^2}}{y+1}\right)}{2\sqrt{A^2}}-\frac{A^2\ln\left(\frac{2A^2-2(y-1)B^2+2\sqrt{A^2}\sqrt{A^2-(y-1)^2B^2-2(y-1)B^2}}{y-1}\right)}{2\sqrt{A^2}}-\frac{B^2\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y)
```

```
[Out] -1/2*(A^2-(y+1)^2*B^2+2*(y+1)*B^2)^(1/2)-1/2/(B^2)^(1/2)*B^2*arctan((B^2)^(
1/2)/(A^2-(y+1)^2*B^2+2*(y+1)*B^2)^(1/2)*y)+1/2/(A^2)^(1/2)*A^2*ln((2*A^2+2
*(y+1)*B^2+2*(A^2)^(1/2)*(A^2-(y+1)^2*B^2+2*(y+1)*B^2)^(1/2))/(y+1))+1/2*(A
^2-(y-1)^2*B^2-2*(y-1)*B^2)^(1/2)-1/2/(B^2)^(1/2)*B^2*arctan((B^2)^(1/2)/(A
^2-(y-1)^2*B^2-2*(y-1)*B^2)^(1/2)*y)-1/2/(A^2)^(1/2)*A^2*ln((2*A^2-2*(y-1)*
B^2+2*(A^2)^(1/2)*(A^2-(y-1)^2*B^2-2*(y-1)*B^2)^(1/2))/(y-1))
```

maxima [B] time = 1.49, size = 123, normalized size = 2.32

$$-B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2+B^4}}\right)+\frac{1}{2}A\log\left(B^2+\frac{2A^2}{|2y+2|}+\frac{2\sqrt{-B^2y^2+A^2+B^2}A}{|2y+2|}\right)-\frac{1}{2}A\log\left(-B^2+\frac{2A^2}{|2y-2|}+\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] -B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) + 1/2*A*log(B^2 + 2*A^2/abs(2*y + 2) +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y + 2)) - 1/2*A*log(-B^2 + 2*A^2/abs(
2*y - 2) + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \int \frac{\sqrt{-B^2y^2}}{y^2-1}\,dy & \text{if } A^2+B^2=0 \\ \ln\left(2y\sqrt{-B^2}+2\sqrt{A^2-B^2y^2+B^2}\right)\sqrt{-B^2}+\operatorname{atan}\left(\frac{y\sqrt{A^2}\,1i}{\sqrt{A^2-B^2y^2+B^2}}\right)\sqrt{A^2}\,1i & \text{if } A^2+B^2\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2 - B^2*(y^2 - 1))^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
0, atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i + lo
g(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{A^2-B^2y^2+B^2}}{(y-1)(y+1)}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

3.70 $\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)-Bz$$

[Out] -B*z-A*arctanh(A*tan(z)/B)

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {12, 3191, 391, 203, 206}

$$-A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)-Bz$$

Antiderivative was successfully verified.

[In] Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]

[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{(A^2+B^2)\int \frac{\cos^2(z)}{1-\frac{(A^2+B^2)\sin^2(z)}{B^2}}\,dz}{B}$$

$$= -\frac{(A^2+B^2)\,\text{Subst}\left(\int \frac{1}{(1+z^2)\left(1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2\right)}\,dz, z, \tan(z)\right)}{B}$$

$$= -\frac{A^2\,\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2}\,dz, z, \tan(z)\right)}{B} - B\,\text{Subst}\left(\int \frac{1}{1+z^2}\,dz, z, \tan(z)\right)$$

$$= -Bz - A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)$$

Mathematica [B] time = 0.10, size = 35, normalized size = 2.19

$$-\frac{B(A^2+B^2)\left(A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)+Bz\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

fricas [B] time = 0.49, size = 67, normalized size = 4.19

$$-Bz-\frac{1}{4}A\log\left(2AB\cos(z)\sin(z)-(A^2-B^2)\cos(z)^2+A^2\right)+\frac{1}{4}A\log\left(-2AB\cos(z)\sin(z)-(A^2-B^2)\cos(z$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="fr
icas")
```

```
[Out] -B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2) + 1/4*A*
log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

giac [B] time = 1.23, size = 83, normalized size = 5.19

$$-\frac{\left(\frac{A^3B\log(|A\tan(z)+B|)}{A^4+A^2B^2}-\frac{A^3B\log(|A\tan(z)-B|)}{A^4+A^2B^2}+\frac{2B^2z}{A^2+B^2}\right)(A^2+B^2)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="gi
ac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*tan(z)
 - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B
```

maple [B] time = 0.07, size = 127, normalized size = 7.94

$$\frac{A^3\ln(A\tan(z)-B)}{2A^2+2B^2}-\frac{A^3\ln(A\tan(z)+B)}{2(A^2+B^2)}-\frac{A^2B\arctan(\tan(z))}{A^2+B^2}+\frac{AB^2\ln(A\tan(z)-B)}{2A^2+2B^2}-\frac{AB^2\ln(A\tan(z)}{2(A^2+B^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z)
```

```
[Out] -1/2*A^3/(A^2+B^2)*ln(A*tan(z)+B)-1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)+B)+1/2*A^
3/(A^2+B^2)*ln(A*tan(z)-B)+1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)-B)-B/(A^2+B^2)*a
rctan(tan(z))*A^2-1/(A^2+B^2)*arctan(tan(z))*B^3
```

maxima [B] time = 1.51, size = 69, normalized size = 4.31

$$-\frac{\left(A^2+B^2\right)\left(\frac{2\,B^2 z}{A^2+B^2}+\frac{AB\log(A\tan(z)+B)}{A^2+B^2}-\frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="ma
xima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^2 + B^2) -
 A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

mupad [B] time = 0.50, size = 360, normalized size = 22.50

$$-A\,\mathrm{atanh}\left(\frac{2\,A^{13}\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{2\,A^7\,B^6\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A}{2\,A^{12}\,B+6\,A^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(z)^2*(A^2 + B^2))/(B*((sin(z)^2*(A^2 + B^2))/B^2 - 1)),z)
```

```
[Out] - A*atanh((2*A^13*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) +
 (2*A^7*B^6*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^
9*B^4*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2
*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B
^9*tan(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*ta
n(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*tan(z))
/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*tan(z))/(2*
A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] Timed out
```

3.71 $\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

[Out] -B*arctan(w)-A*arctanh(A*w/B)

Rubi [A] time = 0.12, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 6688, 391, 203, 208}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

[In] Int[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))), w]

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int -\frac{A^2+B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw = -\frac{\left(A^2+B^2\right)\int \frac{1}{\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw}{B}$$

$$= -\frac{\left(A^2+B^2\right)\int \frac{B^2}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw}{B}$$

$$= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw\right)$$

$$= -\left(B\int \frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw$$

$$= -B\tan^{-1}(w)-A\tanh^{-1}\left(\frac{Aw}{B}\right)$$

Mathematica [B] time = 0.02, size = 35, normalized size = 2.19

$$-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)
))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

fricas [A] time = 0.43, size = 26, normalized size = 1.62

$$-B\arctan(w)-\frac{1}{2}A\log\left(Aw+B\right)+\frac{1}{2}A\log\left(Aw-B\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

giac [B] time = 1.16, size = 82, normalized size = 5.12

$$-\frac{\left(\frac{A^3B\log(|Aw+B|)}{A^4+A^2B^2}-\frac{A^3B\log(|Aw-B|)}{A^4+A^2B^2}+\frac{2\,B^2\arctan(w)}{A^2+B^2}\right)\left(A^2+B^2\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="giac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*w - B))/(A^
4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

maple [B] time = 0.01, size = 121, normalized size = 7.56

$$\frac{A^3\ln\left(Aw-B\right)}{2A^2+2B^2}-\frac{A^3\ln\left(Aw+B\right)}{2\left(A^2+B^2\right)}-\frac{A^2B\arctan(w)}{A^2+B^2}+\frac{A\,B^2\ln\left(Aw-B\right)}{2A^2+2B^2}-\frac{A\,B^2\ln\left(Aw+B\right)}{2\left(A^2+B^2\right)}-\frac{B^3\arctan(w)}{A^2+B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w)
```

```
[Out] -1/2*A^3/(A^2+B^2)*ln(A*w+B)-1/2*A*B^2/(A^2+B^2)*ln(A*w+B)+1/2*A^3/(A^2+B^2
)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-B/(A^2+B^2)*arctan(w)*A^2-1/(A^2+
B^2)*arctan(w)*B^3
```

maxima [B] time = 1.41, size = 68, normalized size = 4.25

$$-\frac{\left(A^2+B^2\right)\left(\frac{2\,B^2\arctan(w)}{A^2+B^2}+\frac{AB\log(Aw+B)}{A^2+B^2}-\frac{AB\log(Aw-B)}{A^2+B^2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/(A^2 + B^2
) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

mupad [B] time = 0.24, size = 352, normalized size = 22.00

$$-A\,\mathrm{atanh}\left(\frac{2\,A^{13}\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{2\,A^7\,B^6\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{}{2\,A^{12}\,B+6\,A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2 + B^2)/(B*(w^2 + 1)^2*((w^2*(A^2 + B^2))/(B^2*(w^2 + 1)) - 1)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

sympy [C] time = 1.93, size = 422, normalized size = 26.38

$$\left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B\left(A^2+B^2\right)^3}-\frac{A^7B}{\left(A^2+B^2\right)^3}+\frac{A^5B^3}{\left(A^2+B^2\right)^3}+\frac{A^5}{B\left(A^2+B^2\right)}+\frac{A^3B^5}{\left(A^2+B^2\right)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{A\log\left(w+\frac{\frac{A^9}{B\left(A^2+B^2\right)^3}+\frac{A^7B}{\left(A^2+B^2\right)^3}-}{}\right.}{2B\left(A\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

3.72 $\int -\frac{B\left(A^2+B^2\right)}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

```
[Out] -B*arctan(w)-A*arctanh(A*w/B)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {12, 391, 203, 208}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

```
[In] Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int -\frac{B\left(A^2+B^2\right)}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw &= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw\right)\\ &= -\left(B\int \frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw\\ &= -B\tan^{-1}(w)-A\tanh^{-1}\left(\frac{Aw}{B}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 35, normalized size = 2.19

$$-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

fricas [A] time = 0.43, size = 26, normalized size = 1.62

$$-B\arctan(w) - \frac{1}{2}A\log(Aw + B) + \frac{1}{2}A\log(Aw - B)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

giac [B] time = 1.04, size = 79, normalized size = 4.94

$$-\frac{1}{2}\left(\frac{A^3\log(|Aw + B|)}{A^4B + A^2B^3} - \frac{A^3\log(|Aw - B|)}{A^4B + A^2B^3} + \frac{2\arctan(w)}{A^2 + B^2}\right)\left(A^2 + B^2\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="giac")
```

```
[Out] -1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4*
B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

maple [B] time = 0.01, size = 121, normalized size = 7.56

$$\frac{A^3\ln(Aw - B)}{2A^2 + 2B^2} - \frac{A^3\ln(Aw + B)}{2\left(A^2 + B^2\right)} - \frac{A^2B\arctan(w)}{A^2 + B^2} + \frac{AB^2\ln(Aw - B)}{2A^2 + 2B^2} - \frac{AB^2\ln(Aw + B)}{2\left(A^2 + B^2\right)} - \frac{B^3\arctan(w)}{A^2 + B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w)
```

```
[Out] 1/2/(A^2+B^2)*A^3*ln(A*w-B)-1/2/(A^2+B^2)*A^3*ln(A*w+B)-1/(A^2+B^2)*A^2*B*a
rctan(w)+1/2/(A^2+B^2)*A*B^2*ln(A*w-B)-1/2/(A^2+B^2)*A*B^2*ln(A*w+B)-1/(A^2
+B^2)*B^3*arctan(w)
```

maxima [B] time = 1.42, size = 65, normalized size = 4.06

$$-\frac{1}{2}\left(A^2 + B^2\right)B\left(\frac{A\log(Aw + B)}{A^2B + B^3} - \frac{A\log(Aw - B)}{A^2B + B^3} + \frac{2\arctan(w)}{A^2 + B^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B +
B^3) + 2*arctan(w)/(A^2 + B^2))
```

mupad [B] time = 0.06, size = 352, normalized size = 22.00

$$-A\,\mathrm{atanh}\left(\frac{2\,A^{13}\,w}{2\,A^{12}\,B + 6\,A^{10}\,B^3 + 6\,A^8\,B^5 + 2\,A^6\,B^7} + \frac{2\,A^7\,B^6\,w}{2\,A^{12}\,B + 6\,A^{10}\,B^3 + 6\,A^8\,B^5 + 2\,A^6\,B^7} + \frac{}{2\,A^{12}\,B + 6\,A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(A^2 + B^2))/((w^2 + 1)*(B^2 - A^2*w^2)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

sympy [C] time = 1.90, size = 422, normalized size = 26.38

$$\left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B\left(A^2+B^2\right)^3}-\frac{A^7B}{\left(A^2+B^2\right)^3}+\frac{A^5B^3}{\left(A^2+B^2\right)^3}+\frac{A^5}{B\left(A^2+B^2\right)}+\frac{A^3B^5}{\left(A^2+B^2\right)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{A\log\left(w+\frac{\frac{A^9}{B\left(A^2+B^2\right)^3}+\frac{A^7B}{\left(A^2+B^2\right)^3}-\cdots\right.}{2B\left(A^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

3.73 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

fricas [B] time = 0.41, size = 63, normalized size = 1.80

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

giac [A] time = 1.05, size = 29, normalized size = 0.83

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}\,x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.86

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3/(-x^2+1)^(3/2)*x^3-1/(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A] time = 1.27, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

mupad [B] time = 0.00, size = 91, normalized size = 2.60

$$\mathrm{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

sympy [B] time = 2.07, size = 105, normalized size = 3.00

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.74 $\int \tan^4(y)\,dy$

Optimal. Leaf size=14

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

```
[Out] y-tan(y)+1/3*tan(y)^3
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[y]^4,y]
[Out] y - Tan[y] + Tan[y]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(y)\,dy &= \frac{\tan^3(y)}{3}-\int \tan^2(y)\,dy\\ &= -\tan(y)+\frac{\tan^3(y)}{3}+\int 1\,dy\\ &= y-\tan(y)+\frac{\tan^3(y)}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.29

$$y-\frac{4\tan(y)}{3}+\frac{1}{3}\tan(y)\sec^2(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[y]^4,y]
[Out] y - (4*Tan[y])/3 + (Sec[y]^2*Tan[y])/3
```

fricas [B] time = 0.45, size = 26, normalized size = 1.86

$$\frac{3\,y\cos(y)^3-\left(4\,\cos(y)^2-1\right)\sin(y)}{3\,\cos(y)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
```

```
[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

giac [A] time = 0.99, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
[Out] 1/3*tan(y)^3 + y - tan(y)
```

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$\frac{\left(\tan^3(y)\right)}{3} + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)^4/cos(y)^4,y)
[Out] y-tan(y)+1/3*tan(y)^3
```

maxima [A] time = 1.33, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
[Out] 1/3*tan(y)^3 + y - tan(y)
```

mupad [B] time = 0.07, size = 12, normalized size = 0.86

$$\frac{\tan(y)^3}{3} - \tan(y) + y$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)^4/cos(y)^4,y)
[Out] y - tan(y) + tan(y)^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.36

$$y + \frac{\sin^3(y)}{3\cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)**4/cos(y)**4,y)
[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

3.75 $\int \frac{z^4}{1+z^2} dz$

Optimal. Leaf size=13

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

```
[Out] -z+1/3*z^3+arctan(z)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {302, 203}

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In] Int[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{z^4}{1+z^2} dz &= \int \left(-1+z^2+\frac{1}{1+z^2}\right) dz\\ &= -z+\frac{z^3}{3}+\int \frac{1}{1+z^2} dz\\ &= -z+\frac{z^3}{3}+\tan^{-1}(z)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In] Integrate[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

fricas [A] time = 0.39, size = 11, normalized size = 0.85

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="fricas")
[Out] 1/3*z^3 - z + arctan(z)
```

giac [A] time = 0.89, size = 11, normalized size = 0.85

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="giac")
[Out] 1/3*z^3 - z + arctan(z)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{z^3}{3} - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^4/(z^2+1),z)
[Out] -z+1/3*z^3+arctan(z)
```

maxima [A] time = 1.47, size = 11, normalized size = 0.85

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="maxima")
[Out] 1/3*z^3 - z + arctan(z)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.85

$$\mathrm{atan}(z) - z + \frac{z^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^4/(z^2 + 1),z)
[Out] atan(z) - z + z^3/3
```

sympy [A] time = 0.09, size = 8, normalized size = 0.62

$$\frac{z^3}{3} - z + \mathrm{atan}\,(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**4/(z**2+1),z)
[Out] z**3/3 - z + atan(z)
```

3.76 $\int e^{x^2}\left(1+2x^2\right)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2226, 2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2}\left(1+2x^2\right)\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx\\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)-\int e^{x^2}\,dx\\
&= e^{x^2}x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

fricas [A] time = 0.41, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
[Out] x*e^(x^2)
```

giac [A] time = 1.08, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
[Out] x*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 1.00

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x)
[Out] x*exp(x^2)
```

maxima [A] time = 0.67, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
[Out] x*e^(x^2)
```

mupad [B] time = 0.00, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
[Out] x*exp(x^2)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
[Out] x*exp(x**2)
```

3.77 $\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=24

$$e^{x^2}x+\frac{e^{x^2}}{2\left(x^2+1\right)}$$

```
[Out] exp(x^2)*x+1/2*exp(x^2)/(x^2+1)
```

Rubi [A] time = 0.36, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6742, 2204, 2212, 6715, 2177, 2178}

$$e^{x^2}x+\frac{e^{x^2}}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2}\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2-\frac{e^{x^2}x}{\left(1+x^2\right)^2}+\frac{e^{x^2}x}{1+x^2}\right)dx \\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx-\int \frac{e^{x^2}x}{\left(1+x^2\right)^2}\,dx+\int \frac{e^{x^2}x}{1+x^2}\,dx \\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)-\frac{1}{2}\text{Subst}\left(\int \frac{e^x}{(1+x)^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{e}{1-}\right. \\
&= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}+\frac{\text{Ei}\left(1+x^2\right)}{2e}-\frac{1}{2}\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x^2\right) \\
&= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 20, normalized size = 0.83

$$\frac{1}{2}e^{x^2}\left(\frac{1}{x^2+1}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
[Out] (E^x^2*(2*x + (1 + x^2)^(-1)))/2
```

fricas [A] time = 0.43, size = 23, normalized size = 0.96

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="fricas
")
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

giac [A] time = 1.06, size = 30, normalized size = 1.25

$$\frac{2\,x^3e^{\left(x^2\right)}+2\,xe^{\left(x^2\right)}+e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\left(2x^3+2x+1\right)e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x)
```

```
[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

maxima [A] time = 1.80, size = 23, normalized size = 0.96

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

mupad [B] time = 0.22, size = 24, normalized size = 1.00

$$\frac{\mathrm{e}^{x^2}\,\left(2\,x^3+2\,x+1\right)}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x^2)*(4*x^2 + x^3 + 5*x^4 + 2*x^6 + 1))/(x^2 + 1)^2,x)
```

```
[Out] (exp(x^2)*(2*x + 2*x^3 + 1))/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 20, normalized size = 0.83

$$\frac{\left(2x^3+2x+1\right)e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

```
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

3.78 $\int e^{-1-x}\,dx$

Optimal. Leaf size=9

$$-e^{-x-1}$$

```
[Out] -exp(-1-x)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2194}

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-1 - x),x]
```

```
[Out] -E^(-1 - x)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{-1-x}\,dx = -e^{-1-x}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-1 - x),x]
```

```
[Out] -E^(-1 - x)
```

fricas [A] time = 0.41, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="fricas")
```

```
[Out] -e^(-x - 1)
```

giac [A] time = 1.15, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="giac")
```

```
[Out] -e^(-x - 1)
```

maple [A] time = 0.00, size = 9, normalized size = 1.00

$$-\mathrm{e}^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x-1),x)
[Out] -exp(-x-1)
```

maxima [A] time = 0.47, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="maxima")
[Out] -e^(-x - 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.89

$$-\mathrm{e}^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(- x - 1),x)
[Out] -exp(- x - 1)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x)
[Out] -exp(-x - 1)
```

3.79 $\int \left(\frac{1}{x}+x\right)\log(x)\,dx$

Optimal. Leaf size=25

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1593, 14, 2351, 2301, 2304}

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + x)*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$\begin{aligned}\int\left(\frac{1}{x}+x\right)\log(x)\,dx &= \int\frac{(1+x^2)\log(x)}{x}\,dx\\ &= \int\left(\frac{\log(x)}{x}+x\log(x)\right)dx\\ &= \int\frac{\log(x)}{x}\,dx+\int x\log(x)\,dx\\ &= -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + x)*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

fricas [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2+\frac{1}{2}\,\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

giac [A] time = 0.96, size = 19, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x)-\frac{1}{4}\,x^2+\frac{1}{2}\,\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{x^2\ln(x)}{2}-\frac{x^2}{4}+\frac{\ln(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+x)*ln(x),x)
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

maxima [A] time = 0.57, size = 24, normalized size = 0.96

$$-\frac{1}{4}\,x^2+\frac{1}{2}\,\left(x^2+2\,\log(x)\right)\log(x)-\frac{1}{2}\,\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

mupad [B] time = 0.23, size = 19, normalized size = 0.76

$$\frac{x^2\,\ln(x)}{2}-\frac{x^2}{4}+\frac{\ln(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(x + 1/x),x)
```

```
[Out] (x^2*log(x))/2 + log(x)^2/2 - x^2/4
```

sympy [A] time = 0.10, size = 19, normalized size = 0.76

$$\frac{x^2\log(x)}{2}-\frac{x^2}{4}+\frac{\log(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

3.80 $\int \frac{x}{1+x^4} dx$

Optimal. Leaf size=8

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

[Out] 1/2*arctan(x^2)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {275, 203}

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^4),x]

[Out] ArcTan[x^2]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^4} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, x^2\right)\\ &= \frac{1}{2}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x^4),x]

[Out] ArcTan[x^2]/2

fricas [A] time = 0.40, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4+1),x, algorithm="fricas")

[Out] 1/2*arctan(x^2)

giac [A] time = 1.06, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+1),x)
[Out] 1/2*arctan(x^2)
```

maxima [A] time = 1.42, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="maxima")
[Out] 1/2*arctan(x^2)
```

mupad [B] time = 0.06, size = 6, normalized size = 0.75

$$\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 1),x)
[Out] atan(x^2)/2
```

sympy [A] time = 0.09, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+1),x)
[Out] atan(x**2)/2
```

3.81 $\int \frac{x^5}{1+x^4}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {275, 321, 203}

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4),x]
[Out] x^2/2 - ArcTan[x^2]/2
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

giac [A] time = 1.03, size = 12, normalized size = 0.75

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^2}{2} - \frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1),x)
[Out] 1/2*x^2-1/2*arctan(x^2)
```

maxima [A] time = 1.47, size = 12, normalized size = 0.75

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + 1),x)
[Out] x^2/2 - atan(x^2)/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.62

$$\frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1),x)
[Out] x**2/2 - atan(x**2)/2
```

3.82 $\int \frac{1}{1+\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3657, 2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\tan^2(x)}\,dx &= \int \cos^2(x)\,dx\\ &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + Sin[2*x]/4
```

fricas [**A**] time = 0.44, size = 20, normalized size = 1.43

$$\frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

giac [**A**] time = 0.92, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

maple [**A**] time = 0.02, size = 17, normalized size = 1.21

$$\frac{x}{2} + \frac{\tan(x)}{2\left(\tan^2(x)\right) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2+1),x)
[Out] 1/2/(tan(x)^2+1)*tan(x)+1/2*x
```

maxima [**A**] time = 1.36, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

mupad [**B**] time = 0.18, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2 + 1),x)
[Out] x/2 + sin(2*x)/4
```

sympy [**B**] time = 0.39, size = 36, normalized size = 2.57

$$\frac{x\tan^2(x)}{2\tan^2(x) + 2} + \frac{x}{2\tan^2(x) + 2} + \frac{\tan(x)}{2\tan^2(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2),x)
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 +
 2)
```

3.83 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

fricas [B]   time = 0.39, size = 63, normalized size = 1.80

$$-\frac{6\left(x^4-2\,x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4\,x^3-3\,x\right)\sqrt{-x^2+1}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

giac [A]   time = 1.10, size = 29, normalized size = 0.83

$$\frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}\,x}{3\left(x^2-1\right)^2}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

maple [A]   time = 0.00, size = 30, normalized size = 0.86

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
[Out] 1/3/(-x^2+1)^(3/2)*x^3-1/(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A]   time = 1.40, size = 44, normalized size = 1.26

$$\frac{1}{3}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

mupad [B]   time = 0.00, size = 91, normalized size = 2.60

$$\mathrm{asin}(x)+\frac{3\,\sqrt{1-x^2}}{4\,\left(x-1\right)}+\frac{3\,\sqrt{1-x^2}}{4\,\left(x+1\right)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,\left(x-1\right)}-\frac{1}{12\,{\left(x-1\right)}^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,\left(x+1\right)}+\frac{1}{12\,{\left(x+1\right)}^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

sympy [**B**] time = 2.09, size = 105, normalized size = 3.00

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2 \operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3 \operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.84 $\int -\frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=17

$$\sin^{-1}(x)-\frac{x}{\sqrt{1-x^2}}$$

```
[Out] arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {288, 216}

$$\sin^{-1}(x)-\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int -\frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx &= -\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.88

$$\frac{\sqrt{1-x^2}\,x+x^2\sin^{-1}(x)-\sin^{-1}(x)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 - x^2] - ArcSin[x] + x^2*ArcSin[x])/(-1 + x^2)
```

fricas [B] time = 0.43, size = 45, normalized size = 2.65

$$-\frac{2\left(x^2-1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*(x^2 - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 - 1)
```

giac [A] time = 1.07, size = 21, normalized size = 1.24

$$\frac{\sqrt{-x^2+1}\,x}{x^2-1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(-x^2+1)^(3/2),x)
```

```
[Out] arcsin(x)-1/(-x^2+1)^(1/2)*x
```

maxima [A] time = 1.53, size = 15, normalized size = 0.88

$$-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.16, size = 37, normalized size = 2.18

$$\mathrm{asin}(x)+\frac{\sqrt{1-x^2}}{2\,(x-1)}+\frac{\sqrt{1-x^2}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(1 - x^2)^(3/2),x)
```

```
[Out] asin(x) + (1 - x^2)^(1/2)/(2*(x - 1)) + (1 - x^2)^(1/2)/(2*(x + 1))
```

sympy [B] time = 0.51, size = 34, normalized size = 2.00

$$\frac{x^2\,\mathrm{asin}\,(x)}{x^2-1}+\frac{x\sqrt{1-x^2}}{x^2-1}-\frac{\mathrm{asin}\,(x)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2/(-x**2+1)**(3/2),x)
```

```
[Out] x**2*asin(x)/(x**2 - 1) + x*sqrt(1 - x**2)/(x**2 - 1) - asin(x)/(x**2 - 1)
```

3.85 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.68

$$-\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

giac [A] time = 0.95, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{\cos(x)e^x}{2}+\frac{e^x\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -1/2*cos(x)*exp(x)+1/2*exp(x)*sin(x)
```

maxima [A] time = 0.82, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x)-\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

mupad [B] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{e^x\,(\cos(x)-\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

sympy [A] time = 0.30, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.86 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

fricas [A] time = 0.39, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

giac [A] time = 0.95, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

maxima [A] time = 0.51, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

Optimal. Leaf size=45

$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

```
[Out] -1/12*ln(cos(t)-sin(t))-1/4*ln(cos(t)+sin(t))+1/3*ln(2*cos(t)+sin(t))-1/2/(
1+tan(t))
```

Rubi [A] time = 0.12, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {709, 800}

$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]]/
3 - 1/(2*(1 + Tan[t]))
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt &= \text{Subst}\left(\int \frac{1}{(1+t)^2\left(2-t-t^2\right)}\,dt,t,\tan(t)\right)\\
&= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \frac{t}{(1+t)\left(2-t-t^2\right)}\,dt,t,\tan(t)\right)\\
&= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{6(-1+t)}-\frac{1}{2(1+t)}+\frac{2}{3(2+t)}\right)dt,t,\tan(t)\right)\\
&= -\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t))+\frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{}{2(}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 73, normalized size = 1.62

$$-\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\sin(t)+\cos(t))-4\log(\sin(t)+2\cos(t)))+\sin(t)(\log(\cos(t)-\sin(t))+}{12(\sin(t)+\cos(t))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -1/12*(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[
t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log
[2*Cos[t] + Sin[t]])*Sin[t])/(Cos[t] + Sin[t])
```

fricas [A] time = 0.47, size = 71, normalized size = 1.58

$$\frac{4\left(\cos(t)+\sin(t)\right)\log\left(\frac{3}{4}\cos(t)^2+\cos(t)\sin(t)+\frac{1}{4}\right)-3\left(\cos(t)+\sin(t)\right)\log\left(2\cos(t)\sin(t)+1\right)-\left(\cos(t)+\sin\right.}{24\left(\cos(t)+\sin(t)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
```

```
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos(
t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*sin
(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

giac [A] time = 1.04, size = 33, normalized size = 0.73

$$-\frac{1}{2\left(\tan(t)+1\right)}+\frac{1}{3}\log\left(|\tan(t)+2|\right)-\frac{1}{4}\log\left(|\tan(t)+1|\right)-\frac{1}{12}\log\left(|\tan(t)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) - 1
/12*log(abs(tan(t) - 1))
```

maple [A] time = 0.27, size = 31, normalized size = 0.69

$$-\frac{\ln\left(\tan(t)+1\right)}{4}-\frac{\ln\left(\tan(t)-1\right)}{12}+\frac{\ln\left(\tan(t)+2\right)}{3}-\frac{1}{2\left(\tan(t)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t)
```

```
[Out] -1/2/(1+tan(t))-1/4*ln(1+tan(t))+1/3*ln(tan(t)+2)-1/12*ln(tan(t)-1)
```

maxima [B] time = 1.98, size = 256, normalized size = 5.69

$$\frac{3\left(\cos\left(2\,t\right)^2+\sin\left(2\,t\right)^2+2\,\sin\left(2\,t\right)+1\right)\log\left(953674316406250\left(3\,\cos\left(2\,t\right)+\sin\left(2\,t\right)+4\right)\cos\left(4\,t\right)+23841857\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")
```

```
[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*c
os(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 9536743164
06250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t) + 2
384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218750*
cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + s
in(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) +
 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 + 5*sin
(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t)^2 - 2*sin(2*t
) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)
```

mupad [B] time = 0.72, size = 32, normalized size = 0.71

$$\frac{\ln(\tan(t)+2)}{3}-\frac{\ln(\tan(t)+1)}{4}-\frac{\ln(\tan(t)-1)}{12}-\frac{1}{2\,(\tan(t)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*t)*(3*tan(t) + 1/cos(t)^2 + 1)),t)
```

```
[Out] log(tan(t) + 2)/3 - log(tan(t) + 1)/4 - log(tan(t) - 1)/12 - 1/(2*(tan(t) +
 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(2t)}{3\tan(t)+\sec^2(t)+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

fricas [A] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.91, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x)
```

```
[Out] 1/2*cos(x)*sin(x)+1/2*x
```

maxima [A] time = 0.67, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B] time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.89 $\int \frac{1+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

```
[Out] 2/5*x^(5/2)+2*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1+x^2}{\sqrt{x}} \, dx = \int \left(\frac{1}{\sqrt{x}} + x^{3/2}\right) dx$$
$$= 2\sqrt{x} + \frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{2}{5}\sqrt{x}\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(5 + x^2))/5
```

fricas [A] time = 0.41, size = 10, normalized size = 0.59

$$\frac{2}{5}\left(x^2+5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 5)*sqrt(x)
```

giac [A] time = 0.95, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

maple [A] time = 0.01, size = 11, normalized size = 0.65

$$\frac{2\left(x^{2}+5\right)\sqrt{x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x^(1/2),x)
[Out] 2/5*x^(1/2)*(x^2+5)
```

maxima [A] time = 0.52, size = 11, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

mupad [B] time = 0.02, size = 10, normalized size = 0.59

$$\frac{2\,\sqrt{x}\,\left(x^2+5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/x^(1/2),x)
[Out] (2*x^(1/2)*(x^2 + 5))/5
```

sympy [A] time = 0.22, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x**(1/2),x)
[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}} \, dx$

Optimal. Leaf size=23

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {640, 619, 215}

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{5+2x+x^2}} \, dx &= \sqrt{5+2x+x^2}-\int \frac{1}{\sqrt{5+2x+x^2}} \, dx\\ &= \sqrt{5+2x+x^2}-\frac{1}{4} \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}} \, dx,x,2+2x\right)\\ &= \sqrt{5+2x+x^2}-\sinh^{-1}\left(\frac{1+x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.09

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{1}{4}(2x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[5 + 2*x + x^2],x]
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(2 + 2*x)/4]
```

fricas [A] time = 0.41, size = 27, normalized size = 1.17

$$\sqrt{x^2 + 2x + 5} + \log\left(-x + \sqrt{x^2 + 2x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

giac [A] time = 1.02, size = 27, normalized size = 1.17

$$\sqrt{x^2 + 2x + 5} + \log\left(-x + \sqrt{x^2 + 2x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-\operatorname{arcsinh}\left(\frac{x}{2} + \frac{1}{2}\right) + \sqrt{x^2 + 2x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+2*x+5)^(1/2),x)
[Out] -arcsinh(1/2*x+1/2)+(x^2+2*x+5)^(1/2)
```

maxima [A] time = 1.27, size = 19, normalized size = 0.83

$$\sqrt{x^2 + 2x + 5} - \operatorname{arsinh}\left(\frac{1}{2}x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

mupad [B] time = 0.08, size = 27, normalized size = 1.17

$$\sqrt{x^2 + 2x + 5} - \ln\left(x + \sqrt{x^2 + 2x + 5} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x + x^2 + 5)^(1/2),x)
[Out] (2*x + x^2 + 5)^(1/2) - log(x + (2*x + x^2 + 5)^(1/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 + 2x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+5)**(1/2),x)
[Out] Integral(x/sqrt(x**2 + 2*x + 5), x)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^3(x)}{3}$$

[Out] 1/3*sin(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2564, 30}

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x]^2,x]

[Out] Sin[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^2(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)\\ &= \frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x]^2,x]

[Out] Sin[x]^3/3

fricas [A] time = 0.42, size = 10, normalized size = 1.25

$$-\frac{1}{3}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^2,x, algorithm="fricas")

[Out] -1/3*(cos(x)^2 - 1)*sin(x)

giac [A] time = 0.99, size = 6, normalized size = 0.75

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sin^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] 1/3*sin(x)^3
```

maxima [A] time = 0.51, size = 6, normalized size = 0.75

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] sin(x)^3/3
```

sympy [A] time = 0.06, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2,x)
```

```
[Out] sin(x)**3/3
```

3.92 $\int \frac{e^x}{1+e^x} dx$

Optimal. Leaf size=6

$$\log(e^x + 1)$$

```
[Out] ln(1+exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2246, 31}

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+e^x}\, dx &= \text{Subst}\left(\int \frac{1}{1+x}\, dx, x, e^x\right)\\ &= \log(1+e^x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

fricas [A] time = 0.41, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

giac [A] time = 0.97, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
[Out] log(e^x + 1)
```

maple [A] time = 0.00, size = 6, normalized size = 1.00

$$\ln\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x)+1),x)
[Out] ln(exp(x)+1)
```

maxima [A] time = 0.55, size = 5, normalized size = 0.83

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
[Out] log(e^x + 1)
```

mupad [B] time = 0.00, size = 5, normalized size = 0.83

$$\ln\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x) + 1),x)
[Out] log(exp(x) + 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.83

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
[Out] log(exp(x) + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x}\,dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2x}}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, e^x\right)\\ &= e^x - \log(1+e^x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
[Out] e^x - log(e^x + 1)
```

giac [A] time = 0.98, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
[Out] e^x - log(e^x + 1)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\mathrm{e}^{x} - \ln(\mathrm{e}^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x)+1),x)
[Out] exp(x)-ln(exp(x)+1)
```

maxima [A] time = 0.69, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
[Out] e^x - log(e^x + 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.83

$$\mathrm{e}^{x} - \ln(\mathrm{e}^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x) + 1),x)
[Out] exp(x) - log(exp(x) + 1)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
[Out] exp(x) - log(exp(x) + 1)
```

3.94 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 0.87, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.02, size = 9, normalized size = 0.75

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
[Out] -1/tan(1/2*x)
```

maxima [A] time = 0.76, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

mupad [B] time = 0.26, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

sympy [A] time = 0.34, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

fricas [A] time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

giac [A] time = 0.86, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sec^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x)
```

```
[Out] 1/2*sec(x)^2
```

maxima [A] time = 0.67, size = 6, normalized size = 0.75

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

mupad [B] time = 0.00, size = 6, normalized size = 0.75

$$\frac{\tan(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^2,x)
```

```
[Out] tan(x)^2/2
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

3.96 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2304}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2} \, x^2 \log(x) - \frac{1}{4} \, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

giac [A] time = 0.97, size = 13, normalized size = 0.76

$$\frac{1}{2} \, x^2 \log(x) - \frac{1}{4} \, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x^2 \ln(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
[Out] 1/2*x^2*ln(x)-1/4*x^2
```

maxima [A] time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{2}\, x^2 \log(x) - \frac{1}{4}\, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

mupad [B] time = 0.00, size = 9, normalized size = 0.53

$$\frac{x^2 \left(\ln(x) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^2 \log{(x)}}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

3.97 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] 1/2*sin(x)^2

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x],x]

[Out] -1/2*Cos[x]^2

fricas [A] time = 0.44, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)^2

giac [A] time = 0.90, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sin^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] 1/2*sin(x)^2
```

maxima [A] time = 0.54, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

mupad [B] time = 0.00, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

sympy [A] time = 0.06, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}} \, dx$

Optimal. Leaf size=24

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

```
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {640, 619, 216}

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\sqrt{2x-x^2}}\,dx &= -\sqrt{2x-x^2}+2\int \frac{1}{\sqrt{2x-x^2}}\,dx\\ &= -\sqrt{2x-x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,2-2x\right)\\ &= -\sqrt{2x-x^2}-2\sin^{-1}(1-x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.12

$$-\sqrt{-((x-2)x)}-4\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)] - 4*ArcSin[Sqrt[1 - x/2]]
```

fricas [A] time = 0.41, size = 32, normalized size = 1.33

$$-\sqrt{-x^2+2x}-4\arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

giac [A] time = 1.03, size = 20, normalized size = 0.83

$$-\sqrt{-x^2+2x}+2\arcsin(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$2\arcsin(x-1)-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(-x^2+2*x)^(1/2),x)
[Out] 2*arcsin(x-1)-(-x^2+2*x)^(1/2)
```

maxima [A] time = 1.20, size = 22, normalized size = 0.92

$$-\sqrt{-x^2+2x}-2\arcsin(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

mupad [B] time = 0.27, size = 20, normalized size = 0.83

$$2\operatorname{asin}(x-1)-\sqrt{2x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x - x^2)^(1/2),x)
[Out] 2*asin(x - 1) - (2*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{-x(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+2*x)**(1/2),x)
[Out] Integral((x + 1)/sqrt(-x*(x - 2)), x)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

```
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {12, 2249, 203}

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*E^x)/(2 + 3*E^(2*x)),x]
```

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2e^x}{2+3e^{2x}}\,dx &= 2\int \frac{e^x}{2+3e^{2x}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)\\ &= \sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

fricas [A] time = 0.42, size = 19, normalized size = 0.95

$$\frac{1}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

giac [A] time = 0.98, size = 13, normalized size = 0.65

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

maple [A] time = 0.00, size = 14, normalized size = 0.70

$$\frac{\sqrt{6}\arctan\left(\frac{\sqrt{6}\,{\mathrm e}^{x}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)/(2+3*exp(2*x)),x)
[Out] 1/3*6^(1/2)*arctan(1/2*6^(1/2)*exp(x))
```

maxima [A] time = 1.34, size = 13, normalized size = 0.65

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

mupad [B] time = 0.09, size = 13, normalized size = 0.65

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,{\mathrm{e}}^x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*exp(x))/(3*exp(2*x) + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/3
```

sympy [A] time = 0.12, size = 15, normalized size = 0.75

$$\operatorname{RootSum}{\left(6 z^{2} + 1, \left(i \mapsto i \log{\left(2 i + e^{x} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

3.100 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

fricas [B] time = 0.42, size = 63, normalized size = 1.80

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

giac [A] time = 0.91, size = 29, normalized size = 0.83

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}\,x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.86

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3/(-x^2+1)^(3/2)*x^3-1/(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A] time = 1.33, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

mupad [B] time = 0.00, size = 91, normalized size = 2.60

$$\mathrm{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

sympy [B] time = 2.14, size = 105, normalized size = 3.00

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3\sqrt{1-x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x\sqrt{1-x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}}\,dx$

Optimal. Leaf size=20

$$\frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

```
[Out] 1/2*exp(2*x)-1/2*arctan(exp(2*x))
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2248, 321, 203}

$$\frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(6*x)/(1 + E^(4*x)),x]
```

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{6x}}{1+e^{4x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,e^{2x}\right)\\ &= \frac{e^{2x}}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^{2x}\right)\\ &= \frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$\frac{1}{2}\left(e^{2x}-\tan^{-1}\left(e^{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(6*x)/(1 + E^(4*x)),x]
[Out] (E^(2*x) - ArcTan[E^(2*x)])/2
```

fricas [A] time = 0.42, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="fricas")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

giac [A] time = 0.92, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.75

$$-\frac{\arctan\left(e^{2x}\right)}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6*x)/(1+exp(4*x)),x)
[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="maxima")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

mupad [B] time = 0.07, size = 14, normalized size = 0.70

$$\frac{e^{2x}}{2} - \frac{\mathrm{atan}\left(e^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6*x)/(exp(4*x) + 1),x)
[Out] exp(2*x)/2 - atan(exp(2*x))/2
```

sympy [A] time = 0.12, size = 24, normalized size = 1.20

$$\frac{e^{2x}}{2} + \operatorname{RootSum}\left(16z^2 + 1, \left(i \mapsto i\log\left(-4i + e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x)
```

```
[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))
```

3.102 $\int \log\left(2+3x^2\right)\,dx$

Optimal. Leaf size=33

$$x \log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x \log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(2+3x^2\right)\,dx &= x\log\left(2+3x^2\right)-6\int \frac{x^2}{2+3x^2}\,dx\\ &= -2x+x\log\left(2+3x^2\right)+4\int \frac{1}{2+3x^2}\,dx\\ &= -2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$x \log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

fricas [A] time = 0.42, size = 32, normalized size = 0.97

$$\frac{2}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)+x\log\left(3x^2+2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

giac [A] time = 1.12, size = 26, normalized size = 0.79

$$x\log\left(3x^2+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

maple [A] time = 0.00, size = 27, normalized size = 0.82

$$x\ln\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\arctan\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x)
[Out] x*ln(3*x^2+2)-2*x+2/3*6^(1/2)*arctan(1/2*6^(1/2)*x)
```

maxima [A] time = 1.27, size = 26, normalized size = 0.79

$$x\log\left(3x^2+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

mupad [B] time = 0.00, size = 26, normalized size = 0.79

$$\frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}-2x+x\ln\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(3*x^2 + 2),x)
[Out] (2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.94

$$x\log\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {8}

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

fricas [A] time = 0.41, size = 31, normalized size = 1.48

$$\frac{\sqrt{2\,Hr^2-a^2}\,x}{2\,Hr^3-a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)
```

giac [A] time = 0.96, size = 19, normalized size = 0.90

$$\frac{x}{\sqrt{2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

maple [A] time = 0.00, size = 20, normalized size = 0.95

$$\frac{x}{\sqrt{2H\,r^2-a^2}\;r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2)^(1/2),x)
```

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

maxima [A] time = 0.50, size = 19, normalized size = 0.90

$$\frac{x}{\sqrt{2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

mupad [B] time = 0.00, size = 19, normalized size = 0.90

$$\frac{x}{r\,\sqrt{2\,H\,r^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - a^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - a^2)^(1/2))
```

sympy [A] time = 0.06, size = 15, normalized size = 0.71

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2))
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

fricas [A] time = 0.39, size = 40, normalized size = 1.54

$$\frac{\sqrt{2\,Hr^2-a^2-e^2}\,x}{2\,Hr^3-\left(a^2+e^2\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - (a^2 + e^2)*r)
```

giac [A] time = 0.92, size = 23, normalized size = 0.88

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

maple [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{x}{\sqrt{2H\,r^2-a^2-e^2}\;r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x)
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

maxima [A] time = 0.68, size = 24, normalized size = 0.92

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

mupad [B] time = 0.00, size = 24, normalized size = 0.92

$$\frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - a^2 - e^2)^(1/2))
```

sympy [A] time = 0.06, size = 19, normalized size = 0.73

$$\frac{x}{r\sqrt{2Hr^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

fricas [A] time = 0.39, size = 43, normalized size = 1.59

$$-\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}\,x}{2\,Kr^5-2\,Hr^3+a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)
```

giac [A] time = 0.88, size = 25, normalized size = 0.93

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{x}{\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}}\;r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x)
```

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

maxima [A] time = 0.79, size = 25, normalized size = 0.93

$$\frac{x}{\sqrt{-2\,Kr^{4}+2\,Hr^{2}-a^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

mupad [B] time = 0.00, size = 25, normalized size = 0.93

$$\frac{x}{r\,\sqrt{-a^{2}-2\,K\,r^{4}+2\,H\,r^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2))
```

sympy [A] time = 0.06, size = 22, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^{2}-2Kr^{4}-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=32

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

fricas [A] time = 0.40, size = 52, normalized size = 1.62

$$-\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}\,x}{2\,Kr^5-2\,Hr^3+\left(a^2+e^2\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + (a^2 + e^2)*r)
```

giac [A] time = 0.89, size = 29, normalized size = 0.91

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

maple [A] time = 0.00, size = 31, normalized size = 0.97

$$\frac{x}{\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}\;r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x)
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

maxima [A] time = 0.54, size = 30, normalized size = 0.94

$$\frac{x}{\sqrt{-2\,Kr^{4}+2\,Hr^{2}-a^{2}-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

mupad [B] time = 0.00, size = 30, normalized size = 0.94

$$\frac{x}{r\,\sqrt{-a^{2}-e^{2}-2\,K\,r^{4}+2\,H\,r^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2))
```

sympy [A] time = 0.06, size = 26, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^{2}-2Kr^{4}-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[Out] x/r/(-a^2-2*r*(-H*r+K))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.04

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

fricas [A] time = 0.41, size = 41, normalized size = 1.71

$$\frac{\sqrt{2\,Hr^2-a^2-2\,Kr}\,x}{2\,Hr^3-a^2r-2\,Kr^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)
```

giac [A] time = 0.94, size = 23, normalized size = 0.96

$$\frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

maple [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x}{\sqrt{2H r^2 - 2Kr - a^2}\ r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x)
[Out] 1/r/(2*H*r^2-2*K*r-a^2)^(1/2)*x
```

maxima [A] time = 0.61, size = 23, normalized size = 0.96

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

mupad [B] time = 0.00, size = 23, normalized size = 0.96

$$\frac{x}{r\,\sqrt{-a^2 + 2\,H\,r^2 - 2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2))
```

sympy [A] time = 0.06, size = 20, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}} \, dx$

Optimal. Leaf size=29

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{8\}$

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}} \, dx = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.03

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

fricas [A] time = 0.41, size = 50, normalized size = 1.72

$$\frac{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}\,x}{2\,Hr^3-2\,Kr^2-\left(a^2+e^2\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*x/(2*H*r^3 - 2*K*r^2 - (a^2 + e^2)*r)
```

giac [A] time = 0.97, size = 27, normalized size = 0.93

$$\frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr-e^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)
```

maple [A] time = 0.00, size = 29, normalized size = 1.00

$$\frac{x}{\sqrt{2H r^2-2Kr-a^2-e^2}\ r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

maxima [A] time = 0.58, size = 28, normalized size = 0.97

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

mupad [B] time = 0.00, size = 28, normalized size = 0.97

$$\frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2))
```

sympy [A] time = 0.06, size = 24, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx$

Optimal. Leaf size=19

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

```
[Out] r*x/(2*E*r^2-a^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {8}

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2}}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

fricas [A] time = 0.39, size = 17, normalized size = 0.89

$$\frac{rx}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="fricas")
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

giac [A] time = 0.98, size = 17, normalized size = 0.89

$$\frac{rx}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{rx}{\sqrt{2E\,r^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*E*r^2-a^2)^(1/2),x)
[Out] r*x/(2*E*r^2-a^2)^(1/2)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.89

$$\frac{rx}{\sqrt{2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

mupad [B] time = 0.00, size = 18, normalized size = 0.95

$$\frac{r\,x}{\sqrt{2\,r^2\,\mathrm{e} - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - a^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - a^2)^(1/2)
```

sympy [A] time = 0.05, size = 17, normalized size = 0.89

$$\frac{rx}{\sqrt{-a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r**2-a**2)**(1/2),x)
[Out] r*x/sqrt(-a**2 + 2*E*r**2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

Optimal. Leaf size=24

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[Out] r*x/(2*E*r^2-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

fricas [A] time = 0.40, size = 22, normalized size = 0.92

$$\frac{rx}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

giac [A] time = 0.98, size = 21, normalized size = 0.88

$$\frac{rx}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{rx}{\sqrt{2E\,r^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*E*r^2-a^2-e^2)^(1/2),x)
[Out] r*x/(2*E*r^2-a^2-e^2)^(1/2)
```

maxima [A] time = 0.73, size = 22, normalized size = 0.92

$$\frac{rx}{\sqrt{2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

mupad [B] time = 0.00, size = 23, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2 - e^2 + 2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - a^2 - e^2)^(1/2)
```

sympy [A] time = 0.06, size = 20, normalized size = 0.83

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r**2-a**2-e**2)**(1/2),x)
[Out] r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=25

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

```
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

fricas [A] time = 0.40, size = 42, normalized size = 1.68

$$-\frac{\sqrt{-2\,Kr^4+2\,Er^2-a^2}\,rx}{2\,Kr^4-2\,Er^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*E*r^2 - a^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2)
```

giac [A] time = 0.92, size = 23, normalized size = 0.92

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

maple [A]   time = 0.00, size = 24, normalized size = 0.96

$$\frac{rx}{\sqrt{-2K\,r^4 + 2E\,r^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x)
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2)^(1/2)
```

maxima [A]   time = 0.47, size = 23, normalized size = 0.92

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

mupad [B]   time = 0.00, size = 24, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2 - 2\,K\,r^4 + 2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2)
```

sympy [A]   time = 0.06, size = 24, normalized size = 0.96

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r**4+2*E*r**2-a**2)**(1/2),x)
[Out] r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}} \, dx$

Optimal. Leaf size=30

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

```
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}} \, dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

fricas [A] time = 0.39, size = 50, normalized size = 1.67

$$-\frac{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}\,rx}{2\,Kr^4-2\,Er^2+a^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2 + e^2)
```

giac [A] time = 1.08, size = 27, normalized size = 0.90

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$\frac{rx}{\sqrt{-2K\,r^4 + 2E\,r^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x)
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2)
```

maxima [A] time = 0.55, size = 28, normalized size = 0.93

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

mupad [B] time = 0.00, size = 29, normalized size = 0.97

$$\frac{r\,x}{\sqrt{-a^2 - e^2 - 2\,K\,r^4 + 2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2)
```

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r**4+2*E*r**2-a**2-e**2)**(1/2),x)
[Out] r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=27

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.04

$$\frac{rx}{\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.96

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

giac [A] time = 1.28, size = 25, normalized size = 0.93

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-2\,Kr-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

maple [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

maxima [A] time = 0.65, size = 26, normalized size = 0.96

$$\frac{rx}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

mupad [B] time = 0.00, size = 26, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2 - e^2 + 2\,H\,r^2 - 2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)
```

sympy [A] time = 0.06, size = 24, normalized size = 0.89

$$\frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
[Out] r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```