Computer algebra independent integration tests

0-Independent-test-suites/Hearn-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [284]. This is test number [5].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.24 (279)	% 1.76 (5)
Mathematica	% 99.65 (283)	% 0.35 (1)
Maple	% 99.30 (282)	% 0.70 (2)
Maxima	% 88.73 (252)	% 11.27 (32)
Fricas	% 98.59 (280)	% 1.41 (4)
Sympy	% 87.68 (249)	% 12.32 (35)
Giac	% 94.72 (269)	% 5.28 (15)
Mupad	% 95.07 (270)	% 4.93 (14)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

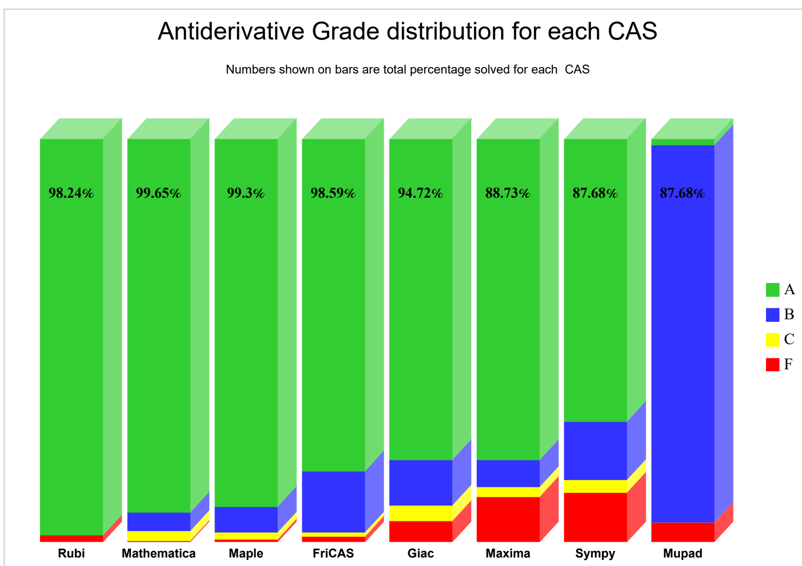
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

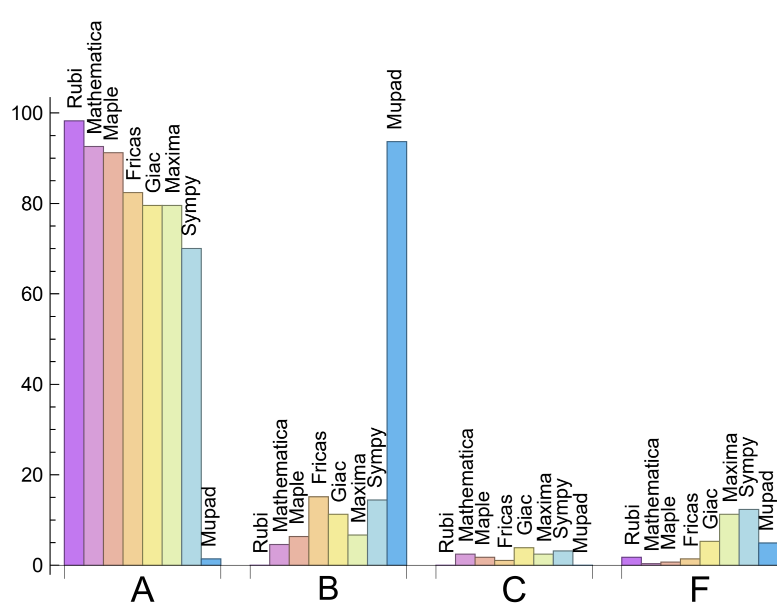
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.24	0.00	0.00	1.76
Mathematica	92.61	4.58	2.46	0.35
Maple	91.20	6.34	1.76	0.70
Maxima	79.58	6.69	2.46	11.27
Fricas	82.39	15.14	1.06	1.41
Sympy	70.07	14.44	3.17	12.32
Giac	79.58	11.27	3.87	5.28
Mupad	1.41	93.66	0.00	4.93

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	5	100.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	32	68.75 %	0.00 %	31.25 %
Fricas	4	75.00 %	0.00 %	25.00 %
Sympy	35	80.00 %	17.14 %	2.86 %
Giac	15	100.00 %	0.00 %	0.00 %
Mupad	14	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

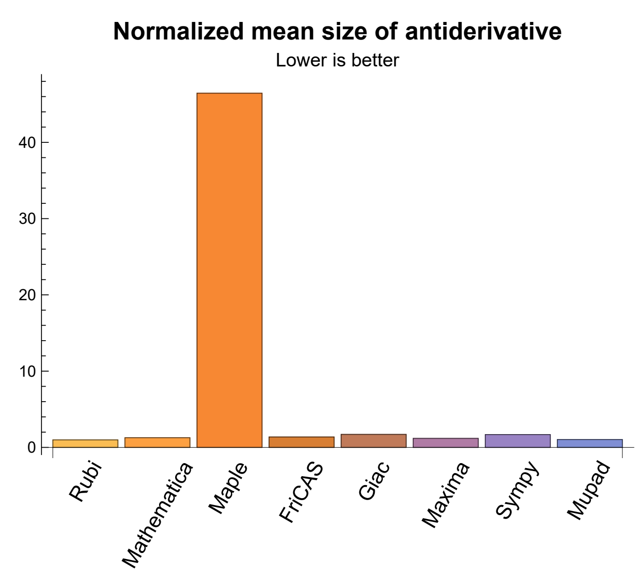
1.3 Performance

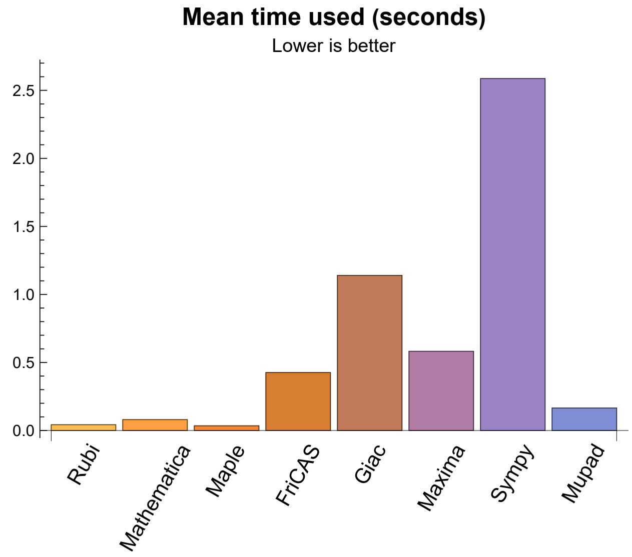
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	39.11	0.99	22.00	1.00
Mathematica	0.08	65.92	1.27	23.00	1.00
Maple	0.03	4314.29	46.45	21.00	0.94
Maxima	0.58	35.04	1.18	19.00	0.88
Fricas	0.43	49.70	1.37	22.00	1.00
Sympy	2.59	74.24	1.68	20.00	1.00
Giac	1.14	104.49	1.70	22.00	0.96
Mupad	0.17	35.62	1.03	18.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{75, 145, 170, 273}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {20, 279, 281}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

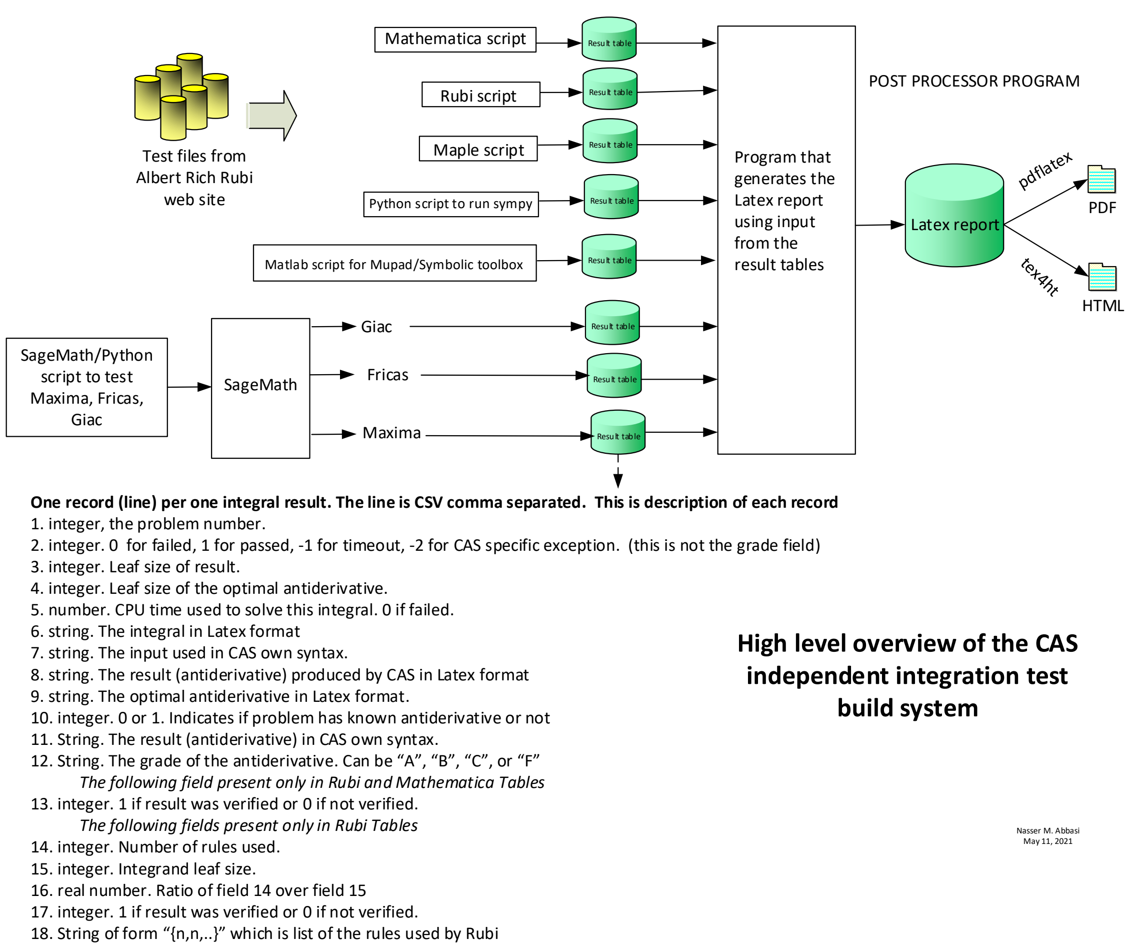
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283 }

B grade: { }

C grade: { }

F grade: { 169, 278, 279, 281, 284 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 52, 81, 82, 108, 111, 120, 121, 190, 202, 235, 236, 256, 280 }

C grade: { 20, 44, 45, 51, 278, 279, 281 }

F grade: { 60 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 281, 283, 284 }

B grade: { 32, 44, 45, 60, 61, 110, 124, 126, 134, 135, 176, 204, 237, 242, 256, 257, 278, 280 }

C grade: { 49, 51, 203, 279, 282 }

F grade: { 86, 251 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 164, 165, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 212, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 283 }

B grade: { 32, 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }

C grade: { 102, 103, 104, 105, 166, 168, 197 }

F grade: { 8, 39, 40, 42, 43, 44, 45, 49, 51, 86, 122, 123, 147, 160, 161, 163, 176, 196, 203, 211, 213, 214, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 43, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 279, 282, 283 }

B grade: { 36, 37, 39, 40, 41, 42, 44, 45, 49, 78, 79, 80, 81, 82, 90, 103, 110, 111, 112, 127, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

C grade: { 128, 129, 197 }

F grade: { 86, 174, 257, 281 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 130, 132, 134, 135, 136, 137, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 181, 182, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 205, 206, 209, 210, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 273, 274, 276, 277, 282, 283, 284 }

B grade: { 7, 8, 13, 32, 39, 41, 44, 80, 81, 82, 90, 103, 104, 131, 133, 138, 139, 140, 141, 142, 143, 175, 178, 179, 180, 183, 184, 189, 204, 220, 222, 228, 229, 233, 234, 239, 246, 247, 254, 256, 272 }

C grade: { 9, 14, 31, 50, 70, 72, 73, 203, 224 }

F grade: { 12, 86, 123, 128, 129, 145, 146, 147, 160, 162, 163, 176, 196, 197, 198, 207, 208, 211, 212, 213, 214, 235, 236, 237, 238, 242, 249, 251, 257, 265, 275, 278, 279, 280, 281 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 11, 23, 32, 67, 79, 81, 82, 110, 111, 112, 118, 127, 164, 176, 178, 179, 195, 198, 199, 220, 222, 228, 235, 236, 245, 246, 247, 256, 260, 268, 269, 280 }

C grade: { 134, 135, 136, 137, 138, 139, 140, 141, 160, 161, 166 }

F grade: { 56, 63, 86, 128, 129, 162, 163, 197, 200, 201, 203, 257, 278, 279, 281 }

2.1.8 Mupad

A grade: { 75, 145, 170, 273 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

C grade: { }

F grade: { 102, 103, 104, 105, 128, 129, 163, 169, 203, 249, 257, 278, 279, 281 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.002	0.000	0.002	0.418	0.389	0.053	1.235	0.021

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	15
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.68
time (sec)	N/A	0.010	0.001	0.001	0.421	0.372	0.058	1.011	0.039

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	15
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.68
time (sec)	N/A	0.004	0.001	0.001	0.420	0.377	0.058	1.068	0.029

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.000	0.417	0.401	0.057	1.120	0.009

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	32	46	29	23	22
normalized size	1	1.00	0.67	0.75	0.89	1.28	0.81	0.64	0.61
time (sec)	N/A	0.013	0.012	0.008	0.423	0.398	0.116	1.009	0.118

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	33	24	34	26
normalized size	1	1.00	0.88	0.78	0.75	1.03	0.75	1.06	0.81
time (sec)	N/A	0.012	0.012	0.010	0.445	0.437	0.149	1.150	0.034

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	66	40	34	144	42	40
normalized size	1	1.00	0.85	1.65	1.00	0.85	3.60	1.05	1.00
time (sec)	N/A	0.027	0.017	0.008	0.452	0.411	0.879	1.182	0.250

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
normalized size	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.030	0.011	0.008	0.000	0.415	0.208	1.348	0.198

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	26	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.88	1.62	0.88	0.88
time (sec)	N/A	0.006	0.006	0.002	0.970	0.396	0.161	1.191	0.038

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	17	14	14	22	14	14
normalized size	1	1.00	0.84	0.89	0.74	0.74	1.16	0.74	0.74
time (sec)	N/A	0.010	0.005	0.005	0.950	0.423	0.108	1.211	0.031

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	39	71	41	80	49
normalized size	1	1.00	0.71	0.73	0.80	1.45	0.84	1.63	1.00
time (sec)	N/A	0.029	0.023	0.013	0.952	0.415	0.184	1.039	0.126

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	78	81	0	81	87
normalized size	1	1.00	0.91	1.01	1.15	1.19	0.00	1.19	1.28
time (sec)	N/A	0.052	0.032	0.010	0.422	0.495	0.000	1.124	0.568

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	44	43	32	121	43	256
normalized size	1	1.00	0.72	0.94	0.91	0.68	2.57	0.91	5.45
time (sec)	N/A	0.020	0.010	0.008	0.425	0.414	0.705	1.055	0.304

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	40	30	393	40	191
normalized size	1	1.00	0.75	1.02	1.00	0.75	9.82	1.00	4.78
time (sec)	N/A	0.019	0.015	0.012	0.968	0.430	1.257	1.166	0.175

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.74	0.70	0.70	0.70	0.74	0.93
time (sec)	N/A	0.018	0.005	0.005	0.950	0.409	0.128	1.159	0.046

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	34	34	41	35	46
normalized size	1	1.00	0.98	0.85	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.020	0.009	0.008	0.948	0.421	0.134	1.192	0.206

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	31	40	31	36	33
normalized size	1	1.00	0.79	0.74	0.72	0.93	0.72	0.84	0.77
time (sec)	N/A	0.130	0.021	0.010	0.950	0.417	0.158	1.046	0.043

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	72	95	73	72	33
normalized size	1	1.00	0.75	0.68	0.85	1.12	0.86	0.85	0.39
time (sec)	N/A	0.041	0.019	0.000	0.973	0.428	0.155	1.308	0.002

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	72	95	73	72	33
normalized size	1	1.00	0.75	0.68	0.85	1.12	0.86	0.85	0.39
time (sec)	N/A	0.041	0.010	0.003	0.977	0.425	0.152	0.993	0.205

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	53	53	70	53	47
normalized size	1	1.00	1.09	0.81	0.79	0.79	1.04	0.79	0.70
time (sec)	N/A	0.036	0.056	0.003	0.951	0.425	0.193	1.016	0.179

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	18	18
normalized size	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.004	0.010	0.001	0.415	0.430	0.061	0.992	0.220

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	76	94
normalized size	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	2.41
time (sec)	N/A	0.013	0.019	0.003	0.431	0.431	0.651	1.237	0.392

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	140	192
normalized size	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	3.20
time (sec)	N/A	0.021	0.030	0.007	0.431	0.441	1.235	1.219	0.601

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.001	0.001	0.411	0.394	0.061	1.094	0.026

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
normalized size	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.002	0.003	0.000	0.418	0.396	0.127	1.059	0.118

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
normalized size	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.010	0.003	0.003	0.415	0.397	0.099	1.191	0.035

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
normalized size	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.016	0.003	0.004	0.413	0.392	0.111	0.795	0.127

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	20	15
normalized size	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.004	0.004	0.006	0.412	0.428	0.141	1.201	0.134

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.013	0.005	0.010	0.412	0.409	0.175	1.090	0.054

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	45
normalized size	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07
time (sec)	N/A	0.023	0.039	0.013	0.417	0.401	0.256	1.148	0.188

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.007	0.952	0.400	0.110	1.174	0.037

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	22	21	18	15	23	10
normalized size	1	1.00	1.00	2.20	2.10	1.80	1.50	2.30	1.00
time (sec)	N/A	0.003	0.003	0.007	0.431	0.399	0.119	0.946	0.159

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	58	66	63	78	57	72
normalized size	1	1.00	0.85	0.74	0.85	0.81	1.00	0.73	0.92
time (sec)	N/A	0.044	0.022	0.005	0.959	0.409	0.298	1.012	0.277

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	54	56	68	71	57	72
normalized size	1	1.00	0.88	0.73	0.76	0.92	0.96	0.77	0.97
time (sec)	N/A	0.034	0.018	0.004	0.959	0.405	0.296	1.277	0.356

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	97	300	20	104	101
normalized size	1	1.00	0.77	0.80	0.84	2.61	0.17	0.90	0.88
time (sec)	N/A	0.066	0.026	0.003	0.961	0.426	0.154	1.269	0.315

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	35	34	63	46	39	20
normalized size	1	1.00	1.23	1.00	0.97	1.80	1.31	1.11	0.57
time (sec)	N/A	0.011	0.019	0.003	0.978	0.438	0.312	1.294	0.158

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	41	58	48	39	18
normalized size	1	1.00	1.23	1.03	1.17	1.66	1.37	1.11	0.51
time (sec)	N/A	0.014	0.015	0.004	0.979	0.440	0.317	1.084	0.175

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	120	111	151	163	151	95	45
normalized size	1	1.00	0.70	0.65	0.88	0.95	0.88	0.56	0.26
time (sec)	N/A	0.108	0.049	0.006	0.969	0.434	0.403	1.168	0.111

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	146	74	93
normalized size	1	1.00	0.93	0.77	0.00	1.81	2.00	1.01	1.27
time (sec)	N/A	0.074	0.053	0.055	0.000	0.417	0.450	1.275	0.224

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	24	74	93
normalized size	1	1.00	0.93	0.77	0.00	1.81	0.33	1.01	1.27
time (sec)	N/A	0.017	0.027	0.043	0.000	0.434	0.354	1.465	0.135

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	75	91	158	81	67
normalized size	1	1.00	1.15	0.75	1.04	1.26	2.19	1.12	0.93
time (sec)	N/A	0.065	0.037	0.006	0.961	0.421	0.355	1.282	0.098

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	123	24	101	98
normalized size	1	1.00	1.00	0.90	0.00	1.84	0.36	1.51	1.46
time (sec)	N/A	0.052	0.031	0.044	0.000	0.430	0.367	1.232	0.231

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	87	92	51	117
normalized size	1	1.00	1.00	0.90	0.00	1.30	1.37	0.76	1.75
time (sec)	N/A	0.012	0.020	0.041	0.000	0.440	0.215	1.153	0.197

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	297	994	248	61
normalized size	1	1.00	0.46	1.97	0.00	1.52	5.07	1.27	0.31
time (sec)	N/A	0.148	0.053	0.118	0.000	0.433	1.138	1.812	0.225

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	275	24	252	132
normalized size	1	1.00	0.46	1.97	0.00	1.40	0.12	1.29	0.67
time (sec)	N/A	0.139	0.083	0.110	0.000	0.444	0.545	1.999	0.110

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	66	65	65	83	67	88
normalized size	1	1.00	1.03	0.90	0.89	0.89	1.14	0.92	1.21
time (sec)	N/A	0.100	0.017	0.009	0.949	0.417	0.254	1.124	0.091

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	122	111	112	175	14	114	140
normalized size	1	1.00	0.88	0.80	0.81	1.27	0.10	0.83	1.01
time (sec)	N/A	0.192	0.039	0.156	0.955	0.437	0.566	1.306	0.210

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	115	95	107	177	14	107	135
normalized size	1	1.00	0.83	0.69	0.78	1.28	0.10	0.78	0.98
time (sec)	N/A	0.281	0.026	0.194	0.975	0.430	0.278	1.250	0.129

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	1027	14	239	288
normalized size	1	1.00	0.62	0.06	0.00	3.03	0.04	0.71	0.85
time (sec)	N/A	0.235	0.007	0.004	0.000	0.431	2.824	1.035	0.304

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	66	88	111	44	90	45
normalized size	1	1.00	1.01	0.68	0.91	1.14	0.45	0.93	0.46
time (sec)	N/A	0.052	0.024	0.004	0.954	0.438	161.868	0.959	0.165

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	215	165	205	53
normalized size	1	1.00	0.15	0.11	0.00	0.78	0.60	0.75	0.19
time (sec)	N/A	0.262	0.012	0.012	0.000	0.451	0.222	1.060	0.100

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	40	52
normalized size	1	1.00	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.037	0.115	0.007	0.961	0.429	0.172	1.134	0.219

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75
time (sec)	N/A	0.001	0.001	0.001	0.409	0.439	0.080	1.070	0.023

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.004	0.001	0.000	0.408	0.406	0.088	1.104	0.034

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.006	0.001	0.007	0.445	0.440	0.091	1.106	0.032

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	34	26	25	56	0	32
normalized size	1	1.00	0.73	1.31	1.00	0.96	2.15	0.00	1.23
time (sec)	N/A	0.010	0.008	0.016	0.419	0.437	0.737	0.000	0.239

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
normalized size	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.004	0.001	0.000	0.433	0.420	0.093	0.914	0.030

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	104	71	103	133	103	71
normalized size	1	1.00	1.00	0.82	0.56	0.81	1.05	0.81	0.56
time (sec)	N/A	0.141	0.004	0.003	0.422	0.426	0.289	1.024	0.177

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.012	0.001	0.001	0.417	0.426	0.086	1.071	0.060

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	A	A	A	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	0	9	3	2	2	3	2
normalized size	1	1.00	0.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.002	0.002	0.007	0.545	0.413	0.459	1.175	0.004

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	5	11	5	4	3	5	4
normalized size	1	1.00	1.25	2.75	1.25	1.00	0.75	1.25	1.00
time (sec)	N/A	0.004	0.011	0.005	0.529	0.410	0.493	1.096	0.016

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.012	0.004	0.000	0.411	0.404	0.096	1.230	0.120

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	6	19	14	0	17
normalized size	1	1.00	1.00	0.88	0.35	1.12	0.82	0.00	1.00
time (sec)	N/A	0.035	0.028	0.012	0.519	0.402	0.653	0.000	0.030

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	22
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.83
time (sec)	N/A	0.016	0.003	0.004	0.411	0.439	0.881	1.227	0.146

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	28	25	25	25	22	24	21
normalized size	1	1.04	1.00	0.89	0.89	0.89	0.79	0.86	0.75
time (sec)	N/A	0.011	0.001	0.006	0.416	0.437	0.114	1.184	0.141

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	48	47	47	44	47	47
normalized size	1	1.00	0.98	0.89	0.87	0.87	0.81	0.87	0.87
time (sec)	N/A	0.028	0.010	0.003	0.418	0.405	0.137	1.102	0.170

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	138	35
normalized size	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.21
time (sec)	N/A	0.013	0.012	0.010	0.414	0.431	0.358	1.194	0.232

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	47	44	39	42	58	66
normalized size	1	1.00	1.00	1.02	0.96	0.85	0.91	1.26	1.43
time (sec)	N/A	0.023	0.015	0.003	0.414	0.405	0.176	1.316	0.189

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	58	57	49	54	94	75
normalized size	1	1.00	1.00	0.98	0.97	0.83	0.92	1.59	1.27
time (sec)	N/A	0.032	0.019	0.005	0.414	0.421	0.191	1.266	0.182

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	36	23	23
normalized size	1	1.00	1.00	1.04	1.00	1.00	1.57	1.00	1.00
time (sec)	N/A	0.007	0.002	0.010	0.958	0.429	0.161	1.167	0.066

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	29	28	23	31	28	51
normalized size	1	1.00	0.96	1.07	1.04	0.85	1.15	1.04	1.89
time (sec)	N/A	0.020	0.003	0.001	0.420	0.423	0.160	1.072	0.041

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	36	53	36	65
normalized size	1	1.00	1.00	0.84	0.82	0.82	1.20	0.82	1.48
time (sec)	N/A	0.023	0.002	0.006	0.965	0.431	0.194	1.036	0.036

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	63	44	73
normalized size	1	1.00	1.00	0.83	0.81	0.81	1.17	0.81	1.35
time (sec)	N/A	0.025	0.003	0.005	0.957	0.404	0.209	0.996	0.128

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	31	31	29	33	25
normalized size	1	1.00	1.00	1.28	1.24	1.24	1.16	1.32	1.00
time (sec)	N/A	0.010	0.002	0.013	0.423	0.427	0.163	1.225	0.074

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12
time (sec)	N/A	0.006	0.029	0.027	0.000	0.441	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.002	0.001	0.000	0.424	0.419	0.061	1.031	0.021

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.002	0.414	0.419	0.060	0.958	0.027

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	5
normalized size	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.00
time (sec)	N/A	0.002	0.002	0.001	0.429	0.434	0.065	1.045	0.029

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	16	3	17	13
normalized size	1	1.00	1.00	1.33	1.00	5.33	1.00	5.67	4.33
time (sec)	N/A	0.002	0.002	0.002	0.417	0.431	0.068	1.130	0.197

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	26	25	37	75	26	27
normalized size	1	1.00	1.29	1.24	1.19	1.76	3.57	1.24	1.29
time (sec)	N/A	0.026	0.042	0.029	0.961	0.441	0.360	1.174	0.246

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	15	17	15	17	11
normalized size	1	1.00	11.00	2.33	5.00	5.67	5.00	5.67	3.67
time (sec)	N/A	0.003	0.005	0.016	0.416	0.430	0.108	1.129	0.129

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	15	19	15	17	5
normalized size	1	1.00	3.40	1.80	3.00	3.80	3.00	3.40	1.00
time (sec)	N/A	0.002	0.003	0.017	0.415	0.421	0.109	1.181	0.045

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.005	0.002	0.000	0.432	0.435	0.063	1.197	0.032

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.016	0.002	0.006	0.423	0.438	0.569	1.037	0.158

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
normalized size	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.005	0.002	0.000	0.412	0.444	0.065	0.944	0.034

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	35
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.008	0.039	0.527	0.000	0.462	0.000	0.000	0.315

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	18	17	15	15
normalized size	1	1.00	1.00	0.84	0.79	0.95	0.89	0.79	0.79
time (sec)	N/A	0.019	0.004	0.025	0.415	0.417	1.148	1.050	0.045

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.001	0.418	0.419	0.062	1.068	0.026

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	9	9
normalized size	1	1.00	1.36	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.006	0.002	0.001	0.412	0.452	0.066	1.082	0.031

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	2
normalized size	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	1.00
time (sec)	N/A	0.006	0.002	0.121	0.418	0.433	0.061	1.313	0.023

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	6	6
normalized size	1	1.00	1.00	0.47	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.007	0.005	0.029	0.415	0.441	0.566	1.141	0.028

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.008	0.003	0.000	0.422	0.428	0.179	1.007	0.021

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	15	15	17	15	15
normalized size	1	1.00	0.88	1.06	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.020	0.016	0.000	0.427	0.443	0.321	1.319	0.028

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
normalized size	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.013	0.011	0.000	0.421	0.438	0.334	1.155	0.055

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
normalized size	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.030	0.037	0.000	0.427	0.448	0.629	1.006	0.155

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	23	25
normalized size	1	1.00	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.022	0.009	0.000	0.422	0.431	0.600	1.051	0.066

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.002	0.013	0.418	0.429	0.175	1.243	0.019

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	14	14	17	14	14
normalized size	1	1.00	0.88	1.06	0.88	0.88	1.06	0.88	0.88
time (sec)	N/A	0.021	0.013	0.013	0.428	0.429	0.319	1.215	0.026

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
normalized size	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.014	0.012	0.016	0.424	0.439	0.337	1.065	0.151

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
normalized size	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.032	0.035	0.036	0.421	0.441	0.619	1.396	0.065

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	25	39	23	25
normalized size	1	1.00	0.94	0.70	0.70	0.76	1.18	0.70	0.76
time (sec)	N/A	0.023	0.009	0.025	0.422	0.444	0.613	1.381	0.169

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	-1
normalized size	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	-0.50
time (sec)	N/A	0.011	0.012	0.000	0.569	0.412	0.614	1.393	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	11	12	2	-1
normalized size	1	1.00	1.00	1.50	6.50	5.50	6.00	1.00	-0.50
time (sec)	N/A	0.012	0.002	0.015	0.566	0.450	0.998	1.302	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	15	20	17	13	-1
normalized size	1	1.00	1.00	1.10	1.50	2.00	1.70	1.30	-0.10
time (sec)	N/A	0.025	0.002	0.013	0.553	0.432	1.452	1.276	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	17	17	10	11	-1
normalized size	1	1.00	1.00	0.80	1.13	1.13	0.67	0.73	-0.07
time (sec)	N/A	0.038	0.013	0.023	0.563	0.430	1.085	1.131	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16
normalized size	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33
time (sec)	N/A	0.006	0.003	0.004	0.439	0.417	0.094	0.994	0.021

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11
normalized size	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00
time (sec)	N/A	0.004	0.007	0.005	0.413	0.420	0.139	0.958	0.021

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	10	10
normalized size	1	1.00	2.10	1.10	1.00	1.00	1.20	1.00	1.00
time (sec)	N/A	0.004	0.009	0.016	0.414	0.419	0.139	0.921	0.022

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16
normalized size	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33
time (sec)	N/A	0.004	0.009	0.003	0.416	0.451	0.133	1.232	0.234

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	29	11	27	29	56	28
normalized size	1	1.00	1.73	2.64	1.00	2.45	2.64	5.09	2.55
time (sec)	N/A	0.004	0.010	0.006	0.419	0.455	0.334	1.341	0.190

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	21	26	30	17	51	12
normalized size	1	1.00	3.17	1.75	2.17	2.50	1.42	4.25	1.00
time (sec)	N/A	0.004	0.014	0.020	0.422	0.460	0.533	1.163	0.117

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	26	28	34	28	11
normalized size	1	1.00	1.00	1.73	2.36	2.55	3.09	2.55	1.00
time (sec)	N/A	0.004	0.002	0.020	0.415	0.452	0.610	1.393	0.024

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	23	46	18	18
normalized size	1	1.00	0.92	1.08	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.009	0.027	0.019	0.418	0.422	0.227	1.197	0.209

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	22	22	37	25	24
normalized size	1	1.00	1.07	0.81	0.81	0.81	1.37	0.93	0.89
time (sec)	N/A	0.010	0.011	0.019	0.417	0.417	0.468	1.195	0.147

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	22	22	46	18	18
normalized size	1	1.00	0.92	1.08	0.88	0.88	1.84	0.72	0.72
time (sec)	N/A	0.009	0.017	0.027	0.428	0.442	0.225	1.064	0.177

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	24
normalized size	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.92
time (sec)	N/A	0.011	0.007	0.028	0.420	0.445	0.482	1.043	0.025

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	58	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.80	5.80	1.00	1.00
time (sec)	N/A	0.008	0.003	0.019	0.420	0.397	1.198	0.964	0.145

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
normalized size	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.007	0.004	0.017	0.413	0.441	0.191	1.172	0.171

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
normalized size	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.008	0.009	0.020	0.417	0.414	0.342	0.983	0.166

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	10
normalized size	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.00
time (sec)	N/A	0.006	0.009	0.017	0.416	0.408	0.364	1.037	0.024

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	10
normalized size	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	0.91
time (sec)	N/A	0.008	0.012	0.035	0.417	0.418	0.372	1.243	0.034

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	148	133	48	45
normalized size	1	1.00	1.00	0.98	0.00	3.70	3.32	1.20	1.12
time (sec)	N/A	0.040	0.038	0.028	0.000	0.441	8.694	1.083	0.383

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	43	0	287	0	60	58
normalized size	1	1.00	0.94	0.91	0.00	6.11	0.00	1.28	1.23
time (sec)	N/A	0.056	0.062	0.068	0.000	0.469	0.000	1.194	0.260

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	117	54	105	45	52
normalized size	1	1.00	0.64	2.16	1.60	0.74	1.44	0.62	0.71
time (sec)	N/A	0.043	0.121	0.018	0.439	0.433	1.128	1.178	0.137

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.007	0.005	0.045	0.440	0.436	0.556	0.972	0.025

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	113	54	105	45	52
normalized size	1	1.00	0.64	2.16	1.55	0.74	1.44	0.62	0.71
time (sec)	N/A	0.043	0.122	0.025	0.459	0.443	1.112	0.984	0.103

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	14	31	14	29	21
normalized size	1	1.00	1.00	1.57	1.00	2.21	1.00	2.07	1.50
time (sec)	N/A	0.008	0.003	0.009	0.415	0.443	0.094	1.187	0.168

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	138	485	182	0	0	-1
normalized size	1	1.00	0.97	1.33	4.66	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.154	0.088	1.258	0.461	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	133	237	766	212	0	0	-1
normalized size	1	1.00	0.87	1.55	5.01	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.407	0.332	0.102	2.589	0.448	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	23	13
normalized size	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	0.87
time (sec)	N/A	0.014	0.014	0.013	0.979	0.443	0.174	1.204	0.022

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	11	13
normalized size	1	1.00	1.00	0.80	0.73	0.87	1.73	0.73	0.87
time (sec)	N/A	0.008	0.006	0.064	0.414	0.423	0.520	1.019	0.044

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	10	18
normalized size	1	1.00	0.58	0.79	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.025	0.005	0.009	0.415	0.431	0.070	1.119	0.044

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	9	18	12	9	6
normalized size	1	1.00	0.86	2.14	1.29	2.57	1.71	1.29	0.86
time (sec)	N/A	0.021	0.007	0.024	0.416	0.420	0.075	1.115	0.064

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	69	25	22	104	328	22
normalized size	1	1.00	0.69	2.16	0.78	0.69	3.25	10.25	0.69
time (sec)	N/A	0.011	0.020	0.038	0.430	0.438	1.042	1.328	0.024

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	71	24	20	107	329	20
normalized size	1	1.00	0.65	2.29	0.77	0.65	3.45	10.61	0.65
time (sec)	N/A	0.008	0.018	0.033	0.433	0.423	1.030	1.179	0.017

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	50	137	60	60	308	1166	57
normalized size	1	1.00	0.60	1.63	0.71	0.71	3.67	13.88	0.68
time (sec)	N/A	0.049	0.052	0.041	0.486	0.432	3.279	1.278	0.285

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	142	58	58	304	1165	55
normalized size	1	1.00	0.59	1.71	0.70	0.70	3.66	14.04	0.66
time (sec)	N/A	0.046	0.048	0.043	0.466	0.440	3.244	1.288	0.213

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	225	107	115	665	2631	133
normalized size	1	1.00	0.58	1.39	0.66	0.71	4.10	16.24	0.82
time (sec)	N/A	0.176	0.089	0.058	0.524	0.444	8.457	1.775	0.392

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	93	231	105	111	668	2631	132
normalized size	1	1.00	0.58	1.43	0.65	0.69	4.15	16.34	0.82
time (sec)	N/A	0.171	0.076	0.054	0.515	0.437	8.474	1.519	0.348

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	169	431	186	203	1355	5079	231
normalized size	1	1.00	0.65	1.65	0.71	0.78	5.19	19.46	0.89
time (sec)	N/A	0.435	0.152	0.078	0.578	0.457	22.171	1.725	0.627

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	168	441	184	202	1352	5075	232
normalized size	1	1.00	0.65	1.70	0.71	0.78	5.20	19.52	0.89
time (sec)	N/A	0.416	0.128	0.072	0.572	0.452	22.235	1.745	0.640

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	14
normalized size	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	0.56
time (sec)	N/A	0.031	0.011	0.117	0.433	0.439	12.669	1.108	0.166

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	112	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.73	0.73	0.73
time (sec)	N/A	0.032	0.010	0.089	0.422	0.436	12.701	1.178	0.276

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	55	64	55	59	100	60	67
normalized size	1	1.00	0.65	0.75	0.65	0.69	1.18	0.71	0.79
time (sec)	N/A	0.066	0.077	0.053	0.433	0.432	2.100	1.139	0.230

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	240	0	0	0	-1
normalized size	1	0.00	0.00	0.00	17.14	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.435	0.132	1.207	0.474	0.435	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	41	16	0	18	10
normalized size	1	1.00	1.00	0.92	3.42	1.33	0.00	1.50	0.83
time (sec)	N/A	0.030	0.013	0.101	0.433	0.429	0.000	1.243	0.175

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	143	0	312	0	98	133
normalized size	1	1.00	0.99	1.86	0.00	4.05	0.00	1.27	1.73
time (sec)	N/A	0.098	0.256	0.102	0.000	0.489	0.000	1.053	0.485

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	12	13	15	13	13
normalized size	1	1.00	1.00	0.82	0.71	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.003	0.001	0.447	0.426	0.380	1.053	0.133

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	13	13
normalized size	1	1.00	1.00	0.82	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.003	0.002	0.424	0.432	0.390	1.008	0.148

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.001	0.413	0.399	0.038	1.033	0.007

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.001	0.005	0.417	0.424	0.088	1.155	0.155

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.002	0.001	0.002	0.414	0.400	0.050	1.039	0.027

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
normalized size	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.011	0.006	0.011	0.576	0.402	0.777	1.172	0.014

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	23	22	15	26	22
normalized size	1	1.00	1.00	1.29	0.96	0.92	0.62	1.08	0.92
time (sec)	N/A	0.015	0.005	0.007	0.425	0.441	0.124	1.306	0.093

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.021	0.009	0.010	0.417	0.437	0.088	1.099	0.045

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	14	12	14	14	14
normalized size	1	1.00	1.00	1.15	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.008	0.004	0.002	0.422	0.401	0.088	1.125	0.052

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	23	85	26	21	21
normalized size	1	1.00	1.00	0.71	0.74	2.74	0.84	0.68	0.68
time (sec)	N/A	0.029	0.010	0.007	0.994	0.433	0.180	1.044	0.228

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	14	13	13	19	13	13
normalized size	1	1.00	0.67	0.67	0.62	0.62	0.90	0.62	0.62
time (sec)	N/A	0.009	0.006	0.002	0.443	0.410	0.094	0.993	0.027

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	102	102	101	101	102	101	101
normalized size	1	1.00	0.63	0.63	0.62	0.62	0.63	0.62	0.62
time (sec)	N/A	0.242	0.011	0.004	0.424	0.417	0.129	1.209	0.362

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	18	0	216	18
normalized size	1	1.00	1.00	1.06	0.00	1.00	0.00	12.00	1.00
time (sec)	N/A	0.021	0.011	0.006	0.000	0.424	0.000	1.441	0.204

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	14	24	237	14
normalized size	1	1.00	1.00	1.07	0.00	1.00	1.71	16.93	1.00
time (sec)	N/A	0.015	0.007	0.006	0.000	0.426	0.571	1.504	0.167

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	10	19	0	0	19
normalized size	1	1.00	1.00	1.24	0.59	1.12	0.00	0.00	1.12
time (sec)	N/A	0.019	0.012	0.043	0.589	0.415	0.000	0.000	0.131

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	54	0	0	-1
normalized size	1	1.00	0.67	1.23	0.00	0.84	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.135	0.057	0.000	0.432	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	16	16	12	45	15
normalized size	1	1.00	1.00	1.00	1.00	1.00	0.75	2.81	0.94
time (sec)	N/A	0.032	0.047	0.004	0.429	0.403	0.117	1.185	0.196

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	17	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	1.31	0.85	0.85
time (sec)	N/A	0.007	0.002	0.007	0.430	0.416	0.098	1.050	0.026

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
normalized size	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.003	0.002	0.008	0.421	0.433	0.199	1.230	0.017

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.007	0.001	0.001	0.416	0.412	0.082	1.139	0.018

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	17	18	17	17	14	24	17
normalized size	1	1.00	0.63	0.67	0.63	0.63	0.52	0.89	0.63
time (sec)	N/A	0.105	0.012	0.003	0.526	0.404	0.098	1.006	0.158

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	22	36	31	36	-1
normalized size	1	0.00	1.00	0.92	0.88	1.44	1.24	1.44	-0.04
time (sec)	N/A	0.982	0.167	0.124	0.755	0.419	0.349	1.015	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.046	0.018	0.039	0.000	0.419	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	10	10	8	10	10
normalized size	1	1.00	1.00	1.09	0.91	0.91	0.73	0.91	0.91
time (sec)	N/A	0.016	0.006	0.014	0.528	0.426	1.954	1.191	0.026

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	15	15	17	15	28
normalized size	1	1.00	0.77	0.95	0.68	0.68	0.77	0.68	1.27
time (sec)	N/A	0.052	0.012	0.017	0.525	0.424	3.342	1.149	0.219

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	17	28	11	11	10	11	11
normalized size	1	1.00	0.74	1.22	0.48	0.48	0.43	0.48	0.48
time (sec)	N/A	0.013	0.005	0.015	0.436	0.411	0.096	1.224	0.042

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	0	12	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.00	0.75	0.75	0.75
time (sec)	N/A	0.002	0.000	0.001	0.947	0.000	0.059	1.068	0.030

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	48	58	89	920	48	49
normalized size	1	1.00	0.75	0.84	1.02	1.56	16.14	0.84	0.86
time (sec)	N/A	0.030	0.024	0.017	0.962	0.448	4.146	1.171	0.094

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	305	0	300	0	232	88
normalized size	1	1.00	1.02	2.63	0.00	2.59	0.00	2.00	0.76
time (sec)	N/A	0.068	0.290	0.016	0.000	0.443	0.000	1.222	0.169

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.005	0.002	0.420	0.402	0.060	0.967	0.025

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	30	202	66	25
normalized size	1	1.00	0.71	0.62	0.76	0.88	5.94	1.94	0.74
time (sec)	N/A	0.009	0.012	0.002	0.423	0.412	1.141	1.183	0.030

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	42	666	93	37
normalized size	1	1.00	0.66	0.60	0.77	0.79	12.57	1.75	0.70
time (sec)	N/A	0.014	0.017	0.005	0.420	0.421	1.749	1.031	0.148

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	32	27
normalized size	1	1.00	1.00	0.80	1.20	2.09	1.94	0.91	0.77
time (sec)	N/A	0.012	0.009	0.007	0.967	0.422	1.408	1.098	0.146

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	37	47	93	44	41	31
normalized size	1	1.00	1.21	0.95	1.21	2.38	1.13	1.05	0.79
time (sec)	N/A	0.011	0.027	0.011	0.961	0.422	1.871	1.286	0.054

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.001	0.003	0.003	0.420	0.396	0.059	1.179	0.018

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	26	19	162	23	25
normalized size	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.78
time (sec)	N/A	0.008	0.010	0.006	0.419	0.406	1.109	1.211	0.029

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	41	31	600	37	37
normalized size	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.73
time (sec)	N/A	0.013	0.015	0.007	0.434	0.415	1.701	1.267	0.038

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
normalized size	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.007	0.004	0.007	0.953	0.410	1.066	1.014	0.144

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	40	60	93	44	47	33
normalized size	1	1.00	1.15	0.98	1.46	2.27	1.07	1.15	0.80
time (sec)	N/A	0.012	0.064	0.008	0.990	0.440	2.212	1.082	0.159

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	21	25	26	21	21
normalized size	1	1.00	1.04	1.09	0.91	1.09	1.13	0.91	0.91
time (sec)	N/A	0.004	0.013	0.003	0.424	0.438	0.065	1.041	0.251

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	45	58	216	86	94
normalized size	1	1.00	0.79	0.90	0.94	1.21	4.50	1.79	1.96
time (sec)	N/A	0.014	0.021	0.004	0.432	0.423	0.694	1.280	0.448

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	42	52	37	230	52	43
normalized size	1	1.00	0.87	0.76	0.95	0.67	4.18	0.95	0.78
time (sec)	N/A	0.028	0.035	0.007	0.969	0.456	1.046	1.137	0.162

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	14	14	2	15	10
normalized size	1	1.00	3.17	0.92	1.17	1.17	0.17	1.25	0.83
time (sec)	N/A	0.002	0.003	0.004	0.412	0.413	0.138	1.054	0.197

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	50	71	34	119	39	30
normalized size	1	1.00	0.72	1.16	1.65	0.79	2.77	0.91	0.70
time (sec)	N/A	0.005	0.012	0.005	0.418	0.406	2.566	1.029	0.217

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.014	0.017	0.446	0.420	0.310	0.928	0.245

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	11	18	8	11	11
normalized size	1	1.00	1.00	1.38	0.85	1.38	0.62	0.85	0.85
time (sec)	N/A	0.002	0.002	0.003	0.413	0.401	0.969	1.005	0.348

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	16	18	19	6	16
normalized size	1	1.00	1.00	0.88	2.00	2.25	2.38	0.75	2.00
time (sec)	N/A	0.003	0.003	0.011	0.955	0.400	0.921	1.227	0.312

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	25	10
normalized size	1	1.00	1.00	0.79	1.79	1.79	0.57	1.79	0.71
time (sec)	N/A	0.006	0.002	0.011	0.438	0.402	0.919	1.191	0.186

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	0	22	0	22	18
normalized size	1	1.00	1.00	0.78	0.00	1.22	0.00	1.22	1.00
time (sec)	N/A	0.016	0.003	0.012	0.000	0.415	0.000	0.941	0.359

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	17	55	0	0	32
normalized size	1	1.00	0.93	0.77	0.57	1.83	0.00	0.00	1.07
time (sec)	N/A	0.021	0.004	0.019	1.011	0.414	0.000	0.000	0.538

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	14	17	0	35	14
normalized size	1	1.00	0.94	0.88	0.82	1.00	0.00	2.06	0.82
time (sec)	N/A	0.009	0.007	0.005	0.954	0.403	0.000	1.197	0.151

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	14	2	14	2
normalized size	1	1.00	1.00	1.50	1.00	7.00	1.00	7.00	1.00
time (sec)	N/A	0.001	0.003	0.003	0.949	0.392	0.140	1.077	0.028

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	14	14	17	0	14
normalized size	1	1.00	1.50	0.75	0.70	0.70	0.85	0.00	0.70
time (sec)	N/A	0.913	0.018	0.005	0.416	0.404	1.165	0.000	0.345

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	30	20	19	0	20
normalized size	1	1.00	1.00	0.88	1.25	0.83	0.79	0.00	0.83
time (sec)	N/A	0.324	0.167	0.005	1.191	0.409	1.338	0.000	0.310

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	71	24	31	29	8	31	23
normalized size	1	1.00	2.63	0.89	1.15	1.07	0.30	1.15	0.85
time (sec)	N/A	0.008	0.008	0.003	0.416	0.407	0.178	1.083	0.687

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	216	51	0	-1
normalized size	1	1.00	1.55	0.30	0.00	2.63	0.62	0.00	-0.01
time (sec)	N/A	0.074	0.070	0.079	0.000	0.458	1.316	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	39	40	29	48	10	169
normalized size	1	1.00	1.00	3.25	3.33	2.42	4.00	0.83	14.08
time (sec)	N/A	0.053	0.022	0.011	0.985	0.425	154.693	1.294	0.061

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	36	85	66	34	32
normalized size	1	1.00	1.00	0.82	0.90	2.12	1.65	0.85	0.80
time (sec)	N/A	0.011	0.010	0.005	0.422	0.427	1.145	1.146	0.479

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	57	41	42	38	40
normalized size	1	1.00	1.00	1.43	1.24	0.89	0.91	0.83	0.87
time (sec)	N/A	0.034	0.011	0.011	0.975	0.422	1.249	1.020	0.665

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	52	40	52	0	40	51
normalized size	1	1.00	1.05	1.41	1.08	1.41	0.00	1.08	1.38
time (sec)	N/A	0.018	0.019	0.008	0.961	0.433	0.000	1.307	0.105

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	65	74	77	97	0	51	72
normalized size	1	1.00	1.07	1.21	1.26	1.59	0.00	0.84	1.18
time (sec)	N/A	0.024	0.030	0.007	0.969	0.445	0.000	1.739	0.246

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	29	19	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	1.26	0.83	0.83
time (sec)	N/A	0.004	0.004	0.004	0.415	0.426	0.450	1.041	0.289

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	36	16	24
normalized size	1	1.00	1.00	0.89	0.86	0.86	1.29	0.57	0.86
time (sec)	N/A	0.005	0.005	0.006	0.416	0.410	0.588	1.216	0.272

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	0	152	0	60	50
normalized size	1	1.00	1.00	0.84	0.00	2.71	0.00	1.07	0.89
time (sec)	N/A	0.040	0.023	0.016	0.000	0.421	0.000	1.216	0.979

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	82	70	68	190	0	72	67
normalized size	1	1.00	1.01	0.86	0.84	2.35	0.00	0.89	0.83
time (sec)	N/A	0.028	0.102	0.008	0.422	0.438	0.000	1.343	0.237

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	56	0	58	0	45	54
normalized size	1	1.00	1.11	1.27	0.00	1.32	0.00	1.02	1.23
time (sec)	N/A	0.039	0.008	0.017	0.000	0.409	0.000	1.150	0.400

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	78	0	106	0	45	72
normalized size	1	1.00	1.04	1.15	0.00	1.56	0.00	0.66	1.06
time (sec)	N/A	0.049	0.025	0.016	0.000	0.441	0.000	1.250	0.442

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	22	10	11	11
normalized size	1	1.00	1.00	0.92	0.85	1.69	0.77	0.85	0.85
time (sec)	N/A	0.017	0.006	0.014	0.423	0.423	0.321	1.036	0.089

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	11	17	15	11
normalized size	1	1.00	1.00	0.84	0.79	0.58	0.89	0.79	0.58
time (sec)	N/A	0.006	0.001	0.013	0.437	0.417	0.099	1.193	0.165

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	21	20	12	10	12	12
normalized size	1	1.00	1.00	1.75	1.67	1.00	0.83	1.00	1.00
time (sec)	N/A	0.009	0.026	0.017	0.437	0.435	0.179	1.329	0.083

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	10	7	10	10
normalized size	1	1.00	1.00	1.00	0.91	0.91	0.64	0.91	0.91
time (sec)	N/A	0.025	0.008	0.019	0.576	0.412	0.272	0.991	0.216

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.018	0.005	0.004	0.416	0.399	0.135	1.138	0.154

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	16	20	50	165	66	15
normalized size	1	1.00	0.64	0.48	0.61	1.52	5.00	2.00	0.45
time (sec)	N/A	0.003	0.006	0.004	0.425	0.422	4.659	1.401	0.299

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	11	8
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.78	1.22	0.89
time (sec)	N/A	0.001	0.002	0.005	0.420	0.410	0.092	1.163	0.197

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	28	28	39	32	13
normalized size	1	1.00	1.00	0.74	1.47	1.47	2.05	1.68	0.68
time (sec)	N/A	0.005	0.003	0.004	0.974	0.423	0.311	1.267	0.075

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	11	14	15	11	12
normalized size	1	1.00	0.84	0.68	0.58	0.74	0.79	0.58	0.63
time (sec)	N/A	0.003	0.003	0.004	0.416	0.400	1.094	1.115	0.024

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	112	85	86	104	153	103	119
normalized size	1	1.00	1.27	0.97	0.98	1.18	1.74	1.17	1.35
time (sec)	N/A	0.039	0.042	0.013	0.972	0.408	1.693	2.430	0.181

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
normalized size	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.011	0.002	0.014	0.429	0.403	0.190	1.186	0.021

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
normalized size	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.011	0.002	0.014	0.432	0.409	0.188	1.198	0.026

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	26	7	13	7
normalized size	1	1.00	1.00	0.89	0.78	2.89	0.78	1.44	0.78
time (sec)	N/A	0.004	0.005	0.015	0.431	0.433	0.153	1.180	0.186

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	13	26	22	23	13
normalized size	1	1.00	1.00	1.33	1.08	2.17	1.83	1.92	1.08
time (sec)	N/A	0.033	0.087	0.025	0.425	0.447	0.392	1.371	0.321

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	32	75	22	22
normalized size	1	1.00	1.00	0.82	0.79	1.14	2.68	0.79	0.79
time (sec)	N/A	0.023	0.016	0.018	0.470	0.445	2.200	1.437	0.332

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14
normalized size	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.008	0.003	0.000	0.948	0.428	0.249	1.469	0.019

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	19	15
normalized size	1	1.00	1.00	0.76	0.71	0.62	0.71	0.90	0.71
time (sec)	N/A	0.008	0.003	0.007	0.958	0.429	0.251	1.424	0.048

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	22	19	26	22	41
normalized size	1	1.00	0.96	1.00	0.96	0.83	1.13	0.96	1.78
time (sec)	N/A	0.019	0.003	0.005	0.415	0.415	0.154	1.356	0.144

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	0.78
time (sec)	N/A	0.016	0.019	0.054	0.418	0.436	0.536	1.037	0.026

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	0.78
time (sec)	N/A	0.013	0.013	0.046	0.419	0.438	0.538	1.030	0.024

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	22	16
normalized size	1	1.00	3.33	1.42	0.00	2.00	0.00	1.83	1.33
time (sec)	N/A	0.006	0.012	0.058	0.000	0.417	0.000	1.160	0.139

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	0	26	0	35	18
normalized size	1	1.00	3.00	1.64	0.00	1.86	0.00	2.50	1.29
time (sec)	N/A	0.009	0.013	0.068	0.000	0.431	0.000	1.196	0.136

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	9	10	0	14	10
normalized size	1	1.00	1.33	1.83	0.75	0.83	0.00	1.17	0.83
time (sec)	N/A	0.007	0.007	0.052	1.164	0.422	0.000	1.039	0.145

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	20	18	0	23	12
normalized size	1	1.00	1.29	1.57	1.43	1.29	0.00	1.64	0.86
time (sec)	N/A	0.009	0.008	0.062	0.972	0.419	0.000	1.073	0.033

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	13	63	13	21
normalized size	1	1.00	0.81	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.008	0.019	0.005	0.000	0.404	0.385	1.035	0.193

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	18	18	20	19	18
normalized size	1	1.00	1.00	1.29	0.75	0.75	0.83	0.79	0.75
time (sec)	N/A	0.010	0.010	0.006	0.423	0.409	0.139	1.215	0.106

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	33	16	7	16	8
normalized size	1	1.00	1.00	0.75	2.75	1.33	0.58	1.33	0.67
time (sec)	N/A	0.004	0.002	0.007	0.415	0.416	0.897	1.087	0.037

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	24	24	0	24	24
normalized size	1	1.00	1.00	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.012	0.012	0.029	0.420	0.423	0.000	1.171	0.030

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	26	26
normalized size	1	1.00	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.009	0.013	0.009	0.971	0.439	0.138	1.035	0.060

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3
normalized size	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.002	0.000	0.413	0.438	0.065	1.164	0.025

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	16	48	19	34	10
normalized size	1	1.00	1.50	1.17	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.010	0.003	0.000	0.971	0.412	0.073	1.151	0.028

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3
normalized size	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00
time (sec)	N/A	0.003	0.002	0.001	0.423	0.433	0.130	1.086	0.011

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	12	12	3
normalized size	1	1.00	1.00	1.33	1.00	6.00	4.00	4.00	1.00
time (sec)	N/A	0.003	0.002	0.004	0.417	0.425	0.320	0.935	0.137

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.009	0.415	0.409	0.088	1.028	0.165

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	32	10	10	0	11	-1
normalized size	1	1.00	0.59	0.65	0.20	0.20	0.00	0.22	-0.02
time (sec)	N/A	0.015	0.009	0.008	0.986	0.398	0.000	1.120	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	12	12	13	10
normalized size	1	1.00	1.06	0.81	0.75	0.75	0.75	0.81	0.62
time (sec)	N/A	0.006	0.003	0.003	0.419	0.402	0.076	1.000	0.145

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	30	0	40	32
normalized size	1	1.00	0.90	0.00	0.00	0.75	0.00	1.00	0.80
time (sec)	N/A	0.016	0.010	0.009	0.000	0.412	0.000	0.938	0.042

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	58	55	109	57	69
normalized size	1	1.00	0.52	0.49	0.45	0.43	0.85	0.45	0.54
time (sec)	N/A	0.190	0.173	0.020	0.467	0.429	4.949	1.100	0.278

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	17	17	27	15	17
normalized size	1	1.00	0.60	0.67	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.037	0.027	0.016	0.435	0.437	0.828	0.938	0.171

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	7	17	17	7	7
normalized size	1	1.00	0.82	0.73	0.64	1.55	1.55	0.64	0.64
time (sec)	N/A	0.001	0.001	0.000	0.414	0.410	0.107	1.079	0.066

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	32	32	41	33	46
normalized size	1	1.00	1.00	0.82	0.80	0.80	1.02	0.82	1.15
time (sec)	N/A	0.021	0.009	0.003	0.962	0.431	0.133	1.072	0.115

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	17	17	15	18	6
normalized size	1	1.00	2.88	2.75	2.12	2.12	1.88	2.25	0.75
time (sec)	N/A	0.003	0.003	0.003	0.414	0.428	0.097	1.013	0.067

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	194	1210	0	0	0	0	-1
normalized size	1	1.00	0.85	5.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.275	0.029	0.000	0.437	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	26	25	24	37	25	51
normalized size	1	1.00	1.25	1.08	1.04	1.00	1.54	1.04	2.12
time (sec)	N/A	0.009	0.003	0.004	0.418	0.434	1.200	1.038	0.443

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	71	54	57	69	73	58	70
normalized size	1	1.00	0.91	0.69	0.73	0.88	0.94	0.74	0.90
time (sec)	N/A	0.046	0.023	0.004	0.979	0.427	0.303	0.949	0.272

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	14	2	14	2
normalized size	1	1.00	1.00	1.50	1.00	7.00	1.00	7.00	1.00
time (sec)	N/A	0.001	0.003	0.000	0.956	0.437	0.141	0.928	0.002

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	20	25	24	25	20
normalized size	1	1.00	1.00	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.004	0.007	0.003	0.961	0.444	0.209	0.994	0.048

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.004	0.003	0.008	0.415	0.413	0.078	0.948	0.031

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	0.88
time (sec)	N/A	0.004	0.003	0.002	0.951	0.442	0.130	1.073	0.220

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	38	24
normalized size	1	1.00	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.023	0.012	0.003	1.008	0.454	0.351	0.856	0.029

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	15	9
normalized size	1	1.00	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.115	0.030	0.126	0.424	0.474	0.000	0.958	0.810

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.006	0.002	0.024	0.417	0.452	0.067	1.156	0.002

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	18	14	18	16
normalized size	1	1.00	1.00	0.94	0.89	1.00	0.78	1.00	0.89
time (sec)	N/A	0.014	0.004	0.008	0.423	0.414	0.120	1.106	0.152

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	16	7	16	6
normalized size	1	1.00	1.00	0.70	0.60	1.60	0.70	1.60	0.60
time (sec)	N/A	0.001	0.005	0.005	0.967	0.408	0.151	1.074	0.038

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	14	3	14	4
normalized size	1	1.00	1.00	0.83	0.67	2.33	0.50	2.33	0.67
time (sec)	N/A	0.001	0.004	0.003	0.965	0.423	0.143	1.067	0.029

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	22	16	16
normalized size	1	1.00	1.00	0.81	0.76	0.76	1.05	0.76	0.76
time (sec)	N/A	0.017	0.009	0.005	0.951	0.391	0.117	1.053	0.142

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	47	73	46	46	45
normalized size	1	1.00	0.96	0.79	0.89	1.38	0.87	0.87	0.85
time (sec)	N/A	0.031	0.021	0.013	0.957	0.407	0.176	1.120	0.061

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	66	41	41	762	44	46
normalized size	1	1.00	1.00	1.35	0.84	0.84	15.55	0.90	0.94
time (sec)	N/A	0.078	0.027	0.009	0.425	0.448	86.228	1.152	0.210

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.004	0.238	0.046	0.000	0.422	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	27	44	26	27	27
normalized size	1	1.00	1.00	1.00	0.96	1.57	0.93	0.96	0.96
time (sec)	N/A	0.211	0.174	0.045	0.560	0.428	0.475	1.113	0.338

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	136	127	160	145	0	114	140
normalized size	1	1.00	0.68	0.64	0.80	0.73	0.00	0.57	0.70
time (sec)	N/A	0.101	0.236	0.102	0.540	0.445	0.000	1.006	1.111

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	20	15	15	15
normalized size	1	1.00	1.00	0.89	0.83	1.11	0.83	0.83	0.83
time (sec)	N/A	0.005	0.007	0.008	0.419	0.413	0.310	1.067	0.155

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	21	37	36	37	21
normalized size	1	1.00	1.00	0.92	0.88	1.54	1.50	1.54	0.88
time (sec)	N/A	0.008	0.030	0.005	0.418	0.408	0.514	1.153	0.066

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	F	B	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	5137	1197351	0	179	0	0	-1
normalized size	1	0.00	54.65	12737.78	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	1.840	6.443	1.432	0.000	0.548	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	A	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	630	352	0	223	0	0	-1
normalized size	1	0.00	4.44	2.48	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	3.324	1.569	0.315	0.000	0.564	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	97	1088	171	162	0	76	172
normalized size	1	1.00	4.62	51.81	8.14	7.71	0.00	3.62	8.19
time (sec)	N/A	0.285	1.351	0.216	0.734	0.434	0.000	1.375	2.373

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	4030	0	3168	4640	0	0	0	0	-1
normalized size	1	0.00	0.79	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.025	6.043	1.158	0.000	0.491	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	181	1356	0	269	330	472	265
normalized size	1	1.00	0.55	4.11	0.00	0.82	1.00	1.43	0.80
time (sec)	N/A	0.872	0.145	0.224	0.000	0.444	21.736	1.096	0.767

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	5	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.012	0.001	0.030	0.442	0.416	0.758	1.089	0.020

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	B	A	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	71	102	0	137	76	94	103
normalized size	1	0.00	1.00	1.44	0.00	1.93	1.07	1.32	1.45
time (sec)	N/A	0.754	0.039	0.034	0.000	0.435	0.221	1.292	0.280

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	0	1.00	6	0.000
2	A	3	2	1.00	13	0.154
3	A	2	1	1.00	10	0.100
4	A	1	1	1.00	3	0.333
5	A	2	1	1.00	11	0.091
6	A	2	1	1.00	14	0.071
7	A	2	1	1.00	20	0.050

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
8	A	2	2	1.00	12	0.167
9	A	3	3	1.00	13	0.231
10	A	2	2	1.00	10	0.200
11	A	6	5	1.00	13	0.385
12	A	2	1	1.00	23	0.043
13	A	4	3	1.00	20	0.150
14	A	3	2	1.00	22	0.091
15	A	5	4	1.00	14	0.286
16	A	6	6	1.00	9	0.667
17	A	3	2	1.00	16	0.125
18	A	9	6	1.00	7	0.857
19	A	9	6	1.00	11	0.546
20	A	9	5	1.00	10	0.500
21	A	1	1	1.00	7	0.143
22	A	2	1	1.00	9	0.111
23	A	2	1	1.00	11	0.091
24	A	1	1	1.00	7	0.143
25	A	1	1	1.00	7	0.143
26	A	2	1	1.00	9	0.111
27	A	2	1	1.00	11	0.091
28	A	3	3	1.00	11	0.273
29	A	2	1	1.00	11	0.091
30	A	2	1	1.00	11	0.091
31	A	1	1	1.00	9	0.111
32	A	1	1	1.00	11	0.091
33	A	6	6	1.00	9	0.667
34	A	6	6	1.00	7	0.857
35	A	6	6	1.00	11	0.546
36	A	3	3	1.00	7	0.429
37	A	3	3	1.00	9	0.333
38	A	9	6	1.00	9	0.667
39	A	3	3	1.00	12	0.250
40	A	3	3	1.00	12	0.250
41	A	3	2	1.00	12	0.167
42	A	3	2	1.00	12	0.167
43	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
44	A	9	5	1.00	10	0.500
45	A	9	5	1.00	12	0.417
46	A	10	6	1.00	7	0.857
47	A	10	6	1.00	7	0.857
48	A	10	6	1.00	7	0.857
49	A	19	6	1.00	7	0.857
50	A	13	10	1.00	7	1.429
51	A	19	6	1.00	12	0.500
52	A	7	7	1.00	11	0.636
53	A	1	1	1.00	2	0.500
54	A	1	1	1.00	4	0.250
55	A	1	1	1.00	6	0.167
56	A	1	1	1.00	6	0.167
57	A	2	2	1.00	4	0.500
58	A	11	2	1.00	8	0.250
59	A	2	2	1.00	8	0.250
60	A	1	1	1.00	4	0.250
61	A	2	2	1.00	6	0.333
62	A	2	2	1.00	8	0.250
63	A	3	3	1.00	8	0.375
64	A	2	2	1.00	8	0.250
65	A	2	1	1.04	8	0.125
66	A	4	4	1.00	10	0.400
67	A	2	2	1.00	10	0.200
68	A	3	2	1.00	8	0.250
69	A	3	2	1.00	10	0.200
70	A	3	3	1.00	8	0.375
71	A	3	3	1.00	10	0.300
72	A	4	3	1.00	12	0.250
73	A	4	3	1.00	12	0.250
74	A	3	3	1.00	10	0.300
75	A	0	0	0.00	0	0.000
76	A	1	1	1.00	2	0.500
77	A	1	1	1.00	2	0.500
78	A	1	1	1.00	2	0.500
79	A	1	1	1.00	2	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	2	2	1.00	6	0.333
81	A	1	1	1.00	2	0.500
82	A	1	1	1.00	2	0.500
83	A	2	2	1.00	4	0.500
84	A	3	3	1.00	8	0.375
85	A	2	1	1.00	4	0.250
86	A	1	1	1.00	4	0.250
87	A	3	2	1.00	11	0.182
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	4	0.250
90	A	2	2	1.00	4	0.500
91	A	1	1	1.00	7	0.143
92	A	2	2	1.00	4	0.500
93	A	3	2	1.00	6	0.333
94	A	2	2	1.00	6	0.333
95	A	4	4	1.00	8	0.500
96	A	3	3	1.00	6	0.500
97	A	2	2	1.00	4	0.500
98	A	3	2	1.00	6	0.333
99	A	2	2	1.00	6	0.333
100	A	4	4	1.00	8	0.500
101	A	3	3	1.00	6	0.500
102	A	1	1	1.00	6	0.167
103	A	1	1	1.00	6	0.167
104	A	2	2	1.00	6	0.333
105	A	3	2	1.00	8	0.250
106	A	2	2	1.00	4	0.500
107	A	1	1	1.00	6	0.167
108	A	1	1	1.00	6	0.167
109	A	1	1	1.00	6	0.167
110	A	1	1	1.00	6	0.167
111	A	1	1	1.00	6	0.167
112	A	1	1	1.00	6	0.167
113	A	2	2	1.00	8	0.250
114	A	2	1	1.00	8	0.125
115	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	2	1	1.00	8	0.125
117	A	2	2	1.00	8	0.250
118	A	1	1	1.00	6	0.167
119	A	1	1	1.00	8	0.125
120	A	1	1	1.00	6	0.167
121	A	1	1	1.00	8	0.125
122	A	3	3	1.00	8	0.375
123	A	3	3	1.00	10	0.300
124	A	4	4	1.00	12	0.333
125	A	1	1	1.00	7	0.143
126	A	4	4	1.00	12	0.333
127	A	2	2	1.00	4	0.500
128	A	17	8	1.00	8	1.000
129	A	34	9	1.00	8	1.125
130	A	3	3	1.00	6	0.500
131	A	1	1	1.00	9	0.111
132	A	3	3	1.00	9	0.333
133	A	3	2	1.00	9	0.222
134	A	1	1	1.00	6	0.167
135	A	1	1	1.00	6	0.167
136	A	4	3	1.00	7	0.429
137	A	4	3	1.00	7	0.429
138	A	11	5	1.00	9	0.556
139	A	11	5	1.00	9	0.556
140	A	25	5	1.00	9	0.556
141	A	25	5	1.00	9	0.556
142	A	5	2	1.00	11	0.182
143	A	5	2	1.00	11	0.182
144	A	6	4	1.00	10	0.400
145	A	0	0	0.00	0	0.000
146	A	4	3	1.00	9	0.333
147	A	5	5	1.00	15	0.333
148	A	1	1	1.00	3	0.333
149	A	1	1	1.00	3	0.333
150	A	1	1	1.00	3	0.333
151	A	1	1	1.00	3	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	1	1	1.00	5	0.200
153	A	1	1	1.00	9	0.111
154	A	4	4	1.00	11	0.364
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	9	0.222
157	A	2	2	1.00	18	0.111
158	A	2	2	1.00	7	0.286
159	A	21	2	1.00	7	0.286
160	A	2	2	1.00	9	0.222
161	A	2	2	1.00	7	0.286
162	A	2	2	1.00	7	0.286
163	A	5	3	1.00	12	0.250
164	A	1	1	1.00	14	0.071
165	A	1	1	1.00	7	0.143
166	A	1	1	1.00	5	0.200
167	A	1	1	1.00	7	0.143
168	A	7	3	1.00	12	0.250
169	F	0	0	N/A	0	N/A
170	A	0	0	0.00	0	0.000
171	A	2	3	1.00	6	0.500
172	A	5	5	1.00	7	0.714
173	A	3	3	1.00	10	0.300
174	A	1	0	1.00	13	0.000
175	A	4	4	1.00	12	0.333
176	A	5	4	1.00	19	0.210
177	A	1	1	1.00	9	0.111
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	13	0.077
180	A	3	3	1.00	13	0.231
181	A	3	3	1.00	13	0.231
182	A	1	1	1.00	9	0.111
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	13	0.077
185	A	2	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	2	1	1.00	13	0.077
189	A	6	6	1.00	18	0.333
190	A	2	2	1.00	9	0.222
191	A	4	3	1.00	13	0.231
192	A	3	3	1.00	6	0.500
193	A	1	1	1.00	13	0.077
194	A	2	2	1.00	13	0.154
195	A	3	3	1.00	13	0.231
196	A	3	3	1.00	14	0.214
197	A	3	3	1.00	18	0.167
198	A	1	1	1.00	20	0.050
199	A	1	1	1.00	9	0.111
200	A	3	2	1.00	65	0.031
201	A	5	4	1.00	68	0.059
202	A	5	2	1.00	21	0.095
203	A	6	6	1.00	21	0.286
204	A	4	3	1.00	23	0.130
205	A	2	2	1.00	16	0.125
206	A	3	3	1.00	25	0.120
207	A	2	2	1.00	24	0.083
208	A	2	2	1.00	29	0.069
209	A	1	1	1.00	18	0.056
210	A	1	1	1.00	23	0.043
211	A	3	3	1.00	24	0.125
212	A	3	3	1.00	22	0.136
213	A	3	3	1.00	26	0.115
214	A	3	3	1.00	31	0.097
215	A	3	3	1.00	16	0.188
216	A	1	1	1.00	8	0.125
217	A	2	2	1.00	6	0.333
218	A	1	1	1.00	20	0.050
219	A	3	2	1.00	13	0.154
220	A	2	2	1.00	13	0.154
221	A	3	3	1.00	9	0.333
222	A	2	2	1.00	13	0.154
223	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	5	5	1.00	13	0.385
225	A	2	2	1.00	4	0.500
226	A	2	2	1.00	4	0.500
227	A	1	1	1.00	4	0.250
228	A	1	1	1.00	28	0.036
229	A	4	2	1.00	11	0.182
230	A	3	3	1.00	4	0.750
231	A	3	3	1.00	4	0.750
232	A	3	3	1.00	8	0.375
233	A	3	2	1.00	7	0.286
234	A	3	2	1.00	7	0.286
235	A	1	1	1.00	8	0.125
236	A	1	1	1.00	10	0.100
237	A	1	1	1.00	8	0.125
238	A	1	1	1.00	10	0.100
239	A	3	3	1.00	17	0.176
240	A	4	3	1.00	13	0.231
241	A	2	2	1.00	11	0.182
242	A	4	3	1.00	13	0.231
243	A	3	3	1.00	8	0.375
244	A	1	1	1.00	2	0.500
245	A	3	2	1.00	4	0.500
246	A	1	1	1.00	2	0.500
247	A	1	1	1.00	2	0.500
248	A	1	1	1.00	3	0.333
249	A	4	3	1.00	12	0.250
250	A	2	1	1.00	13	0.077
251	A	3	3	1.00	14	0.214
252	A	14	8	1.00	12	0.667
253	A	4	3	1.00	7	0.429
254	A	1	1	1.00	5	0.200
255	A	6	6	1.00	9	0.667
256	A	2	2	1.00	9	0.222
257	A	10	3	1.00	15	0.200
258	A	6	2	1.00	11	0.182
259	A	6	6	1.00	7	0.857

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	1	1	1.00	9	0.111
261	A	2	2	1.00	9	0.222
262	A	2	1	1.00	7	0.143
263	A	2	2	1.00	2	1.000
264	A	4	3	1.00	6	0.500
265	A	4	2	1.00	17	0.118
266	A	2	2	1.00	4	0.500
267	A	2	1	1.00	19	0.053
268	A	1	1	1.00	11	0.091
269	A	1	1	1.00	9	0.111
270	A	2	2	1.00	12	0.167
271	A	3	2	1.00	24	0.083
272	A	2	1	1.00	29	0.034
273	A	0	0	0.00	0	0.000
274	A	9	7	1.00	54	0.130
275	A	4	3	1.00	21	0.143
276	A	1	1	1.00	2	0.500
277	A	1	1	1.00	4	0.250
278	F	0	0	N/A	0	N/A
279	F	0	0	N/A	0	N/A
280	A	1	1	1.00	85	0.012
281	F	0	0	N/A	0	N/A
282	A	20	8	1.00	107	0.075
283	A	2	2	1.00	7	0.286
284	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int \left(1+x+x^2\right) dx$

Optimal. Leaf size=16

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

```
[Out] x+1/2*x^2+1/3*x^3
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + x + x^2,x]
```

```
[Out] x + x^2/2 + x^3/3
```

Rubi steps

$$\int \left(1+x+x^2\right) dx = x+\frac{x^2}{2}+\frac{x^3}{3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x + x^2,x]
```

```
[Out] x + x^2/2 + x^3/3
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

giac [A] time = 1.24, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{1}{3}x^{3} + \frac{1}{2}x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2+x+1,x)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

maxima [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + x^2 + 1,x)
```

```
[Out] (x*(3*x + 2*x^2 + 6))/6
```

sympy [A] time = 0.05, size = 10, normalized size = 0.62

$$\frac{x^{3}}{3} + \frac{x^{2}}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2+x+1,x)
```

```
[Out] x**3/3 + x**2/2 + x
```

3.2 $\int x^2 \left(x+2x^2\right)^2 dx$

Optimal. Leaf size=22

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

```
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {647, 43}

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(x+2x^2\right)^2 dx &= \int x^4(1+2x)^2 dx\\ &= \int \left(x^4+4x^5+4x^6\right) dx\\ &= \frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

fricas [A] time = 0.37, size = 16, normalized size = 0.73

$$\frac{4}{7}x^7+\frac{2}{3}x^6+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="fricas")
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

giac [A] time = 1.01, size = 16, normalized size = 0.73

$$\frac{4}{7}\,x^{7} + \frac{2}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="giac")
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{4}{7}x^{7} + \frac{2}{3}x^{6} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^2+x)^2,x)
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

maxima [A] time = 0.42, size = 16, normalized size = 0.73

$$\frac{4}{7}\,x^{7} + \frac{2}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

mupad [B] time = 0.04, size = 15, normalized size = 0.68

$$\frac{x^5\,\left(60\,x^2+70\,x+21\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 2*x^2)^2,x)
[Out] (x^5*(70*x + 60*x^2 + 21))/105
```

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{4x^{7}}{7} + \frac{2x^{6}}{3} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**2+x)**2,x)
[Out] 4*x**7/7 + 2*x**6/3 + x**5/5
```

3.3 $\int x\left(1+2x+x^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {14}

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(1+2x+x^2\right)\,dx &= \int \left(x+2x^2+x^3\right)\,dx\\ &= \frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

fricas [A] time = 0.38, size = 16, normalized size = 0.73

$$\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

giac [A] time = 1.07, size = 16, normalized size = 0.73

$$\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+1),x)
```

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

maxima [A] time = 0.42, size = 16, normalized size = 0.73

$$\frac{1}{4}\,x^4 + \frac{2}{3}\,x^3 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

mupad [B] time = 0.03, size = 15, normalized size = 0.68

$$\frac{x^2\,\left(3\,x^2+8\,x+6\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x + x^2 + 1),x)
```

```
[Out] (x^2*(8*x + 3*x^2 + 6))/12
```

sympy [A] time = 0.06, size = 15, normalized size = 0.68

$$\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+1),x)
```

```
[Out] x**4/4 + 2*x**3/3 + x**2/2
```

3.4 $\int \frac{1}{x} dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} dx = \log(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

fricas [A] time = 0.40, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

giac [A] time = 1.12, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

3.5 $\int \frac{(1+x)^3}{(-1+x)^4} \, dx$

Optimal. Leaf size=36

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

```
[Out] 8/3/(1-x)^3-6/(1-x)^2+6/(1-x)+ln(1-x)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(-1 + x)^4,x]
[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^3}{(-1+x)^4} \, dx &= \int \left(\frac{8}{(-1+x)^4} + \frac{12}{(-1+x)^3} + \frac{6}{(-1+x)^2} + \frac{1}{-1+x}\right) dx \\ &= \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.67

$$\log(x-1) - \frac{2\left(9x^2-9x+4\right)}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(-1 + x)^4,x]
[Out] (-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]
```

fricas [A] time = 0.40, size = 46, normalized size = 1.28

$$-\frac{18\,x^2 - 3\left(x^3 - 3\,x^2 + 3\,x - 1\right)\log\left(x-1\right) - 18\,x + 8}{3\left(x^3 - 3\,x^2 + 3\,x - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")
[Out] -1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(x^3 - 3*x^
2 + 3*x - 1)
```

giac [A] time = 1.01, size = 23, normalized size = 0.64

$$-\frac{2\left(9\,x^2-9\,x+4\right)}{3\,(x-1)^3}+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="giac")
[Out] -2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + log(abs(x - 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\ln\left(x-1\right)-\frac{6}{(x-1)^2}-\frac{6}{x-1}-\frac{8}{3\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^3/(x-1)^4,x)
[Out] -6/(x-1)^2-6/(x-1)+ln(x-1)-8/3/(x-1)^3
```

maxima [A] time = 0.42, size = 32, normalized size = 0.89

$$-\frac{2\left(9\,x^2-9\,x+4\right)}{3\left(x^3-3\,x^2+3\,x-1\right)}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")
[Out] -2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)
```

mupad [B] time = 0.12, size = 22, normalized size = 0.61

$$\ln\left(x-1\right)-\frac{6\,x^2-6\,x+\frac{8}{3}}{(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(x - 1)^4,x)
[Out] log(x - 1) - (6*x^2 - 6*x + 8/3)/(x - 1)^3
```

sympy [A] time = 0.12, size = 29, normalized size = 0.81

$$\frac{-18x^2+18x-8}{3x^3-9x^2+9x-3}+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-1+x)**4,x)
[Out] (-18*x**2 + 18*x - 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)
```

3.6 $\int \frac{1}{(-1+x)x(1+x)^2} dx$

Optimal. Leaf size=32

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

```
[Out] -1/2/(1+x)+1/4*ln(1-x)-ln(x)+3/4*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {72}

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] -1/(2*(1 + x)) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)x(1+x)^2}\,dx &= \int \left(\frac{1}{4(-1+x)}-\frac{1}{x}+\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right) dx\\ &= -\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-\frac{2}{x+1}+\log(1-x)-4\log(x)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] (-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4
```

fricas [A] time = 0.44, size = 33, normalized size = 1.03

$$\frac{3\,(x+1)\log\,(x+1)+(x+1)\log\,(x-1)-4\,(x+1)\log(x)-2}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x) - 2)/(x +
 1)
```

giac [A] time = 1.15, size = 34, normalized size = 1.06

$$-\frac{1}{2(x+1)} - \log\left(\left|-\frac{1}{x+1}+1\right|\right) + \frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="giac")
[Out] -1/2/(x + 1) - log(abs(-1/(x + 1) + 1)) + 1/4*log(abs(-2/(x + 1) + 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.78

$$-\ln(x) + \frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} - \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)/x/(x+1)^2,x)
[Out] -1/2/(x+1)+3/4*ln(x+1)+1/4*ln(x-1)-ln(x)
```

maxima [A] time = 0.45, size = 24, normalized size = 0.75

$$-\frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")
[Out] -1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.81

$$\frac{\ln(x-1)}{4} + \frac{3\ \ln(x+1)}{4} - \ln(x) - \frac{1}{2\ (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - 1)*(x + 1)^2),x)
[Out] log(x - 1)/4 + (3*log(x + 1))/4 - log(x) - 1/(2*(x + 1))
```

sympy [A] time = 0.15, size = 24, normalized size = 0.75

$$-\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)**2,x)
[Out] -log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)
```

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)}\,dx$

Optimal. Leaf size=40

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

```
[Out] (a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {72}

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax}{(-p+x)(-q+x)}\,dx &= \int \left(\frac{-b-ap}{(p-q)(p-x)}+\frac{b+aq}{(p-q)(q-x)}\right)dx\\ &= \frac{(b+ap)\log(p-x)}{p-q}-\frac{(b+aq)\log(q-x)}{p-q}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.85

$$\frac{(ap+b)\log(x-p)-(aq+b)\log(x-q)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)
```

fricas [A] time = 0.41, size = 34, normalized size = 0.85

$$\frac{\left(ap+b\right)\log\left(-p+x\right)-\left(aq+b\right)\log\left(-q+x\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fricas")
```

```
[Out] ((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)
```

giac [A] time = 1.18, size = 42, normalized size = 1.05

$$\frac{(ap+b)\log(|-p+x|)}{p-q}-\frac{(aq+b)\log(|-q+x|)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="giac")
```

```
[Out] (a*p + b)*log(abs(-p + x))/(p - q) - (a*q + b)*log(abs(-q + x))/(p - q)
```

maple [A] time = 0.01, size = 66, normalized size = 1.65

$$\frac{ap\ln(-p+x)}{p-q}-\frac{aq\ln(-q+x)}{p-q}+\frac{b\ln(-p+x)}{p-q}-\frac{b\ln(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] -1/(p-q)*ln(-q+x)*a*q-1/(p-q)*ln(-q+x)*b+1/(p-q)*ln(-p+x)*a*p+1/(p-q)*ln(-p
+x)*b
```

maxima [A] time = 0.45, size = 40, normalized size = 1.00

$$\frac{(ap+b)\log(-p+x)}{p-q}-\frac{(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")
```

```
[Out] (a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)
```

mupad [B] time = 0.25, size = 40, normalized size = 1.00

$$\frac{\ln(x-p)\,(b+a\,p)}{p-q}-\frac{\ln(x-q)\,(b+a\,q)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x)/((p - x)*(q - x)),x)
```

```
[Out] (log(x - p)*(b + a*p))/(p - q) - (log(x - q)*(b + a*q))/(p - q)
```

sympy [B] time = 0.88, size = 144, normalized size = 3.60

$$\frac{(ap+b)\log\left(x+\frac{-2apq-bp-bq-\frac{p^2(ap+b)}{p-q}+\frac{2pq(ap+b)}{p-q}-\frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q}-\frac{(aq+b)\log\left(x+\frac{-2apq-bp-bq+\frac{p^2(aq+b)}{p-q}-\frac{2pq(aq+b)}{p-q}+\frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] (a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(a
*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b))/(p - q) - (a*q
 + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*q +
 b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b))/(p - q)
```

3.8 $\int \frac{1}{c+bx+ax^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(-1),x]
[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c+bx+ax^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(-1),x]
[Out] (2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 0.41, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="fricas")
```

```
[Out] [log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a
*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [**A**] time = 1.35, size = 34, normalized size = 1.00

$$\frac{2 \arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="giac")
```

```
[Out] 2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [**A**] time = 0.01, size = 35, normalized size = 1.03

$$\frac{2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x+c),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 0.20, size = 46, normalized size = 1.35

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,a\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + b*x + a*x^2),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*a*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

sympy [**B**] time = 0.21, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x+c),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))
```

3.9 $\int \frac{b+ax}{1+x^2} dx$

Optimal. Leaf size=16

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

```
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {635, 203, 260}

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax}{1+x^2}\,dx &= a\int \frac{x}{1+x^2}\,dx+b\int \frac{1}{1+x^2}\,dx\\ &= b\tan^{-1}(x)+\frac{1}{2}a\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{2}a\log\left(x^2+1\right)+b\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

fricas [A] time = 0.40, size = 14, normalized size = 0.88

$$b\arctan(x)+\frac{1}{2}a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="fricas")
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

giac [A] time = 1.19, size = 14, normalized size = 0.88

$$b\arctan(x) + \frac{1}{2}\,a\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="giac")
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{a\ln\left(x^2 + 1\right)}{2} + b\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(x^2+1),x)
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

maxima [A] time = 0.97, size = 14, normalized size = 0.88

$$b\arctan(x) + \frac{1}{2}\,a\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="maxima")
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$\frac{a\,\ln\left(x^2 + 1\right)}{2} + b\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x)/(x^2 + 1),x)
[Out] (a*log(x^2 + 1))/2 + b*atan(x)
```

sympy [C] time = 0.16, size = 26, normalized size = 1.62

$$\left(\frac{a}{2} - \frac{ib}{2}\right)\log\left(x - i\right) + \left(\frac{a}{2} + \frac{ib}{2}\right)\log\left(x + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x**2+1),x)
[Out] (a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)
```

3.10 $\int \frac{1}{3-2x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(1/2*(1-x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3-2x+x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+2x\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.42, size = 14, normalized size = 0.74

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

giac [A] time = 1.21, size = 14, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{\sqrt{2}\arctan\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(2*x-2)*2^(1/2))
```

maxima [A] time = 0.95, size = 14, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

mupad [B] time = 0.03, size = 14, normalized size = 0.74

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}(x-1)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x + 3),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1))/2))/2
```

sympy [A] time = 0.11, size = 22, normalized size = 1.16

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}-\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2
```

3.11 $\int \frac{1}{(-1+x)^2(1+x^2)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4(x^2+1)}+\frac{1}{4}\log(x^2+1)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

```
[Out] 1/4/(1-x)-1/4/(x^2+1)+1/4*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {741, 801, 635, 203, 260}

$$-\frac{1}{4(x^2+1)}+\frac{1}{4}\log(x^2+1)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Log[1 + x^2]
/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-1+x)^2\left(1+x^2\right)^2}\,dx &= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \frac{-4+2x}{(-1+x)^2\left(1+x^2\right)}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}+\frac{-1-2x}{1+x^2}\right)dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)-\frac{1}{4}\int \frac{-1-2x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)-\frac{1}{2}\log(1-x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.71

$$\frac{1}{4}\left(-\frac{1}{x^2+1}+\log\left(x^2+1\right)+\frac{1}{1-x}-2\log(x-1)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] ((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[1 + x^2])/
4
```

fricas [A] time = 0.42, size = 71, normalized size = 1.45

$$-\frac{x^2-\left(x^3-x^2+x-1\right)\arctan(x)-\left(x^3-x^2+x-1\right)\log\left(x^2+1\right)+2\left(x^3-x^2+x-1\right)\log(x-1)+x}{4\left(x^3-x^2+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*log(x^2 + 1
) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 + x - 1)
```

giac [B] time = 1.04, size = 80, normalized size = 1.63

$$\frac{1}{16}\pi-\frac{1}{4}\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{\frac{2}{x-1}+1}{8\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)}-\frac{1}{4(x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/(x - 1) +
1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*arctan(x) + 1/4*log(2/
(x - 1) + 2/(x - 1)^2 + 1)
```

maple [A] time = 0.01, size = 36, normalized size = 0.73

$$\frac{\arctan(x)}{4}-\frac{\ln(x-1)}{2}+\frac{\ln\left(x^2+1\right)}{4}-\frac{1}{4(x-1)}-\frac{1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^2/(x^2+1)^2,x)
```

```
[Out] -1/4/(x-1)-1/2*ln(x-1)-1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)
```

maxima [A] time = 0.95, size = 39, normalized size = 0.80

$$-\frac{x^2+x}{4\left(x^3-x^2+x-1\right)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2 + 1) - 1/2
*log(x - 1)
```

mupad [B] time = 0.13, size = 49, normalized size = 1.00

$$-\frac{\ln\left(x-1\right)}{2}-\frac{\frac{x^2}{4}+\frac{x}{4}}{x^3-x^2+x-1}+\ln\left(x-\mathrm{i}\right)\left(\frac{1}{4}-\frac{1}{8}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x - 1)^2),x)
```

```
[Out] log(x - 1i)*(1/4 - 1i/8) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/8) - (x/4 +
 x^2/4)/(x - x^2 + x^3 - 1)
```

sympy [A] time = 0.18, size = 41, normalized size = 0.84

$$\frac{-x^2-x}{4x^3-4x^2+4x-4}-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

```
[Out] (-x**2 - x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2 + 1)/4 +
atan(x)/4
```

3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx$

Optimal. Leaf size=68

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

```
[Out] a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {148}

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Lo
g[c - x])/((a - c)*(b - c))
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx &= \int \left(-\frac{a}{(a-b)(a-c)(a-x)}+\frac{b}{(a-b)(b-c)(b-x)}+\frac{c}{(a-c)(-b+c)(c-x)}\right) \\ &= \frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.91

$$\frac{a(b-c)\log(x-a)+b(c-a)\log(x-b)+c(a-b)\log(x-c)}{(a-b)(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/((
a - b)*(a - c)*(b - c))
```

fricas [A] time = 0.49, size = 81, normalized size = 1.19

$$\frac{(a-b)c\log(-c+x)+(ab-ac)\log(-a+x)-(ab-bc)\log(-b+x)}{a^2b-ab^2+(a-b)c^2-\left(a^2-b^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")
```

```
[Out] ((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x))
/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)
```

giac [A] time = 1.12, size = 81, normalized size = 1.19

$$\frac{a\log\left(\left|-a+x\right|\right)}{a^2-ab-ac+bc}-\frac{b\log\left(\left|-b+x\right|\right)}{ab-b^2-ac+bc}+\frac{c\log\left(\left|-c+x\right|\right)}{ab-ac-bc+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="giac")
```

```
[Out] a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2
- a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)
```

maple [A] time = 0.01, size = 69, normalized size = 1.01

$$\frac{a\ln\left(-a+x\right)}{\left(a-b\right)\left(a-c\right)}-\frac{b\ln\left(-b+x\right)}{\left(a-b\right)\left(b-c\right)}+\frac{c\ln\left(-c+x\right)}{\left(b-c\right)\left(a-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] c/(b-c)/(a-c)*ln(-c+x)+a/(a-b)/(a-c)*ln(-a+x)-b/(a-b)/(b-c)*ln(-b+x)
```

maxima [A] time = 0.42, size = 78, normalized size = 1.15

$$\frac{a\log\left(-a+x\right)}{a^2-ab-(a-b)c}-\frac{b\log\left(-b+x\right)}{ab-b^2-(a-b)c}+\frac{c\log\left(-c+x\right)}{ab-(a+b)c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")
```

```
[Out] a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)*
c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)
```

mupad [B] time = 0.57, size = 87, normalized size = 1.28

$$\ln\left(x-a\right)\left(\frac{b}{\left(a-b\right)\,\left(b-c\right)}-\frac{c}{\left(a-c\right)\,\left(b-c\right)}\right)-\frac{b\,\ln\left(x-b\right)}{\left(a-b\right)\,\left(b-c\right)}+\frac{c\,\ln\left(x-c\right)}{\left(a-c\right)\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - x)*(b - x)*(c - x)),x)
```

```
[Out] log(x - a)*(b/((a - b)*(b - c)) - c/((a - c)*(b - c))) - (b*log(x - b))/((a
 - b)*(b - c)) + (c*log(x - c))/((a - c)*(b - c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] Timed out
```

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=47

$$\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}$$

```
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {444, 36, 31}

$$\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] -Log[a^2 + x^2]/(2*(a^2 - b^2)) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a^2+x^2\right)\left(b^2+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a^2+x\right)\left(b^2+x\right)}\,dx,x,x^2\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{a^2+x}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{b^2+x}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}\\
&= -\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.72

$$\frac{\log\left(b^2+x^2\right)-\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))
```

fricas [A] time = 0.41, size = 32, normalized size = 0.68

$$-\frac{\log\left(a^2+x^2\right)-\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

giac [A] time = 1.05, size = 43, normalized size = 0.91

$$-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

```
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

maple [A] time = 0.01, size = 44, normalized size = 0.94

$$-\frac{\ln\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\ln\left(b^2+x^2\right)}{2a^2-2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+x^2)/(b^2+x^2),x)
```

```
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

maxima [A] time = 0.42, size = 43, normalized size = 0.91

$$-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

mupad [B] time = 0.30, size = 256, normalized size = 5.45

$$\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x^2\left(8a^2+8b^2\right)+16a^2b^2}{2\left(a^2-b^2\right)}-4x^2\right)1\mathrm{i}}{2\left(a^2-b^2\right)}-\frac{\left(\frac{x^2\left(8a^2+8b^2\right)+16a^2b^2}{2\left(a^2-b^2\right)}+4x^2\right)1\mathrm{i}}{2\left(a^2-b^2\right)}}{\frac{\frac{x^2\left(8a^2+8b^2\right)+16a^2b^2}{2\left(a^2-b^2\right)}-4x^2}{2\left(a^2-b^2\right)}+\frac{\frac{x^2\left(8a^2+8b^2\right)+16a^2b^2}{2\left(a^2-b^2\right)}+4x^2}{2\left(a^2-b^2\right)}}\right)1\mathrm{i}}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a^2 + x^2)*(b^2 + x^2)),x)
```

```
[Out] (atan(((((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)*1i)/(2
*(a^2 - b^2)) - (((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^
2)*1i)/(2*(a^2 - b^2)))/(((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)
) - 4*x^2)/(2*(a^2 - b^2)) + ((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 -
b^2)) + 4*x^2)/(2*(a^2 - b^2))))*1i)/(a^2 - b^2)
```

sympy [B] time = 0.70, size = 121, normalized size = 2.57

$$\frac{\log\left(-\frac{a^4}{2(a-b)(a+b)} + \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} - \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x^2\right)}{2\,(a-b)\,(a+b)} - \frac{\log\left(\frac{a^4}{2(a-b)(a+b)} - \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} + \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x\right.}{2\,(a-b)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**4
/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - log(a**4/(2*(a
- b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(a +
 b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))
```

3.14 $\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=40

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

```
[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {481, 203}

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+x^2\right)\left(b^2+x^2\right)}\,dx &= \frac{a^2\int \frac{1}{a^2+x^2}\,dx}{a^2-b^2}-\frac{b^2\int \frac{1}{b^2+x^2}\,dx}{a^2-b^2}\\ &= \frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.75

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)-b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)
```

fricas [A] time = 0.43, size = 30, normalized size = 0.75

$$\frac{a\arctan\left(\frac{x}{a}\right)-b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

```
[Out] (a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)
```

giac [A] time = 1.17, size = 40, normalized size = 1.00

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

maple [A] time = 0.01, size = 41, normalized size = 1.02

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+x^2)/(b^2+x^2),x)
```

```
[Out] a*arctan(1/a*x)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

maxima [A] time = 0.97, size = 40, normalized size = 1.00

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

mupad [B] time = 0.18, size = 191, normalized size = 4.78

$$-\frac{a\,\mathrm{atan}\left(\frac{x\,\left(2\,a^4+2\,b^4\right)-\frac{a^2\,x\,\left(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6\right)}{{\left(2\,a^2-2\,b^2\right)}^2}}{a\,b^2\,\left(2\,a^2-2\,b^2\right)}\right)}{a^2-b^2}-\frac{b\,\mathrm{atan}\left(\frac{x\,\left(2\,a^4+2\,b^4\right)-\frac{b^2\,x\,\left(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6\right)}{{\left(2\,a^2-2\,b^2\right)}^2}}{a^2\,b\,\left(2\,a^2-2\,b^2\right)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a^2 + x^2)*(b^2 + x^2)),x)
```

```
[Out] - (a*atan((x*(2*a^4 + 2*b^4) - (a^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^
2))/(2*a^2 - 2*b^2)^2)/(a*b^2*(2*a^2 - 2*b^2))))/(a^2 - b^2) - (b*atan((x*(
2*a^4 + 2*b^4) - (b^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^2))/(2*a^2 - 2
*b^2)^2)/(a^2*b*(2*a^2 - 2*b^2))))/(a^2 - b^2)
```

sympy [C] time = 1.26, size = 393, normalized size = 9.82

$$-\frac{ia\log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3}+\frac{4ia^5b^2}{(a-b)^3(a+b)^3}-\frac{2ia^3b^4}{(a-b)^3(a+b)^3}+\frac{ia^3}{(a-b)(a+b)}+\frac{iab^2}{(a-b)(a+b)}+x\right)}{2\,(a-b)\,(a+b)}+\frac{ia\log\left(\frac{2ia^7}{(a-b)^3(a+b)^3}-\frac{4ia^5b^2}{(a-b)^3(a+b)^3}+\right.}{2\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] -I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a - b)**3*(a +
 b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**3/((a - b)*(a + b))
+ I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*a*log(2*I*a**7/((
a - b)**3*(a + b)**3) - 4*I*a**5*b**2/((a - b)**3*(a + b)**3) + 2*I*a**3*b*
*4/((a - b)**3*(a + b)**3) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a - b)*(
a + b)) + x)/(2*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3*(a +
b)**3) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(a + b))
 - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b)) + x)/(2*(a -
 b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(a + b)**3) - 4*I*a**2*b**
5/((a - b)**3*(a + b)**3) - I*a**2*b/((a - b)*(a + b)) + 2*I*b**7/((a - b)*
*3*(a + b)**3) - I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b))
```

3.15 $\int \frac{x}{(-1+x)\left(1+x^2\right)} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1-x)-1/4*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {801, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(-1+x)\left(1+x^2\right)} \, dx &= \int \left(\frac{1}{2(-1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) dx\\
&= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1-x}{1+x^2} \, dx\\
&= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{1+x^2} \, dx-\frac{1}{2}\int \frac{x}{1+x^2} \, dx\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1-x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

fricas [A] time = 0.41, size = 19, normalized size = 0.70

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

giac [A] time = 1.16, size = 20, normalized size = 0.74

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{\arctan(x)}{2}+\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-1)/(x^2+1),x)
[Out] 1/2*ln(x-1)-1/4*ln(x^2+1)+1/2*arctan(x)
```

maxima [A] time = 0.95, size = 19, normalized size = 0.70

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.93

$$\frac{\ln\left(x-1\right)}{2}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x - 1)),x)
[Out] log(x - 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy [**A**] time = 0.13, size = 19, normalized size = 0.70

$$\frac{\log(x-1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x**2+1),x)
```

```
[Out] log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.16 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

fricas [A] time = 0.42, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

giac [A] time = 1.19, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x+1\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x)
```

```
[Out] -1/3*ln(x+1)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 0.95, size = 34, normalized size = 0.83

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^{2}-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

mupad [B] time = 0.21, size = 46, normalized size = 1.12

$$-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

sympy [A] time = 0.13, size = 41, normalized size = 1.00

$$-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^{2}-x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

3.17 $\int \frac{x^3}{(-1+x)^2\left(1+x^3\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

```
[Out] 1/2/(1-x)+3/4*ln(1-x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6725, 628}

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(-1+x)^2\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{2(-1+x)^2}+\frac{3}{4(-1+x)}-\frac{1}{12(1+x)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx\\
&= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.79

$$\frac{1}{12}\left(-\frac{6}{x-1}+9\log(x-1)-\log(x+1)-4\log\left((x-1)^2+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] (-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12
```

fricas [A] time = 0.42, size = 40, normalized size = 0.93

$$-\frac{4\,(x-1)\log\left(x^2-x+1\right)+(x-1)\log\left(x+1\right)-9\,(x-1)\log\left(x-1\right)+6}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1)*log(x -
1) + 6)/(x - 1)
```

giac [A] time = 1.05, size = 36, normalized size = 0.84

$$-\frac{1}{2(x-1)}-\frac{1}{3}\log\left(\frac{1}{x-1}+\frac{1}{(x-1)^2}+1\right)-\frac{1}{12}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2/(x - 1) - 1/3*log(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*log(abs(-2/(x -
1) - 1))
```

maple [A] time = 0.01, size = 32, normalized size = 0.74

$$\frac{3\ln(x-1)}{4}-\frac{\ln(x+1)}{12}-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{1}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x-1)^2/(x^3+1),x)
```

```
[Out] -1/12*ln(x+1)-1/3*ln(x^2-x+1)-1/2/(x-1)+3/4*ln(x-1)
```

maxima [A] time = 0.95, size = 31, normalized size = 0.72

$$-\frac{1}{2(x-1)}-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{12}\log(x+1)+\frac{3}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x - 1)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.77

$$\frac{3\ \ln(x-1)}{4}-\frac{\ln(x+1)}{12}-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{1}{2\ (x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x^3 + 1)*(x - 1)^2),x)
```

```
[Out] (3*log(x - 1))/4 - log(x + 1)/12 - log(x^2 - x + 1)/3 - 1/(2*(x - 1))
```

sympy [A] time = 0.16, size = 31, normalized size = 0.72

$$\frac{3\log(x-1)}{4}-\frac{\log(x+1)}{12}-\frac{\log\left(x^2-x+1\right)}{3}-\frac{1}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-1+x)**2/(x**3+1),x)
```

```
[Out] 3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2)
```

3.18 $\int \frac{1}{1+x^4} \, dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}$$

$$= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,d\right.}{2\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

fricas [A] time = 0.43, size = 95, normalized size = 1.12

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

giac [A] time = 1.31, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.00, size = 58, normalized size = 0.68

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{4}+\frac{\sqrt{2}\ \ln\left(\frac{x^{2}+\sqrt{2}\ x+1}{x^{2}-\sqrt{2}\ x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x)
```

```
[Out] 1/4*2^(1/2)*arctan(2^(1/2)*x-1)+1/4*2^(1/2)*arctan(2^(1/2)*x+1)+1/8*2^(1/2)
*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))
```

maxima [A] time = 0.97, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^{2}+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.00, size = 33, normalized size = 0.39

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

sympy [A] time = 0.16, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^{2}-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^{2}+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.19 $\int \frac{x^2}{1+x^4} dx$

Optimal. Leaf size=85

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\ &= \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{2}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.75

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)-\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqrt[2]*x + x
^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

fricas [A] time = 0.43, size = 95, normalized size = 1.12

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{8}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

giac [A] time = 0.99, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.00, size = 58, normalized size = 0.68

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{4}+\frac{\sqrt{2}\ \ln\left(\frac{x^{2}-\sqrt{2}\ x+1}{x^{2}+\sqrt{2}\ x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1),x)
```

```
[Out] 1/8*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))+1/4*2^(1/2)*arctan(2^(1
/2)*x-1)+1/4*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 0.98, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^{2}+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.21, size = 33, normalized size = 0.39

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 + 1i/4)
```

sympy [A] time = 0.15, size = 73, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^{2}-\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^{2}+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 +
 sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.20 $\int \frac{1}{1+x^2+x^4} \, dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx$$
$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subst}\Big($$
$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)$$

Mathematica [C] time = 0.06, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

fricas [A] time = 0.42, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

giac [A] time = 1.02, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 54, normalized size = 0.81

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2+x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+1),x)
```

```
[Out] -1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1/
6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 0.95, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

mupad [B] time = 0.18, size = 47, normalized size = 0.70

$$\operatorname{atanh}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atanh}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^4 + 1),x)
```

```
[Out] atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

sympy [A] time = 0.19, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.21 $\int (a+bx)^p \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{p+1}}{bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b + b*p)
```

fricas [A] time = 0.43, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^p}{bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^p/(b*p + b)
```

giac [A] time = 0.99, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="giac")
```

```
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{(bx+a)^{p+1}}{(p+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^p,x)
```

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

maxima [A] time = 0.41, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 0.22, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{p+1}}{b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^p,x)
```

```
[Out] (a + b*x)^(p + 1)/(b*(p + 1))
```

sympy [A] time = 0.06, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b
```

3.22 $\int x(a+bx)^p\,dx$

Optimal. Leaf size=39

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

```
[Out] -a*(b*x+a)^(1+p)/b^2/(1+p)+(b*x+a)^(2+p)/b^2/(2+p)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^p,x]
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^p\,dx &= \int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx\\ &= -\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{p+1}(b(p+1)x-a)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^p,x]
[Out] ((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

fricas [A] time = 0.43, size = 53, normalized size = 1.36

$$\frac{\left(abpx+\left(b^2p+b^2\right)x^2-a^2\right)(bx+a)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="fricas")
[Out] (a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

giac [A] time = 1.24, size = 76, normalized size = 1.95

$$\frac{(bx+a)^p b^2 p x^2 + (bx+a)^p abpx + (bx+a)^p b^2 x^2 - (bx+a)^p a^2}{b^2 p^2 + 3\,b^2 p + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x +
 a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$-\frac{\left(-xpb-bx+a\right)(bx+a)^{p+1}}{\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^p,x)
```

```
[Out] -(b*x+a)^(p+1)*(-b*p*x-b*x+a)/b^2/(p^2+3*p+2)
```

maxima [A] time = 0.43, size = 42, normalized size = 1.08

$$\frac{\left(b^2\left(p+1\right)x^2+abpx-a^2\right)(bx+a)^p}{\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)
```

mupad [B] time = 0.39, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if}\ \ p=-1\\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if}\ \ p=-2\\ \frac{2\left(\frac{(a+b\,x)^{p+2}}{2\,p+4}-\frac{a\,(a+b\,x)^{p+1}}{2\,p+2}\right)}{b^2} & \text{if}\ \ p\neq -1\wedge p\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a/
(a + b*x))/b^2, p ~= -1 & p ~= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(a
+ b*x)^(p + 1))/(2*p + 2)))/b^2)
```

sympy [A] time = 0.65, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b=0\\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ p=-2\\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for}\ p=-1\\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p +
 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x*
*2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b
**2*p**2 + 3*b**2*p + 2*b**2), True))
```

3.23 $\int x^2(a+bx)^p\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

```
[Out] a^2*(b*x+a)^(1+p)/b^3/(1+p)-2*a*(b*x+a)^(2+p)/b^3/(2+p)+(b*x+a)^(3+p)/b^3/(
3+p)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^p,x]
```

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 + p
)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^p\,dx&=\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{p+1}\left(2a^2-2ab(p+1)x+b^2\left(p^2+3p+2\right)x^2\right)}{b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b^
3*(1 + p)*(2 + p)*(3 + p))
```

fricas [A] time = 0.44, size = 96, normalized size = 1.60

$$-\frac{\left(2\,a^2bpx-\left(b^3p^2+3\,b^3p+2\,b^3\right)x^3-2\,a^3-\left(ab^2p^2+ab^2p\right)x^2\right)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b^
2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

giac [B] time = 1.22, size = 140, normalized size = 2.33

$$\frac{(bx+a)^p b^3 p^2 x^3 + (bx+a)^p ab^2 p^2 x^2 + 3\,(bx+a)^p b^3 p x^3 + (bx+a)^p ab^2 p x^2 + 2\,(bx+a)^p b^3 x^3 - 2\,(bx+a)^p a^2 bpx +}{b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p*
x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2*b
*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

maple [A] time = 0.01, size = 73, normalized size = 1.22

$$\frac{\left(b^2p^2x^2 + 3b^2p\,x^2 - 2abpx + 2x^2b^2 - 2axb + 2a^2\right)(bx+a)^{p+1}}{\left(p^3 + 6p^2 + 11p + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^p,x)
```

```
[Out] (b*x+a)^(p+1)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(p^3+6*p^2+11*p+6)
```

maxima [A] time = 0.43, size = 68, normalized size = 1.13

$$\frac{\left(\left(p^2 + 3\,p + 2\right)b^3x^3 + \left(p^2 + p\right)ab^2x^2 - 2\,a^2bpx + 2\,a^3\right)(bx+a)^p}{\left(p^3 + 6\,p^2 + 11\,p + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x
+ a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

mupad [B] time = 0.60, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } p=-1 \\ \frac{x}{b^2} - \frac{a^2}{b^3\,(a+b\,x)} - \frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } p=-2 \\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } p=-3 \\ \frac{2\,(a+b\,x)^{p+1}\left(8\,a^2-8\,a\,b\,p\,x-8\,a\,b\,x+4\,b^2\,p^2\,x^2+12\,b^2\,p\,x^2+8\,b^2\,x^2\right)}{b^3\left(8\,p^3+48\,p^2+88\,p+48\right)} & \text{if } p\neq -1 \land p\neq -2 \land p\neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p ~=
```

```
-3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x + 4*b
^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

sympy [A] time = 1.24, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^p x^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2
 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2
 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*
p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3
*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*b*
*3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p**3 + 6*b*
*3*p**2 + 11*b**3*p + 6*b**3), True))
```

3.24 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

fricas [A] time = 0.39, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

giac [A] time = 1.09, size = 11, normalized size = 1.10

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

maxima [A] time = 0.41, size = 10, normalized size = 1.00

$$\frac{\log (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\frac{\ln (a + b\, x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log (a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

3.25 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

fricas [A] time = 0.40, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 1.06, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

maxima [A] time = 0.42, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

mupad [B] time = 0.12, size = 12, normalized size = 1.00

$$-\frac{1}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

sympy [A] time = 0.13, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

3.26 $\int \frac{x}{a+bx} dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx} dx &= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx \\ &= \frac{x}{b} - \frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

fricas [A] time = 0.40, size = 17, normalized size = 0.94

$$\frac{bx - a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

giac [A] time = 1.19, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a\log(|bx+a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$-\frac{a\ln\left(bx+a\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x)
```

```
[Out] 1/b*x-a*ln(b*x+a)/b^2
```

maxima [A] time = 0.42, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

mupad [B] time = 0.03, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

sympy [A] time = 0.10, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

3.27 $\int \frac{x^2}{a+bx}\,dx$

Optimal. Leaf size=31

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

fricas [A] time = 0.39, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

giac [A] time = 0.79, size = 30, normalized size = 0.97

$$\frac{a^2\log\left(|bx+a|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.97

$$\frac{x^2}{2b} + \frac{a^2 \ln(bx+a)}{b^3} - \frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x)
```

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

maxima [A] time = 0.41, size = 29, normalized size = 0.94

$$\frac{a^2 \log(bx+a)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

mupad [B] time = 0.13, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln(a+b\,x) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

sympy [A] time = 0.11, size = 26, normalized size = 0.84

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.28 $\int \frac{1}{x(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a}-\frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

fricas [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

giac [A] time = 1.20, size = 20, normalized size = 1.11

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

maxima [A] time = 0.41, size = 18, normalized size = 1.00

$$-\frac{\log(bx+a)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

mupad [B] time = 0.13, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

sympy [A] time = 0.14, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

3.29 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x)),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x)),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2

fricas [A] time = 0.41, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")

[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)

giac [A] time = 1.09, size = 30, normalized size = 1.07

$$\frac{b\log(|bx+a|)}{a^2}-\frac{b\log(|x|)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x)
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

maxima [A] time = 0.41, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.17, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.30 $\int \frac{1}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

fricas [A] time = 0.40, size = 63, normalized size = 1.50

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\left(b^2x^2+abx\right)\log(x)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

giac [A] time = 1.15, size = 52, normalized size = 1.24

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 1.02

$$-\frac{b}{(bx+a)\,a^2}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

maxima [A] time = 0.42, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log(bx+a)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

mupad [B] time = 0.19, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3}-\frac{1}{a\,x\,\left(a+b\,x\right)}-\frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

sympy [A] time = 0.26, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.31 $\int \frac{1}{c^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] arctan(x/c)/c
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/c]/c
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/c]/c
```

fricas [A] time = 0.40, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/c)/c
```

giac [A] time = 1.17, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="giac")
[Out] arctan(x/c)/c
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2+x^2),x)
[Out] arctan(x/c)/c
```

maxima [A] time = 0.95, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="maxima")
[Out] arctan(x/c)/c
```

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2 + x^2),x)
[Out] atan(x/c)/c
```

sympy [C] time = 0.11, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ic+x\right)}{2}+\frac{i\log\left(ic+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2+x**2),x)
[Out] (-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c
```

3.32 $\int \frac{1}{c^2-x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] arctanh(x/c)/c
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {206}

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2-x^2}\,dx = \frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

fricas [A] time = 0.40, size = 18, normalized size = 1.80

$$\frac{\log\left(c+x\right)-\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(c + x) - log(-c + x))/c
```

giac [B] time = 0.95, size = 23, normalized size = 2.30

$$\frac{\log\left(|c+x|\right)}{2\,c}-\frac{\log\left(|-c+x|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="giac")
[Out] 1/2*log(abs(c + x))/c - 1/2*log(abs(-c + x))/c
```

maple [B] time = 0.01, size = 22, normalized size = 2.20

$$-\frac{\ln\left(-c+x\right)}{2c}+\frac{\ln\left(c+x\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2-x^2),x)
[Out] -1/2/c*ln(-c+x)+1/2/c*ln(x+c)
```

maxima [B] time = 0.43, size = 21, normalized size = 2.10

$$\frac{\log\left(c+x\right)}{2\,c}-\frac{\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="maxima")
[Out] 1/2*log(c + x)/c - 1/2*log(-c + x)/c
```

mupad [B] time = 0.16, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2 - x^2),x)
[Out] atanh(x/c)/c
```

sympy [B] time = 0.12, size = 15, normalized size = 1.50

$$-\frac{\frac{\log\left(-c+x\right)}{2}-\frac{\log\left(c+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2-x**2),x)
[Out] -(log(-c + x)/2 - log(c + x)/2)/c
```

3.33 $\int \frac{1}{-1+2x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/6*ln(1-2^(1/3)*x)*2^(2/3)-1/12*ln(1+2^(1/3)*x+2^(2/3)*x^2)*2^(2/3)-1/6*ar
ctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 - 2^(1/3)*x]
/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+2x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+\sqrt[3]{2}\,x}\,dx+\frac{1}{3}\int \frac{-2-\sqrt[3]{2}\,x}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx\\ &= \frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{1}{2}\int \frac{1}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx-\frac{\int \frac{\sqrt[3]{2}+2\,2^{2/3}x}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx}{6\sqrt[3]{2}}\\ &= \frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{2}\,x\right)}{\sqrt[3]{2}}\\ &= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}\,x}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.85

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}\,x+1\right)-2\log\left(1-\sqrt[3]{2}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{2}\,x+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)^(-1),x]
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)*x] +
Log[1 + 2^(1/3)*x + 2^(2/3)*x^2])/2^(1/3)
```

fricas [A] time = 0.41, size = 63, normalized size = 0.81

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2\,x^2+2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(2\,x-2^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="fricas")
[Out] -1/6*sqrt(6)*2^(1/6)*arctan(1/6*sqrt(6)*2^(1/6)*(2*2^(2/3)*x + 2^(1/3))) -
1/12*2^(2/3)*log(2*x^2 + 2^(2/3)*x + 2^(1/3)) + 1/6*2^(2/3)*log(2*x - 2^(2/
3))
```

giac [A] time = 1.01, size = 57, normalized size = 0.73

$$-\frac{1}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(2\,x+\left(\frac{1}{2}\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 4^{\frac{1}{3}}\log\left(x^2+\left(\frac{1}{2}\right)^{\frac{1}{3}}x+\left(\frac{1}{2}\right)^{\frac{2}{3}}\right)+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{1}{2}\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*(1/2)^(1/3)*arctan(2/3*sqrt(3)*(1/2)^(2/3)*(2*x + (1/2)^(1/3))
) - 1/12*4^(1/3)*log(x^2 + (1/2)^(1/3)*x + (1/2)^(2/3)) + 1/3*(1/2)^(1/3)*l
og(abs(x - (1/2)^(1/3)))
```

maple [A] time = 0.00, size = 58, normalized size = 0.74

$$-\frac{2^{\frac{2}{3}}\sqrt{3}\ \arctan\left(\frac{\left(2\,2^{\frac{1}{3}}x+1\right)\sqrt{3}}{3}\right)}{6}+\frac{2^{\frac{2}{3}}\ln\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\ln\left(x^{2}+\frac{2^{\frac{2}{3}}x}{2}+\frac{2^{\frac{1}{3}}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^3-1),x)
```

```
[Out] 1/6*2^(2/3)*ln(x-1/2*2^(2/3))-1/12*2^(2/3)*ln(x^2+1/2*2^(2/3)*x+1/2*2^(1/3)
)-1/6*arctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)
```

maxima [A] time = 0.96, size = 66, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}x+1\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^(1/3))) -
1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(2/3)*log(1/2*2^(2/3)
*(2^(1/3)*x - 1))
```

mupad [B] time = 0.28, size = 72, normalized size = 0.92

$$\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}}{2}\right)}{6}+\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^3 - 1),x)
```

```
[Out] (2^(2/3)*log(x - 2^(2/3)/2))/6 + (2^(2/3)*log(x - (2^(2/3)*(3^(1/2)*1i - 1)
)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 1))/4)*
(3^(1/2)*1i + 1))/12
```

sympy [A] time = 0.30, size = 78, normalized size = 1.00

$$\frac{2^{\frac{2}{3}}\log\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\log\left(x^{2}+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)}{12}-\frac{2^{\frac{2}{3}}\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt[3]{2}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**3-1),x)
```

```
[Out] 2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2 + 2**(1/3
)/2)/12 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.34 $\int \frac{1}{-2+x^3}\,dx$

Optimal. Leaf size=74

$$-\frac{\log\left(x^2+\sqrt[3]{2}x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] 1/6*ln(2^(1/3)-x)*2^(1/3)-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan
(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(x^2+\sqrt[3]{2}x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3) - x]/(3*
2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-2+x^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{2}+x}\,dx}{3\ 2^{2/3}}+\frac{\int \frac{-2\sqrt[3]{2}-x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{3\ 2^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\int \frac{\sqrt[3]{2}+2x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{6\ 2^{2/3}}-\frac{\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{2\sqrt[3]{2}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}x+x^2\right)}{6\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}x\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}x+x^2\right)}{6\ 2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.88

$$-\frac{\log\left(\sqrt[3]{2}x^2+2^{2/3}x+2\right)-2\log\left(2-2^{2/3}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x] + Lo
g[2 + 2^(2/3)*x + 2^(1/3)*x^2])/2^(2/3)
```

fricas [A] time = 0.41, size = 68, normalized size = 0.92

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}x+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(2x^2+4^{\frac{2}{3}}x+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(2x-4^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*x + 4^(1/3)*sqrt(3
))) - 1/24*4^(2/3)*log(2*x^2 + 4^(2/3)*x + 2*4^(1/3)) + 1/12*4^(2/3)*log(2*
x - 4^(2/3))
```

giac [A] time = 1.28, size = 57, normalized size = 0.77

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|x-2^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(
1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(abs(x - 2^(1/3)))
```

maple [A] time = 0.00, size = 54, normalized size = 0.73

$$-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\left(2^{\frac{2}{3}}x+1\right)\sqrt{3}}{3}\right)}{6}+\frac{2^{\frac{1}{3}}\ln\left(x-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{1}{3}}\ln\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-2),x)
```

```
[Out] 1/6*2^(1/3)*ln(x-2^(1/3))-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan
(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

maxima [A] time = 0.96, size = 56, normalized size = 0.76

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2\,x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(
1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x - 2^(1/3))
```

mupad [B] time = 0.36, size = 72, normalized size = 0.97

$$\frac{2^{1/3}\ln\left(x-2^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(x-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(x+\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 2),x)
```

```
[Out] (2^(1/3)*log(x - 2^(1/3)))/6 + (2^(1/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 1))/
2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(x + (2^(1/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/12
```

sympy [A] time = 0.30, size = 71, normalized size = 0.96

$$\frac{\sqrt[3]{2}\log\left(x-\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\log\left(x^2+\sqrt[3]{2}\,x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-2),x)
```

```
[Out] 2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2**(2/3))/1
2 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.35 $\int \frac{1}{-b+ax^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}\,x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] 1/3*ln(b^(1/3)-a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)+a^(1/3)*b^(1/3)*x+
a^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*a^(1/3)*x)/b^(1/3)*3
^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}\,x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) + a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-b+ax^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{-2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\int \frac{1}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,b^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-2\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3)
 - a^(1/3)*x] + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(a^(1/3)*b^
(2/3))
```

fricas [A] time = 0.43, size = 300, normalized size = 2.61

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx+b^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2-\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3-b}\right)-\left(ab^2\right)^{\frac{2}{3}}\log\left(abx^2+\left(ab^2\right)^{\frac{2}{3}}x+\right.}{6\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1/
3)*b*x + b^2 - 3*sqrt(1/3)*(2*a*b*x^2 - (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*
```

```
sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 - b)) - (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^2)^
(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3)))/(a
*b^2), -1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b
^2)^(2/3)*x + (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) + (a*b^2)^(2/3)*l
og(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a*b^2)^(2/3)*log(a*b*x
 - (a*b^2)^(2/3)))/(a*b^2)]
```

giac [A] time = 1.27, size = 104, normalized size = 0.90

$$\frac{\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{\sqrt{3}\left(a^{2}b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}-\frac{\left(a^{2}b\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="giac")
```

```
[Out] 1/3*(b/a)^(1/3)*log(abs(x - (b/a)^(1/3)))/b - 1/3*sqrt(3)*(a^2*b)^(1/3)*arc
tan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*b) - 1/6*(a^2*b)^(1/3)*
log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*b)
```

maple [A] time = 0.00, size = 92, normalized size = 0.80

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a}+\frac{\ln\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a}-\frac{\ln\left(x^{2}+\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3-b),x)
```

```
[Out] 1/3/a/(b/a)^(2/3)*ln(x-(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(x^2+(b/a)^(1/3)*x+
(b/a)^(2/3))-1/3/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*x+
1))
```

maxima [A] time = 0.96, size = 97, normalized size = 0.84

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{2}+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(
2/3)) - 1/6*log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*lo
g(x - (b/a)^(1/3))/(a*(b/a)^(2/3))
```

mupad [B] time = 0.31, size = 101, normalized size = 0.88

$$\frac{\ln\left(a^{1/3}\,x-b^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(3\,a^2\,x-\frac{3\,a^{5/3}\,b^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}-\frac{\ln\left(3\,a^2\,x+\frac{3\,a^{5/3}\,b^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b - a*x^3),x)
```

```
[Out] log(a^(1/3)*x - b^(1/3))/(3*a^(1/3)*b^(2/3)) + (log(3*a^2*x - (3*a^(5/3)*b^
(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(3*a
^2*x + (3*a^(5/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(1/3)
*b^(2/3))
```

sympy [A] time = 0.15, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3-b),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))
```

3.36 $\int \frac{1}{-2+x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}$$

```
[Out] -1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/4*arctanh(1/2*x*2^(3/4))*2^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/2^(1/4)]/(2*2^(3/4)) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2+x^4}\,dx &= -\frac{\int \frac{1}{\sqrt{2}-x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}+x^2}\,dx}{2\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\,2^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.23

$$-\frac{-\log\left(2-2^{3/4}x\right)+\log\left(2^{3/4}x+2\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)^(-1),x]
```

```
[Out] -1/4*(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/2^(3/4
)
```

fricas [B] time = 0.44, size = 63, normalized size = 1.80

$$\frac{1}{8}\cdot 8^{\frac{3}{4}} \arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2x^2+2\sqrt{2}} - \frac{1}{2}\cdot 8^{\frac{1}{4}}x\right) - \frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4x+8^{\frac{3}{4}}\right) + \frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4x-8^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="fricas")
```

```
[Out] 1/8*8^(3/4)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(2*x^2 + 2*sqrt(2)) - 1/2*8^(1/4
)*x) - 1/32*8^(3/4)*log(4*x + 8^(3/4)) + 1/32*8^(3/4)*log(4*x - 8^(3/4))
```

giac [A] time = 1.29, size = 39, normalized size = 1.11

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right) - \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x+2^{\frac{1}{4}}\right|\right) + \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x-2^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="giac")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) - 1/8*2^(1/4)*log(abs(x + 2^(1/4))) + 1/
8*2^(1/4)*log(abs(x - 2^(1/4)))
```

maple [A] time = 0.00, size = 35, normalized size = 1.00

$$-\frac{2^{\frac{1}{4}} \arctan\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4} - \frac{2^{\frac{1}{4}}\ln\left(\frac{x+2^{\frac{1}{4}}}{x-2^{\frac{1}{4}}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2),x)
```

```
[Out] -1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/8*2^(1/4)*ln((x+2^(1/4))/(x-2^(1/4)))
```

maxima [A] time = 0.98, size = 34, normalized size = 0.97

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right) + \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x-2^{\frac{1}{4}}}{x+2^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="maxima")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log((x - 2^(1/4))/(x + 2^(
1/4)))
```

mupad [B] time = 0.16, size = 20, normalized size = 0.57

$$-\frac{2^{1/4}\left(\mathrm{atan}\left(\frac{2^{3/4}x}{2}\right) + \mathrm{atanh}\left(\frac{2^{3/4}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 2),x)
```

```
[Out] -(2^(1/4)*(atan((2^(3/4)*x)/2) + atanh((2^(3/4)*x)/2)))/4
```

sympy [A] time = 0.31, size = 46, normalized size = 1.31

$$\frac{\sqrt[4]{2}\log\left(x-\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\log\left(x+\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2),x)
```

```
[Out] 2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**(1/4)*atan
(2**(3/4)*x/2)/4
```

3.37 $\int \frac{1}{-1+5x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

```
[Out] -1/10*arctan(5^(1/4)*x)*5^(3/4)-1/10*arctanh(5^(1/4)*x)*5^(3/4)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -ArcTan[5^(1/4)*x]/(2*5^(1/4)) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+5x^4} dx &= -\left(\frac{1}{2}\int \frac{1}{1-\sqrt{5}x^2} dx\right)-\frac{1}{2}\int \frac{1}{1+\sqrt{5}x^2} dx\\ &= -\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.23

$$-\frac{-\log\left(1-\sqrt[4]{5}x\right)+\log\left(\sqrt[4]{5}x+1\right)+2\tan^{-1}\left(\sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -1/4*(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/5^(1/4
)
```

fricas [B] time = 0.44, size = 58, normalized size = 1.66

$$\frac{1}{5} \cdot 5^{\frac{3}{4}} \arctan\left(\frac{1}{5} \cdot 5^{\frac{3}{4}} \sqrt{5 x^2 + \sqrt{5}} - 5^{\frac{1}{4}} x\right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5 x + 5^{\frac{3}{4}}\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5 x - 5^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*arctan(1/5*5^(3/4)*sqrt(5*x^2 + sqrt(5)) - 5^(1/4)*x) - 1/20*5^
(3/4)*log(5*x + 5^(3/4)) + 1/20*5^(3/4)*log(5*x - 5^(3/4))
```

giac [A] time = 1.08, size = 39, normalized size = 1.11

$$-\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan\left(5 \left(\frac{1}{5}\right)^{\frac{3}{4}} x\right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x + \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x - \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="giac")
```

```
[Out] -1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/4
))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))
```

maple [A] time = 0.00, size = 36, normalized size = 1.03

$$-\frac{5^{\frac{3}{4}} \arctan\left(5^{\frac{1}{4}} x\right)}{10} - \frac{5^{\frac{3}{4}} \ln\left(\frac{x+\frac{5^{\frac{3}{4}}}{5}}{x-\frac{5^{\frac{3}{4}}}{5}}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4-1),x)
```

```
[Out] -1/10*arctan(5^(1/4)*x)*5^(3/4)-1/20*5^(3/4)*ln((x+1/5*5^(3/4))/(x-1/5*5^(3
/4)))
```

maxima [A] time = 0.98, size = 41, normalized size = 1.17

$$-\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan\left(5^{\frac{1}{4}} x\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\frac{\sqrt{5} x - 5^{\frac{1}{4}}}{\sqrt{5} x + 5^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="maxima")
```

```
[Out] -1/10*5^(3/4)*arctan(5^(1/4)*x) + 1/20*5^(3/4)*log((sqrt(5)*x - 5^(1/4))/(s
qrt(5)*x + 5^(1/4)))
```

mupad [B] time = 0.18, size = 18, normalized size = 0.51

$$-\frac{5^{3/4} \left(\mathrm{atan}\left(5^{1/4} x\right) + \mathrm{atanh}\left(5^{1/4} x\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4 - 1),x)
```

```
[Out] -(5^(3/4)*(atan(5^(1/4)*x) + atanh(5^(1/4)*x)))/10
```

sympy [A] time = 0.32, size = 48, normalized size = 1.37

$$\frac{5^{\frac{3}{4}} \log \left(x - \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \log \left(x + \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \operatorname{atan}\left(\sqrt[4]{5}\, x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4-1),x)
```

```
[Out] 5**(3/4)*log(x - 5**(3/4)/5)/20 - 5**(3/4)*log(x + 5**(3/4)/5)/20 - 5**(3/4
)*atan(5**(1/4)*x)/10
```

3.38 $\int \frac{1}{7+3x^4}\,dx$

Optimal. Leaf size=171

$$-\frac{\log\left(3x^2-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\,\sqrt{2}\,x\right)}{2\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\,\sqrt{2}\,x+1\right)}{2\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}$$

```
[Out] 1/84*arctan(-1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/84*
arctan(1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)-1/168*ln(3*
x^2-3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/168*ln(3*
x^2+3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(3x^2-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\,\sqrt{2}\,x\right)}{2\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\,\sqrt{2}\,x+1\right)}{2\sqrt{2}\,\sqrt[4]{3}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 3*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) + ArcTan[1 +
 (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - Log[Sqrt[21] - Sqrt[2
]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4)) + Log[Sqrt[21] + S
qrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{7+3x^4}\,dx = \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{7+3x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+\sqrt{3}x^2}{7+3x^4}\,dx}{2\sqrt{7}}$$

$$= -\frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}+2x}{-\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}-2x}{-\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2}\,dx}{4\sqrt{21}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2}\,dx}{4\sqrt{21}}$$

$$= -\frac{\log\left(\sqrt{21}-\sqrt{2}\,3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}\,3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x,\right.}{2\sqrt{2}\sqrt[4]{3}\,7}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}} - \frac{\log\left(\sqrt{21}-\sqrt{2}\,3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\log\left(\sqrt{2}\right.}{}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 0.70

$$\frac{-\log\left(\sqrt{21}x^2-\sqrt{2}\sqrt[4]{3}\,7^{3/4}x+7\right)+\log\left(\sqrt{21}x^2+\sqrt{2}\sqrt[4]{3}\,7^{3/4}x+7\right)-2\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\right.}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 3*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]
 - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^(1
/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

fricas [A] time = 0.43, size = 163, normalized size = 0.95

$$-\frac{1}{2058}\cdot 1029^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{147}\cdot 1029^{\frac{1}{4}}\sqrt{3}\sqrt{2}\sqrt{1029^{\frac{3}{4}}\sqrt{2}x+147x^2+49\sqrt{21}}-\frac{1}{7}\cdot 1029^{\frac{1}{4}}\sqrt{2}x-1\right)-\frac{1}{2058}\cdot 10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="fricas")
```

```
[Out] -1/2058*1029^(3/4)*sqrt(2)*arctan(1/147*1029^(1/4)*sqrt(3)*sqrt(2)*sqrt(102
9^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(21)) - 1/7*1029^(1/4)*sqrt(2)*x - 1)
- 1/2058*1029^(3/4)*sqrt(2)*arctan(1/147*1029^(1/4)*sqrt(3)*sqrt(2)*sqrt(-1
029^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(21)) - 1/7*1029^(1/4)*sqrt(2)*x + 1
) + 1/8232*1029^(3/4)*sqrt(2)*log(1029^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(
21)) - 1/8232*1029^(3/4)*sqrt(2)*log(-1029^(3/4)*sqrt(2)*x + 147*x^2 + 49*s
qrt(21))
```

giac [A] time = 1.17, size = 95, normalized size = 0.56

$$\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 756^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="giac")
```

```
[Out] 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)*sqrt(2)))
 + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x - (7/3)^(1/4)*sqrt(2
))) + 1/168*756^(1/4)*log(x^2 + (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3)) - 1/168*
756^(1/4)*log(x^2 - (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3))
```

maple [A] time = 0.01, size = 111, normalized size = 0.65

$$\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,21^{\frac{3}{4}}x}{21}-1\right)}{84}+\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,21^{\frac{3}{4}}x}{21}+1\right)}{84}+\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{21}}{3}}{x^{2}-\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{21}}{3}}\right)}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+7),x)
```

```
[Out] 1/84*3^(1/2)*21^(1/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4)*x-1)+1/1
68*3^(1/2)*21^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*21^(
1/2))/(x^2-1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*21^(1/2)))+1/84*3^(1/2)*21^(1
/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4)*x+1)
```

maxima [A] time = 0.97, size = 151, normalized size = 0.88

$$\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\,\sqrt{3}x+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\,\sqrt{3}x-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="maxima")
```

```
[Out] 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)
*x + 7^(1/4)*3^(1/4)*sqrt(2))) + 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7
^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)*x - 7^(1/4)*3^(1/4)*sqrt(2))) + 1/168*7^(
1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 + 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt(7))
 - 1/168*7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 - 7^(1/4)*3^(1/4)*sqrt(2)*
x + sqrt(7))
```

mupad [B] time = 0.11, size = 45, normalized size = 0.26

$$\sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}-\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}+\frac{1}{84}\mathrm{i}\right)+\sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}+\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}-\frac{1}{84}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 7),x)
```

```
[Out] 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 - 1i/126))*(1/84 + 1i/84)
 + 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 + 1i/126))*(1/84 - 1i/
84)
```

sympy [A] time = 0.40, size = 151, normalized size = 0.88

$$-\frac{\sqrt[4]{189}\,\sqrt{2}\,\log\left(x^2-\frac{\sqrt[4]{189}\,\sqrt{2}\,x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt[4]{189}\,\sqrt{2}\,\log\left(x^2+\frac{\sqrt[4]{189}\,\sqrt{2}\,x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-\right.}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+7),x)
```

```
[Out] -189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + 1
89**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + sqr
t(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)
*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84
```

3.39 $\int \frac{1}{-1+3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}\,x\right)$$

```
[Out] -1/13*arctan(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/26
*arctanh(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1093, 207, 203}

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[(3 +
 Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+3x^2+x^4}\,dx &= \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}-\frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}\\ &= -\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.93

$$-\frac{\sqrt{\sqrt{13}-3}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)+\sqrt{3+\sqrt{13}}\,\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}\,x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x] + Sqrt[3 + Sqrt[13]
]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x])/Sqrt[26])
```

fricas [B] time = 0.42, size = 132, normalized size = 1.81

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}-3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}+3}\sqrt{\sqrt{13}-3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}-3}\right)+\frac{1}{52}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) - 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) + 3)*sqrt(sqrt(13) - 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) - 3)) + 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) -
 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(-sqr
t(26)*(3*sqrt(13) - 13)*sqrt(sqrt(13) + 3) + 52*x)
```

giac [A] time = 1.27, size = 74, normalized size = 1.01

$$-\frac{1}{26}\sqrt{26\sqrt{13}-78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}+78}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) - 1/52*sqrt
(26*sqrt(13) + 78)*log(abs(x + sqrt(1/2*sqrt(13) - 3/2))) + 1/52*sqrt(26*sq
rt(13) + 78)*log(abs(x - sqrt(1/2*sqrt(13) - 3/2)))
```

maple [A] time = 0.06, size = 56, normalized size = 0.77

$$-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctanh(2*x/(-6+2*13^(1/2))^(1/2))-2/1
3*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctan(2*x/(6+2*13^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4+3x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 3*x^2 - 1), x)
```

mupad [B] time = 0.22, size = 93, normalized size = 1.27

$$-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\,\sqrt{\sqrt{13}+3}}+\frac{3\,\sqrt{13}\,\sqrt{26}\,x}{26\,\sqrt{\sqrt{13}+3}}\right)\sqrt{\sqrt{13}+3}}{26}-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\,\sqrt{3-\sqrt{13}}}-\frac{3\,\sqrt{13}\,\sqrt{26}\,x}{26\,\sqrt{3-\sqrt{13}}}\right)\sqrt{3-\sqrt{13}}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 - 1),x)
```

```
[Out] - (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) + 3)^(1/2)) + (3*13^(1/2)*26^(1
/2)*x)/(26*(13^(1/2) + 3)^(1/2)))*(13^(1/2) + 3)^(1/2))/26 - (26^(1/2)*atan
h((26^(1/2)*x)/(2*(3 - 13^(1/2))^(1/2)) - (3*13^(1/2)*26^(1/2)*x)/(26*(3 -
13^(1/2))^(1/2)))*(3 - 13^(1/2))^(1/2))/26
```

sympy [B] time = 0.45, size = 146, normalized size = 2.00

$$\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-22\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}+312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}\right)-\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2-1),x)
```

```
[Out] sqrt(3/104 + sqrt(13)/104)*log(x - 22*sqrt(3/104 + sqrt(13)/104) + 312*(3/1
04 + sqrt(13)/104)**(3/2)) - sqrt(3/104 + sqrt(13)/104)*log(x - 312*(3/104
+ sqrt(13)/104)**(3/2) + 22*sqrt(3/104 + sqrt(13)/104)) - 2*sqrt(-3/104 + s
qrt(13)/104)*atan(2*sqrt(2)*x/(3*sqrt(-3 + sqrt(13)) + sqrt(13)*sqrt(-3 + s
qrt(13))))
```

3.40 $\int \frac{1}{-1-3x^2+x^4} \, dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

```
[Out] -1/13*arctanh(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/2
6*arctan(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1093, 207, 203}

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2/(13*(
3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1-3x^2+x^4} \, dx &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2} \, dx}{\sqrt{13}}-\frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2} \, dx}{\sqrt{13}}\\ &= -\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.93

$$-\frac{\sqrt{3+\sqrt{13}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)+\sqrt{\sqrt{13}-3}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x] + Sqrt[-3 + Sqrt[13
]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x])/Sqrt[26])
```

fricas [B] time = 0.43, size = 132, normalized size = 1.81

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}+3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}-3}\sqrt{\sqrt{13}+3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}+3}\right)-\frac{1}{52}\sqrt{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) + 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) - 3)*sqrt(sqrt(13) + 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) + 3)) - 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) +
 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(-sqr
t(26)*(3*sqrt(13) + 13)*sqrt(sqrt(13) - 3) + 52*x)
```

giac [A] time = 1.47, size = 74, normalized size = 1.01

$$-\frac{1}{26}\sqrt{26\sqrt{13}+78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}-78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}-78}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqrt
(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*sq
rt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))
```

maple [A] time = 0.04, size = 56, normalized size = 0.77

$$-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctanh(2/(6+2*13^(1/2))^(1/2)*x)-2/13*
13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctan(2/(-6+2*13^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4-3x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 3*x^2 - 1), x)
```

mupad [B] time = 0.13, size = 93, normalized size = 1.27

$$-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\,\sqrt{\sqrt{13}-3}}-\frac{3\,\sqrt{13}\,\sqrt{26}\,x}{26\,\sqrt{\sqrt{13}-3}}\right)\,\sqrt{\sqrt{13}-3}}{26}-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\,\sqrt{-\sqrt{13}-3}}+\frac{3\,\sqrt{13}\,\sqrt{26}\,x}{26\,\sqrt{-\sqrt{13}-3}}\right)\,\sqrt{-\sqrt{13}-3}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*x^2 - x^4 + 1),x)
```

```
[Out] - (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) - 3)^(1/2)) - (3*13^(1/2)*26^(1
/2)*x)/(26*(13^(1/2) - 3)^(1/2)))*(13^(1/2) - 3)^(1/2))/26 - (26^(1/2)*atan
h((26^(1/2)*x)/(2*(- 13^(1/2) - 3)^(1/2)) + (3*13^(1/2)*26^(1/2)*x)/(26*(-
13^(1/2) - 3)^(1/2)))*(- 13^(1/2) - 3)^(1/2))/26
```

sympy [A] time = 0.35, size = 24, normalized size = 0.33

$$\mathrm{RootSum}\left(2704t^4+156t^2-1,\left(t\mapsto t\log\left(-312t^3-22t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t +
x)))
```

3.41 $\int \frac{1}{1-3x^2+x^4} \, dx$

Optimal. Leaf size=72

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -1/5*arctanh(x*2^(1/2)/(3+5^(1/2))^(1/2))*10^(1/2)/(3+5^(1/2))^(1/2)+arctan
h(x*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1093, 207}

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-3x^2+x^4} \, dx &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} \, dx}{\sqrt{5}}-\frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} \, dx}{\sqrt{5}}\\ &= -\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 1.15

$$\frac{1}{20}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-2x+\sqrt{5}-1\right)\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5]
- 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 + Sqrt[5])*Log[1 + Sqr
t[5] + 2*x])/20
```

fricas [B] time = 0.42, size = 91, normalized size = 1.26

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x-1)-2x+3}{x^2-x-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
0*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*lo
g(x^2 + x - 1) + 1/4*log(x^2 - x - 1)
```

giac [A] time = 1.28, size = 81, normalized size = 1.12

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}-1\right|}{\left|2x+\sqrt{5}-1\right|}\right)-\frac{1}{4}\log\left(\left|x^2+x-1\right|\right)+\frac{1}{4}\log\left(\left|x^2-x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/20*sqr
t(5)*log(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) - 1/4*log(abs(x^2 +
 x - 1)) + 1/4*log(abs(x^2 - x - 1))
```

maple [A] time = 0.01, size = 54, normalized size = 0.75

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{10}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{10}+\frac{\ln\left(x^2-x-1\right)}{4}-\frac{\ln\left(x^2+x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2+1),x)
```

```
[Out] -1/4*ln(x^2+x-1)+1/10*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/4*ln(x^2-x-1)+
1/10*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))
```

maxima [A] time = 0.96, size = 75, normalized size = 1.04

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{2x-\sqrt{5}-1}{2x+\sqrt{5}-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x^2 + x - 1) + 1/4*lo
g(x^2 - x - 1)
```

mupad [B] time = 0.10, size = 67, normalized size = 0.93

$$\operatorname{atanh}\left(\frac{4x}{\sqrt{5}-3}-\frac{2\sqrt{5}\,x}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{10}-\frac{1}{2}\right)+\operatorname{atanh}\left(\frac{4x}{\sqrt{5}+3}+\frac{2\sqrt{5}\,x}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 3*x^2 + 1),x)
```

```
[Out] atanh((4*x)/(5^(1/2) - 3) - (2*5^(1/2)*x)/(5^(1/2) - 3))*(5^(1/2)/10 - 1/2)
 + atanh((4*x)/(5^(1/2) + 3) + (2*5^(1/2)*x)/(5^(1/2) + 3))*(5^(1/2)/10 + 1
/2)
```

sympy [B] time = 0.35, size = 158, normalized size = 2.19

$$\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}-\frac{7\sqrt{5}}{10}+120\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^{3}\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7}{2}+120\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^{3}+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2+1),x)
```

```
[Out] (sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 + 1/4)**3)
+ (1/4 - sqrt(5)/20)*log(x - 7/2 + 120*(1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10
) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/10 + 120*(-1/4 + sqrt(5)/20)**3 +
 7/2) + (-1/4 - sqrt(5)/20)*log(x + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/
10 + 7/2)
```

3.42 $\int \frac{1}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/2*arctanh(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arct
anh(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-4x^2+x^4}\,dx &= \frac{\int \frac{1}{-2-\sqrt{3}+x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{1}{-2+\sqrt{3}+x^2}\,dx}{2\sqrt{3}}\\ &= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

fricas [B] time = 0.43, size = 123, normalized size = 1.84

$$-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)+\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)-\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) +
1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(-sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) -
1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x) +
 1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log(-(sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x)
```

giac [A] time = 1.23, size = 101, normalized size = 1.51

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2))) + 1/24*(
sqrt(6) + 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2))) - 1/24*(sqrt(6
) + 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) + 1/2*sqrt(2))) - 1/24*(sqrt(6) - 3*
sqrt(2))*log(abs(x - 1/2*sqrt(6) - 1/2*sqrt(2)))
```

maple [A] time = 0.04, size = 60, normalized size = 0.90

$$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6
^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4-4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 4*x^2 + 1), x)
```

mupad [B] time = 0.23, size = 98, normalized size = 1.46

$$\operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}+4}+\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}+4}\right)\left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{12}\right)-\operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}-4}-\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}-4}\right)\left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 4*x^2 + 1),x)
```

```
[Out] atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) + 4) + (3*6^(1/2)*x)/(2^(1/2)*6^(1/2)
+ 4))*(2^(1/2)/4 + 6^(1/2)/12) - atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) - 4)
- (3*6^(1/2)*x)/(2^(1/2)*6^(1/2) - 4))*(2^(1/2)/4 - 6^(1/2)/12)
```

sympy [**A**] time = 0.37, size = 24, normalized size = 0.36

$$\text{RootSum}\left(2304t^4 - 192t^2 + 1, \left(t \mapsto t\log\left(384t^3 - 28t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-4*x**2+1),x)
```

```
[Out] RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t + x
)))
```

3.43 $\int \frac{1}{1+4x^2+x^4} \, dx$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arcta
n(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+4x^2+x^4} \, dx &= \frac{\int \frac{1}{2-\sqrt{3}+x^2} \, dx}{2\sqrt{3}}-\frac{\int \frac{1}{2+\sqrt{3}+x^2} \, dx}{2\sqrt{3}}\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

fricas [A] time = 0.44, size = 87, normalized size = 1.30

$$-\frac{1}{3}\sqrt{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x-\sqrt{x^2-\sqrt{3}+2}\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{3}\sqrt{3}\sqrt{-\sqrt{3}+2}\arctan\left(-x\sqrt{-\sqrt{3}+2}+\sqrt{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*sqrt(sqrt(3) + 2)*arctan(-(x - sqrt(x^2 - sqrt(3) + 2))*sqrt(s
qrt(3) + 2)) + 1/3*sqrt(3)*sqrt(-sqrt(3) + 2)*arctan(-x*sqrt(-sqrt(3) + 2)
+ sqrt(x^2 + sqrt(3) + 2)*sqrt(-sqrt(3) + 2))
```

giac [A] time = 1.15, size = 51, normalized size = 0.76

$$\frac{1}{12}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12*(sqrt(6)
+ 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))
```

maple [A] time = 0.04, size = 60, normalized size = 0.90

$$\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}-\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+1),x)
```

```
[Out] -1/3*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2/(6^(1/2)+2^(1/2))*x)+1/3*3^(1/2)/(6
^(1/2)-2^(1/2))*arctan(2/(6^(1/2)-2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4+4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 4*x^2 + 1), x)
```

mupad [B] time = 0.20, size = 117, normalized size = 1.75

$$2\,\mathrm{atanh}\left(\frac{24x\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\sqrt{3}-4}-\frac{16\sqrt{3}\,x\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\sqrt{3}-4}\right)\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}-2\,\mathrm{atanh}\left(\frac{24x\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\sqrt{3}+4}+\frac{16\sqrt{3}\,x\sqrt{-\frac{\sqrt{3}}{48}}}{2\sqrt{3}+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((24*x*(3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4) - (16*3^(1/2)*x*(3
^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4))*(3^(1/2)/48 - 1/24)^(1/2) - 2*ata
nh((24*x*(- 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4) + (16*3^(1/2)*x*(- 3^
(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4))*(- 3^(1/2)/48 - 1/24)^(1/2)
```

sympy [A] time = 0.22, size = 92, normalized size = 1.37

$$-2\sqrt{\frac{1}{24}-\frac{\sqrt{3}}{48}}\operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right)-2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+1),x)
```

```
[Out] -2*sqrt(1/24 - sqrt(3)/48)*atan(x/(sqrt(3)*sqrt(2 - sqrt(3)) + 2*sqrt(2 - s
qrt(3)))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqrt(sqrt(3) + 2) + sqrt(3
)*sqrt(sqrt(3) + 2)))
```

3.44 $\int \frac{1}{2+x^2+x^4} dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)$$

```
[Out] -1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-14+28*2^(1/
2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-14+
28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/2))
^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/2))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*
Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*
x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}+2}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x}}{4\sqrt{2\left(-1+2\right.}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,d\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}\right.}{4}$$

Mathematica [C] time = 0.05, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}} - \frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*A
rcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]
```

fricas [B] time = 0.43, size = 297, normalized size = 1.52

$$-\frac{1}{56}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{-4\sqrt{2}+16}\arctan\left(-\frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{-4\sqrt{2}+16}+\frac{1}{112}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4x^2+8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] -1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(2) + 16)*arctan(-1/56*8^(3/4)*sq
rt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/112*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(
4*x^2 + 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))*sqrt(-4*sqrt(2) + 16)
- 1/7*sqrt(7)*(2*sqrt(2) - 1)) - 1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(
2) + 16)*arctan(-1/56*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/1
12*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4
*sqrt(2))*sqrt(-4*sqrt(2) + 16) + 1/7*sqrt(7)*(2*sqrt(2) - 1)) + 1/224*8^(1
```

```
/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*log(4*x^2 + 8^(1/4)*x*sqrt(-4*sqrt(
2) + 16) + 4*sqrt(2)) - 1/224*8^(1/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*l
og(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))
```

giac [A] time = 1.81, size = 248, normalized size = 1.27

$$\frac{1}{224}\sqrt{7}\left(2\sqrt{7}2^{\frac{3}{4}}\sqrt{\sqrt{2}+4}-2^{\frac{1}{4}}\sqrt{-8\sqrt{2}+32}\right)\arctan\left(\frac{2\cdot 2^{\frac{3}{4}}\sqrt{\frac{1}{2}}\left(x+2^{\frac{1}{4}}\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right)}{\sqrt{\sqrt{2}+4}}\right)+\frac{1}{224}\sqrt{7}\left(2\sqrt{7}2^{\frac{3}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] 1/224*sqrt(7)*(2*sqrt(7)*2^(3/4)*sqrt(sqrt(2) + 4) - 2^(1/4)*sqrt(-8*sqrt(2
) + 32))*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/
sqrt(sqrt(2) + 4)) + 1/224*sqrt(7)*(2*sqrt(7)*2^(3/4)*sqrt(sqrt(2) + 4) - 2
^(1/4)*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(
-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/448*sqrt(7)*(2*2^(3/4)*sqrt(sqr
t(2) + 4) + sqrt(7)*2^(1/4)*sqrt(-8*sqrt(2) + 32))*log(x^2 + 2*2^(1/4)*x*sq
rt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/448*sqrt(7)*(2*2^(3/4)*sqrt(sqrt(2) +
 4) + sqrt(7)*2^(1/4)*sqrt(-8*sqrt(2) + 32))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/
8*sqrt(2) + 1/2) + sqrt(2))
```

maple [B] time = 0.12, size = 386, normalized size = 1.97

$$-\frac{\left(-1+2\sqrt{2}\right)\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{28\sqrt{1+2\sqrt{2}}}-\frac{\left(-1+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{7\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{1+2\sqrt{2}}}-\frac{(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)+1/
14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)-1/28/(1+2*2^
(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2
^(1/2))*2^(1/2)-1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(
1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+
2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)-1/56*ln(x^2+2^(1/2)-x*(-1+2*
2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^
(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1
+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)-1/7/(1+2*2^(
1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^
(1/2))+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/
2))^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + x^2 + 2), x)
```

mupad [B] time = 0.22, size = 61, normalized size = 0.31

$$\frac{\mathrm{atan}\left(\frac{\sqrt{7}\,x\,\sqrt{7-\sqrt{7}\,7\mathrm{i}}}{14}\right)\sqrt{7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{x\,\sqrt{1+\sqrt{7}\,1\mathrm{i}}}{2}\right)\sqrt{1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^4 + 2),x)
```

```
[Out] (atan((7^(1/2)*x*(7 - 7^(1/2)*7i)^(1/2))/14)*(7 - 7^(1/2)*7i)^(1/2)*1i)/14
- (7^(1/2)*atan((x*(7^(1/2)*1i + 1)^(1/2))/2)*(7^(1/2)*1i + 1)^(1/2)*1i)/14
```

sympy [B] time = 1.14, size = 994, normalized size = 5.07

$$\sqrt{\frac{1}{224}+\frac{\sqrt{2}}{112}}\log\left(x^2+x\left(-\frac{4\sqrt{7}\sqrt{1+2\sqrt{2}}}{7}+\frac{5\sqrt{14}\sqrt{1+2\sqrt{2}}}{28}+\frac{3\sqrt{14}\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{28}\right)-\frac{33\sqrt{4\sqrt{2}+}}{28}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+2),x)
```

```
[Out] sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7 +
5*sqrt(14)*sqrt(1 + 2*sqrt(2))/28 + 3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*s
qrt(2) + 9)/28) - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqr
t(2) + 9)/28 + 83*sqrt(2)/28) - sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-3*
sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/28 - 5*sqrt(14)*sqrt(1 + 2
*sqrt(2))/28 + 4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7) - 33*sqrt(4*sqrt(2) + 9)/28
 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28) + 2*sqrt(-sqr
t(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqr
t(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*s
qrt(2) + 9) + 1 + 6*sqrt(2))) - 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(
2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqr
t(2) + 9) + 1 + 6*sqrt(2))) + 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sq
rt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9)
+ 1 + 6*sqrt(2))) + 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(
2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqr
t(2) + 9) + 1 + 6*sqrt(2)))) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*
sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2)
+ 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 3
*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(
4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqr
t(2))) - 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2)
+ 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 8
*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) +
9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))))
```

3.45 $\int \frac{1}{2-x^2+x^4} dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)$$

```
[Out] -1/28*arctan((-2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28*2^(1/2
))^(1/2)+1/28*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28
*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/
2)+1/4*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqrt[-1 + 2*S
qrt[2]]])/2 + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] + 2*x)/
Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sq
rt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt
[2*(1 + 2*Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2-x^2+x^4}\,dx = \frac{\int \frac{\sqrt{1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2}\,d}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{2}-x^2}\,dx, x,\right.}{2\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.08, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}} - \frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (I
*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]
```

fricas [B] time = 0.44, size = 275, normalized size = 1.40

$$-\frac{1}{28}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{\sqrt{2}+4}\,\arctan\left(-\frac{1}{28}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{\sqrt{2}+4} + \frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4x^2+2\cdot 8^{\frac{1}{4}}x\sqrt{\sqrt{2}+4}+4\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="fricas")
```

```
[Out] -1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/28*8^(3/4)*sqrt(7
)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/56*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 + 2
*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2))*sqrt(sqrt(2) + 4) - 1/7*sqrt(7)*(
2*sqrt(2) + 1)) - 1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/
28*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/56*8^(3/4)*sqrt(7)*sqrt(
2)*sqrt(4*x^2 - 2*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2))*sqrt(sqrt(2) + 4
) + 1/7*sqrt(7)*(2*sqrt(2) + 1)) - 1/112*8^(1/4)*sqrt(sqrt(2) + 4)*(sqrt(2)
```

```
 - 4)*log(4*x^2 + 2*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2)) + 1/112*8^(1/4
)*sqrt(sqrt(2) + 4)*(sqrt(2) - 4)*log(4*x^2 - 2*8^(1/4)*x*sqrt(sqrt(2) + 4)
 + 4*sqrt(2))
```

giac [A] time = 2.00, size = 252, normalized size = 1.29

$$\frac{1}{224}\sqrt{7}\left(2\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+4}+\sqrt{7}\,2^{\frac{1}{4}}\sqrt{-8\sqrt{2}+32}\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}+4}+2x\right)}{4\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}}\right)+\frac{1}{224}\sqrt{7}\left(2\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="giac")
```

```
[Out] 1/224*sqrt(7)*(2*2^(3/4)*sqrt(sqrt(2) + 4) + sqrt(7)*2^(1/4)*sqrt(-8*sqrt(2
) + 32))*arctan(1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*sqrt(sqrt(2) + 4) + 2*x)/sqr
t(-1/8*sqrt(2) + 1/2)) + 1/224*sqrt(7)*(2*2^(3/4)*sqrt(sqrt(2) + 4) + sqrt(
7)*2^(1/4)*sqrt(-8*sqrt(2) + 32))*arctan(-1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*sq
rt(sqrt(2) + 4) - 2*x)/sqrt(-1/8*sqrt(2) + 1/2)) + 1/448*sqrt(7)*(2*sqrt(7)
*2^(3/4)*sqrt(sqrt(2) + 4) - 2^(1/4)*sqrt(-8*sqrt(2) + 32))*log(2^(1/4)*sqr
t(1/2)*x*sqrt(sqrt(2) + 4) + x^2 + sqrt(2)) - 1/448*sqrt(7)*(2*sqrt(7)*2^(3
/4)*sqrt(sqrt(2) + 4) - 2^(1/4)*sqrt(-8*sqrt(2) + 32))*log(-2^(1/4)*sqrt(1/
2)*x*sqrt(sqrt(2) + 4) + x^2 + sqrt(2))
```

maple [B] time = 0.11, size = 386, normalized size = 1.97

$$\frac{\left(1+2\sqrt{2}\right)\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{28\sqrt{-1+2\sqrt{2}}}-\frac{\left(1+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{7\sqrt{-1+2\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{-1+2\sqrt{2}}}+\frac{\left(1+2\sqrt{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2)*2^(1/2)-1/14
*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2)+1/28/(-1+2*2^(1/
2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/
2))*2^(1/2)-1/7/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2
*2^(1/2))^(1/2))*(1+2*2^(1/2))+1/2/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^
(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*2^(1/2)-1/56*ln(x^2+2^(1/2)+x*(1+2*2^(1
/2))^(1/2))*(1+2*2^(1/2))^(1/2)*2^(1/2)+1/14*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))
^(1/2))*(1+2*2^(1/2))^(1/2)+1/28/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1+2*2^(1
/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))*2^(1/2)-1/7/(-1+2*2^(1/2))^
(1/2)*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))+
1/2/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1
/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4-x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

mupad [B] time = 0.11, size = 132, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{4\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}+\frac{\sqrt{7}\,x\sqrt{-7-\sqrt{7}\,7\mathrm{i}}}{28\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{x\sqrt{-1+\sqrt{7}\,1\mathrm{i}}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}-\frac{\sqrt{7}\,x\sqrt{-1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\sqrt{-1+\sqrt{7}\,1\mathrm{i}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2 + 2),x)
```

```
[Out] (atan((x*(- 7^(1/2)*7i - 7)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 + 1/2)) + (7^(1/2)
*x*(- 7^(1/2)*7i - 7)^(1/2))/(28*((7^(1/2)*1i)/2 + 1/2)))*(- 7^(1/2)*7i - 7
)^(1/2)*1i)/14 + (7^(1/2)*atan((x*(7^(1/2)*1i - 1)^(1/2))/(4*((7^(1/2)*1i)/
2 - 1/2)) - (7^(1/2)*x*(7^(1/2)*1i - 1)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 - 1/2)
))*(7^(1/2)*1i - 1)^(1/2)*1i)/14
```

sympy [A] time = 0.54, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^4+28t^2+1,\left(t\mapsto t\log\left(-112t^3+6t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 + 6*_t + x)
))
```

3.46 $\int \frac{1}{-1+x^6} \, dx$

Optimal. Leaf size=73

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/3*arctanh(x)+1/12*ln(x^2-x+1)-1/12*ln(x^2+x+1)+1/6*arctan(1/3*(1-2*x)*3^
(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\
&= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 75, normalized size = 1.03

$$
\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

fricas [A] time = 0.42, size = 65, normalized size = 0.89

$$
-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

giac [A] time = 1.12, size = 67, normalized size = 0.92

$$
-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)+\frac{1}{6}\log\left(|x-1|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 0.90

$$
-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{12}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x)
```

```
[Out] 1/6*ln(x-1)-1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/6*ln
(x+1)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 0.95, size = 65, normalized size = 0.89

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

mupad [B] time = 0.09, size = 88, normalized size = 1.21

$$-\frac{\operatorname{atanh}(x)}{3}-\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}+\frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}-\frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 1),x)
```

```
[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))
*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1
i - 1))*(3^(1/2)/6 - 1i/6)
```

sympy [A] time = 0.25, size = 83, normalized size = 1.14

$$\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

3.47 $\int \frac{1}{-2+x^6}\,dx$

Optimal. Leaf size=138

$$\frac{\log\left(x^2-\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

```
[Out] -1/6*arctanh(1/2*x*2^(5/6))*2^(1/6)+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6)-
1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)
*x*3^(1/2))*2^(1/6)*3^(1/2)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*
2^(1/6)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{\log\left(x^2-\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)^(-1),x]
```

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan[1/Sqrt
[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTanh[x/2^(1/6)]/(3*2^(5
/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(5/6)) - Log[2^(1/3) + 2^(1/6)*
x + x^2]/(12*2^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-2+x^6}\,dx = -\frac{\int \frac{\sqrt[6]{2}-\frac{x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{3\ 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+\frac{x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{3\ 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-x^2}\,dx}{3\ 2^{2/3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\int \frac{-\sqrt[6]{2}+2x}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{12\ 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+2x}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{12\ 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{4\ 2^{2/3}} - \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{4\ 2^{2/3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{2\ 2^{5/6}}$$

$$= \frac{\tan^{-1}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}x\right.}{12\ 2^{5/6}}$$

Mathematica [A] time = 0.04, size = 122, normalized size = 0.88

$$-\frac{-\log\left(2^{2/3}x^2-2^{5/6}x+2\right)+\log\left(2^{2/3}x^2+2^{5/6}x+2\right)-2\log\left(2-2^{5/6}x\right)+2\log\left(2^{5/6}x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{5/6}}{\right.}}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)^(-1),x]
```

```
[Out] -1/12*(2*Sqrt[3]*ArcTan[(-1 + 2^(5/6)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2
^(5/6)*x)/Sqrt[3]] - 2*Log[2 - 2^(5/6)*x] + 2*Log[2 + 2^(5/6)*x] - Log[2 -
2^(5/6)*x + 2^(2/3)*x^2] + Log[2 + 2^(5/6)*x + 2^(2/3)*x^2])/2^(5/6)
```

fricas [A] time = 0.44, size = 175, normalized size = 1.27

$$\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32^{\frac{1}{6}}\sqrt{3}x+\frac{1}{12}\cdot 32^{\frac{1}{6}}\sqrt{3}\sqrt{16x^2+32^{\frac{5}{6}}x+8\cdot 4^{\frac{2}{3}}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="fricas")
```

```
[Out] 1/96*32^(5/6)*sqrt(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3
)*sqrt(16*x^2 + 32^(5/6)*x + 8*4^(2/3)) - 1/3*sqrt(3)) + 1/96*32^(5/6)*sqrt
(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3)*sqrt(16*x^2 - 32
^(5/6)*x + 8*4^(2/3)) + 1/3*sqrt(3)) - 1/384*32^(5/6)*log(16*x^2 + 32^(5/6)
*x + 8*4^(2/3)) + 1/384*32^(5/6)*log(16*x^2 - 32^(5/6)*x + 8*4^(2/3)) - 1/1
92*32^(5/6)*log(16*x + 32^(5/6)) + 1/192*32^(5/6)*log(16*x - 32^(5/6))
```

giac [A] time = 1.31, size = 114, normalized size = 0.83

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sq
rt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*lo
g(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3))
- 1/12*2^(1/6)*log(abs(x + 2^(1/6))) + 1/12*2^(1/6)*log(abs(x - 2^(1/6)))
```

maple [A] time = 0.16, size = 111, normalized size = 0.80

$$-\frac{2^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{2^{\frac{5}{6}}\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{12}-\frac{2^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{2^{\frac{5}{6}}\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{2^{\frac{1}{6}}\ln\left(x-2^{\frac{1}{6}}\right)}{12}-\frac{2^{\frac{1}{6}}\ln\left(x+2^{\frac{1}{6}}\right)}{12}+\frac{2^{\frac{1}{6}}\ln\left(x^{2}-2^{\frac{1}{6}}x\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-2),x)
```

```
[Out] 1/12*2^(1/6)*ln(x-2^(1/6))-1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arct
an(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)-2^(1/
6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3
^(1/2)-1/12*2^(1/6)*ln(x+2^(1/6))
```

maxima [A] time = 0.95, size = 112, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^{2}+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{24}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sq
rt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*lo
g(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3))
- 1/12*2^(1/6)*log(x + 2^(1/6)) + 1/12*2^(1/6)*log(x - 2^(1/6))
```

mupad [B] time = 0.21, size = 140, normalized size = 1.01

$$-\frac{2^{1/6}\,\mathrm{atanh}\left(\frac{2^{5/6}\,x}{2}\right)}{6}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1\mathrm{i}}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}-\frac{2^{1/6}\sqrt{3}\,x}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)1\mathrm{i}}{12}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1\mathrm{i}}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 2),x)
```

```
[Out] (2^(1/6)*atan((2^(1/6)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)) - (2^
(1/6)*3^(1/2)*x)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)))*(3^(1/2)*1i + 1)
*1i)/12 - (2^(1/6)*atanh((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x*1i)/(
2*((2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x)/(2*((2^(1/3)*
3^(1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2)*1i - 1)*1i)/12
```

sympy [A] time = 0.57, size = 14, normalized size = 0.10

$$\mathrm{RootSum}\left(1492992t^{6}-1,\left(t\mapsto t\log\left(-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-2),x)
```

```
[Out] RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

3.48 $\int \frac{1}{2+x^6}\,dx$

Optimal. Leaf size=138

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}}{6\ 2^{5/6}}$$

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/
12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))
*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {209, 634, 618, 204, 628, 203}

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}}{6\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)^(-1),x]
```

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^(5/6)) + A
rcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x +
x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6
)*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+x^6}\,dx = \frac{\int \frac{\sqrt[6]{2}-\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\,2^{5/6}} + \frac{\int \frac{\sqrt[6]{2}+\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\,2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}+x^2}\,dx}{3\,2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\,2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\,2^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\,2^{2/3}} - \frac{\int \frac{-\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{4\,2^{5/6}\sqrt{3}} + \frac{\int \frac{\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}}{4\,2^{5/6}\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\,2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\,2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\,2^{5/6}\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1 -\right.}{6\,2^{5/6}\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\,2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\,2^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3}+2^{5/6}x\right)}{6\,2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\,2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}\right.}{}$$

Mathematica [A] time = 0.03, size = 115, normalized size = 0.83

$$\frac{-\sqrt{3}\log\left(2^{2/3}x^2-2^{5/6}\sqrt{3}x+2\right)+\sqrt{3}\log\left(2^{2/3}x^2+2^{5/6}\sqrt{3}x+2\right)+4\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)-2\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)+2\,\text{ta}}{12\,2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)^(-1),x]
```

```
[Out] (4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[Sqrt[3] + 2
^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2] + Sqrt[3]*Log[
2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(5/6))
```

fricas [A] time = 0.43, size = 177, normalized size = 1.28

$$\frac{1}{384}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(32^{\frac{5}{6}}\sqrt{3}x+16x^2+8\cdot 4^{\frac{2}{3}}\right)-\frac{1}{384}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(-32^{\frac{5}{6}}\sqrt{3}x+16x^2+8\cdot 4^{\frac{2}{3}}\right)-\frac{1}{48}\cdot 32^{\frac{5}{6}}\arctan\left(\frac{1}{4}\cdot 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/384*32^(5/6)*sqrt(3)*log(32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) - 1/384
*32^(5/6)*sqrt(3)*log(-32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) - 1/48*32^(
5/6)*arctan(1/4*32^(1/6)*sqrt(2)*sqrt(2*x^2 + 4^(2/3)) - 1/2*32^(1/6)*x) -
1/96*32^(5/6)*arctan(-32^(1/6)*x + 1/4*32^(1/6)*sqrt(32^(5/6)*sqrt(3)*x + 1
6*x^2 + 8*4^(2/3)) - sqrt(3)) - 1/96*32^(5/6)*arctan(-32^(1/6)*x + 1/4*32^(
1/6)*sqrt(-32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) + sqrt(3))
```

giac [A] time = 1.25, size = 107, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x+\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{12}\cdot 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*
2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^
(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqr
t(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

maple [A] time = 0.19, size = 95, normalized size = 0.69

$$\frac{2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{5}{6}}x}{2}\right)}{6}+\frac{2^{\frac{1}{6}}\arctan\left(2^{\frac{5}{6}}x-\sqrt{3}\right)}{12}+\frac{2^{\frac{1}{6}}\arctan\left(2^{\frac{5}{6}}x+\sqrt{3}\right)}{12}-\frac{2^{\frac{1}{6}}\sqrt{3}\,\ln\left(x^{2}-2^{\frac{1}{6}}\sqrt{3}\,x+2^{\frac{1}{3}}\right)}{24}+\frac{2^{\frac{1}{6}}\sqrt{3}\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2),x)
```

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/
12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))
*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

maxima [A] time = 0.98, size = 107, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^{2}+\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^{2}-\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x+\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*
2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^
(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqr
t(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

mupad [B] time = 0.13, size = 135, normalized size = 0.98

$$\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{5/6}\,x}{2}\right)}{6}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}+\frac{2^{1/6}\,\sqrt{3}\,x\,1\mathrm{i}}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(\sqrt{3}-\mathrm{i}\right)1\mathrm{i}}{12}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1\mathrm{i}}{2}\right)}-\frac{}{2(}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 + 2),x)
```

```
[Out] (2^(1/6)*atan((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^(1/3)*3^
(1/2)*1i)/2 - 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)
/2 - 2^(1/3)/2)))*(3^(1/2) - 1i)*1i)/12 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^
(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) - (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(
1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2) + 1i)*1i)/12
```

sympy [A] time = 0.28, size = 14, normalized size = 0.10

$$\mathrm{RootSum}\left(1492992t^{6}+1,\left(t\mapsto t\log\left(12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2),x)
```

```
[Out] RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))
```

3.49 $\int \frac{1}{1+x^8} dx$

Optimal. Leaf size=339

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

```
[Out] -1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2
))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arctan((-
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2
*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {213, 1169, 634, 618, 204, 628}

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2]
)]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^8}\,dx = \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\ldots}{1+\sqrt{2+\sqrt{2}}\ldots}}{4\sqrt{2\left(2+\ldots\right.}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx - \ldots$$

$$= -\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}\sqrt{2+\ldots}$$

$$= -\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan\ldots$$

Mathematica [A] time = 0.01, size = 209, normalized size = 0.62

$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)^(-1),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Si
n[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*
Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x
^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])
/8
```

fricas [B] time = 0.43, size = 1027, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*s
qrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(
2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqr
t(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + s
qrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))
) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2
*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqr
t(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(s
qrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) -
 sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2
*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt
(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2)
)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 + 1/
2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/6
4*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/64*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2
)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/8*sqrt(sq
rt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sq
rt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqr
t(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
- 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) +
1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan
(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) -
1/32*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*sqrt(-sqrt
(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(
x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

giac [A] time = 1.03, size = 239, normalized size = 0.71

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{}}{\sqrt{-\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*
sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) +
 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.00, size = 22, normalized size = 0.06

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^8+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+1),x)
[Out] 1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="maxima")
[Out] integrate(1/(x^8 + 1), x)
```

mupad [B] time = 0.30, size = 288, normalized size = 0.85

$$\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\mathrm{atan}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 + 1),x)
[Out] atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2
) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) -
 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*
1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^
(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/
2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)
*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2
)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i - ata
n(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2
)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 2.82, size = 14, normalized size = 0.04

$$\mathrm{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+1),x)
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

3.50 $\int \frac{1}{-1+x^8} dx$

Optimal. Leaf size=97

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/4*arctan(x)-1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1
+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4
*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + S
qrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-1+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{1+x^4}\,dx\\
&= -\left(\frac{1}{4}\int \frac{1}{1-x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{1+x^2}\,dx-\frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{1}{4}\tan^{-1}(x)-\frac{1}{4}\tanh^{-1}(x)-\frac{1}{8}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,d\cdots}{8\sqrt{2}}\\
&= -\frac{1}{4}\tan^{-1}(x)-\frac{1}{4}\tanh^{-1}(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,d\cdots\right.}{4\sqrt{2}\cdots}\\
&= -\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}}-\cdots
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+2\log(1-x)-2\log(x+1)-4\tan^{-1}(x)+2\sqrt{2}\tan^{-1}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)^(-1),x]
```

```
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt
[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.44, size = 111, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{16}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/
4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/16
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) -
 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

giac [A] time = 0.96, size = 90, normalized size = 0.93

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8*lo
g(abs(x - 1))
```

maple [A] time = 0.00, size = 66, normalized size = 0.68

$$-\frac{\operatorname{arctanh}(x)}{4}-\frac{\arctan(x)}{4}-\frac{\sqrt{2}\arctan\left(\sqrt{2}x-1\right)}{8}-\frac{\sqrt{2}\arctan\left(\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-1),x)
```

```
[Out] -1/4*arctanh(x)-1/4*arctan(x)-1/16*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2
)*x+1))-1/8*2^(1/2)*arctan(2^(1/2)*x-1)-1/8*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 0.95, size = 88, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x -
 1)
```

mupad [B] time = 0.16, size = 45, normalized size = 0.46

$$\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\operatorname{atan}(x)}{4}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 1),x)
```

```
[Out] (atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

sympy [C] time = 161.87, size = 44, normalized size = 0.45

$$\frac{\log(x-1)}{8}-\frac{\log(x+1)}{8}+\frac{i\log(x-i)}{8}-\frac{i\log(x+i)}{8}+\operatorname{RootSum}\left(4096t^4+1,\left(t\mapsto t\log(-8t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-1),x)
```

```
[Out] log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(409
6*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

3.51 $\int \frac{1}{1-x^4+x^8} dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+}}{1+\sqrt{}}}{4\sqrt{3}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \int$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \text{lo}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

fricas [A] time = 0.45, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

giac [A] time = 1.06, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\text{ arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 30, normalized size = 0.11

$$\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+3\,\mathrm{RootOf}\left(9\_Z^4+1\right)x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^{3}-4x^{2}+2\sqrt{6}\,x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^{3}+4x^{2}+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

3.52 $\int \frac{x^7}{1+x^{12}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right)+\frac{1}{12}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)\end{aligned}$$

Mathematica [B] time = 0.11, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}x+1\right)-2\log\left(x^2+\sqrt{2}x+1\right)+\log\left(2x^2-\sqrt{6}x+\sqrt{2}x+2\right)+\log\left(2x^2+\sqrt{2}\left(\sqrt{3}-1\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

fricas [A] time = 0.43, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

giac [A] time = 1.13, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

maple [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2
))
```

maxima [A] time = 0.96, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

mupad [B] time = 0.22, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^4+1\right)}{12}-\ln\left(x^4-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)+\ln\left(x^4+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12 + 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

sympy [A] time = 0.17, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

3.53 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x) - x$$

```
[Out] -x+x*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2295}

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x],x]
[Out] -x + x*Log[x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log(x)\,dx = -x + x\log(x)$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x],x]
[Out] -x + x*Log[x]
```

fricas [A] time = 0.44, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
[Out] x*log(x) - x
```

giac [A] time = 1.07, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
[Out] x*log(x) - x
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$x\ln(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x)
```

```
[Out] -x+x*ln(x)
```

maxima [A] time = 0.41, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$x\,\left(\ln(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x),x)
```

```
[Out] x*(log(x) - 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.62

$$x\log{\left(x \right)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

3.54 $\int x \log(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2304}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

giac [A] time = 1.10, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x^2\ln(x)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] 1/2*x^2*ln(x)-1/4*x^2
```

maxima [A] time = 0.41, size = 13, normalized size = 0.76

$$\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

mupad [B] time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^2\left(\ln(x)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^2\log{(x)}}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

3.55 $\int x^2 \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

```
[Out] -1/9*x^3+1/3*x^3*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[x],x]
[Out] -x^3/9 + (x^3*Log[x])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log(x) \, dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[x],x]
[Out] -1/9*x^3 + (x^3*Log[x])/3
```

fricas [A] time = 0.44, size = 13, normalized size = 0.76

$$\frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="fricas")
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

giac [A] time = 1.11, size = 13, normalized size = 0.76

$$\frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x),x)
```

```
[Out] -1/9*x^3+1/3*x^3*ln(x)
```

maxima [A] time = 0.45, size = 13, normalized size = 0.76

$$\frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

mupad [B] time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^3 \left(\ln(x) - \frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x),x)
```

```
[Out] (x^3*(log(x) - 1/3))/3
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x),x)
```

```
[Out] x**3*log(x)/3 - x**3/9
```

3.56 $\int x^p \log(x)\,dx$

Optimal. Leaf size=26

$$\frac{x^{p+1}\log(x)}{p+1}-\frac{x^{p+1}}{(p+1)^2}$$

```
[Out] -x^(1+p)/(1+p)^2+x^(1+p)*ln(x)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{x^{p+1}\log(x)}{p+1}-\frac{x^{p+1}}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*Log[x],x]
[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^p \log(x)\,dx=-\frac{x^{1+p}}{(1+p)^2}+\frac{x^{1+p}\log(x)}{1+p}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^{p+1}((p+1)\log(x)-1)}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*Log[x],x]
[Out] (x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2
```

fricas [A] time = 0.44, size = 25, normalized size = 0.96

$$\frac{\left((p+1)x\log(x)-x\right)x^p}{p^2+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="fricas")
[Out] ((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^p \log(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^p*log(x), x)
```

maple [A] time = 0.02, size = 34, normalized size = 1.31

$$\frac{x\,\mathrm{e}^{p\ln(x)}\ln(x)}{p+1}-\frac{x\,\mathrm{e}^{p\ln(x)}}{p^2+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*ln(x),x)
```

```
[Out] 1/(p+1)*x*ln(x)*exp(ln(x)*p)-1/(p^2+2*p+1)*x*exp(ln(x)*p)
```

maxima [A] time = 0.42, size = 26, normalized size = 1.00

$$\frac{x^{p+1}\log(x)}{p+1}-\frac{x^{p+1}}{(p+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="maxima")
```

```
[Out] x^(p + 1)*log(x)/(p + 1) - x^(p + 1)/(p + 1)^2
```

mupad [B] time = 0.24, size = 32, normalized size = 1.23

$$\begin{cases} \frac{\ln(x)^2}{2} & \text{if } p=-1 \\ \frac{x^{p+1}\left(\ln(x)\,(p+1)-1\right)}{(p+1)^2} & \text{if } p\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*log(x),x)
```

```
[Out] piecewise(p == -1, log(x)^2/2, p ~= -1, (x^(p + 1)*(log(x)*(p + 1) - 1))/(p
 + 1)^2)
```

sympy [A] time = 0.74, size = 56, normalized size = 2.15

$$\begin{cases} \frac{pxx^p\log(x)}{p^2+2p+1}+\frac{xx^p\log(x)}{p^2+2p+1}-\frac{xx^p}{p^2+2p+1} & \text{for } p\neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*ln(x),x)
```

```
[Out] Piecewise((p*x*x**p*log(x)/(p**2 + 2*p + 1) + x*x**p*log(x)/(p**2 + 2*p + 1
) - x*x**p/(p**2 + 2*p + 1), Ne(p, -1)), (log(x)**2/2, True))
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2,x]
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2,x]
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

fricas [A] time = 0.42, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

giac [A] time = 0.91, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$x\ln(x)^2 - 2x\ln(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] x*ln(x)^2-2*x*ln(x)+2*x
```

maxima [A] time = 0.43, size = 12, normalized size = 0.80

$$\left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

mupad [B] time = 0.03, size = 12, normalized size = 0.80

$$x\,\left(\ln(x)^2 - 2\,\ln(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2,x)
```

```
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

sympy [A] time = 0.09, size = 15, normalized size = 1.00

$$x\log{(x)}^2 - 2x\log{(x)} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

3.58 $\int x^9 \log^{11}(x)\,dx$

Optimal. Leaf size=127

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x}{12500}$$

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+207
9/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/
12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln
(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2305, 2304}

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x}{12500}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\int x^9\log^{11}(x)\,dx = \frac{1}{10}x^{10}\log^{11}(x) - \frac{11}{10}\int x^9\log^{10}(x)\,dx$$
$$= -\frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{11}{10}\int x^9\log^9(x)\,dx$$
$$= \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \frac{99}{100}\int x^9\log^8(x)\,dx$$
$$= -\frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{99}{125}\int x^9\log^7(x)\,dx$$
$$= \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \frac{693}{}\int$$
$$= -\frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x$$
$$= \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x$$
$$= -\frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{1}{1}$$
$$= \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - $$
$$= -\frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500}$$
$$= -\frac{6237x^{10}}{156250000} + \frac{6237x^{10}\log(x)}{15625000} - \frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + 2$$

Mathematica [A] time = 0.00, size = 127, normalized size = 1.00

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^9(x)-\frac{99x^{10}\log^8(x)}{1000}+\frac{99x^{10}\log^7(x)}{1250}-\frac{693x^{10}\log^6(x)}{12500}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

fricas [A] time = 0.43, size = 103, normalized size = 0.81

$$\frac{1}{10}x^{10}\log(x)^{11}-\frac{11}{100}x^{10}\log(x)^{10}+\frac{11}{100}x^{10}\log(x)^9-\frac{99}{1000}x^{10}\log(x)^8+\frac{99}{1250}x^{10}\log(x)^7-\frac{693}{12500}x^{10}\log(x)^6+\frac{207}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

giac [A] time = 1.02, size = 103, normalized size = 0.81

$$\frac{1}{10}x^{10}\log(x)^{11}-\frac{11}{100}x^{10}\log(x)^{10}+\frac{11}{100}x^{10}\log(x)^9-\frac{99}{1000}x^{10}\log(x)^8+\frac{99}{1250}x^{10}\log(x)^7-\frac{693}{12500}x^{10}\log(x)^6+\frac{207}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="giac")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

maple [A] time = 0.00, size = 104, normalized size = 0.82

$$\frac{x^{10}\ln(x)^{11}}{10}-\frac{11x^{10}\ln(x)^{10}}{100}+\frac{11x^{10}\ln(x)^{9}}{100}-\frac{99x^{10}\ln(x)^{8}}{1000}+\frac{99x^{10}\ln(x)^{7}}{1250}-\frac{693x^{10}\ln(x)^{6}}{12500}+\frac{2079x^{10}\ln(x)^{5}}{62500}-\frac{2079x^{1}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*ln(x)^11,x)
```

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+207
9/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/
12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln
(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

maxima [A] time = 0.42, size = 71, normalized size = 0.56

$$\frac{1}{156250000}\left(15625000\,\log(x)^{11}-17187500\,\log(x)^{10}+17187500\,\log(x)^{9}-15468750\,\log(x)^{8}+12375000\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="maxima")
```

```
[Out] 1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*log(x)^9 -
15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^6 + 5197500*log(x)^5
 - 2598750*log(x)^4 + 1039500*log(x)^3 - 311850*log(x)^2 + 62370*log(x) - 6
237)*x^10
```

mupad [B] time = 0.18, size = 71, normalized size = 0.56

$$\frac{6237\,x^{10}\left(\frac{15625000\ln(x)^{11}}{6237}-\frac{1562500\ln(x)^{10}}{567}+\frac{1562500\ln(x)^{9}}{567}-\frac{156250\ln(x)^{8}}{63}+\frac{125000\ln(x)^{7}}{63}-\frac{12500\ln(x)^{6}}{9}+\frac{2500\ln(x)^{5}}{3}-\frac{12}{}\right.}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*log(x)^11,x)
```

```
[Out] (6237*x^10*(10*log(x) - 50*log(x)^2 + (500*log(x)^3)/3 - (1250*log(x)^4)/3
+ (2500*log(x)^5)/3 - (12500*log(x)^6)/9 + (125000*log(x)^7)/63 - (156250*l
og(x)^8)/63 + (1562500*log(x)^9)/567 - (1562500*log(x)^10)/567 + (15625000*
log(x)^11)/6237 - 1))/156250000
```

sympy [A] time = 0.29, size = 133, normalized size = 1.05

$$\frac{x^{10}\log{(x)}^{11}}{10}-\frac{11x^{10}\log{(x)}^{10}}{100}+\frac{11x^{10}\log{(x)}^{9}}{100}-\frac{99x^{10}\log{(x)}^{8}}{1000}+\frac{99x^{10}\log{(x)}^{7}}{1250}-\frac{693x^{10}\log{(x)}^{6}}{12500}+\frac{2079x^{10}\log{(}}{62500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*ln(x)**11,x)
```

```
[Out] x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**9/100 - 99
*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x**10*log(x)**6/12500
 + 2079*x**10*log(x)**5/62500 - 2079*x**10*log(x)**4/125000 + 2079*x**10*lo
g(x)**3/312500 - 6237*x**10*log(x)**2/3125000 + 6237*x**10*log(x)/15625000
- 6237*x**10/156250000
```

3.59 $\int \frac{\log^2(x)}{x} \, dx$

Optimal. Leaf size=8

$$\frac{\log^3(x)}{3}$$

```
[Out] 1/3*ln(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 30}

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^2(x)}{x} \, dx &= \text{Subst}\left(\int x^2 \, dx, x, \log(x)\right)\\ &= \frac{\log^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

fricas [A] time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*log(x)^3
```

giac [A] time = 1.07, size = 6, normalized size = 0.75

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="giac")
[Out] 1/3*log(x)^3
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\ln(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2/x,x)
[Out] 1/3*ln(x)^3
```

maxima [A] time = 0.42, size = 6, normalized size = 0.75

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="maxima")
[Out] 1/3*log(x)^3
```

mupad [B] time = 0.06, size = 6, normalized size = 0.75

$$\frac{\ln(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2/x,x)
[Out] log(x)^3/3
```

sympy [A] time = 0.09, size = 5, normalized size = 0.62

$$\frac{\log{(x)}^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2/x,x)
[Out] log(x)**3/3
```

3.60 $\int \frac{1}{\log(x)} dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] Li(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2298}

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
[Out] LogIntegral[x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(x)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]^(-1),x]
[Out] Integrate[Log[x]^(-1), x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
[Out] log_integral(x)
```

giac [A] time = 1.17, size = 3, normalized size = 1.50

$$\text{Ei}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
[Out] Ei(log(x))
```

maple [**B**] time = 0.01, size = 9, normalized size = 4.50

$$-\operatorname{Ei}(1, -\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x)
[Out] -Ei(1,-ln(x))
```

maxima [**A**] time = 0.55, size = 3, normalized size = 1.50

$$\operatorname{Ei}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
[Out] Ei(log(x))
```

mupad [**B**] time = 0.00, size = 2, normalized size = 1.00

$$\operatorname{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x),x)
[Out] logint(x)
```

sympy [**A**] time = 0.46, size = 2, normalized size = 1.00

$$\operatorname{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
[Out] li(x)
```

3.61 $\int \frac{1}{\log(1+x)}\,dx$

Optimal. Leaf size=4

$$\text{LogIntegral}(x+1)$$

```
[Out] Li(1+x)
```

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2298}

$$\text{LogIntegral}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]^(-1),x]
```

```
[Out] LogIntegral[1 + x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\log(1+x)}\,dx = \text{Subst}\left(\int \frac{1}{\log(x)}\,dx, x, 1+x\right)$$
$$= \text{li}(1+x)$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.25

$$\text{ExpIntegralEi}(\log(x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]^(-1),x]
```

```
[Out] ExpIntegralEi[Log[1 + x]]
```

fricas [A] time = 0.41, size = 4, normalized size = 1.00

$$\text{log\_integral}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="fricas")
```

```
[Out] log_integral(x + 1)
```

giac [A] time = 1.10, size = 5, normalized size = 1.25

$$\text{Ei}\left(\log\,(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="giac")
[Out] Ei(log(x + 1))
```

maple [B] time = 0.00, size = 11, normalized size = 2.75

$$-\operatorname{Ei}(1, -\ln(x+1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x+1),x)
[Out] -Ei(1,-ln(x+1))
```

maxima [A] time = 0.53, size = 5, normalized size = 1.25

$$\operatorname{Ei}\left(\log(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="maxima")
[Out] Ei(log(x + 1))
```

mupad [B] time = 0.02, size = 4, normalized size = 1.00

$$\operatorname{logint}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x + 1),x)
[Out] logint(x + 1)
```

sympy [A] time = 0.49, size = 3, normalized size = 0.75

$$\operatorname{li}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(1+x),x)
[Out] li(x + 1)
```

3.62 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

fricas [A] time = 0.40, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

giac [A] time = 1.23, size = 4, normalized size = 1.33

$$\log\left(|\log(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
[Out] log(abs(log(x)))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x)
[Out] ln(ln(x))
```

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

mupad [B] time = 0.12, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)),x)
[Out] log(log(x))
```

sympy [A] time = 0.10, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

3.63 $\int \frac{1}{x^2 \log^2(x)} \, dx$

Optimal. Leaf size=17

$$-\text{Ei}(-\log(x))-\frac{1}{x\log(x)}$$

```
[Out] -Ei(-ln(x))-1/x/ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2306, 2309, 2178}

$$-\text{ExpIntegralEi}(-\log(x))-\frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Log[x]^2),x]
```

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\log^2(x)}\,dx &= -\frac{1}{x\log(x)}-\int \frac{1}{x^2\log(x)}\,dx\\
&= -\frac{1}{x\log(x)}-\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx,x,\log(x)\right)\\
&= -\text{Ei}(-\log(x))-\frac{1}{x\log(x)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 17, normalized size = 1.00

$$-\text{Ei}(-\log(x))-\frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Log[x]^2),x]
```

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

fricas [A] time = 0.40, size = 19, normalized size = 1.12

$$-\frac{x\log(x)\,\mathrm{log\_integral}\left(\frac{1}{x}\right)+1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="fricas")
[Out] -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\log(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="giac")
[Out] integrate(1/(x^2*log(x)^2), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.88

$$\mathrm{Ei}\left(1,\ln(x)\right)-\frac{1}{x\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(x)^2,x)
[Out] -1/x/ln(x)+Ei(1,ln(x))
```

maxima [A] time = 0.52, size = 6, normalized size = 0.35

$$-\Gamma\left(-1,\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="maxima")
[Out] -gamma(-1, log(x))
```

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$-\mathrm{ei}\left(-\ln(x)\right)-\frac{1}{x\,\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(x)^2),x)
[Out] - ei(-log(x)) - 1/(x*log(x))
```

sympy [A] time = 0.65, size = 14, normalized size = 0.82

$$-\mathrm{Ei}\left(-\log\left(x\right)\right)-\frac{1}{x\log\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(x)**2,x)
[Out] -Ei(-log(x)) - 1/(x*log(x))
```

3.64 $\int \frac{\log^p(x)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{\log^{p+1}(x)}{p+1}$$

```
[Out] ln(x)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 30}

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^p(x)}{x}\,dx &= \text{Subst}\left(\int x^p\,dx, x, \log(x)\right)\\ &= \frac{\log^{1+p}(x)}{1+p}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

fricas [A] time = 0.44, size = 12, normalized size = 1.00

$$\frac{\log(x)^p\log(x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="fricas")
```

```
[Out] log(x)^p*log(x)/(p + 1)
```

giac [A]   time = 1.23, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="giac")
[Out] log(x)^(p + 1)/(p + 1)
```

maple [A]   time = 0.00, size = 13, normalized size = 1.08

$$\frac{\ln(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^p/x,x)
[Out] ln(x)^(p+1)/(p+1)
```

maxima [A]   time = 0.41, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="maxima")
[Out] log(x)^(p + 1)/(p + 1)
```

mupad [B]   time = 0.15, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\ln(x)) & \text{if } p = -1 \\ \frac{\ln(x)^{p+1}}{p+1} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^p/x,x)
[Out] piecewise(p == -1, log(log(x)), p ~= -1, log(x)^(p + 1)/(p + 1))
```

sympy [A]   time = 0.88, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(\log(x)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**p/x,x)
[Out] Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))
```

3.65 $\int (b+ax)\log(x)\,dx$

Optimal. Leaf size=28

$$-\frac{ax^2}{4}+\frac{1}{2}ax^2\log(x)-bx+bx\log(x)$$

```
[Out] -b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2313}

$$\frac{1}{2}\log(x)\left(ax^2+2bx\right)-\frac{ax^2}{4}-bx$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)*Log[x],x]
[Out] -(b*x) - (a*x^2)/4 + ((2*b*x + a*x^2)*Log[x])/2
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (b+ax)\log(x)\,dx &= \frac{1}{2}\left(2bx+ax^2\right)\log(x)-\int \left(b+\frac{ax}{2}\right)dx\\ &= -bx-\frac{ax^2}{4}+\frac{1}{2}\left(2bx+ax^2\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{ax^2}{4}+\frac{1}{2}ax^2\log(x)-bx+bx\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)*Log[x],x]
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

fricas [A] time = 0.44, size = 25, normalized size = 0.89

$$-\frac{1}{4}\,ax^2-bx+\frac{1}{2}\left(ax^2+2\,bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="fricas")
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

giac [A] time = 1.18, size = 24, normalized size = 0.86

$$\frac{1}{2}\,ax^2\log(x)-\frac{1}{4}\,ax^2+bx\log(x)-bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="giac")
[Out] 1/2*a*x^2*log(x) - 1/4*a*x^2 + b*x*log(x) - b*x
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{a\,x^2 \ln(x)}{2} - \frac{a\,x^2}{4} + bx\ln(x) - bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)*ln(x),x)
[Out] -b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

maxima [A] time = 0.42, size = 25, normalized size = 0.89

$$-\frac{1}{4}\,ax^2 - bx + \frac{1}{2}\left(ax^2 + 2\,bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="maxima")
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

mupad [B] time = 0.14, size = 21, normalized size = 0.75

$$-\frac{x\,\left(4\,b + a\,x - 4\,b\,\ln(x) - 2\,a\,x\,\ln(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(b + a*x),x)
[Out] -(x*(4*b + a*x - 4*b*log(x) - 2*a*x*log(x)))/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.79

$$-\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*ln(x),x)
[Out] -a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)
```

3.66 $\int (b+ax)^2 \log(x)\,dx$

Optimal. Leaf size=54

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

```
[Out] -b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {32, 2313, 12, 43}

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b + a*x)^3*Lo
g[x])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int (b+ax)^2\log(x)\,dx &= \frac{(b+ax)^3\log(x)}{3a}-\int \frac{(b+ax)^3}{3ax}\,dx\\
&= \frac{(b+ax)^3\log(x)}{3a}-\frac{\int \frac{(b+ax)^3}{x}\,dx}{3a}\\
&= \frac{(b+ax)^3\log(x)}{3a}-\frac{\int \left(3ab^2+\frac{b^3}{x}+3a^2bx+a^3x^2\right)dx}{3a}\\
&= -b^2x-\frac{1}{2}abx^2-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}+\frac{(b+ax)^3\log(x)}{3a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.98

$$-\frac{1}{9}a^2x^3+\frac{1}{3}a^2x^3\log(x)-\frac{1}{2}abx^2+abx^2\log(x)-b^2x+b^2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)^2*Log[x],x]
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^2
*x^3*Log[x])/3
```

fricas [A] time = 0.40, size = 47, normalized size = 0.87

$$-\frac{1}{9}a^2x^3-\frac{1}{2}abx^2-b^2x+\frac{1}{3}\left(a^2x^3+3abx^2+3b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="fricas")
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

giac [A] time = 1.10, size = 47, normalized size = 0.87

$$\frac{1}{3}a^2x^3\log(x)-\frac{1}{9}a^2x^3+abx^2\log(x)-\frac{1}{2}abx^2+b^2x\log(x)-b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="giac")
[Out] 1/3*a^2*x^3*log(x) - 1/9*a^2*x^3 + a*b*x^2*log(x) - 1/2*a*b*x^2 + b^2*x*log
(x) - b^2*x
```

maple [A] time = 0.00, size = 48, normalized size = 0.89

$$\frac{a^2x^3\ln(x)}{3}-\frac{a^2x^3}{9}+abx^2\ln(x)-\frac{abx^2}{2}+b^2x\ln(x)-b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^2*ln(x),x)
[Out] 1/3*a^2*x^3*ln(x)-1/9*a^2*x^3+a*b*x^2*ln(x)-1/2*a*b*x^2+ln(x)*x*b^2-b^2*x
```

maxima [A] time = 0.42, size = 47, normalized size = 0.87

$$-\frac{1}{9}a^2x^3-\frac{1}{2}abx^2-b^2x+\frac{1}{3}\left(a^2x^3+3abx^2+3b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="maxima")
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

mupad [B] time = 0.17, size = 47, normalized size = 0.87

$$b^2\,x\,\ln(x)-\frac{a^2\,x^3}{9}-b^2\,x+\frac{a^2\,x^3\,\ln(x)}{3}-\frac{a\,b\,x^2}{2}+a\,b\,x^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(b + a*x)^2,x)
```

```
[Out] b^2*x*log(x) - (a^2*x^3)/9 - b^2*x + (a^2*x^3*log(x))/3 - (a*b*x^2)/2 + a*b
*x^2*log(x)
```

sympy [A] time = 0.14, size = 44, normalized size = 0.81

$$-\frac{a^2x^3}{9} - \frac{abx^2}{2} - b^2x + \left(\frac{a^2x^3}{3} + abx^2 + b^2x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**2*ln(x),x)
```

```
[Out] -a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*log(
x)
```

3.67 $\int \frac{\log(x)}{(b+ax)^2}\,dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{b(ax+b)}-\frac{\log(ax+b)}{ab}$$

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2314, 31}

$$\frac{x\log(x)}{b(ax+b)}-\frac{\log(ax+b)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(b + a*x)^2,x]
```

```
[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{(b+ax)^2}\,dx &= \frac{x\log(x)}{b(b+ax)}-\frac{\int \frac{1}{b+ax}\,dx}{b}\\ &= \frac{x\log(x)}{b(b+ax)}-\frac{\log(b+ax)}{ab}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{ax+b}-\frac{\log(ax+b)}{a}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(b + a*x)^2,x]
```

```
[Out] ((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b
```

fricas [A] time = 0.43, size = 34, normalized size = 1.17

$$\frac{ax\log(x)-(ax+b)\log(ax+b)}{a^2bx+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)
```

giac [B] time = 1.19, size = 138, normalized size = 4.76

$$a^2\left(\frac{\log\left(\frac{(ax+b)^2|a|\left|\frac{b}{ax+b}-1\right|}{a^2|ax+b|}\right)}{a^3b}+\frac{\log\left(-\frac{b+\frac{(ax+b)a\left(\frac{b}{ax+b}-1\right)-ab}{a}}{a}\right)}{\left((ax+b)\left(\frac{b}{ax+b}-1\right)-b\right)a^3}-\frac{\log\left(\left|-(ax+b)\left(\frac{b}{ax+b}-1\right)+b\right|\right)}{a^3b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="giac")
```

```
[Out] a^2*(log((a*x + b)^2*abs(a)*abs(b/(a*x + b) - 1)/(a^2*abs(a*x + b)))/(a^3*b
) + log(-(b + ((a*x + b)*a*(b/(a*x + b) - 1) - a*b)/a)/a)/(((a*x + b)*(b/(a
*x + b) - 1) - b)*a^3) - log(abs(-(a*x + b)*(b/(a*x + b) - 1) + b))/(a^3*b)
)
```

maple [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{x\ln(x)}{(ax+b)\,b}-\frac{\ln(ax+b)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^2,x)
```

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

maxima [A] time = 0.41, size = 38, normalized size = 1.31

$$-\frac{\frac{\log(ax+b)}{b}-\frac{\log(x)}{b}}{a}-\frac{\log(x)}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")
```

```
[Out] -(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)
```

mupad [B] time = 0.23, size = 35, normalized size = 1.21

$$\frac{x^2\,\ln(x)}{b\,\left(a\,x^2+b\,x\right)}-\frac{\ln(b+a\,x)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(b + a*x)^2,x)
```

```
[Out] (x^2*log(x))/(b*(b*x + a*x^2)) - log(b + a*x)/(a*b)
```

sympy [A] time = 0.36, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{a^2x+ab}+\frac{\log(x)-\log\left(x+\frac{b}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**2,x)
```

```
[Out] -log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)
```

3.68 $\int x \log(b + ax) \, dx$

Optimal. Leaf size=46

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

```
[Out] 1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 43}

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x \log(b+ax) \, dx &= \frac{1}{2}x^2 \log(b+ax) - \frac{1}{2}a \int \frac{x^2}{b+ax} \, dx \\
&= \frac{1}{2}x^2 \log(b+ax) - \frac{1}{2}a \int \left(-\frac{b}{a^2} + \frac{x}{a} + \frac{b^2}{a^2(b+ax)}\right) dx \\
&= \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b+ax)}{2a^2} + \frac{1}{2}x^2 \log(b+ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

fricas [A] time = 0.40, size = 39, normalized size = 0.85

$$-\frac{a^2x^2 - 2\,abx - 2\left(a^2x^2 - b^2\right)\log\left(ax + b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2
```

giac [A] time = 1.32, size = 58, normalized size = 1.26

$$\frac{(ax + b)^2\log\left(ax + b\right)}{2\,a^2} - \frac{(ax + b)b\log\left(ax + b\right)}{a^2} - \frac{(ax + b)^2}{4\,a^2} + \frac{(ax + b)b}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x
+ b)^2/a^2 + (a*x + b)*b/a^2
```

maple [A] time = 0.00, size = 47, normalized size = 1.02

$$\frac{x^2\ln\left(ax + b\right)}{2} - \frac{x^2}{4} + \frac{bx}{2a} - \frac{b^2\ln\left(ax + b\right)}{2a^2} + \frac{3b^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a*x+b),x)
```

```
[Out] -1/2*b^2*ln(a*x+b)/a^2+1/2*b*x/a+3/4*b^2/a^2+1/2*x^2*ln(a*x+b)-1/4*x^2
```

maxima [A] time = 0.41, size = 44, normalized size = 0.96

$$\frac{1}{2}\,x^2\log\left(ax + b\right) - \frac{1}{4}\,a\left(\frac{2\,b^2\log\left(ax + b\right)}{a^3} + \frac{ax^2 - 2\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2)
```

mupad [B] time = 0.19, size = 66, normalized size = 1.43

$$\begin{cases} \frac{x^2\left(\ln(a\,x)-\frac{1}{2}\right)}{2} & \text{if } b = 0 \\ \frac{\ln(b+a\,x)\left(x^2-\frac{b^2}{a^2}\right)}{2} - \frac{b^2\left(\frac{a^2\,x^2}{2\,b^2}-\frac{a\,x}{b}\right)}{2\,a^2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(b + a*x),x)
```

```
[Out] piecewise(b == 0, (x^2*(log(a*x) - 1/2))/2, b ~= 0, (log(b + a*x)*(x^2 - b^
2/a^2))/2 - (b^2*((a^2*x^2)/(2*b^2) - (a*x)/b))/(2*a^2))
```

sympy [A] time = 0.18, size = 42, normalized size = 0.91

$$-a\left(\frac{x^2}{4a} - \frac{bx}{2a^2} + \frac{b^2\log\left(ax + b\right)}{2a^3}\right) + \frac{x^2\log\left(ax + b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a*x+b),x)
```

```
[Out] -a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x
+ b)/2
```

3.69 $\int x^2 \log(b + ax) \, dx$

Optimal. Leaf size=59

$$\frac{b^3 \log(ax + b)}{3a^3} - \frac{b^2 x}{3a^2} + \frac{1}{3} x^3 \log(ax + b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

```
[Out] -1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2395, 43}

$$-\frac{b^2 x}{3a^2} + \frac{b^3 \log(ax + b)}{3a^3} + \frac{bx^2}{6a} + \frac{1}{3} x^3 \log(ax + b) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[b + a*x],x]
```

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log(b + ax) \, dx &= \frac{1}{3} x^3 \log(b + ax) - \frac{1}{3} a \int \frac{x^3}{b + ax} \, dx \\
&= \frac{1}{3} x^3 \log(b + ax) - \frac{1}{3} a \int \left(\frac{b^2}{a^3} - \frac{bx}{a^2} + \frac{x^2}{a} - \frac{b^3}{a^3(b + ax)} \right) dx \\
&= -\frac{b^2 x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b + ax)}{3a^3} + \frac{1}{3} x^3 \log(b + ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$\frac{b^3 \log(ax + b)}{3a^3} - \frac{b^2 x}{3a^2} + \frac{1}{3} x^3 \log(ax + b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[b + a*x],x]
```

```
[Out] -1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

fricas [A] time = 0.42, size = 49, normalized size = 0.83

$$-\frac{2\,a^{3}x^{3}-3\,a^{2}bx^{2}+6\,ab^{2}x-6\left(a^{3}x^{3}+b^{3}\right)\log\left(ax+b\right)}{18\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b)
)/a^3
```

giac [A] time = 1.27, size = 94, normalized size = 1.59

$$\frac{(ax+b)^{3}\log\left(ax+b\right)}{3\,a^{3}}-\frac{(ax+b)^{2}b\log\left(ax+b\right)}{a^{3}}+\frac{(ax+b)b^{2}\log\left(ax+b\right)}{a^{3}}-\frac{(ax+b)^{3}}{9\,a^{3}}+\frac{(ax+b)^{2}b}{2\,a^{3}}-\frac{(ax+b)b^{2}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x +
b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (a*
x + b)*b^2/a^3
```

maple [A] time = 0.00, size = 58, normalized size = 0.98

$$\frac{x^{3}\ln\left(ax+b\right)}{3}-\frac{x^{3}}{9}+\frac{b\,x^{2}}{6a}-\frac{b^{2}x}{3a^{2}}+\frac{b^{3}\ln\left(ax+b\right)}{3a^{3}}-\frac{11b^{3}}{18a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a*x+b),x)
```

```
[Out] 1/3*x^3*ln(a*x+b)+1/3*b^3*ln(a*x+b)/a^3-1/9*x^3+1/6*b*x^2/a-1/3*b^2*x/a^2-1
1/18*b^3/a^3
```

maxima [A] time = 0.41, size = 57, normalized size = 0.97

$$\frac{1}{3}\,x^{3}\log\left(ax+b\right)+\frac{1}{18}\,a\left(\frac{6\,b^{3}\log\left(ax+b\right)}{a^{4}}-\frac{2\,a^{2}x^{3}-3\,abx^{2}+6\,b^{2}x}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b*
x^2 + 6*b^2*x)/a^3)
```

mupad [B] time = 0.18, size = 75, normalized size = 1.27

$$\begin{cases}\frac{x^{3}\left(\ln(a\,x)-\frac{1}{3}\right)}{3} & \text{if } b=0\\ \frac{\ln(b+a\,x)\left(x^{3}+\frac{b^{3}}{a^{3}}\right)}{3}-\frac{b^{3}\left(\frac{a^{3}\,x^{3}}{3\,b^{3}}-\frac{a^{2}\,x^{2}}{2\,b^{2}}+\frac{a\,x}{b}\right)}{3\,a^{3}} & \text{if } b\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(b + a*x),x)
```

```
[Out] piecewise(b == 0, (x^3*(log(a*x) - 1/3))/3, b ~= 0, (log(b + a*x)*(x^3 + b^
3/a^3))/3 - (b^3*(- (a^2*x^2)/(2*b^2) + (a^3*x^3)/(3*b^3) + (a*x)/b))/(3*a^
3))
```

sympy [A] time = 0.19, size = 54, normalized size = 0.92

$$-a\left(\frac{x^3}{9a}-\frac{bx^2}{6a^2}+\frac{b^2x}{3a^3}-\frac{b^3\log\left(ax+b\right)}{3a^4}\right)+\frac{x^3\log\left(ax+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x+b),x)
```

```
[Out] -a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*a
**4)) + x**3*log(a*x + b)/3
```

3.70 $\int \log\left(a^2+x^2\right)\,dx$

Optimal. Leaf size=23

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a^2+x^2\right)\,dx &= x\log\left(a^2+x^2\right)-2\int \frac{x^2}{a^2+x^2}\,dx\\
&= -2x+x\log\left(a^2+x^2\right)+\left(2a^2\right)\int \frac{1}{a^2+x^2}\,dx\\
&= -2x+2a\tan^{-1}\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

fricas [A] time = 0.43, size = 23, normalized size = 1.00

$$2\,a\arctan\left(\frac{x}{a}\right) + x\log\left(a^2 + x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="fricas")
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

giac [A] time = 1.17, size = 23, normalized size = 1.00

$$2\,a\arctan\left(\frac{x}{a}\right) + x\log\left(a^2 + x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="giac")
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$2a\arctan\left(\frac{x}{a}\right) + x\ln\left(a^2 + x^2\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a^2+x^2),x)
[Out] -2*x+2*a*arctan(1/a*x)+x*ln(a^2+x^2)
```

maxima [A] time = 0.96, size = 23, normalized size = 1.00

$$2\,a\arctan\left(\frac{x}{a}\right) + x\log\left(a^2 + x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="maxima")
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

mupad [B] time = 0.07, size = 23, normalized size = 1.00

$$x\,\ln\left(a^2 + x^2\right) - 2\,x + 2\,a\,\mathrm{atan}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a^2 + x^2),x)
[Out] x*log(a^2 + x^2) - 2*x + 2*a*atan(x/a)
```

sympy [C] time = 0.16, size = 36, normalized size = 1.57

$$-2a\left(\frac{i\log\left(-ia + x\right)}{2} - \frac{i\log\left(ia + x\right)}{2}\right) + x\log\left(a^2 + x^2\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a**2+x**2),x)
[Out] -2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x
```

3.71 $\int x \log\left(a^2+x^2\right) dx$

Optimal. Leaf size=27

$$\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)-\frac{x^2}{2}$$

```
[Out] -1/2*x^2+1/2*(a^2+x^2)*ln(a^2+x^2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2454, 2389, 2295}

$$\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a^2 + x^2],x]
```

```
[Out] -x^2/2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\log\left(a^2+x^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int \log\left(a^2+x\right)dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \log(x)\,dx, x, a^2+x^2\right)\\
&= -\frac{x^2}{2}+\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{2}\left(\left(a^2+x^2\right)\log\left(a^2+x^2\right)-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a^2 + x^2],x]
```

```
[Out] (-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2
```

fricas [A] time = 0.42, size = 23, normalized size = 0.85

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

giac [A] time = 1.07, size = 28, normalized size = 1.04

$$-\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="giac")
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

maple [A] time = 0.00, size = 29, normalized size = 1.07

$$-\frac{a^2}{2} - \frac{x^2}{2} + \frac{\left(a^2 + x^2\right)\ln\left(a^2 + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a^2+x^2),x)
[Out] 1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*x^2-1/2*a^2
```

maxima [A] time = 0.42, size = 28, normalized size = 1.04

$$-\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="maxima")
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

mupad [B] time = 0.04, size = 51, normalized size = 1.89

$$\frac{a^2\,\ln\left(x - \sqrt{-a^2}\right)}{2} + \frac{x^2\,\ln\left(a^2 + x^2\right)}{2} - \frac{x^2}{2} + \frac{a^2\,\ln\left(x + \sqrt{-a^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a^2 + x^2),x)
[Out] (a^2*log(x - (-a^2)^(1/2)))/2 + (x^2*log(a^2 + x^2))/2 - x^2/2 + (a^2*log(x
 + (-a^2)^(1/2)))/2
```

sympy [A] time = 0.16, size = 31, normalized size = 1.15

$$\frac{a^2\log\left(a^2 + x^2\right)}{2} + \frac{x^2\log\left(a^2 + x^2\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a**2+x**2),x)
[Out] a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

3.72 $\int x^2 \log\left(a^2+x^2\right)\,dx$

Optimal. Leaf size=44

$$-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

```
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 302, 203}

$$\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(a^2+x^2\right)\,dx &= \frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{2}{3}\int\frac{x^4}{a^2+x^2}\,dx\\ &= \frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{2}{3}\int\left(-a^2+x^2+\frac{a^4}{a^2+x^2}\right)dx\\ &= \frac{2a^2x}{3}-\frac{2x^3}{9}+\frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{1}{3}\left(2a^4\right)\int\frac{1}{a^2+x^2}\,dx\\ &= \frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a^2 + x^2],x]
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

fricas [A] time = 0.43, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="fricas")
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

giac [A] time = 1.04, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="giac")
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

maple [A] time = 0.01, size = 37, normalized size = 0.84

$$-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a^2+x^2),x)
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(1/a*x)+1/3*x^3*ln(a^2+x^2)
```

maxima [A] time = 0.96, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="maxima")
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

mupad [B] time = 0.04, size = 65, normalized size = 1.48

$$\frac{2\,a^2\,x}{3}-\frac{\ln\left(x-\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{3/2}}{3}+\frac{x^3\,\ln\left(a^2+x^2\right)}{3}+\frac{\ln\left(x+\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{3/2}}{3}-\frac{2\,x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(a^2 + x^2),x)
[Out] (2*a^2*x)/3 - (log(x - (-a^2)^(1/2))*(-a^2)^(3/2))/3 + (x^3*log(a^2 + x^2))
/3 + (log(x + (-a^2)^(1/2))*(-a^2)^(3/2))/3 - (2*x^3)/9
```

sympy [C] time = 0.19, size = 53, normalized size = 1.20

$$-2a^3\left(-\frac{i\log\left(-ia+x\right)}{6}+\frac{i\log\left(ia+x\right)}{6}\right)+\frac{2a^2x}{3}+\frac{x^3\log\left(a^2+x^2\right)}{3}-\frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a**2+x**2),x)
```

```
[Out] -2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x**3*log(a**
2 + x**2)/3 - 2*x**3/9
```

3.73 $\int x^4 \log\left(a^2+x^2\right) dx$

Optimal. Leaf size=54

$$\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

```
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 302, 203}

$$\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2a^4x}{5}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\log\left(a^2+x^2\right)dx&=\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int\frac{x^6}{a^2+x^2}dx\\&=\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int\left(a^4-a^2x^2+x^4-\frac{a^6}{a^2+x^2}\right)dx\\&=-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{1}{5}x^5\log\left(a^2+x^2\right)+\frac{1}{5}\left(2a^6\right)\int\frac{1}{a^2+x^2}dx\\&=-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

fricas [A] time = 0.40, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^{5}\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^{5}\log\left(a^{2}+x^{2}\right)-\frac{2}{5}\,a^{4}x+\frac{2}{15}\,a^{2}x^{3}-\frac{2}{25}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

giac [A] time = 1.00, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^{5}\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^{5}\log\left(a^{2}+x^{2}\right)-\frac{2}{5}\,a^{4}x+\frac{2}{15}\,a^{2}x^{3}-\frac{2}{25}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{2a^{5}\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^{5}\ln\left(a^{2}+x^{2}\right)}{5}-\frac{2a^{4}x}{5}+\frac{2a^{2}x^{3}}{15}-\frac{2x^{5}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(a^2+x^2),x)
```

```
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(1/a*x)+1/5*x^5*ln(a^2+x^2)
```

maxima [A] time = 0.96, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^{5}\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^{5}\log\left(a^{2}+x^{2}\right)-\frac{2}{5}\,a^{4}x+\frac{2}{15}\,a^{2}x^{3}-\frac{2}{25}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

mupad [B] time = 0.13, size = 73, normalized size = 1.35

$$\frac{x^5\,\ln\left(a^2+x^2\right)}{5}-\frac{2\,a^4\,x}{5}-\frac{\ln\left(x-\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{5/2}}{5}+\frac{\ln\left(x+\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{5/2}}{5}-\frac{2\,x^5}{25}+\frac{2\,a^2\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(a^2 + x^2),x)
```

```
[Out] (x^5*log(a^2 + x^2))/5 - (2*a^4*x)/5 - (log(x - (-a^2)^(1/2))*(-a^2)^(5/2))
/5 + (log(x + (-a^2)^(1/2))*(-a^2)^(5/2))/5 - (2*x^5)/25 + (2*a^2*x^3)/15
```

<u>**sympy**</u> **[C]** time = 0.21, size = 63, normalized size = 1.17

$$-2a^5\left(\frac{i\log\left(-ia+x\right)}{10}-\frac{i\log\left(ia+x\right)}{10}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{x^5\log\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(a**2+x**2),x)
```

```
[Out] -2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**3
/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25
```

3.74 $\int \log\left(-a^2+x^2\right)\,dx$

Optimal. Leaf size=25

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x+2*a*arctanh(x/a)+x*ln(-a^2+x^2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2448, 321, 207}

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(-a^2+x^2\right)\,dx &= x\log\left(-a^2+x^2\right)-2\int \frac{x^2}{-a^2+x^2}\,dx\\ &= -2x+x\log\left(-a^2+x^2\right)-\left(2a^2\right)\int \frac{1}{-a^2+x^2}\,dx\\ &= -2x+2a\tanh^{-1}\left(\frac{x}{a}\right)+x\log\left(-a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

fricas [A] time = 0.43, size = 31, normalized size = 1.24

$$x \log\left(-a^2 + x^2\right) + a \log\left(a + x\right) - a \log\left(-a + x\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="fricas")
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

giac [A] time = 1.22, size = 33, normalized size = 1.32

$$x \log\left(-a^2 + x^2\right) + a \log\left(|a + x|\right) - a \log\left(|-a + x|\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="giac")
[Out] x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x
```

maple [A] time = 0.01, size = 32, normalized size = 1.28

$$-a \ln\left(-a + x\right) + a \ln\left(a + x\right) + x \ln\left(-a^2 + x^2\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-a^2+x^2),x)
[Out] x*ln(-a^2+x^2)-2*x-a*ln(-a+x)+a*ln(a+x)
```

maxima [A] time = 0.42, size = 31, normalized size = 1.24

$$x \log\left(-a^2 + x^2\right) + a \log\left(a + x\right) - a \log\left(-a + x\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="maxima")
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

mupad [B] time = 0.07, size = 25, normalized size = 1.00

$$x\,\ln\left(x^2 - a^2\right) - 2\,x + 2\,a\,\mathrm{atanh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2 - a^2),x)
[Out] x*log(x^2 - a^2) - 2*x + 2*a*atanh(x/a)
```

sympy [A] time = 0.16, size = 29, normalized size = 1.16

$$-2a\left(\frac{\log\left(-a + x\right)}{2} - \frac{\log\left(a + x\right)}{2}\right) + x \log\left(-a^2 + x^2\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-a**2+x**2),x)
[Out] -2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x
```

3.75 $\int \log(\log(\log(\log(x)))) \, dx$

Optimal. Leaf size=8

$$\text{Int}(\log(\log(\log(\log(x)))), x)$$

```
[Out] CannotIntegrate(ln(ln(ln(ln(x)))),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \log(\log(\log(\log(x)))) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[Log[Log[x]]]],x]
[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]
```

Rubi steps

$$\int \log(\log(\log(\log(x)))) \, dx = \int \log(\log(\log(\log(x)))) \, dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \log(\log(\log(\log(x)))) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[Log[Log[x]]]],x]
[Out] Integrate[Log[Log[Log[Log[x]]]], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\log\left(\log\left(\log(x)\right)\right)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="fricas")
[Out] integral(log(log(log(log(x)))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\log\left(\log\left(\log(x)\right)\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="giac")
[Out] integrate(log(log(log(log(x)))), x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \ln\left(\ln\left(\ln\left(\ln(x)\right)\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(ln(ln(x)))),x)
```

```
[Out] int(ln(ln(ln(ln(x)))),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\log\left(\log\left(\log(x)\right)\right)\right)-\int\frac{1}{\log(x)\log\left(\log(x)\right)\log\left(\log\left(\log(x)\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="maxima")
```

```
[Out] x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x))))
, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.12

$$\int\ln\left(\ln\left(\ln\left(\ln(x)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(log(log(x)))),x)
```

```
[Out] int(log(log(log(log(x)))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\log\left(\log\left(\log\left(x\right)\right)\right)\right)-\int\frac{1}{\log\left(x\right)\log\left(\log\left(x\right)\right)\log\left(\log\left(\log\left(x\right)\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(ln(ln(x)))),x)
```

```
[Out] x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x)))),
 x)
```

3.76 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

[Out] -cos(x)

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2638}

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x],x]

[Out] -Cos[x]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x)\,dx = -\cos(x)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x],x]

[Out] -Cos[x]

fricas [A] time = 0.42, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x),x, algorithm="fricas")

[Out] -cos(x)

giac [A] time = 1.03, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x),x, algorithm="giac")

[Out] -cos(x)

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
[Out] -cos(x)
```

maxima [A] time = 0.42, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="maxima")
[Out] -cos(x)
```

mupad [B] time = 0.02, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
[Out] -cos(x)
```

sympy [A] time = 0.06, size = 3, normalized size = 0.75

$$-\cos{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
[Out] -cos(x)
```

3.77 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] sin(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

fricas [A] time = 0.42, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

giac [A] time = 0.96, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
[Out] sin(x)
```

maxima [A] time = 0.41, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
[Out] sin(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\sin{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

3.78 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(x)\,dx = -\log(\cos(x))$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

fricas [B] time = 0.43, size = 11, normalized size = 2.20

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 1.04, size = 6, normalized size = 1.20

$$-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="giac")
```

```
[Out] -log(abs(cos(x)))
```

maple [A] time = 0.00, size = 6, normalized size = 1.20

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
```

```
[Out] -ln(cos(x))
```

maxima [A] time = 0.43, size = 3, normalized size = 0.60

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x))
```

mupad [B] time = 0.03, size = 5, normalized size = 1.00

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
```

```
[Out] -log(cos(x))
```

sympy [A] time = 0.06, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

3.79 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

fricas [B] time = 0.43, size = 16, normalized size = 5.33

$$\frac{1}{2}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

giac [B] time = 1.13, size = 17, normalized size = 5.67

$$-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)
```

maple [**A**] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x)
[Out] ln(sin(x))
```

maxima [**A**] time = 0.42, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="maxima")
[Out] log(sin(x))
```

mupad [**B**] time = 0.20, size = 13, normalized size = 4.33

$$\ln(\tan(x)) - \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x)
[Out] log(tan(x)) - log(tan(x)^2 + 1)/2
```

sympy [**A**] time = 0.07, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x)
[Out] log(sin(x))
```

3.80 $\int \frac{1}{(1+\tan(x))^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

```
[Out] 1/2*ln(cos(x)+sin(x))-1/2/(1+tan(x))
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3530}

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x])^(-2),x]
```

```
[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\tan(x))^2}\,dx &= -\frac{1}{2(1+\tan(x))}+\frac{1}{2}\int \frac{1-\tan(x)}{1+\tan(x)}\,dx\\ &= \frac{1}{2}\log(\cos(x)+\sin(x))-\frac{1}{2(1+\tan(x))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 1.29

$$\frac{\tan(x)+\log(\sin(x)+\cos(x))+\tan(x)\log(\sin(x)+\cos(x))}{2\tan(x)+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x])^(-2),x]
```

```
[Out] (Log[Cos[x] + Sin[x]] + Tan[x] + Log[Cos[x] + Sin[x]]*Tan[x])/(2 + 2*Tan[x]
)
```

fricas [B] time = 0.44, size = 37, normalized size = 1.76

$$\frac{(\tan(x)+1)\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)+\tan(x)-1}{4(\tan(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) -
1)/(tan(x) + 1)
```

giac [A] time = 1.17, size = 26, normalized size = 1.24

$$-\frac{1}{2\left(\tan(x)+1\right)}-\frac{1}{4}\,\log\left(\tan(x)^{2}+1\right)+\frac{1}{2}\,\log\left(\left|\tan(x)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))
```

maple [A] time = 0.03, size = 26, normalized size = 1.24

$$-\frac{\ln\left(\tan^{2}(x)+1\right)}{4}+\frac{\ln\left(\tan(x)+1\right)}{2}-\frac{1}{2\left(\tan(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)+1)^2,x)
```

```
[Out] -1/2/(tan(x)+1)+1/2*ln(tan(x)+1)-1/4*ln(tan(x)^2+1)
```

maxima [A] time = 0.96, size = 25, normalized size = 1.19

$$-\frac{1}{2\left(\tan(x)+1\right)}-\frac{1}{4}\,\log\left(\tan(x)^{2}+1\right)+\frac{1}{2}\,\log\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)
```

mupad [B] time = 0.25, size = 27, normalized size = 1.29

$$\frac{\ln\left(\tan(x)+1\right)}{2}-\frac{\ln\left({\tan(x)}^{2}+1\right)}{4}-\frac{1}{2\,\left(\tan(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1)^2,x)
```

```
[Out] log(tan(x) + 1)/2 - log(tan(x)^2 + 1)/4 - 1/(2*(tan(x) + 1))
```

sympy [B] time = 0.36, size = 75, normalized size = 3.57

$$\frac{2\log\left(\tan\left(x\right)+1\right)\tan\left(x\right)}{4\tan\left(x\right)+4}+\frac{2\log\left(\tan\left(x\right)+1\right)}{4\tan\left(x\right)+4}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)\tan\left(x\right)}{4\tan\left(x\right)+4}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{4\tan\left(x\right)+4}-\frac{2}{4\tan\left(x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))**2,x)
```

```
[Out] 2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4)
- log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x) +
 4) - 2/(4*tan(x) + 4)
```

3.81 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\tanh^{-1}(\sin(x))$$

```
[Out] arctanh(sin(x))
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3770}

$$\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(x)\,dx = \tanh^{-1}(\sin(x))$$

Mathematica [B] time = 0.00, size = 33, normalized size = 11.00

$$\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

fricas [B] time = 0.43, size = 17, normalized size = 5.67

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

giac [B] time = 1.13, size = 17, normalized size = 5.67

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

maple [**A**]   time = 0.02, size = 7, normalized size = 2.33

$$\ln\left(\sec(x)+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
[Out] ln(sec(x)+tan(x))
```

maxima [**B**]   time = 0.42, size = 15, normalized size = 5.00

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="maxima")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

mupad [**B**]   time = 0.13, size = 11, normalized size = 3.67

$$\ln\left(\frac{1}{\cos(x)}\right)+\ln\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
[Out] log(1/cos(x)) + log(sin(x) + 1)
```

sympy [**B**]   time = 0.11, size = 15, normalized size = 5.00

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x)
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

3.82 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))
```

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3770}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(x)\,dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.00, size = 17, normalized size = 3.40

$$\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.42, size = 19, normalized size = 3.80

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 1.18, size = 17, normalized size = 3.40

$$-\frac{1}{2}\log\left(\cos(x)+1\right)+\frac{1}{2}\log\left(-\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

maple [A] time = 0.02, size = 9, normalized size = 1.80

$$\ln(-\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x)
[Out] ln(csc(x)-cot(x))
```

maxima [B] time = 0.42, size = 15, normalized size = 3.00

$$-\frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="maxima")
[Out] -1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

mupad [B] time = 0.04, size = 5, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x)
[Out] log(tan(x/2))
```

sympy [B] time = 0.11, size = 15, normalized size = 3.00

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

3.83 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

fricas [A] time = 0.43, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

giac [**A**] time = 1.20, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

maple [**A**] time = 0.00, size = 11, normalized size = 0.79

$$-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
[Out] -1/2*cos(x)*sin(x)+1/2*x
```

maxima [**A**] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

mupad [**B**] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

sympy [**A**] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
[Out] x/2 - sin(x)*cos(x)/2
```

3.84 $\int x^3 \sin\left(x^2\right) dx$

Optimal. Leaf size=20

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3\sin\left(x^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx,x,x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\text{Subst}\left(\int\cos(x)\,dx,x,x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

fricas [A] time = 0.44, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

giac [A] time = 1.04, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

maple [A] time = 0.01, size = 17, normalized size = 0.85

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

maxima [A] time = 0.42, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

sympy [A] time = 0.57, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

3.85 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

fricas [A] time = 0.44, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.94, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{\left(\sin^2(x)+2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] -1/3*(sin(x)^2+2)*cos(x)
```

maxima [A] time = 0.41, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.77

$$\frac{\cos(x)\left(\cos(x)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

3.86 $\int \sin^p(x)\,dx$

Optimal. Leaf size=44

$$\frac{\cos(x)\sin^{p+1}(x)\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

```
[Out] cos(x)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(x)^2)*sin(x)^(1+p)/(1+p)/
(cos(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2643}

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^p,x]
```

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 +
p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^p(x)\,dx = \frac{\cos(x)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(x)\right)\sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.00

$$-\cos(x)\sin^{p+1}(x)\sin^2(x)^{\frac{1}{2}(-p-1)}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3}{2};\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^p,x]
```

```
[Out] -(Cos[x]*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cos[x]^2]*Sin[x]^(1 + p)*(S
in[x]^2)^((-1 - p)/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin(x)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="fricas")
```

```
[Out] integral(sin(x)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="giac")
```

```
[Out] integrate(sin(x)^p, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \sin^p(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^p,x)
```

```
[Out] int(sin(x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^p, x)
```

mupad [B] time = 0.32, size = 35, normalized size = 0.80

$$-\frac{\cos(x) \sin(x)^{p+1} \, {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{p}{2}; \frac{3}{2}; \cos(x)^2\right)}{\left(\sin(x)^2\right)^{\frac{p}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^p,x)
```

```
[Out] -(cos(x)*sin(x)^(p + 1)*hypergeom([1/2, 1/2 - p/2], 3/2, cos(x)^2))/(sin(x)
^2)^(p/2 + 1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^p(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**p,x)
```

```
[Out] Integral(sin(x)**p, x)
```

3.87 $\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3190, 194}

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx &= \text{Subst}\left(\int \left(1+x^2\right)^2\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,\sin(x)\right)\\ &= \sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

fricas [A] time = 0.42, size = 18, normalized size = 0.95

$$\frac{1}{15}\left(3\cos(x)^4-16\cos(x)^2+28\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

giac [A] time = 1.05, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.02, size = 16, normalized size = 0.84

$$\frac{\left(\sin^5(x)\right)}{5} + \frac{2\left(\sin^3(x)\right)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^2)^2,x)
```

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

maxima [A] time = 0.41, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.79

$$\frac{\sin(x)^5}{5} + \frac{2\sin(x)^3}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sin(x)^2 + 1)^2,x)
```

```
[Out] sin(x) + (2*sin(x)^3)/3 + sin(x)^5/5
```

sympy [A] time = 1.15, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**2)**2,x)
```

```
[Out] sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

fricas [A] time = 0.42, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [**A**] time = 1.07, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

maple [**A**] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
[Out] 1/2*cos(x)*sin(x)+1/2*x
```

maxima [**A**] time = 0.42, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [**B**] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

sympy [**A**] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

3.89 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] sin(x)-1/3*sin(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

fricas [A] time = 0.45, size = 10, normalized size = 0.91

$$\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

giac [A] time = 1.08, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
[Out] -1/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{\left(\cos^2(x)+2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
[Out] 1/3*(cos(x)^2+2)*sin(x)
```

maxima [A] time = 0.41, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
[Out] -1/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$\sin(x)-\frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
[Out] sin(x) - sin(x)^3/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
[Out] -sin(x)**3/3 + sin(x)
```

3.90 $\int \sec^2(x)\, dx$

Optimal. Leaf size=2

$$\tan(x)$$

```
[Out] tan(x)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8}

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\, dx &= -\text{Subst}\left(\int 1\, dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

fricas [B] time = 0.43, size = 7, normalized size = 3.50

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

giac [A] time = 1.31, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

maple [A] time = 0.12, size = 3, normalized size = 1.50

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

maxima [A] time = 0.42, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

mupad [B] time = 0.02, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

sympy [B] time = 0.06, size = 5, normalized size = 2.50

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

3.91 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

```
[Out] 1/2*sin(x)-1/6*sin(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

fricas [A] time = 0.44, size = 10, normalized size = 0.67

$$-\frac{2}{3}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

giac [A] time = 1.14, size = 6, normalized size = 0.40

$$\frac{2}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] 2/3*sin(x)^3
```

maple [A] time = 0.03, size = 7, normalized size = 0.47

$$\frac{2\left(\sin^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x)
```

```
[Out] 2/3*sin(x)^3
```

maxima [A] time = 0.42, size = 11, normalized size = 0.73

$$-\frac{1}{6}\sin\left(3x\right)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

mupad [B] time = 0.03, size = 6, normalized size = 0.40

$$\frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(x),x)
```

```
[Out] (2*sin(x)^3)/3
```

sympy [A] time = 0.57, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

3.92 $\int x \sin(x)\,dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

fricas [A] time = 0.43, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

giac [A] time = 1.01, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
[Out] -x*cos(x) + sin(x)
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
[Out] -x*cos(x)+sin(x)
```

maxima [A] time = 0.42, size = 8, normalized size = 1.00

$$-x\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
[Out] -x*cos(x) + sin(x)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\sin(x)-x\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
[Out] sin(x) - x*cos(x)
```

sympy [A] time = 0.18, size = 7, normalized size = 0.88

$$-x\cos{\left(x\right)}+\sin{\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
[Out] -x*cos(x) + sin(x)
```

3.93 $\int x^2 \sin(x)\,dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin(x)\,dx &= -x^2\cos(x) + 2\int x\cos(x)\,dx\\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\,dx\\ &= 2\cos(x) - x^2\cos(x) + 2x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.88

$$2x\sin(x) - \left(x^2-2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

fricas [A] time = 0.44, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

giac [A] time = 1.32, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

maple [A] time = 0.00, size = 18, normalized size = 1.06

$$-x^2\cos(x)+2x\sin(x)+2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
[Out] -x^2*cos(x)+2*x*sin(x)+2*cos(x)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x)+2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2\,x\,\sin(x)-\cos(x)\,\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
[Out] 2*x*sin(x) - cos(x)*(x^2 - 2)
```

sympy [A] time = 0.32, size = 17, normalized size = 1.00

$$-x^2\cos(x)+2x\sin(x)+2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

3.94 $\int x\sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\,dx&=-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\&=\frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

fricas [A] time = 0.44, size = 19, normalized size = 0.76

$$-\frac{1}{2}\,x\cos(x)\sin(x)+\frac{1}{4}\,x^2-\frac{1}{4}\,\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

giac [A] time = 1.16, size = 19, normalized size = 0.76

$$\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{x^2}{4} + \frac{\left(\sin^2(x)\right)}{4} + \left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] -1/4*x^2+1/4*sin(x)^2+(-1/2*cos(x)*sin(x)+1/2*x)*x
```

maxima [A] time = 0.42, size = 19, normalized size = 0.76

$$\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.76

$$\frac{\sin(x)^2}{4} - \frac{x\,\sin(2x)}{4} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

sympy [A] time = 0.33, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4} + \frac{x^2\cos^2(x)}{4} - \frac{x\sin(x)\cos(x)}{2} + \frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)**2/4
```

3.95 $\int x^2 \sin^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int\sin^2(x)\,dx\\ &= \frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

fricas [A] time = 0.45, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 - \frac{1}{2}x\cos(x)^2 - \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

giac [A] time = 1.01, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.90

$$-\frac{x^3}{3} - \frac{x\left(\cos^2(x)\right)}{2} + \left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right)x^2 + \frac{\cos(x)\sin(x)}{4} + \frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] -1/3*x^3-1/2*x*cos(x)^2+(-1/2*cos(x)*sin(x)+1/2*x)*x^2+1/4*cos(x)*sin(x)+1/
4*x
```

maxima [A] time = 0.43, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

mupad [B] time = 0.16, size = 28, normalized size = 0.68

$$\frac{\sin(2x)}{8} - \frac{x\,\cos(2x)}{4} - \frac{x^2\,\sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

sympy [A] time = 0.63, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} - \frac{x^2\sin(x)\cos(x)}{2} + \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.96 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] -2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3310, 3296, 2637}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx&=-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\&=-\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\&=-\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

fricas [A] time = 0.43, size = 23, normalized size = 0.70

$$\frac{1}{3}\,x\cos(x)^3 - x\cos(x) - \frac{1}{9}\left(\cos(x)^2 - 7\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

giac [A] time = 1.05, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos(x) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.70

$$\frac{\left(\sin^3(x)\right)}{9} - \frac{\left(\sin^2(x) + 2\right)x\cos(x)}{3} + \frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
```

```
[Out] 1/9*sin(x)^3-1/3*(sin(x)^2+2)*x*cos(x)+2/3*sin(x)
```

maxima [A] time = 0.42, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos(x) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

mupad [B] time = 0.07, size = 25, normalized size = 0.76

$$\frac{x\cos(x)^3}{3} - \frac{\sin(x)\cos(x)^2}{9} - x\,\cos(x) + \frac{7\,\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
```

```
[Out] (7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

sympy [A] time = 0.60, size = 39, normalized size = 1.18

$$-x\sin^2\left(x\right)\cos\left(x\right) - \frac{2x\cos^3\left(x\right)}{3} + \frac{7\sin^3\left(x\right)}{9} + \frac{2\sin\left(x\right)\cos^2\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

3.97 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x)+\cos(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2638}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int \sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

giac [A] time = 1.24, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
[Out] x*sin(x) + cos(x)
```

maple [A] time = 0.01, size = 8, normalized size = 1.14

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
[Out] cos(x)+x*sin(x)
```

maxima [A] time = 0.42, size = 7, normalized size = 1.00

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

mupad [B] time = 0.02, size = 7, normalized size = 1.00

$$\cos(x) + x\ \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
[Out] cos(x) + x*sin(x)
```

sympy [A] time = 0.18, size = 7, normalized size = 1.00

$$x \sin{(x)} + \cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
[Out] x*sin(x) + cos(x)
```

3.98 $\int x^2 \cos(x) \, dx$

Optimal. Leaf size=16

$$x^2 \sin(x) - 2\sin(x) + 2x\cos(x)$$

```
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^2 \sin(x) - 2\sin(x) + 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x],x]
[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \cos(x)\,dx &= x^2\sin(x) - 2\int x\sin(x)\,dx\\ &= 2x\cos(x) + x^2\sin(x) - 2\int \cos(x)\,dx\\ &= 2x\cos(x) - 2\sin(x) + x^2\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$\left(x^2-2\right)\sin(x) + 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x],x]
[Out] 2*x*Cos[x] + (-2 + x^2)*Sin[x]
```

fricas [A] time = 0.43, size = 14, normalized size = 0.88

$$2\,x\cos(x) + \left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="fricas")
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

giac [A] time = 1.21, size = 14, normalized size = 0.88

$$2\,x\cos(x) + \left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="giac")
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$x^2\sin(x) + 2x\cos(x) - 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x),x)
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

maxima [A] time = 0.43, size = 14, normalized size = 0.88

$$2\,x\cos(x) + \left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="maxima")
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\sin(x)\,\left(x^2 - 2\right) + 2\,x\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x),x)
[Out] sin(x)*(x^2 - 2) + 2*x*cos(x)
```

sympy [A] time = 0.32, size = 17, normalized size = 1.06

$$x^2\sin{(x)} + 2x\cos{(x)} - 2\sin{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x),x)
[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

3.99 $\int x\cos^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] 1/4*x^2+1/4*cos(x)^2+1/2*x*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(x)\,dx&=\frac{\cos^2(x)}{4}+\frac{1}{2}x\cos(x)\sin(x)+\frac{\int x\,dx}{2}\\&=\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4}+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4
```

fricas [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{2}\,x\cos(x)\sin(x)+\frac{1}{4}\,x^2+\frac{1}{4}\,\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="fricas")
[Out] 1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2
```

giac [A] time = 1.06, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^{2}+\frac{1}{4}\,x\sin\left(2\,x\right)+\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="giac")
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

maple [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{x^{2}}{4}-\frac{\left(\sin^{2}(x)\right)}{4}+\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2,x)
[Out] x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2
```

maxima [A] time = 0.42, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^{2}+\frac{1}{4}\,x\sin\left(2\,x\right)+\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="maxima")
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

mupad [B] time = 0.15, size = 19, normalized size = 0.76

$$\frac{x\,\sin\left(2\,x\right)}{4}-\frac{\sin(x)^{2}}{4}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2,x)
[Out] (x*sin(2*x))/4 - sin(x)^2/4 + x^2/4
```

sympy [A] time = 0.34, size = 36, normalized size = 1.44

$$\frac{x^{2}\sin^{2}\left(x\right)}{4}+\frac{x^{2}\cos^{2}\left(x\right)}{4}+\frac{x\sin\left(x\right)\cos\left(x\right)}{2}-\frac{\sin^{2}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2,x)
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 - sin(x)**2/4
```

3.100 $\int x^2 \cos^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -1/4*x+1/6*x^3+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)+1/2*x^2*cos(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^2(x)\,dx &= \frac{1}{2}x\cos^2(x)+\frac{1}{2}x^2\cos(x)\sin(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int\cos^2(x)\,dx\\ &= \frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(6x^2-3\right)\sin(2x)+6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2,x]
[Out] (4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24
```

fricas [A] time = 0.44, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 + \frac{1}{2}x\cos(x)^2 + \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) - \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x
```

giac [A] time = 1.40, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 + \frac{1}{4}x\cos(2x) + \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="giac")
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.90

$$-\frac{x^3}{3} + \frac{x\left(\cos^2(x)\right)}{2} + \left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right)x^2 - \frac{\cos(x)\sin(x)}{4} - \frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2,x)
[Out] x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^
3
```

maxima [A] time = 0.42, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 + \frac{1}{4}x\cos(2x) + \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="maxima")
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

mupad [B] time = 0.06, size = 28, normalized size = 0.68

$$\frac{x\,\cos(2x)}{4} - \frac{\sin(2x)}{8} + \frac{x^2\,\sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2,x)
[Out] (x*cos(2*x))/4 - sin(2*x)/8 + (x^2*sin(2*x))/4 + x^3/6
```

sympy [A] time = 0.62, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} + \frac{x^2\sin(x)\cos(x)}{2} - \frac{x\sin^2(x)}{4} + \frac{x\cos^2(x)}{4} - \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*sin(x)**2/4
+ x*cos(x)**2/4 - sin(x)*cos(x)/4
```

3.101 $\int x\cos^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

```
[Out] 2/3*cos(x)+1/9*cos(x)^3+2/3*x*sin(x)+1/3*x*cos(x)^2*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3310, 3296, 2638}

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^3,x]
```

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\cos^3(x)\,dx &= \frac{\cos^3(x)}{9}+\frac{1}{3}x\cos^2(x)\sin(x)+\frac{2}{3}\int x\cos(x)\,dx\\ &= \frac{\cos^3(x)}{9}+\frac{2}{3}x\sin(x)+\frac{1}{3}x\cos^2(x)\sin(x)-\frac{2}{3}\int \sin(x)\,dx\\ &= \frac{2\cos(x)}{3}+\frac{\cos^3(x)}{9}+\frac{2}{3}x\sin(x)+\frac{1}{3}x\cos^2(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$\frac{3}{4}x\sin(x)+\frac{1}{12}x\sin(3x)+\frac{3\cos(x)}{4}+\frac{1}{36}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^3,x]
```

```
[Out] (3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12
```

fricas [A] time = 0.44, size = 25, normalized size = 0.76

$$\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(x\cos(x)^2 + 2x\right)\sin(x) + \frac{2}{3}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="fricas")
[Out] 1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)
```

giac [A] time = 1.38, size = 23, normalized size = 0.70

$$\frac{1}{12}x\sin(3x) + \frac{3}{4}x\sin(x) + \frac{1}{36}\cos(3x) + \frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="giac")
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

maple [A] time = 0.02, size = 23, normalized size = 0.70

$$\frac{\left(\cos^3(x)\right)}{9} + \frac{\left(\cos^2(x)+2\right)x\sin(x)}{3} + \frac{2\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x)
[Out] 1/3*x*(cos(x)^2+2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)
```

maxima [A] time = 0.42, size = 23, normalized size = 0.70

$$\frac{1}{12}x\sin(3x) + \frac{3}{4}x\sin(x) + \frac{1}{36}\cos(3x) + \frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="maxima")
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

mupad [B] time = 0.17, size = 25, normalized size = 0.76

$$\frac{\cos(x)^3}{9} + \frac{x\,\sin(x)\cos(x)^2}{3} + \frac{2\,\cos(x)}{3} + \frac{2\,x\,\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x)
[Out] (2*cos(x))/3 + cos(x)^3/9 + (2*x*sin(x))/3 + (x*cos(x)^2*sin(x))/3
```

sympy [A] time = 0.61, size = 39, normalized size = 1.18

$$\frac{2x\sin^3(x)}{3} + x\sin(x)\cos^2(x) + \frac{2\sin^2(x)\cos(x)}{3} + \frac{7\cos^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**3,x)
[Out] 2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*cos(x)**3/9
```

3.102 $\int \frac{\sin(x)}{x} \, dx$

Optimal. Leaf size=2

$$\text{Si}(x)$$

```
[Out] Si(x)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3299}

$$\text{Si}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/x,x]
```

```
[Out] SinIntegral[x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{x} \, dx = \text{Si}(x)$$

Mathematica [A] time = 0.01, size = 2, normalized size = 1.00

$$\text{Si}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/x,x]
```

```
[Out] SinIntegral[x]
```

fricas [A] time = 0.41, size = 2, normalized size = 1.00

$$\text{Si}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="fricas")
```

```
[Out] sin_integral(x)
```

giac [A] time = 1.39, size = 2, normalized size = 1.00

$$\text{Si}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="giac")
```

```
[Out] sin_integral(x)
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\text{Si}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x,x)
```

```
[Out] Si(x)
```

maxima [C] time = 0.57, size = 13, normalized size = 6.50

$$-\frac{1}{2}i\,\mathrm{Ei}\,(i\,x)+\frac{1}{2}i\,\mathrm{Ei}\,(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\mathrm{sinint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x,x)
```

```
[Out] sinint(x)
```

sympy [A] time = 0.61, size = 2, normalized size = 1.00

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x)
```

```
[Out] Si(x)
```

3.103 $\int \frac{\cos(x)}{x}\,dx$

Optimal. Leaf size=2

$$\text{CosIntegral}(x)$$

```
[Out] Ci(x)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3302}

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(x)}{x}\,dx = \text{Ci}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

fricas [B] time = 0.45, size = 11, normalized size = 5.50

$$\frac{1}{2}\,\text{Ci}\,(-x) + \frac{1}{2}\,\text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*cos_integral(-x) + 1/2*cos_integral(x)
```

giac [A] time = 1.30, size = 2, normalized size = 1.00

$$\text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="giac")
```

```
[Out] cos_integral(x)
```

maple **[A]** time = 0.02, size = 3, normalized size = 1.50

$$\text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x)
[Out] Ci(x)
```

maxima **[C]** time = 0.57, size = 13, normalized size = 6.50

$$\frac{1}{2}\,\text{Ei}\left(i\,x\right) + \frac{1}{2}\,\text{Ei}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="maxima")
[Out] 1/2*Ei(I*x) + 1/2*Ei(-I*x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.50

$$\text{cosint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x)
[Out] cosint(x)
```

sympy **[B]** time = 1.00, size = 12, normalized size = 6.00

$$-\log\left(x\right) + \frac{\log\left(x^{2}\right)}{2} + \text{Ci}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x)
[Out] -log(x) + log(x**2)/2 + Ci(x)
```

3.104 $\int \frac{\sin(x)}{x^2} \, dx$

Optimal. Leaf size=10

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

```
[Out] Ci(x)-sin(x)/x
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3297, 3302}

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{x^2} \, dx &= -\frac{\sin(x)}{x} + \int \frac{\cos(x)}{x} \, dx\\ &= \text{Ci}(x) - \frac{\sin(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

fricas [A] time = 0.43, size = 20, normalized size = 2.00

$$\frac{x \, \text{Ci}\left(-x\right) + x \, \text{Ci}\left(x\right) - 2 \, \sin\left(x\right)}{2 \, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(-x) + x*cos_integral(x) - 2*sin(x))/x
```

giac [A] time = 1.28, size = 13, normalized size = 1.30

$$\frac{x\,\mathrm{Ci}(x)-\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="giac")
[Out] (x*cos_integral(x) - sin(x))/x
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\mathrm{Ci}(x)-\frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x^2,x)
[Out] Ci(x)-sin(x)/x
```

maxima [C] time = 0.55, size = 15, normalized size = 1.50

$$\frac{1}{2}\,\Gamma\left(-1,i\,x\right)+\frac{1}{2}\,\Gamma\left(-1,-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="maxima")
[Out] 1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\mathrm{cosint}(x)-\frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x^2,x)
[Out] cosint(x) - sin(x)/x
```

sympy [B] time = 1.45, size = 17, normalized size = 1.70

$$-\log\left(x\right)+\frac{\log\left(x^{2}\right)}{2}+\mathrm{Ci}\left(x\right)-\frac{\sin\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x**2,x)
[Out] -log(x) + log(x**2)/2 + Ci(x) - sin(x)/x
```

3.105 $\int \frac{\sin^2(x)}{x} \, dx$

Optimal. Leaf size=15

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

```
[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3312, 3302}

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/x,x]
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{x} \, dx &= \int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right) dx \\ &= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos(2x)}{x} \, dx \\ &= -\frac{\text{Ci}(2x)}{2} + \frac{\log(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/x,x]
[Out] -1/2*CosIntegral[2*x] + Log[x]/2
```

fricas [A] time = 0.43, size = 17, normalized size = 1.13

$$-\frac{1}{4}\,\text{Ci}\,(2\,x) - \frac{1}{4}\,\text{Ci}\,(-2\,x) + \frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*cos_integral(2*x) - 1/4*cos_integral(-2*x) + 1/2*log(x)
```

giac [A] time = 1.13, size = 11, normalized size = 0.73

$$-\frac{1}{2}\,\mathrm{Ci}\left(2\,x\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*cos_integral(2*x) + 1/2*log(x)
```

maple [A] time = 0.02, size = 12, normalized size = 0.80

$$-\frac{\mathrm{Ci}\left(2x\right)}{2}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/x,x)
```

```
[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

maxima [C] time = 0.56, size = 17, normalized size = 1.13

$$-\frac{1}{4}\,\mathrm{Ei}\left(2i\,x\right)-\frac{1}{4}\,\mathrm{Ei}\left(-2i\,x\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\frac{\ln(x)}{2}-\frac{\mathrm{cosint}\left(2\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/x,x)
```

```
[Out] log(x)/2 - cosint(2*x)/2
```

sympy [A] time = 1.08, size = 10, normalized size = 0.67

$$\frac{\log\left(x\right)}{2}-\frac{\mathrm{Ci}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/x,x)
```

```
[Out] log(x)/2 - Ci(2*x)/2
```

3.106 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

```
[Out] ln(cos(x))+1/2*tan(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \tan^3(x)\,dx &= \frac{\tan^2(x)}{2} - \int \tan(x)\,dx \\ &= \log(\cos(x)) + \frac{\tan^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{\sec^2(x)}{2} + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

fricas [A] time = 0.42, size = 18, normalized size = 1.50

$$\frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 0.99, size = 16, normalized size = 1.33

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="giac")
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{\left(\tan^2(x)\right)}{2} - \frac{\ln\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
[Out] 1/2*tan(x)^2-1/2*ln(tan(x)^2+1)
```

maxima [A] time = 0.44, size = 20, normalized size = 1.67

$$-\frac{1}{2\left(\sin(x)^2 - 1\right)} + \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 0.02, size = 16, normalized size = 1.33

$$\ln\left(\cos(x)\right) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
[Out] log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)
```

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$\log\left(\cos\left(x\right)\right) + \frac{1}{2\cos^2\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3,x)
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

3.107 $\int \sin(a + bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

[Out] -cos(b*x+a)/b

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x],x]

[Out] -(Cos[a + b*x]/b)

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 2.00

$$\frac{\sin(a)\sin(bx)}{b} - \frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b*x],x]

[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b

fricas [A] time = 0.42, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a),x, algorithm="fricas")

[Out] -cos(b*x + a)/b

giac [A] time = 0.96, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a),x, algorithm="giac")

```
[Out] -cos(b*x + a)/b
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a),x)
[Out] -cos(b*x+a)/b
```

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
[Out] -cos(b*x + a)/b
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{\cos(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x),x)
[Out] -cos(a + b*x)/b
```

sympy [A] time = 0.14, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x)
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

3.108 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

```
[Out] sin(b*x+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x],x]
```

```
[Out] Sin[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{\sin(a)\cos(bx)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

fricas [A] time = 0.42, size = 10, normalized size = 1.00

$$\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

giac [A] time = 0.92, size = 10, normalized size = 1.00

$$\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

maple [A] time = 0.02, size = 11, normalized size = 1.10

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a),x)
```

```
[Out] sin(b*x+a)/b
```

maxima [A] time = 0.41, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\sin(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x),x)
```

```
[Out] sin(a + b*x)/b
```

sympy [A] time = 0.14, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for}\ b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

3.109 $\int \tan(a + bx)\, dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a + bx))}{b}$$

[Out] -ln(cos(b*x+a))/b

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Tan[a + b*x],x]

[Out] -(Log[Cos[a + b*x]]/b)

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a + bx)\, dx = -\frac{\log(\cos(a + bx))}{b}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b*x],x]

[Out] -(Log[Cos[a + b*x]]/b)

fricas [A] time = 0.45, size = 18, normalized size = 1.50

$$-\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a),x, algorithm="fricas")

[Out] -1/2*log(1/(tan(b*x + a)^2 + 1))/b

giac [A] time = 1.23, size = 13, normalized size = 1.08

$$-\frac{\log\left(|\cos\left(bx + a\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a),x, algorithm="giac")

```
[Out] -log(abs(cos(b*x + a)))/b
```

maple [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{\ln\left(\tan^2(bx+a)+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a),x)
```

```
[Out] 1/2/b*ln(1+tan(b*x+a)^2)
```

maxima [A] time = 0.42, size = 11, normalized size = 0.92

$$\frac{\log(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a))/b
```

mupad [B] time = 0.23, size = 16, normalized size = 1.33

$$\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x),x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/(2*b)
```

sympy [A] time = 0.13, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log\left(\tan^2(a+bx)+1\right)}{2b} & \text{for } b \neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))
```

3.110 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

```
[Out] ln(sin(b*x+a))/b
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.73

$$\frac{\log(\tan(a+bx)) + \log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b
```

fricas [B] time = 0.45, size = 27, normalized size = 2.45

$$\frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b
```

giac [B] time = 1.34, size = 56, normalized size = 5.09

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [B] time = 0.01, size = 29, normalized size = 2.64

$$-\frac{\ln\left(\tan^2(bx+a)+1\right)}{2b}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(b*x+a),x)
```

```
[Out] -1/2/b*ln(tan(b*x+a)^2+1)+1/b*ln(tan(b*x+a))
```

maxima [A] time = 0.42, size = 11, normalized size = 1.00

$$\frac{\log(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

mupad [B] time = 0.19, size = 28, normalized size = 2.55

$$\frac{\ln(\tan(a+b\,x))}{b}-\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(a + b*x),x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b)
```

sympy [A] time = 0.33, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log\left(\tan^2(a+bx)+1\right)}{2b}+\frac{\log(\tan(a+bx))}{b} & \text{for } b\neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0))
, (x/tan(a), True))
```

3.111 $\int \csc(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x],x]
[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)\,dx = -\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Mathematica [B] time = 0.01, size = 38, normalized size = 3.17

$$\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x],x]
[Out] -(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b
```

fricas [B] time = 0.46, size = 30, normalized size = 2.50

$$-\frac{\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

giac [B] time = 1.16, size = 51, normalized size = 4.25

$$\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}+\frac{1}{|b|}\right|\right)}{2\,|b|}-\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}-\frac{1}{|b|}\right|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*log(abs(-cos(b*x + a)
/b - 1/abs(b)))/abs(b)
```

maple [A] time = 0.02, size = 21, normalized size = 1.75

$$\frac{\ln\left(-\cot\left(bx+a\right)+\csc\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a),x)
```

```
[Out] 1/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.42, size = 26, normalized size = 2.17

$$-\frac{\log\left(\cos\left(bx+a\right)+1\right)-\log\left(\cos\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.12, size = 12, normalized size = 1.00

$$-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x),x)
```

```
[Out] -atanh(cos(a + b*x))/b
```

sympy [A] time = 0.53, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\sin\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))
```

3.112 $\int \sec(a + bx)\, dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

```
[Out] arctanh(sin(b*x+a))/b
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a + bx)\, dx = \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

fricas [B] time = 0.45, size = 28, normalized size = 2.55

$$\frac{\log\left(\sin\left(bx + a\right) + 1\right) - \log\left(-\sin\left(bx + a\right) + 1\right)}{2\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

giac [B] time = 1.39, size = 28, normalized size = 2.55

$$\frac{\log\left(|\sin\left(bx + a\right) + 1|\right) - \log\left(|\sin\left(bx + a\right) - 1|\right)}{2\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.02, size = 19, normalized size = 1.73

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.41, size = 26, normalized size = 2.36

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x),x)
```

```
[Out] atanh(sin(a + b*x))/b
```

sympy [A] time = 0.61, size = 34, normalized size = 3.09

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b} & \text{for}\ b\neq 0 \\ \frac{x}{\cos\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(b
, 0)), (x/cos(a), True))
```

3.113 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

fricas [A] time = 0.42, size = 23, normalized size = 0.92

$$\frac{bx-\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

giac [A]   time = 1.20, size = 18, normalized size = 0.72

$$\frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

maple [A]   time = 0.02, size = 27, normalized size = 1.08

$$\frac{\frac{bx}{2} - \frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A]   time = 0.42, size = 24, normalized size = 0.96

$$\frac{2\,bx + 2\,a - \sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

mupad [B]   time = 0.21, size = 18, normalized size = 0.72

$$\frac{x}{2} - \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2,x)
```

```
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

sympy [A]   time = 0.23, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2\left(a+bx\right)}{2} + \frac{x\cos^2\left(a+bx\right)}{2} - \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} & \text{for}\ b \neq 0 \\ x\sin^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

3.114 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{\cos(3(a+bx))}{12b}-\frac{3\cos(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

fricas [A] time = 0.42, size = 22, normalized size = 0.81

$$\frac{\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

giac [A] time = 1.19, size = 25, normalized size = 0.93

$$\frac{\cos\left(bx+a\right)^3}{3\,b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

maple [A] time = 0.02, size = 22, normalized size = 0.81

$$-\frac{\left(\sin^{2}\left(bx+a\right)+2\right)\cos\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x)
```

```
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

maxima [A] time = 0.42, size = 22, normalized size = 0.81

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

mupad [B] time = 0.15, size = 24, normalized size = 0.89

$$-\frac{3\,\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3,x)
```

```
[Out] -(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

sympy [A] time = 0.47, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2\cos^{3}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0 \\ x\sin^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

3.115 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] 1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2,x]
```

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\,dx &= \frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

fricas [A] time = 0.44, size = 22, normalized size = 0.88

$$\frac{bx+\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

giac [A] time = 1.06, size = 18, normalized size = 0.72

$$\frac{1}{2}\,x + \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.03, size = 27, normalized size = 1.08

$$\frac{\frac{bx}{2} + \frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A] time = 0.43, size = 22, normalized size = 0.88

$$\frac{2\,bx + 2\,a + \sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

mupad [B] time = 0.18, size = 18, normalized size = 0.72

$$\frac{x}{2} + \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2,x)
```

```
[Out] x/2 + sin(2*a + 2*b*x)/(4*b)
```

sympy [A] time = 0.23, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2\left(a+bx\right)}{2} + \frac{x\cos^2\left(a+bx\right)}{2} + \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} & \text{for } b \neq 0 \\ x\cos^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

3.116 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

fricas [A] time = 0.45, size = 21, normalized size = 0.81

$$\frac{\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

giac [A] time = 1.04, size = 22, normalized size = 0.85

$$-\frac{\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

maple [A] time = 0.03, size = 22, normalized size = 0.85

$$\frac{\left(\cos^2(bx+a)+2\right)\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3,x)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

maxima [A] time = 0.42, size = 22, normalized size = 0.85

$$-\frac{\sin(bx+a)^3-3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

mupad [B] time = 0.03, size = 24, normalized size = 0.92

$$\frac{3\sin(a+bx)-\sin(a+bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3,x)
```

```
[Out] (3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)
```

sympy [A] time = 0.48, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b\neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
0)), (x*cos(a)**3, True))
```

3.117 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

```
[Out] tan(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767, 8}

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

fricas [A] time = 0.40, size = 18, normalized size = 1.80

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

giac [**A**] time = 0.96, size = 10, normalized size = 1.00

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

maple [**A**] time = 0.02, size = 11, normalized size = 1.10

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^2,x)
```

```
[Out] tan(b*x+a)/b
```

maxima [**A**] time = 0.42, size = 10, normalized size = 1.00

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

mupad [**B**] time = 0.14, size = 10, normalized size = 1.00

$$\frac{\tan(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^2,x)
```

```
[Out] tan(a + b*x)/b
```

sympy [**A**] time = 1.20, size = 58, normalized size = 5.80

$$\begin{cases} \tilde{\infty}x & \text{for } \left(a=-\frac{\pi}{2} \vee a=-bx-\frac{\pi}{2}\right) \wedge \left(a=-bx-\frac{\pi}{2} \vee b=0\right) \\ \frac{x}{\cos^2(a)} & \text{for } b=0 \\ -\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)-b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a, -
b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 + b
*x/2)**2 - b), True))
```

3.118 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] sin(x)/(1+cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

fricas [A] time = 0.44, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

giac [B] time = 1.17, size = 30, normalized size = 3.33

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

maple [A] time = 0.02, size = 5, normalized size = 0.56

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+1),x)
```

```
[Out] tan(1/2*x)
```

maxima [A] time = 0.41, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

mupad [B] time = 0.17, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1),x)
```

```
[Out] tan(x/2)
```

sympy [A] time = 0.19, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

3.119 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 0.98, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.02, size = 9, normalized size = 0.75

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
[Out] -1/tan(1/2*x)
```

maxima [A] time = 0.42, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

mupad [B] time = 0.17, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

sympy [A] time = 0.34, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

3.120 $\int \frac{1}{1+\sin(x)}\, dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\sin(x)}\, dx = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.30

$$\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

fricas [A] time = 0.41, size = 18, normalized size = 1.80

$$-\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

giac [A] time = 1.04, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

maple [A] time = 0.02, size = 11, normalized size = 1.10

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+1),x)
```

```
[Out] -2/(tan(1/2*x)+1)
```

maxima [A] time = 0.42, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 1),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

sympy [A] time = 0.36, size = 8, normalized size = 0.80

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

3.121 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

fricas [A] time = 0.42, size = 17, normalized size = 1.55

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

giac [A] time = 1.24, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

maple [A] time = 0.04, size = 11, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(1/2*x)-1)
```

maxima [A] time = 0.42, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

sympy [A] time = 0.37, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

3.122 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] 2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)\right)\\
&= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

fricas [A] time = 0.44, size = 148, normalized size = 3.70

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\left(a^2-b^2\right)},-\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

giac [A] time = 1.08, size = 48, normalized size = 1.20

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.38, size = 45, normalized size = 1.12

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^
(1/2)
```

sympy [A] time = 8.69, size = 133, normalized size = 3.32

$$\begin{cases} \frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((2*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2)))
, (-2*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(t
an(x/2))/b, Eq(a, 0)), (log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a*
*2 + b**2) - log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2),
 True))
```

3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

```
[Out] -2*arctanh((b-(1-a)*tan(1/2*x))/(-a^2+b^2+1)^(1/2))/(-a^2+b^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 - a^2 + b^2
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+a+2bx+(-1+a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(1-a^2+b^2\right)-x^2}\,dx, x, 2b+2(-1+a)\tan\left(\frac{x}{2}\right)\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.94

$$\frac{2\tan^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b^
2]
```

fricas [A] time = 0.47, size = 287, normalized size = 6.11

$$\left[-\frac{\sqrt{-a^2+b^2+1}\log\left(-\frac{b^4+\left(a^2+3\right)b^2-\left(2a^2b^2-b^4-2a^2+1\right)\cos(x)^2-a^2+2\left(ab^2+a\right)\cos(x)+2\left(ab^3+ab-\left(b^3-\left(2a^2-1\right)b\right)\cos(x)\right)\sin(x)-2\left(2ab\right.}{\left(b^2-1\right)\cos(x)^2-a^2-b^2-2a\cos(x)-2\left(ab+b\cos(x\right.}\right.}{2\left(a^2-b^2-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + 1)*log(-(b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 - 2
*a^2 + 1)*cos(x)^2 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 - (
2*a^2 - 1)*b)*cos(x))*sin(x) - 2*(2*a*b*cos(x)^2 - a*b + (b^3 + b)*cos(x) -
 (b^2 - (a*b^2 - a)*cos(x) + 1)*sin(x))*sqrt(-a^2 + b^2 + 1) + 2)/((b^2 - 1
)*cos(x)^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/(a^2 - b^
2 - 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2 - b^2 - 1)/((b^3
 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x)))/sqrt(a^2 - b^2 - 1)]
```

giac [A] time = 1.19, size = 60, normalized size = 1.28

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2a-2\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b-\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2-1}}\right)\right)}{\sqrt{a^2-b^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(1
/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)
```

maple [A] time = 0.07, size = 43, normalized size = 0.91

$$\frac{2\arctan\left(\frac{2b+2(a-1)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-1)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-a^2+1>0)', see `assume?` for mo
re details)Is b^2-a^2+1 positive or negative?
```

mupad [B] time = 0.26, size = 58, normalized size = 1.23

$$\begin{cases} \frac{\ln\left(b\tan\left(\frac{x}{2}\right)+1\right)}{b} & \text{if } a=1 \\ \frac{2\,\mathrm{atan}\left(\frac{b+\tan\left(\frac{x}{2}\right)(a-1)}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}} & \text{if } a\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + cos(x) + b*sin(x)),x)
```

```
[Out] piecewise(a == 1, log(b*tan(x/2) + 1)/b, a ~= 1, (2*atan((b + tan(x/2)*(a -
 1))/(a^2 - b^2 - 1)^(1/2)))/(a^2 - b^2 - 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.124 $\int x^2 \sin^2(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

```
[Out] -1/4*x/b^2+1/6*x^3+1/4*cos(b*x+a)*sin(b*x+a)/b^3-1/2*x^2*cos(b*x+a)*sin(b*x
+a)/b+1/2*x*sin(b*x+a)^2/b^2
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$\frac{x\sin^2(a+bx)}{2b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*Cos[a + b*x
]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^2(a+bx)\,dx &= -\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}+\frac{\int x^2\,dx}{2}-\frac{\int \sin^2(a+bx)\,dx}{2b^2}\\ &= \frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{\int 1\,dx}{4b^2}\\ &= -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.64

$$\frac{\left(3-6b^2x^2\right)\sin(2(a+bx))-6bx\cos(2(a+bx))+4b^3x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(24
*b^3)
```

fricas [A] time = 0.43, size = 54, normalized size = 0.74

$$\frac{2\,b^3x^3-6\,bx\cos\left(bx+a\right)^2-3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) + 3*b*x)/b^3
```

giac [A] time = 1.18, size = 45, normalized size = 0.62

$$\frac{1}{6}\,x^3-\frac{x\cos\left(2\,bx+2\,a\right)}{4\,b^2}-\frac{\left(2\,b^2x^2-1\right)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

maple [B] time = 0.02, size = 158, normalized size = 2.16

$$\frac{\left(\frac{bx}{2}-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{a}{2}\right)a^2+\frac{bx}{4}-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{a}{4}-2\left(\frac{\left(\sin^2(bx+a)\right)}{4}+(bx+a)\left(\frac{bx}{2}-\frac{\cos(b}{}\right.\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(1/2*b*x-1/2*cos(b*x+a)*sin(b*x+a)+1/2*a)-1/2*(b*x+a)*cos(
b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)
*(1/2*b*x-1/2*cos(b*x+a)*sin(b*x+a)+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+
a^2*(1/2*b*x-1/2*cos(b*x+a)*sin(b*x+a)+1/2*a))
```

maxima [A] time = 0.44, size = 117, normalized size = 1.60

$$\frac{4\left(bx+a\right)^3+6\left(2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)\right)a^2-6\left(2\left(bx+a\right)^2-2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\right)a-}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a - 6*(b*x + a)*cos
(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

mupad [B] time = 0.14, size = 52, normalized size = 0.71

$$\frac{x^3}{6}+\frac{\sin\left(2\,a+2\,b\,x\right)}{8\,b^3}-\frac{x\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*x)^2,x)
```

```
[Out] x^3/6 + sin(2*a + 2*b*x)/(8*b^3) - (x*cos(2*a + 2*b*x))/(4*b^2) - (x^2*sin(
2*a + 2*b*x))/(4*b)
```

sympy [A] time = 1.13, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{4b^2}-\frac{x\cos^2(a+bx)}{4b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b\neq 0 \\ \frac{x^3\sin^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(4*
b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3, T
rue))
```

3.125 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

[Out] 1/2*sin(x)+1/6*sin(3*x)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

fricas [A] time = 0.44, size = 12, normalized size = 0.80

$$\frac{1}{3}\left(2\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")

[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)

giac [A] time = 0.97, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

maple [A] time = 0.04, size = 12, normalized size = 0.80

$$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x)
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

maxima [A] time = 0.44, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.60

$$\sin(x) - \frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(x),x)
[Out] sin(x) - (2*sin(x)^3)/3
```

sympy [A] time = 0.56, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

3.126 $\int x^2 \cos^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

```
[Out] -1/4*x/b^2+1/6*x^3+1/2*x*cos(b*x+a)^2/b^2-1/4*cos(b*x+a)*sin(b*x+a)/b^3+1/2
*x^2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$\frac{x\cos^2(a+bx)}{2b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*Sin[a + b*x
])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^2(a+bx)\,dx &= \frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int x^2\,dx}{2}-\frac{\int \cos^2(a+bx)\,dx}{2b^2}\\ &= \frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}-\frac{\int 1\,dx}{4b^2}\\ &= -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.64

$$\frac{(6b^2x^2-3)\sin(2(a+bx))+6bx\cos(2(a+bx))+4b^3x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(2
4*b^3)
```

fricas [A] time = 0.44, size = 54, normalized size = 0.74

$$\frac{2\,b^3x^3+6\,bx\cos\left(bx+a\right)^2+3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) - 3*b*x)/b^3
```

giac [A] time = 0.98, size = 45, normalized size = 0.62

$$\frac{1}{6}\,x^3+\frac{x\cos\left(2\,bx+2\,a\right)}{4\,b^2}+\frac{\left(2\,b^2x^2-1\right)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

maple [B] time = 0.02, size = 158, normalized size = 2.16

$$\frac{\left(\frac{bx}{2}+\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{a}{2}\right)a^2-\frac{bx}{4}+\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}-\frac{\cos(bx+a)\sin(bx+a)}{4}-\frac{a}{4}-2\left(-\frac{\left(\sin^2(bx+a)\right)}{4}+(bx+a)\left(\frac{bx}{2}+\right.\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(1/2*b*x+1/2*cos(b*x+a)*sin(b*x+a)+1/2*a)+1/2*(b*x+a)*cos(
b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)
*(1/2*b*x+1/2*cos(b*x+a)*sin(b*x+a)+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+
a^2*(1/2*b*x+1/2*cos(b*x+a)*sin(b*x+a)+1/2*a))
```

maxima [A] time = 0.46, size = 113, normalized size = 1.55

$$\frac{4\left(bx+a\right)^3+6\left(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)\right)a^2-6\left(2\left(bx+a\right)^2+2\left(bx+a\right)\sin\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a + 6*(b*x + a)*cos
(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

mupad [B] time = 0.10, size = 52, normalized size = 0.71

$$\frac{x^3}{6}-\frac{\sin(2\,a+2\,b\,x)}{8\,b^3}+\frac{x\,\cos(2\,a+2\,b\,x)}{4\,b^2}+\frac{x^2\,\sin(2\,a+2\,b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x)^2,x)
```

```
[Out] x^3/6 - sin(2*a + 2*b*x)/(8*b^3) + (x*cos(2*a + 2*b*x))/(4*b^2) + (x^2*sin(
2*a + 2*b*x))/(4*b)
```

sympy [A] time = 1.11, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x\sin^2(a+bx)}{4b^2}+\frac{x\cos^2(a+bx)}{4b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\cos^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(4*
b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3, T
rue))
```

3.127 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

[Out] -1/2*cot(x)^2-ln(sin(x))

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3475}

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3,x]

[Out] -Cot[x]^2/2 - Log[Sin[x]]

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\,dx &= -\frac{1}{2}\cot^2(x)-\int \cot(x)\,dx\\ &= -\frac{1}{2}\cot^2(x)-\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^3,x]

[Out] -1/2*Csc[x]^2 - Log[Sin[x]]

fricas [B] time = 0.44, size = 31, normalized size = 2.21

$$-\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2+\tan(x)^2+1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x, algorithm="fricas")

[Out] -1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2

giac [B] time = 1.19, size = 29, normalized size = 2.07

$$\frac{\tan(x)^2 - 1}{2\tan(x)^2} + \frac{1}{2}\log\left(\tan(x)^2 + 1\right) - \frac{1}{2}\log\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x, algorithm="giac")

[Out] 1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)

maple [A] time = 0.01, size = 22, normalized size = 1.57

$$\frac{\ln\left(\tan^2(x) + 1\right)}{2} - \ln(\tan(x)) - \frac{1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^3,x)

[Out] 1/2*ln(tan(x)^2+1)-1/2/tan(x)^2-ln(tan(x))

maxima [A] time = 0.41, size = 14, normalized size = 1.00

$$-\frac{1}{2\sin(x)^2} - \frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x, algorithm="maxima")

[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)

mupad [B] time = 0.17, size = 21, normalized size = 1.50

$$\frac{\ln\left(\tan(x)^2 + 1\right)}{2} - \ln(\tan(x)) - \frac{1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^3,x)

[Out] log(tan(x)^2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)^2)

sympy [A] time = 0.09, size = 14, normalized size = 1.00

$$-\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)**3,x)

[Out] -log(sin(x)) - 1/(2*sin(x)**2)

3.128 $\int x^3 \tan^4(x)\,dx$

Optimal. Leaf size=104

$$4ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-2\,\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)-\frac{1}{2}x^2\,\text{ta}$$

```
[Out] -1/2*x^2+4/3*I*x^3+1/4*x^4-4*x^2*ln(1+exp(2*I*x))+ln(cos(x))+4*I*x*polylog(
2,-exp(2*I*x))-2*polylog(3,-exp(2*I*x))+x*tan(x)-x^3*tan(x)-1/2*x^2*tan(x)^
2+1/3*x^3*tan(x)^3
```

Rubi [A] time = 0.23, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3720, 3475, 30, 3719, 2190, 2531, 2282, 6589}

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)+\frac{1}{3}x^3\tan^3(x)-\frac{1}{2}x^2\tan^2(x)-x^3\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^4,x]
```

```
[Out] -x^2/2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] +
 (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] + x*Tan[x] -
 x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3\tan^4(x)\,dx &= \frac{1}{3}x^3\tan^3(x) - \int x^3\tan^2(x)\,dx - \int x^2\tan^3(x)\,dx\\
&= -x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) + 3\int x^2\tan(x)\,dx + \int x^3\,dx + \int x^2\tan(x)\,dx + \int x\\
&= \frac{4ix^3}{3} + \frac{x^4}{4} + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) - 2i\int\frac{e^{2ix}x^2}{1+e^{2ix}}\,dx - 6i\int\frac{e^{2ix}x}{1+e^{}}\\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x\\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan(x) - \frac{1}{2}\\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan(x) - \frac{1}{2}\\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) - 2\mathrm{Li}_3\left(-e^{2ix}\right) + x\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.15, size = 101, normalized size = 0.97

$$4ix\,\mathrm{PolyLog}\left(2,-e^{2ix}\right) - 2\,\mathrm{PolyLog}\left(3,-e^{2ix}\right) + \frac{x^4}{4} + \frac{4ix^3}{3} - \frac{4}{3}x^3\tan(x) + \frac{1}{3}x^3\tan(x)\sec^2(x) - 4x^2\log\left(1+e^{2ix}\right) - \frac{1}{2}x^2\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^4,x]
```

```
[Out] ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x*
PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2 +
x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3
```

fricas [C] time = 0.46, size = 182, normalized size = 1.75

$$\frac{1}{3}x^3\tan(x)^3 + \frac{1}{4}x^4 - \frac{1}{2}x^2\tan(x)^2 - \frac{1}{2}x^2 - 2ix\,\mathrm{Li}_2\left(\frac{2(i\tan(x)-1)}{\tan(x)^2+1}+1\right) + 2ix\,\mathrm{Li}_2\left(\frac{2(-i\tan(x)-1)}{\tan(x)^2+1}+1\right) - \frac{1}{2}\left(4x^2-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*tan(x)^3 + 1/4*x^4 - 1/2*x^2*tan(x)^2 - 1/2*x^2 - 2*I*x*dilog(2*(I*
tan(x) - 1)/(tan(x)^2 + 1) + 1) + 2*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 +
 1) + 1) - 1/2*(4*x^2 - 1)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*(4*x
^2 - 1)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - (x^3 - x)*tan(x) - polylog
(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) - polylog(3, (tan(x)^2 - 2*
I*tan(x) - 1)/(tan(x)^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan(x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^4, x)
```

maple [A] time = 0.09, size = 138, normalized size = 1.33

$$\frac{x^4}{4}+\frac{8ix^3}{3}-4x^2\ln\left(e^{2ix}+1\right)+4ix\operatorname{polylog}\left(2,-e^{2ix}\right)-\frac{2i\left(6x^2e^{2ix}+6x^2e^{4ix}+4x^2-3ix\,e^{2ix}-3ix\,e^{4ix}-6\,e^{2ix}-3\right.}{3\left(e^{2ix}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^4,x)
```

```
[Out] 1/4*x^4-2/3*I*x*(6*x^2*exp(4*I*x)+6*x^2*exp(2*I*x)-3*exp(4*I*x)-3*I*x*exp(4
*I*x)+4*x^2-6*exp(2*I*x)-3*I*x*exp(2*I*x)-3)/(exp(2*I*x)+1)^3-2*ln(exp(I*x)
)+ln(exp(2*I*x)+1)+8/3*I*x^3-4*x^2*ln(exp(2*I*x)+1)+4*I*x*polylog(2,-exp(2*
I*x))-2*polylog(3,-exp(2*I*x))
```

maxima [B] time = 1.26, size = 485, normalized size = 4.66

$$-\frac{3i\,x^4+\left(48\,x^2+12\left(4\,x^2-1\right)\cos\left(6\,x\right)+36\left(4\,x^2-1\right)\cos\left(4\,x\right)+36\left(4\,x^2-1\right)\cos\left(2\,x\right)+\left(48i\,x^2-12i\right)\sin\left(6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -(3*I*x^4 + (48*x^2 + 12*(4*x^2 - 1)*cos(6*x) + 36*(4*x^2 - 1)*cos(4*x) + 3
6*(4*x^2 - 1)*cos(2*x) + (48*I*x^2 - 12*I)*sin(6*x) + (144*I*x^2 - 36*I)*si
n(4*x) + (144*I*x^2 - 36*I)*sin(2*x) - 12)*arctan2(sin(2*x), cos(2*x) + 1)
+ (3*I*x^4 - 32*x^3 + 24*x)*cos(6*x) + (9*I*x^4 - 48*x^3 - 24*I*x^2 + 48*x)
*cos(4*x) + (9*I*x^4 - 48*x^3 - 24*I*x^2 + 24*x)*cos(2*x) - (48*x*cos(6*x)
+ 144*x*cos(4*x) + 144*x*cos(2*x) + 48*I*x*sin(6*x) + 144*I*x*sin(4*x) + 14
4*I*x*sin(2*x) + 48*x)*dilog(-e^(2*I*x)) + (-24*I*x^2 + (-24*I*x^2 + 6*I)*c
os(6*x) + (-72*I*x^2 + 18*I)*cos(4*x) + (-72*I*x^2 + 18*I)*cos(2*x) + 6*(4*
x^2 - 1)*sin(6*x) + 18*(4*x^2 - 1)*sin(4*x) + 18*(4*x^2 - 1)*sin(2*x) + 6*I
)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (-24*I*cos(6*x) - 72*I*co
s(4*x) - 72*I*cos(2*x) + 24*sin(6*x) + 72*sin(4*x) + 72*sin(2*x) - 24*I)*po
lylog(3, -e^(2*I*x)) - (3*x^4 + 32*I*x^3 - 24*I*x)*sin(6*x) - (9*x^4 + 48*I
*x^3 - 24*x^2 - 48*I*x)*sin(4*x) - (9*x^4 + 48*I*x^3 - 24*x^2 - 24*I*x)*sin
(2*x))/(-12*I*cos(6*x) - 36*I*cos(4*x) - 36*I*cos(2*x) + 12*sin(6*x) + 36*s
in(4*x) + 36*sin(2*x) - 12*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\mathrm{tan}\left(x\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^4,x)
```

```
[Out] int(x^3*tan(x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^4(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**4,x)
```

```
[Out] Integral(x**3*tan(x)**4, x)
```

3.129 $\int x^3 \tan^6(x)\,dx$

Optimal. Leaf size=153

$$-\frac{23}{5} ix \operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10} \operatorname{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{19x^2}{20}+\frac{23}{5}$$

```
[Out] 19/20*x^2-23/15*I*x^3-1/4*x^4+23/5*x^2*ln(1+exp(2*I*x))-2*ln(cos(x))-23/5*I
*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))-19/10*x*tan(x)+x^3*t
an(x)-1/20*tan(x)^2+4/5*x^2*tan(x)^2+1/10*x*tan(x)^3-1/3*x^3*tan(x)^3-3/20*
x^2*tan(x)^4+1/5*x^3*tan(x)^5
```

Rubi [A] time = 0.41, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3720, 3473, 3475, 30, 3719, 2190, 2531, 2282, 6589}

$$-\frac{23}{5} ix\operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\operatorname{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)+\frac{1}{5}x^3\tan^5(x)-\frac{3}{20}x^2\ \text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^6,x]
```

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*x)])/5 - 2
*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] + (23*PolyLog[3, -E^((
2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x] - Tan[x]^2/20 + (4*x^2*Tan[x]^
2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[
x]^5)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3\tan^6(x)\,dx &= \frac{1}{5}x^3\tan^5(x)-\frac{3}{5}\int x^2\tan^5(x)\,dx-\int x^3\tan^4(x)\,dx\\
&= -\frac{1}{3}x^3\tan^3(x)-\frac{3}{20}x^2\tan^4(x)+\frac{1}{5}x^3\tan^5(x)+\frac{3}{10}\int x\tan^4(x)\,dx+\frac{3}{5}\int x^2\tan^3(x)\,dx+\int x^3\\
&= x^3\tan(x)+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{1}{3}x^3\tan^3(x)-\frac{3}{20}x^2\tan^4(x)+\frac{1}{5}x^3\tan^5(x)-\frac{1}{10}\int \mathrm{ta}\\
&= -\frac{23ix^3}{15}-\frac{x^4}{4}-\frac{19}{10}x\tan(x)+x^3\tan(x)-\frac{\tan^2(x)}{20}+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{1}{3}x^3\tan^3(x)\\
&= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{19}{10}x\tan(x)+x^3\tan(x)-\frac{\tan^2(x)}{20}\\
&= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\mathrm{Li}_2\left(-e^{2ix}\right)-\frac{19}{10}x\tan(x)+x^3\\
&= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\mathrm{Li}_2\left(-e^{2ix}\right)-\frac{19}{10}x\tan(x)+x^3\\
&= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\mathrm{Li}_2\left(-e^{2ix}\right)+\frac{23}{10}\mathrm{Li}_3\left(-e^{2ix}\right)-
\end{aligned}
$$

Mathematica [A] time = 0.33, size = 133, normalized size = 0.87

$$\frac{1}{60}\left(-276ix\,\mathrm{PolyLog}\left(2,-e^{2ix}\right)+138\,\mathrm{PolyLog}\left(3,-e^{2ix}\right)-15x^4-92ix^3+92x^3\tan(x)+12x^3\tan(x)\sec^4(x)-44$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^6,x]
```

```
[Out] ((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Cos[x]] - (2
76*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((2*I)*x)] - 3*Sec[x]^
2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*Tan[x] + 92*x^3*Tan[x] + 6*x*S
ec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Tan[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

fricas [C] time = 0.45, size = 212, normalized size = 1.39

$$\frac{1}{5}\,x^{3}\tan(x)^{5}-\frac{3}{20}\,x^{2}\tan(x)^{4}-\frac{1}{4}\,x^{4}-\frac{1}{30}\left(10\,x^{3}-3\,x\right)\tan(x)^{3}+\frac{1}{20}\left(16\,x^{2}-1\right)\tan(x)^{2}+\frac{19}{20}\,x^{2}+\frac{23}{10}i\,x\mathrm{Li}_2\left(\frac{2\,(i\,\tan(}{\tan(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x)
^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x) -
 1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1)
+ 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(23*
x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x)*tan
(x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) + 23/20*
polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\tan(x)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^6, x)
```

maple [A] time = 0.10, size = 237, normalized size = 1.55

$$-\frac{x^{4}}{4}-\frac{46ix^{3}}{15}+\frac{23x^{2}\ln\left(\mathrm{e}^{2ix}+1\right)}{5}-\frac{23ix\,\mathrm{polylog}\left(2,-\mathrm{e}^{2ix}\right)}{5}+\frac{23\,\mathrm{polylog}\left(3,-\mathrm{e}^{2ix}\right)}{10}-2\ln\left(\mathrm{e}^{2ix}+1\right)+4\ln\left(\mathrm{e}^{ix}\right)+\frac{i\,(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x)
```

```
[Out] -1/4*x^4+1/15*I*(-162*I*x^2*exp(4*I*x)+90*x^3*exp(8*I*x)+3*I*exp(8*I*x)+9*I
*exp(4*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)-66*I*x^2*exp(2*I*x)-162*I*x^
2*exp(6*I*x)+280*x^3*exp(4*I*x)-246*x*exp(6*I*x)+9*I*exp(6*I*x)-66*I*x^2*ex
p(8*I*x)+140*x^3*exp(2*I*x)-354*x*exp(4*I*x)+3*I*exp(2*I*x)+46*x^3-234*x*ex
p(2*I*x)-60*x)/(exp(2*I*x)+1)^5+4*ln(exp(I*x))-2*ln(exp(2*I*x)+1)-46/15*I*x
^3+23/5*x^2*ln(exp(2*I*x)+1)-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(
3,-exp(2*I*x))
```

maxima [B] time = 2.59, size = 766, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="maxima")
```

```
[Out] (15*I*x^4 + (276*x^2 + 12*(23*x^2 - 10)*cos(10*x) + 60*(23*x^2 - 10)*cos(8*
x) + 120*(23*x^2 - 10)*cos(6*x) + 120*(23*x^2 - 10)*cos(4*x) + 60*(23*x^2 -
 10)*cos(2*x) + (276*I*x^2 - 120*I)*sin(10*x) + (1380*I*x^2 - 600*I)*sin(8*
```

```
x) + (2760*I*x^2 - 1200*I)*sin(6*x) + (2760*I*x^2 - 1200*I)*sin(4*x) + (138
0*I*x^2 - 600*I)*sin(2*x) - 120)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^
4 - 184*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x +
12*I)*cos(8*x) + (150*I*x^4 - 1120*x^3 - 648*I*x^2 + 1416*x + 36*I)*cos(6*x
) + (150*I*x^4 - 720*x^3 - 648*I*x^2 + 984*x + 36*I)*cos(4*x) + (75*I*x^4 -
 360*x^3 - 264*I*x^2 + 264*x + 12*I)*cos(2*x) - (276*x*cos(10*x) + 1380*x*c
os(8*x) + 2760*x*cos(6*x) + 2760*x*cos(4*x) + 1380*x*cos(2*x) + 276*I*x*sin
(10*x) + 1380*I*x*sin(8*x) + 2760*I*x*sin(6*x) + 2760*I*x*sin(4*x) + 1380*I
*x*sin(2*x) + 276*x)*dilog(-e^(2*I*x)) + (-138*I*x^2 + (-138*I*x^2 + 60*I)*
cos(10*x) + (-690*I*x^2 + 300*I)*cos(8*x) + (-1380*I*x^2 + 600*I)*cos(6*x)
+ (-1380*I*x^2 + 600*I)*cos(4*x) + (-690*I*x^2 + 300*I)*cos(2*x) + 6*(23*x^
2 - 10)*sin(10*x) + 30*(23*x^2 - 10)*sin(8*x) + 60*(23*x^2 - 10)*sin(6*x) +
 60*(23*x^2 - 10)*sin(4*x) + 30*(23*x^2 - 10)*sin(2*x) + 60*I)*log(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (-138*I*cos(10*x) - 690*I*cos(8*x) - 13
80*I*cos(6*x) - 1380*I*cos(4*x) - 690*I*cos(2*x) + 138*sin(10*x) + 690*sin(
8*x) + 1380*sin(6*x) + 1380*sin(4*x) + 690*sin(2*x) - 138*I)*polylog(3, -e^
(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I*x^3 -
 264*x^2 - 936*I*x + 12)*sin(8*x) - (150*x^4 + 1120*I*x^3 - 648*x^2 - 1416*
I*x + 36)*sin(6*x) - (150*x^4 + 720*I*x^3 - 648*x^2 - 984*I*x + 36)*sin(4*x
) - (75*x^4 + 360*I*x^3 - 264*x^2 - 264*I*x + 12)*sin(2*x))/(-60*I*cos(10*x
) - 300*I*cos(8*x) - 600*I*cos(6*x) - 600*I*cos(4*x) - 300*I*cos(2*x) + 60*
sin(10*x) + 300*sin(8*x) + 600*sin(6*x) + 600*sin(4*x) + 300*sin(2*x) - 60*
I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(x)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x)
```

```
[Out] int(x^3*tan(x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^6(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**6,x)
```

```
[Out] Integral(x**3*tan(x)**6, x)
```

3.130 $\int x \tan^2(x)\,dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

```
[Out] -1/2*x^2+ln(cos(x))+x*tan(x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3475, 30}

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tan^2(x)\,dx &= x\tan(x)-\int x\,dx-\int \tan(x)\,dx\\ &= -\frac{x^2}{2}+\log(\cos(x))+x\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

fricas [A] time = 0.44, size = 21, normalized size = 1.40

$$-\frac{1}{2}\,x^2+x\tan(x)+\frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

giac [A] time = 1.20, size = 23, normalized size = 1.53

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 1.33

$$-\frac{x^2}{2} + x\tan(x) - \frac{\ln\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(tan(x)^2+1)
```

maxima [B] time = 0.98, size = 107, normalized size = 7.13

$$-\frac{x^2\cos(2x)^2 + x^2\sin(2x)^2 + 2x^2\cos(2x) + x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2x)^2 + \sin}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(\cos(x)) + x\tan(x) - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
```

```
[Out] log(cos(x)) + x*tan(x) - x^2/2
```

sympy [A] time = 0.17, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

3.131 $\int \cos(3x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4284}

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(2x)\,dx=\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

fricas [A] time = 0.42, size = 13, normalized size = 0.87

$$-\frac{8}{5}\cos(x)^5+2\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 2*cos(x)^3
```

giac [A] time = 1.02, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos(5x)+\frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

maple [A] time = 0.06, size = 12, normalized size = 0.80

$$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(2*x),x)
```

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

maxima [A] time = 0.41, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos(5\,x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$2\cos(x)^3 - \frac{8\cos(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(2*x),x)
```

```
[Out] 2*cos(x)^3 - (8*cos(x)^5)/5
```

sympy [B] time = 0.52, size = 26, normalized size = 1.73

$$\frac{3\sin(2x)\sin(3x)}{5} + \frac{2\cos(2x)\cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x)
```

```
[Out] 3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

3.132 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x)+\frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

fricas [A] time = 0.43, size = 19, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos(x)^3-\cos(x)\right)\sin(x)+\frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

giac [A] time = 1.12, size = 10, normalized size = 0.42

$$\frac{1}{8}x-\frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$-\frac{\left(\cos^3(x)\right)\sin(x)}{4}+\frac{\cos(x)\sin(x)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

maxima [A] time = 0.42, size = 10, normalized size = 0.42

$$\frac{1}{8}x-\frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos(x)\sin(x)^3}{4}-\frac{\cos(x)\sin(x)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

sympy [A] time = 0.07, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

3.133 $\int \csc^2(x) \sec^2(x) \, dx$

Optimal. Leaf size=7

$$\tan(x) - \cot(x)$$

```
[Out] -cot(x)+tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2620, 14}

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -Cot[x] + Tan[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned} \int \csc^2(x) \sec^2(x) \, dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2} \, dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{x^2}\right) \, dx, x, \tan(x)\right)\\ &= -\cot(x)+\tan(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -2*Cot[2*x]
```

fricas [B] time = 0.42, size = 18, normalized size = 2.57

$$-\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

giac [**A**] time = 1.12, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
[Out] -1/tan(x) + tan(x)
```

maple [**A**] time = 0.02, size = 15, normalized size = 2.14

$$-2\cot(x) + \frac{1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x)
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

maxima [**A**] time = 0.42, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/tan(x) + tan(x)
```

mupad [**B**] time = 0.06, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*sin(x)^2),x)
[Out] -2*cot(2*x)
```

sympy [**B**] time = 0.08, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
[Out] -2*cos(2*x)/sin(2*x)
```

3.134 $\int d^x \sin(x)\,dx$

Optimal. Leaf size=32

$$\frac{d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x\cos(x)}{\log^2(d)+1}$$

```
[Out] -d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Sin[x],x]
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x\sin(x)\,dx = -\frac{d^x\cos(x)}{1+\log^2(d)} + \frac{d^x\log(d)\sin(x)}{1+\log^2(d)}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.69

$$\frac{d^x(\log(d)\sin(x)-\cos(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Sin[x],x]
[Out] (d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)
```

fricas [A] time = 0.44, size = 22, normalized size = 0.69

$$\frac{\left(\log(d)\sin(x)-\cos(x)\right)d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="fricas")
[Out] (log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)
```

giac [C] time = 1.33, size = 328, normalized size = 10.25

$$|d|^x\left(\frac{\left(\pi-\pi\mathrm{sgn}(d)-2\right)\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)}{\left(\pi-\pi\mathrm{sgn}(d)-2\right)^2+4\log\left(|d|\right)^2}+\frac{2\log\left(|d|\right)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)}{\left(\pi-\pi\mathrm{sgn}(d)-2\right)^2+4\log\left(|d|\right)^2}\right)-|d|^x\left(\frac{(\pi-\pi\mathrm{s}}{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*((pi - pi*sgn(d) - 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - p
i*sgn(d) - 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*((pi - p
i*sgn(d) + 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 +
 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) + 1/2*abs(d)^x*(2*I*e^(1/2*I*pi*x*sg
n(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) +
2*I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*l
og(abs(d)) - 4*I)) + 1/2*abs(d)^x*(-2*I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x -
 I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) - 2*I*e^(-1/2*I*pi*x*
sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

maple [B] time = 0.04, size = 69, normalized size = 2.16

$$\frac{\frac{2\,\mathrm{e}^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}+\frac{\mathrm{e}^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}-\frac{\mathrm{e}^{x\ln(d)}}{\ln(d)^2+1}}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*exp(x*ln(d))+2*ln(d)
/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.43, size = 25, normalized size = 0.78

$$\frac{d^x\log(d)\sin(x)-d^x\cos(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="maxima")
```

```
[Out] (d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)
```

mupad [B] time = 0.02, size = 22, normalized size = 0.69

$$-\frac{d^x\,\left(\cos(x)-\ln(d)\,\sin(x)\right)}{{\ln(d)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*sin(x),x)
```

```
[Out] -(d^x*(cos(x) - log(d)*sin(x)))/(log(d)^2 + 1)
```

sympy [A] time = 1.04, size = 104, normalized size = 3.25

$$\begin{cases} \frac{xe^{-ix}\sin(x)}{2}-\frac{ixe^{-ix}\cos(x)}{2}-\frac{e^{-ix}\cos(x)}{2} & \text{for}\ d=e^{-i} \\ \frac{xe^{ix}\sin(x)}{2}+\frac{ixe^{ix}\cos(x)}{2}-\frac{e^{ix}\cos(x)}{2} & \text{for}\ d=e^{i} \\ \frac{d^x\log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x\cos(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*sin(x),x)

[Out] Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x)
/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I*x
)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(
x)/(log(d)**2 + 1), True))
```

3.135 $\int d^x \cos(x)\, dx$

Optimal. Leaf size=31

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

```
[Out] d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Cos[x],x]
```

```
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \cos(x)\, dx=\frac{d^x \cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x \sin(x)}{1+\log^2(d)}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.65

$$\frac{d^x(\log(d)\cos(x)+\sin(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)
```

fricas [A] time = 0.42, size = 20, normalized size = 0.65

$$\frac{\left(\cos(x)\log(d)+\sin(x)\right)d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)
```

giac [C] time = 1.18, size = 329, normalized size = 10.61

$$|d|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)\log\left(|d|\right)}{\left(\pi-\pi\mathrm{sgn}(d)-2\right)^2+4\log\left(|d|\right)^2}-\frac{\left(\pi-\pi\mathrm{sgn}(d)-2\right)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)}{\left(\pi-\pi\mathrm{sgn}(d)-2\right)^2+4\log\left(|d|\right)^2}\right)+|d|^x\left(\frac{2\cos\left(\frac{1}{2}\pi\right.}{\left(\pi-\pi s\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)*log(abs(d))/((pi - pi*sgn(d
) - 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) - 2)*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) + abs(d)^x*(2*cos(1
/2*pi*x*sgn(d) - 1/2*pi*x - x)*log(abs(d))/((pi - pi*sgn(d) + 2)^2 + 4*log(
abs(d))^2) - (pi - pi*sgn(d) + 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) - 1/2*I*abs(d)^x*(-2*I*e^(1/2*I*pi*x
*sgn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I)
 + 2*I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) +
4*log(abs(d)) - 4*I)) - 1/2*I*abs(d)^x*(-2*I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*p
i*x - I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) + 2*I*e^(-1/2*I*
pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4
*I))
```

maple [B] time = 0.03, size = 71, normalized size = 2.29

$$\frac{-\frac{e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}+\frac{e^{x\ln(d)}\ln(d)}{\ln(d)^2+1}+\frac{2\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*cos(x),x)
```

```
[Out] (ln(d)/(ln(d)^2+1)*exp(x*ln(d))+2/(ln(d)^2+1)*exp(x*ln(d))*tan(1/2*x)-ln(d)
/(ln(d)^2+1)*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.43, size = 24, normalized size = 0.77

$$\frac{d^x\cos(x)\log(d)+d^x\sin(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="maxima")
```

```
[Out] (d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)
```

mupad [B] time = 0.02, size = 20, normalized size = 0.65

$$\frac{d^x\,\left(\sin(x)+\ln(d)\,\cos(x)\right)}{{\ln(d)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*cos(x),x)
```

```
[Out] (d^x*(sin(x) + log(d)*cos(x)))/(log(d)^2 + 1)
```

sympy [A] time = 1.03, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2}+\frac{xe^{-ix}\cos(x)}{2}+\frac{ie^{-ix}\cos(x)}{2} & \text{for}\ d=e^{-i}\\ -\frac{ixe^{ix}\sin(x)}{2}+\frac{xe^{ix}\cos(x)}{2}-\frac{ie^{ix}\cos(x)}{2} & \text{for}\ d=e^{i}\\ \frac{d^x\log(d)\cos(x)}{\log(d)^2+1}+\frac{d^x\sin(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(
x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*ex
p(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x
*sin(x)/(log(d)**2 + 1), True))
```

3.136 $\int d^x x \sin(x)\, dx$

Optimal. Leaf size=84

$$\frac{xd^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2} + \frac{d^x \sin(x)}{\left(\log^2(d)+1\right)^2} - \frac{xd^x \cos(x)}{\log^2(d)+1} + \frac{2d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] 2*d^x*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x*cos(x)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(
d)^2)^2-d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4432, 4465, 4433}

$$\frac{xd^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2} + \frac{d^x \sin(x)}{\left(\log^2(d)+1\right)^2} - \frac{xd^x \cos(x)}{\log^2(d)+1} + \frac{2d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[d]^2) + (d
^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x
*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x \sin(x)\, dx &= -\frac{d^x x \cos(x)}{1+\log^2(d)} + \frac{d^x x \log(d)\sin(x)}{1+\log^2(d)} - \int \left(-\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d)\sin(x)}{1+\log^2(d)}\right) dx \\
&= -\frac{d^x x \cos(x)}{1+\log^2(d)} + \frac{d^x x \log(d)\sin(x)}{1+\log^2(d)} + \frac{\int d^x \cos(x)\, dx}{1+\log^2(d)} - \frac{\log(d)\int d^x \sin(x)\, dx}{1+\log^2(d)} \\
&= \frac{2d^x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x \cos(x)}{1+\log^2(d)} + \frac{d^x \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{d^x \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x \log(d)\sin(x)}{1+\log^2(d)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.60

$$\frac{d^x \left(\sin(x) \left(x \log^3(d) + x \log(d) - \log^2(d) + 1\right) - \cos(x) \left(x \log^2(d) - 2 \log(d) + x\right)\right)}{\left(\log^2(d) + 1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x*
Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

fricas [A] time = 0.43, size = 60, normalized size = 0.71

$$-\frac{\left(x \cos(x) \log(d)^2 + x \cos(x) - 2 \cos(x) \log(d) - \left(x \log(d)^3 + x \log(d) - \log(d)^2 + 1\right) \sin(x)\right) d^x}{\log(d)^4 + 2 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="fricas")
```

```
[Out] -(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 + x*log(d) -
 log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

giac [C] time = 1.28, size = 1166, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*x
*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*
sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(abs(
d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2
 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*p
i*x*sgn(d) - 1/2*pi*x + x) + 2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*
sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*lo
g(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)
) + 2*log(abs(d)))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi
*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*lo
g(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((2*pi
- pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi*x*sgn(d) - pi
*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^
2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi*l
og(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2
*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log
(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) -
 1/2*pi*x - x) - 2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*
log(abs(d)) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2
- 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(ab
s(d)))^2) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)
*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*s
gn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/2*abs(d)^x*((2*pi*x*sg
n(d) - 2*pi*x - 4*I*x*log(abs(d)) + 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I
*pi*x + I*x)/(8*pi + 4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) - 4*pi^2 - 8
*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) - 8) -
```

```
(2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(abs(d)) + 4*x - 4*I)*e^(-1/2*I*pi*x*sg
n(d) + 1/2*I*pi*x - I*x)/(8*pi + 4*pi^2*sgn(d) - 8*I*pi*log(abs(d))*sgn(d)
- 4*pi^2 + 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(ab
s(d)) - 8)) - 1/2*abs(d)^x*((2*pi*x*sgn(d) - 2*pi*x - 4*I*x*log(abs(d)) - 4
*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(8*pi - 4*pi^2*sgn(d) -
8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 + 8*I*pi*log(abs(d)) - 8*log(abs(d))^2 -
 8*pi*sgn(d) + 16*I*log(abs(d)) + 8) - (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(
abs(d)) - 4*x - 4*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(8*pi - 4*pi
^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 - 8*I*pi*log(abs(d)) - 8*log
(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(abs(d)) + 8))
```

maple [A] time = 0.04, size = 137, normalized size = 1.63

$$\frac{\frac{2x\,e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}+\frac{x\,e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}-\frac{2\,e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(\ln(d)^2+1\right)^2}-\frac{x\,e^{x\ln(d)}}{\ln(d)^2+1}+\frac{2\,e^{x\ln(d)}\ln(d)}{\left(\ln(d)^2+1\right)^2}-\frac{2\left(\ln(d)^2-1\right)e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(\ln(d)^2+1\right)^2}}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*sin(x),x)
```

```
[Out] (1/(ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)^2+2/(ln(d)^2+1)^2*ln(d)*exp(x*ln(d
))-1/(ln(d)^2+1)*x*exp(x*ln(d))-2/(ln(d)^2+1)^2*ln(d)*exp(x*ln(d))*tan(1/2*
x)^2-2*(ln(d)^2-1)/(ln(d)^2+1)^2*exp(x*ln(d))*tan(1/2*x)+2*ln(d)/(ln(d)^2+1
)*x*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.49, size = 60, normalized size = 0.71

$$-\frac{\left(\left(\log(d)^2+1\right)x-2\log(d)\right)d^x\cos(x)-\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\sin(x)}{\log(d)^4+2\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))*x - log(d
)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

mupad [B] time = 0.28, size = 57, normalized size = 0.68

$$\frac{d^x\left(\sin(x)+2\ln(d)\cos(x)-\ln(d)^2\sin(x)-x\cos(x)+x\ln(d)\sin(x)-x\ln(d)^2\cos(x)+x\ln(d)^3\sin(x)\right)}{\left(\ln(d)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*sin(x),x)
```

```
[Out] (d^x*(sin(x) + 2*log(d)*cos(x) - log(d)^2*sin(x) - x*cos(x) + x*log(d)*sin(
x) - x*log(d)^2*cos(x) + x*log(d)^3*sin(x)))/(log(d)^2 + 1)^2
```

sympy [A] time = 3.28, size = 308, normalized size = 3.67

$$\begin{cases} \frac{x^2e^{-ix}\sin(x)}{4}-\frac{ix^2e^{-ix}\cos(x)}{4}+\frac{ixe^{-ix}\sin(x)}{4}-\frac{xe^{-ix}\cos(x)}{4}+\frac{ie^{-ix}\cos(x)}{4} \\ \frac{x^2e^{ix}\sin(x)}{4}+\frac{ix^2e^{ix}\cos(x)}{4}-\frac{ixe^{ix}\sin(x)}{4}-\frac{xe^{ix}\cos(x)}{4}-\frac{ie^{ix}\cos(x)}{4} \\ \frac{d^xx\log(d)^3\sin(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^xx\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1}+\frac{d^xx\log(d)\sin(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^xx\cos(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1}+\frac{2d^x\log(d)\text{co}}{\log(d)^4+2\log(}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*sin(x),x)
```

```
[Out] Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-I
*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I)))
, (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(x)/
4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(
d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d
)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1
) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x)/(lo
g(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2
+ 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

3.137 $\int d^x x \cos(x) \, dx$

Optimal. Leaf size=83

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] d^x*cos(x)/(1+ln(d)^2)^2-d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x*cos(x)*ln(d
)/(1+ln(d)^2)-2*d^x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Cos[x],x]
```

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d
^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x\cos(x)\,dx &= \frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\int\left(\frac{d^x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x\sin(x)}{1+\log^2(d)}\right)dx\\
&=\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\frac{\int d^x\sin(x)\,dx}{1+\log^2(d)}-\frac{\log(d)\int d^x\cos(x)\,dx}{1+\log^2(d)}\\
&=\frac{d^x\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\sin(x)}{1+\log^2(d)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.59

$$\frac{d^x\left(\sin(x)\left(x\log^2(d)-2\log(d)+x\right)+\cos(x)\left(x\log^3(d)+x\log(d)-\log^2(d)+1\right)\right)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[d] + x*Log
[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

fricas [A] time = 0.44, size = 58, normalized size = 0.70

$$\frac{\left(x\cos(x)\log(d)^3+x\cos(x)\log(d)-\cos(x)\log(d)^2+\left(x\log(d)^2+x-2\,\log(d)\right)\sin(x)+\cos(x)\right)d^x}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="fricas")
```

```
[Out] (x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d)^2 + x -
2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

giac [C] time = 1.29, size = 1165, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d))
+ 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d
) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2) +
(2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(x*log(abs(
d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2
 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*p
i*x*sgn(d) - 1/2*pi*x + x) - ((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2
- 2*pi*sgn(d) - 2)*(pi*x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 +
 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(a
bs(d)) + 2*log(abs(d)))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*
log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*lo
g(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(2*((pi*
x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(
d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4
*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) - (2*pi - pi^2
*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(x*log(abs(d)) - 1)/((2
*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log
(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) -
 1/2*pi*x - x) + ((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d
) + 2)*(pi*x*sgn(d) - pi*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d
))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*l
og(abs(d)))^2) + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))
*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*s
gn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/2*I*abs(d)^x*((2*pi*x*
sgn(d) - 2*pi*x - 4*I*x*log(abs(d)) + 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2
*I*pi*x + I*x)/(8*pi + 4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) - 4*pi^2 -
 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) - 8)
```

```
 + (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(abs(d)) + 4*x - 4*I)*e^(-1/2*I*pi*x*
sgn(d) + 1/2*I*pi*x - I*x)/(8*pi + 4*pi^2*sgn(d) - 8*I*pi*log(abs(d))*sgn(d
) - 4*pi^2 + 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(
abs(d)) - 8)) + 1/2*I*abs(d)^x*((2*pi*x*sgn(d) - 2*pi*x - 4*I*x*log(abs(d))
 - 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(8*pi - 4*pi^2*sgn(d
) - 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 + 8*I*pi*log(abs(d)) - 8*log(abs(d))
^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) + 8) + (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*
log(abs(d)) - 4*x - 4*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(8*pi -
4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 - 8*I*pi*log(abs(d)) - 8
*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(abs(d)) + 8))
```

maple [A] time = 0.04, size = 142, normalized size = 1.71

$$\frac{-\frac{x\,e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}+\frac{x\,e^{x\ln(d)}\ln(d)}{\ln(d)^2+1}+\frac{2x\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}-\frac{4\,e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\left(\ln(d)^2+1\right)^2}+\frac{\left(\ln(d)^2-1\right)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(\ln(d)^2+1\right)^2}-\frac{\left(\ln(d)^2-1\right)e^{x\ln(d)}}{\left(\ln(d)^2+1\right)^2}}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*cos(x),x)
```

```
[Out] ((ln(d)^2-1)/(ln(d)^2+1)^2*exp(x*ln(d))*tan(1/2*x)^2+ln(d)/(ln(d)^2+1)*x*ex
p(x*ln(d))-(ln(d)^2-1)/(ln(d)^2+1)^2*exp(x*ln(d))-4/(ln(d)^2+1)^2*ln(d)*exp
(x*ln(d))*tan(1/2*x)+2/(ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)-ln(d)/(ln(d)^2
+1)*x*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.47, size = 58, normalized size = 0.70

$$\frac{\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\cos(x)+\left(\left(\log(d)^2+1\right)x-2\,\log(d)\right)d^x\sin(x)}{\log(d)^4+2\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 + 1)*x - 2*
log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

mupad [B] time = 0.21, size = 55, normalized size = 0.66

$$\frac{d^x\,\left(\cos(x)-2\,\ln(d)\,\sin(x)-\ln(d)^2\,\cos(x)+x\,\sin(x)+x\,\ln(d)\,\cos(x)+x\ln(d)^3\,\cos(x)+x\ln(d)^2\,\sin(x)\right)}{\left(\ln(d)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*cos(x),x)
```

```
[Out] (d^x*(cos(x) - 2*log(d)*sin(x) - log(d)^2*cos(x) + x*sin(x) + x*log(d)*cos(
x) + x*log(d)^3*cos(x) + x*log(d)^2*sin(x)))/(log(d)^2 + 1)^2
```

sympy [A] time = 3.24, size = 304, normalized size = 3.66

$$\begin{cases} \frac{ix^2e^{-ix}\sin(x)}{4}+\frac{x^2e^{-ix}\cos(x)}{4}+\frac{xe^{-ix}\sin(x)}{4}+\frac{ixe^{-ix}\cos(x)}{4}+\frac{e^{-ix}\cos(x)}{4} \\ -\frac{ix^2e^{ix}\sin(x)}{4}+\frac{x^2e^{ix}\cos(x)}{4}+\frac{xe^{ix}\sin(x)}{4}-\frac{ixe^{ix}\cos(x)}{4}+\frac{e^{ix}\cos(x)}{4} \\ \frac{d^xx\log(d)^3\cos(x)}{\log(d)^4+2\log(d)^2+1}+\frac{d^xx\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1}+\frac{d^xx\log(d)\cos(x)}{\log(d)^4+2\log(d)^2+1}+\frac{d^xx\sin(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^x\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1}-\frac{2d^x\log(d)\sin}{\log(d)^4+2\log(}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*cos(x),x)
```

```
[Out] Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x
)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))),
(-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 -
 I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)*
*3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(d)**
4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) +
 d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(log(d
)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1
) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

3.138 $\int d^x x^2 \sin(x)\,dx$

Optimal. Leaf size=162

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+$$

```
[Out] 2*d^x*cos(x)/(1+ln(d)^2)^3-6*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+4*d^x*x*cos(x
)*ln(d)/(1+ln(d)^2)^2-d^x*x^2*cos(x)/(1+ln(d)^2)-6*d^x*ln(d)*sin(x)/(1+ln(d
)^2)^3+2*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+2*d^x*x*sin(x)/(1+ln(d)^2)^2-2*d^
x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3
+ (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*Cos[x])/(1 + Log[d]^2
) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*Log[d]^3*Sin[x])/(1 + L
og[d]^2)^3 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/
(1 + Log[d]^2)^2 + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^2 \sin(x)\,dx = -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int x\left(-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}\right)dx$$
$$= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int\left(-\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}\right)dx$$
$$= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}+\frac{2\int d^x x\cos(x)\,dx}{1+\log^2(d)}-\frac{(2\log(d))\int d^x x\sin(x)\,dx}{1+\log^2(d)}$$
$$= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)\sin(x}{1+\log^2(d)}$$
$$= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)\sin(x}{1+\log^2(d)}$$
$$= \frac{2d^x\cos(x)}{\left(1+\log^2(d)\right)^3}-\frac{2d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3}+\frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^3}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\log^3(d)+\left(x^2-6\right)\log(d)-2x\log^4(d)+2x\right)-\cos(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\right.\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[d]^3
 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*L
og[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^2)^3
```

fricas [A] time = 0.44, size = 115, normalized size = 0.71

$$-\frac{\left(x^2\cos(x)\log(d)^4-4\,x\cos(x)\log(d)^3+2\left(x^2+3\right)\cos(x)\log(d)^2-4\,x\cos(x)\log(d)+\left(x^2-2\right)\cos(x)-\left(x^2\log\right.\right.}{\log(d)^6+3\,\log(d)^4+3\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*log(d)^2 -
 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 - 2*x*log(d)^4 + 2*(x
^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*sin(x))*d^x/(log(d)^6 + 3*log(d)
^4 + 3*log(d)^2 + 1)
```

giac [C] time = 1.78, size = 2631, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi^2
*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 -
 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn
(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2
- 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*
log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))
^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d))
 - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(ab
s(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 +
(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*lo
g(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d
) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi
^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*s
gn(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(
d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*s
gn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^
2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)
))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(
d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sg
n(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d
))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(
1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*l
og(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((
3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2
- 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*l
og(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))
*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(
d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)*(3*
pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(a
bs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(
d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - (2*(3*pi - p
i^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2
*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*s
gn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/
((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^
2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2
*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d
))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^
3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi
^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*
sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d)
)*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))
*abs(d)^x + 1/2*abs(d)^x*((4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d
) - 4*I*pi^2*x^2 + 8*pi*x^2*log(abs(d)) + 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^
```

```
2*sgn(d) + 8*I*pi*x^2 - 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x - 8*I*x
^2 - 16*I*x*log(abs(d)) + 16*x + 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x +
I*x)/(24*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(
abs(d))^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 +
 16*log(abs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2
 + 48*pi*log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d))
 - 16*I) + (4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) - 4*I*pi^2*x^
2 - 8*pi*x^2*log(abs(d)) + 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*
pi*x^2 + 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x - 8*I*x^2 - 16*I*x*log
(abs(d)) - 16*x + 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(-24*I*pi
 + 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn
(d) - 8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d
))^3 - 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 + 48*pi*log(
abs(d)) - 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I)) + 1
/2*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2
*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8
*I*pi*x^2 - 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I*x^2 + 16*I*x*
log(abs(d)) + 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(24*I*p
i - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))^2*sg
n(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log(abs(
d))^3 - 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 - 48*pi*log
(abs(d)) - 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I) + (
-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2*x^2 + 8*pi*x^
2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*pi*x^2 + 16
*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*log(abs(d)) -
16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(-24*I*pi + 8*I*pi^3
*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn(d) - 8*I*p
i^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d))^3 + 24*I
*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 - 48*pi*log(abs(d)) + 4
8*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I))
```

maple [A] time = 0.06, size = 225, normalized size = 1.39

$$\frac{\frac{2x^{2}e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\ln(d)^{2}+1}+\frac{x^{2}e^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\ln(d)^{2}+1}-\frac{4x\,e^{x\ln(d)}\ln(d)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(\ln(d)^{2}+1\right)^{2}}-\frac{x^{2}e^{x\ln(d)}}{\ln(d)^{2}+1}+\frac{4x\,e^{x\ln(d)}\ln(d)}{\left(\ln(d)^{2}+1\right)^{2}}-\frac{4\left(\ln(d)^{2}-1\right)x\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(\ln(d)^{2}+1\right)^{2}}+4(\ln(\cdots}{\tan^{2}\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x)
```

```
[Out] (1/(ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)^2-1/(ln(d)^2+1)*x^2*exp(x*ln(d))
-2*(3*ln(d)^2-1)/(ln(d)^2+1)^3*exp(x*ln(d))+4/(ln(d)^2+1)^2*ln(d)*x*exp(x*l
n(d))+2*(3*ln(d)^2-1)/(ln(d)^2+1)^3*exp(x*ln(d))*tan(1/2*x)^2-4/(ln(d)^2+1)
^2*ln(d)*x*exp(x*ln(d))*tan(1/2*x)^2-4*(ln(d)^2-1)/(ln(d)^2+1)^2*x*exp(x*ln
(d))*tan(1/2*x)+2*ln(d)/(ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)+4*ln(d)*(ln
(d)^2-3)/(ln(d)^2+1)^3*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.52, size = 107, normalized size = 0.66

$$-\frac{\left(\left(\log(d)^{4}+2\log(d)^{2}+1\right)x^{2}-4\left(\log(d)^{3}+\log(d)\right)x+6\log(d)^{2}-2\right)d^{x}\cos(x)-\left(\left(\log(d)^{5}+2\log(d)^{3}+\log(\cdots\right.\right.}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 -
 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(lo
g(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
 1)
```

mupad [B] time = 0.39, size = 133, normalized size = 0.82

$$\frac{d^x\left(2\cos(x)-x^2\cos(x)+2x\sin(x)\right)+d^x\ln(d)^3\left(2\sin(x)+2x^2\sin(x)+4x\cos(x)\right)-d^x\ln(d)^2\left(6\cos(x)\right.}{(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x)
```

```
[Out] (d^x*(2*cos(x) - x^2*cos(x) + 2*x*sin(x)) + d^x*log(d)^3*(2*sin(x) + 2*x^2*
sin(x) + 4*x*cos(x)) - d^x*log(d)^2*(6*cos(x) + 2*x^2*cos(x)) + d^x*log(d)*
(x^2*sin(x) - 6*sin(x) + 4*x*cos(x)) - d^x*log(d)^4*(x^2*cos(x) + 2*x*sin(x
)) + d^x*x^2*log(d)^5*sin(x))/(log(d)^2 + 1)^3
```

sympy [B] time = 8.46, size = 665, normalized size = 4.10

$$\begin{cases} \frac{x^3e^{-ix}\sin(x)}{6}-\frac{ix^3e^{-ix}\cos(x)}{6}+\frac{ix^2e^{-ix}\sin(x)}{4}-\frac{x^2e^{-ix}\cos(x)}{4}+\frac{xe^{-ix}\sin(x)}{4}+\frac{ixe^{-ix}\cos(x)}{4}+\frac{e^{-ix}\cos(x)}{4} \\ \frac{x^3e^{ix}\sin(x)}{6}+\frac{ix^3e^{ix}\cos(x)}{6}-\frac{ix^2e^{ix}\sin(x)}{4}-\frac{x^2e^{ix}\cos(x)}{4}+\frac{xe^{ix}\sin(x)}{4}-\frac{ixe^{ix}\cos(x)}{4}+\frac{e^{ix}\cos(x)}{4} \\ \frac{d^xx^2\log(d)^5\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{d^xx^2\log(d)^4\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx^2\log(d)^3\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{2d^xx^2\log(d)^2\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*sin(x),x)
```

```
[Out] Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*exp
(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(
-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin(x)
/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*co
s(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, E
q(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log
(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
+ 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**
x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x
*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*lo
g(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*sin(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*sin(x)/(log(d
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*cos(x)/(log(d)**6
+ 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(log(d)**6 + 3*log(
d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1), True))
```

3.139 $\int d^x x^2 \cos(x)\,dx$

Optimal. Leaf size=161

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+-$$

```
[Out] -6*d^x*cos(x)*ln(d)/(1+ln(d)^2)^3+2*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^3+2*d^x*
x*cos(x)/(1+ln(d)^2)^2-2*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x^2*cos(x)*
ln(d)/(1+ln(d)^2)-2*d^x*sin(x)/(1+ln(d)^2)^3+6*d^x*ln(d)^2*sin(x)/(1+ln(d)^
2)^3-4*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x^2*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4433, 4466, 14, 4432, 4465}

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+-$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)/(1 + Log[
d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Cos[x]*Log[d]^2)/(1
+ Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Sin[x])/(1
+ Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*
Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^2\cos(x)\,dx = \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2\sin(x)}{1+\log^2(d)} - 2\int x\left(\frac{d^x\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x\sin(x)}{1+\log^2(d)}\right)dx$$
$$= \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2\sin(x)}{1+\log^2(d)} - 2\int\left(\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x\sin(x)}{1+\log^2(d)}\right)dx$$
$$= \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2\sin(x)}{1+\log^2(d)} - \frac{2\int d^x x\sin(x)\,dx}{1+\log^2(d)} - \frac{(2\log(d))\int d^x x\cos(x)\,dx}{1+\log^2(d)}$$
$$= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2 \mathrm{si}}{1+\log}$$
$$= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2 \mathrm{si}}{1+\log}$$
$$= -\frac{2d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2 \mathrm{c}}{1-}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\log^2(d)-4x\log^3(d)-4x\log(d)+x^2-2\right)+\cos(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\right.\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^4
 + x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[
d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^3
```

fricas [A] time = 0.44, size = 111, normalized size = 0.69

$$\frac{\left(x^2\cos(x)\log(d)^5-2\,x\cos(x)\log(d)^4+2\left(x^2+1\right)\cos(x)\log(d)^3+\left(x^2-6\right)\cos(x)\log(d)+2\,x\cos(x)+\left(x^2\mathrm{l}\right.\right.}{\log(d)^6+3\,\log(d)^4+3\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="fricas")
```

```
[Out] (x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*log(d)^3 +
(x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 - 4*x*log(d)^3 + 2*(x^
2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x))*d^x/(log(d)^6 + 3*log(d)^4
+ 3*log(d)^2 + 1)
```

giac [C] time = 1.52, size = 2631, normalized size = 16.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi*
x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*sgn(
d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d)
 - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x
^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2
*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
 + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs
(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (
3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log
(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d)
 - 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(
abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*l
og(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log
(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6
*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^
2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d
)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn
(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d)
)^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1
/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*((2*(3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*
log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/((3*pi - pi^3*s
gn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(
d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sg
n(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*p
i*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log
(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*p
i^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(ab
s(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d
)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((3*pi - pi^3
*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sg
n(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*l
og(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^
2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*p
i*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log
(abs(d))*sgn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi
*x + 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^
3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*s
gn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))
*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*
abs(d)^x - 1/2*I*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn
(d) + 4*I*pi^2*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 + 8*I*pi*
```

```
x^2*sgn(d) - 8*I*pi*x^2 + 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I
*x^2 + 16*I*x*log(abs(d)) - 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x
+ I*x)/(24*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*lo
g(abs(d))^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2
 + 16*log(abs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi
^2 + 48*pi*log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d
)) - 16*I) - (-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2
*x^2 + 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 + 8*I*pi*x^2*sgn(d) - 8
*I*pi*x^2 - 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*
log(abs(d)) + 16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(-24*I
*pi + 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*
sgn(d) - 8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(ab
s(d))^3 - 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 + 48*pi*l
og(abs(d)) - 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I))
- 1/2*I*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) + 4*I
*pi^2*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d
) + 8*I*pi*x^2 - 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I*x^2 + 16
*I*x*log(abs(d)) + 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(2
4*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))
^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log
(abs(d))^3 - 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 - 48*p
i*log(abs(d)) - 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I
) - (-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2*x^2 + 8*
pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*pi*x^2
 + 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*log(abs(d
)) - 16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(-24*I*pi + 8*I
*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn(d) -
8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d))^3 +
 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 - 48*pi*log(abs(d)
) + 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I))
```

maple [A] time = 0.05, size = 231, normalized size = 1.43

$$\frac{-\frac{x^2 e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}+\frac{x^2 e^{x\ln(d)}\ln(d)}{\ln(d)^2+1}+\frac{2x^2 e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}-\frac{8x\,e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\left(\ln(d)^2+1\right)^2}+\frac{2\left(\ln(d)^2-1\right)x\,e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(\ln(d)^2+1\right)^2}-\frac{2\left(\ln(d)^2-3\right)e^{\cdots}}{(\ln\cdots}}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x)
```

```
[Out] (ln(d)/(ln(d)^2+1)*x^2*exp(x*ln(d))+2/(ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*
x)-2*(ln(d)^2-1)/(ln(d)^2+1)^2*x*exp(x*ln(d))+4*(3*ln(d)^2-1)/(ln(d)^2+1)^3
*exp(x*ln(d))*tan(1/2*x)+2*ln(d)*(ln(d)^2-3)/(ln(d)^2+1)^3*exp(x*ln(d))-8/(
ln(d)^2+1)^2*ln(d)*x*exp(x*ln(d))*tan(1/2*x)+2*(ln(d)^2-1)/(ln(d)^2+1)^2*x*
exp(x*ln(d))*tan(1/2*x)^2-ln(d)/(ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)^2-2
*ln(d)*(ln(d)^2-3)/(ln(d)^2+1)^3*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1
)
```

maxima [A] time = 0.52, size = 105, normalized size = 0.65

$$\frac{\left(\left(\log(d)^5+2\,\log(d)^3+\log(d)\right)x^2+2\,\log(d)^3-2\left(\log(d)^4-1\right)x-6\,\log(d)\right)d^x\cos(x)+\left(\left(\log(d)^4+2\,\log(d)\cdots\right.\right.}{\log(d)^6+3\,\log(d)^4+3\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x -
6*log(d))*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log
```

```
(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
1)
```

mupad [B] time = 0.35, size = 132, normalized size = 0.82

$$\frac{d^x\left(x^2\sin(x)-2\sin(x)+2x\cos(x)\right)+d^x\ln(d)^3\left(2\cos(x)+2x^2\cos(x)-4x\sin(x)\right)+d^x\ln(d)^2\left(6\sin(x)+2\ldots\right.}{\left(\ln(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x)
```

```
[Out] (d^x*(x^2*sin(x) - 2*sin(x) + 2*x*cos(x)) + d^x*log(d)^3*(2*cos(x) + 2*x^2*
cos(x) - 4*x*sin(x)) + d^x*log(d)^2*(6*sin(x) + 2*x^2*sin(x)) - d^x*log(d)*
(6*cos(x) - x^2*cos(x) + 4*x*sin(x)) + d^x*log(d)^4*(x^2*sin(x) - 2*x*cos(x
)) + d^x*x^2*log(d)^5*cos(x))/(log(d)^2 + 1)^3
```

sympy [B] time = 8.47, size = 668, normalized size = 4.15

$$\begin{cases} \frac{ix^3e^{-ix}\sin(x)}{6}+\frac{x^3e^{-ix}\cos(x)}{6}+\frac{x^2e^{-ix}\sin(x)}{4}+\frac{ix^2e^{-ix}\cos(x)}{4}-\frac{ixe^{-ix}\sin(x)}{4}+\frac{xe^{-ix}\cos(x)}{4}-\frac{ie^{-ix}\cos(x)}{4} \\ -\frac{ix^3e^{ix}\sin(x)}{6}+\frac{x^3e^{ix}\cos(x)}{6}+\frac{x^2e^{ix}\sin(x)}{4}-\frac{ix^2e^{ix}\cos(x)}{4}+\frac{ixe^{ix}\sin(x)}{4}+\frac{xe^{ix}\cos(x)}{4}+\frac{ie^{ix}\cos(x)}{4} \\ \frac{d^xx^2\log(d)^5\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{d^xx^2\log(d)^4\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx^2\log(d)^3\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx^2\log(d)^2\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*cos(x),x)
```

```
[Out] Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 + x**2*exp(-
I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*x)*sin(x)/4 + x*exp(
-I*x)*cos(x)/4 - I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (-I*x**3*exp(I*x)*s
in(x)/6 + x**3*exp(I*x)*cos(x)/6 + x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(I*x)
*cos(x)/4 + I*x*exp(I*x)*sin(x)/4 + x*exp(I*x)*cos(x)/4 + I*exp(I*x)*cos(x)
/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)**4 +
3*log(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) + d**x*x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) + d**x*x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) -
2*d**x*x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d
**x*x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x
*x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*cos
(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*cos(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)**2*sin(x)/(log(d
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3
*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1), True))
```

3.140 $\int d^x x^3 \sin(x)\,dx$

Optimal. Leaf size=261

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{18xd^x \log(d)\sin}{\left(\log^2(d)+1\right)^3}$$

```
[Out] -24*d^x*cos(x)*ln(d)/(1+ln(d)^2)^4+24*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^4+6*d^
x*x*cos(x)/(1+ln(d)^2)^3-18*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+6*d^x*x^2*co
s(x)*ln(d)/(1+ln(d)^2)^2-d^x*x^3*cos(x)/(1+ln(d)^2)-6*d^x*sin(x)/(1+ln(d)^2
)^4+36*d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^4-6*d^x*ln(d)^4*sin(x)/(1+ln(d)^2)^4-
18*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^3+6*d^x*x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+3*
d^x*x^2*sin(x)/(1+ln(d)^2)^2-3*d^x*x^2*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^3
*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.43, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{6xd^x \log^3(d)\sin}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Sin[x],x]
```

```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^3)/(1 + Lo
g[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x*x*Cos[x]*Log[d]^2)/
(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^3*Co
s[x])/(1 + Log[d]^2) - (6*d^x*Sin[x])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*S
in[x])/(1 + Log[d]^2)^4 - (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^
x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]
^2)^3 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[x])/(
1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^3 \sin(x)\,dx &= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int x^2\left(-\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int\left(-\frac{d^x x^2 \cos(x)}{1+\log^2(d)} + \frac{d^x x^2 \log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} + \frac{3\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)}\\
&= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin}{1+\log^2(d)}\\
&= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin}{1+\log^2(d)}\\
&= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin}{1+\log^2(d)}\\
&= \frac{6d^x x\cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \frac{6d^x x\log(d)\sin}{\left(1+\log^2(d)\right)}\\
&= \frac{6d^x x\cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \frac{6d^x x\log(d)\sin}{\left(1+\log^2(d)\right)}\\
&= -\frac{12d^x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^4} + \frac{6d^x x\cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 c}{(1+}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 169, normalized size = 0.65

$$d^x\left(\sin(x)\left(x^3\log^7(d) - 3x^2\log^6(d) + 3x\left(x^2+2\right)\log^5(d) - 3\left(x^2+2\right)\log^4(d) + 3x\left(x^2-4\right)\log^3(d) + 3\left(x^2+12\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2
 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Lo
g[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3
*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*
x^2*Log[d]^6 + x^3*Log[d]^7)*Sin[x]))/(1 + Log[d]^2)^4
```

fricas [A] time = 0.46, size = 203, normalized size = 0.78

$$-\frac{\left(x^3\cos(x)\log(d)^6 - 6\,x^2\cos(x)\log(d)^5 + 3\left(x^3+6\,x\right)\cos(x)\log(d)^4 - 12\left(x^2+2\right)\cos(x)\log(d)^3 + 3\left(x^3+4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos(x)*log(d)
^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x)*log(d)^2 - 6*(x^2
- 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^3*log(d)^7 - 3*x^2*log(d)^6 +
3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3 + 3*
(x^2 + 12)*log(d)^2 + 3*x^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(log(d)^
8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

giac [C] time = 1.73, size = 5079, normalized size = 19.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log
(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) +
 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 -
 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x
^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(ab
s(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x
^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x
)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(ab
s(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*
pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*
pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*lo
g(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d
)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d
)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))
^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(
abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d)
)^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*
pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sgn
(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs
(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(ab
s(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(a
bs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))*
cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs
(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(
d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn
(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*
sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d)
)*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*lo
g(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x
*log(abs(d)) - 12)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^
4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(
d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*l
og(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(ab
```

s(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*s
gn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*
pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(
d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi
^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d
) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*s
gn(d) + 6*pi*x - 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
 pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*l
og(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
 2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 +
 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))
^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(
abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d
))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(
d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*
log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d
)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi
^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sg
n(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d
))^2 - 4*pi*sgn(d) + 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) -
pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*
log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 -
6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x
+ 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*
log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d
) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^
2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn
(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(
d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log
(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log
(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(a
bs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^
3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(
abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^
3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))/((4*pi - pi
^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*l
og(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*
log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2
)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(
d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2
*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d))
)^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - ((4*pi - pi^4*sgn(d) + 6*pi^2*l
og(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^
3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi
^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)*(3*pi^2*x^3*log(ab
s(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(
abs(d))*sgn(d) - 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*
x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2
+ 12*x*log(abs(d)) - 12)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d)
 + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*lo
g(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2
- 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*
log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs
(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d
) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*lo
g(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d)
- 3*pi^2*x^3 + 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))
*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*

```
pi*x*sgn(d) + 6*pi*x + 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sg
n(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*
pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs
(d)) - 2*log(abs(d)))/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) +
pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(a
bs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 1
2*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log
(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d)
)*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) -
 3*pi*log(abs(d)) - 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))
*abs(d)^x - 1/2*abs(d)^x*((8*pi^3*x^3*sgn(d) + 24*I*pi^2*x^3*log(abs(d))*sg
n(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 - 24*I*pi^2*x^3*log(abs(
d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d)
 - 48*I*pi*x^3*log(abs(d))*sgn(d) + 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)) -
 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2
*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) - 4
8*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I*pi
*x^2 + 16*x^3 + 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x + 48*I*x^2 +
96*I*x*log(abs(d)) - 96*x - 96*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/
(64*pi + 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d)
)^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) - 16*pi^4 - 64*I*pi^3*log(abs(d))
 + 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi
^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 6
4*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 - 128*I*log(abs(d))^
3 + 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) - 96*pi^2 - 192*I*pi*log(a
bs(d)) + 192*log(abs(d))^2 - 64*pi*sgn(d) + 128*I*log(abs(d)) - 32) - (8*pi
^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*
sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 -
 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(
d) + 24*pi^2*x^3 - 48*I*pi*x^3*log(abs(d)) - 48*x^3*log(abs(d))^2 + 24*I*pi
^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) - 24*I*pi^2
*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*I*x^3*log(abs(d)) + 48*I*x^2*
log(abs(d))^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 + 16*x^3 + 96*x^2*log(abs(
d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96*I*x*log(abs(d)) - 96*x + 96*
I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(64*pi + 16*pi^4*sgn(d) - 64*I
*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d
))^3*sgn(d) - 16*pi^4 + 64*I*pi^3*log(abs(d)) + 96*pi^2*log(abs(d))^2 - 64*
I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi^3*sgn(d) + 192*I*pi^2*log(abs
(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 - 192*I*pi^2*log(abs(d)
) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3 + 96*pi^2*sgn(d) - 192*I*pi*
log(abs(d))*sgn(d) - 96*pi^2 + 192*I*pi*log(abs(d)) + 192*log(abs(d))^2 - 6
4*pi*sgn(d) - 128*I*log(abs(d)) - 32)) - 1/2*abs(d)^x*((8*pi^3*x^3*sgn(d) +
 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3
*x^3 - 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(a
bs(d))^3 + 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^
3 - 48*I*pi*x^3*log(abs(d)) + 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) +
 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^
3 - 48*pi*x^2*log(abs(d)) - 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 -
 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*s
gn(d) + 48*pi*x + 48*I*x^2 + 96*I*x*log(abs(d)) + 96*x - 96*I)*e^(1/2*I*pi*
x*sgn(d) - 1/2*I*pi*x - I*x)/(64*pi - 16*pi^4*sgn(d) - 64*I*pi^3*log(abs(d)
)*sgn(d) + 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d))^3*sgn(d) + 16
*pi^4 + 64*I*pi^3*log(abs(d)) - 96*pi^2*log(abs(d))^2 - 64*I*pi*log(abs(d))
^3 + 32*log(abs(d))^4 - 64*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 19
2*pi*log(abs(d))^2*sgn(d) + 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(a
bs(d))^2 - 128*I*log(abs(d))^3 - 96*pi^2*sgn(d) - 192*I*pi*log(abs(d))*sgn(
d) + 96*pi^2 + 192*I*pi*log(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) + 12
8*I*log(abs(d)) + 32) - (8*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(
d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)
```

```
) + 24*pi*x^3*log(abs(d))^2 - 16*I*x^3*log(abs(d))^3 + 24*pi^2*x^3*sgn(d) -
 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)) + 4
8*x^3*log(abs(d))^2 + 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*l
og(abs(d))*sgn(d) - 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*
I*x^3*log(abs(d)) + 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I*pi*x
^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96
*I*x*log(abs(d)) + 96*x + 96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(
64*pi - 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) + 96*pi^2*log(abs(d))
^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) + 16*pi^4 - 64*I*pi^3*log(abs(d))
- 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 + 32*log(abs(d))^4 - 64*pi^
3*sgn(d) + 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64
*pi^3 - 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3
 - 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) + 96*pi^2 - 192*I*pi*log(ab
s(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) - 128*I*log(abs(d)) + 32))
```

maple [A] time = 0.08, size = 431, normalized size = 1.65

$$\frac{\frac{2x^3e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}+\frac{x^3e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}-\frac{6x^2e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^4+2\ln(d)^2+1}-\frac{x^3e^{x\ln(d)}}{\ln(d)^2+1}+\frac{6x^2e^{x\ln(d)}\ln(d)}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(\ln(d)^2-1\right)x^2e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^4+2\ln(d)^2+1}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*sin(x),x)
```

```
[Out] (1/(ln(d)^2+1)*x^3*exp(x*ln(d))*tan(1/2*x)^2-1/(ln(d)^2+1)*x^3*exp(x*ln(d))
+6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))-6*(ln(d)^2-1)/(ln(d)^4+2*ln
(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)-6*(3*ln(d)^2-1)/(ln(d)^2+1)/(ln(d)^4+2
*ln(d)^2+1)*x*exp(x*ln(d))-12*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^4+2*ln(d)^2+1)/(
ln(d)^2+1)^2*exp(x*ln(d))*tan(1/2*x)+24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(
d)*(ln(d)^2-1)/(ln(d)^2+1)*exp(x*ln(d))+2*ln(d)/(ln(d)^2+1)*x^3*exp(x*ln(d)
)*tan(1/2*x)-6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)^2+6*
(3*ln(d)^2-1)/(ln(d)^2+1)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)^2
-24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(d)*(ln(d)^2-1)/(ln(d)^2+1)*exp(x*ln(
d))*tan(1/2*x)^2+12*ln(d)*(ln(d)^2-3)/(ln(d)^2+1)/(ln(d)^4+2*ln(d)^2+1)*x*e
xp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.58, size = 186, normalized size = 0.71

$$-\frac{\left(\left(\log(d)^6+3\log(d)^4+3\log(d)^2+1\right)x^3-6\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2-24\log(d)^3+6\left(3\log(d)^4+2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3
+ log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*log(d)
)*d^x*cos(x) - ((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d
)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3
 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*lo
g(d)^4 + 4*log(d)^2 + 1)
```

mupad [B] time = 0.63, size = 231, normalized size = 0.89

$$-\frac{d^x\left(6\sin(x)+x^3\cos(x)-3x^2\sin(x)-6x\cos(x)\right)-d^x\ln(d)^5\left(6x^2\cos(x)+3x^3\sin(x)+6x\sin(x)\right)+d^x\ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*sin(x),x)
```

```
[Out] -(d^x*(6*sin(x) + x^3*cos(x) - 3*x^2*sin(x) - 6*x*cos(x)) - d^x*log(d)^5*(6
*x^2*cos(x) + 3*x^3*sin(x) + 6*x*sin(x)) + d^x*log(d)^4*(6*sin(x) + 3*x^3*c
os(x) + 3*x^2*sin(x) + 18*x*cos(x)) - d^x*log(d)^3*(24*cos(x) + 12*x^2*cos(
x) + 3*x^3*sin(x) - 12*x*sin(x)) - d^x*log(d)^2*(36*sin(x) - 3*x^3*cos(x) +
 3*x^2*sin(x) - 12*x*cos(x)) + d^x*log(d)^6*(x^3*cos(x) + 3*x^2*sin(x)) + d
^x*log(d)*(24*cos(x) - 6*x^2*cos(x) - x^3*sin(x) + 18*x*sin(x)) - d^x*x^3*l
og(d)^7*sin(x))/(log(d)^2 + 1)^4
```

sympy [B] time = 22.17, size = 1355, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*sin(x),x)
```

```
[Out] Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*exp
(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3*I
*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(x)/
8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x*
*4*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 +
3*x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(
x)/8 + 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x
*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 +
 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x
)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*l
og(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
 d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)*
*2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d
)**2 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d
)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d
)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d
)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x**2*log(d)**
3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x
*x**2*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
 + 1) + 6*d**x*x**2*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
4*log(d)**2 + 1) + 3*d**x*x**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**
4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)**4*cos(x)/(log(d)**8 + 4
*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*sin(x)/(l
og(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)*
*2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d*
*x*x*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1
) + 6*d**x*x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) - 6*d**x*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log
(d)**2 + 1) + 24*d**x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)*
*4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*sin(x)/(log(d)**8 + 4*log(d
)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

3.141 $\int d^x x^3 \cos(x)\, dx$

Optimal. Leaf size=260

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18xd^x \log^2(d)\sin(x}{\left(\log^2(d)+1\right)^3}$$

```
[Out] -6*d^x*cos(x)/(1+ln(d)^2)^4+36*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^4-6*d^x*cos(x
)*ln(d)^4/(1+ln(d)^2)^4-18*d^x*x*cos(x)*ln(d)/(1+ln(d)^2)^3+6*d^x*x*cos(x)*
ln(d)^3/(1+ln(d)^2)^3+3*d^x*x^2*cos(x)/(1+ln(d)^2)^2-3*d^x*x^2*cos(x)*ln(d)
^2/(1+ln(d)^2)^2+d^x*x^3*cos(x)*ln(d)/(1+ln(d)^2)+24*d^x*ln(d)*sin(x)/(1+ln
(d)^2)^4-24*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^4-6*d^x*x*sin(x)/(1+ln(d)^2)^3+1
8*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^3-6*d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)^2+d
^x*x^3*sin(x)/(1+ln(d)^2)
```

Rubi [A] time = 0.42, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4433, 4466, 14, 4432, 4465}

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18xd^x \log^2(d)\sin(x}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^
4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*x*Cos[x]*Log[d])/(1
+ Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 + (3*d^x*x^2*Cos
[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x
*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^
4 - (24*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d
]^2)^3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[d]*Si
n[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^3 \cos(x)\,dx = \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int x^2 \left(\frac{d^x \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x \sin(x)}{1+\log^2(d)}\right) dx$$

$$= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int \left(\frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)}\right) dx$$

$$= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - \frac{3\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3}{1+\mathrm{l}}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3}{1+\mathrm{l}}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3}{1+\mathrm{l}}$$

$$= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x}{}$$

$$= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x}{}$$

$$= -\frac{6d^x \cos(x)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x \cos(x)\log^4(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x}{(1}$$

Mathematica [A] time = 0.13, size = 168, normalized size = 0.65

$$d^x \left(\sin(x)\left(x^3\log^6(d) - 6x^2\log^5(d) + 3x\left(x^2+6\right)\log^4(d) - 12\left(x^2+2\right)\log^3(d) + 3x\left(x^2+4\right)\log^2(d) - 6\left(x^2 -\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 +
 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 -
3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*
(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2
*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4
```

fricas [A] time = 0.45, size = 202, normalized size = 0.78

$$\frac{\left(x^3\cos(x)\log(d)^7 - 3\,x^2\cos(x)\log(d)^6 + 3\left(x^3 + 2\,x\right)\cos(x)\log(d)^5 - 3\left(x^2 + 2\right)\cos(x)\log(d)^4 + 3\left(x^3 - 4\,x\right)\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

```
[Out] (x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)^
5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2 +
12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) + (x^
3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^
3 + x^3 + 3*(x^3 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(l
og(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

giac [C] time = 1.75, size = 5075, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

```
[Out] -1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*lo
g(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d)
+ 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2
- 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d))
+ 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d))
- 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2
+ 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi + pi^4
*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log
(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*lo
g(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^
2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)
) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*l
og(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x
^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3
*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(
d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)*(pi^3*log
(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d
))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3
*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sg
n(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(ab
s(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(a
bs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
 pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(
abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))
*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) + ((4*pi + pi^4*sgn(d) - 6*pi^2*log(ab
s(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn
(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sg
n(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi
*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*
sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(
abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d
)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d
))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d)
 + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d
) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*s
```

```
gn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*p
i^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(ab
s(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs
(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(a
bs(d))*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x
^3*log(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 +
 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d)
- pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*
log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d))
+ 2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4
+ 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d)
)^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log
(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(
d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn
(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi
*log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(
d)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*p
i^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*s
gn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(
d))^2 - 4*pi*sgn(d) + 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(ab
s(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log(ab
s(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(
d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi
- pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 +
 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12
*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d)
 + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(
abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
 + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs
(d)))^2) + 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 +
3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*log(abs(d))
^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*lo
g(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 12*x)*(pi
^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log
(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))
^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))/((4*pi - p
i^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*
log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi
*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) +
2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs
(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)
))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((4*pi - pi^4*sgn(d) + 6*pi^2*
log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi
^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*p
i^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)*(pi^3*x^3*sgn(d)
- 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^
2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^
2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log
(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log
(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*
sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2
*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs
(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3
 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*
log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*l
og(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3
*log(abs(d))*sgn(d) - 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2
 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6
```

```
*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*s
gn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3
*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(ab
s(d)) - 2*log(abs(d)))/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) +
 pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(
abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 -
12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*lo
g(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d
))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d)
- 3*pi*log(abs(d)) - 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)
)*abs(d)^x - 1/2*I*abs(d)^x*((8*pi^3*x^3*sgn(d) + 24*I*pi^2*x^3*log(abs(d))
*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 - 24*I*pi^2*x^3*log(a
bs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn
(d) - 48*I*pi*x^3*log(abs(d))*sgn(d) + 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)
) - 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*
x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d))
- 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I
*pi*x^2 + 16*x^3 + 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x + 48*I*x^2
 + 96*I*x*log(abs(d)) - 96*x - 96*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*
x)/(64*pi + 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs
(d))^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) - 16*pi^4 - 64*I*pi^3*log(abs(
d)) + 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64
*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d)
+ 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 - 128*I*log(abs(d
))^3 + 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) - 96*pi^2 - 192*I*pi*lo
g(abs(d)) + 192*log(abs(d))^2 - 64*pi*sgn(d) + 128*I*log(abs(d)) - 32) + (8
*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))
^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^
2 - 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*s
gn(d) + 24*pi^2*x^3 - 48*I*pi*x^3*log(abs(d)) - 48*x^3*log(abs(d))^2 + 24*I
*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) - 24*I*p
i^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*I*x^3*log(abs(d)) + 48*I*x
^2*log(abs(d))^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 + 16*x^3 + 96*x^2*log(a
bs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96*I*x*log(abs(d)) - 96*x +
96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(64*pi + 16*pi^4*sgn(d) - 6
4*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(ab
s(d))^3*sgn(d) - 16*pi^4 + 64*I*pi^3*log(abs(d)) + 96*pi^2*log(abs(d))^2 -
64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi^3*sgn(d) + 192*I*pi^2*log(
abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 - 192*I*pi^2*log(abs
(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3 + 96*pi^2*sgn(d) - 192*I*
pi*log(abs(d))*sgn(d) - 96*pi^2 + 192*I*pi*log(abs(d)) + 192*log(abs(d))^2
- 64*pi*sgn(d) - 128*I*log(abs(d)) - 32)) + 1/2*I*abs(d)^x*((8*pi^3*x^3*sgn
(d) + 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8
*pi^3*x^3 - 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*
log(abs(d))^3 + 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi
^2*x^3 - 48*I*pi*x^3*log(abs(d)) + 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn
(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*
pi*x^3 - 48*pi*x^2*log(abs(d)) - 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d)
)^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*p
i*x*sgn(d) + 48*pi*x + 48*I*x^2 + 96*I*x*log(abs(d)) + 96*x - 96*I)*e^(1/2*
I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(64*pi - 16*pi^4*sgn(d) - 64*I*pi^3*log(a
bs(d))*sgn(d) + 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d))^3*sgn(d)
 + 16*pi^4 + 64*I*pi^3*log(abs(d)) - 96*pi^2*log(abs(d))^2 - 64*I*pi*log(ab
s(d))^3 + 32*log(abs(d))^4 - 64*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d)
 + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*
log(abs(d))^2 - 128*I*log(abs(d))^3 - 96*pi^2*sgn(d) - 192*I*pi*log(abs(d))
*sgn(d) + 96*pi^2 + 192*I*pi*log(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d)
 + 128*I*log(abs(d)) + 32) + (8*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))
*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(a
```

```
bs(d)) + 24*pi*x^3*log(abs(d))^2 - 16*I*x^3*log(abs(d))^3 + 24*pi^2*x^3*sgn
(d) - 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)
) + 48*x^3*log(abs(d))^2 + 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*
x^2*log(abs(d))*sgn(d) - 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d))
+ 48*I*x^3*log(abs(d)) + 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I
*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2
 - 96*I*x*log(abs(d)) + 96*x + 96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I
*x)/(64*pi - 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) + 96*pi^2*log(ab
s(d))^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) + 16*pi^4 - 64*I*pi^3*log(abs
(d)) - 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 + 32*log(abs(d))^4 - 6
4*pi^3*sgn(d) + 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d)
 + 64*pi^3 - 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(
d))^3 - 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) + 96*pi^2 - 192*I*pi*l
og(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) - 128*I*log(abs(d)) + 32))
```

maple [A] time = 0.07, size = 441, normalized size = 1.70

$$-\frac{x^3 e^{x\ln(d)}\ln(d)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^2+1}+\frac{x^3 e^{x\ln(d)}\ln(d)}{\ln(d)^2+1}+\frac{2x^3 e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^2+1}-\frac{12x^2 e^{x\ln(d)}\ln(d)\tan\left(\frac{x}{2}\right)}{\ln(d)^4+2\ln(d)^2+1}+\frac{3\left(\ln(d)^2-1\right)x^2 e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(\ln(d)^2-3\right.}{\left(\ln(d)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x)
```

```
[Out] (ln(d)/(ln(d)^2+1)*x^3*exp(x*ln(d))+2/(ln(d)^2+1)*x^3*exp(x*ln(d))*tan(1/2*
x)-3*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))-6/(ln(d)^6+3*ln(d)^
4+3*ln(d)^2+1)*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^2+1)*exp(x*ln(d))+3*(ln(d)^2-1)
/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)^2+6/(ln(d)^6+3*ln(d)^4+3
*ln(d)^2+1)*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^2+1)*exp(x*ln(d))*tan(1/2*x)^2-ln(
d)/(ln(d)^2+1)*x^3*exp(x*ln(d))*tan(1/2*x)^2-12*ln(d)/(ln(d)^4+2*ln(d)^2+1)
*x^2*exp(x*ln(d))*tan(1/2*x)-48*(ln(d)^2-1)*ln(d)/(ln(d)^4+2*ln(d)^2+1)/(ln
(d)^2+1)^2*exp(x*ln(d))*tan(1/2*x)+12*(3*ln(d)^2-1)/(ln(d)^2+1)/(ln(d)^4+2*
ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)+6*ln(d)*(ln(d)^2-3)/(ln(d)^2+1)/(ln(d)
^4+2*ln(d)^2+1)*x*exp(x*ln(d))-6*ln(d)*(ln(d)^2-3)/(ln(d)^2+1)/(ln(d)^4+2*l
n(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.57, size = 184, normalized size = 0.71

$$\frac{\left(\left(\log(d)^7+3\log(d)^5+3\log(d)^3+\log(d)\right)x^3-6\log(d)^4-3\left(\log(d)^6+\log(d)^4-\log(d)^2-1\right)x^2+6\left(\log(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d
)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x
 + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*
x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4
+ 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log
(d)^4 + 4*log(d)^2 + 1)
```

mupad [B] time = 0.64, size = 232, normalized size = 0.89

$$-\frac{d^x\left(6\cos(x)-3x^2\cos(x)-x^3\sin(x)+6x\sin(x)\right)-d^x\ln(d)^5\left(3x^3\cos(x)-6x^2\sin(x)+6x\cos(x)\right)+d\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x)
```

```
[Out] -(d^x*(6*cos(x) - 3*x^2*cos(x) - x^3*sin(x) + 6*x*sin(x)) - d^x*log(d)^5*(3
*x^3*cos(x) - 6*x^2*sin(x) + 6*x*cos(x)) + d^x*log(d)^4*(6*cos(x) + 3*x^2*c
os(x) - 3*x^3*sin(x) - 18*x*sin(x)) + d^x*log(d)^3*(24*sin(x) - 3*x^3*cos(x
) + 12*x^2*sin(x) + 12*x*cos(x)) - d^x*log(d)^2*(36*cos(x) + 3*x^2*cos(x) +
 3*x^3*sin(x) + 12*x*sin(x)) + d^x*log(d)^6*(3*x^2*cos(x) - x^3*sin(x)) - d
^x*log(d)*(24*sin(x) + x^3*cos(x) - 6*x^2*sin(x) - 18*x*cos(x)) - d^x*x^3*l
og(d)^7*cos(x))/(log(d)^2 + 1)^4
```

sympy [B] time = 22.24, size = 1352, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*cos(x),x)
```

```
[Out] Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(-
I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 + 3
*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(x)/
8 - 3*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x**
4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 + 3
*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x)/
8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x*x*
*3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*
log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*
log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8 + 4*
log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(x)/(
log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(
d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d*
*x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
+ 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**
4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**
6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(d)**
8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x**2*log(d)**3*s
in(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x*
*2*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) - 6*d**x*x**2*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*l
og(d)**2 + 1) + 3*d**x*x**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*
log(d)**4 + 4*log(d)**2 + 1) + 18*d**x*x*log(d)**4*sin(x)/(log(d)**8 + 4*lo
g(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*cos(x)/(log(
d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x*log(d)**2*
sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*
x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) -
 6*d**x*x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
- 6*d**x*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)
**2 + 1) - 24*d**x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
+ 4*log(d)**2 + 1) + 36*d**x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*
log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)*sin(x)/(log(d)**8 + 4*log(d)*
*6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*cos(x)/(log(d)**8 + 4*log(d)**
6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\
&= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\
&= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

fricas [A] time = 0.44, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

giac [A] time = 1.11, size = 13, normalized size = 0.52

$$-\frac{4}{3}\sin(x)^6+\frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{24}\cos(6x)-\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

mupad [B] time = 0.17, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\left(8\sin(x)^2-9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

sympy [B] time = 12.67, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{5\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

3.143 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

fricas [A] time = 0.44, size = 25, normalized size = 0.83

$$\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

giac [A] time = 1.18, size = 22, normalized size = 0.73

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

maple [A] time = 0.09, size = 23, normalized size = 0.77

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

maxima [A] time = 0.42, size = 22, normalized size = 0.73

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

mupad [B] time = 0.28, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

sympy [B] time = 12.70, size = 112, normalized size = 3.73

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} + \frac{\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
24 - sin(2*x)*cos(x)*cos(3*x)/8 + sin(3*x)*cos(x)*cos(2*x)/3
```

3.144 $\int x^2 \sin^3(kx)\, dx$

Optimal. Leaf size=85

$$-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

```
[Out] 14/9*cos(k*x)/k^3-2/3*x^2*cos(k*x)/k-2/27*cos(k*x)^3/k^3+4/3*x*sin(k*x)/k^2
-1/3*x^2*cos(k*x)*sin(k*x)^2/k+2/9*x*sin(k*x)^3/k^2
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[k*x]^3,x]
```

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(27*k^3) +
(4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) + (2*x*Sin[k*x]^3)
/(9*k^2)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\sin^3(kx)\,dx = -\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2}{3}\int x^2\sin(kx)\,dx-\frac{2\int\sin^3(kx)\,dx}{9k^2}$$
$$= -\frac{2x^2\cos(kx)}{3k}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2\,\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(kx)\right)}{9k^3}+\frac{4}{\cdot}$$
$$= \frac{2\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}-\frac{4}{\cdot}$$
$$= \frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.65

$$\frac{-81\left(k^2x^2-2\right)\cos(kx)+\left(9k^2x^2-2\right)\cos(3kx)-6kx(\sin(3kx)-27\sin(kx))}{108k^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[k*x]^3,x]
```

```
[Out] (-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Sin
[k*x] + Sin[3*k*x]))/(108*k^3)
```

fricas [A] time = 0.43, size = 59, normalized size = 0.69

$$\frac{\left(9\,k^2x^2-2\right)\cos\left(kx\right)^3-3\left(9\,k^2x^2-14\right)\cos\left(kx\right)-6\left(kx\cos\left(kx\right)^2-7\,kx\right)\sin\left(kx\right)}{27\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) - 6*(k*x*cos
(k*x)^2 - 7*k*x)*sin(k*x))/k^3
```

giac [A] time = 1.14, size = 60, normalized size = 0.71

$$-\frac{x\sin\left(3\,kx\right)}{18\,k^2}+\frac{3\,x\sin\left(kx\right)}{2\,k^2}+\frac{\left(9\,k^2x^2-2\right)\cos\left(3\,kx\right)}{108\,k^3}-\frac{3\left(k^2x^2-2\right)\cos\left(kx\right)}{4\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="giac")
```

```
[Out] -1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*k
*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3
```

maple [A] time = 0.05, size = 64, normalized size = 0.75

$$\frac{-\frac{\left(\sin^2(kx)+2\right)k^2x^2\cos(kx)}{3}+\frac{2kx\left(\sin^3(kx)\right)}{9}+\frac{4kx\sin(kx)}{3}+\frac{4\cos(kx)}{3}+\frac{2\left(\sin^2(kx)+2\right)\cos(kx)}{27}}{k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(k*x)^3,x)
```

```
[Out] 1/k^3*(-1/3*k^2*x^2*(2+sin(k*x)^2)*cos(k*x)+4/3*cos(k*x)+4/3*k*x*sin(k*x)+2
/9*k*x*sin(k*x)^3+2/27*(2+sin(k*x)^2)*cos(k*x))
```

maxima [A] time = 0.43, size = 55, normalized size = 0.65

$$-\frac{6\,kx\sin(3\,kx)-162\,kx\sin(kx)-\left(9\,k^2x^2-2\right)\cos(3\,kx)+81\left(k^2x^2-2\right)\cos(kx)}{108\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos(3*k*x) +
81*(k^2*x^2 - 2)*cos(k*x))/k^3
```

mupad [B] time = 0.23, size = 67, normalized size = 0.79

$$\frac{\frac{14\cos(k\,x)}{9}-\frac{2\cos(k\,x)^3}{27}+k\left(\frac{14\,x\,\sin(k\,x)}{9}-\frac{2\,x\cos(k\,x)^2\,\sin(k\,x)}{9}\right)+k^2\left(\frac{x^2\cos(k\,x)^3}{3}-x^2\,\cos(k\,x)\right)}{k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(k*x)^3,x)
```

```
[Out] ((14*cos(k*x))/9 - (2*cos(k*x)^3)/27 + k*((14*x*sin(k*x))/9 - (2*x*cos(k*x)
^2*sin(k*x))/9) + k^2*((x^2*cos(k*x)^3)/3 - x^2*cos(k*x)))/k^3
```

sympy [A] time = 2.10, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{x^2\sin^2(kx)\cos(kx)}{k}-\frac{2x^2\cos^3(kx)}{3k}+\frac{14x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)\cos^2(kx)}{3k^2}+\frac{14\sin^2(kx)\cos(kx)}{9k^3}+\frac{40\cos^3(kx)}{27k^3} & \text{for } k\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(k*x)**3,x)
```

```
[Out] Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*s
in(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*co
s(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))
```

3.145 $\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$

Optimal. Leaf size=14

$$\text{Int}(x\cot(x)\csc(x)\cos(k\csc(x)),x)$$

```
[Out] CannotIntegrate(x*cos(k*csc(x))*cot(x)*csc(x),x)
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi steps

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx=\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\cos(x)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2, x)
```

maple [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

```
[Out] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

maxima [A] time = 0.47, size = 240, normalized size = 17.14

$$-\frac{\left(xe^{\left(\frac{4\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}+\frac{4\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}+xe^{\left(\frac{4\,k\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}\right)e^{\left(-\frac{2\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}-\cdots\right.}}{2\,k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) +
 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k
*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)))*e^(-2*k*cos(2*x)*cos(x
)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x
)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 -
2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(
cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x\,\cos\left(\frac{k}{\sin(x)}\right)\,\cos(x)}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(k/sin(x))*cos(x))/sin(x)^2,x)
```

```
[Out] int((x*cos(k/sin(x))*cos(x))/sin(x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

```
[Out] Timed out
```

3.146 $\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$

Optimal. Leaf size=12

$$-x-\cot\left(\frac{x}{2}\right)$$

```
[Out] -x-cot(1/2*x)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 453, 203}

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x/2]*Cot[x],x]
```

```
[Out] -x - Cot[x/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx &= 2\,\text{Subst}\left(\int \frac{1-x^2}{2x^2\left(1+x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \text{Subst}\left(\int \frac{1-x^2}{x^2\left(1+x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\cot\left(\frac{x}{2}\right)-2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -x-\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x/2]*Cot[x],x]
[Out] -x - Cot[x/2]
```

fricas [A] time = 0.43, size = 16, normalized size = 1.33

$$-\frac{x\tan\left(\frac{1}{2}x\right)+1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")
[Out] -(x*tan(1/2*x) + 1)/tan(1/2*x)
```

giac [A] time = 1.24, size = 18, normalized size = 1.50

$$-x-\frac{1}{2\tan\left(\frac{1}{4}x\right)}+\frac{1}{2}\tan\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="giac")
[Out] -x - 1/2/tan(1/4*x) + 1/2*tan(1/4*x)
```

maple [A] time = 0.10, size = 11, normalized size = 0.92

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)/tan(1/2*x),x)
[Out] -x-cot(1/2*x)
```

maxima [B] time = 0.43, size = 41, normalized size = 3.42

$$-\frac{x\cos(x)^2+x\sin(x)^2-2\,x\cos(x)+x+2\,\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")
[Out] -(x*cos(x)^2 + x*sin(x)^2 - 2*x*cos(x) + x + 2*sin(x))/(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

mupad [B] time = 0.18, size = 10, normalized size = 0.83

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(tan(x/2)*sin(x)),x)
[Out] - x - cot(x/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sin(x)\tan\left(\frac{x}{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x)
```

```
[Out] Integral(cos(x)/(sin(x)*tan(x/2)), x)
```

3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(c\sin(ax)+b)}$$

```
[Out] -2*c*arctan((c+b*tan(1/2*a*x))/(b^2-c^2)^(1/2))/a/(b^2-c^2)^(3/2)-b*cos(a*x
)/a/(b^2-c^2)/(b+c*sin(a*x))
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(c\sin(ax)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2)^(3/2)) -
 (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx = -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{\int \frac{c}{b+c\sin(ax)}\,dx}{-b^2+c^2}$$
$$= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{c\int \frac{1}{b+c\sin(ax)}\,dx}{b^2-c^2}$$
$$= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+2cx+bx^2}\,dx,x,\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}$$
$$= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{(4c)\,\text{Subst}\left(\int \frac{1}{-4\left(b^2-c^2\right)-x^2}\,dx,x,2c+2b\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}$$
$$= -\frac{2c\tan^{-1}\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}$$

Mathematica [A] time = 0.26, size = 76, normalized size = 0.99

$$-\frac{\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{b\cos(ax)}{(b-c)(b+c)(c\sin(ax)+b)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] -(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)
```

fricas [A] time = 0.49, size = 312, normalized size = 4.05

$$\left[\frac{\left(c^2\sin(ax)+bc\right)\sqrt{-b^2+c^2}\,\log\left(\frac{\left(2b^2-c^2\right)\cos(ax)^2-2bc\sin(ax)-b^2-c^2+2\left(b\cos(ax)\sin(ax)+c\cos(ax)\right)\sqrt{-b^2+c^2}}{c^2\cos(ax)^2-2bc\sin(ax)-b^2-c^2}\right)-2\left(b^3-bc^2\right)\text{cc}}{2\left(ab^5-2ab^3c^2+abc^4+\left(ab^4c-2ab^2c^3+ac^5\right)\sin(ax)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x)^2 -
 2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-b^
2 + c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^2)*
cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*
sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) + c)/(
sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2)*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 +
 a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]
```

giac [A] time = 1.05, size = 98, normalized size = 1.27

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ax}{2\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^2-c^2}}\right)\right)c}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{c\tan\left(\frac{1}{2}ax\right)+b}{\left(b\tan\left(\frac{1}{2}ax\right)^2+2c\tan\left(\frac{1}{2}ax\right)+b\right)\left(b^2-c^2\right)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*a*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*a*x) + c)/sqrt(b
^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)/((b*tan(1/2*a*x)^2 +
 2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

maple [A] time = 0.10, size = 143, normalized size = 1.86

$$-\frac{8c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{\left(4b^2-4c^2\right)\sqrt{b^2-c^2}\,a}-\frac{8c\tan\left(\frac{ax}{2}\right)}{\left(4b^2-4c^2\right)\left(b\left(\tan^2\left(\frac{ax}{2}\right)\right)+2c\tan\left(\frac{ax}{2}\right)+b\right)a}-\frac{8b}{\left(4b^2-4c^2\right)\left(b\left(\tan^2\left(\frac{ax}{2}\right)\right)+2c\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)/(b+c*sin(a*x))^2,x)
```

```
[Out] -8/a/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*c*tan(1/2*a*x)-8/a
/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*b-8/a*c/(4*b^2-4*c^2)/
(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 0.49, size = 133, normalized size = 1.73

$$-\frac{\frac{2\,b}{b^2-c^2}+\frac{2\,c\tan\left(\frac{a\,x}{2}\right)}{b^2-c^2}}{a\,\left(b\tan\left(\frac{a\,x}{2}\right)^2+2\,c\tan\left(\frac{a\,x}{2}\right)+b\right)}-\frac{2\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c^2}{{(b+c)}^{3/2}\,{(b-c)}^{3/2}}+\frac{2\,b\,c\tan\left(\frac{a\,x}{2}\right)}{{(b+c)}^{3/2}\,{(b-c)}^{3/2}}\right)\left(b^2-c^2\right)}{2\,c}\right)}{a\,{(b+c)}^{3/2}\,{(b-c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)/(b + c*sin(a*x))^2,x)
```

```
[Out] - ((2*b)/(b^2 - c^2) + (2*c*tan((a*x)/2))/(b^2 - c^2))/(a*(b + 2*c*tan((a*x
)/2) + b*tan((a*x)/2)^2)) - (2*c*atan((((2*c^2)/((b + c)^(3/2)*(b - c)^(3/2
)) + (2*b*c*tan((a*x)/2))/((b + c)^(3/2)*(b - c)^(3/2)))*(b^2 - c^2))/(2*c)
))/(a*(b + c)^(3/2)*(b - c)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

3.148 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4475}

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4475

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Sin[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] - Simp[(b*d*n*x*Cos[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]],x]
```

```
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,x\cos\left(\log(x)\right) + \frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

giac [A] time = 1.05, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,x\cos\left(\log(x)\right) + \frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{x\cos\left(\ln(x)\right)}{2}+\frac{x\sin\left(\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

maxima [A] time = 0.45, size = 12, normalized size = 0.71

$$-\frac{1}{2}\,x\left(\cos\left(\log(x)\right)-\sin\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

mupad [B] time = 0.13, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2}\,x\,\cos\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x)),x)
```

```
[Out] -(2^(1/2)*x*cos(pi/4 + log(x)))/2
```

sympy [A] time = 0.38, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}-\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

3.149 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx=\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

giac [A] time = 1.01, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log(x)\right)+\frac{1}{2}\,x\sin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x)
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

maxima [A] time = 0.42, size = 10, normalized size = 0.59

$$\frac{1}{2}\,x\left(\cos\left(\log(x)\right)+\sin\left(\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

mupad [B] time = 0.15, size = 13, normalized size = 0.76

$$\frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(x)),x)
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

sympy [A] time = 0.39, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log(x)\right)}{2}+\frac{x\cos\left(\log(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

3.150 $\int e^x\,dx$

Optimal. Leaf size=3

$$e^x$$

[Out] exp(x)

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$e^x$$

Antiderivative was successfully verified.

[In] Int[E^x,x]

[Out] E^x

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x\,dx = e^x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Antiderivative was successfully verified.

[In] Integrate[E^x,x]

[Out] E^x

fricas [A] time = 0.40, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x),x, algorithm="fricas")

[Out] e^x

giac [A] time = 1.03, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x),x, algorithm="giac")

[Out] e^x

maple [A] time = 0.00, size = 3, normalized size = 1.00

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
[Out] exp(x)
```

maxima [A] time = 0.41, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
[Out] e^x
```

mupad [B] time = 0.01, size = 2, normalized size = 0.67

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
[Out] exp(x)
```

sympy [A] time = 0.04, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
[Out] exp(x)
```

3.151 $\int a^x\,dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/ln(a)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

fricas [A] time = 0.42, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

giac [A] time = 1.15, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/log(a)
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/ln(a)
```

maxima [A] time = 0.42, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

mupad [B] time = 0.16, size = 8, normalized size = 1.00

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/log(a)
```

sympy [A] time = 0.09, size = 8, normalized size = 1.00

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

3.152 $\int e^{ax} \, dx$

Optimal. Leaf size=9

$$\frac{e^{ax}}{a}$$

[Out] exp(a*x)/a

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194}

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(a*x),x]

[Out] E^(a*x)/a

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{ax} \, dx = \frac{e^{ax}}{a}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a*x),x]

[Out] E^(a*x)/a

fricas [A] time = 0.40, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x),x, algorithm="fricas")

[Out] e^(a*x)/a

giac [A] time = 1.04, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x),x, algorithm="giac")

```
[Out] e^(a*x)/a
```

maple [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x),x)
```

```
[Out] exp(a*x)/a
```

maxima [A] time = 0.41, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="maxima")
```

```
[Out] e^(a*x)/a
```

mupad [B] time = 0.03, size = 8, normalized size = 0.89

$$\frac{e^{a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x),x)
```

```
[Out] exp(a*x)/a
```

sympy [A] time = 0.05, size = 7, normalized size = 0.78

$$\begin{cases} \frac{e^{ax}}{a} & \text{for}\: a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x)
```

```
[Out] Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))
```

3.153 $\int \frac{e^{ax}}{x} dx$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(ax)$$

```
[Out] Ei(a*x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2178}

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)/x,x]
[Out] ExpIntegralEi[a*x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{ax}}{x} dx = \text{Ei}(ax)$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)/x,x]
[Out] ExpIntegralEi[a*x]
```

fricas [A] time = 0.40, size = 4, normalized size = 1.00

$$\text{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="fricas")
[Out] Ei(a*x)
```

giac [A] time = 1.17, size = 4, normalized size = 1.00

$$\text{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="giac")
[Out] Ei(a*x)
```

maple [A] time = 0.01, size = 9, normalized size = 2.25

$$-\mathrm{Ei}\left(1,-ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x)
[Out] -Ei(1,-a*x)
```

maxima [A] time = 0.58, size = 4, normalized size = 1.00

$$\mathrm{Ei}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="maxima")
[Out] Ei(a*x)
```

mupad [B] time = 0.01, size = 4, normalized size = 1.00

$$\mathrm{ei}\left(a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x)
[Out] ei(a*x)
```

sympy [A] time = 0.78, size = 3, normalized size = 0.75

$$\mathrm{Ei}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x)
[Out] Ei(a*x)
```

3.154 $\int \frac{1}{a+be^{mx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{a}-\frac{\log \left(a+b e^{m x}\right)}{a m}$$

```
[Out] x/a-ln(a+b*exp(m*x))/a/m
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 36, 29, 31}

$$\frac{x}{a}-\frac{\log \left(a+b e^{m x}\right)}{a m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(m*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b e^{m x}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+b x)}\,dx,x,e^{m x}\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{m x}\right)}{a m}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+b x}\,dx,x,e^{m x}\right)}{a m}\\
&= \frac{x}{a}-\frac{\log \left(a+b e^{m x}\right)}{a m}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{x}{a}-\frac{\log \left(a+b e^{m x}\right)}{a m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(m*x))^(-1),x]
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

fricas [A] time = 0.44, size = 22, normalized size = 0.92

$$\frac{mx - \log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")
[Out] (m*x - log(b*e^(m*x) + a))/(a*m)
```

giac [A] time = 1.31, size = 26, normalized size = 1.08

$$\frac{\frac{mx}{a} - \frac{\log\left(\left|be^{(mx)}+a\right|\right)}{a}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="giac")
[Out] (m*x/a - log(abs(b*e^(m*x) + a))/a)/m
```

maple [A] time = 0.01, size = 31, normalized size = 1.29

$$-\frac{\ln\left(b\,e^{mx}+a\right)}{am}+\frac{\ln\left(e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(m*x)),x)
[Out] -ln(a+b*exp(m*x))/a/m+1/m/a*ln(exp(m*x))
```

maxima [A] time = 0.42, size = 23, normalized size = 0.96

$$\frac{x}{a} - \frac{\log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")
[Out] x/a - log(b*e^(m*x) + a)/(a*m)
```

mupad [B] time = 0.09, size = 22, normalized size = 0.92

$$-\frac{\ln\left(a+b\,e^{m\,x}\right)-m\,x}{a\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(m*x)),x)
[Out] -(log(a + b*exp(m*x)) - m*x)/(a*m)
```

sympy [A] time = 0.12, size = 15, normalized size = 0.62

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x)
```

```
[Out] x/a - log(a/b + exp(m*x))/(a*m)
```

3.155 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{x}{1+x} \, dx, x, e^x\right) \\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) \, dx, x, e^x\right) \\ &= e^x - \log(1+e^x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

fricas [A] time = 0.44, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
[Out] e^x - log(e^x + 1)
```

giac [A] time = 1.10, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
[Out] e^x - log(e^x + 1)
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x)+1),x)
[Out] exp(x)-ln(exp(x)+1)
```

maxima [A] time = 0.42, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
[Out] e^x - log(e^x + 1)
```

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x) + 1),x)
[Out] exp(x) - log(exp(x) + 1)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
[Out] exp(x) - log(exp(x) + 1)
```

3.156 $\int e^{2x+ax}\,dx$

Optimal. Leaf size=13

$$\frac{e^{(a+2)x}}{a+2}$$

```
[Out] exp((2+a)*x)/(2+a)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2227, 2194}

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x + a*x),x]
[Out] E^((2 + a)*x)/(2 + a)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rubi steps

$$\begin{aligned} \int e^{2x+ax}\,dx &= \int e^{(2+a)x}\,dx \\ &= \frac{e^{(2+a)x}}{2+a} \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x + a*x),x]
[Out] E^((2 + a)*x)/(2 + a)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.92

$$\frac{e^{((a+2)x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="fricas")
[Out] e^((a + 2)*x)/(a + 2)
```

giac [**A**] time = 1.12, size = 14, normalized size = 1.08

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="giac")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

maple [**A**] time = 0.00, size = 15, normalized size = 1.15

$$\frac{e^{ax+2x}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x+2*x),x)
```

```
[Out] 1/(2+a)*exp(a*x+2*x)
```

maxima [**A**] time = 0.42, size = 14, normalized size = 1.08

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="maxima")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

mupad [**B**] time = 0.05, size = 14, normalized size = 1.08

$$\frac{e^{2x+ax}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x + a*x),x)
```

```
[Out] exp(2*x + a*x)/(a + 2)
```

sympy [**A**] time = 0.09, size = 14, normalized size = 1.08

$$\begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a+2 \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x)
```

```
[Out] Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a + 2, 0)), (x, True))
```

3.157 $\int \frac{1}{be^{-mx}+ae^{mx}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

```
[Out] arctan(exp(m*x)*a^(1/2)/b^(1/2))/m/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{be^{-mx}+ae^{mx}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, e^{mx}\right)}{m}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

fricas [A] time = 0.43, size = 85, normalized size = 2.74

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ae^{(2\,mx)}-2\,\sqrt{-ab}\,e^{(mx)}-b}{ae^{(2\,mx)}+b}\right)}{2\,abm}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,e^{(-mx)}}{a}\right)}{abm}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((a*e^(2*m*x) - 2*sqrt(-a*b)*e^(m*x) - b)/(a*e^(2*m*x)
+ b))/(a*b*m), -sqrt(a*b)*arctan(sqrt(a*b)*e^(-m*x)/a)/(a*b*m)]
```

giac [A] time = 1.04, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ae^{(mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="giac")
```

```
[Out] arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

maple [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{\arctan\left(\frac{a\,e^{mx}}{\sqrt{ab}}\right)}{\sqrt{ab}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] 1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))
```

maxima [A] time = 0.99, size = 23, normalized size = 0.74

$$-\frac{\arctan\left(\frac{be^{(-mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")
```

```
[Out] -arctan(b*e^(-m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

mupad [B] time = 0.23, size = 21, normalized size = 0.68

$$\frac{\mathrm{atan}\left(\frac{a\,e^{m\,x}}{\sqrt{a\,b}}\right)}{m\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(m*x) + b*exp(-m*x)),x)
```

```
[Out] atan((a*exp(m*x))/(a*b)^(1/2))/(m*(a*b)^(1/2))
```

sympy [A] time = 0.18, size = 26, normalized size = 0.84

$$\frac{\mathrm{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(-2ia+e^{-mx}\right)\right)\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(-2*_i*a + exp(-m*x))))/m
```

3.158 $\int e^{ax} x \, dx$

Optimal. Leaf size=21

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

```
[Out] -exp(a*x)/a^2+exp(a*x)*x/a
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*x,x]
```

```
[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{ax} x \, dx &= \frac{e^{ax} x}{a} - \frac{\int e^{ax} \, dx}{a} \\
&= -\frac{e^{ax}}{a^2} + \frac{e^{ax} x}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.67

$$\frac{e^{ax}(ax-1)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*x,x]
```

```
[Out] (E^(a*x)*(-1 + a*x))/a^2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="fricas")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

giac [A] time = 0.99, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="giac")
[Out] (a*x - 1)*e^(a*x)/a^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{(ax-1)\,\mathrm{e}^{ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x,x)
[Out] (a*x-1)*exp(a*x)/a^2
```

maxima [A] time = 0.44, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="maxima")
[Out] (a*x - 1)*e^(a*x)/a^2
```

mupad [B] time = 0.03, size = 13, normalized size = 0.62

$$\frac{\mathrm{e}^{a\,x}\,\left(a\,x-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(a*x),x)
[Out] (exp(a*x)*(a*x - 1))/a^2
```

sympy [A] time = 0.09, size = 19, normalized size = 0.90

$$\begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x)
[Out] Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

3.159 $\int e^x x^{20}\,dx$

Optimal. Leaf size=163

$e^x x^{20} - 20 e^x x^{19} + 380 e^x x^{18} - 6840 e^x x^{17} + 116280 e^x x^{16} - 1860480 e^x x^{15} + 27907200 e^x x^{14} - 390700800 e^x x^{13} + 507911040$

```
[Out] 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+1216451004088320000
*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-202
74183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x
)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*exp(x
)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13
+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^
17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

Rubi [A] time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2176, 2194}

$e^x x^{20} - 20 e^x x^{19} + 380 e^x x^{18} - 6840 e^x x^{17} + 116280 e^x x^{16} - 1860480 e^x x^{15} + 27907200 e^x x^{14} - 390700800 e^x x^{13} + 507911040$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^20,x]
```

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 1216451004088320000*E
^x*x^2 - 405483668029440000*E^x*x^3 + 101370917007360000*E^x*x^4 - 20274183
401472000*E^x*x^5 + 3379030566912000*E^x*x^6 - 482718652416000*E^x*x^7 + 60
339831552000*E^x*x^8 - 6704425728000*E^x*x^9 + 670442572800*E^x*x^10 - 6094
9324800*E^x*x^11 + 5079110400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*
x^14 - 1860480*E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 -
20*E^x*x^19 + E^x*x^20
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x x^{20}\,dx = e^x x^{20} - 20\int e^x x^{19}\,dx$$
$$= -20e^x x^{19} + e^x x^{20} + 380\int e^x x^{18}\,dx$$
$$= 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 6840\int e^x x^{17}\,dx$$
$$= -6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 116280\int e^x x^{16}\,dx$$
$$= 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 1860480\int e^x x^{15}\,dx$$
$$= -1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 27907200\int e^x x^{14}\,d$$
$$= 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 390$$
$$= -390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 2$$
$$= 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x$$
$$= -60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} +$$
$$= 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x$$
$$= -6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 39070$$
$$= 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 507$$
$$= -482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} -$$
$$= 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x$$
$$= -20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 60339831552$$
$$= 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 4827186524$$
$$= -405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 337903$$
$$= 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 202741$$
$$= -2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 1013$$
$$= 2432902008176640000e^x - 2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 405483$$

Mathematica [A] time = 0.01, size = 102, normalized size = 0.63

$$e^x\left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 507911040\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^20,x]
```

```
[Out] E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2
- 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 +
 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 67044257
28000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700
800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^1
8 - 20*x^19 + x^20)
```

fricas [A] time = 0.42, size = 101, normalized size = 0.62

$$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 50791104\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="fricas")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

giac [A] time = 1.21, size = 101, normalized size = 0.62

$$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="giac")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

maple [A] time = 0.00, size = 102, normalized size = 0.63

$$\left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^20,x)
```

```
[Out] (x^20-20*x^19+380*x^18-6840*x^17+116280*x^16-1860480*x^15+27907200*x^14-390
700800*x^13+5079110400*x^12-60949324800*x^11+670442572800*x^10-670442572800
0*x^9+60339831552000*x^8-482718652416000*x^7+3379030566912000*x^6-202741834
01472000*x^5+101370917007360000*x^4-405483668029440000*x^3+1216451004088320
000*x^2-2432902008176640000*x+2432902008176640000)*exp(x)
```

maxima [A] time = 0.42, size = 101, normalized size = 0.62

$$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="maxima")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

mupad [B] time = 0.36, size = 101, normalized size = 0.62

$$e^x\,\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 507911040$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*exp(x),x)
```

```
[Out] exp(x)*(1216451004088320000*x^2 - 2432902008176640000*x - 40548366802944000
0*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x
^6 - 482718652416000*x^7 + 60339831552000*x^8 - 6704425728000*x^9 + 6704425
72800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200
*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20
+ 2432902008176640000)
```

sympy [A] time = 0.13, size = 102, normalized size = 0.63

$$\left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**20,x)
```

```
[Out] (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15 +
 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11 +
670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 48271865241
6000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 10137091700736
0000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 2432902008
176640000*x + 2432902008176640000)*exp(x)
```

3.160 $\int a^x b^{-x} \, dx$

Optimal. Leaf size=18

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2287, 2194}

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/b^x,x]
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int a^x b^{-x} \, dx &= \int e^{x(\log(a)-\log(b))} \, dx\\ &= \frac{a^x b^{-x}}{\log(a) - \log(b)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/b^x,x]
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

fricas [A] time = 0.42, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x \left(\log(a) - \log(b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="fricas")
```

[Out] a^x/(b^x*(log(a) - log(b)))

giac [C] time = 1.44, size = 216, normalized size = 12.00

$$2\left(\frac{2\left(\log\left(|a|\right)-\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\pi x\mathrm{sgn}(b)\right)}{\left(\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)^2+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^2}-\frac{\left(\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\pi x\mathrm{s}\right.}{\left(\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)^2+4\left(\log\left(|a|\right)-\log\left(|b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) -
1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*s
gn(b) + 2*log(abs(a)) - 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*
pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*
e^(x*(log(abs(a)) - log(abs(b))))
```

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$\frac{a^x b^{-x}}{\ln(a)-\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/(b^x),x)
```

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?` f
or more details)Is -log(b)/log(a) equal to -1?
```

mupad [B] time = 0.20, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x\,\left(\ln(a)-\ln(b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/b^x,x)
```

```
[Out] a^x/(b^x*(log(a) - log(b)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/(b**x),x)
```

```
[Out] Exception raised: TypeError
```

3.161 $\int a^x b^x \, dx$

Optimal. Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

```
[Out] a^x*b^x/(ln(a)+ln(b))
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2287, 2194}

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int a^x b^x \, dx &= \int e^{x(\log(a)+\log(b))} \, dx\\ &= \frac{a^x b^x}{\log(a) + \log(b)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

fricas [A] time = 0.43, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="fricas")
```

```
[Out] a^x*b^x/(log(a) + log(b))
```

giac [C] time = 1.50, size = 237, normalized size = 16.93

$$2\left(\frac{2\left(\log\left(|a|\right)+\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)-\frac{1}{2}\pi x\mathrm{sgn}(b)+\pi x\right)}{\left(2\pi-\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)^{2}+4\left(\log\left(|a|\right)+\log\left(|b|\right)\right)^{2}}+\frac{\left(2\pi-\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi x\mathrm{sgn}\right.}{\left(2\pi-\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)^{2}+4\left(\log\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + p
i*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2) +
 (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi
*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2))*e
^(x*(log(abs(a)) + log(abs(b)))) - 1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) + 1/2*I
*pi*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a))
 + 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)
/(2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x
*(log(abs(a)) + log(abs(b))))
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$\frac{a^{x}b^{x}}{\ln(a)+\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*b^x,x)
```

```
[Out] a^x*b^x/(ln(a)+ln(b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(log(b)/log(a)>0)', see `assume?` fo
r more details)Is log(b)/log(a) equal to -1?
```

mupad [B] time = 0.17, size = 14, normalized size = 1.00

$$\frac{a^{x}\,b^{x}}{\ln(a)+\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*b^x,x)
```

```
[Out] (a^x*b^x)/(log(a) + log(b))
```

sympy [A] time = 0.57, size = 24, normalized size = 1.71

$$\begin{cases} \frac{a^{x}b^{x}}{\log(a)+\log(b)} & \text{for}\ a\neq\frac{1}{b} \\ \tilde{\infty}b^{x}\left(\frac{1}{b}\right)^{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*b**x,x)
```

```
[Out] Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (zoo*b**x*(1/b)**x, Tr
ue))
```

3.162 $\int \frac{a^x}{x^2} \, dx$

Optimal. Leaf size=17

$$\log(a)\text{Ei}(x\log(a)) - \frac{a^x}{x}$$

```
[Out] -a^x/x+Ei(x*ln(a))*ln(a)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2177, 2178}

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned} \int \frac{a^x}{x^2} \, dx &= -\frac{a^x}{x} + \log(a) \int \frac{a^x}{x} \, dx \\ &= -\frac{a^x}{x} + \text{Ei}(x\log(a))\log(a) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\log(a)\text{Ei}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

fricas [A] time = 0.41, size = 19, normalized size = 1.12

$$\frac{x\text{Ei}\left(x\log(a)\right)\log(a) - a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="fricas")
[Out] (x*Ei(x*log(a))*log(a) - a^x)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="giac")
[Out] integrate(a^x/x^2, x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.24

$$-\operatorname{Ei}(1, -x\ln(a))\ln(a) - \frac{a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/x^2,x)
[Out] -a^x/x-ln(a)*Ei(1,-x*ln(a))
```

maxima [A] time = 0.59, size = 10, normalized size = 0.59

$$\Gamma\left(-1, -x\log(a)\right)\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="maxima")
[Out] gamma(-1, -x*log(a))*log(a)
```

mupad [B] time = 0.13, size = 19, normalized size = 1.12

$$-\ln(a)\operatorname{expint}(-x\ \ln(a)) - \frac{a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/x^2,x)
[Out] - log(a)*expint(-x*log(a)) - a^x/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/x**2,x)
[Out] Integral(a**x/x**2, x)
```

3.163 $\int \frac{a^x x}{(1+bx)^2} \, dx$

Optimal. Leaf size=64

$$-\frac{a^{-1/b}\log(a)\text{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^3}+\frac{a^{-1/b}\text{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^2}+\frac{a^x}{b^2(bx+1)}$$

```
[Out] a^x/b^2/(b*x+1)+Ei((b*x+1)*ln(a)/b)/(a^(1/b))/b^2-Ei((b*x+1)*ln(a)/b)*ln(a)
/(a^(1/b))/b^3
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2199, 2177, 2178}

$$\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^x}{b^2(bx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) -
(ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*b^3)
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{a^x x}{(1+bx)^2}\,dx = \int \left(-\frac{a^x}{b(1+bx)^2}+\frac{a^x}{b(1+bx)}\right)dx$$
$$= -\frac{\int \frac{a^x}{(1+bx)^2}\,dx}{b}+\frac{\int \frac{a^x}{1+bx}\,dx}{b}$$
$$= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{\log(a)\int \frac{a^x}{1+bx}\,dx}{b^2}$$
$$= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)\log(a)}{b^3}$$

Mathematica [A] time = 0.13, size = 43, normalized size = 0.67

$$\frac{a^{-1/b}(b-\log(a))\mathrm{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)+\frac{ba^x}{bx+1}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^b
^(-1))/b^3
```

fricas [A] time = 0.43, size = 54, normalized size = 0.84

$$\frac{a^x b+\frac{\left(b^2x-(bx+1)\log(a)+b\right)\mathrm{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4x+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="fricas")
```

```
[Out] (a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^(1/b))/(b^
4*x + b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(a^x*x/(b*x + 1)^2, x)
```

maple [A] time = 0.06, size = 79, normalized size = 1.23

$$-\frac{a^{-\frac{1}{b}}\,\mathrm{Ei}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)}{b^2}+\frac{a^{-\frac{1}{b}}\,\mathrm{Ei}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)\ln(a)}{b^3}+\frac{a^x\ln(a)}{\left(x\ln(a)+\frac{\ln(a)}{b}\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*x/(b*x+1)^2,x)
```

```
[Out] -1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(a)/b)+ln(a
)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{x}x}{b^{2}x^{2}\log(a) + 2\,bx\log(a) + \log(a)} + \int \frac{(bx-1)a^{x}}{b^{3}x^{3}\log(a) + 3\,b^{2}x^{2}\log(a) + 3\,bx\log(a) + \log(a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")
```

```
[Out] a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x - 1)*a^x/(b
^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a^{x}\,x}{{\left(b\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x*x)/(b*x + 1)^2,x)
```

```
[Out] int((a^x*x)/(b*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{x}x}{(bx+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*x/(b*x+1)**2,x)
```

```
[Out] Integral(a**x*x/(b*x + 1)**2, x)
```

3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$

Optimal. Leaf size=16

$$\frac{e^{ax}}{a^2(ax+1)}$$

[Out] exp(a*x)/a^2/(a*x+1)

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2197}

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^(a*x)*x)/(1 + a*x)^2,x]

[Out] E^(a*x)/(a^2*(1 + a*x))

Rule 2197

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[(g*u^(m + 1)*F^(c*v))/(b*c*e*Lo
g[F]), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{ax}}{a^2(1+ax)}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]

[Out] E^(a*x)/(a^2*(1 + a*x))

fricas [A] time = 0.40, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")

[Out] e^(a*x)/(a^3*x + a^2)

giac [B] time = 1.19, size = 45, normalized size = 2.81

$$-\frac{e^{\left(-(ax+1)\left(\frac{1}{ax+1}-1\right)\right)}}{(ax+1)a^2\left(\frac{1}{ax+1}-1\right)-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="giac")
```

```
[Out] -e^(-(a*x + 1)*(1/(a*x + 1) - 1))/((a*x + 1)*a^2*(1/(a*x + 1) - 1) - a^2)
```

maple [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{e^{ax}}{(ax+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x/(a*x+1)^2,x)
```

```
[Out] exp(a*x)/a^2/(a*x+1)
```

maxima [A] time = 0.43, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")
```

```
[Out] e^(a*x)/(a^3*x + a^2)
```

mupad [B] time = 0.20, size = 15, normalized size = 0.94

$$\frac{e^{a\,x}}{a^2\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(a*x))/(a*x + 1)^2,x)
```

```
[Out] exp(a*x)/(a^2*(a*x + 1))
```

sympy [A] time = 0.12, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)**2,x)
```

```
[Out] exp(a*x)/(a**3*x + a**2)
```

3.165 $\int k^{x^2} x\,dx$

Optimal. Leaf size=13

$$\frac{k^{x^2}}{2\log(k)}$$

[Out] 1/2*k^(x^2)/ln(k)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

[In] Int[k^x^2*x,x]

[Out] k^x^2/(2*Log[k])

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int k^{x^2} x\,dx = \frac{k^{x^2}}{2\log(k)}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

[In] Integrate[k^x^2*x,x]

[Out] k^x^2/(2*Log[k])

fricas [A] time = 0.42, size = 11, normalized size = 0.85

$$\frac{k^{\left(x^2\right)}}{2\,\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(k^(x^2)*x,x, algorithm="fricas")

[Out] 1/2*k^(x^2)/log(k)

giac [A] time = 1.05, size = 11, normalized size = 0.85

$$\frac{k^{\left(x^2\right)}}{2\,\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="giac")
[Out] 1/2*k^(x^2)/log(k)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\frac{k^{x^2}}{2\ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(k^(x^2)*x,x)
[Out] 1/2*k^(x^2)/ln(k)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{k^{(x^2)}}{2\,\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="maxima")
[Out] 1/2*k^(x^2)/log(k)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.85

$$\frac{k^{x^2}}{2\,\ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(k^(x^2)*x,x)
[Out] k^(x^2)/(2*log(k))
```

sympy [A] time = 0.10, size = 17, normalized size = 1.31

$$\begin{cases} \frac{k^{x^2}}{2\log(k)} & \text{for}\ 2\log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k**(x**2)*x,x)
[Out] Piecewise((k**(x**2)/(2*log(k)), Ne(2*log(k), 0)), (x**2/2, True))
```

3.166 $\int e^{x^2} \, dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\ \text{Erfi}(x)$$

[Out] 1/2*erfi(x)*Pi^(1/2)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2204}

$$\frac{1}{2}\sqrt{\pi}\,\text{Erfi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2} \, dx = \frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\sqrt{\pi}\ \text{Erfi}(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

fricas [A] time = 0.43, size = 7, normalized size = 0.64

$$\frac{1}{2}\,\sqrt{\pi}\ \text{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2),x, algorithm="fricas")

[Out] 1/2*sqrt(pi)*erfi(x)

giac [C] time = 1.23, size = 9, normalized size = 0.82

$$\frac{1}{2}i\,\sqrt{\pi}\ \text{erf}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2),x, algorithm="giac")

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

maple [A]   time = 0.01, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\ \mathrm{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
[Out] 1/2*erfi(x)*Pi^(1/2)
```

maxima [C]   time = 0.42, size = 9, normalized size = 0.82

$$-\frac{1}{2}i\sqrt{\pi}\ \mathrm{erf}\left(i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

mupad [B]   time = 0.02, size = 7, normalized size = 0.64

$$\frac{\sqrt{\pi}\ \mathrm{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
[Out] (pi^(1/2)*erfi(x))/2
```

sympy [A]   time = 0.20, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\ \mathrm{erfi}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
[Out] sqrt(pi)*erfi(x)/2
```

3.167 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] 1/2*exp(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

fricas [A] time = 0.41, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

giac [A] time = 1.14, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
[Out] 1/2*e^(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
[Out] 1/2*exp(x^2)
```

maxima [A] time = 0.42, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.67

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
[Out] exp(x^2)/2
```

sympy [A] time = 0.08, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
[Out] exp(x**2)/2
```

3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} \, dx$

Optimal. Leaf size=27

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

```
[Out] -exp(1/x)-exp(1/x)/x^2+exp(1/x)/x
```

Rubi [A] time = 0.11, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 2212, 2209}

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^(-1)*(1 + x))/x^4,x]
```

```
[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} \, dx &= \int \left(\frac{e^{\frac{1}{x}}}{x^4}+\frac{e^{\frac{1}{x}}}{x^3}\right) dx\\
&= \int \frac{e^{\frac{1}{x}}}{x^4} \, dx+\int \frac{e^{\frac{1}{x}}}{x^3} \, dx\\
&= -\frac{e^{\frac{1}{x}}}{x^2}-\frac{e^{\frac{1}{x}}}{x}-2 \int \frac{e^{\frac{1}{x}}}{x^3} \, dx-\int \frac{e^{\frac{1}{x}}}{x^2} \, dx\\
&= e^{\frac{1}{x}}-\frac{e^{\frac{1}{x}}}{x^2}+\frac{e^{\frac{1}{x}}}{x}+2 \int \frac{e^{\frac{1}{x}}}{x^2} \, dx\\
&= -e^{\frac{1}{x}}-\frac{e^{\frac{1}{x}}}{x^2}+\frac{e^{\frac{1}{x}}}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.63

$$\frac{e^{\frac{1}{x}}\left(-x^2+x-1\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^(-1)*(1 + x))/x^4,x]
[Out] (E^x^(-1)*(-1 + x - x^2))/x^2
```

fricas [A] time = 0.40, size = 17, normalized size = 0.63

$$-\frac{\left(x^2-x+1\right)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")
[Out] -(x^2 - x + 1)*e^(1/x)/x^2
```

giac [A] time = 1.01, size = 24, normalized size = 0.89

$$\frac{e^{\frac{1}{x}}}{x}-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="giac")
[Out] e^(1/x)/x - e^(1/x)/x^2 - e^(1/x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.67

$$-\frac{\left(x^2-x+1\right)\mathrm{e}^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/x)*(x+1)/x^4,x)
[Out] -(x^2-x+1)*exp(1/x)/x^2
```

maxima [C] time = 0.53, size = 17, normalized size = 0.63

$$-\Gamma\left(3,-\frac{1}{x}\right)+\Gamma\left(2,-\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")
[Out] -gamma(3, -1/x) + gamma(2, -1/x)
```

mupad [B] time = 0.16, size = 17, normalized size = 0.63

$$-\frac{\mathrm{e}^{1/x}\left(x^2-x+1\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(1/x)*(x + 1))/x^4,x)
```

```
[Out] -(exp(1/x)*(x^2 - x + 1))/x^2
```

sympy [A] time = 0.10, size = 14, normalized size = 0.52

$$\frac{\left(-x^2+x-1\right)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x**4,x)
```

```
[Out] (-x**2 + x - 1)*exp(1/x)/x**2
```

3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

```
[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

Rubi [F] time = 0.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] Defer[Int][(E^(1 - E^x^2*x + 2*x^2)*x)/(-1 + E^x^2*x)^2, x] + 2*Defer[Int][
(E^(1 - E^x^2*x + 2*x^2)*x^3)/(-1 + E^x^2*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx &= \int \frac{e^{1-e^{x^2}x+2x^2}x\left(1+2x^2\right)}{\left(1-e^{x^2}x\right)^2}\,dx\\
&= \int \left(\frac{e^{1-e^{x^2}x+2x^2}x}{\left(-1+e^{x^2}x\right)^2}+\frac{2e^{1-e^{x^2}x+2x^2}x^3}{\left(-1+e^{x^2}x\right)^2}\right)dx\\
&= 2\int \frac{e^{1-e^{x^2}x+2x^2}x^3}{\left(-1+e^{x^2}x\right)^2}\,dx+\int \frac{e^{1-e^{x^2}x+2x^2}x}{\left(-1+e^{x^2}x\right)^2}\,dx
\end{aligned}$$

Mathematica [A] time = 0.17, size = 25, normalized size = 1.00

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

fricas [A] time = 0.42, size = 36, normalized size = 1.44

$$-\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
fricas")
```

[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))

giac [A] time = 1.02, size = 36, normalized size = 1.44

$$-\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="giac")

[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))

maple [A] time = 0.12, size = 23, normalized size = 0.92

$$-\frac{e^{-x\,e^{x^2}+1}}{x\,e^{x^2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x)

[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)

maxima [A] time = 0.75, size = 22, normalized size = 0.88

$$-\frac{e^{\left(-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(x^2\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="maxima")

[Out] -e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e^{2\,x^2-x\,e^{x^2}+1}\,\left(2\,x^3+x\right)}{{\left(x\,e^{x^2}-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2,x)

[Out] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2, x)

sympy [A] time = 0.35, size = 31, normalized size = 1.24

$$-\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)

[Out] -exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))

3.170 $\int e^{e^{e^{e^x}}}\,dx$

Optimal. Leaf size=12

$$\text{Int}\left(e^{e^{e^{e^x}}},x\right)$$

[Out] CannotIntegrate(exp(exp(exp(exp(x)))),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification is Not applicable to the result.

[In] Int[E^E^E^E^x,x]

[Out] Defer[Subst][Defer[Int][E^E^E^x/x, x], x, E^x]

Rubi steps

$$\int e^{e^{e^{e^x}}}\,dx = \text{Subst}\left(\int \frac{e^{e^{e^x}}}{x}\,dx,x,e^x\right)$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[E^E^E^E^x,x]

[Out] Integrate[E^E^E^E^x, x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="fricas")

[Out] integral(e^(e^(e^(e^x))), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="giac")

```
[Out] integrate(e^(e^(e^(e^x))), x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x)))),x)
[Out] int(exp(exp(exp(exp(x)))),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="maxima")
[Out] integrate(e^(e^(e^(e^x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x)))),x)
[Out] int(exp(exp(exp(exp(x)))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x)
[Out] Integral(exp(exp(exp(exp(x)))), x)
```

3.171 $\int e^x \log(x)\,dx$

Optimal. Leaf size=11

$$e^x \log(x) - \mathrm{Ei}(x)$$

```
[Out] -Ei(x)+exp(x)*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2194, 2554, 2178}

$$e^x \log(x) - \mathrm{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[x],x]
```

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int e^x \log(x)\,dx &= e^x \log(x) - \int \frac{e^x}{x}\,dx \\ &= -\mathrm{Ei}(x) + e^x \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$e^x \log(x) - \mathrm{Ei}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[x],x]
```

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

fricas [A] time = 0.43, size = 10, normalized size = 0.91

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="fricas")
```

```
[Out] e^x*log(x) - Ei(x)
```

giac [A] time = 1.19, size = 10, normalized size = 0.91

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="giac")
[Out] e^x*log(x) - Ei(x)
```

maple [A] time = 0.01, size = 12, normalized size = 1.09

$$e^x \ln(x) + \text{Ei}(1, -x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(x),x)
[Out] exp(x)*ln(x)+Ei(1,-x)
```

maxima [A] time = 0.53, size = 10, normalized size = 0.91

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="maxima")
[Out] e^x*log(x) - Ei(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.91

$$e^x \ln(x) - \text{ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*log(x),x)
[Out] exp(x)*log(x) - ei(x)
```

sympy [A] time = 1.95, size = 8, normalized size = 0.73

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(x),x)
[Out] exp(x)*log(x) - Ei(x)
```

3.172 $\int e^x x \log(x)\, dx$

Optimal. Leaf size=22

$$\text{Ei}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

```
[Out] -exp(x)+Ei(x)-exp(x)*ln(x)+exp(x)*x*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2176, 2194, 2554, 2199, 2178}

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !$UseGamma === True
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x x \log(x)\, dx &= -e^x \log(x) + e^x x \log(x) - \int \frac{e^x(-1+x)}{x}\, dx \\ &= -e^x \log(x) + e^x x \log(x) - \int \left(e^x - \frac{e^x}{x}\right) dx \\ &= -e^x \log(x) + e^x x \log(x) - \int e^x\, dx + \int \frac{e^x}{x}\, dx \\ &= -e^x + \mathrm{Ei}(x) - e^x \log(x) + e^x x \log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.77

$$\mathrm{Ei}(x) - e^x + e^x(x-1)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Log[x],x]
[Out] -E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]
```

fricas [A] time = 0.42, size = 15, normalized size = 0.68

$$(x-1)e^x \log(x) + \mathrm{Ei}(x) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="fricas")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

giac [A] time = 1.15, size = 15, normalized size = 0.68

$$(x-1)e^x \log(x) + \mathrm{Ei}(x) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="giac")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

maple [A] time = 0.02, size = 21, normalized size = 0.95

$$(x-1)\, \mathrm{e}^x \ln(x) - \mathrm{Ei}\,(1, -x) - \mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*ln(x),x)
[Out] (x-1)*exp(x)*ln(x)-Ei(1,-x)-exp(x)
```

maxima [A] time = 0.52, size = 15, normalized size = 0.68

$$(x-1)e^x \log(x) + \mathrm{Ei}(x) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="maxima")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

mupad [B] time = 0.22, size = 28, normalized size = 1.27

$$\mathrm{ei}(x) - \frac{x\, \mathrm{e}^x + x\, \mathrm{e}^x\, \ln(x) - x^2\, \mathrm{e}^x\, \ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*log(x),x)
```

```
[Out] ei(x) - (x*exp(x) + x*exp(x)*log(x) - x^2*exp(x)*log(x))/x
```

sympy [**A**] time = 3.34, size = 17, normalized size = 0.77

$$(xe^x - e^x)\log(x) - e^x + \operatorname{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*ln(x),x)
```

```
[Out] (x*exp(x) - exp(x))*log(x) - exp(x) + Ei(x)
```

3.173 $\int e^{2x}\log(e^x)\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}e^{2x}\log(e^x)-\frac{e^{2x}}{4}$$

```
[Out] -1/4*exp(2*x)+1/2*exp(2*x)*ln(exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2194, 2554, 12}

$$\frac{1}{2}e^{2x}\log(e^x)-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Log[E^x],x]
[Out] -E^(2*x)/4 + (E^(2*x)*Log[E^x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\log(e^x)\,dx&=\frac{1}{2}e^{2x}\log(e^x)-\int\frac{e^{2x}}{2}\,dx\\&=\frac{1}{2}e^{2x}\log(e^x)-\frac{1}{2}\int e^{2x}\,dx\\&=-\frac{e^{2x}}{4}+\frac{1}{2}e^{2x}\log(e^x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.74

$$\frac{1}{4}e^{2x}\left(2\log(e^x)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Log[E^x],x]
[Out] (E^(2*x)*(-1 + 2*Log[E^x]))/4
```

fricas [A] time = 0.41, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

giac [A] time = 1.22, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

maple [A] time = 0.02, size = 28, normalized size = 1.22

$$\frac{x\,e^{2x}}{2}-\frac{e^{2x}}{4}+\frac{(-x+\ln(e^{x}))\,e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*ln(exp(x)),x)
```

```
[Out] 1/2*exp(2*x)*x-1/4*exp(2*x)+1/2*exp(2*x)*(ln(exp(x))-x)
```

maxima [A] time = 0.44, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.48

$$\frac{e^{2x}\,(2x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x))*exp(2*x),x)
```

```
[Out] (exp(2*x)*(2*x - 1))/4
```

sympy [A] time = 0.10, size = 10, normalized size = 0.43

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*ln(exp(x)),x)
```

```
[Out] (2*x - 1)*exp(2*x)/4
```

3.174 $\int \left(2x+\sqrt{2}\, x^2\right)\, dx$

Optimal. Leaf size=16

$$\frac{\sqrt{2}\, x^3}{3}+x^2$$

```
[Out] x^2+1/3*x^3*2^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{\sqrt{2}\, x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi steps

$$\int \left(2x+\sqrt{2}\, x^2\right)\, dx = x^2+\frac{\sqrt{2}\, x^3}{3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\sqrt{2}\, x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: SyntaxError >> Malformed expression
```

giac [A] time = 1.07, size = 12, normalized size = 0.75

$$\frac{1}{3}\,\sqrt{2}\, x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{2}\, x^3}{3}+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x+x^2*2^(1/2),x)
```

```
[Out] x^2+1/3*x^3*2^(1/2)
```

maxima [A] time = 0.95, size = 12, normalized size = 0.75

$$\frac{1}{3}\sqrt{2}\,x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{x^2\left(\sqrt{2}\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 2^(1/2)*x^2,x)
```

```
[Out] (x^2*(2^(1/2)*x + 3))/3
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{\sqrt{2}\,x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x**2*2**(1/2),x)
```

```
[Out] sqrt(2)*x**3/3 + x**2
```

3.175 $\int \frac{\log(x)}{\sqrt{b+ax}}\,dx$

Optimal. Leaf size=57

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

```
[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b
)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2319, 50, 63, 208}

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/a + (2*Sq
rt[b + a*x]*Log[x])/a
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(x)}{\sqrt{b+ax}}\,dx &= \frac{2\sqrt{b+ax}\,\log(x)}{a} - \frac{2\int \frac{\sqrt{b+ax}}{x}\,dx}{a} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax}\,\log(x)}{a} - \frac{(2b)\int \frac{1}{x\sqrt{b+ax}}\,dx}{a} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax}\,\log(x)}{a} - \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax}\right)}{a^2} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax}\,\log(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.75

$$\frac{2(\log(x)-2)\sqrt{ax+b} + 4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))/
a
```

fricas [A] time = 0.45, size = 89, normalized size = 1.56

$$\left[\frac{2\left(\sqrt{ax+b}\left(\log(x)-2\right)+\sqrt{b}\log\left(\frac{ax+2\sqrt{ax+b}\sqrt{b}+2b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}\left(\log(x)-2\right)-2\sqrt{-b}\arctan\left(\frac{\sqrt{ax+b}\sqrt{-b}}{b}\right)\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(a*x + b)*(log(x) - 2) + sqrt(b)*log((a*x + 2*sqrt(a*x + b)*sqrt(b)
 + 2*b)/x))/a, 2*(sqrt(a*x + b)*(log(x) - 2) - 2*sqrt(-b)*arctan(sqrt(a*x +
 b)*sqrt(-b)/b))/a]
```

giac [A] time = 1.17, size = 48, normalized size = 0.84

$$-\frac{2\left(\frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{ax+b}\,\log(x) + 2\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2*
sqrt(a*x + b))/a
```

maple [A] time = 0.02, size = 48, normalized size = 0.84

$$\frac{4\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{ax+b}\,\ln(x)}{a} - \frac{4\sqrt{ax+b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^(1/2),x)
```

```
[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b
)^(1/2)/a
```

maxima [A] time = 0.96, size = 58, normalized size = 1.02

$$\frac{2\left(\sqrt{ax+b}\log(x)-\sqrt{b}\log\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right)-2\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(a*x + b)*log(x) - sqrt(b)*log((sqrt(a*x + b) - sqrt(b))/(sqrt(a*x +
 b) + sqrt(b))) - 2*sqrt(a*x + b))/a
```

mupad [B] time = 0.09, size = 49, normalized size = 0.86

$$\frac{2\sqrt{b}\,\ln\left(\frac{2\,b+a\,x+2\,\sqrt{b}\,\sqrt{b+a\,x}}{x}\right)}{a}+\frac{2\,\left(\ln(x)-2\right)\,\sqrt{b+a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(b + a*x)^(1/2),x)
```

```
[Out] (2*b^(1/2)*log((2*b + a*x + 2*b^(1/2)*(b + a*x)^(1/2))/x))/a + (2*(log(x) -
 2)*(b + a*x)^(1/2))/a
```

sympy [B] time = 4.15, size = 920, normalized size = 16.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**(1/2),x)
```

```
[Out] Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x +
b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sq
rt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2
*I*pi*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1
)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*l
og(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x +
 b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(x + b
/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x +
b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sq
rt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2
*I*pi*sqrt(x + b/a)/sqrt(a), (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) >
 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)
*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x
 + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), 1/Abs(x
 + b/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(
x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x +
b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,),
 (0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (
0,)), x + b/a)/sqrt(a) + meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*l
og(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqr
t(a), Abs(b/(a*(x + b/a))) > 1), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x +
 b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*
```

```
log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt(x
+ b/a)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b/a)/
sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a) + m
eijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi*mei
jerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqrt(a), True))
```

3.176 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

```
[Out] -1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3
/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2)*
(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{8bd}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,d\right.}{4b^2d}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{4b^2d}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.29, size = 118, normalized size = 1.02

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)(ad+b(c+2dx)) - (bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4b^2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(a*d + b*(c + 2*d*x)) - (b*c - a*d)^(5/2
)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(4*b^2*d^(3/2)*Sqrt[c + d*x])
```

fricas [A] time = 0.44, size = 300, normalized size = 2.59

$$\left[\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx + bc + ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}\right.}{16\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

giac [B] time = 1.22, size = 232, normalized size = 2.00

$$-\frac{\frac{4\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}} - \sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a|b|}{b^2} - \frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c}{}\right.}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

maple [B] time = 0.02, size = 305, normalized size = 2.63

$$-\frac{\sqrt{(bx+a)(dx+c)}\,a^2 d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{\sqrt{(bx+a)(dx+c)}\,ac\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{\right.}{4\sqrt{dx+c}\,\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a-1/4/d
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*a^2+1/4*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x
+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^
(1/2))/(b*d)^(1/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^
(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2
))/(b*d)^(1/2)*c^2*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.17, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2)/(8*b^(3/2)*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

3.177 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

giac [A] time = 0.97, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

maxima [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{2\left(a+b\,x\right)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\left(a+bx\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

3.178 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx&=\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{3/2}(3bx-2a)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-2*a + 3*b*x))/(15*b^2)
```

fricas [A] time = 0.41, size = 30, normalized size = 0.88

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

giac [B] time = 1.18, size = 66, normalized size = 1.94

$$\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a}{b}+\frac{3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\,(-3bx+2a)}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

maxima [A] time = 0.42, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

sympy [B] time = 1.14, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

3.179 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx &= \int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

fricas [A] time = 0.42, size = 42, normalized size = 0.79

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

giac [B] time = 1.03, size = 93, normalized size = 1.75

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{3}{2}}\left(15b^2x^2-12axb+8a^2\right)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

maxima [A] time = 0.42, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

mupad [B] time = 0.15, size = 37, normalized size = 0.70

$$\frac{30\,(a+b\,x)^{7/2}-84\,a\,(a+b\,x)^{5/2}+70\,a^2\,(a+b\,x)^{3/2}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

sympy [B] time = 1.75, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{4(}{105a^8b^3+315a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
```

```
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

3.180 $\int \frac{\sqrt{a+bx}}{x} \, dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x} \, dx &= 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} \, dx \\
&= 2\sqrt{a+bx} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} \, dx, x, \sqrt{a+bx}\right)}{b} \\
&= 2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 0.42, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a},2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

giac [A] time = 1.10, size = 32, normalized size = 0.91

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

maxima [A] time = 0.97, size = 42, normalized size = 1.20

$$\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

mupad [B] time = 0.15, size = 27, normalized size = 0.77

$$2\sqrt{a+b\,x}-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
```

```
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

sympy [B] time = 1.41, size = 68, normalized size = 1.94

$$-2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

3.181 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^2}\,dx &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= -\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.21

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x*Sqrt[a +
b*x]))
```

fricas [A] time = 0.42, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2\,ax}, \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

giac [A] time = 1.29, size = 41, normalized size = 1.05

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

maple [A] time = 0.01, size = 37, normalized size = 0.95

$$2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 0.96, size = 47, normalized size = 1.21

$$\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.87, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

3.182 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

[Out] 2*(b*x+a)^(1/2)/b

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

fricas [A] time = 0.40, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(b*x + a)/b

giac [A] time = 1.18, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*x + a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x)
[Out] 2*(b*x+a)^(1/2)/b
```

maxima [A] time = 0.42, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(1/2))/b
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
[Out] 2*sqrt(a + b*x)/b
```

3.183 $\int \frac{x}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^2-2*a*(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

fricas [A] time = 0.41, size = 19, normalized size = 0.59

$$\frac{2\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

giac [A] time = 1.21, size = 23, normalized size = 0.72

$$\frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

maple [A] time = 0.01, size = 21, normalized size = 0.66

$$-\frac{2\sqrt{bx+a}\,\left(-bx+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x)
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

maxima [A] time = 0.42, size = 26, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2} - \frac{2\sqrt{bx+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x} - 2\,(a+b\,x)^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

sympy [B] time = 1.11, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

3.184 $\int \frac{x^2}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

```
[Out] -4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {43}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}} \, dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right) dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

fricas [A] time = 0.42, size = 31, normalized size = 0.61

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

giac [A] time = 1.27, size = 37, normalized size = 0.73

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

maple [A] time = 0.01, size = 32, normalized size = 0.63

$$\frac{2\sqrt{bx+a}\,\left(3b^2x^2-4axb+8a^2\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x)
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

maxima [A] time = 0.43, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,(a+b\,x)^{5/2}-20\,a\,(a+b\,x)^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

sympy [B] time = 1.70, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{40a^{\frac{19}{2}}bx\sqrt{}}{15a^8b^3+45a^7b^4x+45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
```

```
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

3.185 $\int \frac{1}{x\sqrt{a+bx}} \, dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}} \, dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} \, dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.41, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

giac [A] time = 1.01, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 0.95, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 0.14, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [**A**] time = 1.07, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

```
[Out] b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{ax}-\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}\\
&= -\frac{\sqrt{a+bx}}{ax}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a}\\
&= -\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.15

$$\frac{\sqrt{a+bx}\left(\frac{b\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{\frac{bx}{a}+1}}-\frac{a}{x}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(-(a/x) + (b*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[1 + (b*x)/a]))
/a^2
```

fricas [A] time = 0.44, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x}, -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

giac [A] time = 1.08, size = 47, normalized size = 1.15

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 0.99, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 0.16, size = 33, normalized size = 0.80

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

sympy [A] time = 2.21, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

3.187 $\int (a+bx)^{p/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

```
[Out] 2*(b*x+a)^(1+1/2*p)/b/(2+p)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32}

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{p/2}\,dx=\frac{2(a+bx)^{\frac{2+p}{2}}}{b(2+p)}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{2(a+bx)^{\frac{p}{2}+1}}{bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2))/(2*b + b*p)
```

fricas [A] time = 0.44, size = 25, normalized size = 1.09

$$\frac{2\,(bx+a)\sqrt{bx+a}^{\,p}}{bp+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)
```

giac [A] time = 1.04, size = 21, normalized size = 0.91

$$\frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$\frac{2\,(bx+a)\,(bx+a)^{\frac{p}{2}}}{\left(p+2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^(1/2))^p,x)
```

```
[Out] 2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)
```

maxima [A] time = 0.42, size = 21, normalized size = 0.91

$$\frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

mupad [B] time = 0.25, size = 21, normalized size = 0.91

$$\frac{2\,{\left(a+b\,x\right)}^{\frac{p}{2}+1}}{b\,\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(p/2),x)
```

```
[Out] (2*(a + b*x)^(p/2 + 1))/(b*(p + 2))
```

sympy [A] time = 0.06, size = 26, normalized size = 1.13

$$\frac{\begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for}\ \frac{p}{2}\neq -1 \\ \log{(a+bx)} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p/2, -1)), (log(a + b*x), Tru
e))/b
```

3.188 $\int x(a+bx)^{p/2}\,dx$

Optimal. Leaf size=48

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

```
[Out] -2*a*(b*x+a)^(1+1/2*p)/b^2/(2+p)+2*(b*x+a)^(2+1/2*p)/b^2/(4+p)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(p/2),x]
```

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^2
*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{p/2}\,dx &= \int\left(\frac{(a+bx)^{1+\frac{p}{2}}}{b}-\frac{a(a+bx)^{p/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{\frac{2+p}{2}}}{b^2(2+p)}+\frac{2(a+bx)^{\frac{4+p}{2}}}{b^2(4+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.79

$$\frac{2(a+bx)^{\frac{p}{2}+1}(b(p+2)x-2a)}{b^2(p+2)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))
```

fricas [A] time = 0.42, size = 58, normalized size = 1.21

$$\frac{2\left(abpx+\left(b^2p+2\,b^2\right)x^2-2\,a^2\right)\sqrt{bx+a}^{\,p}}{b^2p^2+6\,b^2p+8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2*
p + 8*b^2)
```

giac [A] time = 1.28, size = 86, normalized size = 1.79

$$\frac{2\left((bx+a)^{\frac{1}{2}p}b^2px^2+(bx+a)^{\frac{1}{2}p}abpx+2\,(bx+a)^{\frac{1}{2}p}b^2x^2-2\,(bx+a)^{\frac{1}{2}p}a^2\right)}{b^2p^2+6\,b^2p+8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(1
/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)
```

maple [A] time = 0.00, size = 43, normalized size = 0.90

$$-\frac{2\left(-xpb-2bx+2a\right)(bx+a)\,(bx+a)^{\frac{p}{2}}}{\left(p^2+6p+8\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^(1/2))^p,x)
```

```
[Out] -2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)
```

maxima [A] time = 0.43, size = 45, normalized size = 0.94

$$\frac{2\left(b^2\left(p+2\right)x^2+abpx-2\,a^2\right)(bx+a)^{\frac{1}{2}p}}{\left(p^2+6\,p+8\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b^
2)
```

mupad [B] time = 0.45, size = 94, normalized size = 1.96

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if } p=-2\\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } p=-4\\ \frac{2\left(\frac{(a+b\,x)^{\frac{p}{2}+2}}{p+4}-\frac{a\,(a+b\,x)^{\frac{p}{2}+1}}{p+2}\right)}{b^2} & \text{if } p\neq -2\land p\neq -4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(p/2),x)
```

```
[Out] piecewise(p == -2, -(a*log(a + b*x) - b*x)/b^2, p == -4, (log(a + b*x) + a/
(a + b*x))/b^2, p ~= -2 & p ~= -4, (2*((a + b*x)^(p/2 + 2)/(p + 4) - (a*(a
+ b*x)^(p/2 + 1))/(p + 2)))/b^2)
```

sympy [A] time = 0.69, size = 216, normalized size = 4.50

$$\begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b = 0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -4 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) +
a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a*l
og(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2 +
6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b
**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b
**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))
```

3.189 $\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

Optimal. Leaf size=55

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}\,x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] x*arctan(-1+x*2^(1/2))-1/2*arctan(-1+x*2^(1/2))*2^(1/2)-1/4*ln(1+x^2-x*2^(1
/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5203, 12, 634, 617, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}\,x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 - Sqrt[2]*x
 + x^2]/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)dx = -x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\int \frac{x}{\sqrt{2}\left(1-\sqrt{2}\,x+x^2\right)}\,dx$$
$$= -x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\frac{\int \frac{x}{1-\sqrt{2}\,x+x^2}\,dx}{\sqrt{2}}$$
$$= -x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{\int \frac{-\sqrt{2}+2x}{1-\sqrt{2}\,x+x^2}\,dx}{2\sqrt{2}}$$
$$= -x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,x\right)}{\sqrt{2}}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.87

$$\frac{1}{4}\left(2\left(\sqrt{2}-2x\right)\tan^{-1}\left(1-\sqrt{2}\,x\right)-\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] (2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
)/4
```

fricas [A] time = 0.46, size = 37, normalized size = 0.67

$$\frac{1}{2}\left(2x-\sqrt{2}\right)\arctan\left(\sqrt{2}\,x-1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - sqrt(2))*arctan(sqrt(2)*x - 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x
 + 1)
```

giac [A] time = 1.14, size = 52, normalized size = 0.95

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

maple [A] time = 0.01, size = 42, normalized size = 0.76

$$x\arctan\left(\sqrt{2}\,x-1\right)-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{2}-\frac{\sqrt{2}\,\ln\left(\left(\sqrt{2}\,x-1\right)^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x)
```

```
[Out] x*arctan(2^(1/2)*x-1)-1/2*2^(1/2)*arctan(2^(1/2)*x-1)-1/4*2^(1/2)*ln((2^(1/
2)*x-1)^2+1)
```

maxima [A] time = 0.97, size = 52, normalized size = 0.95

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

mupad [B] time = 0.16, size = 43, normalized size = 0.78

$$\operatorname{atan}\left(\frac{\sqrt{2}\,\left(2x-\sqrt{2}\right)}{2}\right)\left(x-\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{2}\,\ln\left(\left(2x-\sqrt{2}\right)^{2}+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((2^(1/2)*(2*x - 2^(1/2)))/2),x)
```

```
[Out] atan((2^(1/2)*(2*x - 2^(1/2)))/2)*(x - 2^(1/2)/2) - (2^(1/2)*log((2*x - 2^(
1/2))^2 + 2))/4
```

sympy [B] time = 1.05, size = 230, normalized size = 4.18

$$\frac{4x^{3}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^{2}-4\sqrt{2}x+4}-\frac{\sqrt{2}x^{2}\log\left(x^{2}-\sqrt{2}x+1\right)}{4x^{2}-4\sqrt{2}x+4}-\frac{6\sqrt{2}x^{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^{2}-4\sqrt{2}x+4}+\frac{2x\log\left(x^{2}-\sqrt{2}x+1\right)}{4x^{2}-4\sqrt{2}x+4}+\frac{8x\operatorname{atan}\left(\sqrt{2}x\right.}{4x^{2}-4\sqrt{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

```
[Out] 4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x**2*log(x*
*2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt(2)*x**2*atan(sqrt(2
)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log(x**2 - sqrt(2)*x + 1)/(4*x**2
 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) -
sqrt(2)*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(2)*at
an(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4)
```

3.190 $\int \frac{1}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=12

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) \end{aligned}$$

Mathematica [B] time = 0.00, size = 38, normalized size = 3.17

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + x^2],x]
```

```
[Out] -1/2*Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]/2
```

fricas [A] time = 0.41, size = 14, normalized size = 1.17

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

giac [A] time = 1.05, size = 15, normalized size = 1.25

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2),x)
[Out] ln(x+(x^2-1)^(1/2))
```

maxima [A] time = 0.41, size = 14, normalized size = 1.17

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

mupad [B] time = 0.20, size = 10, normalized size = 0.83

$$\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^(1/2),x)
[Out] log(x + (x^2 - 1)^(1/2))
```

sympy [A] time = 0.14, size = 2, normalized size = 0.17

$$\operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2),x)
[Out] acosh(x)
```

3.191 $\int \sqrt{x}\sqrt{1+x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\sqrt{x+1}\,x^{3/2}+\frac{1}{4}\sqrt{x+1}\,\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/4*arcsinh(x^(1/2))+1/2*x^(3/2)*(1+x)^(1/2)+1/4*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\frac{1}{2}\sqrt{x+1}\,x^{3/2}+\frac{1}{4}\sqrt{x+1}\,\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[x]]/4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{1+x}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1+x}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{8}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}\,(2x+1)-\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(1 + 2*x) - ArcSinh[Sqrt[x]])/4
```

fricas [A] time = 0.41, size = 34, normalized size = 0.79

$$\frac{1}{4}(2x+1)\sqrt{x+1}\sqrt{x}+\frac{1}{8}\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1
)
```

giac [A] time = 1.03, size = 39, normalized size = 0.91

$$\frac{1}{4}(2x-3)\sqrt{x+1}\sqrt{x}+\sqrt{x+1}\sqrt{x}+\frac{1}{4}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 1/4*log(sqrt(x +
1) - sqrt(x))
```

maple [A] time = 0.00, size = 50, normalized size = 1.16

$$\frac{(x+1)^{\frac{3}{2}}\sqrt{x}}{2}-\frac{\sqrt{x+1}\sqrt{x}}{4}-\frac{\sqrt{(x+1)x}\ \ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{x+1}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+1)^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(x+1)^(3/2)-1/4*x^(1/2)*(x+1)^(1/2)-1/8*(x*(x+1))^(1/2)/(x+1)^(
1/2)/x^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [B] time = 0.42, size = 71, normalized size = 1.65

$$\frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2}-\frac{2(x+1)}{x}+1\right)}-\frac{1}{8}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{8}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1
)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) -
 1)
```

mupad [B] time = 0.22, size = 30, normalized size = 0.70

$$\sqrt{x}\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x+1}-\frac{\ln\left(x+\sqrt{x}\sqrt{x+1}+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/4)*(x + 1)^(1/2) - log(x + x^(1/2)*(x + 1)^(1/2) + 1/2)/8
```

sympy [A] time = 2.57, size = 119, normalized size = 2.77

$$\begin{cases} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4} + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}} - \frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}} + \frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{i \operatorname{asin}\left(\sqrt{x+1}\right)}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)**
(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqrt(
x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-x))
 - I*sqrt(x + 1)/(4*sqrt(-x)), True))
```

3.192 $\int \sin\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3361, 3296, 2637}

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\sqrt{x}\right) dx &= 2\,\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

fricas [A] time = 0.42, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

giac [A] time = 0.93, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

maple [A] time = 0.02, size = 17, normalized size = 0.77

$$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.45, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

mupad [B] time = 0.25, size = 16, normalized size = 0.73

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

sympy [A] time = 0.31, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.193 $\int \frac{x}{(1-x^2)^{9/8}} \, dx$

Optimal. Leaf size=13

$$\frac{4}{\sqrt[8]{1-x^2}}$$

[Out] 4/(-x^2+1)^(1/8)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^2)^(9/8),x]

[Out] 4/(1 - x^2)^(1/8)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(1-x^2)^{9/8}} \, dx = \frac{4}{\sqrt[8]{1-x^2}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - x^2)^(9/8),x]

[Out] 4/(1 - x^2)^(1/8)

fricas [A] time = 0.40, size = 18, normalized size = 1.38

$$-\frac{4\left(-x^2+1\right)^{\frac{7}{8}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")

[Out] -4*(-x^2 + 1)^(7/8)/(x^2 - 1)

giac [A] time = 1.00, size = 11, normalized size = 0.85

$$\frac{4}{\left(-x^2+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="giac")
[Out] 4/(-x^2 + 1)^(1/8)
```

maple [A] time = 0.00, size = 18, normalized size = 1.38

$$-\frac{4\left(x-1\right)\left(x+1\right)}{\left(-x^{2}+1\right)^{\frac{9}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(9/8),x)
[Out] -4*(x-1)*(x+1)/(-x^2+1)^(9/8)
```

maxima [A] time = 0.41, size = 11, normalized size = 0.85

$$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")
[Out] 4/(-x^2 + 1)^(1/8)
```

mupad [B] time = 0.35, size = 11, normalized size = 0.85

$$\frac{4}{\left(1-x^{2}\right)^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^2)^(9/8),x)
[Out] 4/(1 - x^2)^(1/8)
```

sympy [A] time = 0.97, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{1-x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(9/8),x)
[Out] 4/(1 - x**2)**(1/8)
```

3.194 $\int \frac{x}{\sqrt{1-x^4}}\, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] 1/2*arcsin(x^2)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 216}

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^4}}\, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, x^2\right)\\ &= \frac{1}{2}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

fricas [B] time = 0.40, size = 18, normalized size = 2.25

$$-\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

giac [**A**] time = 1.23, size = 6, normalized size = 0.75

$$\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x^2)
```

maple [**A**] time = 0.01, size = 7, normalized size = 0.88

$$\frac{\arcsin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x^2)
```

maxima [**B**] time = 0.96, size = 16, normalized size = 2.00

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

mupad [**B**] time = 0.31, size = 16, normalized size = 2.00

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^4)^(1/2),x)
```

```
[Out] atan(x^2/(1 - x^4)^(1/2))/2
```

sympy [**A**] time = 0.92, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i \operatorname{acosh}\left(x^2\right)}{2} & \text{for}\ \left|x^4\right| > 1 \\ \frac{\operatorname{asin}\left(x^2\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

3.195 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] -1/2*arctanh((x^4+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{1+x^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\
&= -\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]
```

fricas [B] time = 0.40, size = 25, normalized size = 1.79

$$-\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,+1\right)+\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

giac [B] time = 1.19, size = 25, normalized size = 1.79

$$-\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,+1\right)+\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(1/2),x)
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [B] time = 0.44, size = 25, normalized size = 1.79

$$-\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,+1\right)+\frac{1}{4}\,\log\left(\sqrt{x^4+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

mupad [B] time = 0.19, size = 10, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)^(1/2)),x)
[Out] -atanh((x^4 + 1)^(1/2))/2
```

sympy [A] time = 0.92, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1)**(1/2),x)
[Out] -asinh(x**(-2))/2
```

3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=18

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

```
[Out] 1/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 619, 215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x^2+x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}} \, dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} \, dx, x, 1+2x^2\right)}{2\sqrt{3}}\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

fricas [A] time = 0.41, size = 22, normalized size = 1.22

$$-\frac{1}{2} \log\left(-2 x^2 + 2 \sqrt{x^4 + x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

giac [A] time = 0.94, size = 22, normalized size = 1.22

$$-\frac{1}{2} \log\left(-2 x^2 + 2 \sqrt{x^4 + x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

maple [A] time = 0.01, size = 14, normalized size = 0.78

$$\frac{\operatorname{arcsinh}\left(\frac{2 \sqrt{3} \left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1)^(1/2),x)
[Out] 1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + x^2 + 1), x)
```

mupad [B] time = 0.36, size = 18, normalized size = 1.00

$$\frac{\ln\left(\sqrt{x^4 + x^2 + 1} + x^2 + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1)^(1/2),x)
[Out] log((x^2 + x^4 + 1)^(1/2) + x^2 + 1/2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1)**(1/2),x)
[Out] Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

```
[Out] -1/2*arctan(1/2*(-x^2+2)/(-x^4+x^2-1)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1114, 724, 204}

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] -ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x-x^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-2+x^2}{\sqrt{-1+x^2-x^4}}\right)\\
&= \frac{1}{2}\tan^{-1}\left(\frac{-2+x^2}{2\sqrt{-1+x^2-x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 0.93

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] ArcTan[(-2 + x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

fricas [C] time = 0.41, size = 55, normalized size = 1.83

$$\frac{1}{4} i \log\left(\frac{x^2 + 2i \sqrt{-x^4 + x^2 - 1} - 2}{2 x^2}\right) - \frac{1}{4} i \log\left(\frac{x^2 - 2i \sqrt{-x^4 + x^2 - 1} - 2}{2 x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*log(1/2*(x^
2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + x^2 - 1}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 - 1)*x), x)
```

maple [A] time = 0.02, size = 23, normalized size = 0.77

$$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+x^2-1)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

maxima [C] time = 1.01, size = 17, normalized size = 0.57

$$-\frac{1}{2} i \operatorname{arsinh}\left(-\frac{1}{3} \sqrt{3} + \frac{2\sqrt{3}}{3 x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)
```

mupad [B] time = 0.54, size = 32, normalized size = 1.07

$$\frac{\ln\left(\frac{1}{x^2}\right) 1\mathrm{i}}{2} + \frac{\ln\left(x^2 - 2 + \sqrt{-x^4 + x^2 - 1}\, 2\mathrm{i}\right) 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - x^4 - 1)^(1/2)),x)
```

```
[Out] (log(1/x^2)*1i)/2 + (log((x^2 - x^4 - 1)^(1/2)*2i + x^2 - 2)*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-x^4 + x^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)
```

3.198 $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x^2+1}}{1-x}$$

```
[Out] (x^2+1)^(1/2)/(1-x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {803}

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

```
[Out] Sqrt[1 + x^2]/(1 - x)
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && N
eQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g,
 0]
```

Rubi steps

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx=\frac{\sqrt{1+x^2}}{1-x}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.94

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

```
[Out] -(Sqrt[1 + x^2]/(-1 + x))
```

fricas [A] time = 0.40, size = 17, normalized size = 1.00

$$-\frac{x+\sqrt{x^2+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x^2 + 1) - 1)/(x - 1)
```

giac [**B**] time = 1.20, size = 35, normalized size = 2.06

$$-\frac{\sqrt{\frac{2}{x-1}+\frac{2}{(x-1)^2}+1}}{\operatorname{sgn}\left(\frac{1}{x-1}\right)}+\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(2/(x - 1) + 2/(x - 1)^2 + 1)/sgn(1/(x - 1)) + sgn(1/(x - 1))
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.88

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(-x+1)^2/(x^2+1)^(1/2),x)
[Out] -(x^2+1)^(1/2)/(x-1)
```

maxima [**A**] time = 0.95, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 + 1)/(x - 1)
```

mupad [**B**] time = 0.15, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 1)^(1/2)*(x - 1)^2),x)
[Out] -(x^2 + 1)^(1/2)/(x - 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)
[Out] Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)
```

3.199 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

fricas [B] time = 0.39, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

giac [B] time = 1.08, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
[Out] arcsinh(x)
```

maxima [A] time = 0.95, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x)
```

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
[Out] asinh(x)
```

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
[Out] asinh(x)
```

3.200 $\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+x}}{2\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {12, 6688}

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(
2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+x}}{2\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+x}}{\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\,dx\\
&= \frac{1}{2}\int \left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{1+x}}+\frac{1}{\sqrt{2+x}}\right)dx\\
&= \sqrt{x}+\sqrt{1+x}+\sqrt{2+x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.50

$$\frac{1}{2}\left(2\sqrt{x}+2\sqrt{x+1}+2\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 +
 x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] (2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

fricas [A] time = 0.40, size = 14, normalized size = 0.70

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+2}\sqrt{x+1} + \sqrt{x+2}\sqrt{x} + \sqrt{x+1}\sqrt{x}}{2\sqrt{x+2}\sqrt{x+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*
sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$\sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*((x+1)^(1/2)*x^(1/2)+x^(1/2)*(x+2)^(1/2)+(x+1)^(1/2)*(x+2)^(1/2))/x
^(1/2)/(x+1)^(1/2)/(x+2)^(1/2),x)
```

```
[Out] x^(1/2)+(x+1)^(1/2)+(x+2)^(1/2)
```

maxima [A] time = 0.42, size = 14, normalized size = 0.70

$$\sqrt{x+2} + \sqrt{x+1} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

mupad [B] time = 0.34, size = 14, normalized size = 0.70

$$\sqrt{x+1} + \sqrt{x+2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2)*(x + 1)^(1/2))/2 + (x^(1/2)*(x + 2)^(1/2))/2 + ((x + 1)^(1/2)
*(x + 2)^(1/2))/2)/(x^(1/2)*(x + 1)^(1/2)*(x + 2)^(1/2)),x)
```

```
[Out] (x + 1)^(1/2) + (x + 2)^(1/2) + x^(1/2)
```

sympy [A] time = 1.17, size = 17, normalized size = 0.85

$$\sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+
x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)
```

```
[Out] sqrt(x) + sqrt(x + 1) + sqrt(x + 2)
```

3.201 $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$

Optimal. Leaf size=24

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 68, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {12, 6688, 261, 2099}

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2
*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2099

```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[(Coeff[Pm, x, m]*Qn^(p + 1))/(n*(p + 1)*Coeff[Qn, x, n]), x] + Dist
[Simplify[Pm - (Coeff[Pm, x, m]*D[Qn, x])/(n*Coeff[Qn, x, n])], Int[Qn^p, x
], x] /; EqQ[m, n - 1] && EqQ[D[Simplify[Pm - (Coeff[Pm, x, m]*D[Qn, x])/(n
*Coeff[Qn, x, n])], x], 0]] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x]
&& NeQ[p, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx &= \frac{1}{2}\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx\\
&= \frac{1}{2}\int \left(-\frac{3x^2}{\sqrt{1+x^3}}-\frac{2}{\sqrt{1-2x+x^5}}+\frac{5x^4}{\sqrt{1-2x+x^5}}\right)dx\\
&= -\left(\frac{3}{2}\int \frac{x^2}{\sqrt{1+x^3}}\,dx\right)+\frac{5}{2}\int \frac{x^4}{\sqrt{1-2x+x^5}}\,dx-\int \frac{}{\sqrt{1}}\\
&= -\sqrt{1+x^3}+\sqrt{1-2x+x^5}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 24, normalized size = 1.00

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^
5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

fricas [A] time = 0.41, size = 20, normalized size = 0.83

$$\sqrt{x^5-2\,x+1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,\sqrt{x^3+1}\,x^4-3\,\sqrt{x^5-2\,x+1}\,x^2-2\,\sqrt{x^3+1}}{2\,\sqrt{x^5-2\,x+1}\,\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3
 + 1))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3
+1)^(1/2)/(x^5-2*x+1)^(1/2),x)
```

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

maxima [A] time = 1.19, size = 30, normalized size = 1.25

$$\sqrt{x^4+x^3+x^2+x-1}\,\sqrt{x-1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)
```

mupad [B] time = 0.31, size = 20, normalized size = 0.83

$$\sqrt{x^5-2\,x+1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(1/2) + (3*x^2*(x^5 - 2*x + 1)^(1/2))/2 - (5*x^4*(x^3 + 1)^
(1/2))/2)/((x^3 + 1)^(1/2)*(x^5 - 2*x + 1)^(1/2)),x)
```

```
[Out] (x^5 - 2*x + 1)^(1/2) - (x^3 + 1)^(1/2)
```

sympy [**A**] time = 1.34, size = 19, normalized size = 0.79

$$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+1
)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

```
[Out] -sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)
```

3.202 $\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx$

Optimal. Leaf size=27

$$10\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] 10*arctanh(x/(x^2-4)^(1/2))+arctanh(x/(x^2-1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {217, 206}

$$10\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx &= 10\int \frac{1}{\sqrt{-4+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx\\ &= 10\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-4+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= 10\tanh^{-1}\left(\frac{x}{\sqrt{-4+x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 71, normalized size = 2.63

$$-5\log\left(1-\frac{x}{\sqrt{x^2-4}}\right)+5\log\left(\frac{x}{\sqrt{x^2-4}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] -5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[1 - x/Sqrt
[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

fricas [A] time = 0.41, size = 29, normalized size = 1.07

$$-\log\left(-x+\sqrt{x^2-1}\right)-10\,\log\left(-x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))
```

giac [A] time = 1.08, size = 31, normalized size = 1.15

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)-10\,\log\left(\left|-x+\sqrt{x^2-4}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1))) - 10*log(abs(-x + sqrt(x^2 - 4)))
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$10\ln\left(x+\sqrt{x^2-4}\right)+\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x)
```

```
[Out] ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))
```

maxima [A] time = 0.42, size = 31, normalized size = 1.15

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)+10\,\log\left(2\,x+2\,\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))
```

mupad [B] time = 0.69, size = 23, normalized size = 0.85

$$\ln\left(x+\sqrt{x^2-1}\right)+10\,\ln\left(x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^(1/2) + 10/(x^2 - 4)^(1/2),x)
```

```
[Out] log(x + (x^2 - 1)^(1/2)) + 10*log(x + (x^2 - 4)^(1/2))
```

sympy [A] time = 0.18, size = 8, normalized size = 0.30

$$10\,\mathrm{acosh}\left(\frac{x}{2}\right)+\mathrm{acosh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)
```

```
[Out] 10*acosh(x/2) + acosh(x)
```

3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2+x^2}+x} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}+\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}+\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}-(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)-(2a)\,\text{Subst}\left(\int \frac{}{a}\right.\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 127, normalized size = 1.55

$$-\frac{2\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] (-2*Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqr
t[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[x + S
qrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
```

fricas [A] time = 0.46, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
```

```
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

maple [C] time = 0.08, size = 25, normalized size = 0.30

$$2\sqrt{2}\,\sqrt{x}\ \mathrm{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

sympy [C] time = 1.32, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\right|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

3.204 $\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx$

Optimal. Leaf size=12

$$\log\left(\sqrt{x^3+1}+1\right)$$

```
[Out] ln(1+(x^3+1)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {12, 2155, 31}

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx &= \frac{3}{2}\int \frac{x^2}{1+x^3+\sqrt{1+x^3}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+\sqrt{1+x}}\,dx, x, x^3\right)\\
&= \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1+x^3}\right)\\
&= \log\left(1+\sqrt{1+x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
[Out] Log[1 + Sqrt[1 + x^3]]
```

fricas [B] time = 0.42, size = 29, normalized size = 2.42

$$\frac{3}{2}\log(x)+\frac{1}{2}\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{2}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fricas")
[Out] 3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) - 1)
```

giac [A] time = 1.29, size = 10, normalized size = 0.83

$$\log\left(\sqrt{x^3+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="giac")
[Out] log(sqrt(x^3 + 1) + 1)
```

maple [B] time = 0.01, size = 39, normalized size = 3.25

$$\operatorname{arctanh}\left(\sqrt{x^3+1}\right)+\frac{3\ln(x)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln\left(x^3+1\right)}{2}-\frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x)
[Out] -1/2*ln(x+1)-1/2*ln(x^2-x+1)+3/2*ln(x)+1/2*ln(x^3+1)+arctanh((x^3+1)^(1/2))
```

maxima [B] time = 0.98, size = 40, normalized size = 3.33

$$-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(\frac{x^3+\sqrt{x^2-x+1}\sqrt{x+1}+1}{\sqrt{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")
[Out] -1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1) + 1)/sqrt(
x + 1))
```

mupad [B] time = 0.06, size = 169, normalized size = 14.08

$$\frac{3\ln(x)}{2}+\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2)/(2*((x^3 + 1)^(1/2) + x^3 + 1)),x)
[Out] (3*log(x))/2 + (3*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1
i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3
```

```
/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/(
(3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [B] time = 154.69, size = 48, normalized size = 4.00

$$-\frac{\log\left(2\sqrt{x^3+1}\right)}{2}+\frac{\log\left(2\sqrt{x^3+1}+2\right)}{2}+\frac{\log\left(3x^3+3\sqrt{x^3+1}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)
```

```
[Out] -log(2*sqrt(x**3 + 1))/2 + log(2*sqrt(x**3 + 1) + 2)/2 + log(3*x**3 + 3*sqr
t(x**3 + 1) + 3)/2
```

3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

```
[Out] 1/2*arctanh(r*2^(1/2)*h^(1/2)/(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr = \text{Subst}\left(\int \frac{1}{1-2hr^2}\,dr, r, \frac{r}{\sqrt{-\alpha^2+2hr^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

fricas [A] time = 0.43, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{2}\log\left(4\,hr^2+2\sqrt{2}\sqrt{2\,hr^2-\alpha^2}\sqrt{h}\,r-\alpha^2\right)}{4\sqrt{h}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{h}}\arctan\left(\frac{\sqrt{2}\,hr\sqrt{-\frac{1}{h}}}{\sqrt{2\,hr^2-\alpha^2}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - al
pha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqrt(
2*h*r^2 - alpha^2))]
```

giac [A] time = 1.15, size = 34, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(\left|-\sqrt{2}\sqrt{h}r+\sqrt{2hr^2-\alpha^2}\right|\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))/sqrt(h)
```

maple [A] time = 0.00, size = 33, normalized size = 0.82

$$\frac{\sqrt{2}\ \ln\left(\sqrt{2}\ \sqrt{h}\ r+\sqrt{2hr^2-\alpha^2}\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

maxima [A] time = 0.42, size = 36, normalized size = 0.90

$$\frac{\sqrt{2}\log\left(4hr+2\sqrt{2}\sqrt{2hr^2-\alpha^2}\sqrt{h}\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(4*h*r + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h))/sqrt(h)
```

mupad [B] time = 0.48, size = 32, normalized size = 0.80

$$\frac{\sqrt{2}\ \ln\left(\sqrt{2hr^2-\alpha^2}+\sqrt{2}\ \sqrt{h}\ r\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*h*r^2 - alpha^2)^(1/2),r)
```

```
[Out] (2^(1/2)*log((2*h*r^2 - alpha^2)^(1/2) + 2^(1/2)*h^(1/2)*r))/(2*h^(1/2))
```

sympy [A] time = 1.14, size = 66, normalized size = 1.65

$$\begin{cases} \frac{\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{for } 2\left|\frac{hr^2}{\alpha^2}\right|>1 \\ -\frac{\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), 2*Abs(h*r**2
/alpha**2) > 1), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Tru
e))
```

3.206 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha^
2+epsilon^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {266, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr}}\,dr, r, r^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{-\alpha^2-\epsilon^2}{2h}+\frac{r^2}{2h}}\,dr, r, \sqrt{-\alpha^2-\epsilon^2+2hr^2}\right)}{2h}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

fricas [A] time = 0.42, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilon^2))/sqrt
(alpha^2 + epsilon^2)
```

giac [A] time = 1.02, size = 38, normalized size = 0.83

$$\frac{1.00000000000000 \times 10^{12}\,\arctan\left(\frac{1.00000000000000\times 10^{12}\,\sqrt{2.00000000000000\,hr^2-1.00000000000000\,\alpha^2-1.00000000000000\times 10^{-24}}}{\sqrt{1.00000000000000\times 10^{24}\,\alpha^2+1.00000000000000}}\right)}{\sqrt{1.00000000000000\times 10^{24}\,\alpha^2+1.00000000000000}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1.00000000000000e12*arctan(1.00000000000000e12*sqrt(2.00000000000000*h*r^2
- 1.00000000000000*alpha^2 - 1.00000000000000e-24)/sqrt(1.00000000000000e24
*alpha^2 + 1.00000000000000))/sqrt(1.00000000000000e24*alpha^2 + 1.00000000
000000)
```

maple [A] time = 0.01, size = 66, normalized size = 1.43

$$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*(-alpha^2-epsilo
n^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2)^(1/2))/r)
```

maxima [A] time = 0.98, size = 57, normalized size = 1.24

$$-\frac{\arcsin\left(\frac{\sqrt{2}\,\alpha^2}{2\,\sqrt{\left(\alpha^2+\epsilon^2\right)h}\,r}+\frac{\sqrt{2}\,\epsilon^2}{2\,\sqrt{\left(\alpha^2+\epsilon^2\right)h}\,r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] -arcsin(1/2*sqrt(2)*alpha^2/(sqrt((alpha^2 + epsilon^2)*h)*r) + 1/2*sqrt(2)
*epsilon^2/(sqrt((alpha^2 + epsilon^2)*h)*r))/sqrt(alpha^2 + epsilon^2)
```

mupad [**B**] time = 0.66, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2\,h\,r^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] atan((2*h*r^2 - alpha^2 - epsilon^2)^(1/2)/(alpha^2 + epsilon^2)^(1/2))/(al
pha^2 + epsilon^2)^(1/2)
```

sympy [**A**] time = 1.25, size = 42, normalized size = 0.91

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}{2\,\sqrt{h}\,r}\right)}{\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] -asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqrt
(polar_lift(-alpha**2 - epsilon**2))
```

3.207 $\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

```
[Out] -arctan((alpha^2+k*r)/alpha/(2*h*r^2-alpha^2-2*k*r)^(1/2))/alpha
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr &= -\left(2\,\text{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{-2\alpha^2-2kr}{\sqrt{-\alpha^2-2kr+2hr^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-kr}{\alpha\sqrt{2r(hr-k)-\alpha^2}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] ArcTan[(-alpha^2 - k*r)/(alpha*Sqrt[-alpha^2 + 2*r*(-k + h*r)])]/alpha
```

fricas [A] time = 0.43, size = 52, normalized size = 1.41

$$-\frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-2\,kr}\,(\alpha^2+kr)}{2\,\alpha hr^2-\alpha^3-2\,\alpha kr}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - al
pha^3 - 2*alpha*k*r))/alpha
```

giac [A] time = 1.31, size = 40, normalized size = 1.08

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{h}r-\sqrt{2hr^2-\alpha^2-2kr}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alph
a
```

maple [A] time = 0.01, size = 52, normalized size = 1.41

$$-\frac{\ln\left(\frac{-2\alpha^2-2kr+2\sqrt{-\alpha^2}\sqrt{2hr^2-\alpha^2-2kr}}{r}\right)}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha^
2-2*k*r)^(1/2))/r)
```

maxima [A] time = 0.96, size = 40, normalized size = 1.08

$$-\frac{\arcsin\left(\frac{k}{\sqrt{2\alpha^2h+k^2}}+\frac{\alpha^2}{\sqrt{2\alpha^2h+k^2}\,r}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
[Out] -arcsin(k/sqrt(2*alpha^2*h + k^2) + alpha^2/(sqrt(2*alpha^2*h + k^2)*r))/al
pha
```

mupad [B] time = 0.10, size = 51, normalized size = 1.38

$$-\frac{\ln\left(\frac{\sqrt{-\alpha^2}\sqrt{-\alpha^2+2hr^2-2kr}}{r}-\frac{\alpha^2}{r}-k\right)}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r - alpha^2)^(1/2)),r)
```

```
[Out] -log(((-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2)^(1/2))/r - alpha^2/r - k
)/(-alpha^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)
```

3.208 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -arctan((alpha^2+epsilon^2+k*r)/(alpha^2+epsilon^2)^(1/2)/(2*h*r^2-alpha^2-
epsilon^2-2*k*r)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha
^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr &= -\left(2\,\text{Subst}\left(\int \frac{1}{4\left(-\alpha^2-\epsilon^2\right)-r^2}\,dr, r, \frac{2\left(-\alpha^2-\epsilon^2\right)-2kr}{\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2-kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2r(hr-k)}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

[Out] ArcTan[(-alpha^2 - epsilon^2 - k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha^
2 - epsilon^2 + 2*r*(-k + h*r)])]/Sqrt[alpha^2 + epsilon^2]

fricas [A] time = 0.45, size = 97, normalized size = 1.59

$$-\frac{\arctan\left(-\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\left(\alpha^2+\epsilon^2+kr\right)\sqrt{\alpha^2+\epsilon^2}}{\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\left(\alpha^2+\epsilon^2\right)hr^2+2\left(\alpha^2+\epsilon^2\right)kr}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fricas"
)
```

```
[Out] -arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2 +
 k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4
- 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alpha^
2 + epsilon^2)
```

giac [A] time = 1.74, size = 51, normalized size = 0.84

$$\frac{2.00000000000000\times10^{12}\,\arctan\left(\frac{\left(6.55360000000000\times10^{-8}\right)\left(-2.15791864375777\times10^{19}\,\sqrt{h}\,r+1.52587890625000\times10^{19}\,\sqrt{2.0000000000000}\cdots\right.}{\sqrt{1.00000000000000\times10^{24}\,\alpha^2+1.000000000000\cdots}}\right.}{\sqrt{1.00000000000000\times10^{24}\,\alpha^2+1.00000000000000}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2.00000000000000e12*arctan((6.55360000000000e-8)*(-2.15791864375777e19*sqrt
(h)*r + 1.52587890625000e19*sqrt(2.00000000000000*h*r^2 - alpha^2 - 2.00000
000000000*k*r - 1.00000000000000e-24))/sqrt(1.00000000000000e24*alpha^2 + 1
.00000000000000))/sqrt(1.00000000000000e24*alpha^2 + 1.00000000000000)
```

maple [A] time = 0.01, size = 74, normalized size = 1.21

$$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2-2kr+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2-2kr}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2-
epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)
```

maxima [A] time = 0.97, size = 77, normalized size = 1.26

$$-\frac{\arcsin\left(\frac{k}{\sqrt{2\left(\alpha^2+\epsilon^2\right)h+k^2}}+\frac{\alpha^2}{\sqrt{2\left(\alpha^2+\epsilon^2\right)h+k^2}\,r}+\frac{\epsilon^2}{\sqrt{2\left(\alpha^2+\epsilon^2\right)h+k^2}\,r}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima"
)
```

```
[Out] -arcsin(k/sqrt(2*(alpha^2 + epsilon^2)*h + k^2) + alpha^2/(sqrt(2*(alpha^2
+ epsilon^2)*h + k^2)*r) + epsilon^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)
*r))/sqrt(alpha^2 + epsilon^2)
```

mupad [B] time = 0.25, size = 72, normalized size = 1.18

$$-\frac{\ln\left(\frac{\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-\alpha^2-\epsilon^2+2\,h\,r^2-2\,k\,r}}{r}-\frac{\alpha^2+\epsilon^2}{r}-k\right)}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] -log(((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)
^(1/2))/r - (alpha^2 + epsilon^2)/r - k)/(- alpha^2 - epsilon^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)
```

3.209 $\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr$

Optimal. Leaf size=23

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {261}

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2+2er^2}}{2e}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

fricas [A] time = 0.43, size = 19, normalized size = 0.83

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

giac [A] time = 1.04, size = 19, normalized size = 0.83

$$\frac{1}{2}\sqrt{2\,r^2e-\alpha^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{\sqrt{2e\,r^2-\alpha^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

maxima [A] time = 0.42, size = 19, normalized size = 0.83

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

mupad [B] time = 0.29, size = 19, normalized size = 0.83

$$\frac{\sqrt{2\,e\,r^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - alpha^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - alpha^2)^(1/2)/(2*e)
```

sympy [A] time = 0.45, size = 29, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{-\alpha^2+2er^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alph
a**2)), True))
```

3.210 $\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr$

Optimal. Leaf size=28

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {261}

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$\frac{\sqrt{2\,er^2-\alpha^2-\epsilon^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

giac [A] time = 1.22, size = 16, normalized size = 0.57

$$0.183939720586000\,\sqrt{-1.00000000000000\,\alpha^2+5.43656365692000\,r^2-1.00000000000000\times 10^{-24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] 0.183939720586000*sqrt(-1.00000000000000*alpha^2 + 5.43656365692000*r^2 - 1
.00000000000000e-24)
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

maxima [A] time = 0.42, size = 24, normalized size = 0.86

$$\frac{\sqrt{2\,er^{2}-\alpha^{2}-\epsilon^{2}}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

mupad [B] time = 0.27, size = 24, normalized size = 0.86

$$\frac{\sqrt{-\alpha^{2}-\epsilon^{2}+2\,e\,r^{2}}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - alpha^2 - epsilon^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - alpha^2 - epsilon^2)^(1/2)/(2*e)
```

sympy [A] time = 0.59, size = 36, normalized size = 1.29

$$\begin{cases} \frac{\sqrt{-\alpha^{2}+2er^{2}-\epsilon^{2}}}{2e} & \text{for}\: e \neq 0 \\ \frac{r^{2}}{2\sqrt{-\alpha^{2}-\epsilon^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2/
(2*sqrt(-alpha**2 - epsilon**2)), True))
```

3.211 $\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

```
[Out] -1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1/
2))*2^(1/2)/k^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]
/(2*Sqrt[2]*Sqrt[k])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-\alpha^2+2er-2kr^2}}\,dr, r, r^2\right)\\
&= \text{Subst}\left(\int \frac{1}{-8k-r^2}\,dr, r, \frac{2\left(e-2kr^2\right)}{\sqrt{-\alpha^2+2er^2-2kr^4}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -1/2*ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^
4])]/(Sqrt[2]*Sqrt[k])
```

fricas [A] time = 0.42, size = 152, normalized size = 2.71

$$\left[-\frac{\sqrt{2}\sqrt{-k}\log\left(-8\,k^2r^4+8\,ekr^2-2\,\alpha^2k+2\,\sqrt{2}\,\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\left(2\,kr^2-e\right)\sqrt{-k}\,-e^2\right)}{8\,k},-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(2)*sqrt(-k)*log(-8*k^2*r^4 + 8*e*k*r^2 - 2*alpha^2*k + 2*sqrt(2)
*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*sqrt(-k) - e^2)/k, -1/4*s
qrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*
sqrt(k)/(2*k^2*r^4 - 2*e*k*r^2 + alpha^2*k))/sqrt(k)]
```

giac [A] time = 1.22, size = 60, normalized size = 1.07

$$-\frac{\sqrt{2}\log\left(\left|\sqrt{2}\left(\sqrt{2}\,\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,r^2e-\alpha^2}\right)\sqrt{-k}\,+e\right|\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*r^2*
e - alpha^2))*sqrt(-k) + e))/sqrt(-k)
```

maple [A] time = 0.02, size = 47, normalized size = 0.84

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(r^2-\frac{e}{2k}\right)\sqrt{k}}{\sqrt{-2k\,r^4+2e\,r^2-\alpha^2}}\right)}{4\sqrt{k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/4*2^(1/2)/k^(1/2)*arctan(2^(1/2)*k^(1/2)*(r^2-1/2*e/k)/(-2*k*r^4+2*e*r^2-
alpha^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-e^2>0)', see `assume?`
for more details)Is 2*alpha^2*k-e^2 zero or nonzero?
```

mupad [B] time = 0.98, size = 50, normalized size = 0.89

$$\frac{\sqrt{2}\ \ln\left(\sqrt{-\alpha^2 - 2\,k\,r^4 + 2\,e\,r^2} + \frac{\sqrt{2}\,\left(e-2\,k\,r^2\right)}{2\,\sqrt{-k}}\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2),r)
```

```
[Out] (2^(1/2)*log((2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2) + (2^(1/2)*(e - 2*k*r^2))/
(2*(-k)^(1/2))))/(4*(-k)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\, dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)
```

3.212 $\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$

Optimal. Leaf size=81

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}\,e^{3/2}}$$

```
[Out] -1/4*k*arctanh(1/2*(-2*e*r+k)*2^(1/2)/e^(1/2)/(2*e*r^2-alpha^2-2*k*r)^(1/2)
)/e^(3/2)*2^(1/2)+1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e
```

Rubi [A] time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {640, 621, 206}

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sq
rt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr &= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\int \frac{1}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr}{2e}\\ &= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\,\text{Subst}\left(\int \frac{1}{8e-r^2}\,dr, r, \frac{-2k+4er}{\sqrt{-\alpha^2-2kr+2er^2}}\right)}{e}\\ &= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2-2kr+2er^2}}\right)}{2\sqrt{2}\,e^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 82, normalized size = 1.01

$$\frac{1}{4}\left(\frac{\sqrt{2}k\tanh^{-1}\left(\frac{2er-k}{\sqrt{2}\sqrt{e}\sqrt{2r(er-k)-\alpha^2}}\right)}{e^{3/2}}+\frac{2\sqrt{2r(er-k)-\alpha^2}}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] ((2*Sqrt[-alpha^2 + 2*r*(-k + e*r)])/e + (Sqrt[2]*k*ArcTanh[(-k + 2*e*r)/(S
qrt[2]*Sqrt[e]*Sqrt[-alpha^2 + 2*r*(-k + e*r)])])/e^(3/2))/4
```

fricas [A] time = 0.44, size = 190, normalized size = 2.35

$$\left[\frac{\sqrt{2}\sqrt{e}k\log\left(8e^2r^2-2\alpha^2e-8ekr+2\sqrt{2}\sqrt{2er^2-\alpha^2-2kr}(2er-k)\sqrt{e}+k^2\right)+4\sqrt{2er^2-\alpha^2-2kr}e}{8e^2}, -\frac{\sqrt{2}\sqrt{\cdot}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*sqrt(e)*k*log(8*e^2*r^2 - 2*alpha^2*e - 8*e*k*r + 2*sqrt(2)*s
qrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(e) + k^2) + 4*sqrt(2*e*r^2
- alpha^2 - 2*k*r)*e)/e^2, -1/4*(sqrt(2)*sqrt(-e)*k*arctan(1/2*sqrt(2)*sqrt
(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(-e)/(2*e^2*r^2 - alpha^2*e - 2
*e*k*r)) - 2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*e)/e^2]
```

giac [A] time = 1.34, size = 72, normalized size = 0.89

$$-\frac{1}{4}\sqrt{2}ke^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{2}\left(\sqrt{2}re^{\frac{1}{2}}-\sqrt{2r^2e-\alpha^2-2kr}\right)e^{\frac{1}{2}}+k\right|\right)+\frac{1}{2}\sqrt{2r^2e-\alpha^2-2kr}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*k*e^(-3/2)*log(abs(-sqrt(2)*(sqrt(2)*r*e^(1/2) - sqrt(2*r^2*e
- alpha^2 - 2*k*r))*e^(1/2) + k)) + 1/2*sqrt(2*r^2*e - alpha^2 - 2*k*r)*e^(
-1)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$\frac{\sqrt{2}k\ln\left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}}+\sqrt{2er^2-\alpha^2-2kr}\right)}{4e^{\frac{3}{2}}}+\frac{\sqrt{2er^2-\alpha^2-2kr}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k)*2^(1/2)/
e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

maxima [A] time = 0.42, size = 68, normalized size = 0.84

$$\frac{\sqrt{2}k\log\left(4er+2\sqrt{2}\sqrt{2er^2-\alpha^2-2kr}\sqrt{e}-2k\right)}{4e^{\frac{3}{2}}}+\frac{\sqrt{2er^2-\alpha^2-2kr}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*k*log(4*e*r + 2*sqrt(2)*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*sqrt(e)
 - 2*k)/e^(3/2) + 1/2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)/e
```

mupad [B] time = 0.24, size = 67, normalized size = 0.83

$$\frac{\sqrt{-\alpha^2+2\,e\,r^2-2\,k\,r}}{2\,e}+\frac{\sqrt{2}\,k\,\ln\left(\sqrt{-\alpha^2+2\,e\,r^2-2\,k\,r}-\frac{\sqrt{2}\,(k-2\,e\,r)}{2\,\sqrt{e}}\right)}{4\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - 2*k*r - alpha^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - 2*k*r - alpha^2)^(1/2)/(2*e) + (2^(1/2)*k*log((2*e*r^2 - 2*k*r -
 alpha^2)^(1/2) - (2^(1/2)*(k - 2*e*r))/(2*e^(1/2)))))/(4*e^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r), r)
```

3.213 $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

```
[Out] -1/2*arctan((-h*r^2+alpha^2)/alpha/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/alpha
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/(2*al
pha)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2+2hr-2kr^2}}\,dr, r, r^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{2\left(-\alpha^2+hr^2\right)}{\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{-\alpha^2+hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{2hr^2-2\alpha^2}{2\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ArcTan[(-2*alpha^2 + 2*h*r^2)/(2*alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]
/(2*alpha)
```

fricas [A] time = 0.41, size = 58, normalized size = 1.32

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\left(hr^2-\alpha^2\right)}{2\,\alpha kr^4-2\,\alpha hr^2+\alpha^3}\right)}{2\,\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*(h*r^2 - alpha^2)/(2*alpha*k
*r^4 - 2*alpha*h*r^2 + alpha^3))/alpha
```

giac [A] time = 1.15, size = 45, normalized size = 1.02

$$\frac{\arctan\left(-\frac{\sqrt{2}\,\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/
alpha
```

maple [A] time = 0.02, size = 56, normalized size = 1.27

$$-\frac{\ln\left(\frac{2h\,r^2-2\alpha^2+2\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+2
*h*r^2-alpha^2)^(1/2))/r^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-h^2>0)', see `assume?`
for more details)Is 2*alpha^2*k-h^2 positive, negative or zero?
```

mupad [B] time = 0.40, size = 54, normalized size = 1.23

$$-\frac{\ln\left(\frac{1}{r^2}\right)+\ln\left(h\,r^2-\alpha^2+\sqrt{-\alpha^2}\,\sqrt{-\alpha^2-2\,k\,r^4+2\,h\,r^2}\right)}{2\,\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)),r)
```

```
[Out] -(log(1/r^2) + log(h*r^2 - alpha^2 + (-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r^4 -
alpha^2)^(1/2)))/(2*(-alpha^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}\, dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)
```

3.214 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=68

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -1/2*arctan((-h*r^2+alpha^2+epsilon^2)/(alpha^2+epsilon^2)^(1/2)/(-2*k*r^4+
2*h*r^2-alpha^2-epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2 + epsilon^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr-2kr^2}}\,dr, r, r^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4\left(-\alpha^2-\epsilon^2\right)-r^2}\,dr, r, \frac{2\left(-\alpha^2-\epsilon^2+hr^2\right)}{\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2+hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.04

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2+hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ArcTan[(-alpha^2 - epsilon^2 + h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2 + epsilon^2])
```

fricas [A] time = 0.44, size = 106, normalized size = 1.56

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\left(hr^2-\alpha^2-\epsilon^2\right)\sqrt{\alpha^2+\epsilon^2}}{2\left(\alpha^2+\epsilon^2\right)kr^4+\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\left(\alpha^2+\epsilon^2\right)hr^2}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fric
as")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^2
 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 + al
pha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2))/s
qrt(alpha^2 + epsilon^2)
```

giac [A] time = 1.25, size = 45, normalized size = 0.66

$$\frac{\arctan\left(-\frac{\sqrt{2}\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac
")
```

```
[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/
alpha
```

maple [A] time = 0.02, size = 78, normalized size = 1.15

$$-\frac{\ln\left(\frac{2h\,r^2-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alph
a^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*epsilon^2*k+2*alpha^2*k>0)', see
`assume?` for more details)Is 2*epsilon^2*k+2*alpha^2*k
                -h^2 positive, negative or zero?
```

mupad [B] time = 0.44, size = 72, normalized size = 1.06

$$-\frac{\ln\left(h-\frac{\alpha^2+\epsilon^2}{r^2}+\frac{\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-\alpha^2-\epsilon^2-2\,k\,r^4+2\,h\,r^2}}{r^2}\right)}{2\,\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] -log(h - (alpha^2 + epsilon^2)/r^2 + ((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^
2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2))/r^2)/(2*(- alpha^2 - epsilon^2)^(
1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)
```

3.215 $\int a\cos(5+3x)\sin^2(5+3x)\,dx$

Optimal. Leaf size=13

$$\frac{1}{9}a\sin^3(3x+5)$$

```
[Out] 1/9*a*sin(5+3*x)^3
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {12, 2564, 30}

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int a\cos(5+3x)\sin^2(5+3x)\,dx &= a\int\cos(5+3x)\sin^2(5+3x)\,dx\\ &= \frac{1}{3}a\,\text{Subst}\left(\int x^2\,dx, x, \sin(5+3x)\right)\\ &= \frac{1}{9}a\sin^3(5+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

fricas [**A**] time = 0.42, size = 22, normalized size = 1.69

$$-\frac{1}{9}\left(a\cos\left(3x+5\right)^{2}-a\right)\sin\left(3x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")
[Out] -1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

giac [**A**] time = 1.04, size = 11, normalized size = 0.85

$$\frac{1}{9}a\sin\left(3x+5\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="giac")
[Out] 1/9*a*sin(3*x + 5)^3
```

maple [**A**] time = 0.01, size = 12, normalized size = 0.92

$$\frac{a\left(\sin^{3}\left(3x+5\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(5+3*x)*sin(5+3*x)^2,x)
[Out] 1/9*a*sin(5+3*x)^3
```

maxima [**A**] time = 0.42, size = 11, normalized size = 0.85

$$\frac{1}{9}a\sin\left(3x+5\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")
[Out] 1/9*a*sin(3*x + 5)^3
```

mupad [**B**] time = 0.09, size = 11, normalized size = 0.85

$$\frac{a\sin\left(3x+5\right)^{3}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(3*x + 5)*sin(3*x + 5)^2,x)
[Out] (a*sin(3*x + 5)^3)/9
```

sympy [**A**] time = 0.32, size = 10, normalized size = 0.77

$$\frac{a\sin^{3}\left(3x+5\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)
[Out] a*sin(3*x + 5)**3/9
```

3.216 $\int \frac{\log(x^2)}{x^3}\, dx$

Optimal. Leaf size=19

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

```
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2]/x^3,x]
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(x^2\right)}{x^3}\, dx=-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2]/x^3,x]
[Out] -1/2*1/x^2 - Log[x^2]/(2*x^2)
```

fricas [A] time = 0.42, size = 11, normalized size = 0.58

$$-\frac{\log\left(x^2\right)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="fricas")
[Out] -1/2*(log(x^2) + 1)/x^2
```

giac [A] time = 1.19, size = 15, normalized size = 0.79

$$-\frac{\log\left(x^2\right)}{2\,x^2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="giac")
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{\ln\left(x^2\right)}{2x^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2)/x^3,x)
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

maxima [A] time = 0.44, size = 15, normalized size = 0.79

$$-\frac{\log\left(x^2\right)}{2\,x^2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="maxima")
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

mupad [B] time = 0.16, size = 11, normalized size = 0.58

$$-\frac{\ln\left(x^2\right)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2)/x^3,x)
[Out] -(log(x^2) + 1)/(2*x^2)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.89

$$-\frac{\log\left(x^2\right)}{2x^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2)/x**3,x)
[Out] -log(x**2)/(2*x**2) - 1/(2*x**2)
```

3.217 $\int x \sin(a+x) \, dx$

Optimal. Leaf size=12

$$\sin(a+x) - x\cos(a+x)$$

```
[Out] -x*cos(a+x)+sin(a+x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$\sin(a+x) - x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \sin(a+x) \, dx &= -x\cos(a+x) + \int \cos(a+x) \, dx \\ &= -x\cos(a+x) + \sin(a+x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$\sin(a+x) - x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

fricas [A] time = 0.43, size = 12, normalized size = 1.00

$$-x\cos(a+x) + \sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="fricas")
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

giac [A] time = 1.33, size = 12, normalized size = 1.00

$$-x\cos(a+x) + \sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="giac")
[Out] -x*cos(a + x) + sin(a + x)
```

maple [A] time = 0.02, size = 21, normalized size = 1.75

$$a\cos(a+x)-(a+x)\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+x),x)
[Out] a*cos(a+x)+sin(a+x)-(a+x)*cos(a+x)
```

maxima [A] time = 0.44, size = 20, normalized size = 1.67

$$-(a+x)\cos(a+x)+a\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="maxima")
[Out] -(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)
```

mupad [B] time = 0.08, size = 12, normalized size = 1.00

$$\sin(a+x)-x\ \cos(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + x),x)
[Out] sin(a + x) - x*cos(a + x)
```

sympy [A] time = 0.18, size = 10, normalized size = 0.83

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x)
[Out] -x*cos(a + x) + sin(a + x)
```

3.218 $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{e^{-x}x}{\log(x)}$$

```
[Out] x/exp(x)/ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2201}

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

Rule 2201

```
Int[Log[(d_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((e_) + Log[(d_
.)*(x_)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[(e*x*F^(c*(a + b*x)
)*Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x]
 && EqQ[e - f*h*(n + 1), 0] && EqQ[g*h*(n + 1) - b*c*e*Log[F], 0] && NeQ[n,
 -1]
```

Rubi steps

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{e^{-x}x}{\log(x)}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

fricas [A] time = 0.41, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")
```

```
[Out] x*e^(-x)/log(x)
```

giac [A] time = 0.99, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="giac")
```

```
[Out] x*e^(-x)/log(x)
```

maple [A] time = 0.02, size = 11, normalized size = 1.00

$$\frac{x\,\mathrm{e}^{-x}}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-x+1)*ln(x))/exp(x)/ln(x)^2,x)
```

```
[Out] x/exp(x)/ln(x)
```

maxima [A] time = 0.58, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")
```

```
[Out] x*e^(-x)/log(x)
```

mupad [B] time = 0.22, size = 10, normalized size = 0.91

$$\frac{x\,\mathrm{e}^{-x}}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(-x)*(log(x)*(x - 1) + 1))/log(x)^2,x)
```

```
[Out] (x*exp(-x))/log(x)
```

sympy [A] time = 0.27, size = 7, normalized size = 0.64

$$\frac{xe^{-x}}{\log{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

```
[Out] x*exp(-x)/log(x)
```

3.219 $\int \frac{x^3}{b+ax^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a} - \frac{b \log\left(ax^2+b\right)}{2a^2}$$

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{x^2}{2a} - \frac{b \log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b + a*x^2),x]
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{b+ax^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x}{b+ax} \, dx, x, x^2\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right) \, dx, x, x^2\right) \\ &= \frac{x^2}{2a} - \frac{b \log\left(b+ax^2\right)}{2a^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2a} - \frac{b \log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b + a*x^2),x]
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.81

$$\frac{ax^2 - b \log\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="fricas")
[Out] 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

giac [A] time = 1.14, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,a} - \frac{b\log\left(\left|ax^2 + b\right|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="giac")
[Out] 1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2a} - \frac{b\ln\left(a\,x^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2+b),x)
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

maxima [A] time = 0.42, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,a} - \frac{b\log\left(ax^2 + b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="maxima")
[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

mupad [B] time = 0.15, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(a\,x^2 + b\right) - a\,x^2}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b + a*x^2),x)
[Out] -(b*log(b + a*x^2) - a*x^2)/(2*a^2)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**2+b),x)
[Out] x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

3.220 $\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx$

Optimal. Leaf size=33

$$\frac{4x^{3/2}}{15(x+1)^{3/2}}+\frac{2x^{3/2}}{5(x+1)^{5/2}}$$

```
[Out] 2/5*x^(3/2)/(1+x)^(5/2)+4/15*x^(3/2)/(1+x)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {45, 37}

$$\frac{4x^{3/2}}{15(x+1)^{3/2}}+\frac{2x^{3/2}}{5(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x)^(7/2),x]
[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx &= \frac{2x^{3/2}}{5(1+x)^{5/2}}+\frac{2}{5}\int \frac{\sqrt{x}}{(1+x)^{5/2}} \, dx\\ &= \frac{2x^{3/2}}{5(1+x)^{5/2}}+\frac{4x^{3/2}}{15(1+x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.64

$$\frac{2x^{3/2}(2x+5)}{15(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x)^(7/2),x]
[Out] (2*x^(3/2)*(5 + 2*x))/(15*(1 + x)^(5/2))
```

fricas [B] time = 0.42, size = 50, normalized size = 1.52

$$\frac{2\left(2x^{3}+\left(2x^{2}+5x\right)\sqrt{x+1}\sqrt{x}+6x^{2}+6x+2\right)}{15\left(x^{3}+3x^{2}+3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 + 3
*x^2 + 3*x + 1)
```

giac [B] time = 1.40, size = 66, normalized size = 2.00

$$\frac{8\left(15\left(\sqrt{x+1}-\sqrt{x}\right)^{6}-5\left(\sqrt{x+1}-\sqrt{x}\right)^{4}+5\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)}{15\left(\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4 + 5*(sqrt(
x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1)^5
```

maple [A] time = 0.00, size = 16, normalized size = 0.48

$$\frac{2\left(2x+5\right)x^{\frac{3}{2}}}{15\left(x+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+1)^(7/2),x)
```

```
[Out] 2/15*x^(3/2)*(5+2*x)/(x+1)^(5/2)
```

maxima [A] time = 0.42, size = 20, normalized size = 0.61

$$\frac{2x^{\frac{5}{2}}\left(\frac{5(x+1)}{x}-3\right)}{15\left(x+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)
```

mupad [B] time = 0.30, size = 15, normalized size = 0.45

$$\frac{2x^{3/2}\left(2x+5\right)}{15\left(x+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1)^(7/2),x)
```

```
[Out] (2*x^(3/2)*(2*x + 5))/(15*(x + 1)^(5/2))
```

sympy [B] time = 4.66, size = 165, normalized size = 5.00

$$\begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}}{15} + \frac{2i\sqrt{-1+\frac{1}{x+1}}}{15(x+1)} - \frac{2i\sqrt{-1+\frac{1}{x+1}}}{5(x+1)^2} & \text{for } \frac{1}{|x+1|} > 1 \\ \frac{4\sqrt{1-\frac{1}{x+1}}\,(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2\sqrt{1-\frac{1}{x+1}}\,(x+1)}{-15x+15(x+1)^2-15} - \frac{8\sqrt{1-\frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6\sqrt{1-\frac{1}{x+1}}}{(x+1)\left(-15x+15(x+1)^2-15\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(7/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 1/(x + 1))/15 + 2*I*sqrt(-1 + 1/(x + 1))/(15*(x +
1)) - 2*I*sqrt(-1 + 1/(x + 1))/(5*(x + 1)**2), 1/Abs(x + 1) > 1), (4*sqrt(1
 - 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15) - 2*sqrt(1 - 1/(x + 1
))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*sqrt(1 - 1/(x + 1))/(-15*x + 15
*(x + 1)**2 - 15) + 6*sqrt(1 - 1/(x + 1))/((x + 1)*(-15*x + 15*(x + 1)**2 -
 15)), True))
```

3.221 $\int \frac{1}{x(1+x)} \, dx$

Optimal. Leaf size=9

$$\log(x) - \log(x+1)$$

[Out] ln(x)-ln(1+x)

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {36, 29, 31}

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x)} \, dx &= \int \frac{1}{x} \, dx - \int \frac{1}{1+x} \, dx \\ &= \log(x) - \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]

fricas [A] time = 0.41, size = 9, normalized size = 1.00

$$-\log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+x),x, algorithm="fricas")

[Out] -log(x + 1) + log(x)

giac [**A**] time = 1.16, size = 11, normalized size = 1.22

$$-\log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="giac")
[Out] -log(abs(x + 1)) + log(abs(x))
```

maple [**A**] time = 0.00, size = 10, normalized size = 1.11

$$\ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1),x)
[Out] ln(x)-ln(x+1)
```

maxima [**A**] time = 0.42, size = 9, normalized size = 1.00

$$-\log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="maxima")
[Out] -log(x + 1) + log(x)
```

mupad [**B**] time = 0.20, size = 8, normalized size = 0.89

$$-\ln\left(\frac{1}{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)),x)
[Out] -log(1/x + 1)
```

sympy [**A**] time = 0.09, size = 7, normalized size = 0.78

$$\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x)
[Out] log(x) - log(x + 1)
```

3.222 $\int \frac{1}{\sqrt{x}(-1+2x)}\,dx$

Optimal. Leaf size=19

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

```
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {63, 207}

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-1 + 2*x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(-1+2x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{x}\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

fricas [B] time = 0.42, size = 28, normalized size = 1.47

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{x}-2x-1}{2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))
```

giac [B] time = 1.27, size = 32, normalized size = 1.68

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sqrt{2}+\sqrt{x}\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2)
+ sqrt(x)))
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\sqrt{2}\ \operatorname{arctanh}\left(\sqrt{2}\ \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(2*x-1),x)
```

```
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

maxima [B] time = 0.97, size = 28, normalized size = 1.47

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{x}}{\sqrt{2}+2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))
```

mupad [B] time = 0.08, size = 13, normalized size = 0.68

$$-\sqrt{2}\ \operatorname{atanh}\left(\sqrt{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 1)),x)
```

```
[Out] -2^(1/2)*atanh((2*x)^(1/2))
```

sympy [B] time = 0.31, size = 39, normalized size = 2.05

$$\frac{\sqrt{2}\log\left(\sqrt{x}-\frac{\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\log\left(\sqrt{x}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+2*x),x)
```

```
[Out] sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2
```

3.223 $\int \sqrt{x} \left(1+x^2\right) \, dx$

Optimal. Leaf size=19

$$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}$$

```
[Out] 2/3*x^(3/2)+2/7*x^(7/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} \left(1+x^2\right) \, dx &= \int \left(\sqrt{x}+x^{5/2}\right) \, dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{7/2}}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{2}{21} x^{3/2} \left(3x^2+7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*(7 + 3*x^2))/21
```

fricas [A] time = 0.40, size = 14, normalized size = 0.74

$$\frac{2}{21} \left(3\,x^3+7\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 7*x)*sqrt(x)
```

giac [A] time = 1.11, size = 11, normalized size = 0.58

$$\frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{2\left(3x^2+7\right)x^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2+1),x)
```

```
[Out] 2/21*x^(3/2)*(3*x^2+7)
```

maxima [A] time = 0.42, size = 11, normalized size = 0.58

$$\frac{2}{7}x^{\frac{7}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.63

$$\frac{2\,x^{3/2}\,\left(3\,x^2+7\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2 + 1),x)
```

```
[Out] (2*x^(3/2)*(3*x^2 + 7))/21
```

sympy [A] time = 1.09, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x**2+1),x)
```

```
[Out] 2*x**(7/2)/7 + 2*x**(3/2)/3
```

3.224 $\int \frac{\sqrt[3]{-a+x}}{x} \, dx$

Optimal. Leaf size=88

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

```
[Out] 3*(-a+x)^(1/3)+1/2*a^(1/3)*ln(x)-3/2*a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+a^(1/
3)*arctan(1/3*(a^(1/3)-2*(-a+x)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(1/3))/(Sqr
t[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^(1/3) + (-a + x)^(1/
3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{-a+x}}{x}\,dx = 3\sqrt[3]{-a+x} - a\int \frac{1}{x(-a+x)^{2/3}}\,dx$$
$$= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+x}\right) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int\right.$$
$$= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right) - \left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.$$
$$= 3\sqrt[3]{-a+x} + \sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)$$

Mathematica [A] time = 0.04, size = 112, normalized size = 1.27

$$\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{x-a}+(x-a)^{2/3}\right)+3\sqrt[3]{x-a}-\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))/
Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(1/3)*Log[a^(2/3) - a
^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2
```

fricas [A] time = 0.41, size = 104, normalized size = 1.18

$$\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,a-2\,\sqrt{3}\,(-a)^{\frac{2}{3}}\,(-a+x)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^(
1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a
+ x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/
3)
```

giac [A] time = 2.43, size = 103, normalized size = 1.17

$$-\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}}+2\,(-a+x)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a)
^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a +
 x)^(2/3)) + (-a)^(1/3)*log(abs(-(-a)^(1/3) + (-a + x)^(1/3))) + 3*(-a + x)
^(1/3)
```

maple [A] time = 0.01, size = 85, normalized size = 0.97

$$-\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2(-a+x)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)-a^{\frac{1}{3}} \ln\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+\frac{a^{\frac{1}{3}} \ln\left(a^{\frac{2}{3}}-(-a+x)^{\frac{1}{3}} a^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)}{2}+3(-a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)^(1/3)/x,x)
```

```
[Out] 3*(-a+x)^(1/3)-a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+1/2*a^(1/3)*ln((-a+x)^(2/3)
-a^(1/3)*(-a+x)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3
)*(-a+x)^(1/3)-1))
```

maxima [A] time = 0.97, size = 86, normalized size = 0.98

$$-\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}\left(a^{\frac{1}{3}}-2(-a+x)^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)+\frac{1}{2}\, a^{\frac{1}{3}} \log\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)-a^{\frac{1}{3}} \log\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(-1/3*sqrt(3)*(a^(1/3) - 2*(-a + x)^(1/3))/a^(1/3))
+ 1/2*a^(1/3)*log(a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) - a^(1
/3)*log(a^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)
```

mupad [B] time = 0.18, size = 119, normalized size = 1.35

$$(-a)^{1/3} \ln\left(-9(-a)^{4/3}-9\,a\,(x-a)^{1/3}\right)+3\,(x-a)^{1/3}+\frac{(-a)^{1/3} \ln\left(\frac{9(-a)^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,a\,(x-a)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - a)^(1/3)/x,x)
```

```
[Out] (-a)^(1/3)*log(- 9*(-a)^(4/3) - 9*a*(x - a)^(1/3)) + 3*(x - a)^(1/3) + ((-a
)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i - 1))/2 + 9*a*(x - a)^(1/3))*(3^(1/2)
*1i - 1))/2 - ((-a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i + 1))/2 - 9*a*(x -
a)^(1/3))*(3^(1/2)*1i + 1))/2
```

sympy [C] time = 1.69, size = 153, normalized size = 1.74

$$\frac{4\sqrt[3]{a}\, e^{-\frac{i\pi}{3}} \log\left(1-\frac{\sqrt[3]{-a+x}\, e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{a} \log\left(1-\frac{\sqrt[3]{-a+x}\, e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\, e^{\frac{i\pi}{3}} \log\left(1-\frac{\sqrt[3]{-a+x}\, e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-a+}}{\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))
*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar(I
*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1 - (
-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*
(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

3.225 $\int x \sinh(x) \, dx$

Optimal. Leaf size=9

$$x \cosh(x) - \sinh(x)$$

```
[Out] x*cosh(x)-sinh(x)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2637}

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x \sinh(x) \, dx &= x \cosh(x) - \int \cosh(x) \, dx\\ &= x \cosh(x) - \sinh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

fricas [A] time = 0.40, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="fricas")
```

```
[Out] x*cosh(x) - sinh(x)
```

giac [A] time = 1.19, size = 17, normalized size = 1.89

$$\frac{1}{2} (x+1)e^{(-x)} + \frac{1}{2} (x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x),x)
```

```
[Out] x*cosh(x)-sinh(x)
```

maxima [B] time = 0.43, size = 34, normalized size = 3.78

$$\frac{1}{2}\,x^2\sinh(x) + \frac{1}{4}\left(x^2 + 2\,x + 2\right)e^{(-x)} - \frac{1}{4}\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

mupad [B] time = 0.02, size = 9, normalized size = 1.00

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x),x)
```

```
[Out] x*cosh(x) - sinh(x)
```

sympy [A] time = 0.19, size = 7, normalized size = 0.78

$$x\cosh\left(x\right) - \sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x)
```

```
[Out] x*cosh(x) - sinh(x)
```

3.226 $\int x \cosh(x) \, dx$

Optimal. Leaf size=9

$$x\sinh(x) - \cosh(x)$$

```
[Out] -cosh(x)+x*sinh(x)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2638}

$$x\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh(x)\,dx &= x\sinh(x) - \int \sinh(x)\,dx\\ &= -\cosh(x) + x\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$x\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

fricas [A] time = 0.41, size = 9, normalized size = 1.00

$$x\sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="fricas")
```

```
[Out] x*sinh(x) - cosh(x)
```

giac [A] time = 1.20, size = 17, normalized size = 1.89

$$-\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="giac")
[Out] -1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$x \sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x),x)
[Out] -cosh(x)+x*sinh(x)
```

maxima [B] time = 0.43, size = 34, normalized size = 3.78

$$\frac{1}{2} x^2 \cosh(x) - \frac{1}{4} \left(x^2 + 2x + 2\right) e^{(-x)} - \frac{1}{4} \left(x^2 - 2x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="maxima")
[Out] 1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

mupad [B] time = 0.03, size = 9, normalized size = 1.00

$$x \sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x),x)
[Out] x*sinh(x) - cosh(x)
```

sympy [A] time = 0.19, size = 7, normalized size = 0.78

$$x \sinh{\left(x \right)} - \cosh{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x)
[Out] x*sinh(x) - cosh(x)
```

3.227 $\int \tanh(2x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log(\cosh(2x))$$

[Out] 1/2*ln(cosh(2*x))

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3475}

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

[In] Int[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(2x)\,dx = \frac{1}{2}\log(\cosh(2x))$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2

fricas [B] time = 0.43, size = 26, normalized size = 2.89

$$-x + \frac{1}{2}\log\left(\frac{2\cosh(2x)}{\cosh(2x) - \sinh(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")

[Out] -x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))

giac [A] time = 1.18, size = 13, normalized size = 1.44

$$-x + \frac{1}{2}\log\left(e^{(4x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")

```
[Out] -x + 1/2*log(e^(4*x) + 1)
```

maple [**A**] time = 0.02, size = 8, normalized size = 0.89

$$\frac{\ln(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x)
[Out] 1/2*ln(cosh(2*x))
```

maxima [**A**] time = 0.43, size = 7, normalized size = 0.78

$$\frac{1}{2}\log(\cosh(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
[Out] 1/2*log(cosh(2*x))
```

mupad [**B**] time = 0.19, size = 7, normalized size = 0.78

$$\frac{\ln(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x)
[Out] log(cosh(2*x))/2
```

sympy [**A**] time = 0.15, size = 7, normalized size = 0.78

$$\frac{\log(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x)
[Out] log(cosh(2*x))/2
```

3.228 $\int \frac{-1+i\mathbf{eps}\sinh(x)}{ia-x+i\mathbf{eps}\cosh(x)}\,dx$

Optimal. Leaf size=12

$$\log(a+\operatorname{eps}\cosh(x)+ix)$$

```
[Out] ln(a+I*x+eps*cosh(x))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {6684}

$$\log(a+\operatorname{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
[Out] Log[a + I*x + eps*Cosh[x]]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /; !FalseQ[q]]
```

Rubi steps

$$\int \frac{-1+i\operatorname{eps}\sinh(x)}{ia-x+i\operatorname{eps}\cosh(x)}\,dx=\log(a+ix+\operatorname{eps}\cosh(x))$$

Mathematica [A] time = 0.09, size = 12, normalized size = 1.00

$$\log(a+\operatorname{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
[Out] Log[a + I*x + eps*Cosh[x]]
```

fricas [B] time = 0.45, size = 26, normalized size = 2.17

$$-x+\log\left(\frac{epse^{(2x)}+2(a+ix)e^{x}+eps}{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")
[Out] -x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)
```

giac [B] time = 1.37, size = 23, normalized size = 1.92

$$-x+\log\left(epse^{(2x)}+2\,ae^{x}+2i\,xe^{x}+eps\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="giac")
[Out] -x + log(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)
```

maple [**A**] time = 0.02, size = 16, normalized size = 1.33

$$\ln\left(ieps\cosh(x) + ia - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
[Out] ln(I*a-x+I*eps*cosh(x))
```

maxima [**A**] time = 0.43, size = 13, normalized size = 1.08

$$\log\left(i\,eps\cosh(x) + i\,a - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")
[Out] log(I*eps*cosh(x) + I*a - x)
```

mupad [**B**] time = 0.32, size = 13, normalized size = 1.08

$$\ln\left(x - a\,1\mathrm{i} - \mathrm{eps}\cosh(x)\,1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((eps*sinh(x)*1i - 1)/(a*1i - x + eps*cosh(x)*1i),x)
[Out] log(x - a*1i - eps*cosh(x)*1i)
```

sympy [**B**] time = 0.39, size = 22, normalized size = 1.83

$$-x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)\,e^{x}}{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
[Out] -x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)
```

3.229 $\int \cos^2(x) \sin(3+2x)\, dx$

Optimal. Leaf size=28

$$\frac{1}{4} x \sin(3) - \frac{1}{4} \cos(2x+3) - \frac{1}{16} \cos(4x+3)$$

```
[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{1}{4} x \sin(3) - \frac{1}{4} \cos(2x+3) - \frac{1}{16} \cos(4x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[3 + 2*x],x]
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x) \sin(3+2x)\, dx &= \int \left(\frac{\sin(3)}{4} + \frac{1}{2} \sin(3+2x) + \frac{1}{4} \sin(3+4x)\right) dx \\
&= \frac{1}{4} x \sin(3) + \frac{1}{4} \int \sin(3+4x)\, dx + \frac{1}{2} \int \sin(3+2x)\, dx \\
&= -\frac{1}{4} \cos(3+2x) - \frac{1}{16} \cos(3+4x) + \frac{1}{4} x \sin(3)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{1}{4} x \sin(3) - \frac{1}{4} \cos(2x+3) - \frac{1}{16} \cos(4x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[3 + 2*x],x]
[Out] -1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

fricas [A] time = 0.45, size = 32, normalized size = 1.14

$$-\frac{1}{2} \cos(3) \cos(x)^4 + \frac{1}{4} x \sin(3) + \frac{1}{4} \left(2 \cos(x)^3 \sin(3) + \cos(x) \sin(3)\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(3
))*sin(x)
```

giac [A] time = 1.44, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x\sin(3) - \frac{1}{16}\,\cos\left(4\,x+3\right) - \frac{1}{4}\,\cos\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="giac")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

maple [A] time = 0.02, size = 23, normalized size = 0.82

$$\frac{\sin(3)x}{4} - \frac{\cos\left(2x+3\right)}{4} - \frac{\cos\left(4x+3\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(2*x+3),x)
```

```
[Out] -1/4*cos(2*x+3)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

maxima [A] time = 0.47, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x\sin(3) - \frac{1}{16}\,\cos\left(4\,x+3\right) - \frac{1}{4}\,\cos\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

mupad [B] time = 0.33, size = 22, normalized size = 0.79

$$\frac{x\,\sin(3)}{4} - \frac{\cos\left(4\,x+3\right)}{16} - \frac{\cos\left(2\,x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x + 3)*cos(x)^2,x)
```

```
[Out] (x*sin(3))/4 - cos(4*x + 3)/16 - cos(2*x + 3)/4
```

sympy [B] time = 2.20, size = 75, normalized size = 2.68

$$-\frac{x\sin^{2}\left(x\right)\sin\left(2x+3\right)}{4} - \frac{x\sin\left(x\right)\cos\left(x\right)\cos\left(2x+3\right)}{2} + \frac{x\sin\left(2x+3\right)\cos^{2}\left(x\right)}{4} - \frac{\sin\left(x\right)\sin\left(2x+3\right)\cos\left(x\right)}{4} - \frac{\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(3+2*x),x)
```

```
[Out] -x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x +
3)*cos(x)**2/4 - sin(x)*sin(2*x + 3)*cos(x)/4 - cos(x)**2*cos(2*x + 3)/2
```

3.230 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \tan^{-1}(x)\, dx &= \frac{1}{2}x^2 \tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx \\
&= -\frac{x}{2} + \frac{1}{2}x^2 \tan^{-1}(x) + \frac{1}{2}\int \frac{1}{1+x^2}\, dx \\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

giac [A] time = 1.47, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan(x)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{x^2\arctan(x)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
[Out] 1/2*x^2*arctan(x)-1/2*x+1/2*arctan(x)
```

maxima [A] time = 0.95, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan(x)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.67

$$\mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

sympy [A] time = 0.25, size = 15, normalized size = 0.71

$$\frac{x^2\,\mathrm{atan}\,(x)}{2}-\frac{x}{2}+\frac{\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

3.231 $\int x\cot^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4853, 321, 203}

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[x],x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4853

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^p)/(d*(m + 1)), x] + Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}(x)\,dx&=\frac{1}{2}x^2\cot^{-1}(x)+\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\&=\frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\&=\frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[x],x]
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

fricas [A] time = 0.43, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\operatorname{arccot}(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arccot(x) + 1/2*x
```

giac [A] time = 1.42, size = 19, normalized size = 0.90

$$\frac{1}{2}x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{2}x+\frac{1}{2}\arctan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(1/x) + 1/2*x + 1/2*arctan(1/x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.76

$$\frac{x^2\operatorname{arccot}(x)}{2}+\frac{x}{2}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x),x)
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

maxima [A] time = 0.96, size = 15, normalized size = 0.71

$$\frac{1}{2}x^2\operatorname{arccot}(x)+\frac{1}{2}x-\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)
```

mupad [B] time = 0.05, size = 15, normalized size = 0.71

$$\frac{x}{2}-\frac{\operatorname{atan}(x)}{2}+\frac{x^2\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(x),x)
[Out] x/2 - atan(x)/2 + (x^2*acot(x))/2
```

sympy [A] time = 0.25, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{acot}(x)}{2}+\frac{x}{2}+\frac{\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x),x)
[Out] x**2*acot(x)/2 + x/2 + acot(x)/2
```

3.232 $\int x \log \left(a+x^2\right) \, dx$

Optimal. Leaf size=23

$$\frac{1}{2} \left(a+x^2\right) \log \left(a+x^2\right)-\frac{x^2}{2}$$

```
[Out] -1/2*x^2+1/2*(x^2+a)*ln(x^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2454, 2389, 2295}

$$\frac{1}{2} \left(a+x^2\right) \log \left(a+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + x^2],x]
```

```
[Out] -x^2/2 + ((a + x^2)*Log[a + x^2])/2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x \log \left(a+x^2\right) \, dx &= \frac{1}{2} \text{Subst}\left(\int \log (a+x) \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \log (x) \, dx,x,a+x^2\right)\\
&= -\frac{x^2}{2}+\frac{1}{2} \left(a+x^2\right) \log \left(a+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{1}{2} \left(\left(a+x^2\right) \log \left(a+x^2\right)-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + x^2],x]
```

```
[Out] (-x^2 + (a + x^2)*Log[a + x^2])/2
```

fricas [A] time = 0.41, size = 19, normalized size = 0.83

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 + a\right)\log\left(x^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)
```

giac [A] time = 1.36, size = 22, normalized size = 0.96

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 + a\right)\log\left(x^2 + a\right) - \frac{1}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="giac")
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

maple [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{x^2}{2} - \frac{a}{2} + \frac{\left(x^2 + a\right)\ln\left(x^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^2+a),x)
[Out] 1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a
```

maxima [A] time = 0.42, size = 22, normalized size = 0.96

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 + a\right)\log\left(x^2 + a\right) - \frac{1}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="maxima")
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

mupad [B] time = 0.14, size = 41, normalized size = 1.78

$$\frac{a\,\ln\left(x + \sqrt{-a}\right)}{2} + \frac{x^2\,\ln\left(x^2 + a\right)}{2} + \frac{a\,\ln\left(x - \sqrt{-a}\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a + x^2),x)
[Out] (a*log(x + (-a)^(1/2)))/2 + (x^2*log(a + x^2))/2 + (a*log(x - (-a)^(1/2)))/
2 - x^2/2
```

sympy [A] time = 0.15, size = 26, normalized size = 1.13

$$\frac{a\log\left(a + x^2\right)}{2} + \frac{x^2\log\left(a + x^2\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**2+a),x)
[Out] a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

3.233 $\int \cos(x)\sin(a+x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[a + x],x]
[Out] -Cos[a + 2*x]/4 + (x*Sin[a])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(a+x)\,dx &= \int \left(\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\ &= \frac{1}{2}x\sin(a)+\frac{1}{2}\int \sin(a+2x)\,dx\\ &= -\frac{1}{4}\cos(a+2x)+\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{1}{4}(2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[a + x],x]
[Out] (-Cos[a + 2*x] + 2*x*Sin[a])/4
```

fricas [A] time = 0.44, size = 28, normalized size = 1.56

$$-\frac{1}{2}\cos(a+x)^{2}\cos(a)-\frac{1}{2}\cos(a+x)\sin(a+x)\sin(a)+\frac{1}{2}x\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2*x*sin(a)
```

giac [A] time = 1.04, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin(a) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="giac")
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

maple [A] time = 0.05, size = 15, normalized size = 0.83

$$\frac{x\sin(a)}{2} - \frac{\cos\left(a + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(a+x),x)
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

maxima [A] time = 0.42, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin(a) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="maxima")
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.78

$$\frac{x\,\sin(a)}{2} - \frac{\cos\left(a + 2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + x)*cos(x),x)
[Out] (x*sin(a))/2 - cos(a + 2*x)/4
```

sympy [B] time = 0.54, size = 32, normalized size = 1.78

$$-\frac{x\sin\left(x\right)\cos\left(a + x\right)}{2} + \frac{x\sin\left(a + x\right)\cos\left(x\right)}{2} + \frac{\sin\left(x\right)\sin\left(a + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x)
[Out] -x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 + sin(x)*sin(a + x)/2
```

3.234 $\int \cos(a+x)\sin(x)\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + x]*Sin[x],x]
[Out] -Cos[a + 2*x]/4 - (x*Sin[a])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(a+x)\sin(x)\,dx&=\int\left(-\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\&=-\frac{1}{2}x\sin(a)+\frac{1}{2}\int\sin(a+2x)\,dx\\&=-\frac{1}{4}\cos(a+2x)-\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{4}(-2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + x]*Sin[x],x]
[Out] (-Cos[a + 2*x] - 2*x*Sin[a])/4
```

fricas [A] time = 0.44, size = 28, normalized size = 1.56

$$-\frac{1}{2}\cos{(a+x)}^2\cos(a)-\frac{1}{2}\cos{(a+x)}\sin{(a+x)}\sin(a)-\frac{1}{2}x\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2*x*sin(a)
```

giac [A] time = 1.03, size = 14, normalized size = 0.78

$$-\frac{1}{2}\,x\sin(a) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="giac")
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

maple [A] time = 0.05, size = 15, normalized size = 0.83

$$-\frac{x\sin(a)}{2} - \frac{\cos\left(a + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+x)*sin(x),x)
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

maxima [A] time = 0.42, size = 14, normalized size = 0.78

$$-\frac{1}{2}\,x\sin(a) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="maxima")
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$-\frac{\cos\left(a + 2\,x\right)}{4} - \frac{x\,\sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + x)*sin(x),x)
[Out] - cos(a + 2*x)/4 - (x*sin(a))/2
```

sympy [B] time = 0.54, size = 32, normalized size = 1.78

$$\frac{x\sin\left(x\right)\cos\left(a + x\right)}{2} - \frac{x\sin\left(a + x\right)\cos\left(x\right)}{2} + \frac{\sin\left(x\right)\sin\left(a + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x)
[Out] x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 + sin(x)*sin(a + x)/2
```

3.235 $\int \sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

```
[Out] -2*cos(x)/(1+sin(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2646}

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[x]],x]
```

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

Mathematica [B] time = 0.01, size = 40, normalized size = 3.33

$$\frac{2\sqrt{\sin(x)+1}\left(\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

fricas [B] time = 0.42, size = 24, normalized size = 2.00

$$-\frac{2\,(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

giac [B] time = 1.16, size = 22, normalized size = 1.83

$$2\sqrt{2}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)
```

maple [A] time = 0.06, size = 17, normalized size = 1.42

$$\frac{2\left(\sin(x)-1\right)\sqrt{\sin(x)+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+1)^(1/2),x)
```

```
[Out] 2*(sin(x)-1)*(sin(x)+1)^(1/2)/cos(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x) + 1), x)
```

mupad [B] time = 0.14, size = 16, normalized size = 1.33

$$\frac{2\,\left(\sin(x)-1\right)\,\sqrt{\sin(x)+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + 1)^(1/2),x)
```

```
[Out] (2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin{\left(x\right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x) + 1), x)
```

3.236 $\int \sqrt{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

```
[Out] 2*cos(x)/(1-sin(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2646}

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sin[x]],x]
[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Mathematica [B] time = 0.01, size = 42, normalized size = 3.00

$$\frac{2\sqrt{1-\sin(x)}\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sin[x]],x]
[Out] (2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])
```

fricas [B] time = 0.43, size = 26, normalized size = 1.86

$$\frac{2\,(\cos(x)+\sin(x)+1)\sqrt{-\sin(x)+1}}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="fricas")
[Out] 2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)
```

giac [B] time = 1.20, size = 35, normalized size = 2.50

$$-2\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)-\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(cos(-1/4*pi + 1/2*x)*sgn(sin(-1/4*pi + 1/2*x)) - sgn(sin(-1/4*p
i + 1/2*x)))
```

maple [A] time = 0.07, size = 23, normalized size = 1.64

$$-\frac{2\left(\sin(x)-1\right)\left(\sin(x)+1\right)}{\sqrt{-\sin(x)+1}\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x))^(1/2),x)
```

```
[Out] -2*(sin(x)-1)*(sin(x)+1)/cos(x)/(1-sin(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sin(x) + 1), x)
```

mupad [B] time = 0.14, size = 18, normalized size = 1.29

$$\frac{2\sqrt{1-\sin(x)}\,\left(\sin(x)+1\right)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin(x))^(1/2),x)
```

```
[Out] (2*(1 - sin(x))^(1/2)*(sin(x) + 1))/cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\sin{\left(x \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sin(x)), x)
```

3.237 $\int \sqrt{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

```
[Out] 2*sin(x)/(1+cos(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2646}

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cos[x]],x]
```

```
[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{1+\cos(x)}}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.33

$$2\sqrt{\cos(x)+1}\,\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cos[x]],x]
```

```
[Out] 2*Sqrt[1 + Cos[x]]*Tan[x/2]
```

fricas [A] time = 0.42, size = 10, normalized size = 0.83

$$\frac{2\,\sin(x)}{\sqrt{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sin(x)/sqrt(cos(x) + 1)
```

giac [A] time = 1.04, size = 14, normalized size = 1.17

$$2\sqrt{2}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sgn(cos(1/2*x))*sin(1/2*x)
```

maple [B] time = 0.05, size = 22, normalized size = 1.83

$$\frac{4\sqrt{2}\ \cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{2\cos(x)+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+1)^(1/2),x)
```

```
[Out] 2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)
```

maxima [A] time = 1.16, size = 9, normalized size = 0.75

$$2\sqrt{2}\ \sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sin(1/2*x)
```

mupad [B] time = 0.15, size = 10, normalized size = 0.83

$$\frac{2\ \sin(x)}{\sqrt{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)^(1/2),x)
```

```
[Out] (2*sin(x))/(cos(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cos{(x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x) + 1), x)
```

3.238 $\int \sqrt{1-\cos(x)}\, dx$

Optimal. Leaf size=14

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

```
[Out] -2*sin(x)/(1-cos(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2646}

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]],x]
[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\cos(x)}\, dx = -\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.29

$$-2\sqrt{1-\cos(x)}\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]],x]
[Out] -2*Sqrt[1 - Cos[x]]*Cot[x/2]
```

fricas [A] time = 0.42, size = 18, normalized size = 1.29

$$-\frac{2\,(\cos(x)+1)\sqrt{-\cos(x)+1}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="fricas")
[Out] -2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)
```

giac [A] time = 1.07, size = 23, normalized size = 1.64

$$-2\sqrt{2}\left(\cos\left(\frac{1}{2}x\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))
```

maple [A] time = 0.06, size = 22, normalized size = 1.57

$$-\frac{4\sqrt{2}\,\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{-2\cos(x)+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2),x)
[Out] -2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

maxima [A] time = 0.97, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{2}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.86

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x))^(1/2),x)
[Out] -(2*sin(x))/(1 - cos(x))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**(1/2),x)
[Out] Integral(sqrt(1 - cos(x)), x)
```

3.239 $\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

```
[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx &= \int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= \frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.81

$$\frac{2}{3}\left(x^{3/2}+(x-1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*((-1 + x)^(3/2) + x^(3/2)))/3
```

fricas [A] time = 0.40, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

giac [A] time = 1.03, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{2\,(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x-1)^(1/2)+x^(1/2)),x)
```

```
[Out] 2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{x-1}-\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x - 1) - sqrt(x)), x)
```

mupad [B] time = 0.19, size = 21, normalized size = 1.00

$$\frac{2\,x\,\sqrt{x-1}}{3}-\frac{2\,\sqrt{x-1}}{3}+\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x - 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (2*x*(x - 1)^(1/2))/3 - (2*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

sympy [B] time = 0.39, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x-1}}{-3\sqrt{x}\,+3\sqrt{x-1}}-\frac{4x}{-3\sqrt{x}\,+3\sqrt{x-1}}+\frac{2}{-3\sqrt{x}\,+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*sq
rt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))
```

3.240 $\int \frac{1}{1-\sqrt{1+x}}\,dx$

Optimal. Leaf size=24

$$-2\sqrt{x+1} - 2\log\left(1-\sqrt{x+1}\right)$$

```
[Out] -2*ln(1-(1+x)^(1/2))-2*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 190, 43}

$$-2\sqrt{x+1} - 2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[1 + x])^(-1),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1-\sqrt{x}}\,dx, x, 1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)dx, x, \sqrt{1+x}\right)\\
&= -2\sqrt{1+x} - 2\log\left(1-\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-2\sqrt{x+1} - 2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[1 + x])^(-1),x]
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

fricas [A] time = 0.41, size = 18, normalized size = 0.75

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

giac [A] time = 1.21, size = 19, normalized size = 0.79

$$-2\sqrt{x+1}-2\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x + 1) - 2*log(abs(sqrt(x + 1) - 1))
```

maple [A] time = 0.01, size = 31, normalized size = 1.29

$$-\ln(x)-\ln\left(-1+\sqrt{x+1}\right)+\ln\left(1+\sqrt{x+1}\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^(1/2)),x)
[Out] -ln(x)-2*(x+1)^(1/2)-ln(-1+(x+1)^(1/2))+ln(1+(x+1)^(1/2))
```

maxima [A] time = 0.42, size = 18, normalized size = 0.75

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

mupad [B] time = 0.11, size = 18, normalized size = 0.75

$$-2\ln\left(\sqrt{x+1}-1\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^(1/2) - 1),x)
[Out] - 2*log((x + 1)^(1/2) - 1) - 2*(x + 1)^(1/2)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**(1/2)),x)
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

3.241 $\int \frac{x}{\sqrt{36+x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

```
[Out] 1/2*arcsinh(1/6*x^2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[36 + x^4],x]
[Out] ArcSinh[x^2/6]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{36+x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{36+x^2}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[36 + x^4],x]
[Out] ArcSinh[x^2/6]/2
```

fricas [A] time = 0.42, size = 16, normalized size = 1.33

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

giac [A] time = 1.09, size = 16, normalized size = 1.33

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

maple [A] time = 0.01, size = 9, normalized size = 0.75

$$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+36)^(1/2),x)
[Out] 1/2*arcsinh(1/6*x^2)
```

maxima [B] time = 0.41, size = 33, normalized size = 2.75

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 36)^(1/2),x)
[Out] asinh(x^2/6)/2
```

sympy [A] time = 0.90, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+36)**(1/2),x)
[Out] asinh(x**2/6)/2
```

3.242 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]

[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

fricas [A] time = 0.42, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

giac [A] time = 1.17, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

maple [B] time = 0.03, size = 92, normalized size = 2.88

$$-\ln(x-1) - 2\ln\left(x^{\frac{1}{6}} + 1\right) + \ln\left(\sqrt{x} - 1\right) - \ln\left(\sqrt{x} + 1\right) - 2\ln\left(x^{\frac{1}{3}} - 1\right) + 2\ln\left(x^{\frac{1}{6}} - 1\right) + \ln\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - \ln\left(x^{\frac{1}{3}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
[Out] 2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1)
+2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)+6*x^(1/6)-ln(x-1)-2*ln(x^(1/3)-1)+ln
(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

maxima [A] time = 0.42, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln\left(x^{1/6} + 1\right) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.243 $\int \log\left(2+3x^2\right)\,dx$

Optimal. Leaf size=33

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

[In] Int[Log[2 + 3*x^2],x]

[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(2+3x^2\right)\,dx &= x\log\left(2+3x^2\right)-6\int \frac{x^2}{2+3x^2}\,dx\\
&= -2x+x\log\left(2+3x^2\right)+4\int \frac{1}{2+3x^2}\,dx\\
&= -2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

fricas [A] time = 0.44, size = 32, normalized size = 0.97

$$\frac{2}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)+x\log\left(3x^2+2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

giac [A] time = 1.03, size = 26, normalized size = 0.79

$$x\log\left(3x^2+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

maple [A] time = 0.01, size = 27, normalized size = 0.82

$$x\ln\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\arctan\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x)
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

maxima [A] time = 0.97, size = 26, normalized size = 0.79

$$x\log\left(3x^2+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

mupad [B] time = 0.06, size = 26, normalized size = 0.79

$$\frac{2\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}-2x+x\ln\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(3*x^2 + 2),x)
[Out] (2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

sympy [**A**] time = 0.14, size = 31, normalized size = 0.94

$$x\log\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

3.244 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

[Out] ln(sin(x))

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x],x]

[Out] Log[Sin[x]]

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x],x]

[Out] Log[Sin[x]]

fricas [B] time = 0.44, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(2x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x),x, algorithm="fricas")

[Out] 1/2*log(-1/2*cos(2*x) + 1/2)

giac [A] time = 1.16, size = 4, normalized size = 1.33

$$\log\left(|\sin(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x),x, algorithm="giac")

[Out] log(abs(sin(x)))

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
[Out] ln(sin(x))
```

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
[Out] log(sin(x))
```

mupad [B] time = 0.02, size = 3, normalized size = 1.00

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
[Out] log(sin(x))
```

sympy [A] time = 0.06, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
[Out] log(sin(x))
```

3.245 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

fricas [B] time = 0.41, size = 48, normalized size = 4.00

$$\frac{4\cos(2x)^2+3(x\cos(2x)-x)\sin(2x)+2\cos(2x)-2}{3(\cos(2x)-1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

giac [B] time = 1.15, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$-\frac{\left(\cot^3(x)\right)}{3} + x + \cot(x) - \frac{\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+x+cot(x)-1/2*Pi
```

maxima [A] time = 0.97, size = 16, normalized size = 1.33

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

mupad [B] time = 0.03, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] x + cot(x) - cot(x)^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.246 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] ln(cosh(x))
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

fricas [B] time = 0.43, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 1.09, size = 11, normalized size = 3.67

$$-x + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
[Out] ln(cosh(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
[Out] log(cosh(x))
```

mupad [B] time = 0.01, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
[Out] log(cosh(x))
```

sympy [B] time = 0.13, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
[Out] x - log(tanh(x) + 1)
```

3.247 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

[Out] ln(sinh(x))

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x],x]

[Out] Log[Sinh[x]]

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(x)\,dx = \log(\sinh(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x],x]

[Out] Log[Sinh[x]]

fricas [B] time = 0.43, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x),x, algorithm="fricas")

[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))

giac [B] time = 0.94, size = 12, normalized size = 4.00

$$-x + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x),x, algorithm="giac")

[Out] -x + log(abs(e^(2*x) - 1))

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
[Out] ln(sinh(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
[Out] log(sinh(x))
```

mupad [B] time = 0.14, size = 3, normalized size = 1.00

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
[Out] log(sinh(x))
```

sympy [B] time = 0.32, size = 12, normalized size = 4.00

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

3.248 $\int b^x\,dx$

Optimal. Leaf size=8

$$\frac{b^x}{\log(b)}$$

[Out] b^x/ln(b)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

[In] Int[b^x,x]

[Out] b^x/Log[b]

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int b^x\,dx = \frac{b^x}{\log(b)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

[In] Integrate[b^x,x]

[Out] b^x/Log[b]

fricas [A] time = 0.41, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^x,x, algorithm="fricas")

[Out] b^x/log(b)

giac [A] time = 1.03, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^x,x, algorithm="giac")

```
[Out] b^x/log(b)
```

maple [A] time = 0.01, size = 9, normalized size = 1.12

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x)
```

```
[Out] b^x/ln(b)
```

maxima [A] time = 0.41, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="maxima")
```

```
[Out] b^x/log(b)
```

mupad [B] time = 0.16, size = 8, normalized size = 1.00

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x)
```

```
[Out] b^x/log(b)
```

sympy [A] time = 0.09, size = 8, normalized size = 1.00

$$\begin{cases} \frac{b^x}{\log(b)} & \text{for } \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**x,x)
```

```
[Out] Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))
```

3.249 $\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

```
[Out] -x*(2+1/x^4+x^4)^(1/2)/(x^4+1)+1/3*x^5*(2+1/x^4+x^4)^(1/2)/(x^4+1)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1351, 1355, 14}

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^(-4) + x^4],x]
```

```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) + x^4])/(3*(
1 + x^4))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1351

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Dist[(x
^(n*FracPart[p])*(a + b/x^n + c*x^n)^FracPart[p])/(b + a*x^n + c*x^(2*n))^F
racPart[p], Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b, c
, n, p}, x] && EqQ[mn, -n] && !IntegerQ[p] && PosQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx = \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{\sqrt{1+2x^4+x^8}}{x^2}\,dx}{\sqrt{1+2x^4+x^8}}$$
$$= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{1+x^4}{x^2}\,dx}{1+x^4}$$
$$= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \left(\frac{1}{x^2}+x^2\right)dx}{1+x^4}$$
$$= -\frac{x\sqrt{2+\frac{1}{x^4}+x^4}}{1+x^4}+\frac{x^5\sqrt{2+\frac{1}{x^4}+x^4}}{3\left(1+x^4\right)}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.59

$$\frac{x\left(x^4-3\right)\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^(-4) + x^4],x]
[Out] (x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))
```

fricas [A] time = 0.40, size = 10, normalized size = 0.20

$$\frac{x^4-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^4 - 3)/x
```

giac [A] time = 1.12, size = 11, normalized size = 0.22

$$\frac{1}{3}\,x^3-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="giac")
[Out] 1/3*x^3 - 1/x
```

maple [A] time = 0.01, size = 32, normalized size = 0.65

$$\frac{\left(x^4-3\right)\sqrt{\frac{x^8+2x^4+1}{x^4}}\;x}{3x^4+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+1/x^4+x^4)^(1/2),x)
[Out] 1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)
```

maxima [A] time = 0.99, size = 10, normalized size = 0.20

$$\frac{x^4 - 3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^4 - 3)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{x^4} + x^4 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^4 + x^4 + 2)^(1/2),x)
[Out] int((1/x^4 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 + 2 + \frac{1}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x**4+x**4)**(1/2),x)
[Out] Integral(sqrt(x**4 + 2 + x**(-4)), x)
```

3.250 $\int \frac{1+2x}{2+3x} \, dx$

Optimal. Leaf size=16

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

[Out] 2/3*x-1/9*ln(2+3*x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(2 + 3*x),x]

[Out] (2*x)/3 - Log[2 + 3*x]/9

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{2+3x} \, dx &= \int \left(\frac{2}{3} - \frac{1}{3(2+3x)}\right) dx \\ &= \frac{2x}{3} - \frac{1}{9}\log(2+3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{1}{9}(6x - \log(3x+2) + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(2 + 3*x),x]

[Out] (4 + 6*x - Log[2 + 3*x])/9

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{2}{3}x - \frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(2+3*x),x, algorithm="fricas")

[Out] 2/3*x - 1/9*log(3*x + 2)

giac [A] time = 1.00, size = 13, normalized size = 0.81

$$\frac{2}{3}x - \frac{1}{9}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="giac")
[Out] 2/3*x - 1/9*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2x}{3} - \frac{\ln(3x+2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(3*x+2),x)
[Out] 2/3*x-1/9*ln(3*x+2)
```

maxima [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{2}{3}x - \frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="maxima")
[Out] 2/3*x - 1/9*log(3*x + 2)
```

mupad [B] time = 0.14, size = 10, normalized size = 0.62

$$\frac{2x}{3} - \frac{\ln\left(x+\frac{2}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(3*x + 2),x)
[Out] (2*x)/3 - log(x + 2/3)/9
```

sympy [A] time = 0.08, size = 12, normalized size = 0.75

$$\frac{2x}{3} - \frac{\log(3x+2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x)
[Out] 2*x/3 - log(3*x + 2)/9
```

3.251 $\int x \log\left(x+\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=40

$$-\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

```
[Out] 1/4*arcsinh(x)+1/2*x^2*ln(x+(x^2+1)^(1/2))-1/4*x*(x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2536, 321, 215}

$$-\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(x*Sqrt[1 + x^2])/4 + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2536

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]]*((g_.)*(x_))
^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[a + c*x^2]])/
(g*(m + 1)), x] - Dist[(a*c*f^2)/(g*(m + 1)), Int[(g*x)^(m + 1)/(d*e*(a + c
*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g
, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int x\log\left(x+\sqrt{1+x^2}\right)dx&=\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int\frac{x^2}{\sqrt{1+x^2}}\,dx\\&=-\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)+\frac{1}{4}\int\frac{1}{\sqrt{1+x^2}}\,dx\\&=-\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.90

$$\frac{1}{4}\left(-\sqrt{x^2+1}\,x+2x^2\log\left(\sqrt{x^2+1}+x\right)+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + Sqrt[1 + x^2]],x]
[Out] (-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4
```

fricas [A] time = 0.41, size = 30, normalized size = 0.75

$$\frac{1}{4}\left(2\,x^2+1\right)\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,\sqrt{x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

giac [A] time = 0.94, size = 40, normalized size = 1.00

$$\frac{1}{2}\,x^2\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,\sqrt{x^2+1}\,x-\frac{1}{4}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x^
2 + 1))
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x\ln\left(x+\sqrt{x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(x^2+1)^(1/2)),x)
[Out] int(x*ln(x+(x^2+1)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,x^2-\int\frac{x^2}{2\left(x^3+\left(x^2+1\right)^{\frac{3}{2}}+x\right)}\,dx+\frac{1}{4}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 +
1)^(3/2) + x), x) + 1/4*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.80

$$x\,\ln\left(x+\sqrt{x^2+1}\right)\,\left(\frac{x}{2}+\frac{1}{4\,x}\right)-\frac{x\,\sqrt{x^2+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x + (x^2 + 1)^(1/2)),x)
[Out] x*log(x + (x^2 + 1)^(1/2))*(x/2 + 1/(4*x)) - (x*(x^2 + 1)^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(x+\sqrt{x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral(x*log(x + sqrt(x**2 + 1)), x)
```

3.252 $\int x \left(1+e^x \sin(x)\right)^2 \, dx$

Optimal. Leaf size=128

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x$$

```
[Out] -3/32*exp(2*x)+1/8*exp(2*x)*x+1/2*x^2+exp(x)*cos(x)-exp(x)*x*cos(x)-1/32*ex
p(2*x)*cos(2*x)+exp(x)*x*sin(x)+1/16*exp(2*x)*cos(x)*sin(x)-1/4*exp(2*x)*x*
cos(x)*sin(x)-1/16*exp(2*x)*sin(x)^2+1/4*exp(2*x)*x*sin(x)^2+1/32*exp(2*x)*
sin(2*x)
```

Rubi [A] time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6742, 4432, 4465, 4433, 4434, 2194, 4469, 12}

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2
*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*C
os[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)
*Sin[2*x])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(1+e^x\sin(x))^2\,dx=\int\left(x+2e^x x\sin(x)+e^{2x}x\sin^2(x)\right)dx$$
$$=\frac{x^2}{2}+2\int e^x x\sin(x)\,dx+\int e^{2x}x\sin^2(x)\,dx$$
$$=\frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^x x\cos(x)+e^x x\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-2\int\left(-\frac{1}{2}\right.$$
$$=\frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^x x\cos(x)+e^x x\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{8}\int e^{2x}$$
$$=-\frac{e^{2x}}{16}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}x$$
$$=-\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}$$
$$=-\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos$$

Mathematica [A] time = 0.17, size = 67, normalized size = 0.52

$$\frac{1}{8}\left(4x^2+e^{2x}(2x-1)+8e^x x\sin(x)-e^{2x}x\cos(2x)-8e^x(x-1)\cos(x)-e^{2x}(2x-1)\sin(x)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] +
8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

fricas [A] time = 0.43, size = 55, normalized size = 0.43

$$-(x-1)\cos(x)e^x+\frac{1}{2}x^2-\frac{1}{8}\left(2x\cos(x)^2-3x+1\right)e^{(2x)}-\frac{1}{8}\left((2x-1)\cos(x)e^{(2x)}-8xe^x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2*x) - 1/8*
((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)
```

giac [A] time = 1.10, size = 57, normalized size = 0.45

$$\frac{1}{2}x^2 - \frac{1}{16}(2x\cos(2x) + (2x-1)\sin(2x))e^{(2x)} + \frac{1}{8}(2x-1)e^{(2x)} - ((x-1)\cos(x) - x\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*(2*x - 1)*
e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x
```

maple [A] time = 0.02, size = 63, normalized size = 0.49

$$-\frac{x\cos(2x)\,e^{2x}}{8} + x\,e^x\sin(x) + \frac{x^2}{2} + \frac{x\,e^{2x}}{4} + 2\left(-\frac{x}{2} + \frac{1}{2}\right)\cos(x)e^x + \frac{\left(-\frac{x}{4} + \frac{1}{8}\right)e^{2x}\sin(2x)}{2} - \frac{e^{2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+exp(x)*sin(x))^2,x)
```

```
[Out] 1/2*x^2+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/4*exp(x)^2*x-1/8*exp
(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)*sin(2*x)
```

maxima [A] time = 0.47, size = 58, normalized size = 0.45

$$-\frac{1}{8}x\cos(2x)e^{(2x)} - (x-1)\cos(x)e^x - \frac{1}{16}(2x-1)e^{(2x)}\sin(2x) + xe^x\sin(x) + \frac{1}{2}x^2 + \frac{1}{8}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(2
*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)
```

mupad [B] time = 0.28, size = 69, normalized size = 0.54

$$\frac{3\,x\,e^{2\,x}}{8} - \frac{e^{2\,x}}{8} + e^x\cos(x) + \frac{x^2}{2} - \frac{x\,e^{2\,x}\cos(x)^2}{4} + \frac{e^{2\,x}\cos(x)\sin(x)}{8} - x\,e^x\cos(x) + x\,e^x\sin(x) - \frac{x\,e^{2\,x}\cos(x)\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(exp(x)*sin(x) + 1)^2,x)
```

```
[Out] (3*x*exp(2*x))/8 - exp(2*x)/8 + exp(x)*cos(x) + x^2/2 - (x*exp(2*x)*cos(x)^
2)/4 + (exp(2*x)*cos(x)*sin(x))/8 - x*exp(x)*cos(x) + x*exp(x)*sin(x) - (x*
exp(2*x)*cos(x)*sin(x))/4
```

sympy [A] time = 4.95, size = 109, normalized size = 0.85

$$\frac{x^2}{2} + \frac{3xe^{2x}\sin^2(x)}{8} - \frac{xe^{2x}\sin(x)\cos(x)}{4} + \frac{xe^{2x}\cos^2(x)}{8} + xe^x\sin(x) - xe^x\cos(x) - \frac{e^{2x}\sin^2(x)}{8} + \frac{e^{2x}\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))**2,x)
```

```
[Out] x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x)
*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 + e
xp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)
```

3.253 $\int e^x x \cos(x)\, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x \cos(x)\, dx &= \frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x) - \int \left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right) dx \\
&= \frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x) - \frac{1}{2}\int e^x \cos(x)\, dx - \frac{1}{2}\int e^x \sin(x)\, dx \\
&= \frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.60

$$\frac{1}{2}e^x((x-1)\sin(x) + x\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

fricas [A] time = 0.44, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos(x)e^{x} + \frac{1}{2}\,(x-1)e^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

giac [A] time = 0.94, size = 15, normalized size = 0.50

$$\frac{1}{2}\,(x\cos(x) + (x-1)\sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

maple [A] time = 0.02, size = 20, normalized size = 0.67

$$\frac{x\cos(x)\mathrm{e}^{x}}{2} - \left(-\frac{x}{2} + \frac{1}{2}\right)\mathrm{e}^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x)
```

```
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos(x)e^{x} + \frac{1}{2}\,(x-1)e^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

mupad [B] time = 0.17, size = 17, normalized size = 0.57

$$\frac{\mathrm{e}^{x}\,\left(x\,\cos(x) - \sin(x) + x\,\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

sympy [A] time = 0.83, size = 27, normalized size = 0.90

$$\frac{xe^{x}\sin{(x)}}{2} + \frac{xe^{x}\cos{(x)}}{2} - \frac{e^{x}\sin{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

3.254 $\int \frac{1}{(-3+x)^4}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3(3-x)^3}$$

```
[Out] 1/3/(3-x)^3
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {32}

$$\frac{1}{3(3-x)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)^(-4),x]
```

```
[Out] 1/(3*(3 - x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(-3+x)^4}\,dx=\frac{1}{3(3-x)^3}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)^(-4),x]
```

```
[Out] -1/3*1/(-3 + x)^3
```

fricas [B] time = 0.41, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^3-9\,x^2+27\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 9*x^2 + 27*x - 27)
```

giac [A] time = 1.08, size = 7, normalized size = 0.64

$$-\frac{1}{3\,(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 3)^3
```

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{1}{3\left(x-3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-3)^4,x)
```

```
[Out] -1/3/(x-3)^3
```

maxima [A] time = 0.41, size = 7, normalized size = 0.64

$$-\frac{1}{3\left(x-3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x - 3)^3
```

mupad [B] time = 0.07, size = 7, normalized size = 0.64

$$-\frac{1}{3\left(x-3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - 3)^4,x)
```

```
[Out] -1/(3*(x - 3)^3)
```

sympy [B] time = 0.11, size = 17, normalized size = 1.55

$$-\frac{1}{3x^{3}-27x^{2}+81x-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)**4,x)
```

```
[Out] -1/(3*x**3 - 27*x**2 + 81*x - 81)
```

3.255 $\int \frac{x}{-1+x^3} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [**A**] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{-1+x^3}\,dx = \frac{1}{3}\int \frac{1}{-1+x}\,dx - \frac{1}{3}\int \frac{-1+x}{1+x+x^2}\,dx$$
$$= \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx$$
$$= \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^3),x]
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

fricas [A] time = 0.43, size = 32, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

giac [A] time = 1.07, size = 33, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
abs(x - 1))
```

maple [A] time = 0.00, size = 33, normalized size = 0.82

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-1),x)
[Out] 1/3*ln(x-1)-1/6*ln(x^2+x+1)+1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 0.96, size = 32, normalized size = 0.80

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\,\log\left(x^{2}+x+1\right)+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

mupad [B] time = 0.11, size = 46, normalized size = 1.15

$$\frac{\ln\left(x-1\right)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - 1),x)
```

```
[Out] log(x - 1)/3 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.13, size = 41, normalized size = 1.02

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^{2}+x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

3.256 $\int \frac{x}{-1+x^4} \, dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

```
[Out] -1/2*arctanh(x^2)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {275, 207}

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^4),x]
```

```
[Out] -ArcTanh[x^2]/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{-1+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 2.88

$$\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^4),x]
```

```
[Out] Log[1 - x^2]/4 - Log[1 + x^2]/4
```

fricas [B] time = 0.43, size = 17, normalized size = 2.12

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

giac [B] time = 1.01, size = 18, normalized size = 2.25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="giac")
[Out] -1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))
```

maple [B] time = 0.00, size = 22, normalized size = 2.75

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-1),x)
[Out] 1/4*ln(x-1)+1/4*ln(x+1)-1/4*ln(x^2+1)
```

maxima [B] time = 0.41, size = 17, normalized size = 2.12

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="maxima")
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$-\frac{\operatorname{atanh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 1),x)
[Out] -atanh(x^2)/2
```

sympy [B] time = 0.10, size = 15, normalized size = 1.88

$$\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-1),x)
[Out] log(x**2 - 1)/4 - log(x**2 + 1)/4
```

3.257 $\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx$

Optimal. Leaf size=227

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\right.$$

```
[Out] 1/8*(2+I*(-2)^(1/4))*ln(x)*ln(1-(1/2+1/2*I)*x*2^(1/4))+1/16*(4+(1-I)*2^(3/4
))*ln(x)*ln(1+(1/2+1/2*I)*x*2^(1/4))+1/8*(2+(-2)^(1/4))*ln(x)*ln(1-1/2*(-1)
^(3/4)*x*2^(3/4))+1/8*(2-(-2)^(1/4))*ln(x)*ln(1+1/2*(-1)^(3/4)*x*2^(3/4))+1
/16*(4+(1-I)*2^(3/4))*polylog(2,(-1/2-1/2*I)*x*2^(1/4))+1/8*(2+I*(-2)^(1/4)
)*polylog(2,(1/2+1/2*I)*x*2^(1/4))+1/8*(2-(-2)^(1/4))*polylog(2,-1/2*(-1)^(
3/4)*x*2^(3/4))+1/8*(2+(-2)^(1/4))*polylog(2,1/2*(-1)^(3/4)*x*2^(3/4))
```

Rubi [A] time = 0.20, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2357, 2317, 2391}

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((4 + (1 - I)*
2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/16 + ((2 + (-2)^(1/4))*Log[x]
*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1
)^(3/4)*x)/2^(1/4)])/8 + ((4 + (1 - I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(
3/4)])/16 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 -
(-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*Po
lyLog[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \left(\frac{\left(-2+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-x\right)} + \frac{\left(-2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-ix\right)} + \frac{\left(2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}+ix\right)} + \frac{\left(2+\sqrt[4]{-2}\right)\log(\right.}{8\left(\sqrt[4]{-2}+x\right)}\right.$$

$$= \frac{1}{8}\left(-2+\sqrt[4]{-2}\right)\int \frac{\log(x)}{\sqrt[4]{-2}-x}\,dx + \frac{1}{8}\left(-2i+\sqrt[4]{-2}\right)\int \frac{\log(x)}{\sqrt[4]{-2}-ix}\,dx + \frac{1}{8}\left(2i+\sqrt[4]{-2}\right)\int \frac{1}{\sqrt[4]{}}$$

$$= \frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}\left(2\right.$$

$$= \frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}\left(2\right.$$

Mathematica [A] time = 0.28, size = 194, normalized size = 0.85

$$\frac{1}{8}\left(\left(2+\frac{1-i}{\sqrt[4]{2}}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right) + \left(2+\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(1-i)x}{2^{3/4}}\right) - \left(\sqrt[4]{-2}-2\right)\text{PolyLog}\left(2,\frac{(1-i)x}{2^{3/4}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Log[x])/(2 + x^4),x]
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))*
Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I)*
x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2 + (
1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*PolyLog
[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2^(3/4
)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(x^3+1)\log(x)}{x^4+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")
[Out] integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="giac")
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

maple [B] time = 0.03, size = 1210, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*ln(x)/(x^4+2),x)
```

```
[Out] -1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*2^(1/4)*ln(x)*ln((1/2*2^(3/4)+1/2*I*2^(3
/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))-1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*2^(1/
4)*dilog((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))+1/4*I/(
1/2*I*2^(3/4)-1/2*2^(3/4))^3*2^(1/4)*ln(x)*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)
/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4*I/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*2^(1/4)*d
ilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4/(1/2*2^
(3/4)+1/2*I*2^(3/4))^3*ln(x)*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+
1/2*I*2^(3/4)))+1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*dilog((1/2*2^(3/4)+1/2*I*
2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))-1/4*I/(-1/2*2^(3/4)-1/2*I*2^(3/4))^
3*2^(1/4)*dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4))
)-1/4*I/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*2^(1/4)*dilog((-1/2*I*2^(3/4)+1/2*2^
(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*2^
(1/4)*ln(x)*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1
/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*2^(1/4)*dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x
)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*ln(x)*ln((
1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4/(1/2*I*2^(3/4
)-1/2*2^(3/4))^3*dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(
3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*2^(1/4)*ln(x)*ln((-1/2*2^(3/4)-1/
2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4
))^3*2^(1/4)*dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/
4)))+1/4*I/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*2^(1/4)*dilog((1/2*2^(3/4)+1/2*I*2
^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))-1/4*I/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3
*2^(1/4)*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4
)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)
-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*dilog(
(-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))-1/4*I/(-1/2*I*
2^(3/4)+1/2*2^(3/4))^3*2^(1/4)*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/
2*I*2^(3/4)+1/2*2^(3/4)))+1/4*I/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*2^(1/4)*ln(x)
*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))-1/4/(-1/2*I*
2^(3/4)+1/2*2^(3/4))^3*2^(1/4)*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/
2*I*2^(3/4)+1/2*2^(3/4)))-1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*2^(1/4)*dilog(
(-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))+1/4/(-1/2*I*2^
(3/4)+1/2*2^(3/4))^3*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4
)+1/2*2^(3/4)))+1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1/2*I*2^(3/4)+1/
2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(x)\,\left(x^3+1\right)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*(x^3 + 1))/(x^4 + 2),x)
```

```
[Out] int((log(x)*(x^3 + 1))/(x^4 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^2-x+1\right)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

3.258 $\int (\log(x) + \log(1 + x) + \log(2 + x))\, dx$

Optimal. Leaf size=24

$$-3x + x\log(x) + (x + 1)\log(x + 1) + (x + 2)\log(x + 2)$$

```
[Out] -3*x+x*ln(x)+(1+x)*ln(1+x)+(2+x)*ln(2+x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2295, 2389}

$$-3x + x\log(x) + (x + 1)\log(x + 1) + (x + 2)\log(x + 2)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x] + Log[1 + x] + Log[2 + x],x]
```

```
[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int (\log(x) + \log(1 + x) + \log(2 + x))\, dx &= \int \log(x)\, dx + \int \log(1 + x)\, dx + \int \log(2 + x)\, dx\\ &= -x + x\log(x) + \text{Subst}(\int \log(x)\, dx, x, 1 + x) + \text{Subst}(\int \log(x)\, dx,\\ &= -3x + x\log(x) + (1 + x)\log(1 + x) + (2 + x)\log(2 + x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.25

$$-3x + x\log(x) + x\log(x + 1) + x\log(x + 2) + \log(x + 1) + 2\log(x + 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]
```

```
[Out] -3*x + x*Log[x] + Log[1 + x] + x*Log[1 + x] + 2*Log[2 + x] + x*Log[2 + x]
```

fricas [A] time = 0.43, size = 24, normalized size = 1.00

$$(x + 2)\log(x + 2) + (x + 1)\log(x + 1) + x\log(x) - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="fricas")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x
```

giac [A] time = 1.04, size = 25, normalized size = 1.04

$$(x+2)\log(x+2)+(x+1)\log(x+1)+x\log(x)-3x-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="giac")
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

maple [A] time = 0.00, size = 26, normalized size = 1.08

$$x\ln(x)-3x+(x+1)\ln(x+1)+(x+2)\ln(x+2)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)+ln(x+1)+ln(x+2),x)
[Out] x*ln(x)-3*x+(x+1)*ln(x+1)-3+(x+2)*ln(x+2)
```

maxima [A] time = 0.42, size = 25, normalized size = 1.04

$$(x+2)\log(x+2)+(x+1)\log(x+1)+x\log(x)-3x-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="maxima")
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

mupad [B] time = 0.44, size = 51, normalized size = 2.12

$$\ln(x+1)-3x+2\ln(x+2)+x\ln(x+1)+x\ln(x)+\frac{\ln(x+2)\left(x^3+3x^2+2x\right)}{(x+1)(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1) + log(x + 2) + log(x),x)
[Out] log(x + 1) - 3*x + 2*log(x + 2) + x*log(x + 1) + x*log(x) + (log(x + 2)*(2*
x + 3*x^2 + x^3))/((x + 1)*(x + 2))
```

sympy [A] time = 1.20, size = 37, normalized size = 1.54

$$x\log(x)-3x+\left(x+\frac{1}{2}\right)\log(x+1)+(x+1)\log(x+2)+\frac{\log(x+1)}{2}+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)+ln(1+x)+ln(2+x),x)
[Out] x*log(x) - 3*x + (x + 1/2)*log(x + 1) + (x + 1)*log(x + 2) + log(x + 1)/2 +
 log(x + 2)
```

3.259 $\int \frac{1}{5+x^3} dx$

Optimal. Leaf size=78

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}}$$

```
[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)-1/15*arct
an(1/15*(5^(1/3)-2*x)*5^(2/3)*3^(1/2))*5^(1/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) + Log[5^(1/3
) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*5^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{5+x^3}\,dx = \frac{\int \frac{1}{\sqrt[3]{5}+x}\,dx}{3\,5^{2/3}} + \frac{\int \frac{2\sqrt[3]{5}-x}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{3\,5^{2/3}}$$
$$= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\,5^{2/3}} - \frac{\int \frac{-\sqrt[3]{5}+2x}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{6\,5^{2/3}} + \frac{\int \frac{1}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{2\sqrt[3]{5}}$$
$$= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\,5^{2/3}} - \frac{\log\left(5^{2/3}-\sqrt[3]{5}x+x^2\right)}{6\,5^{2/3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{5}}\right)}{5^{2/3}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}} + \frac{\log\left(\sqrt[3]{5}+x\right)}{3\,5^{2/3}} - \frac{\log\left(5^{2/3}-\sqrt[3]{5}x+x^2\right)}{6\,5^{2/3}}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.91

$$\frac{-\log\left(\sqrt[3]{5}x^2-5^{2/3}x+5\right)+2\log\left(5^{2/3}x+5\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\,5^{2/3}x-5}{5\sqrt{3}}\right)}{6\,5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*Sqrt[3])] + 2*Log[5 + 5^(2/3)*x] -
Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))
```

fricas [A] time = 0.43, size = 69, normalized size = 0.88

$$\frac{1}{15}\cdot 25^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{75}\cdot 25^{\frac{1}{6}}\left(2\cdot 25^{\frac{2}{3}}\sqrt{3}x-5\cdot 25^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{150}\cdot 25^{\frac{2}{3}}\log\left(5x^2-25^{\frac{2}{3}}x+5\cdot 25^{\frac{1}{3}}\right)+\frac{1}{75}\cdot 25^{\frac{2}{3}}\log\left(5x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="fricas")
```

```
[Out] 1/15*25^(1/6)*sqrt(3)*arctan(1/75*25^(1/6)*(2*25^(2/3)*sqrt(3)*x - 5*25^(1/
3)*sqrt(3))) - 1/150*25^(2/3)*log(5*x^2 - 25^(2/3)*x + 5*25^(1/3)) + 1/75*2
5^(2/3)*log(5*x + 25^(2/3))
```

giac [A] time = 0.95, size = 58, normalized size = 0.74

$$\frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^2-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(\left|x+5^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="giac")
```

```
[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^
(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(abs(x + 5^(1/3)))
```

maple [A] time = 0.00, size = 54, normalized size = 0.69

$$\frac{5^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,5^{\frac{2}{3}}x}{5}-1\right)}{3}\right)}{15}+\frac{5^{\frac{1}{3}}\ln\left(x+5^{\frac{1}{3}}\right)}{15}-\frac{5^{\frac{1}{3}}\ln\left(x^{2}-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+5),x)
```

```
[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)+1/15*5^(1
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/5*5^(2/3)*x-1))
```

maxima [A] time = 0.98, size = 57, normalized size = 0.73

$$\frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^{2}-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(x+5^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="maxima")
```

```
[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^
(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(x + 5^(1/3))
```

mupad [B] time = 0.27, size = 70, normalized size = 0.90

$$\frac{5^{1/3}\ln\left(x+5^{1/3}\right)}{15}+\frac{5^{1/3}\ln\left(x+\frac{5^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2}\right)\left(-1+\sqrt{3}\,1i\right)}{30}-\frac{5^{1/3}\ln\left(x-\frac{5^{1/3}\left(1+\sqrt{3}\,1i\right)}{2}\right)\left(1+\sqrt{3}\,1i\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 5),x)
```

```
[Out] (5^(1/3)*log(x + 5^(1/3)))/15 + (5^(1/3)*log(x + (5^(1/3)*(3^(1/2)*1i - 1))
/2)*(3^(1/2)*1i - 1))/30 - (5^(1/3)*log(x - (5^(1/3)*(3^(1/2)*1i + 1))/2)*(
3^(1/2)*1i + 1))/30
```

sympy [A] time = 0.30, size = 73, normalized size = 0.94

$$\frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15}-\frac{\sqrt[3]{5}\log\left(x^{2}-\sqrt[3]{5}x+5^{\frac{2}{3}}\right)}{30}+\frac{\sqrt{3}\sqrt[3]{5}\,\operatorname{atan}\left(\frac{2\sqrt{3}\cdot 5^{\frac{2}{3}}x}{15}-\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+5),x)
```

```
[Out] 5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x + 5**(2/3))/
30 + sqrt(3)*5**(1/3)*atan(2*sqrt(3)*5**(2/3)*x/15 - sqrt(3)/3)/15
```

3.260 $\int \frac{1}{\sqrt{1+x^2}} dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

fricas [B] time = 0.44, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

giac [B] time = 0.93, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
```

```
[Out] arcsinh(x)
```

maxima [A] time = 0.96, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

mupad [B] time = 0.00, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asinh}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

3.261 $\int \sqrt{3+x^2}\, dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + x^2],x]
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+x^2}\, dx &= \frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3+x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + x^2],x]
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

fricas [A] time = 0.44, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+3}\,x-\frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

giac [A] time = 0.99, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+3}\,x-\frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

maple [A] time = 0.00, size = 21, normalized size = 0.78

$$\frac{\sqrt{x^2+3}\;x}{2}+\frac{3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)^(1/2),x)
[Out] 3/2*arcsinh(1/3*3^(1/2)*x)+1/2*x*(x^2+3)^(1/2)
```

maxima [A] time = 0.96, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)
```

mupad [B] time = 0.05, size = 20, normalized size = 0.74

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}+\frac{x\,\sqrt{x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)^(1/2),x)
[Out] (3*asinh((3^(1/2)*x)/3))/2 + (x*(x^2 + 3)^(1/2))/2
```

sympy [A] time = 0.21, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+3}}{2}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)**(1/2),x)
[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

3.262 $\int \frac{x}{(1+x)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x+1} + \log(x+1)$$

```
[Out] 1/(1+x)+ln(1+x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)^2} \, dx &= \int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right) \, dx\\
&= \frac{1}{1+x}+\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

fricas [A] time = 0.41, size = 16, normalized size = 1.60

$$\frac{(x+1)\log(x+1)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

giac [A] time = 0.95, size = 11, normalized size = 1.10

$$\frac{1}{x+1} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\ln(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^2,x)
```

```
[Out] ln(x+1)+1/(x+1)
```

maxima [A] time = 0.41, size = 10, normalized size = 1.00

$$\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 1)^2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.80

$$\log(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

3.263 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4619, 261}

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)\,dx &= x\sin^{-1}(x)-\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2}+x\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

fricas [A] time = 0.44, size = 14, normalized size = 0.88

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="fricas")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

giac [A] time = 1.07, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="giac")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x),x)
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

maxima [A] time = 0.95, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="maxima")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

mupad [B] time = 0.22, size = 14, normalized size = 0.88

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x),x)
[Out] x*asin(x) + (1 - x^2)^(1/2)
```

sympy [A] time = 0.13, size = 12, normalized size = 0.75

$$x \operatorname{asin}{(x)} + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x),x)
[Out] x*asin(x) + sqrt(1 - x**2)
```

3.264 $\int x^2 \sin^{-1}(x)\, dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

```
[Out] -1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4627, 266, 43}

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^{-1}(x)\,dx&=\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx\\&=\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int\frac{x}{\sqrt{1-x}}\,dx,x,x^2\right)\\&=\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\operatorname{Subst}\left(\int\left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx,x,x^2\right)\\&=\frac{\sqrt{1-x^2}}{3}-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(3x^3\sin^{-1}(x)+\sqrt{1-x^2}\left(x^2+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

fricas [A] time = 0.45, size = 24, normalized size = 0.60

$$\frac{1}{3}x^3\arcsin(x) + \frac{1}{9}\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

giac [A] time = 0.86, size = 38, normalized size = 0.95

$$\frac{1}{3}\left(x^2-1\right)x\arcsin(x) + \frac{1}{3}x\arcsin(x) - \frac{1}{9}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{3}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.85

$$\frac{x^3\arcsin(x)}{3} + \frac{\sqrt{-x^2+1}\,x^2}{9} + \frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x)
[Out] 1/3*x^3*arcsin(x)+1/9*(-x^2+1)^(1/2)*x^2+2/9*(-x^2+1)^(1/2)
```

maxima [A] time = 1.01, size = 33, normalized size = 0.82

$$\frac{1}{3}x^3\arcsin(x) + \frac{1}{9}\sqrt{-x^2+1}\,x^2 + \frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.60

$$\frac{x^3\,\mathrm{asin}(x)}{3} + \frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x),x)
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

sympy [A] time = 0.35, size = 32, normalized size = 0.80

$$\frac{x^3\,\mathrm{asin}\,(x)}{3} + \frac{x^2\sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

3.265 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

```
[Out] -ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

Rubi [A] time = 0.11, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {616, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\tan(x)\right)\\
&= -\log(1-\tan(x))+\log(2-\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.38

$$2\left(\frac{1}{2}\log(2\cos(x)-\sin(x))-\frac{1}{2}\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
[Out] 2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

fricas [A] time = 0.47, size = 29, normalized size = 1.38

$$\frac{1}{2}\log\left(\frac{3}{4}\cos(x)^2-\cos(x)\sin(x)+\frac{1}{4}\right)-\frac{1}{2}\log\left(-2\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

giac [A] time = 0.96, size = 15, normalized size = 0.71

$$-\log\left(\left|\tan(x)-1\right|\right)+\log\left(\left|\tan(x)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

maple [A] time = 0.13, size = 14, normalized size = 0.67

$$\ln\left(\tan(x)-2\right)-\ln\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)
```

```
[Out] ln(tan(x)-2)-ln(tan(x)-1)
```

maxima [A] time = 0.42, size = 13, normalized size = 0.62

$$-\log\left(\tan(x)-1\right)+\log\left(\tan(x)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

mupad [B] time = 0.81, size = 9, normalized size = 0.43

$$-2\,\mathrm{atanh}\left(2\tan(x)-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
```

```
[Out] -2*atanh(2*tan(x) - 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

3.266 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

fricas [A] time = 0.45, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

giac [A] time = 1.16, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

maple [A] time = 0.02, size = 11, normalized size = 0.79

$$\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x)
```

```
[Out] 1/2*cos(x)*sin(x)+1/2*x
```

maxima [A] time = 0.42, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B] time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.267 $\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

[Out] -1/x+3*ln(2-x)+2*ln(x)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {893}

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]

[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx &= \int \left(\frac{3}{-2+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx\\ &= -\frac{1}{x}+3\log(2-x)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]

[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]

fricas [A] time = 0.41, size = 18, normalized size = 1.00

$$\frac{3\,x\log\left(x-2\right)+2\,x\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")

[Out] (3*x*log(x - 2) + 2*x*log(x) - 1)/x

giac [A] time = 1.11, size = 18, normalized size = 1.00

$$-\frac{1}{x} + 3 \log(|x - 2|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")
[Out] -1/x + 3*log(abs(x - 2)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$2\ln(x) + 3\ln(x - 2) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2-3*x-2)/(x-2)/x^2,x)
[Out] 3*ln(x-2)-1/x+2*ln(x)
```

maxima [A] time = 0.42, size = 16, normalized size = 0.89

$$-\frac{1}{x} + 3 \log(x - 2) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")
[Out] -1/x + 3*log(x - 2) + 2*log(x)
```

mupad [B] time = 0.15, size = 16, normalized size = 0.89

$$3 \ln(x - 2) + 2 \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)
[Out] 3*log(x - 2) + 2*log(x) - 1/x
```

sympy [A] time = 0.12, size = 14, normalized size = 0.78

$$2\log(x) + 3\log(x - 2) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
[Out] 2*log(x) + 3*log(x - 2) - 1/x
```

3.268 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 1/2*arcsinh(2/3*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
[Out] ArcSinh[(2*x)/3]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
[Out] ArcSinh[(2*x)/3]/2
```

fricas [B] time = 0.41, size = 16, normalized size = 1.60

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

giac [B] time = 1.07, size = 16, normalized size = 1.60

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x)
[Out] 1/2*arcsinh(2/3*x)
```

maxima [A] time = 0.97, size = 6, normalized size = 0.60

$$\frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/2*arcsinh(2/3*x)
```

mupad [B] time = 0.04, size = 6, normalized size = 0.60

$$\frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 9)^(1/2),x)
[Out] asinh((2*x)/3)/2
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
[Out] asinh(2*x/3)/2
```

3.269 $\int \frac{1}{\sqrt{4+x^2}} dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] arcsinh(1/2*x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + x^2],x]
[Out] ArcSinh[x/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}} dx = \sinh^{-1}\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + x^2],x]
[Out] ArcSinh[x/2]
```

fricas [B] time = 0.42, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 4))
```

giac [B] time = 1.07, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="giac")
```

[Out] -log(-x + sqrt(x^2 + 4))

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\text{arcsinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+4)^(1/2),x)

[Out] arcsinh(1/2*x)

maxima [A] time = 0.97, size = 4, normalized size = 0.67

$$\text{arsinh}\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")

[Out] arcsinh(1/2*x)

mupad [B] time = 0.03, size = 4, normalized size = 0.67

$$\text{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 4)^(1/2),x)

[Out] asinh(x/2)

sympy [A] time = 0.14, size = 3, normalized size = 0.50

$$\text{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+4)**(1/2),x)

[Out] asinh(x/2)

3.270 $\int \frac{1}{10-12x+9x^2}\,dx$

Optimal. Leaf size=21

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

```
[Out] -1/18*arctan(1/6*(2-3*x)*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 204}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(10 - 12*x + 9*x^2)^(-1),x]
```

```
[Out] -ArcTan[(2 - 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{10-12x+9x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-216-x^2}\,dx, x, -12+18x\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10 - 12*x + 9*x^2)^(-1),x]
```

```
[Out] ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

fricas [A] time = 0.39, size = 16, normalized size = 0.76

$$\frac{1}{18}\,\sqrt{6}\,\arctan\left(\frac{1}{6}\,\sqrt{6}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

giac [A] time = 1.05, size = 16, normalized size = 0.76

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="giac")
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

maple [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{\sqrt{6}\,\arctan\left(\frac{(18x-12)\sqrt{6}}{36}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-12*x+10),x)
[Out] 1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))
```

maxima [A] time = 0.95, size = 16, normalized size = 0.76

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

mupad [B] time = 0.14, size = 16, normalized size = 0.76

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,(3x-2)}{6}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 - 12*x + 10),x)
[Out] (6^(1/2)*atan((6^(1/2)*(3*x - 2))/6))/18
```

sympy [A] time = 0.12, size = 22, normalized size = 1.05

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}-\frac{\sqrt{6}}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-12*x+10),x)
[Out] sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18
```

3.271 $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

```
[Out] 1/2/(1-x)-1/3/x^3-1/x^2-2/x-5/2*ln(1-x)+2*ln(x)+1/4*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2058, 260}

$$-\frac{1}{x^2}-\frac{1}{3x^3}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Lo
g[1 + x^2]/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8} \, dx &= \int \left(\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}+\frac{1}{x^4}+\frac{2}{x^3}+\frac{2}{x^2}+\frac{2}{x}+\frac{x}{2\left(1+x^2\right)}\right) dx\\ &= \frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{2}\int \frac{x}{1+x^2} \, dx\\ &= \frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2(x-1)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] -1/2*1/(-1 + x) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] +
Log[1 + x^2]/4
```

fricas [A] time = 0.41, size = 73, normalized size = 1.38

$$-\frac{30\,x^3-12\,x^2-3\left(x^4-x^3\right)\log\left(x^2+1\right)+30\left(x^4-x^3\right)\log\left(x-1\right)-24\left(x^4-x^3\right)\log(x)-8\,x-4}{12\left(x^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fricas")
```

```
[Out] -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x
- 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

giac [A] time = 1.12, size = 46, normalized size = 0.87

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x-1)x^3}+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{5}{2}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="giac")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log(
abs(x - 1)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.79

$$2\ln(x)-\frac{5\ln(x-1)}{2}+\frac{\ln\left(x^2+1\right)}{4}-\frac{2}{x}-\frac{1}{x^2}-\frac{1}{3x^3}-\frac{1}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x)
```

```
[Out] -1/2/(x-1)-5/2*ln(x-1)+1/4*ln(x^2+1)-1/3/x^3-1/x^2-2/x+2*ln(x)
```

maxima [A] time = 0.96, size = 47, normalized size = 0.89

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\left(x^4-x^3\right)}+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{5}{2}\,\log\left(x-1\right)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x
- 1) + 2*log(x)
```

mupad [B] time = 0.06, size = 45, normalized size = 0.85

$$\frac{\ln\left(x^2+1\right)}{4}-\frac{5\,\ln\left(x-1\right)}{2}+2\,\ln(x)-\frac{-\frac{5\,x^3}{2}+x^2+\frac{2\,x}{3}+\frac{1}{3}}{x^3-x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8),x)
```

```
[Out] log(x^2 + 1)/4 - (5*log(x - 1))/2 + 2*log(x) - ((2*x)/3 + x^2 - (5*x^3)/2 +
 1/3)/(x^3 - x^4)
```

sympy [A] time = 0.18, size = 46, normalized size = 0.87

$$2\log\left(x\right)-\frac{5\log\left(x-1\right)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{-15x^3+6x^2+4x+2}{6x^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 + (-15*x**3 + 6*x**2 + 4*x + 2)
/(6*x**4 - 6*x**3)
```

3.272 $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} \, dx$

Optimal. Leaf size=49

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

```
[Out] a*x+1/12*(27*a+9*b+3*c+d)*ln(3-x)-1/3*d*ln(x)-1/4*(a-b+c-d)*ln(1+x)
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1612}

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c -
 d)*Log[1 + x])/4
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx &= \int \left(a+\frac{27a+9b+3c+d}{12(-3+x)}-\frac{d}{3x}+\frac{-a+b-c+d}{4(1+x)}\right)dx\\ &= ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x)-\frac{1}{3}d\log(x)-\frac{1}{4}(a-b+c-d)\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.00

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)+\frac{1}{4}\log(x+1)(-a+b-c+d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c
+ d)*Log[1 + x])/4
```

fricas [A] time = 0.45, size = 41, normalized size = 0.84

$$ax-\frac{1}{4}(a-b+c-d)\log(x+1)+\frac{1}{12}(27a+9b+3c+d)\log(x-3)-\frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

giac [A] time = 1.15, size = 44, normalized size = 0.90

$$ax - \frac{1}{4}(a - b + c - d)\log(|x + 1|) + \frac{1}{12}(27a + 9b + 3c + d)\log(|x - 3|) - \frac{1}{3}d\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="giac")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*log
(abs(x - 3)) - 1/3*d*log(abs(x))
```

maple [A] time = 0.01, size = 66, normalized size = 1.35

$$ax+\frac{9a\ln(x-3)}{4}-\frac{a\ln(x+1)}{4}+\frac{3b\ln(x-3)}{4}+\frac{b\ln(x+1)}{4}+\frac{c\ln(x-3)}{4}-\frac{c\ln(x+1)}{4}-\frac{d\ln(x)}{3}+\frac{d\ln(x-3)}{12}+\frac{d\ln(x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^2+c*x+d)/(x-3)/x/(x+1),x)
```

```
[Out] a*x+9/4*ln(x-3)*a+3/4*ln(x-3)*b+1/4*ln(x-3)*c+1/12*ln(x-3)*d-1/4*ln(x+1)*a+
1/4*ln(x+1)*b-1/4*ln(x+1)*c+1/4*ln(x+1)*d-1/3*d*ln(x)
```

maxima [A] time = 0.43, size = 41, normalized size = 0.84

$$ax - \frac{1}{4}(a - b + c - d)\log(x + 1) + \frac{1}{12}(27a + 9b + 3c + d)\log(x - 3) - \frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

mupad [B] time = 0.21, size = 46, normalized size = 0.94

$$\ln(x-3)\left(\frac{9a}{4}+\frac{3b}{4}+\frac{c}{4}+\frac{d}{12}\right)-\ln(x+1)\left(\frac{a}{4}-\frac{b}{4}+\frac{c}{4}-\frac{d}{4}\right)+ax-\frac{d\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x + a*x^3 + b*x^2)/(x*(x + 1)*(x - 3)),x)
```

```
[Out] log(x - 3)*((9*a)/4 + (3*b)/4 + c/4 + d/12) - log(x + 1)*(a/4 - b/4 + c/4 -
 d/4) + a*x - (d*log(x))/3
```

sympy [B] time = 86.23, size = 762, normalized size = 15.55

$$ax-\frac{d\log(x)}{3}-\frac{(a-b+c-d)\log\left(x+\frac{-1512a^2d+1134a^2(a-b+c-d)-864abd+648ab(a-b+c-d)-432acd+324ac(a-b+c-d)-144ad^2+81a(a-}{1215a^3-567a^2b+1593a^2c-2691a^2d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a - b
 + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a -
b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**2*(
a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c - d
)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c
 - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*
```

```
d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d -
 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2
- 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27
*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c + d)*log(x
+ (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27
*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d*
*2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c
 + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a + 9*b + 3*c
+ d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c*d**2 + 24*c*d*
(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 136*d**3 + 18*d**2
*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*
a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d +
405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81
*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3
))/12
```

3.273 $\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{1}{\left(2-\log\left(x^2+1\right)\right)^5},x\right)$$

```
[Out] Unintegrable(1/(2-ln(x^2+1))^5,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 - Log[1 + x^2])^(-5),x]
[Out] Defer[Int][(2 - Log[1 + x^2])^(-5), x]
```

Rubi steps

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx=\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx$$

Mathematica [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - Log[1 + x^2])^(-5),x]
[Out] Integrate[(2 - Log[1 + x^2])^(-5), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{1}{\log\left(x^2+1\right)^5-10\log\left(x^2+1\right)^4+40\log\left(x^2+1\right)^3-80\log\left(x^2+1\right)^2+80\log\left(x^2+1\right)-32},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")
[Out] integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*lo
g(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(\log\left(x^2+1\right)-2\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="giac")
```

```
[Out] integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\ln\left(x^2+1\right)+2\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-ln(x^2+1))^5,x)
```

```
[Out] int(1/(2-ln(x^2+1))^5,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{32\,x^8+56\,x^6+120\,x^4+\left(x^8-10\,x^4-24\,x^2-15\right)\log\left(x^2+1\right)^3-2\left(2\,x^8-x^6-33\,x^4-75\,x^2-45\right)\log\left(x^2+1\right.}{384\left(x^7\log\left(x^2+1\right)^4-8\,x^7\log\left(x^2+1\right)^3+24\,x^7\log\left(x^2+1\right)^2-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")
```

```
[Out] 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 + 1
)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4*(
3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 + 1)
^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1) + 1
6*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 + 1) -
 2*x^8), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int -\frac{1}{\left(\ln\left(x^2+1\right)-2\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(log(x^2 + 1) - 2)^5,x)
```

```
[Out] int(-1/(log(x^2 + 1) - 2)^5, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{120x^2}{x^8\log\left(x^2+1\right)-2x^8}\, dx+\int \frac{30x^4}{x^8\log\left(x^2+1\right)-2x^8}\, dx+\int \frac{x^8}{x^8\log\left(x^2+1\right)-2x^8}\, dx+\int \frac{105}{x^8\log\left(x^2+1\right)-2x^8}\, dx}{384}+\frac{\frac{2x^8}{3}+\frac{7x^6}{6}+\frac{5x^4}{2}+\frac{9x}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-ln(x**2+1))**5,x)
```

```
[Out] -(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(x
**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x**
8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3 +
7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(
x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2
 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) +
 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*log(x*
*2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)
```

3.274 $$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx$$

Optimal. Leaf size=28

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

Rubi [A] time = 0.21, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2210, 2209, 2554, 12, 2547, 6742, 2538}

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-
1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2538

```
Int[Log[(c_.)*(x_)^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (
a_.)*(x_)^(m_.))), x_Symbol] :> Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x
] - Dist[(a*m)/(b*n*q), Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /; F
reeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

Rule 2547

```
Int[(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_))/(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.)
+ (a_.)*(x_))^2, x_Symbol] :> -Simp[(e*Log[c*x^n])/(a*(a*x + b*Log[c*x^n]^q
)), x] + Dist[(d + e*n)/a, Int[1/(x*(a*x + b*Log[c*x^n]^q)), x], x] /; Free
Q[{a, b, c, d, e, n, q}, x] && EqQ[d + e*n*q, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int\left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx=2\int e^{x^2}x\log(x)\,dx+\int\frac{e^{x^2}}{x}\,dx+\int\frac{-2+\text{lo}}{\left(x+\log\right.}$$

$$=\frac{\text{Ei}\left(x^2\right)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}-2\int\frac{e^{x^2}}{2x}$$

$$=\frac{\text{Ei}\left(x^2\right)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+2\int\frac{}{x(}$$

$$=e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Mathematica [A] time = 0.17, size = 28, normalized size = 1.00

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1
+ x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

fricas [A] time = 0.43, size = 44, normalized size = 1.57

$$\frac{e^{\left(x^2\right)}\log(x)^3+\left(\log(x)^2+x\right)\log\left(\log(x)^2+x\right)+\left(xe^{\left(x^2\right)}-1\right)\log(x)}{\log(x)^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")
```

```
[Out] (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log(
x))/(log(x)^2 + x)
```

giac [A] time = 1.11, size = 27, normalized size = 0.96

$$e^{\left(x^2\right)}\log(x)-\frac{3\log(x)}{\log(x)^2+x}+\log\left(\log(x)^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="giac")
```

```
[Out] e^(x^2)*log(x) - 3*log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

maple [A] time = 0.04, size = 28, normalized size = 1.00

$$e^{x^2}\ln(x) - \frac{\ln(x)}{\ln(x)^2 + x} + \ln\left(\ln(x)^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/x
)/(x+ln(x)^2),x)
```

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

maxima [A] time = 0.56, size = 27, normalized size = 0.96

$$e^{(x^2)}\log(x) - \frac{\log(x)}{\log(x)^2 + x} + \log\left(\log(x)^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")
```

```
[Out] e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

mupad [B] time = 0.34, size = 27, normalized size = 0.96

$$\ln\left(\ln(x)^2 + x\right) + e^{x^2}\ln(x) - \frac{\ln(x)}{\ln(x)^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) - 2)/(x + log(x)^2)^2 + ((2*log(x))/x + 1/x + 1)/(x + log(x)^2)
 + exp(x^2)/x + 2*x*exp(x^2)*log(x),x)
```

```
[Out] log(x + log(x)^2) + exp(x^2)*log(x) - log(x)/(x + log(x)^2)
```

sympy [A] time = 0.47, size = 26, normalized size = 0.93

$$e^{x^2}\log(x) + \log\left(x + \log(x)^2\right) - \frac{\log(x)}{x + \log(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x
+2*ln(x)/x)/(x+ln(x)**2),x)
```

```
[Out] exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

3.275 $\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$

Optimal. Leaf size=199

$$\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{24}{x^4+2}$$

```
[Out] 24*exp(1/2*x+x*z)*Pi^4*x^3/(64*Pi^4+20*Pi^2*x^2+x^4)-24*exp(1/2*x+x*z)*Pi^3
*x^4*cos(Pi*z)*sin(Pi*z)/(64*Pi^4+20*Pi^2*x^2+x^4)+12*exp(1/2*x+x*z)*Pi^2*x
^5*sin(Pi*z)^2/(64*Pi^4+20*Pi^2*x^2+x^4)-4*exp(1/2*x+x*z)*Pi*x^4*cos(Pi*z)*
sin(Pi*z)^3/(16*Pi^2+x^2)+exp(1/2*x+x*z)*x^5*sin(Pi*z)^4/(16*Pi^2+x^2)
```

Rubi [A] time = 0.10, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 4434, 2194}

$$\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}+\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\text{si}}{x^4+20\pi^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E^(x/2 + x*
z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2 + x^4) + (12*E^(x/2
 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (4*E^(x/2 + x
*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[
Pi*z]^4)/(16*Pi^2 + x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz &= x^4\int e^{\frac{x}{2}+xz}\sin^4(\pi z)\,dz\\
&= -\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2}+\frac{e^{\frac{x}{2}+xz}x^5\sin^4(\pi z)}{16\pi^2+x^2}+\frac{\left(12\pi^2x^4\right)\int e^{\frac{x}{2}+xz}\sin^2(\pi z)\,dz}{16\pi^2+x^2}\\
&= -\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}+\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}-\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi}{16\pi^2+x^2}\\
&= \frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4}-\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}+\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}-4e^{\frac{x}{2}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 136, normalized size = 0.68

$$\frac{x^4e^{x\left(z+\frac{1}{2}\right)}\left(3x^4-8\pi x^3\sin(2\pi z)+4\pi x^3\sin(4\pi z)-4\left(x^2+16\pi^2\right)x^2\cos(2\pi z)+\left(x^2+4\pi^2\right)x^2\cos(4\pi z)+60\pi^2x^2\right.}{8\left(x^5+20\pi^2x^3+64\pi^4x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^2
)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] - 8
*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64
*Pi^4*x + 20*Pi^2*x^3 + x^5))
```

fricas [A] time = 0.44, size = 145, normalized size = 0.73

$$\frac{4\left(\left(4\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)^3-\left(10\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(\pi z\right)+\left(24\,\pi^4x^3+16\,\pi^2x^5+x^7+\left(4\,\pi^2x^5+\right.\right.}{64\,\pi^4+20\,\pi^2x^2+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")
```

```
[Out] (4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*cos(pi*z))*e
^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 + x^7 + (4*pi^2*x^5 +
 x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos(pi*z)^2)*e^(x*z + 1/2*x))/(64
*pi^4 + 20*pi^2*x^2 + x^4)
```

giac [A] time = 1.01, size = 114, normalized size = 0.57

$$\frac{1}{8}\left(\left(\frac{x\cos\left(4\,\pi z\right)}{16\,\pi^2+x^2}+\frac{4\,\pi\sin\left(4\,\pi z\right)}{16\,\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(\frac{x\cos\left(2\,\pi z\right)}{4\,\pi^2+x^2}+\frac{2\,\pi\sin\left(2\,\pi z\right)}{4\,\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}\,x\right)}+\frac{3\,e^{\left(xz+\frac{1}{2}\,x\right)}}{x}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="giac")
```

```
[Out] 1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2))*e^(
x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2 +
 x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)*x^4
```

maple [A] time = 0.10, size = 127, normalized size = 0.64

$$\frac{\left(-\frac{4x\cos(2\pi z)e^{xz+\frac{1}{2}x}}{x^2+4\pi^2}+\frac{x\cos(4\pi z)e^{xz+\frac{1}{2}x}}{x^2+16\pi^2}-\frac{8\pi\,e^{xz+\frac{1}{2}x}\sin(2\pi z)}{x^2+4\pi^2}+\frac{4\pi\,e^{xz+\frac{1}{2}x}\sin(4\pi z)}{x^2+16\pi^2}+\frac{3\,e^{xz+\frac{1}{2}x}}{x}\right)x^4}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z)
```

```
[Out] 1/8*x^4*(3/x*exp(1/2*x+x*z)+x/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)+4*Pi
/(16*Pi^2+x^2)*exp(1/2*x+x*z)*sin(4*Pi*z)-4*x/(4*Pi^2+x^2)*exp(1/2*x+x*z)*c
os(2*Pi*z)-8*Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))
```

maxima [A] time = 0.54, size = 160, normalized size = 0.80

$$\frac{\left(\left(4\,\pi^2x^2+x^4\right)\cos\left(4\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(16\,\pi^2x^2+x^4\right)\cos\left(2\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}+4\left(4\,\pi^3x+\pi x^3\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(4\,\pi z\right)-8\right.}{8\left(64\,\pi^4x+20\,\pi^2x^3+x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")
```

```
[Out] 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4)
*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(4*
pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4 + 2
0*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)
```

mupad [B] time = 1.11, size = 140, normalized size = 0.70

$$\frac{x^3\,e^{\frac{x}{2}+x\,z}\,\left(24\,\Pi^4-\frac{x^4\,\cos(2\,\Pi\,z)}{2}+\frac{x^4\,\cos(4\,\Pi\,z)}{8}+\frac{3\,x^4}{8}+\frac{15\,\Pi^2\,x^2}{2}-\Pi\,x^3\,\sin\left(2\,\Pi\,z\right)-16\,\Pi^3\,x\,\sin\left(2\,\Pi\,z\right)+\frac{\Pi\,x^3\,\sin(4}{2}\right.}{64\,\Pi^4+20\,\Pi^2\,x^2+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(x/2 + x*z)*sin(Pi*z)^4,z)
```

```
[Out] (x^3*exp(x/2 + x*z)*(24*Pi^4 - (x^4*cos(2*Pi*z))/2 + (x^4*cos(4*Pi*z))/8 +
(3*x^4)/8 + (15*Pi^2*x^2)/2 - Pi*x^3*sin(2*Pi*z) - 16*Pi^3*x*sin(2*Pi*z) +
(Pi*x^3*sin(4*Pi*z))/2 + 2*Pi^3*x*sin(4*Pi*z) - 8*Pi^2*x^2*cos(2*Pi*z) + (P
i^2*x^2*cos(4*Pi*z))/2))/(64*Pi^4 + x^4 + 20*Pi^2*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] Timed out
```

3.276 $\int \mathrm{erf}(x)\,dx$

Optimal. Leaf size=18

$$x\mathrm{erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

```
[Out] x*erf(x)+1/exp(x^2)/Pi^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6349}

$$x\mathrm{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[x],x]
```

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{erf}(x)\,dx=\frac{e^{-x^2}}{\sqrt{\pi}}+x\mathrm{erf}(x)$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x\mathrm{erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[x],x]
```

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

fricas [A] time = 0.41, size = 20, normalized size = 1.11

$$\frac{\pi x\,\mathrm{erf}(x)+\sqrt{\pi}\,e^{\left(-x^2\right)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="fricas")
```

```
[Out] (pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi
```

giac [A] time = 1.07, size = 15, normalized size = 0.83

$$x\,\mathrm{erf}(x)+\frac{e^{\left(-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="giac")
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

maple [A] time = 0.01, size = 16, normalized size = 0.89

$$x\,\text{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x)
[Out] x*erf(x)+1/Pi^(1/2)*exp(-x^2)
```

maxima [A] time = 0.42, size = 15, normalized size = 0.83

$$x\,\text{erf}(x) + \frac{e^{\left(-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="maxima")
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

mupad [B] time = 0.15, size = 15, normalized size = 0.83

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x\,\text{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x)
[Out] exp(-x^2)/pi^(1/2) + x*erf(x)
```

sympy [A] time = 0.31, size = 15, normalized size = 0.83

$$x\,\text{erf}\,(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x)
[Out] x*erf(x) + exp(-x**2)/sqrt(pi)
```

3.277 $\int \mathrm{erf}(a+x)\,dx$

Optimal. Leaf size=24

$$(a+x)\mathrm{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

```
[Out] (a+x)*erf(a+x)+1/exp((a+x)^2)/Pi^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6349}

$$(a+x)\mathrm{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[a + x],x]
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{erf}(a+x)\,dx=\frac{e^{-(a+x)^2}}{\sqrt{\pi}}+(a+x)\mathrm{erf}(a+x)$$

Mathematica [A] time = 0.03, size = 24, normalized size = 1.00

$$(a+x)\mathrm{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[a + x],x]
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

fricas [A] time = 0.41, size = 37, normalized size = 1.54

$$\frac{(\pi a+\pi x)\,\mathrm{erf}\,(a+x)+\sqrt{\pi}\,e^{\left(-a^2-2\,ax-x^2\right)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="fricas")
[Out] ((pi*a + pi*x)*erf(a + x) + sqrt(pi)*e^(-a^2 - 2*a*x - x^2))/pi
```

giac [A] time = 1.15, size = 37, normalized size = 1.54

$$x\,\mathrm{erf}\,(a+x)+\frac{\sqrt{\pi}\,a\,\mathrm{erf}\,(a+x)+e^{\left(-a^2-2\,ax-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="giac")
```

```
[Out] x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)
```

maple [A] time = 0.00, size = 22, normalized size = 0.92

$$(a+x)\operatorname{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(a+x),x)
```

```
[Out] (a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)
```

maxima [A] time = 0.42, size = 21, normalized size = 0.88

$$(a+x)\operatorname{erf}(a+x)+\frac{e^{\left(-(a+x)^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="maxima")
```

```
[Out] (a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)
```

mupad [B] time = 0.07, size = 21, normalized size = 0.88

$$\operatorname{erf}(a+x)\ (a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(a + x),x)
```

```
[Out] erf(a + x)*(a + x) + exp(-(a + x)^2)/pi^(1/2)
```

sympy [A] time = 0.51, size = 36, normalized size = 1.50

$$a\operatorname{erf}(a+x)+x\operatorname{erf}(a+x)+\frac{e^{-a^2}e^{-x^2}e^{-2ax}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x)
```

```
[Out] a*erf(a + x) + x*erf(a + x) + exp(-a**2)*exp(-x**2)*exp(-2*a*x)/sqrt(pi)
```

3.278 $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

Optimal. Leaf size=94

$$\frac{(2x+1)\sqrt{x^4+4x^3+2x^2+1}}{2\left(2x^2-1\right)}-\tanh^{-1}\left(\frac{x(x+2)\left(33x^3+27x^2-x+7\right)}{\left(31x^3+37x^2+2\right)\sqrt{x^4+4x^3+2x^2+1}}\right)$$

```
[Out] -arctanh(x*(2+x)*(33*x^3+27*x^2-x+7)/(31*x^3+37*x^2+2)/(x^4+4*x^3+2*x^2+1)^
(1/2))+1/2*(1+2*x)*(x^4+4*x^3+2*x^2+1)^(1/2)/(2*x^2-1)
```

Rubi [F] time = 1.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1
 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] (9*Defer[Int][1/Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x])/4 - (13*Defer[Int][1/((S
qrt[2] - 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/4 + Defer[Int][x/Sqrt[1
 + 2*x^2 + 4*x^3 + x^4], x] + Defer[Int][x^2/Sqrt[1 + 2*x^2 + 4*x^3 + x^4],
 x]/2 - (13*Defer[Int][1/((Sqrt[2] + 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
 x])/4 - (13*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
x])/8 - ((15 + Sqrt[2])*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^
3 + x^4]), x])/8 - (13*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3
 + x^4]), x])/8 - ((15 - Sqrt[2])*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*
x^2 + 4*x^3 + x^4]), x])/8 - (17*Defer[Int][x/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^
2 + 4*x^3 + x^4]), x])/2
```

Rubi steps

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \left(\frac{9}{4\sqrt{1+2x^2+4x^3+x^4}} + \frac{x}{\sqrt{1+2x^2+4x^3+x^4}} + \frac{}{2\sqrt{1+2}}\right.$$

$$= \frac{1}{4}\int \frac{15+2x}{\left(-1+2x^2\right)\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4}}$$

$$= \frac{1}{4}\int \left(-\frac{15+\sqrt{2}}{2\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}} - \frac{15-}{2\left(1+\sqrt{2}\,x\right)\sqrt{1+}}\right.$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx - \frac{13}{8}\int \frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4}}$$

Mathematica [C] time = 6.44, size = 5137, normalized size = 54.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*
Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.55, size = 179, normalized size = 1.90

$$\frac{\left(2\,x^2-1\right)\log\left(\frac{1025\,x^{10}+6138\,x^9+12307\,x^8+10188\,x^7+4503\,x^6+3134\,x^5+1589\,x^4+140\,x^3+176\,x^2-\left(1023\,x^8+4104\,x^7+5084\,x^6+2182\,x^5+805\right.}{32\,x^{10}-80\,x^8+80\,x^6-40\,x^4+10\,x^2-1}\right.}{2\left(2\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^7 + 4503*x
^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^8 + 4104*x^7 + 5084*
x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 + 28*x)*sqrt(x^4 + 4*x^3 + 2*x^
2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x^4 +
 4*x^3 + 2*x^2 + 1)*(2*x + 1))/(2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6+4\,x^5+7\,x^4-3\,x^3-x^2-8\,x-8}{\sqrt{x^4+4\,x^3+2\,x^2+1}\left(2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

maple [B] time = 1.43, size = 1197351, normalized size = 12737.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/
2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6+4\,x^5+7\,x^4-3\,x^3-x^2-8\,x-8}{\sqrt{x^4+4\,x^3+2\,x^2+1}\,\left(2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-2\,x^6-4\,x^5-7\,x^4+3\,x^3+x^2+8\,x+8}{\left(2\,x^2-1\right)^2\,\sqrt{x^4+4\,x^3+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
+ 4*x^3 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
+ 4*x^3 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{\sqrt{(x+1)\left(x^3+3x^2-x+1\right)}\,\left(2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x*
*3+2*x**2+1)**(1/2),x)
```

```
[Out] Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1)
*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)
```

3.279 $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

Optimal. Leaf size=142

$$-\frac{1}{4}\tanh^{-1}\left(\frac{(1-3y)\sqrt{-5y^2-5y+1}}{(1-5y)\sqrt{-y^2-y+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{(3y+4)\sqrt{-5y^2-5y+1}}{(5y+6)\sqrt{-y^2-y+1}}\right)+\frac{9}{4}\tanh^{-1}\left(\frac{(7y+11)\sqrt{-5y^2-5y}}{3(5y+7)\sqrt{-y^2-y}}\right.$$

```
[Out] -1/4*arctanh((1-3*y)*(-5*y^2-5*y+1)^(1/2)/(1-5*y)/(-y^2-y+1)^(1/2))-1/2*arc
tanh((4+3*y)*(-5*y^2-5*y+1)^(1/2)/(6+5*y)/(-y^2-y+1)^(1/2))+9/4*arctanh(1/3
*(11+7*y)*(-5*y^2-5*y+1)^(1/2)/(7+5*y)/(-y^2-y+1)^(1/2))
```

Rubi [F] time = 3.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$

Verification is Not applicable to the result.

```
[In] Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2])
,y]
```

```
[Out] Defer[Int][Sqrt[1 - 5*y - 5*y^2]/(y*Sqrt[1 - y - y^2]), y]/2 + Defer[Int][S
qrt[1 - 5*y - 5*y^2]/((1 + y)*Sqrt[1 - y - y^2]), y] - (3*Defer[Int][Sqrt[1
 - 5*y - 5*y^2]/((2 + y)*Sqrt[1 - y - y^2]), y])/2
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy &= \int \left(\frac{\sqrt{1-5y-5y^2}}{2y\sqrt{1-y-y^2}}+\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}}-\frac{3\sqrt{1-5y-5y^2}}{2(2+y)\sqrt{1-y-y^2}}\right)dy\\ &= \frac{1}{2}\int \frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}}\,dy-\frac{3}{2}\int \frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}}\,dy+\int \frac{\sqrt{1-5y-5}}{(1+y)\sqrt{1-y}}\end{aligned}$$

Mathematica [C] time = 1.57, size = 630, normalized size = 4.44

$$\frac{\left(-1-\frac{2}{\sqrt{5}}\right)\left(2y+\sqrt{5}+1\right)^2\sqrt{\frac{10y+3\sqrt{5}+5}{10y+5\sqrt{5}+5}}\left(20\left(\sqrt{5}\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-4\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y -
 y^2]),y]
```

```
[Out] ((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5*
Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*
Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[
5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*
y)] + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-
3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + S
qrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5
]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)
```

```
/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1
+ Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]
*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqr
t[5] + 2*y)])/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[(-3*(-5 + Sq
rt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt
[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5
 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16])))/(16*Sqrt[1
- 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

fricas [A] time = 0.56, size = 223, normalized size = 1.57

$$\frac{9}{8}\log\left(-\frac{235\,y^4+935\,y^3-3\left(35\,y^2+104\,y+77\right)\sqrt{-y^2-y+1}\,\sqrt{-5\,y^2-5\,y+1}\,+1086\,y^2+131\,y-281}{y^4+8\,y^3+24\,y^2+32\,y+16}\right)+\frac{1}{4}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="fricas")
```

```
[Out] 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*sq
rt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32*y
 + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y +
1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y
 + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*s
qrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-5\,y^2-5\,y+1}\,\left(2\,y+1\right)}{\sqrt{-y^2-y+1}\,\left(y+2\right)\left(y+1\right)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="giac")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

maple [C] time = 0.32, size = 352, normalized size = 2.48

$$-\frac{300\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\,\left(-10y-5+3\sqrt{5}\right)^2\sqrt{\frac{-2y-1+\sqrt{5}}{-10y-5+3\sqrt{5}}}\,\sqrt{5}\,\sqrt{\frac{2y+1+\sqrt{5}}{-10y-5+3\sqrt{5}}}\,\left(-3\,\mathrm{El}\right.}{\sqrt{5y^4+10y^3-y^2-6y+1}\,\sqrt{\left(10y+5+3\sqrt{5}\right)\left(-10y-5\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y)
```

```
[Out] -300*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*
5^(1/2)))^(1/2)*(-10*y-5+3*5^(1/2))^2*((-1+5^(1/2)-2*y)/(-10*y-5+3*5^(1/2))
)^(1/2)*5^(1/2)*((1+5^(1/2)+2*y)/(-10*y-5+3*5^(1/2)))^(1/2)*(2*EllipticPi(2
*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+3*5^(1/2))/(3*5^(1
/2)-5),1/4)-3*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),
-1/4*(5+5^(1/2))/(5^(1/2)-5),1/4)+EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-
5+3*5^(1/2)))^(1/2),-1/4*(3*5^(1/2)-5)/(5+3*5^(1/2)),1/4))/(5*y^4+10*y^3-y^
2-6*y+1)^(1/2)/((10*y+5+3*5^(1/2))*(-10*y-5+3*5^(1/2))*(-1+5^(1/2)-2*y)*(1+
5^(1/2)+2*y))^(1/2)/(5+3*5^(1/2))/(3*5^(1/2)-5)/(5+5^(1/2))/(5^(1/2)-5)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-5y^2 - 5y + 1}\,(2y + 1)}{\sqrt{-y^2 - y + 1}\,(y + 2)(y + 1)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="maxima")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2y+1)\,\sqrt{-5y^2-5y+1}}{y\,(y+1)\,(y+2)\,\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1
/2)),y)
```

```
[Out] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1
/2)), y)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2y+1)\,\sqrt{-5y^2-5y+1}}{y\,(y+1)\,(y+2)\,\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),y
)
```

```
[Out] Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 -
y + 1)), y)
```

3.280 $\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$

Optimal. Leaf size=21

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

```
[Out] ln(1+(x^2-4)^(1/2)+(x^2-1)^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 85, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {6684}

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[
-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]
```

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = \log\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)$$

Mathematica [B] time = 1.35, size = 97, normalized size = 4.62

$$\frac{1}{4}\log\left(-5x^2-4\sqrt{x^2-4}\sqrt{x^2-1}+17\right)+\frac{1}{4}\log\left(-2x^2-2\sqrt{x^2-4}\sqrt{x^2-1}+5\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^2-4}\right)+\frac{1}{2}\tanh^{-1}\Bigg($$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2
*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]))
,x]
```

```
[Out] -1/2*ArcTanh[Sqrt[-4 + x^2]] + ArcTanh[Sqrt[-1 + x^2]/2]/2 + Log[17 - 5*x^2
 - 4*Sqrt[-4 + x^2]*Sqrt[-1 + x^2]]/4 + Log[5 - 2*x^2 - 2*Sqrt[-4 + x^2]*Sq
rt[-1 + x^2]]/4
```

fricas [B] time = 0.43, size = 162, normalized size = 7.71

$$-\frac{1}{4}\log\left(4x^4-\left(4x^2-11\right)\sqrt{x^2-1}\sqrt{x^2-4}-21x^2+23\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x+2)+2x-1\right)+\frac{1}{4}\log\left(x^2-\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^2 + 23) -
1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*log(x^2 - sqrt(x^2 - 4
)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 - 4)*(x - 1) - x - 4) + 1/4*log
(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x - 1) + 1/4*log(x^2 - 5) + 1/4*log(-x^2 +
 sqrt(x^2 - 1)*sqrt(x^2 - 4) + 7)
```

giac [B] time = 1.38, size = 76, normalized size = 3.62

$$-\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}+1\right)-\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}\right)+\frac{1}{2}\log\left(\sqrt{x^2-1}+2\right)+\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x^2 - 1) - sqrt(x^2 - 4) + 1) - 1/2*log(sqrt(x^2 - 1) - sqrt(
x^2 - 4)) + 1/2*log(sqrt(x^2 - 1) + 2) + 1/2*log(abs(-sqrt(x^2 - 1) + sqrt(
x^2 - 4) - 3))
```

maple [B] time = 0.22, size = 1088, normalized size = 51.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/
(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x)
```

```
[Out] -1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/2*(2-2*5^(1/2
)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))+1/(5^(1/2)+1)
/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+
1)^(1/2)-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/2*(2+
2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))+1/(5^
(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x-5^(1/2))^2+2*5^(1/2)*(x-5
^(1/2))+1)^(1/2)-1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x-5
^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+4)^(1/2)+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1
/2))/(5^(1/2)-1)*arctanh(1/4*(8+2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(
1/2)*(x-5^(1/2))+4)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((x-1)^2+2*x-2)^(1/2
)+1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((x-1)^2+2*x-2)^(1/2))+1/2/(5^(1/2)+1)/(
5^(1/2)-1)*((x+1)^2-2*x-2)^(1/2)-1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((x+1)^2-
2*x-2)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))*((x-2)^2+4*x-8)^(1/2)-1/2/(2+5^(
1/2))/(-2+5^(1/2))*ln(x+((x-2)^2+4*x-8)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))
*((x+2)^2-4*x-8)^(1/2)+1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((x+2)^2-4*x-8)^(1
/2))+1/4*ln(x^2-5)+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arcta
nh(1/4*(8-2*5^(1/2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1
/2))-1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^(1/2))^2-2*
5^(1/2)*(x+5^(1/2))+4)^(1/2)+1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1
/2)-1)*5^(1/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))-1/2/(5^(
1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*
5^(1/2)*(x-5^(1/2))+4)^(1/2))-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/
2)-1)*5^(1/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))+1/(5^(1/2
)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(
1/2)*(x-5^(1/2))+1)^(1/2))+7/8*(x^2-4)^(1/2)*(x^2-1)^(1/2)/(x^4-5*x^2+4)^(1
/2)*arctanh(1/4*(5*x^2-17)/(x^4-5*x^2+4)^(1/2))+1/8*(x^2-4)^(1/2)*(x^2-1)^(
1/2)*(2*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))-5*arctanh(1/4*(5*x^2-17)/(x^4-5*x^
2+4)^(1/2)))/(x^4-5*x^2+4)^(1/2)
```

maxima [B] time = 0.73, size = 171, normalized size = 8.14

$$\frac{1}{4}\log(x+1)+\frac{3}{8}\log(x-1)+\frac{1}{8}\log(x-2)+\frac{1}{4}\log\left(\frac{2x^4+4\left(x^2-3\right)\sqrt{x+1}\sqrt{x-1}-7x^2+2\left(\left(x^2-1\right)\sqrt{x+1}\right.}{2\left(\left(x^2-1\right)\sqrt{x+1}\sqrt{x-1}\sqrt{x-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2*x^4 + 4*(
x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)*sqrt(x + 1)*sqrt(x
- 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*sqrt(x + 2) + 3)/((x^2 - 1)*sqr
t(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(((x^
2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

mupad [B] time = 2.37, size = 172, normalized size = 8.19

$$\frac{\ln\left(x-\sqrt{5}\right)}{4}-\operatorname{atanh}\left(\frac{\sqrt{3}-\sqrt{x^2-1}}{\sqrt{x^2-4}}\right)+\frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-1}}{2}\right)}{2}+\frac{\ln\left(x+\sqrt{5}\right)}{4}-\frac{7\operatorname{atanh}\left(\frac{4\left(\sqrt{3}-\sqrt{x^2-1}\right)}{\sqrt{x^2-4}\left(\frac{\left(\sqrt{3}-\sqrt{x^2-1}\right)^2}{x^2-4}+1\right)}\right)}{4}+\frac{5\operatorname{atanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*(x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) - x^2*(x^2 - 1)^(1/2) - x^2*(x
^2 - 4)^(1/2)))/((x^4 - 5*x^2 + 4)*((x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) + 1))
,x)
```

```
[Out] log(x - 5^(1/2))/4 - atanh((3^(1/2) - (x^2 - 1)^(1/2))/(x^2 - 4)^(1/2)) + a
tanh((x^2 - 1)^(1/2)/2)/2 + log(x + 5^(1/2))/4 - (7*atanh((4*(3^(1/2) - (x^
2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((3^(1/2) - (x^2 - 1)^(1/2))^2/(x^2 - 4) +
1))))/4 + (5*atanh((12150*(3^(1/2) - (x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((6
075*(3^(1/2) - (x^2 - 1)^(1/2))^2)/(2*(x^2 - 4)) + 6075/2))))/4 - atanh((x^
2 - 4)^(1/2))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(
x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

```
[Out] Timed out
```

3.281 $\int \left(\sqrt{9-4\sqrt{2}}\,x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)dx$

Optimal. Leaf size=4030

result too large to display

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(-1/3*(x^4+2*x^2+4*x+1)^(1/2)+1/3*(1+x)*(x^4
+2*x^2+4*x+1)^(1/2)+4*I*(-13+3*33^(1/2))^(1/3)*(x^4+2*x^2+4*x+1)^(1/2)/(4*2
^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1/3)
+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))-8*2^(2/3)*EllipticE((26-6*33^(
1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(-26+6
*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)/((39+13*I*3^
(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3))/(39-1
3*I*3^(1/2)+9*I*11^(1/2)-9*33^(1/2)+4*(3+I*3^(1/2))*(-26+6*33^(1/2))^(1/3))
)^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+
3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3))
^(1/2),((84+28*I*3^(1/2)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-3*I*11^(1/2
)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84-28*I*3^(1/2)+12*I*11^(1/2)-12*33^
(1/2)+(3+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2))
*(x^4+2*x^2+4*x+1)^(1/2)*3^(1/2)/(-13+3*33^(1/2)+4*(-26+6*33^(1/2))^(1/3))^
(1/2)*(I*(-19899+x*(59697-10335*33^(1/2))+3445*33^(1/2)+(-26+6*33^(1/2))^(2
/3)*(-2574+466*33^(1/2))+(-26+6*33^(1/2))^(1/3)*(-19899+3445*33^(1/2)))/(-3
9-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(-26+6*33^(1/2))^(
1/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+3*
33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3)))^
(1/2)/(4*2^(2/3)-(-13+3*33^(1/2))^(1/3)+3*x*(-13+3*33^(1/2))^(1/3)-2^(1/3)*
(-13+3*33^(1/2))^(2/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/
2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*
33^(1/2))^(2/3))^(1/2)/(I*(1+x)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13
*I*3^(1/2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))
*(-26+6*33^(1/2))^(2/3))/(104-24*33^(1/2)+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*
33^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))^(
1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*3
3^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/
2)+1/6*EllipticPi(1/6*(13-3*33^(1/2)-2^(1/3)*(-13+3*33^(1/2))^(4/3)+4*(-26+
6*33^(1/2))^(2/3)+x*(-39+9*33^(1/2)))^(1/2)*2^(5/6)*3^(1/2)/(-13+3*33^(1/2)
)^(2/3)/(1+x)^(1/2)/((-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^
(1/2))*(-26+6*33^(1/2))^(1/3))/(104-24*33^(1/2)+(-13+13*I*3^(1/2)-9*I*11^(1
/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2
/3)))^(1/2),2^(1/3)*(4*2^(1/3)*(-3*I+3^(1/2))+(3*I+3^(1/2))*(-13+3*33^(1/2)
)^(2/3))/(4*2^(2/3)*(-I+3^(1/2))-8*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2
)+I)*(-13+3*33^(1/2))^(2/3)),((84-28*I*3^(1/2)+12*I*11^(1/2)-12*33^(1/2)+(3
+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84+28*I*3^(1/2
)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-3*I*11^(1/2)+3*33^(1/2))*(-26+6*33
^(1/2))^(1/3)))^(1/2))*(4*2^(2/3)+2*(-13+3*33^(1/2))^(1/3)-2^(1/3)*(-13+3*3
3^(1/2))^(2/3))*(4*2^(2/3)*(3^(1/2)+I)-4*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(
-I+3^(1/2))*(-13+3*33^(1/2))^(2/3))*(4*2^(2/3)*(-I+3^(1/2))+4*I*(-13+3*33^(
1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))*(1+x)^(1/2)*(x^4+2*
x^2+4*x+1)^(1/2)*((-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^(1/
2))*(-26+6*33^(1/2))^(1/3))/(104-24*33^(1/2)+(-13+13*I*3^(1/2)-9*I*11^(1/2)
+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3)
))^(1/2)*((104-24*33^(1/2)+2*(1+14*I*3^(1/2)-6*I*11^(1/2)+33^(1/2))*(-26+6*
33^(1/2))^(1/3)+(-7-I*3^(1/2)-3*I*11^(1/2)+33^(1/2))*(-26+6*33^(1/2))^(2/3)
+2*x*(-52+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/
3)))/(1+x)/(-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^(1/2))*(-2
6+6*33^(1/2))^(1/3)))^(1/2)*((104-24*33^(1/2)+2*(1-14*I*3^(1/2)+6*I*11^(1/2
)+33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-7+I*3^(1/2)+3*I*11^(1/2)+33^(1/2))*(-2
6+6*33^(1/2))^(2/3)+2*x*(-52+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(
```

```
-26+6*33^(1/2))^(2/3)))/(1+x)/(-39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I
*(3*I+3^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2)*2^(5/6)*3^(1/2)/(4*2^(2/3)*(3
^(1/2)+I)+2*I*(-13+3*33^(1/2))^(1/3)-6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(
-I+3^(1/2))*(-13+3*33^(1/2))^(2/3))/(4*2^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(
1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2
))^(2/3))/(13-3*33^(1/2)-2^(1/3)*(-13+3*33^(1/2))^(4/3)+4*(-26+6*33^(1/2))^
(2/3)+x*(-39+9*33^(1/2)))^(1/2)+1/18*I*EllipticF(1/6*I*(-52+12*33^(1/2)+2^(
1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/3))^(1/2)*(26-6*33^(1/2)+
6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(
1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)*2^(5/6)*3^(1/2)/(
-13+3*33^(1/2))^(2/3)/(1+x)^(1/2)/(39+13*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+
4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3))^(1/2),((84*I-28*3^(1/2)+12*11^(1/2)
-12*I*33^(1/2)+(3*I+3^(1/2)+3*11^(1/2)+3*I*33^(1/2))*(-26+6*33^(1/2))^(1/3)
)/(-56*3^(1/2)+24*11^(1/2)+2*(-26+6*33^(1/2))^(1/3)*(3^(1/2)+3*11^(1/2))))^
(1/2))*(2^(1/3)*(13-13*I*3^(1/2)+9*I*11^(1/2)-3*33^(1/2))+4*2^(2/3)*(1+I*3^
(1/2))*(-13+3*33^(1/2))^(1/3)+20*(-13+3*33^(1/2))^(2/3))*(4*2^(2/3)*(3^(1/2
)+I)+8*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(-I+3^(1/2))*(-13+3*33^(1/2))^(2/3)
)*(x^4+2*x^2+4*x+1)^(1/2)*(1/(-13+3*33^(1/2)+4*(-26+6*33^(1/2))^(1/3))*(-52
+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/3)))^(1/2
)*((-8*I*(-13+3*33^(1/2))+(-43*I-13*3^(1/2)+9*11^(1/2)+5*I*33^(1/2))*(-26+6
*33^(1/2))^(1/3)+(2*I+4*3^(1/2)-2*I*33^(1/2))*(-26+6*33^(1/2))^(2/3)+x*(8*I
*(-13+3*33^(1/2))+(13*I-13*3^(1/2)+9*11^(1/2)-3*I*33^(1/2))*(-26+6*33^(1/2)
)^(1/3)+4*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))/(1+x))^(1/2)*2^(1/3)*3^(1/4)
/(-13+3*33^(1/2))^(1/3)/(4*2^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+
6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))/(1
+x)^(1/2)/(39+13*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*3
3^(1/2))^(1/3))^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)
+9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^
(1/2))^(2/3))^(1/2)/((-104+24*33^(1/2)-(5-3*I*3^(1/2)+3*I*11^(1/2)+33^(1/2)
)*(-26+6*33^(1/2))^(2/3)+(-26+6*33^(1/2))^(1/3)*(-41+15*I*3^(1/2)-3*I*11^(1
/2)+7*33^(1/2))+x*(104-24*33^(1/2)+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2
))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))/(1+x)/(-
39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(-26+6*33^(1/2))^
(1/3)))^(1/2))
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\sqrt{9-4\sqrt{2}}\, x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*Defer[Int][Sqrt[1 + 4*x + 2*x^2 + x^4
], x]
```

Rubi steps

$$\int \left(\sqrt{9-4\sqrt{2}}\, x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx = \frac{1}{2}\sqrt{9-4\sqrt{2}}\, x^2-\sqrt{2}\int \sqrt{1+4x+2x^2+x^4}\, dx$$

Mathematica [C] time = 6.04, size = 3168, normalized size = 0.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]

[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 - (
2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(-(EllipticF[Ar
cSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1
+ Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3
 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1
+ 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*R
oot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi[(1 + Root[1 + 3*#1 - #1^
2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]],
((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2
, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]))*Sq
rt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 +
 #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*(-1 - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))])/(Sqrt[1 + 4*
x + 2*x^2 + x^4]*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #
1^2 + #1^3 & , 3, 0])) + (2*EllipticF[ArcSin[Sqrt[((1 + x)*(-Root[1 + 3*#1
- #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
 0]))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0])*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((-1 - Root[1
+ 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
 1, 0])^2*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*
(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[((1 + x)*(-Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))])/(S
qrt[1 + 4*x + 2*x^2 + x^4]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])) + ((1 + x)*(x - Root[1 + 3*#1 - #1^2 + #1
^3 & , 2, 0])*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])^2*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*
Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*Sqrt[(x -
Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 &
, 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])
)/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 +
#1^3 & , 3, 0])))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*((EllipticE[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))/(1 + Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0]) - (EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-
```

```
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]))]*(1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/(-Root[1 + 3*#1 - #
1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (EllipticF[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 1
, 0]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3
& , 3, 0]) - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 1, 0])*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0]))))/Sqrt[1 + 4*x + 2*x^2 + x^4]))/3
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(2\sqrt{2}\,x - \sqrt{2}\,\sqrt{x^4 + 2\,x^2 + 4\,x + 1} - x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(2\sqrt{2} - 1\right) - \sqrt{2}\,\sqrt{x^4 + 2\,x^2 + 4\,x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1), x)
```

maple [A] time = 1.16, size = 4640, normalized size = 1.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x)
```

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(1/3*x*(x^4+2*x^2+4*x+1)^(1/2)+4/3*(-4/3-1/6
*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(
26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33
^(1/2))^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+
4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))
^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+
```

```
6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)/(1/2*(
26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(
1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26
+6*33^(1/2))^(1/3)+4/3)/((x+1)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-
1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/
6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*EllipticF(((1/2*(26
+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/
2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/
(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(
1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+
6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1
/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/
(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*
3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+4*(
-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(
1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2)
)^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(
1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3
*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2)
)^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1
/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2
)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)
+8/3/(26+6*33^(1/2))^(1/3)+4/3)/((x+1)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1
/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)
*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*((-1/3*(26+
6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)*EllipticF(((1/2*(26+6*33^(
1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/
3)-8/3/(26+6*33^(1/2))^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*3
3^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(
1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^
(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-
1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1
/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
```

```
/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*3
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)
*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+(-4/3+1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))*EllipticPi(((1/2*(26+6*33^
(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*
33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^
(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))
^(1/2),(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/2*(26+6*33^(1
/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3
)-8/3/(26+6*33^(1/2))^(1/3))),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2)
)^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*
(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^
(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1
/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1
/3))))^(1/2)))+2/3*((x+1)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^
(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/
3)))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))+(-4/3-1/6*(26+6*
33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2
))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33
^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))
^(1/3)))*(x+1)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2
*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))^(1/3)+
8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1
/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*33^(1/2)
)^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26
+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*
33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-
1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((1/2*(26+6*3
3^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3))+(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1
/2))^(1/3)+1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))*(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)+(-1/3*
(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)^2)/(1/2*(26+6*33^(1/2)
)^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^
(1/3)+4/3)*EllipticF(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/
2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x+1)/(
1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(
26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33
^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-
1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26
+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+
4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6
*33^(1/2))^(1/3))))^(1/2))+(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/
```

```
2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/
3)))*EllipticE(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x+1)/(1/6*(2
6+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6
*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*3
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3
^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(2
6+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+
6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3))))^(1/2))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+
4/3)))/((x+1)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)*(
x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1
/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/6*(26+6*33^(1/2))
^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\left(2\sqrt{2}-1\right)-\sqrt{2}\int\sqrt{x^3-x^2+3x+1}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="m
axima")
```

```
[Out] 1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt(
x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\,\sqrt{x^4+2x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x\left(-1+2\sqrt{2}\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

```
[Out] Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)
```

3.282 $\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathbf{mc}^8+4\mathbf{mc}^9+24\mathbf{mc}^6x-48\mathbf{mc}^7x-144\mathbf{mc}^5x^2-24\mathbf{mc}^2x^3+176\mathbf{mc}^3\right.\right.}{384x^2}$

Optimal. Leaf size=330

$$\frac{\pi^2(3-4\mathrm{mc})\mathrm{mc}^8\mathrm{Ei}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\mathrm{mc})\mathrm{mc}^6\mathrm{Ei}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\mathrm{mc}^3y\left(-12\mathrm{mc}^2+3\mathrm{mc}-8y\right)\mathrm{Ei}\left(-\frac{x}{y}\right)+\frac{\pi^2(3-4\mathrm{mc})}{384x}$$

```
[Out] 1/384*(3-4*mc)*mc^8*Pi^2/exp(x/y)/x+3/8*mc^5*Pi^2*y/exp(x/y)+1/48*(3-22*mc)
*mc^2*Pi^2*x*y/exp(x/y)-1/128*(1+4*mc)*Pi^2*x^2*y/exp(x/y)+1/48*(3-22*mc)*m
c^2*Pi^2*y^2/exp(x/y)+1/4*mc^3*Pi^2*y^2/exp(x/y)-1/64*(1+4*mc)*Pi^2*x*y^2/e
xp(x/y)-1/64*(1+4*mc)*Pi^2*y^3/exp(x/y)+1/16*(1-2*mc)*mc^6*Pi^2*Ei(-x/y)+1/
384*(3-4*mc)*mc^8*Pi^2*Ei(-x/y)/y+1/32*mc^3*Pi^2*(-12*mc^2+3*mc-8*y)*y*Ei(-
x/y)-1/32*mc^3*Pi^2*(3*(1-4*mc)*mc-8*x)*y*ln(x/mc^2)/exp(x/y)+1/4*mc^3*Pi^2
*y^2*ln(x/mc^2)/exp(x/y)
```

Rubi [A] time = 0.87, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 107, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {12, 6742, 2199, 2194, 2177, 2178, 2176, 2554}

$$\frac{\pi^2(3-4\mathrm{mc})\mathrm{mc}^8\mathrm{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\mathrm{mc})\mathrm{mc}^6\mathrm{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\mathrm{mc}^3y\left(-12\mathrm{mc}^2+3\mathrm{mc}-8\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^
2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 -
8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/y)) + ((3
- 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2*x^2*y)/(128*E^(x/y)
) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y)) + (mc^3*Pi^2*y^2)/(4*E^(x/y))
- ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y))
 + ((1 - 2*mc)*mc^6*Pi^2*ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*
ExpIntegralEi[-(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpInt
egralEi[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/(32
*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !$UseGamma === True
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\text{mc}^8+4\text{mc}^9+24\text{mc}^6x-48\text{mc}^7x-144\text{mc}^5x^2-24\text{mc}^2x^3+176\text{mc}^3x^3+3x^4+12\text{mc}x^4\right)+12\right.}{384x^2}$$

Mathematica [A] time = 0.15, size = 181, normalized size = 0.55

$$\frac{1}{384}\pi^2\left(\frac{e^{-\frac{x}{y}}\left(-4\text{mc}^9+3\text{mc}^8+144\text{mc}^5xy-16\text{mc}^3xy(11x+5y)+24\text{mc}^2xy(x+y)+12\text{mc}^3xy\left(12\text{mc}^2-3\text{mc}+\right.\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 -
24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*m
c^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
[Out] (Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144*
mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8 - 4*mc^9 + 144*mc^5*
x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x + 5*y) - 3*x*y*(x^2 + 2*x*y +
 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y + 2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^2 +
8*(x + y))*Log[x/mc^2])/(E^(x/y)*x)))/384
```

fricas [A] time = 0.44, size = 269, normalized size = 0.82

$$\frac{12\left(8\,\pi^2mc^3xy^3+\left(8\,\pi^2mc^3x^2+3\,\pi^2\left(4\,mc^5-mc^4\right)x\right)y^2\right)e^{\left(-\frac{x}{y}\right)}\log\left(\frac{x}{mc^2}\right)-\left(96\,\pi^2mc^3xy^3+36\,\pi^2\left(4\,mc^5-mc^4\right)xy\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")
```

```
[Out] 1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5 - mc^4)*x)
*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*pi^2*(4*mc^5 - mc^4)*
x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y) -
(6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc + 1)*
x^2 + 4*pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*mc +
1)*x^3 - 8*pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

giac [A] time = 1.10, size = 472, normalized size = 1.43

$$-\frac{4\,\pi^2mc^9x\text{Ei}\left(-\frac{x}{y}\right)+4\,\pi^2mc^9ye^{\left(-\frac{x}{y}\right)}-3\,\pi^2mc^8x\text{Ei}\left(-\frac{x}{y}\right)+48\,\pi^2mc^7xy\text{Ei}\left(-\frac{x}{y}\right)-3\,\pi^2mc^8ye^{\left(-\frac{x}{y}\right)}-144\,\pi^2mc^5xy^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="giac")
```

```
[Out] -1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*mc^8*x*Ei(
-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x/y) - 144*pi^2*mc^5*
x*y^2*e^(-x/y)*log(x/mc^2) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x*y^
2*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)*log
(x/mc^2) - 96*pi^2*mc^3*x^2*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x*y^3*e
^(-x/y)*log(x/mc^2) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^3*Ei(-
x/y) + 176*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x*y^3*e^(-x/y) - 24*pi
^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc*x^3*y^2*e^(-x/y) - 24*pi^2*mc^2*x*y^3
*e^(-x/y) + 24*pi^2*mc*x^2*y^3*e^(-x/y) + 24*pi^2*mc*x*y^4*e^(-x/y) + 3*pi^
2*x^3*y^2*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x*y^4*e^(-x/y))/(x*y)
```

maple [C] time = 0.22, size = 1356, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^
3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/mc
^2))/exp(x/y)/x^2,x)
```

```
[Out] 1/384*(144*Pi^2*mc^5*y-36*Pi^2*mc^4*y+96*Pi^2*mc^3*x*y+96*Pi^2*mc^3*y^2)*ex
p(-x/y)*ln(x)+1/8*I*y^2*Pi^3*mc^3*csgn(I*x)*csgn(I/mc^2*x)^2*exp(-x/y)+1/8*
I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)^2*exp(-x/y)-1/8*I*y*Pi^3*mc^3*c
sgn(I/mc^2*x)^3*x*exp(-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(
I/mc^2*x)^2-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/y)+1/8*I*y
^2*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*m
c^2)^3*x*exp(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)*
x*exp(-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)-1/16*y^3*Pi^2*mc*exp(-x/y)-1/128*y
*Pi^2*exp(-x/y)*x^2-1/64*y^2*Pi^2*x*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5+1/9
6/y*Pi^2*mc^9*Ei(1,x/y)-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/128*Pi^2*mc^8/x*exp(-
x/y)-1/96*Pi^2*mc^9/x*exp(-x/y)-1/2*y*Pi^2*ln(mc)*mc^3*x*exp(-x/y)+1/8*I*y^
2*Pi^3*mc^3*csgn(I*mc^2)^3*exp(-x/y)-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2*x)^3*e
xp(-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc^2)^3+3/64*I*y*Pi^3*exp(-x/y
)*mc^4*csgn(I/mc^2*x)^3+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc^2)^3-3/16*I*
y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)^3-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*
x)*csgn(I/mc^2*x)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x
)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)+3/32*I*y*Pi^3*ex
p(-x/y)*mc^4*csgn(I*mc)*csgn(I*mc^2)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*
mc)^2*csgn(I*mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)*csgn(I*mc^2)^2+3/
16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^2*x)^2-5/24*mc^3*Pi^2*y^2*ex
p(-x/y)+1/8*Pi^2*mc^7*Ei(1,x/y)-1/16*Pi^2*mc^6*Ei(1,x/y)-1/64*y^3*Pi^2*exp(
-x/y)+1/4*Pi^2*mc^3*y^2*Ei(1,x/y)-3/32*Pi^2*mc^4*y*Ei(1,x/y)+3/8*Pi^2*mc^5*
y*Ei(1,x/y)-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2
*x)+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)-1/8*
I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)*exp(-x/y)+1/8*I*y*Pi^
3*mc^3*csgn(I*x)*csgn(I/mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I/mc^2
)*csgn(I/mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*
x*exp(-x/y)-1/4*I*y*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*x*exp(-x/y)-1/2*y^2
*Pi^2*ln(mc)*mc^3*exp(-x/y)+1/16*y*Pi^2*mc^2*x*exp(-x/y)-11/24*y*Pi^2*mc^3*
x*exp(-x/y)-1/32*y*Pi^2*mc*exp(-x/y)*x^2-1/16*y^2*Pi^2*mc*x*exp(-x/y)-3/4*y
*Pi^2*exp(-x/y)*ln(mc)*mc^5+3/16*y*Pi^2*exp(-x/y)*ln(mc)*mc^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\pi^2 mc^9\Gamma\left(-1,\frac{x}{y}\right)}{96\,y}-\frac{1}{8}\,\pi^2 mc^7\mathrm{Ei}\left(-\frac{x}{y}\right)+\frac{\pi^2 mc^8\Gamma\left(-1,\frac{x}{y}\right)}{128\,y}+\frac{3}{8}\,\pi^2 mc^5 y e^{\left(-\frac{x}{y}\right)}\log\left(\frac{x}{mc^2}\right)+\frac{1}{16}\,\pi^2 mc^6\mathrm{Ei}\left(-\frac{x}{y}\right)-\frac{3}{8}\,\pi^2 mc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")
```

```
[Out] -1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128*pi^2*mc^8
*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2) + 1/16*pi^2*mc^6*E
i(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2*mc^5*y*e^(-x/y) - 3/32*pi^2*m
c^4*y*e^(-x/y)*log(x/mc^2) + 3/32*pi^2*mc^4*y*Ei(-x/y) - 11/24*pi^2*(x*y +
y^2)*mc^3*e^(-x/y) + 1/4*pi^2*((x*y + y^2)*e^(-x/y)*log(x) + integrate((2*x
^2*log(mc) - x*y - y^2)*e^(-x/y)/x, x))*mc^3 + 1/16*pi^2*(x*y + y^2)*mc^2*e
^(-x/y) - 1/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi^2*(x^2
*y + 2*x*y^2 + 2*y^3)*e^(-x/y)
```

mupad [B] time = 0.77, size = 265, normalized size = 0.80

$$\mathrm{ei}\left(-\frac{x}{y}\right)\left(\frac{\frac{\Pi^2\,\mathrm{mc}^8}{128}-\frac{\Pi^2\,\mathrm{mc}^9}{96}}{y}+\frac{\Pi^2\,\mathrm{mc}^6}{16}-\frac{\Pi^2\,\mathrm{mc}^7}{8}+y\left(\frac{3\,\Pi^2\,\mathrm{mc}^4}{32}-\frac{3\,\Pi^2\,\mathrm{mc}^5}{8}\right)-\frac{\Pi^2\,\mathrm{mc}^3\,y^2}{4}\right)-\frac{2\,\Pi^2\,x^2\,y\,\mathrm{e}^{-\frac{x}{y}}\,(-7\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x/y)*((Pi^2*(176*mc^3*x^3 - 24*mc^2*x^3 - 144*mc^5*x^2 + 12*mc*x^
4 + 24*mc^6*x - 48*mc^7*x - 3*mc^8 + 4*mc^9 + 3*x^4))/384 - (Pi^2*mc^3*x^2*
log(x/mc^2)*(8*x - 3*mc + 12*mc^2))/32))/x^2,x)
```

```
[Out] ei(-x/y)*(((Pi^2*mc^8)/128 - (Pi^2*mc^9)/96)/y + (Pi^2*mc^6)/16 - (Pi^2*mc^
7)/8 + y*((3*Pi^2*mc^4)/32 - (3*Pi^2*mc^5)/8) - (Pi^2*mc^3*y^2)/4) - (2*Pi^
2*x^2*y*exp(-x/y)*(12*mc*y^2 - 12*mc^2*y + 40*mc^3*y - 72*mc^5 + 3*y^2) + 2
*Pi^2*x^3*y*exp(-x/y)*(3*y + 12*mc*y - 12*mc^2 + 88*mc^3) + Pi^2*mc^8*x*exp
(-x/y)*(4*mc - 3) + 3*Pi^2*x^4*y*exp(-x/y)*(4*mc + 1) - 96*Pi^2*mc^3*x^3*y*
log(x/mc^2)*exp(-x/y) - 12*Pi^2*mc^3*x^2*y*log(x/mc^2)*exp(-x/y)*(8*y - 3*m
c + 12*mc^2))/(384*x^2)
```

sympy [A] time = 21.74, size = 330, normalized size = 1.00

$$-\frac{\pi^2 mc^9\,\mathrm{E}_2\left(\frac{x}{y}\right)}{96x}+\frac{\pi^2 mc^8\,\mathrm{E}_2\left(\frac{x}{y}\right)}{128x}-\frac{\pi^2 mc^7\,\mathrm{Ei}\left(-\frac{x}{y}\right)}{8}+\frac{\pi^2 mc^6\,\mathrm{Ei}\left(-\frac{x}{y}\right)}{16}+\frac{3\pi^2 mc^5 ye^{-\frac{x}{y}}}{8}-\frac{3\pi^2 mc^5\left(y\,\mathrm{Ei}\left(-\frac{x}{y}\right)-ye^{-\frac{x}{y}}\log\left(\cdots\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**
2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc**2
+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

```
[Out] -pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) - p
i**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/y)
/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*mc**
4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp(-x/
y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y) + (-x
*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*exp(-x/y
) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) -
 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y
**3*exp(-x/y))/128
```

3.283 $\int \sec(x)\sin(2x)\,dx$

Optimal. Leaf size=4

$$-2\cos(x)$$

```
[Out] -2*cos(x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 2638}

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sin[2*x],x]
```

```
[Out] -2*Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \sec(x)\sin(2x)\,dx &= 2\int \sin(x)\,dx \\ &= -2\cos(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sin[2*x],x]
```

```
[Out] -2*Cos[x]
```

fricas [A] time = 0.42, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

```
[Out] -2*cos(x)
```

giac [A] time = 1.09, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="giac")
```

```
[Out] -2*cos(x)
```

maple [**A**] time = 0.03, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

maxima [**A**] time = 0.44, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -2*cos(x)
```

mupad [**B**] time = 0.02, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

sympy [**A**] time = 0.76, size = 5, normalized size = 1.25

$$-2\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

3.284 $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$

Optimal. Leaf size=71

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}\,x^2+\sqrt{2}\,x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}\,x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

```
[Out] -1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))*(2^(1/2)-1)+1/2*ln(1+x-x^7+x*2^(1
/2)+x^2*2^(1/2))*(1+2^(1/2))
```

Rubi [F] time = 0.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x -
x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] Log[1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14]/2 + 2*Defer[Int][
(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14)^(-1), x] + 4*Defer[I
nt][x/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 2*Defer[
Int][x^2/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 12*De
fer[Int][x^7/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 1
0*Defer[Int][x^8/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+\frac{1}{14} \\
&= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+\frac{1}{14} \\
&= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2 \\
&= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2 \\
&= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.00

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}\,x^2+\sqrt{2}\,x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}\,x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 +
2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])*
Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

fricas [B] time = 0.44, size = 137, normalized size = 1.93

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^{14}-2x^8-2x^7+2x^4+4x^3+3x^2-2\sqrt{2}\left(x^9+x^8-x^3-2x^2-x\right)+2x+1}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}\right)+\frac{1}{2}\log\left(x^{14}-2x^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*(
x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x
^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 +
 2*x + 1)
```

giac [A] time = 1.29, size = 94, normalized size = 1.32

$$-\frac{1}{2}\sqrt{2}\log\left(\left|x^{7}+\sqrt{2}x^{2}+\sqrt{2}x-x-1\right|\right)+\frac{1}{2}\sqrt{2}\log\left(\left|x^{7}-\sqrt{2}x^{2}-\sqrt{2}x-x-1\right|\right)+\frac{1}{2}\log\left(\left|x^{14}-2x^{8}-2x^{7}-2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(x^7 + sqrt(2)*x^2 + sqrt(2)*x - x - 1)) + 1/2*sqrt(2)*
log(abs(x^7 - sqrt(2)*x^2 - sqrt(2)*x - x - 1)) + 1/2*log(abs(x^14 - 2*x^8
- 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))
```

maple [A] time = 0.03, size = 102, normalized size = 1.44

$$\frac{\ln\left(x^{7}+\sqrt{2}\,x^{2}+\left(\sqrt{2}-1\right)x-1\right)}{2}-\frac{\sqrt{2}\,\ln\left(x^{7}+\sqrt{2}\,x^{2}+\left(\sqrt{2}-1\right)x-1\right)}{2}+\frac{\ln\left(x^{7}-\sqrt{2}\,x^{2}+\left(-1-\sqrt{2}\right)x-1\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4
*x^3-x^2+2*x+1),x)
```

```
[Out] 1/2*ln(x^7-x^2*2^(1/2)+(-1-2^(1/2))*x-1)+1/2*ln(x^7-x^2*2^(1/2)+(-1-2^(1/2)
)*x-1)*2^(1/2)+1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))-1/2*ln(-1+x^7+x*(2^
(1/2)-1)+x^2*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7\,x^{13}+10\,x^{8}+4\,x^{7}-7\,x^{6}-4\,x^{3}-4\,x^{2}+3\,x+3}{x^{14}-2\,x^{8}-2\,x^{7}-2\,x^{4}-4\,x^{3}-x^{2}+2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14
 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

mupad [B] time = 0.28, size = 103, normalized size = 1.45

$$\frac{\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^{2}+x^{7}-1\right)}{2}+\frac{\ln\left(x^{7}-\sqrt{2}\,x-\sqrt{2}\,x^{2}-x-1\right)}{2}-\frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^{2}+x^{7}-1\right)}{2}+\frac{\sqrt{2}\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13 + 3)/(x^2 - 2*x
 + 4*x^3 + 2*x^4 + 2*x^7 + 2*x^8 - x^14 - 1),x)
```

```
[Out] log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1)/2 + log(x^7 - 2^(1/2)*x - 2^(1/2
)*x^2 - x - 1)/2 - (2^(1/2)*log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1))/2 +
 (2^(1/2)*log(x^7 - 2^(1/2)*x - 2^(1/2)*x^2 - x - 1))/2
```

<u>**sympy**</u> **[A]** time = 0.22, size = 76, normalized size = 1.07

$$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^7-\sqrt{2}\,x^2-2x\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)-1\right)+\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)\log\left(x^7+\sqrt{2}\,x^2-2x\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**8
-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

```
[Out] (1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (1
/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```