

Computer algebra independent integration tests

0-Independent-test-suites/Charlwood-Problems

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [50]. This is test number [4].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sageMath 9.3)
5. Fricas 1.3.7 on Linux (via sageMath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sageMath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in the table below reflects the above.

System	solved	Failed
Rubi	% 96.00 (48)	% 4.00 (2)
Mathematica	% 100.00 (50)	% 0.00 (0)
Maple	% 66.00 (33)	% 34.00 (17)
Maxima	% 48.00 (24)	% 52.00 (26)
Fricas	% 96.00 (48)	% 4.00 (2)
Sympy	% 38.00 (19)	% 62.00 (31)
Giac	% 82.00 (41)	% 18.00 (9)
Mupad	% 24.00 (12)	% 76.00 (38)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

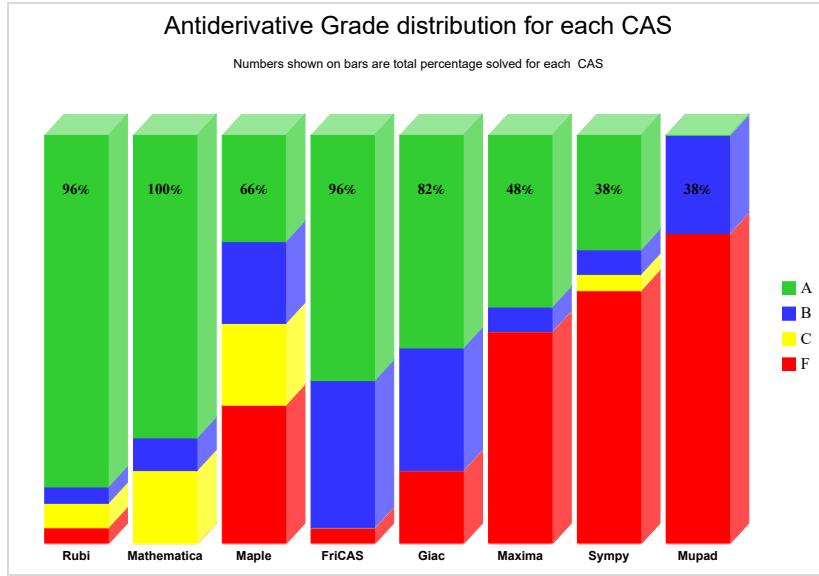
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

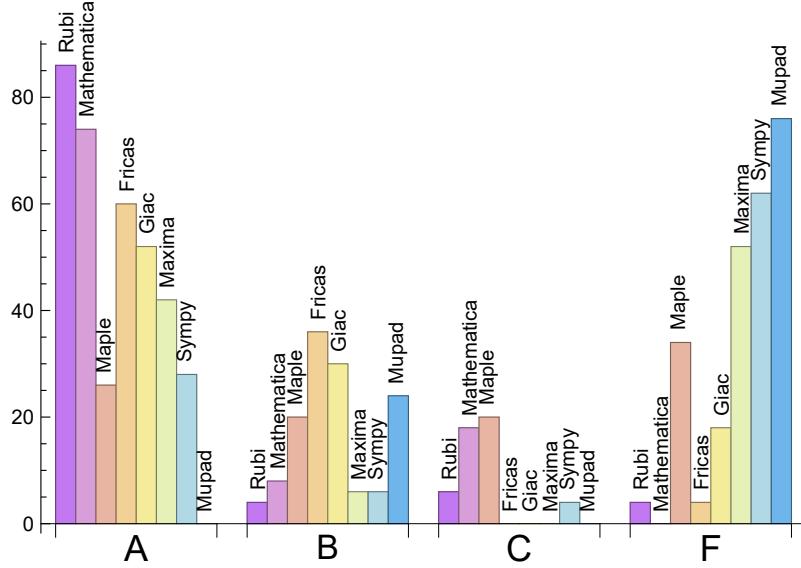
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.00	4.00	6.00	4.00
Mathematica	74.00	8.00	18.00	0.00
Maple	26.00	20.00	20.00	34.00
Maxima	42.00	6.00	0.00	52.00
Fricas	60.00	36.00	0.00	4.00
Sympy	28.00	6.00	4.00	62.00
Giac	52.00	30.00	0.00	18.00
Mupad	0.00	24.00	0.00	76.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sageMath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	17	94.12 %	0.00 %	5.88 %
Maxima	26	92.31 %	0.00 %	7.69 %
Fricas	2	0.00 %	100.00 %	0.00 %
Sympy	31	67.74 %	32.26 %	0.00 %
Giac	9	100.00 %	0.00 %	0.00 %
Mupad	38	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

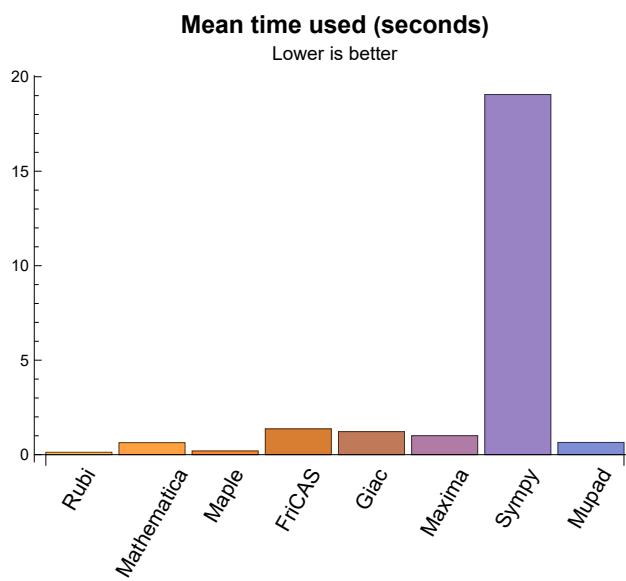
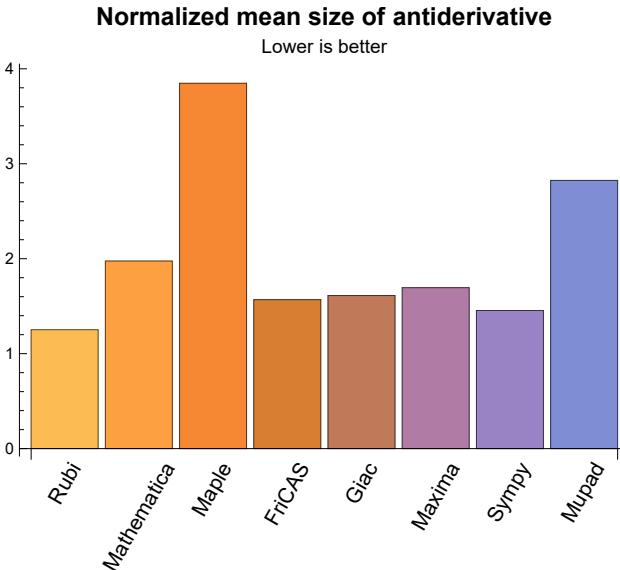
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	66.25	1.25	40.00	1.00
Mathematica	0.63	152.18	1.98	45.50	1.16
Maple	0.19	253.00	3.85	67.00	1.93
Maxima	1.01	53.67	1.69	26.50	0.88
Fricas	1.37	74.44	1.57	48.50	1.35
Sympy	19.06	43.63	1.45	31.00	0.94
Giac	1.22	88.46	1.61	50.00	1.41
Mupad	0.65	223.25	2.82	69.00	1.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {4, 5, 12, 13, 48}

Mathematica {9, 12, 45}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (`SageMath` uses `Maxima`), then any integral where `Maxima` needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore `Maxima` result below is lower than what could result if `Maxima` was run directly and each question `Maxima` asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate if the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail with error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and Xcas syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    """
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

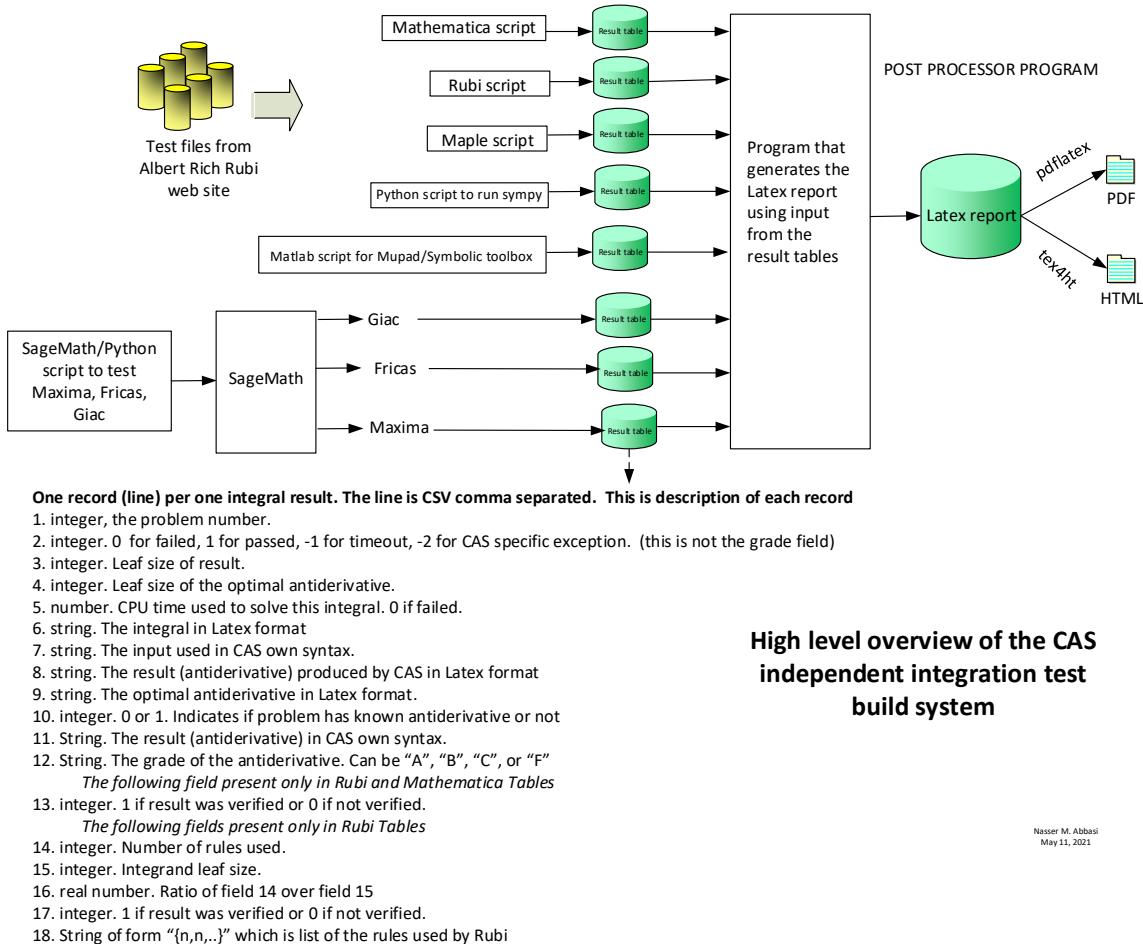
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 48, 49, 50 }

B grade: { 4, 42 }

C grade: { 5, 12, 13 }

F grade: { 3, 45 }

2.1.2 Mathematica

A grade: { 1, 2, 4, 6, 7, 10, 11, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 42, 43, 44, 45, 46, 48, 49, 50 }

B grade: { 3, 17, 21, 41 }

C grade: { 5, 8, 9, 12, 13, 37, 38, 39, 47 }

F grade: { }

2.1.3 Maple

A grade: { 2, 6, 7, 9, 19, 24, 27, 32, 34, 36, 37, 43, 48 }

B grade: { 1, 3, 4, 8, 40, 42, 44, 47, 49, 50 }

C grade: { 5, 12, 18, 28, 31, 33, 35, 38, 39, 46 }

F grade: { 10, 11, 13, 14, 15, 16, 17, 20, 21, 22, 23, 25, 26, 29, 30, 41, 45 }

2.1.4 Maxima

A grade: { 1, 2, 3, 9, 16, 18, 19, 20, 24, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 46, 48 }

B grade: { 7, 40, 43 }

C grade: { }

F grade: { 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 17, 21, 22, 23, 26, 29, 30, 38, 39, 41, 42, 44, 45, 47, 49, 50 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 9, 10, 11, 12, 13, 14, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 46, 48 }

B grade: { 4, 6, 7, 15, 16, 17, 21, 22, 37, 38, 40, 41, 42, 43, 44, 47, 49, 50 }

C grade: { }

F grade: { 8, 45 }

2.1.6 Sympy

A grade: { 1, 2, 10, 19, 20, 24, 25, 31, 33, 34, 35, 36, 46, 48 }

B grade: { 28, 37, 40 }

C grade: { 16, 18 }

F grade: { 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 17, 21, 22, 23, 26, 27, 29, 30, 32, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50 }

2.1.7 Giac

A grade: { 2, 6, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 33, 34, 35, 36, 40, 43, 47, 48, 49, 50 }

B grade: { 1, 4, 7, 8, 12, 13, 27, 29, 30, 31, 32, 37, 41, 42, 44 }

C grade: { }

F grade: { 3, 5, 9, 11, 23, 38, 39, 45, 46 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 9, 12, 19, 24, 30, 37, 40, 46, 47, 48, 50 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 44, 45, 49 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	93	58	54	102	272	-1
normalized size	1	1.00	1.02	1.82	1.14	1.06	2.00	5.33	-0.02
time (sec)	N/A	0.029	0.024	0.062	0.961	0.463	7.755	1.152	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	12	15	-1
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.71	0.88	-0.06
time (sec)	N/A	0.028	0.006	0.077	0.958	0.417	0.210	0.852	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	A	A	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	205	251	4	49	0	0	-1
normalized size	1	0.00	2.97	3.64	0.06	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.666	0.967	3.854	43.869	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F(-1)	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	332	194	426	0	402	0	235	666
normalized size	1	3.42	2.00	4.39	0.00	4.14	0.00	2.42	6.87
time (sec)	N/A	0.669	0.404	0.142	0.000	0.458	0.000	1.245	1.573

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	289	159	312	0	33	0	0	-1
normalized size	1	6.42	3.53	6.93	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.491	2.205	0.604	0.000	0.536	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	74	64	0	88	0	79	-1
normalized size	1	1.00	1.32	1.14	0.00	1.57	0.00	1.41	-0.02
time (sec)	N/A	0.069	0.119	0.079	0.000	0.551	0.000	1.202	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	30	0	28	-1
normalized size	1	1.00	1.00	0.80	1.80	2.00	0.00	1.87	-0.07
time (sec)	N/A	0.031	0.015	0.079	0.422	0.505	0.000	1.229	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	99	1605	0	0	0	495	-1
normalized size	1	1.00	0.72	11.72	0.00	0.00	0.00	3.61	-0.01
time (sec)	N/A	0.180	10.296	0.234	0.000	0.000	0.000	1.436	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	283	42	60	32	0	0	60
normalized size	1	1.00	6.90	1.02	1.46	0.78	0.00	0.00	1.46
time (sec)	N/A	0.022	4.377	0.070	1.012	0.482	0.000	0.000	0.417

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	62	38	0	0	28	56	46	-1
normalized size	1	1.41	0.86	0.00	0.00	0.64	1.27	1.05	-0.02
time (sec)	N/A	0.676	0.172	0.100	0.000	0.470	2.168	1.472	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	0	0	43	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.054	0.092	0.000	0.436	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	B	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	269	1822	439	0	191	0	364	661
normalized size	1	1.91	12.92	3.11	0.00	1.35	0.00	2.58	4.69
time (sec)	N/A	0.746	3.796	0.096	0.000	0.489	0.000	1.467	1.373

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F(-1)	B	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	286	2180	0	0	200	0	373	-1
normalized size	1	1.88	14.34	0.00	0.00	1.32	0.00	2.45	-0.01
time (sec)	N/A	0.457	4.953	0.103	0.000	0.502	0.000	1.568	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	51	44	0	0	63	0	57	-1
normalized size	1	1.13	0.98	0.00	0.00	1.40	0.00	1.27	-0.02
time (sec)	N/A	0.121	0.045	180.000	0.000	0.484	0.000	1.295	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	64	0	0	62	0	36	-1
normalized size	1	1.00	1.88	0.00	0.00	1.82	0.00	1.06	-0.03
time (sec)	N/A	0.035	0.101	0.101	0.000	0.424	0.000	1.383	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	18	56	78	18	-1
normalized size	1	1.00	1.00	0.00	0.82	2.55	3.55	0.82	-0.05
time (sec)	N/A	0.020	0.028	0.191	0.962	0.450	14.501	1.057	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	89	0	0	58	0	36	-1
normalized size	1	1.00	2.78	0.00	0.00	1.81	0.00	1.12	-0.03
time (sec)	N/A	0.041	0.088	0.096	0.000	0.426	0.000	1.119	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	89	41	32	37	62	-1
normalized size	1	1.00	1.00	2.07	0.95	0.74	0.86	1.44	-0.02
time (sec)	N/A	0.045	0.025	0.138	0.956	0.442	159.336	1.161	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	34	34	48	35	174
normalized size	1	1.00	1.00	0.75	1.21	1.21	1.71	1.25	6.21
time (sec)	N/A	0.009	0.005	0.024	0.413	0.417	1.172	1.011	0.135

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	22	22	20	22	-1
normalized size	1	1.00	1.00	0.00	0.85	0.85	0.77	0.85	-0.04
time (sec)	N/A	0.036	0.022	0.115	0.552	0.425	9.120	1.096	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	85	0	0	138	0	38	-1
normalized size	1	1.00	2.24	0.00	0.00	3.63	0.00	1.00	-0.03
time (sec)	N/A	0.052	0.089	0.409	0.000	0.446	0.000	1.101	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	94	88	0	0	110	0	52	-1
normalized size	1	1.34	1.26	0.00	0.00	1.57	0.00	0.74	-0.01
time (sec)	N/A	0.132	0.101	0.740	0.000	0.484	0.000	1.217	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	48	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.142	0.041	0.170	0.000	0.469	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	41	37	36	35	31	36	27
normalized size	1	1.00	0.75	0.67	0.65	0.64	0.56	0.65	0.49
time (sec)	N/A	0.053	0.023	0.006	0.415	0.448	5.992	1.032	0.340

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	22	22	20	22	-1
normalized size	1	1.00	1.00	0.00	0.85	0.85	0.77	0.85	-0.04
time (sec)	N/A	0.033	0.020	0.084	1.059	0.421	4.328	1.001	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	119	0	0	115	0	122	-1
normalized size	1	1.00	1.53	0.00	0.00	1.47	0.00	1.56	-0.01
time (sec)	N/A	0.271	0.064	0.096	0.000	0.445	0.000	1.303	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	35	35	39	0	73	-1
normalized size	1	1.00	0.64	0.90	0.90	1.00	0.00	1.87	-0.03
time (sec)	N/A	0.048	0.033	0.109	0.964	0.445	0.000	1.082	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	54	15	23	29	23	-1
normalized size	1	1.00	1.00	3.18	0.88	1.35	1.71	1.35	-0.06
time (sec)	N/A	0.028	0.016	0.172	0.952	0.463	2.330	0.927	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	0	0	81	0	104	-1
normalized size	1	1.00	1.35	0.00	0.00	1.42	0.00	1.82	-0.02
time (sec)	N/A	0.080	0.069	0.335	0.000	0.473	0.000	1.094	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	69	0	108	37
normalized size	1	1.00	1.00	0.00	0.00	1.53	0.00	2.40	0.82
time (sec)	N/A	0.042	0.036	0.227	0.000	0.453	0.000	1.021	0.030

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	56	22	47	19	54	-1
normalized size	1	1.00	1.14	1.93	0.76	1.62	0.66	1.86	-0.03
time (sec)	N/A	0.048	0.026	0.173	0.953	0.442	7.264	1.104	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	22	0	40	-1
normalized size	1	1.00	1.00	0.95	0.90	1.05	0.00	1.90	-0.05
time (sec)	N/A	0.044	0.015	0.074	0.945	0.440	0.000	1.035	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	27	27	29	28	-1
normalized size	1	1.00	0.79	3.50	0.79	0.79	0.85	0.82	-0.03
time (sec)	N/A	0.034	0.020	0.138	0.973	0.437	2.737	1.011	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	29	29	33	26	58	-1
normalized size	1	1.00	0.64	0.88	0.88	1.00	0.79	1.76	-0.03
time (sec)	N/A	0.042	0.025	0.101	0.970	0.439	5.247	0.975	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	97	15	15	22	22	-1
normalized size	1	1.00	1.40	3.88	0.60	0.60	0.88	0.88	-0.04
time (sec)	N/A	0.029	0.043	0.512	0.465	0.441	19.712	1.094	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	39	25	41	41	44	-1
normalized size	1	1.00	1.18	1.15	0.74	1.21	1.21	1.29	-0.03
time (sec)	N/A	0.035	0.018	0.101	0.962	0.422	4.342	1.115	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	14	24	29	46	27	13
normalized size	1	1.00	2.88	0.88	1.50	1.81	2.88	1.69	0.81
time (sec)	N/A	0.018	0.053	0.028	0.955	0.437	22.867	1.002	0.206

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	36	112	0	42	0	0	-1
normalized size	1	1.00	1.57	4.87	0.00	1.83	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.101	0.086	0.000	0.459	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	40	112	0	18	0	0	-1
normalized size	1	1.00	1.74	4.87	0.00	0.78	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.096	0.025	0.000	0.455	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	39	54	82	93	97	36	55
normalized size	1	1.00	1.77	2.45	3.73	4.23	4.41	1.64	2.50
time (sec)	N/A	0.071	0.051	0.168	0.971	0.453	1.367	1.131	0.381

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	87	0	0	146	0	94	-1
normalized size	1	1.00	2.07	0.00	0.00	3.48	0.00	2.24	-0.02
time (sec)	N/A	0.153	0.094	0.207	0.000	0.489	0.000	1.370	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	59	45	91	0	54	0	92	-1
normalized size	1	2.11	1.61	3.25	0.00	1.93	0.00	3.29	-0.04
time (sec)	N/A	0.184	0.037	0.250	0.000	0.491	0.000	2.218	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	A	F	
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	55	37	565	186	0	50	-1
normalized size	1	1.00	1.62	1.09	16.62	5.47	0.00	1.47	-0.03
time (sec)	N/A	0.045	0.073	0.060	1.348	0.441	0.000	1.065	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	50	65	67	0	63	0	67	-1
normalized size	1	1.28	1.67	1.72	0.00	1.62	0.00	1.72	-0.03
time (sec)	N/A	0.046	0.088	0.409	0.000	0.518	0.000	2.347	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F(-1)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	552	0	0	0	0	0	-1
normalized size	1	0.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.794	2.182	0.261	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	3134	67	52	87	0	78
normalized size	1	1.00	0.75	40.70	0.87	0.68	1.13	0.00	1.01
time (sec)	N/A	0.220	0.022	1.159	0.978	0.470	2.491	0.000	0.257

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	136	508	0	287	0	92	413
normalized size	1	1.00	1.13	4.23	0.00	2.39	0.00	0.77	3.44
time (sec)	N/A	0.131	0.313	0.053	0.000	0.485	0.000	1.151	1.086

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	37	39	28	26	22	29	27	40
normalized size	1	1.19	1.26	0.90	0.84	0.71	0.94	0.87	1.29
time (sec)	N/A	0.009	0.391	0.020	1.126	0.493	89.157	1.062	0.797

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	138	0	60	0	34	-1
normalized size	1	1.00	1.00	4.76	0.00	2.07	0.00	1.17	-0.03
time (sec)	N/A	0.024	0.012	0.124	0.000	0.504	0.000	1.332	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	198	0	164	0	111	455
normalized size	1	1.00	1.00	1.87	0.00	1.55	0.00	1.05	4.29
time (sec)	N/A	0.110	0.183	0.088	0.000	0.512	0.000	1.458	1.160

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [1.400]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.00	5	1.400
2	A	2	2	1.00	15	0.133
3	F	0	0	N/A	0	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules integrand leaf size
4	B	32	13	3.42	14	0.929
5	C	5	4	6.42	19	0.210
6	A	7	7	1.00	13	0.538
7	A	4	4	1.00	13	0.308
8	A	9	7	1.00	14	0.500
9	A	7	7	1.00	12	0.583
10	A	5	5	1.41	19	0.263
11	A	7	8	1.00	29	0.276
12	C	40	15	1.91	14	1.071
13	C	32	16	1.88	27	0.593
14	A	9	11	1.13	18	0.611
15	A	3	4	1.00	24	0.167
16	A	3	4	1.00	12	0.333
17	A	3	4	1.00	22	0.182
18	A	4	4	1.00	15	0.267
19	A	4	4	1.00	13	0.308
20	A	2	3	1.00	23	0.130
21	A	5	6	1.00	17	0.353
22	A	7	8	1.34	15	0.533
23	A	4	9	1.00	25	0.360
24	A	5	5	1.00	27	0.185
25	A	2	3	1.00	23	0.130
26	A	18	11	1.00	27	0.407
27	A	3	3	1.00	17	0.176
28	A	2	2	1.00	13	0.154
29	A	7	6	1.00	17	0.353

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
30	A	5	5	1.00	15	0.333
31	A	4	4	1.00	15	0.267
32	A	2	2	1.00	17	0.118
33	A	5	5	1.00	13	0.385
34	A	3	3	1.00	15	0.200
35	A	2	2	1.00	13	0.154
36	A	5	5	1.00	13	0.385
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	24	0.083
39	A	2	2	1.00	24	0.083
40	A	4	5	1.00	10	0.500
41	A	6	7	1.00	12	0.583
42	B	5	5	2.11	13	0.385
43	A	4	4	1.00	13	0.308
44	A	4	4	1.28	15	0.267
45	F	0	0	N/A	0	N/A
46	A	13	10	1.00	12	0.833
47	A	12	8	1.00	12	0.667
48	A	6	6	1.19	18	0.333
49	A	4	4	1.00	14	0.286
50	A	6	6	1.00	14	0.429

Chapter 3

Listing of integrals

3.1 $\int \sin^{-1}(x) \log(x) dx$

Optimal. Leaf size=51

$$-2\sqrt{1-x^2} + \sqrt{1-x^2} \log(x) + \tanh^{-1}\left(\sqrt{1-x^2}\right) - x(1-\log(x)) \sin^{-1}(x)$$

[Out] $\arctanh((-x^2+1)^{(1/2)}) - x \arcsin(x) * (1 - \ln(x)) - 2(-x^2+1)^{(1/2)} + \ln(x) * (-x^2+1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {4619, 261, 2387, 266, 50, 63, 206}

$$-2\sqrt{1-x^2} + \sqrt{1-x^2} \log(x) + \tanh^{-1}\left(\sqrt{1-x^2}\right) - x \sin^{-1}(x) + x \log(x) \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]*Log[x], x]

[Out] $-2\sqrt{1-x^2} - x \arcsin(x) + \operatorname{ArcTanh}[\sqrt{1-x^2}] + \sqrt{1-x^2} \log(x) + x \arcsin(x) \log(x)$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^((m_.)*(c_.) + (d_.)*(x_))^((n_.), x_Symbol) :> Simp[  
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(  
b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,  
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ  
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{ArcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x]))^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(x) \log(x) dx &= \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x) - \int \left(\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) \right) dx \\
&= \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x) - \int \frac{\sqrt{1-x^2}}{x} dx - \int \sin^{-1}(x) dx \\
&= -x \sin^{-1}(x) + \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{\sqrt{1-x}}{x} dx, x, x^2 \right) + \int \dots \\
&= -2\sqrt{1-x^2} - x \sin^{-1}(x) + \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, x^2 \right) \\
&= -2\sqrt{1-x^2} - x \sin^{-1}(x) + \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x) + \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, x^2 \right) \\
&= -2\sqrt{1-x^2} - x \sin^{-1}(x) + \tanh^{-1} \left(\sqrt{1-x^2} \right) + \sqrt{1-x^2} \log(x) + x \sin^{-1}(x) \log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.02

$$-2\sqrt{1-x^2} + (\sqrt{1-x^2} - 1) \log(x) + \log(\sqrt{1-x^2} + 1) + x(\log(x) - 1) \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[ArcSin[x]*Log[x], x]`

[Out] $-2\sqrt{1-x^2} + x \operatorname{ArcSin}[x](-1 + \operatorname{Log}[x]) + (-1 + \sqrt{1-x^2}) \operatorname{Log}[x] + \operatorname{Log}[1 + \sqrt{1-x^2}]$

fricas [A] time = 0.46, size = 54, normalized size = 1.06

$$x \arcsin(x) \log(x) - x \arcsin(x) + \sqrt{-x^2 + 1} (\log(x) - 2) + \frac{1}{2} \log(\sqrt{-x^2 + 1} + 1) - \frac{1}{2} \log(\sqrt{-x^2 + 1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)*log(x), x, algorithm="fricas")`

[Out] $x \arcsin(x) \log(x) - x \arcsin(x) + \sqrt{-x^2 + 1} (\log(x) - 2) + 1/2 \log(\sqrt{-x^2 + 1} + 1) - 1/2 \log(\sqrt{-x^2 + 1} - 1)$

giac [B] time = 1.15, size = 272, normalized size = 5.33

$$\begin{aligned}
&x \arcsin(x) \log(x) + \sqrt{-x^2 + 1} \log(x) - \frac{2x \arcsin(x)}{\left(\sqrt{-x^2 + 1} + 1 \right) \left(\frac{x^2}{\left(\sqrt{-x^2+1}+1 \right)^2} + 1 \right)} + \frac{x^2 \log \left(\sqrt{-x^2 + 1} + 1 \right)}{\left(\sqrt{-x^2 + 1} + 1 \right)^2 \left(\frac{x^2}{\left(\sqrt{-x^2+1}+1 \right)^2} + 1 \right)} + \dots
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*log(x),x, algorithm="giac")
[Out] x*arcsin(x)*log(x) + sqrt(-x^2 + 1)*log(x) - 2*x*arcsin(x)/((sqrt(-x^2 + 1)
+ 1)*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) + x^2*log(sqrt(-x^2 + 1) + 1)/((sqrt
(-x^2 + 1) + 1)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) + log(sqrt(-x^2 + 1) +
1)/(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1) - x^2*log(abs(x))/((sqrt(-x^2 + 1) + 1
)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) - log(abs(x))/(x^2/(sqrt(-x^2 + 1) +
1)^2 + 1) + 2*x^2/((sqrt(-x^2 + 1) + 1)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1))
- 2/(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)
```

maple [B] time = 0.06, size = 93, normalized size = 1.82

$$\frac{-\ln\left(\tan^2\left(\frac{\arcsin(x)}{2}\right)+1\right)+\frac{2 \arcsin(x) \ln\left(\frac{2 \tan\left(\frac{\arcsin(x)}{2}\right)}{\tan^2\left(\frac{\arcsin(x)}{2}\right)+1}\right) \tan\left(\frac{\arcsin(x)}{2}\right)-2 \ln\left(\frac{2 \tan\left(\frac{\arcsin(x)}{2}\right)}{\tan^2\left(\frac{\arcsin(x)}{2}\right)+1}\right) \left(\tan^2\left(\frac{\arcsin(x)}{2}\right)\right)}}{\tan^2\left(\frac{\arcsin(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)*ln(x),x)
```

```
[Out] 2*(arcsin(x)*tan(1/2*arcsin(x))*ln(2*tan(1/2*arcsin(x))/(tan(1/2*arcsin(x))
^2+1))-tan(1/2*arcsin(x))^2*ln(2*tan(1/2*arcsin(x))/(tan(1/2*arcsin(x))^2+1
))-arcsin(x)*tan(1/2*arcsin(x))-2)/(tan(1/2*arcsin(x))^2+1)-ln(tan(1/2*arcs
in(x))^2+1)
```

maxima [A] time = 0.96, size = 58, normalized size = 1.14

$$(x \log(x) - x) \arcsin(x) + \sqrt{-x^2 + 1} \log(x) - 2 \sqrt{-x^2 + 1} + \log\left(\frac{2 \sqrt{-x^2 + 1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*log(x),x, algorithm="maxima")
```

```
[Out] (x*log(x) - x)*arcsin(x) + sqrt(-x^2 + 1)*log(x) - 2*sqrt(-x^2 + 1) + log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \text{asin}(x) \ln(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(x)*log(x),x)`

[Out] `int(asin(x)*log(x), x)`

sympy [A] time = 7.76, size = 102, normalized size = 2.00

$$x \log(x) \operatorname{asin}(x) - x \operatorname{asin}(x) + \sqrt{1-x^2} \log(x) - \sqrt{1-x^2} - \begin{cases} -\frac{x}{\sqrt{-1+\frac{1}{x^2}}} - \operatorname{acosh}\left(\frac{1}{x}\right) + \frac{1}{x \sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{i x}{\sqrt{1-\frac{1}{x^2}}} + i \operatorname{asin}\left(\frac{1}{x}\right) - \frac{i}{x \sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(x)*ln(x),x)`

[Out] `x*log(x)*asin(x) - x*asin(x) + sqrt(1 - x**2)*log(x) - sqrt(1 - x**2) - Piecewise((-x/sqrt(-1 + x**(-2))) - acosh(1/x) + 1/(x*sqrt(-1 + x**(-2))), 1/Abse(x**2) > 1), (I*x/sqrt(1 - 1/x**2) + I*asin(1/x) - I/(x*sqrt(1 - 1/x**2)), True))`

3.2 $\int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=17

$$x - \sqrt{1 - x^2} \sin^{-1}(x)$$

[Out] $x - \text{arcsin}(x) * (-x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4677, 8}

$$x - \sqrt{1 - x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \text{ArcSin}[x]) / \text{Sqrt}[1 - x^2], x]$

[Out] $x - \text{Sqrt}[1 - x^2] * \text{ArcSin}[x]$

Rule 8

$\text{Int}[a_, x_Symbol] :> \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 4677

$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^n_*(x_*)*((d_.) + (e_.)*(x_.)^2)^p_*, x_Symbol] :> \text{Simp}[((d + e*x^2)^{p+1}*(a + b*\text{ArcSin}[c*x])^n)/(2*e*(p+1)), x] + \text{Dist}[(b*n*d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]})/(2*c*(p+1)*(1 - c^2*x^2)^{\text{FracPart}[p]}), \text{Int}[(1 - c^2*x^2)^{p+1/2}*(a + b*\text{ArcSin}[c*x])^{n-1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&& \text{EqQ}[c^2*d + e, 0] \&& \text{GtQ}[n, 0] \&& \text{NeQ}[p, -1]]$

Rubi steps

$$\begin{aligned} \int \frac{x \sin^{-1}(x)}{\sqrt{1 - x^2}} dx &= -\sqrt{1 - x^2} \sin^{-1}(x) + \int 1 dx \\ &= x - \sqrt{1 - x^2} \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x - \sqrt{1 - x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*ArcSin[x])/Sqrt[1 - x^2], x]`
 [Out] `x - Sqrt[1 - x^2]*ArcSin[x]`

fricas [A] time = 0.42, size = 15, normalized size = 0.88

$$-\sqrt{-x^2 + 1} \arcsin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(x)/(-x^2+1)^(1/2), x, algorithm="fricas")`
 [Out] `-sqrt(-x^2 + 1)*arcsin(x) + x`

giac [A] time = 0.85, size = 15, normalized size = 0.88

$$-\sqrt{-x^2 + 1} \arcsin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(x)/(-x^2+1)^(1/2), x, algorithm="giac")`
 [Out] `-sqrt(-x^2 + 1)*arcsin(x) + x`

maple [A] time = 0.08, size = 16, normalized size = 0.94

$$x - \sqrt{-x^2 + 1} \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsin(x)/(-x^2+1)^(1/2), x)`
 [Out] `x-(-x^2+1)^(1/2)*arcsin(x)`

maxima [A] time = 0.96, size = 15, normalized size = 0.88

$$-\sqrt{-x^2 + 1} \arcsin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(x)/(-x^2+1)^(1/2), x, algorithm="maxima")`
 [Out] `-sqrt(-x^2 + 1)*arcsin(x) + x`

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x \sin^{-1}(x)}{\sqrt{1 - x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*asin(x))/(1 - x^2)^(1/2), x)`

[Out] `int((x*asin(x))/(1 - x^2)^(1/2), x)`

sympy [A] time = 0.21, size = 12, normalized size = 0.71

$$x - \sqrt{1 - x^2} \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asin(x)/(-x**2+1)**(1/2), x)`

[Out] `x - sqrt(1 - x**2)*asin(x)`

$$3.3 \quad \int -\sin^{-1}(\sqrt{x} - \sqrt{1+x}) dx$$

Optimal. Leaf size=69

$$\frac{(\sqrt{x} + 3\sqrt{x+1})\sqrt{\sqrt{x}\sqrt{x+1} - x}}{4\sqrt{2}} - \left(x + \frac{3}{8}\right)\sin^{-1}(\sqrt{x} - \sqrt{x+1})$$

[Out] $-(3/8+x)*\arcsin(x^{(1/2)}-(1+x)^{(1/2)})+1/8*(x^{(1/2)}+3*(1+x)^{(1/2})*(-x+x^{(1/2)})*(1+x)^{(1/2)})^{(1/2)}*2^{(1/2)}$

Rubi [F] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int -\sin^{-1}(\sqrt{x} - \sqrt{1+x}) dx$$

Verification is Not applicable to the result.

[In] Int[-ArcSin[Sqrt[x] - Sqrt[1 + x]], x]

[Out] $-(x*\text{ArcSin}[\text{Sqrt}[x] - \text{Sqrt}[1 + x]]) + \text{Defer}[\text{Subst}[\text{Defer}[\text{Int}][\text{Sqrt}[1 - x^2 + x*\text{Sqrt}[-1 + x^2]], x], x, \text{Sqrt}[1 + x]]/\text{Sqrt}[2]$

Rubi steps

$$\begin{aligned} \int -\sin^{-1}(\sqrt{x} - \sqrt{1+x}) dx &= -x \sin^{-1}(\sqrt{x} - \sqrt{1+x}) + \int \frac{\sqrt{-x + \sqrt{x}\sqrt{1+x}}}{2\sqrt{2}\sqrt{1+x}} dx \\ &= -x \sin^{-1}(\sqrt{x} - \sqrt{1+x}) + \frac{\int \frac{\sqrt{-x + \sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}} dx}{2\sqrt{2}} \\ &= -x \sin^{-1}(\sqrt{x} - \sqrt{1+x}) + \frac{\text{Subst}\left(\int \sqrt{1-x^2 + x\sqrt{-1+x^2}} dx, x, \sqrt{1+x}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [B] time = 0.67, size = 205, normalized size = 2.97

$$\frac{(x+1)(2x - 2\sqrt{x+1}\sqrt{x} + 1)^2 \left(2\sqrt{\sqrt{x}\sqrt{x+1}} - x(-2x + 2\sqrt{x+1}\sqrt{x} - 3) + 3\sqrt{-4x + 4\sqrt{x+1}\sqrt{x} - 2\log(\sqrt{x+1} - \sqrt{x})}\right)}{8\sqrt{2}(\sqrt{x+1} - \sqrt{x})^3(x - \sqrt{x+1}\sqrt{x} + 1)^2}$$

Antiderivative was successfully verified.

[In] `Integrate[-ArcSin[Sqrt[x] - Sqrt[1 + x]], x]`

[Out]
$$\begin{aligned} & -(x \operatorname{ArcSin}[\sqrt{x}] - \sqrt{1+x})) - ((1+x)(1+2x-2\sqrt{x}\sqrt{1+x})^2(2\sqrt{-x+\sqrt{x}}\sqrt{1+x})^2(-3-2x+2\sqrt{x}\sqrt{1+x}) \\ & + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\log[2\sqrt{-x+\sqrt{x}}\sqrt{1+x}])/(8\sqrt{2}(-\sqrt{x}+\sqrt{1+x})^3(1+x-\sqrt{x}\sqrt{1+x})^2) \end{aligned}$$

fricas [A] time = 43.87, size = 49, normalized size = 0.71

$$\frac{1}{8}(8x+3)\arcsin\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{8}\sqrt{2\sqrt{x+1}\sqrt{x}-2x}\left(3\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arcsin(x^(1/2)-(1+x)^(1/2)), x, algorithm="fricas")`

[Out] $1/8*(8*x+3)*\arcsin(\sqrt{x+1}-\sqrt{x})+1/8*\sqrt{2*\sqrt{x+1}*\sqrt{x}-2*x}*(3*\sqrt{x+1}+\sqrt{x})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arcsin(x^(1/2)-(1+x)^(1/2)), x, algorithm="giac")`

[Out] `integrate(-arcsin(-sqrt(x+1)+sqrt(x)), x)`

maple [B] time = 0.97, size = 251, normalized size = 3.64

$$\arcsin\left(\sqrt{x}-\sqrt{x+1}\right)\left(\tan^8\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{x+1}\right)}{2}\right)\right)+2\arcsin\left(\sqrt{x}-\sqrt{x+1}\right)\left(\tan^6\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{x+1}\right)}{2}\right)\right)-2\left(\tan^7\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{x+1}\right)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-arcsin(x^(1/2)-(x+1)^(1/2)), x)`

[Out]
$$\begin{aligned} & -1/16*(\arcsin(x^{1/2}-(x+1)^{1/2})*\tan(1/2*\arcsin(x^{1/2}-(x+1)^{1/2}))^{8+2} \\ & *\arcsin(x^{1/2}-(x+1)^{1/2})*\tan(1/2*\arcsin(x^{1/2}-(x+1)^{1/2}))^{6-2}*\tan(1/2*\arcsin(x^{1/2}-(x+1)^{1/2}))^{7+18}*\arcsin(x^{1/2}-(x+1)^{1/2})*\tan(1/2*\arcsin(x^{1/2}-(x+1)^{1/2}))^{10-2} \end{aligned}$$

$\text{csin}(x^{(1/2)} - (x+1)^{(1/2)})^4 - 6 \tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))^{5+2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}) * \tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))^{2+6 * \tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))^{3+\arcsin(x^{(1/2)} - (x+1)^{(1/2)}) + 2 * \tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))) / (\tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))^{2+1})^{2/\tan(1/2 * \arcsin(x^{(1/2)} - (x+1)^{(1/2)}))^{2}}$

maxima [A] time = 3.85, size = 4, normalized size = 0.06

$$\frac{1}{2} \pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arcsin(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $1/2 * \pi * x$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{asin}\left(\sqrt{x+1} - \sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin((x + 1)^(1/2) - x^(1/2)),x)`

[Out] `int(asin((x + 1)^(1/2) - x^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \text{asin}\left(\sqrt{x} - \sqrt{x+1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-asin(x**(1/2)-(1+x)**(1/2)),x)`

[Out] `-Integral(asin(sqrt(x) - sqrt(x + 1)), x)`

3.4 $\int \log(1 + x\sqrt{1 + x^2}) dx$

Optimal. Leaf size=97

$$x \log(\sqrt{x^2 + 1} x + 1) + \sqrt{2(1 + \sqrt{5})} \tan^{-1}\left(\sqrt{\sqrt{5} - 2} (\sqrt{x^2 + 1} + x)\right) - \sqrt{2(\sqrt{5} - 1)} \tanh^{-1}\left(\sqrt{2 + \sqrt{5}} (\sqrt{x^2 + 1} + x)\right)$$

[Out] $-2*x + x*\ln(1+x*(x^2+1)^(1/2)) - \text{arctanh}((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2) + \text{arctan}((x+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)$

Rubi [B] time = 0.67, antiderivative size = 332, normalized size of antiderivative = 3.42, number of steps used = 32, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {2548, 6742, 261, 1130, 203, 207, 1251, 824, 707, 1093, 1166, 1247, 699}

$$x \log(\sqrt{x^2 + 1} x + 1) + \sqrt{\frac{2}{5}(\sqrt{5} - 1)} \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5} - 1}} \sqrt{x^2 + 1}\right) + \sqrt{\frac{2}{5(\sqrt{5} - 1)}} \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5} - 1}} \sqrt{x^2 + 1}\right) - \sqrt{\frac{2}{5(\sqrt{5} - 1)}} \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5} - 1}} \sqrt{x^2 + 1}\right)$$

Warning: Unable to verify antiderivative.

[In] $\text{Int}[\log[1 + x\sqrt{1 + x^2}], x]$

[Out] $-2*x - \text{Sqrt}[(1 + \text{Sqrt}[5])/10]*\text{ArcTan}[\text{Sqrt}[2/(1 + \text{Sqrt}[5])]*x] + 2*\text{Sqrt}[(2 + \text{Sqrt}[5])/5]*\text{ArcTan}[\text{Sqrt}[2/(1 + \text{Sqrt}[5])]*x] + \text{Sqrt}[2/(5*(-1 + \text{Sqrt}[5]))]*\text{ArcTan}[\text{Sqrt}[2/(-1 + \text{Sqrt}[5])]*\text{Sqrt}[1 + x^2]] + \text{Sqrt}[(2*(-1 + \text{Sqrt}[5]))/5]*\text{ArcTan}[\text{Sqrt}[2/(-1 + \text{Sqrt}[5])]*\text{Sqrt}[1 + x^2]] + 2*\text{Sqrt}[(-2 + \text{Sqrt}[5])/5]*\text{ArcTanh}[\text{Sqrt}[2/(-1 + \text{Sqrt}[5])]*x] + \text{Sqrt}[(-1 + \text{Sqrt}[5])/10]*\text{ArcTanh}[\text{Sqrt}[2/(-1 + \text{Sqrt}[5])]*x] + \text{Sqrt}[2/(5*(1 + \text{Sqrt}[5]))]*\text{ArcTanh}[\text{Sqrt}[2/(1 + \text{Sqrt}[5])]*\text{Sqrt}[1 + x^2]] - \text{Sqrt}[(2*(1 + \text{Sqrt}[5]))/5]*\text{ArcTanh}[\text{Sqrt}[2/(1 + \text{Sqrt}[5])]*\text{Sqrt}[1 + x^2]] + x*\text{Log}[1 + x*\text{Sqrt}[1 + x^2]]$

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_)*(x_)]/((a_.) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_)*(x_)]*((a_.) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_)*(x_))^(m_)*((f_.) + (g_)*(x_))/((a_.) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1093

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 2]
```

Rule 1166

```

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

```

Rule 1247

```

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rule 2548

```

Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
x])/u, x], x] /; InverseFunctionFreeQ[u, x]

```

Rule 6742

```

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

Rubi steps

$$\begin{aligned}
\int \log(1 + x\sqrt{1+x^2}) dx &= x \log(1 + x\sqrt{1+x^2}) - \int \frac{x(1+2x^2)}{x+x^3+\sqrt{1+x^2}} dx \\
&= x \log(1 + x\sqrt{1+x^2}) - \int \left(\frac{x}{x+x^3+\sqrt{1+x^2}} + \frac{2x^3}{x+x^3+\sqrt{1+x^2}} \right) dx \\
&= x \log(1 + x\sqrt{1+x^2}) - 2 \int \frac{x^3}{x+x^3+\sqrt{1+x^2}} dx - \int \frac{x}{x+x^3+\sqrt{1+x^2}} dx \\
&= x \log(1 + x\sqrt{1+x^2}) - 2 \int \left(1 - \frac{x}{\sqrt{1+x^2}} + \frac{1-x^2}{-1+x^2+x^4} - \frac{x\sqrt{1+x^2}}{-1+x^2+x^4} + \frac{x^3\sqrt{1+x^2}}{-1+x^2+x^4} \right) dx \\
&= -2x + x \log(1 + x\sqrt{1+x^2}) + 2 \int \frac{x}{\sqrt{1+x^2}} dx - 2 \int \frac{1-x^2}{-1+x^2+x^4} dx + 2 \int \frac{x\sqrt{1+x^2}}{-1+x^2+x^4} dx \\
&= -2x + \sqrt{1+x^2} + x \log(1 + x\sqrt{1+x^2}) + \frac{1}{2} \text{Subst} \left(\int \frac{x\sqrt{1+x^2}}{-1+x^2+x^4} dx, x, x^2 \right) + \frac{1}{10} \\
&= -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) \\
&= -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) \\
&= -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) \\
&= -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x^2+1} \right) - 2x + \dots
\end{aligned}$$

Mathematica [A] time = 0.40, size = 194, normalized size = 2.00

$$x \log(\sqrt{x^2+1} x + 1) - \frac{\sqrt{2(\sqrt{5}-1)} \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{x^2+1} \right)}{1-\sqrt{5}} - \sqrt{\frac{2}{1+\sqrt{5}}} \tanh^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x^2+1} \right) - 2x + \dots \quad (5)$$

Antiderivative was successfully verified.

[In] `Integrate[Log[1 + x*.Sqrt[1 + x^2]], x]`

[Out] $-2x + ((5 + \sqrt{5})*\text{ArcTan}[\sqrt{2/(1 + \sqrt{5})}*x])/\sqrt{10*(1 + \sqrt{5})}$
 $-(\sqrt{2*(-1 + \sqrt{5})}*\text{ArcTan}[\sqrt{2/(-1 + \sqrt{5})}]*\sqrt{1 + x^2}))/$
 $(1 - \sqrt{5}) - ((-5 + \sqrt{5})*\text{ArcTanh}[\sqrt{2/(-1 + \sqrt{5})}]*x))/\sqrt{10*$
 $(-1 + \sqrt{5})} - \sqrt{2/(1 + \sqrt{5})}*\text{ArcTanh}[\sqrt{2/(1 + \sqrt{5})}]*\sqrt{1 + x^2}] + x*\log[1 + x*\sqrt{1 + x^2}]$

fricas [B] time = 0.46, size = 402, normalized size = 4.14

$$-\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4x^4+4x^2+\sqrt{5}(2x^2+1)-2(2x^3+\sqrt{5}x+x)\sqrt{x^2+1}+1}\right)\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(1+x*(x^2+1)^(1/2)), x, algorithm="fricas")`

[Out] $-\sqrt{2}*\sqrt{\sqrt{5}+1}*\arctan(1/4*\sqrt{2}*\sqrt{4*x^4+4*x^2+\sqrt{5}}*$
 $(2*x^2+1)-2*(2*x^3+\sqrt{5}*x+x)*\sqrt{x^2+1}+1)*(\sqrt{2}*x+\sqrt{2}*\sqrt{x^2+1})*\sqrt{\sqrt{5}+1}-\sqrt{2}*\sqrt{\sqrt{5}+1}*\arctan(1/8*\sqrt{4*x^2+2*\sqrt{5}+2})*(\sqrt{5}*\sqrt{2}-\sqrt{2})*\sqrt{\sqrt{5}+1}-1/4*(\sqrt{5}*\sqrt{2})*x-\sqrt{2})*x)*\sqrt{\sqrt{5}+1}+1/4*\sqrt{2}*\sqrt{\sqrt{5}-1}*\log(4*x^2-4*\sqrt{x^2+1})*x+(\sqrt{5}*\sqrt{2})*x-\sqrt{x^2+1})*(\sqrt{5}*\sqrt{2}+\sqrt{2})+\sqrt{2})*x)*\sqrt{\sqrt{5}-1}+4)-1/4*\sqrt{2}*\sqrt{\sqrt{5}-1}*\log(4*x^2-4*\sqrt{x^2+1})*x-(\sqrt{5}*\sqrt{2})*x-\sqrt{x^2+1})*(\sqrt{5}*\sqrt{2}+\sqrt{2})+\sqrt{2})*x)*\sqrt{\sqrt{5}-1}+4)+x*\log(\sqrt{x^2+1})*x+1/4*\sqrt{2}*\sqrt{\sqrt{5}-1}*\log(2*x+\sqrt{2})*\sqrt{\sqrt{5}-1})-1/4*\sqrt{2}*\sqrt{\sqrt{5}-1}*\log(2*x-\sqrt{2})*\sqrt{\sqrt{5}-1})-2*x$

giac [B] time = 1.25, size = 235, normalized size = 2.42

$$x\log\left(\sqrt{x^2+1}x+1\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(1+x*(x^2+1)^(1/2)), x, algorithm="giac")`

[Out] $x*\log(\sqrt{x^2+1})*x+1)+1/2*\sqrt{2*\sqrt{5}+2}*\arctan(-(x-\sqrt{x^2+1}+1/(x-\sqrt{x^2+1}))/\sqrt{2*\sqrt{5}-2})+1/2*\sqrt{2*\sqrt{5}+2}*\arctan(x/\sqrt{1/2*\sqrt{5}+1/2})-1/4*\sqrt{2*\sqrt{5}-2}*\log(-x+\sqrt{x^2+1}+\sqrt{2*\sqrt{5}+2}-1/(x-\sqrt{x^2+1}))+1/4*\sqrt{2*\sqrt{5}-2}*\log(\abs{x+\sqrt{1/2*\sqrt{5}-1/2}})-1/4*\sqrt{2*\sqrt{5}-2}*\log(\abs{x-\sqrt{1/2*\sqrt{5}-1/2}})+1/4*\sqrt{2*\sqrt{5}-2}*\log(\abs{-x+\sqrt{x^2+1}-\sqrt{2*\sqrt{5}+2}-1/(x-\sqrt{x^2+1}))}-2*x$

maple [B] time = 0.14, size = 426, normalized size = 4.39

$$x \ln\left(\sqrt{x^2 + 1} x + 1\right) - 2x + \frac{\operatorname{arctanh}\left(\frac{-x + \sqrt{x^2 + 1}}{\sqrt{-2 + \sqrt{5}}}\right)}{2\sqrt{-2 + \sqrt{5}}} - \frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{-x + \sqrt{x^2 + 1}}{\sqrt{-2 + \sqrt{5}}}\right)}{10\sqrt{-2 + \sqrt{5}}} - \frac{2\sqrt{-2 + \sqrt{5}} \sqrt{5} \operatorname{arctanh}\left(\frac{-x + \sqrt{x^2 + 1}}{\sqrt{-2 + \sqrt{5}}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(1+x*(x^2+1)^(1/2)),x)`

[Out] $x \ln(1+x*(x^2+1)^(1/2))+1/(2+2*5^(1/2))^(1/2)*\operatorname{arctan}(2/(2+2*5^(1/2))^(1/2)*x)+5^(1/2)/(2+2*5^(1/2))^(1/2)*\operatorname{arctan}(2/(2+2*5^(1/2))^(1/2)*x)-1/(-2+2*5^(1/2))^(1/2)*\operatorname{arctanh}(2/(-2+2*5^(1/2))^(1/2)*x)+5^(1/2)/(-2+2*5^(1/2))^(1/2)*\operatorname{arctanh}(2/(-2+2*5^(1/2))^(1/2)*x)-2*x-3/10*5^(1/2)/(2+5^(1/2))^(1/2)*\operatorname{arctan}((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2)-1/2/(2+5^(1/2))^(1/2)*\operatorname{arctan}(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*\operatorname{arctanh}(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*\operatorname{arctanh}(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2/(2+5^(1/2))^(1/2)*\operatorname{arctanh}(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*\operatorname{arctanh}(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*\operatorname{arctan}(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*\operatorname{arctan}(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+2/5*5^(1/2)*(2+5^(1/2))^(1/2)*\operatorname{arctan}(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))-2/5*(-2+5^(1/2))^(1/2)*\operatorname{arctanh}(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \log\left(\sqrt{x^2 + 1} x + 1\right) - 2x + \operatorname{arctan}(x) + \int \frac{2x^2 + 1}{x^2 + (x^3 + x)\sqrt{x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(1+x*(x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] $x * \log(\sqrt{x^2 + 1} * x + 1) - 2*x + \operatorname{arctan}(x) + \operatorname{integrate}((2*x^2 + 1)/(x^2 + (x^3 + x)*\sqrt{x^2 + 1}) + 1, x)$

mupad [B] time = 1.57, size = 666, normalized size = 6.87

$$x \ln\left(x \sqrt{x^2 + 1} + 1\right) - 2x + \frac{\ln\left(x - \frac{\sqrt{2} \sqrt{\sqrt{5}-1}}{2}\right) \left(\frac{\sqrt{5}}{2} - \frac{5}{2}\right)}{2 \sqrt{\frac{\sqrt{5}}{2} - \frac{1}{2}} + 4 \left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x + \frac{\sqrt{2} \sqrt{\sqrt{5}-1}}{2}\right) \left(\frac{\sqrt{5}}{2} - \frac{5}{2}\right)}{2 \sqrt{\frac{\sqrt{5}}{2} - \frac{1}{2}} + 4 \left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x - \frac{\sqrt{2} \sqrt{-\sqrt{5}-1}}{2}\right) \left(\frac{\sqrt{5}}{2} - \frac{5}{2}\right)}{2 \sqrt{-\frac{\sqrt{5}}{2} - \frac{1}{2}} + 4 \left(-\frac{\sqrt{5}}{2} - \frac{1}{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x*(x^2 + 1)^(1/2) + 1),x)
[Out] x*log(x*(x^2 + 1)^(1/2) + 1) - 2*x + (log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) + (log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) + ((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2) + (2^(1/2)*(x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2) - (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) - ((log((2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2) - (2^(1/2)*x*(- 5^(1/2) - 1)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) - ((log((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2))/2 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x*(x**2+1)**(1/2)),x)
[Out] Timed out
```

$$3.5 \quad \int \frac{\cos^2(x)}{\sqrt{1+\cos^2(x)+\cos^4(x)}} dx$$

Optimal. Leaf size=45

$$\frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{\sin(x) \cos(x) (\cos^2(x) + 1)}{\sqrt{\cos^4(x) + \cos^2(x) + 1} \cos^2(x) + 1} \right)$$

[Out] $1/3*x + 1/3*\arctan(\cos(x)*(1+\cos(x)^2)*\sin(x)/(1+\cos(x)^2*(1+\cos(x)^2+\cos(x)^4)^(1/2)))$

Rubi [C] time = 0.49, antiderivative size = 289, normalized size of antiderivative = 6.42, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6719, 1216, 1103, 1706}

$$\frac{\cos^2(x) \tan^{-1} \left(\frac{\tan(x)}{\sqrt{\tan^4(x) + 3 \tan^2(x) + 3}} \right) \sqrt{\tan^4(x) + 3 \tan^2(x) + 3} (1 + \sqrt{3}) \cos^2(x) (\tan^2(x) + \sqrt{3}) \sqrt{\frac{\tan^4(x) + 3 \tan^2(x) + 3}{(\tan^2(x) + \sqrt{3})^2}}}{2 \sqrt{\cos^4(x) (\tan^4(x) + 3 \tan^2(x) + 3)} 4 \sqrt[4]{3} \sqrt{\cos^4(x) (\tan^4(x) + 3 \tan^2(x) + 3)}}$$

Warning: Unable to verify antiderivative.

[In] Int[Cos[x]^2/Sqrt[1 + Cos[x]^2 + Cos[x]^4], x]

[Out] $(\text{ArcTan}[\text{Tan}[x]/\text{Sqrt}[3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4]] * \text{Cos}[x]^2 * \text{Sqrt}[3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4]) / (2 * \text{Sqrt}[\text{Cos}[x]^4 * (3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4)]) - ((1 + \text{Sqrt}[3]) * \text{Cos}[x]^2 * \text{EllipticF}[2 * \text{ArcTan}[\text{Tan}[x]/3^{(1/4)}], (2 - \text{Sqrt}[3])/4] * (\text{Sqrt}[3] + \text{Tan}[x]^2) * \text{Sqrt}[(3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4) / (\text{Sqrt}[3] + \text{Tan}[x]^2)^2]) / (4 * 3^{(1/4)} * \text{Sqrt}[\text{Cos}[x]^4 * (3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4)]) + ((2 + \text{Sqrt}[3]) * \text{Cos}[x]^2 * \text{EllipticPi}[(3 - 2 * \text{Sqrt}[3])/6, 2 * \text{ArcTan}[\text{Tan}[x]/3^{(1/4)}], (2 - \text{Sqrt}[3])/4] * (\text{Sqrt}[3] + \text{Tan}[x]^2) * \text{Sqrt}[(3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4) / (\text{Sqrt}[3] + \text{Tan}[x]^2)^2]) / (4 * 3^{(1/4)} * \text{Sqrt}[\text{Cos}[x]^4 * (3 + 3 \text{Tan}[x]^2 + \text{Tan}[x]^4)])$

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simpl[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1216

Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int

```
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-b + (c*d)/e + (a*e)/d, 2]), x] + Simpl[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6719

```
Int[(u_)*(a_)*(v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)}{\sqrt{1 + \cos^2(x) + \cos^4(x)}} dx &= \text{Subst} \left(\int \frac{1}{\left(1 + x^2\right)^2 \sqrt{\frac{3+3x^2+x^4}{\left(1+x^2\right)^2}}} dx, x, \tan(x) \right) \\
&= \frac{\left(\cos^2(x)\sqrt{3 + 3\tan^2(x) + \tan^4(x)}\right) \text{Subst} \left(\int \frac{1}{\left(1+x^2\right)\sqrt{3+3x^2+x^4}} dx, x, \tan(x) \right)}{\sqrt{\cos^4(x)(3 + 3\tan^2(x) + \tan^4(x))}} \\
&= \frac{\left((-1 - \sqrt{3})\cos^2(x)\sqrt{3 + 3\tan^2(x) + \tan^4(x)}\right) \text{Subst} \left(\int \frac{1}{\sqrt{3+3x^2+x^4}} dx, x, \tan(x) \right)}{2\sqrt{\cos^4(x)(3 + 3\tan^2(x) + \tan^4(x))}} \\
&= \frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{3+3\tan^2(x)+\tan^4(x)}}\right)\cos^2(x)\sqrt{3 + 3\tan^2(x) + \tan^4(x)}}{2\sqrt{\cos^4(x)(3 + 3\tan^2(x) + \tan^4(x))}} - \frac{(1 + \sqrt{3})\cos^2(x)}{2\sqrt{\cos^4(x)(3 + 3\tan^2(x) + \tan^4(x))}}
\end{aligned}$$

Mathematica [C] time = 2.21, size = 159, normalized size = 3.53

$$\frac{2i \cos^2(x) \sqrt{1 - \frac{2i \tan^2(x)}{\sqrt{3}-3i}} \sqrt{1 + \frac{2i \tan^2(x)}{\sqrt{3}+3i}} \Pi\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}; i \sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\right) \tan(x)\right) |_{\frac{3i-\sqrt{3}}{3i+\sqrt{3}}} }{\sqrt{-\frac{i}{\sqrt{3}-3i}} \sqrt{8 \cos(2x) + \cos(4x) + 15}}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]^2/Sqrt[1 + Cos[x]^2 + Cos[x]^4], x]`

[Out] `((-2*I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(-3*I + Sqrt[3])]*Sqrt[1 - ((2*I)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])])/(Sqrt[-I]/(-3*I + Sqrt[3]))*Sqrt[15 + 8*Cos[2*x] + Cos[4*x]])`

fricas [A] time = 0.54, size = 33, normalized size = 0.73

$$\frac{1}{6} \arctan\left(\frac{2\sqrt{\cos(x)^4 + \cos(x)^2 + 1} \cos(x)^3 \sin(x)}{2 \cos(x)^6 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="fricas")`
[Out] `1/6*arctan(2*sqrt(cos(x)^4 + cos(x)^2 + 1)*cos(x)^3*sin(x)/(2*cos(x)^6 - 1))`
giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="giac")`
[Out] `integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1), x)`
maple [C] time = 0.60, size = 312, normalized size = 6.93

$$\frac{2\sqrt{(\cos^2(2x) + 4\cos(2x) + 7)(\sin^2(2x))} (i\sqrt{3} - 3)\sqrt{\frac{(-1+i\sqrt{3})(\cos(2x)-1)}{(i\sqrt{3}-3)(\cos(2x)+1)}} (\cos(2x) + 1)^2 \sqrt{\frac{\cos(2x)+2+i\sqrt{3}}{(i\sqrt{3}+3)(\cos(2x)+1)}}}{(-1 + i\sqrt{3})\sqrt{(\cos(2x) - 1)(\cos(2x) + 1)(\cos(2x) + 2 + i\sqrt{3})(-\cos(2x) + i\sqrt{3})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x)`
[Out] `-2*((cos(2*x)^2+4*cos(2*x)+7)*sin(2*x)^2)^(1/2)*(I*3^(1/2)-3)*((-1+I*3^(1/2))*((cos(2*x)-1)/(I*3^(1/2)-3)/(cos(2*x)+1))^(1/2)*(cos(2*x)+1)^2*((cos(2*x)+2+I*3^(1/2))/((I*3^(1/2)+3)/(cos(2*x)+1))^(1/2)*((I*3^(1/2)-cos(2*x)-2)/(I*3^(1/2)-3)/(cos(2*x)+1))^(1/2)*EllipticPi((-1+I*3^(1/2))*(cos(2*x)-1)/(I*3^(1/2)-3)/(cos(2*x)+1))^(1/2),(I*3^(1/2)-3)/(-1+I*3^(1/2)),((1+I*3^(1/2))*(I*3^(1/2)-3)/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2)/(-1+I*3^(1/2))/((cos(2*x)-1)*(cos(2*x)+1)*(cos(2*x)+2+I*3^(1/2))*(I*3^(1/2)-cos(2*x)-2))^(1/2)/sin(2*x)/(cos(2*x)^2+4*cos(2*x)+7))^(1/2)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="maxima")`
[Out] `integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(cos(x)^2 + cos(x)^4 + 1)^(1/2),x)

[Out] int(cos(x)^2/(cos(x)^2 + cos(x)^4 + 1)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2/(1+cos(x)**2+cos(x)**4)**(1/2),x)

[Out] Timed out

3.6 $\int \tan(x) \sqrt{1 + \tan^4(x)} \, dx$

Optimal. Leaf size=56

$$\frac{1}{2} \sqrt{\tan^4(x) + 1} - \frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2} \sqrt{\tan^4(x)+1}}\right)}{\sqrt{2}} - \frac{1}{2} \sinh^{-1}(\tan^2(x))$$

[Out] $-1/2*\text{arcsinh}(\tan(x)^2)-1/2*\text{arctanh}(1/2*(1-\tan(x)^2)*2^{(1/2)}/(1+\tan(x)^4)^(1/2))*2^{(1/2)}+1/2*(1+\tan(x)^4)^(1/2)$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3670, 1248, 735, 844, 215, 725, 206}

$$\frac{1}{2} \sqrt{\tan^4(x) + 1} - \frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2} \sqrt{\tan^4(x)+1}}\right)}{\sqrt{2}} - \frac{1}{2} \sinh^{-1}(\tan^2(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\tan[x]*\text{Sqrt}[1 + \tan[x]^4], x]$

[Out] $-\text{ArcSinh}[\tan[x]^2]/2 - \text{ArcTanh}[(1 - \tan[x]^2)/(\text{Sqrt}[2]*\text{Sqrt}[1 + \tan[x]^4])]/\text{Sqrt}[2] + \text{Sqrt}[1 + \tan[x]^4]/2$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*(c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f*x^2)^2, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tan(x) \sqrt{1 + \tan^4(x)} \, dx &= \text{Subst} \left(\int \frac{x \sqrt{1 + x^4}}{1 + x^2} \, dx, x, \tan(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1 + x^2}}{1 + x} \, dx, x, \tan^2(x) \right) \\
&= \frac{1}{2} \sqrt{1 + \tan^4(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{1 - x}{(1 + x)\sqrt{1 + x^2}} \, dx, x, \tan^2(x) \right) \\
&= \frac{1}{2} \sqrt{1 + \tan^4(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1 + x^2}} \, dx, x, \tan^2(x) \right) + \text{Subst} \left(\int \frac{1}{(1 + x)\sqrt{1 + x^2}} \, dx, x, \tan^2(x) \right) \\
&= -\frac{1}{2} \sinh^{-1}(\tan^2(x)) + \frac{1}{2} \sqrt{1 + \tan^4(x)} - \text{Subst} \left(\int \frac{1}{2 - x^2} \, dx, x, \frac{1 - \tan^2(x)}{\sqrt{1 + \tan^4(x)}} \right) \\
&= -\frac{1}{2} \sinh^{-1}(\tan^2(x)) - \frac{\tanh^{-1}\left(\frac{1 - \tan^2(x)}{\sqrt{2}\sqrt{1 + \tan^4(x)}}\right)}{\sqrt{2}} + \frac{1}{2} \sqrt{1 + \tan^4(x)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 74, normalized size = 1.32

$$\frac{\sqrt{\tan^4(x) + 1} \left(\sqrt{\cos(4x) + 3} - 2\sqrt{2} \cos^2(x) \sinh^{-1}(\cos(2x)) - 2 \cos^2(x) \tanh^{-1}\left(\frac{2 \sin^2(x)}{\sqrt{\cos(4x) + 3}}\right) \right)}{2\sqrt{\cos(4x) + 3}}$$

Antiderivative was successfully verified.

[In] `Integrate[Tan[x]*Sqrt[1 + Tan[x]^4], x]`

[Out] $((-2\sqrt{2} \operatorname{ArcSinh}[\cos(2x)] \cos^2(x) - 2\operatorname{ArcTanh}[(2\sin^2(x))/\sqrt{3 + \cos(4x)}]) \cos^2(x) + \sqrt{3 + \cos(4x)}) \sqrt{1 + \tan^4(x)} / (2\sqrt{3 + \cos(4x)})$

fricas [B] time = 0.55, size = 88, normalized size = 1.57

$$\frac{1}{4} \sqrt{2} \log \left(\frac{3 \tan(x)^4 - 2 \tan(x)^2 + 2 \sqrt{\tan(x)^4 + 1} (\sqrt{2} \tan(x)^2 - \sqrt{2}) + 3}{\tan(x)^4 + 2 \tan(x)^2 + 1} \right) + \frac{1}{2} \sqrt{\tan(x)^4 + 1} + \frac{1}{2} \log(-\tan(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+tan(x)^4)^(1/2)*tan(x), x, algorithm="fricas")`

[Out] $\frac{1}{4} \sqrt{2} \log((3 \tan(x)^4 - 2 \tan(x)^2 + 2 \sqrt{\tan(x)^4 + 1}) (\sqrt{2} \tan(x)^2 - \sqrt{2}) + 3) / (\tan(x)^4 + 2 \tan(x)^2 + 1) + \frac{1}{2} \sqrt{\tan(x)^4 + 1} + \frac{1}{2} \log(-\tan(x)^2 + \sqrt{\tan(x)^4 + 1})$

giac [A] time = 1.20, size = 79, normalized size = 1.41

$$\frac{1}{2} \sqrt{2} \log \left(-\frac{\tan(x)^2 + \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1}{\tan(x)^2 - \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1} \right) + \frac{1}{2} \sqrt{\tan(x)^4 + 1} + \frac{1}{2} \log \left(-\tan(x)^2 + \sqrt{\tan(x)^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+tan(x)^4)^(1/2)*tan(x),x, algorithm="giac")`

[Out] $\frac{1}{2} \sqrt{2} \log(-\tan(x)^2 + \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1) / (\tan(x)^2 - \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1) + \frac{1}{2} \sqrt{\tan(x)^4 + 1} + \frac{1}{2} \log(-\tan(x)^2 + \sqrt{\tan(x)^4 + 1})$

maple [A] time = 0.08, size = 64, normalized size = 1.14

$$-\frac{\operatorname{arcsinh}(\tan^2(x))}{2} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(-2(\tan^2(x))+2)\sqrt{2}}{4\sqrt{-2(\tan^2(x))+(\tan^2(x)+1)^2}}\right)}{2} + \frac{\sqrt{-2(\tan^2(x)) + (\tan^2(x)+1)^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+tan(x)^4)^(1/2)*tan(x),x)`

[Out] $\frac{1}{2} ((\tan(x)^2+1)^2 - 2\tan(x)^2)^{(1/2)} - \frac{1}{2} \operatorname{arcsinh}(\tan(x)^2) - \frac{1}{2} 2^{(1/2)} \operatorname{arctanh}\left(\frac{1}{4} (-2\tan(x)^2+2) 2^{(1/2)} / ((\tan(x)^2+1)^2 - 2\tan(x)^2)^{(1/2)}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x)^4 + 1} \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+tan(x)^4)^(1/2)*tan(x),x, algorithm="maxima")`

[Out] `integrate(sqrt(tan(x)^4 + 1)*tan(x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x) \sqrt{\tan(x)^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)*(tan(x)^4 + 1)^(1/2),x)`

[Out] $\int \tan(x) * (\tan(x)^4 + 1)^{1/2} dx$
sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^4(x) + 1} \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1 + \tan(x)^4)^{1/2} \tan(x) dx$
[Out] Integral(sqrt(tan(x)^4 + 1) * tan(x), x)

3.7 $\int \frac{\tan(x)}{\sqrt{1+\sec^3(x)}} dx$

Optimal. Leaf size=15

$$-\frac{2}{3} \tanh^{-1} \left(\sqrt{\sec^3(x) + 1} \right)$$

[Out] $-2/3*\text{arctanh}((1+\sec(x)^3)^{(1/2)})$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4139, 266, 63, 207}

$$-\frac{2}{3} \tanh^{-1} \left(\sqrt{\sec^3(x) + 1} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\tan[x]/\text{Sqrt}[1 + \sec[x]^3], x]$
[Out] $(-2*\text{ArcTanh}[\text{Sqrt}[1 + \sec[x]^3]])/3$

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_)*(c_)*(e_) + (f_)*(x_))^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
```

```
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(x)}{\sqrt{1 + \sec^3(x)}} dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x^3}} dx, x, \sec(x)\right) \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \sec^3(x)\right) \\ &= \frac{2}{3} \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+\sec^3(x)}\right) \\ &= -\frac{2}{3} \tanh^{-1}\left(\sqrt{1+\sec^3(x)}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$-\frac{2}{3} \tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Tan[x]/Sqrt[1 + Sec[x]^3], x]`

[Out] `(-2*ArcTanh[Sqrt[1 + Sec[x]^3]])/3`

fricas [B] time = 0.50, size = 30, normalized size = 2.00

$$\frac{1}{3} \log\left(2 \sqrt{\frac{\cos(x)^3 + 1}{\cos(x)^3}} \cos(x)^3 - 2 \cos(x)^3 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+sec(x)^3)^(1/2), x, algorithm="fricas")`

[Out] `1/3*log(2*sqrt((cos(x)^3 + 1)/cos(x)^3)*cos(x)^3 - 2*cos(x)^3 - 1)`

giac [B] time = 1.23, size = 28, normalized size = 1.87

$$-\frac{1}{3} \log\left(\sqrt{\frac{1}{\cos(x)^3} + 1} + 1\right) + \frac{1}{3} \log\left(\left|\sqrt{\frac{1}{\cos(x)^3} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+sec(x)^3)^(1/2),x, algorithm="giac")`

[Out] $-1/3\log(\sqrt{1/\cos(x)^3 + 1} + 1) + 1/3\log(\sqrt{1/\cos(x)^3 + 1} - 1)$

maple [A] time = 0.08, size = 12, normalized size = 0.80

$$-\frac{2 \operatorname{arctanh}\left(\sqrt{\sec ^3(x)+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(1+sec(x)^3)^(1/2),x)`

[Out] $-2/3\operatorname{arctanh}\left((1+\sec(x)^3)^{(1/2)}\right)$

maxima [B] time = 0.42, size = 27, normalized size = 1.80

$$-\frac{1}{3} \log \left(\sqrt{\frac{1}{\cos(x)^3} + 1} + 1 \right) + \frac{1}{3} \log \left(\sqrt{\frac{1}{\cos(x)^3} + 1} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+sec(x)^3)^(1/2),x, algorithm="maxima")`

[Out] $-1/3\log(\sqrt{1/\cos(x)^3 + 1} + 1) + 1/3\log(\sqrt{1/\cos(x)^3 + 1} - 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(x)}{\sqrt{\frac{1}{\cos(x)^3} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(1/cos(x)^3 + 1)^(1/2),x)`

[Out] `int(tan(x)/(1/cos(x)^3 + 1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{(\sec(x) + 1)(\sec^2(x) - \sec(x) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+sec(x)**3)**(1/2),x)`

[Out] `Integral(tan(x)/sqrt((sec(x) + 1)*(sec(x)**2 - sec(x) + 1)), x)`

3.8 $\int \sqrt{2 + 2 \tan(x) + \tan^2(x)} \, dx$

Optimal. Leaf size=137

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \tan^{-1}\left(\frac{2\sqrt{5} - (5+\sqrt{5})\tan(x)}{\sqrt{10(1+\sqrt{5})}\sqrt{\tan^2(x)+2\tan(x)+2}}\right) - \sqrt{\frac{1}{2}(\sqrt{5}-1)} \tanh^{-1}\left(\frac{(5-\sqrt{5})\tan(x)}{\sqrt{10(\sqrt{5}-1)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)$$

[Out] $\text{arcsinh}(1+\tan(x))-1/2*\text{arctanh}((2*5^{(1/2)}+(5-5^{(1/2)})*\tan(x))/(-10+10*5^{(1/2)}))^{(1/2)}/(2+2*\tan(x)+\tan(x)^2)^{(1/2)}*(-2+2*5^{(1/2)})^{(1/2)}-1/2*\text{arctan}((2*5^{(1/2)}-(5+5^{(1/2)})*\tan(x))/(10+10*5^{(1/2)})^{(1/2)}/(2+2*\tan(x)+\tan(x)^2)^{(1/2)})*(2+2*5^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {990, 619, 215, 1036, 1030, 207, 203}

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \tan^{-1}\left(\frac{2\sqrt{5} - (5+\sqrt{5})\tan(x)}{\sqrt{10(1+\sqrt{5})}\sqrt{\tan^2(x)+2\tan(x)+2}}\right) - \sqrt{\frac{1}{2}(\sqrt{5}-1)} \tanh^{-1}\left(\frac{(5-\sqrt{5})\tan(x)}{\sqrt{10(\sqrt{5}-1)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[2 + 2*\text{Tan}[x] + \text{Tan}[x]^2], x]$

[Out] $\text{ArcSinh}[1 + \text{Tan}[x]] - \text{Sqrt}[(1 + \text{Sqrt}[5])/2]*\text{ArcTan}[(2*\text{Sqrt}[5] - (5 + \text{Sqrt}[5]))*\text{Tan}[x]/(\text{Sqrt}[10*(1 + \text{Sqrt}[5])]*\text{Sqrt}[2 + 2*\text{Tan}[x] + \text{Tan}[x]^2])] - \text{Sqrt}[-1 + \text{Sqrt}[5]]/2]*\text{ArcTanh}[(2*\text{Sqrt}[5] + (5 - \text{Sqrt}[5]))*\text{Tan}[x]/(\text{Sqrt}[10*(-1 + \text{Sqrt}[5])]*\text{Sqrt}[2 + 2*\text{Tan}[x] + \text{Tan}[x]^2])]$

Rule 203

$\text{Int}[((a_) + (b_)*(\text{x}_)^2)^{(-1)}, \text{x_Symbol}] \Rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*\text{x})/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), \text{x}] /; \text{FreeQ}[\{a, b\}, \text{x}] \&& \text{PosQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

Rule 207

$\text{Int}[((a_) + (b_)*(\text{x}_)^2)^{(-1)}, \text{x_Symbol}] \Rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*\text{x})/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), \text{x}] /; \text{FreeQ}[\{a, b\}, \text{x}] \&& \text{NegQ}[a/b] \&& (\text{LtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

Rule 215

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^p_, x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]/((d_) + (f_)*(x_)^2), x_Symbol]
:> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1030

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f
_)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f
_)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2 + 2 \tan(x) + \tan^2(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{2 + 2x + x^2}}{1 + x^2} \, dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{\sqrt{2 + 2x + x^2}} \, dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{-1 - 2x}{(1 + x^2)\sqrt{2 + 2x + x^2}} \, dx, x, \tan(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{4}}} \, dx, x, 2 + 2 \tan(x) \right) - \frac{\text{Subst} \left(\int \frac{5 - \sqrt{5} - 2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}} \, dx, x, \tan(x) \right)}{2\sqrt{5}} \\
&= \sinh^{-1}(1 + \tan(x)) - (2(5 - \sqrt{5})) \text{Subst} \left(\int \frac{1}{20(1 - \sqrt{5}) + 2x^2} \, dx, x, \frac{-2\sqrt{5} - 2\sqrt{5}\tan(x)}{\sqrt{2 + 2\tan(x)}} \right) \\
&= \sinh^{-1}(1 + \tan(x)) - \sqrt{\frac{1}{2}(1 + \sqrt{5})} \tan^{-1} \left(\frac{2\sqrt{5} - (5 + \sqrt{5})\tan(x)}{\sqrt{10(1 + \sqrt{5})}\sqrt{2 + 2\tan(x) + \tan^2(x)}} \right)
\end{aligned}$$

Mathematica [C] time = 10.30, size = 99, normalized size = 0.72

$$\sinh^{-1}(\tan(x)+1)+\frac{1}{2} i \left(\sqrt{1+2 i} \tanh ^{-1}\left(\frac{(1+i) \tan (x)+(2+i)}{\sqrt{1+2 i} \sqrt{\tan ^2(x)+2 \tan (x)+2}}\right)-\sqrt{1-2 i} \tanh ^{-1}\left(\frac{(2-2 i) \tan (x)}{2 \sqrt{1-2 i} \sqrt{\tan ^2(x)+2 \tan (x)+2}}\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[2 + 2*Tan[x] + Tan[x]^2], x]`

[Out] `ArcSinh[1 + Tan[x]] + (I/2)*(Sqrt[1 + 2*I]*ArcTanh[((2 + I) + (1 + I)*Tan[x])/(Sqrt[1 + 2*I]*Sqrt[2 + 2*Tan[x] + Tan[x]^2])] - Sqrt[1 - 2*I]*ArcTanh[((4 - 2*I) + (2 - 2*I)*Tan[x])/(2*Sqrt[1 - 2*I]*Sqrt[2 + 2*Tan[x] + Tan[x]^2])])`

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+2*tan(x)+tan(x)^2)^(1/2), x, algorithm="fricas")`

[Out] Timed out

giac [B] time = 1.44, size = 495, normalized size = 3.61

$$-\frac{1}{4} \sqrt{2 \sqrt{5}-2} \log \left(256 \left(\sqrt{5} \left(\sqrt{\tan (x)^2+2 \tan (x)+2}-\tan (x)\right)+\sqrt{5} \sqrt{\sqrt{5}-2}-\sqrt{5}-2 \sqrt{\tan (x)^2+2 \tan (x)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*tan(x)+tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2*sqrt(5)-2)*log(256*(sqrt(5)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))+sqrt(5)*sqrt(sqrt(5)-2)-sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)+2*sqrt(sqrt(5)-2)+2*tan(x)+2)-2*sqrt(sqrt(5)-2)+2*tan(x)+2)^2+256*(sqrt(5)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))+sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)+sqrt(sqrt(5)-2)+2*tan(x)+2)-tan(x)+sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)-tan(x))-sqrt(5)*sqrt(sqrt(5)-2)-sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)+2*sqrt(sqrt(5)-2)+2*tan(x)+2)^2+256*(sqrt(5)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))+sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)-tan(x))-sqrt(5)*sqrt(sqrt(5)-2)+2*tan(x)+2)^2+1/4*sqrt(2*sqrt(5)-2)*log(256*(sqrt(5)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))-sqrt(5)*sqrt(sqrt(5)-2)-sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)+2*sqrt(sqrt(5)-2)+2*tan(x)+2)-tan(x)+sqrt(5)-2*sqrt(tan(x)^2+2*tan(x)+2)-tan(x))-sqrt(sqrt(5)-2)+2*tan(x)-2)^2)+1/4*(pi+4*arctan(-1/2*(2*sqrt(5)*sqrt(sqrt(5)-2)+sqrt(5)+4*sqrt(sqrt(5)-2)+3)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))+3/2*sqrt(5)*sqrt(sqrt(5)-2)+1/2*sqrt(5)+7/2*sqrt(sqrt(5)-2)+3/2))*sqrt(2*sqrt(5)-2)/(sqrt(5)-1)-1/4*(pi+4*arctan(1/2*(2*sqrt(5)*sqrt(sqrt(5)-2)-sqrt(5)+4*sqrt(sqrt(5)-2)-3)*(sqrt(tan(x)^2+2*tan(x)+2)-tan(x))-3/2*sqrt(5)*sqrt(sqrt(5)-2)+1/2*sqrt(5)-7/2*sqrt(sqrt(5)-2)+3/2))*sqrt(2*sqrt(5)-2)/(sqrt(5)-1)-log(sqrt(tan(x)^2+2*tan(x)+2)-tan(x)-1))
```

maple [B] time = 0.23, size = 1605, normalized size = 11.72

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+2*tan(x)+tan(x)^2)^(1/2),x)
[Out] arcsinh(tan(x)+1)+1/10*(10*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+1+2*5^(1/2))^2*5^(1/2)*(-5*arctan(1/80*(-22+10*5^(1/2)))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+2+5*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+tan(x))/(-1/2*5^(1/2)-1/2-tan(x))*5^(1/2)-5)/((-1/2*5^(1/2)+1/2+tan(x))^4/(-1/2*5^(1/2)-1/2-tan(x))^4+3*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+1)*(-10+10*5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)
```

$$\begin{aligned}
& -3 \arctan(1/80 * (-22+10*5^(1/2))^(1/2) * ((5-5^(1/2)) * (2 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x))^2 + 5^(1/2)+3))^(1/2) * (11*5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x))^2 + 25 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x))^2 + 4 * 5^(1/2)+10) * (-1/2*5^(1/2)+1/2+tan(x)) / (-1/2*5^(1/2)-1/2-tan(x)) * (5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x)))^4 / (-1/2*5^(1/2)-1/2-tan(x)) ^4 + 3 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) * 5^(1/2) * (-10+10*5^(1/2))^(1/2) * (-22+10*5^(1/2))^(1/2) + 20 * \operatorname{arctanh}((10 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 2 * 5^(1/2) * (-1/2*5^(1/2)) * (+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 10 + 2 * 5^(1/2))^(1/2) / (-10+10*5^(1/2))^(1/2) * 5^(1/2) - 60 * \operatorname{arctanh}((10 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 2 * 5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 10 + 2 * 5^(1/2))^(1/2) / (-10+10*5^(1/2))^(1/2) / (-2 * (5^(1/2) * (-1/2*5^(1/2)) * (+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 5 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 - 5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x))) / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) ^2 / ((-1/2*5^(1/2)+1/2+tan(x))) / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) / (5^(1/2)-5) / (-10+10*5^(1/2))^(1/2) + 1/5 * (10 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 2 * 5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 - 5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x))) / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 10 + 2 * 5^(1/2))^(1/2) / (-10+10*5^(1/2))^(1/2) * (-arctan(1/80 * (-22+10*5^(1/2))^(1/2) * ((5-5^(1/2)) * (2 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 + 5^(1/2)+3))^(1/2) * (11*5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x))^2 + 25 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) * (5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x))) ^4 / (-1/2*5^(1/2)-1/2-tan(x)) ^4 + 3 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) * 5^(1/2) * (-10+10*5^(1/2))^(1/2) * (-22+10*5^(1/2))^(1/2) * ((5-5^(1/2)) * (2 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 + 5^(1/2)+3))^(1/2) * (11*5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 + 25 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) * (5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x))) ^4 / (-1/2*5^(1/2)-1/2-tan(x)) ^4 + 3 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) * (-10+10*5^(1/2))^(1/2) * (-22+10*5^(1/2))^(1/2) + 20 * \operatorname{arctanh}((10 * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 2 * 5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 10 + 2 * 5^(1/2))^(1/2) / (-10+10*5^(1/2))^(1/2) / (-2 * (5^(1/2) * (-1/2*5^(1/2)+1/2+tan(x)))^2 / (-1/2*5^(1/2)-1/2-tan(x)))^2 - 5 * (-1/2*5^(1/2)+1/2+tan(x))^2 / (-1/2*5^(1/2)-1/2-tan(x)) ^2 - 5^(1/2)-5) / ((-1/2*5^(1/2)+1/2+tan(x))) / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) ^2 / ((-1/2*5^(1/2)+1/2+tan(x))) / (-1/2*5^(1/2)-1/2-tan(x)) ^2 + 1) / (5^(1/2)-5) / (-10+10*5^(1/2))^(1/2)
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+2*tan(x)+tan(x)^2)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan(x)^2 + 2 \tan(x) + 2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*tan(x) + tan(x)^2 + 2)^(1/2),x)`

[Out] `int((2*tan(x) + tan(x)^2 + 2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^2(x) + 2 \tan(x) + 2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+2*tan(x)+tan(x)**2)**(1/2),x)`

[Out] `Integral(sqrt(tan(x)**2 + 2*tan(x) + 2), x)`

3.9 $\int \tan^{-1}(\sqrt{-1 + \sec(x)}) \sin(x) dx$

Optimal. Leaf size=41

$$\frac{1}{2} \cos(x) \sqrt{\sec(x) - 1} + \frac{1}{2} \tan^{-1}(\sqrt{\sec(x) - 1}) - \cos(x) \tan^{-1}(\sqrt{\sec(x) - 1})$$

[Out] $1/2*\arctan((-1+\sec(x))^{(1/2)})-\arctan((-1+\sec(x))^{(1/2)})*\cos(x)+1/2*\cos(x)*(-1+\sec(x))^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4335, 5203, 12, 242, 51, 63, 203}

$$\frac{1}{2} \cos(x) \sqrt{\sec(x) - 1} + \frac{1}{2} \tan^{-1}(\sqrt{\sec(x) - 1}) - \cos(x) \tan^{-1}(\sqrt{\sec(x) - 1})$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{ArcTan}[\text{Sqrt}[-1 + \text{Sec}[x]]]*\text{Sin}[x], x]$

[Out] $\text{ArcTan}[\text{Sqrt}[-1 + \text{Sec}[x]]]/2 - \text{ArcTan}[\text{Sqrt}[-1 + \text{Sec}[x]]]*\text{Cos}[x] + (\text{Cos}[x]*\text{Sqrt}[-1 + \text{Sec}[x]])/2$

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_.))^(m_)*((c_.) + (d_.)*(x_.))^n_, x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_.))^(m_)*((c_.) + (d_.)*(x_.))^n_, x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 4335

```
Int[(u_)*(F_)[(c_)*(a_.) + (b_.)*(x_)], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]/d, u, x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \sin(x) dx &= -\text{Subst} \left(\int \tan^{-1} \left(\sqrt{-1 + \frac{1}{x}} \right) dx, x, \cos(x) \right) \\
&= -\tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) + \text{Subst} \left(\int -\frac{1}{2\sqrt{-1 + \frac{1}{x}}} dx, x, \cos(x) \right) \\
&= -\tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + \frac{1}{x}}} dx, x, \cos(x) \right) \\
&= -\tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + x^2}} dx, x, \sec(x) \right) \\
&= -\tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) + \frac{1}{2} \cos(x)\sqrt{-1 + \sec(x)} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + x^2}} dx, x, \sec(x) \right) \\
&= -\tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) + \frac{1}{2} \cos(x)\sqrt{-1 + \sec(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sec(x) \right) \\
&= \frac{1}{2} \tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) - \tan^{-1} \left(\sqrt{-1 + \sec(x)} \right) \cos(x) + \frac{1}{2} \cos(x)\sqrt{-1 + \sec(x)}
\end{aligned}$$

Mathematica [C] time = 4.38, size = 283, normalized size = 6.90

$$-\frac{1}{2} (-3 - 2\sqrt{2}) \left((\sqrt{2} - 2) \cos\left(\frac{x}{2}\right) - \sqrt{2} + 1 \right) \cos^2\left(\frac{x}{4}\right) \sqrt{-\tan^2\left(\frac{x}{4}\right) - 2\sqrt{2} + 3} \sqrt{(2\sqrt{2} - 3) \tan^2\left(\frac{x}{4}\right) + 1} \cot\left(\frac{x}{4}\right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[ArcTan[Sqrt[-1 + Sec[x]]]*Sin[x], x]`

[Out] `-(ArcTan[Sqrt[-1 + Sec[x]]]*Cos[x]) + (Cos[x]*Sqrt[-1 + Sec[x]])/2 - ((-3 - 2*Sqrt[2])*Cos[x/4]^2*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[x/2])*Cot[x/4]*(EllipticF[ArcSin[Tan[x/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[x/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[x/2])*Sec[x/4]^2]*Sqrt[(-1 - Sqr[t[2] + (2 + Sqrt[2])*Cos[x/2])*Sec[x/4]^2]*Sqrt[-1 + Sec[x]]*Sec[x]*Sqrt[3 - 2*Sqrt[2] - Tan[x/4]^2]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[x/4]^2])/2`

fricas [A] time = 0.48, size = 32, normalized size = 0.78

$$-\frac{1}{2} (2 \cos(x) - 1) \arctan \left(\sqrt{\sec(x) - 1} \right) + \frac{1}{2} \sqrt{-\frac{\cos(x) - 1}{\cos(x)}} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="fricas")
[Out] -1/2*(2*cos(x) - 1)*arctan(sqrt(sec(x) - 1)) + 1/2*sqrt(-(cos(x) - 1)/cos(x))*cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{undef}$$

```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="giac")
[Out] undef

maple [A] time = 0.07, size = 42, normalized size = 1.02
```

$$\frac{\arctan\left(\sqrt{\sec(x)-1}\right)}{2} - \frac{\arctan\left(\sqrt{-\left(\frac{1}{\sec(x)}-1\right)\sec(x)}\right)}{\sec(x)} + \frac{\sqrt{\sec(x)-1}}{2\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan((-1+sec(x))^(1/2))*sin(x),x)
[Out] -1/sec(x)*arctan((-1/sec(x)-1)*sec(x))^(1/2)+1/2*(-1+sec(x))^(1/2)/sec(x)
+1/2*arctan((-1+sec(x))^(1/2))

maxima [A] time = 1.01, size = 60, normalized size = 1.46
```

$$-\arctan\left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\right)\cos(x) - \frac{\sqrt{-\frac{\cos(x)-1}{\cos(x)}}}{2\left(\frac{\cos(x)-1}{\cos(x)}-1\right)} + \frac{1}{2}\arctan\left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="maxima")
[Out] -arctan(sqrt(-(cos(x) - 1)/cos(x)))*cos(x) - 1/2*sqrt(-(cos(x) - 1)/cos(x))
/((cos(x) - 1)/cos(x) - 1) + 1/2*arctan(sqrt(-(cos(x) - 1)/cos(x)))

mupad [B] time = 0.42, size = 60, normalized size = 1.46
```

$$-\text{atan}\left(\sqrt{\frac{1}{\cos(x)}-1}\right)\cos(x) - \frac{\cos(x)\left(\frac{3 \arcsin(\sqrt{\cos(x)})}{2 \cos(x)^{3/2}} - \frac{3 \sqrt{1-\cos(x)}}{2 \cos(x)}\right)\sqrt{1-\cos(x)}}{3\sqrt{\frac{1}{\cos(x)}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan((1/cos(x) - 1)^(1/2))*sin(x),x)`

[Out] $-\text{atan}((1/\cos(x) - 1)^{(1/2)})\cos(x) - (\cos(x)*((3*\text{asin}(\cos(x)^{(1/2)})))/(2*\cos(x)^{(3/2)}) - (3*(1 - \cos(x))^{(1/2)})/(2*\cos(x)))*(1 - \cos(x))^{(1/2)}/(3*(1/\cos(x) - 1)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \text{atan}\left(\sqrt{\sec(x)-1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan((-1+sec(x))**(1/2))*sin(x),x)`

[Out] `Integral(sin(x)*atan(sqrt(sec(x) - 1)), x)`

3.10 $\int \frac{e^{\sin^{-1}(x)} x^3}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=44

$$\frac{1}{10} \left(x^3 - 3\sqrt{1-x^2} x^2 - 3\sqrt{1-x^2} + 3x \right) e^{\sin^{-1}(x)}$$

[Out] $1/10 * \exp(\arcsin(x)) * (3*x + x^3 - 3*(-x^2 + 1)^{(1/2)} - 3*x^2 * (-x^2 + 1)^{(1/2)})$

Rubi [A] time = 0.68, antiderivative size = 62, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4836, 6741, 6720, 4434, 4432}

$$\frac{1}{10} x^3 e^{\sin^{-1}(x)} - \frac{3}{10} \sqrt{1-x^2} x^2 e^{\sin^{-1}(x)} - \frac{3}{10} \sqrt{1-x^2} e^{\sin^{-1}(x)} + \frac{3}{10} x e^{\sin^{-1}(x)}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcSin[x]*x^3)/Sqrt[1 - x^2], x]

[Out] $(3*E^{\text{ArcSin}[x]}*x)/10 + (E^{\text{ArcSin}[x]}*x^3)/10 - (3*E^{\text{ArcSin}[x]}*\text{Sqrt}[1 - x^2])/10 - (3*E^{\text{ArcSin}[x]}*x^2*\text{Sqrt}[1 - x^2])/10$

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
  Simp[(b*c*Log[F]*F^(c*(a + b*x)))*Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x] -
  Simp[(e*F^(c*(a + b*x)))*Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x] /; F
  FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :>
  Simp[(b*c*Log[F]*F^(c*(a + b*x)))*Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))]*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x)))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4836

```
Int[(u_)*(f_)^(ArcSin[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :>
  Dist[1/b, Subst[Int[(u /. x -> -(a/b) + Sin[x]/b)*f^(c*x^n)*Cos[x], x], x, ArcSi
n[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\sin^{-1}(x)} x^3}{\sqrt{1-x^2}} dx &= \text{Subst}\left(\int \frac{e^x \cos(x) \sin^3(x)}{\sqrt{1-\sin^2(x)}} dx, x, \sin^{-1}(x)\right) \\ &= \text{Subst}\left(\int \frac{e^x \cos(x) \sin^3(x)}{\sqrt{\cos^2(x)}} dx, x, \sin^{-1}(x)\right) \\ &= 1 \text{Subst}\left(\int e^x \sin^3(x) dx, x, \sin^{-1}(x)\right) \\ &= \frac{1}{10} e^{\sin^{-1}(x)} x^3 - \frac{3}{10} e^{\sin^{-1}(x)} x^2 \sqrt{1-x^2} + \frac{3}{5} \text{Subst}\left(\int e^x \sin(x) dx, x, \sin^{-1}(x)\right) \\ &= \frac{3}{10} e^{\sin^{-1}(x)} x + \frac{1}{10} e^{\sin^{-1}(x)} x^3 - \frac{3}{10} e^{\sin^{-1}(x)} \sqrt{1-x^2} - \frac{3}{10} e^{\sin^{-1}(x)} x^2 \sqrt{1-x^2} \end{aligned}$$

Mathematica [A] time = 0.17, size = 38, normalized size = 0.86

$$-\frac{1}{40} e^{\sin^{-1}(x)} \left(15 \left(\sqrt{1-x^2}-x\right)+\sin \left(3 \sin ^{-1}(x)\right)-3 \cos \left(3 \sin ^{-1}(x)\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(E^ArcSin[x]*x^3)/Sqrt[1 - x^2], x]`

[Out] `-1/40*(E^ArcSin[x]*(15*(-x + Sqrt[1 - x^2])) - 3*Cos[3*ArcSin[x]] + Sin[3*ArcSin[x]])`

fricas [A] time = 0.47, size = 28, normalized size = 0.64

$$\frac{1}{10} \left(x^3 - 3(x^2 + 1)\sqrt{-x^2 + 1} + 3x\right) e^{\arcsin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="fricas")`
[Out] $\frac{1}{10}(x^3 - 3x^2 + 1)\sqrt{-x^2 + 1} + 3x\sqrt{-x^2 + 1}$

giac [A] time = 1.47, size = 46, normalized size = 1.05

$$\frac{1}{10}(x^2 - 1)x e^{\arcsin(x)} + \frac{3}{10}(-x^2 + 1)^{\frac{3}{2}} e^{\arcsin(x)} + \frac{2}{5}x e^{\arcsin(x)} - \frac{3}{5}\sqrt{-x^2 + 1} e^{\arcsin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="giac")`
[Out] $\frac{1}{10}(x^2 - 1)x e^{\arcsin(x)} + \frac{3}{10}(-x^2 + 1)^{\frac{3}{2}} e^{\arcsin(x)} + \frac{2}{5}x e^{\arcsin(x)} - \frac{3}{5}\sqrt{-x^2 + 1} e^{\arcsin(x)}$

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x)`
[Out] `int(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="maxima")`
[Out] `integrate(x^3 * e^{\arcsin(x)}) / \sqrt{-x^2 + 1}, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{1 - x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 * exp(asin(x))) / (1 - x^2)^(1/2),x)`
[Out] `int((x^3 * exp(asin(x))) / (1 - x^2)^(1/2), x)`

sympy [A] time = 2.17, size = 56, normalized size = 1.27

$$\frac{x^3 e^{\arcsin(x)}}{10} - \frac{3x^2 \sqrt{1-x^2} e^{\arcsin(x)}}{10} + \frac{3x e^{\arcsin(x)}}{10} - \frac{3\sqrt{1-x^2} e^{\arcsin(x)}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(asin(x))*x**3/(-x**2+1)**(1/2),x)

[Out] $x^{*3} \exp(\arcsin(x))/10 - 3*x^{*2} \sqrt{1 - x^{*2}} \exp(\arcsin(x))/10 + 3*x \exp(\arcsin(x))/10 - 3 \sqrt{1 - x^{*2}} \exp(\arcsin(x))/10$

3.11
$$\int \frac{x \log(1+x^2) \log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=68

$$x(-\log(x^2+1))+\sqrt{x^2+1} \log(x^2+1) \log(\sqrt{x^2+1}+x)-2\sqrt{x^2+1} \log(\sqrt{x^2+1}+x)+4x-2 \tan^{-1}(x)$$

[Out] $4*x - 2*\arctan(x) - x*\ln(x^2+1) - 2*\ln(x+(x^2+1)^{(1/2)})*(x^2+1)^{(1/2)} + \ln(x^2+1)*\ln(x+(x^2+1)^{(1/2)})*(x^2+1)^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {261, 2554, 8, 2557, 12, 2448, 321, 203}

$$x(-\log(x^2+1))+\sqrt{x^2+1} \log(x^2+1) \log(\sqrt{x^2+1}+x)-2\sqrt{x^2+1} \log(\sqrt{x^2+1}+x)+4x-2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*\text{Log}[1+x^2]*\text{Log}[x+\text{Sqrt}[1+x^2]])/\text{Sqrt}[1+x^2], x]$

[Out] $4*x - 2*\text{ArcTan}[x] - x*\text{Log}[1+x^2] - 2*\text{Sqrt}[1+x^2]*\text{Log}[x+\text{Sqrt}[1+x^2]] + \text{Sqrt}[1+x^2]*\text{Log}[1+x^2]*\text{Log}[x+\text{Sqrt}[1+x^2]]$

Rule 8

$\text{Int}[a_, x_\text{Symbol}] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 12

$\text{Int}[(a_)*(u_), x_\text{Symbol}] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&& \text{!MatchQ}[u, (b_)*(v_) /; \text{FreeQ}[b, x]]$

Rule 203

$\text{Int}[(a_) + (b_)*(x_)^2^{(-1)}, x_\text{Symbol}] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{PosQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

Rule 261

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_)*(x_)^{(n_.)})^{(p_)}, x_\text{Symbol}] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)/(b*n*(p+1))}, x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&& \text{EqQ}[m, n-1] \&& \text{NeQ}[p, -1]$

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_)*(d_) + (e_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x \log(1+x^2) \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx &= \sqrt{1+x^2} \log(1+x^2) \log\left(x+\sqrt{1+x^2}\right) - \int \log(1+x^2) dx - \int \frac{2x \log(1+x^2)}{\sqrt{1+x^2}} dx \\ &= -x \log(1+x^2) + \sqrt{1+x^2} \log(1+x^2) \log\left(x+\sqrt{1+x^2}\right) + 2 \int \frac{x^2}{1+x^2} dx \\ &= 2x - x \log(1+x^2) - 2\sqrt{1+x^2} \log\left(x+\sqrt{1+x^2}\right) + \sqrt{1+x^2} \log(1+x^2) \\ &= 4x - 2 \tan^{-1}(x) - x \log(1+x^2) - 2\sqrt{1+x^2} \log\left(x+\sqrt{1+x^2}\right) + \sqrt{1+x^2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.94

$$-2\sqrt{x^2+1} \log\left(\sqrt{x^2+1} + x\right) + \log(x^2+1) \left(\sqrt{x^2+1} \log\left(\sqrt{x^2+1} + x\right) - x\right) + 4x - 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 + x^2]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2], x]
[Out] 4*x - 2*ArcTan[x] - 2*Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]] + Log[1 + x^2]*(-x + Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]])
```

fricas [A] time = 0.44, size = 43, normalized size = 0.63

$$\sqrt{x^2+1} \left(\log(x^2+1) - 2 \right) \log\left(x + \sqrt{x^2+1}\right) - x \log(x^2+1) + 4x - 2 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="fricas")
[Out] sqrt(x^2 + 1)*(log(x^2 + 1) - 2)*log(x + sqrt(x^2 + 1)) - x*log(x^2 + 1) + 4*x - 2*arctan(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log(x^2+1) \log\left(x + \sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="giac")
[Out] integrate(x*log(x^2 + 1)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x \ln\left(x + \sqrt{x^2+1}\right) \ln(x^2+1)}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^2+1)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x)
[Out] int(x*ln(x^2+1)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2x^2 - (x^2 + 1)\log(x^2 + 1) + 2\right)\log\left(x + \sqrt{x^2 + 1}\right)}{\sqrt{x^2 + 1}} + \int \frac{\log(x^2 + 1) - 2}{x^2 + \sqrt{x^2 + 1}x} dx - \int -\frac{2x^2 - (x^2 + 1)\log(x^2 + 1)}{\sqrt{x^2 + 1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="maxima")`

[Out] $-(2*x^2 - (x^2 + 1)*\log(x^2 + 1) + 2)*\log(x + \sqrt{x^2 + 1})/\sqrt{x^2 + 1}$
 $+ \text{integrate}((\log(x^2 + 1) - 2)/(x^2 + \sqrt{x^2 + 1})*x, x) - \text{integrate}(-(2*x^2 - (x^2 + 1)*\log(x^2 + 1) + 2)/(\sqrt{x^2 + 1})*x, x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \ln(x^2 + 1) \ln(x + \sqrt{x^2 + 1})}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*log(x^2 + 1)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2), x)`

[Out] `int((x*log(x^2 + 1)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x**2+1)*ln(x+(x**2+1)**(1/2))/(x**2+1)**(1/2), x)`

[Out] Timed out

$$3.12 \quad \int \tan^{-1} \left(x + \sqrt{1 - x^2} \right) dx$$

Optimal. Leaf size=141

$$\frac{1}{4} \sqrt{3} \tan^{-1} \left(\frac{\sqrt{3}x - 1}{\sqrt{1-x^2}} \right) + \frac{1}{4} \sqrt{3} \tan^{-1} \left(\frac{\sqrt{3}x + 1}{\sqrt{1-x^2}} \right) - \frac{1}{4} \sqrt{3} \tan^{-1} \left(\frac{2x^2 - 1}{\sqrt{3}} \right) + x \tan^{-1} \left(\sqrt{1-x^2} + x \right) - \frac{1}{4} \tanh^{-1} \left(x \sqrt{1-x^2} \right)$$

[Out] $-1/2*\arcsin(x)+x*\arctan(x+(-x^2+1)^(1/2))-1/4*\operatorname{arctanh}(x*(-x^2+1)^(1/2))-1/8*\ln(x^4-x^2+1)-1/4*\arctan(1/3*(2*x^2-1)*3^(1/2))*3^(1/2)+1/4*\arctan((-1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(1/2)+1/4*\arctan((1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(1/2)$

Rubi [C] time = 0.75, antiderivative size = 269, normalized size of antiderivative = 1.91, number of steps used = 40, number of rules used = 15, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5203, 12, 6742, 216, 1114, 634, 618, 204, 628, 1174, 402, 377, 205, 1293, 1107}

$$-\frac{1}{8} \log \left(x^4 - x^2 + 1\right) + \frac{1}{4} \sqrt{3} \tan^{-1} \left(\frac{1 - 2x^2}{\sqrt{3}}\right) + \frac{1}{12} (-\sqrt{3} + 3i) \tan^{-1} \left(\frac{x}{\sqrt{-\frac{-\sqrt{3} + i}{\sqrt{3} + i}} \sqrt{1 - x^2}}\right) + \frac{\tan^{-1} \left(\frac{x}{\sqrt{-\frac{-\sqrt{3} + i}{\sqrt{3} + i}} \sqrt{1 - x^2}}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

[In] Int[ArcTan[x + Sqrt[1 - x^2]], x]

[Out] $-\operatorname{ArcSin}[x]/2 + (\operatorname{Sqrt}[3]*\operatorname{ArcTan}[(1 - 2*x^2)/\operatorname{Sqrt}[3]])/4 + \operatorname{ArcTan}[x/(\operatorname{Sqrt}[-((I - \operatorname{Sqrt}[3])/(I + \operatorname{Sqrt}[3]))]*\operatorname{Sqrt}[1 - x^2])]/\operatorname{Sqrt}[3] + ((3*I - \operatorname{Sqrt}[3])* \operatorname{ArcTan}[x/(\operatorname{Sqrt}[-((I - \operatorname{Sqrt}[3])/(I + \operatorname{Sqrt}[3]))]*\operatorname{Sqrt}[1 - x^2])])/12 + \operatorname{ArcTan}[(\operatorname{Sqrt}[-((I - \operatorname{Sqrt}[3])/(I + \operatorname{Sqrt}[3]))]*x)/\operatorname{Sqrt}[1 - x^2]]/\operatorname{Sqrt}[3] - ((3*I + \operatorname{Sqr}t[3])* \operatorname{ArcTan}[(\operatorname{Sqrt}[-((I - \operatorname{Sqrt}[3])/(I + \operatorname{Sqrt}[3]))]*x)/\operatorname{Sqrt}[1 - x^2]])/12 + x*\operatorname{ArcTan}[x + \operatorname{Sqrt}[1 - x^2]] - \operatorname{Log}[1 - x^2 + x^4]/8$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr[t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_)*(x_)^2)^(p_.)/((c_) + (d_)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dis-
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free-
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1174

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symb-
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x],
x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rule 1293

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 +
(c_)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1))*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1]
&& LeQ[m, 3]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1} \left(x + \sqrt{1-x^2} \right) dx &= x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \int \frac{x \left(1 - \frac{x}{\sqrt{1-x^2}} \right)}{2 \left(1 + x \sqrt{1-x^2} \right)} dx \\
&= x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{2} \int \frac{x \left(1 - \frac{x}{\sqrt{1-x^2}} \right)}{1 + x \sqrt{1-x^2}} dx \\
&= x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{2} \int \left(\frac{x^2}{-x + x^3 - \sqrt{1-x^2}} + \frac{x}{1 + x \sqrt{1-x^2}} \right) dx \\
&= x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{2} \int \frac{x^2}{-x + x^3 - \sqrt{1-x^2}} dx - \frac{1}{2} \int \frac{x}{1 + x \sqrt{1-x^2}} dx \\
&= x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{2} \int \left(\frac{x}{1-x^2+x^4} - \frac{x^2 \sqrt{1-x^2}}{1-x^2+x^4} \right) dx - \frac{1}{2} \int \left(-\frac{1}{\sqrt{1-x^2}} + \right. \\
&\quad \left. x \tan^{-1} \left(x + \sqrt{1-x^2} \right) + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx - \frac{1}{2} \int \frac{x}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{x^3}{1-x^2+x^4} \right. \\
&\quad \left. = \frac{1}{2} \sin^{-1}(x) + x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) - \frac{1}{4} \text{Subst} \left(\right. \right. \\
&\quad \left. \left. = \frac{1}{2} \sin^{-1}(x) + x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) - \frac{1}{8} \text{Subst} \left(\right. \right. \\
&\quad \left. \left. = \frac{1}{2} \sin^{-1}(x) + \frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{2\sqrt{3}} + x \tan^{-1} \left(x + \sqrt{1-x^2} \right) - \frac{1}{8} \log(1-x^2+x^4) + \frac{1}{4} \text{Subst} \left(\right. \right. \\
&\quad \left. \left. = \frac{1}{2} \sin^{-1}(x) + \frac{1}{4}\sqrt{3} \tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right) + \frac{1}{12} (3i - \sqrt{3}) \tan^{-1} \left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}} \sqrt{1-x^2}}} \right) - \frac{1}{12} \right)
\end{aligned}$$

Mathematica [C] time = 3.80, size = 1822, normalized size = 12.92

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[x + Sqrt[1 - x^2]], x]

[Out] $x \operatorname{ArcTan}[x + \sqrt{1 - x^2}] + \frac{(-8 \operatorname{ArcSin}[x] + 2 \sqrt{2 + (2 I) \sqrt{3}}) \operatorname{ArcTan}[((1 + I \sqrt{3} - 2 x^2) (-1 + x^2)) / (-3 I - \sqrt{3} + 2 \sqrt{3}) x^4 + x^3 (-6 - (2 I) \sqrt{3} - 2 \sqrt{2 - (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + x (6 + (2 I) \sqrt{3} - 2 \sqrt{2 - (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + x^2 (3 I - \sqrt{3} + 2 \sqrt{6 - (6 I) \sqrt{3}}) \sqrt{1 - x^2})] - 2 \sqrt{2 + (2 I) \sqrt{3}} \operatorname{ArcTan}[(1 + I \sqrt{3} - 2 x^2) (-1 + x^2)) / (-3 I - \sqrt{3} + 2 \sqrt{3}) x^4 + 2 x (-3 - I \sqrt{3} + \sqrt{2 - (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + 2 x^3 (3 + I \sqrt{3} + \sqrt{2 - (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + x^2 (3 I - \sqrt{3} + 2 \sqrt{6 - (6 I) \sqrt{3}}) \sqrt{1 - x^2})] - 2 \sqrt{2 - (2 I) \sqrt{3}} A \operatorname{rcTan}((-1 + x^2) (-1 + I \sqrt{3} + 2 x^2)) / (3 I - \sqrt{3} + 2 \sqrt{3}) x^4 + x (6 - (2 I) \sqrt{3} - 2 \sqrt{2 + (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + x^3 (-6 + (2 I) \sqrt{3} - 2 \sqrt{2 + (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + x^2 (-3 I - \sqrt{3} + 2 \sqrt{6 + (6 I) \sqrt{3}}) \sqrt{1 - x^2})] + 2 \sqrt{2 - (2 I) \sqrt{3}} \operatorname{ArcTan}((-1 + x^2) (-1 + I \sqrt{3} + 2 x^2)) / (3 I - \sqrt{3} + 2 \sqrt{3}) x^4 + 2 x^3 (3 - I \sqrt{3} + \sqrt{2 + (2 I) \sqrt{3}}) \sqrt{1 - x^2}) + 2 x^2 (-3 I - \sqrt{3} + 2 \sqrt{6 + (6 I) \sqrt{3}}) \sqrt{1 - x^2})] - 2 \operatorname{Log}[-1/2 - (I/2) \sqrt{3} + x^2] + (2 I) \sqrt{3} \operatorname{Log}[-1/2 - (I/2) \sqrt{3} + x^2] - 2 \operatorname{Log}[(I/2) (I + \sqrt{3}) + x^2] - (2 I) \sqrt{3} \operatorname{Log}[(I/2) (I + \sqrt{3}) + x^2] - I \sqrt{2 - (2 I) \sqrt{3}} \operatorname{Log}[16 (1 + \sqrt{3}) x + x^2]^2 + I \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Log}[16 (1 + \sqrt{3}) x + x^2]^2 + I \sqrt{2 - (2 I) \sqrt{3}} \operatorname{Log}[(4 - 4 \sqrt{3}) x + 4 x^2]^2 - I \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Log}[3 I + \sqrt{3} - (-I + \sqrt{3}) x^4 + (2 I) \sqrt{3} \operatorname{Log}[2 - (2 I) \sqrt{3}] \operatorname{Log}[(4 - 4 \sqrt{3}) x + 4 x^2]^2 - I \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Log}[3 I + \sqrt{3} - (-I + \sqrt{3}) x^4 + (2 I) \sqrt{3} \operatorname{Log}[2 - (2 I) \sqrt{3}] \operatorname{Log}[1 - x^2] + (5 I) x^2 (2 + \sqrt{2 - (2 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + x (3 + (5 I) \sqrt{3} + (3 I) \sqrt{6 - (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2] + I x^3 (3 I + 3 \sqrt{3} + \sqrt{6 - (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + I \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Log}[3 I + \sqrt{3} - (-I + \sqrt{3}) x^4 + (2 I) \sqrt{3} \operatorname{Sqrt}[2 - (2 I) \sqrt{3}] \operatorname{Sqrt}[1 - x^2] + (5 I) x^2 (2 + \sqrt{2 - (2 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + x^3 (3 - (3 I) \sqrt{3} - I \sqrt{6 - (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2] - I x^4 (3 I + 5 \sqrt{3} + 3 \sqrt{6 - (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + I \sqrt{2 - (2 I) \sqrt{3}} \operatorname{Log}[-3 I + \sqrt{3} - (I + \sqrt{3}) x^4 - (2 I) \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Sqrt}[1 - x^2] - (5 I) x^2 (2 + \sqrt{2 + (2 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + x (3 - (5 I) \sqrt{3} - (3 I) \sqrt{6 + (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2] - I x^3 (-3 I + 3 \sqrt{3} + \sqrt{6 + (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) - I \sqrt{2 - (2 I) \sqrt{3}} \operatorname{Log}[-3 I + \sqrt{3} - (I + \sqrt{3}) x^4 - (2 I) \sqrt{2 + (2 I) \sqrt{3}} \operatorname{Sqrt}[1 - x^2] - (5 I) x^2 (2 + \sqrt{2 + (2 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2]) + x^3 (3 + (3 I) \sqrt{3} + I \sqrt{6 + (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2] + I x^4 (3 I + 5 \sqrt{3} + 3 \sqrt{6 + (6 I) \sqrt{3}}) \operatorname{Sqrt}[1 - x^2])] / 16$

fricas [A] time = 0.49, size = 191, normalized size = 1.35

$$x \arctan\left(x + \sqrt{-x^2 + 1}\right) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2 - 1)\right) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} \sqrt{-x^2 + 1} x + \sqrt{3}}{3 (2 x^2 - 1)}\right) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} \sqrt{-x^2 + 1} x - \sqrt{3}}{3 (2 x^2 - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x+(-x^2+1)^(1/2)),x, algorithm="fricas")`

[Out] $x \arctan(x + \sqrt{-x^2 + 1}) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \sqrt{2x^2 - 1}\right)$
 $- \frac{1}{8} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{4} \sqrt{3} \sqrt{-x^2 + 1} x + \sqrt{3}}{(2x^2 - 1)}\right)$
 $) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{4} \sqrt{3} \sqrt{-x^2 + 1} x - \sqrt{3}}{(2x^2 - 1)}\right)$
 $+ \frac{1}{2} \arctan\left(\frac{\sqrt{-x^2 + 1} x}{(x^2 - 1)}\right) - \frac{1}{8} \log(x^4 - x^2 + 1) - \frac{1}{16} \log(-x^4 + x^2 + 2\sqrt{-x^2 + 1} x + 1) + \frac{1}{16} \log(-x^4 + x^2 - 2\sqrt{-x^2 + 1} x + 1)$

giac [B] time = 1.47, size = 364, normalized size = 2.58

$$x \arctan\left(x + \sqrt{-x^2 + 1}\right) - \frac{1}{4} \pi \operatorname{sgn}(x) + \frac{1}{8} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan\left(-\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2 + 1} - 1}{x} + \frac{(\sqrt{-x^2 + 1} - 1)^2}{x^2} - 1 \right)}{3 (\sqrt{-x^2 + 1} - 1)} \right) \right) + \frac{1}{8} \sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x+(-x^2+1)^(1/2)),x, algorithm="giac")`

[Out] $x \arctan(x + \sqrt{-x^2 + 1}) - \frac{1}{4} \pi \operatorname{sgn}(x) + \frac{1}{8} \sqrt{3} (\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{1}{3} \sqrt{3} \sqrt{2x^2 - 1}\right))$
 $+ \frac{1}{8} \sqrt{3} (\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{1}{3} \sqrt{4} \sqrt{3} \sqrt{-x^2 + 1} x + \sqrt{3}}{(2x^2 - 1)}\right))$
 $+ \frac{1}{8} \sqrt{3} (\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{1}{3} \sqrt{4} \sqrt{3} \sqrt{-x^2 + 1} x - \sqrt{3}}{(2x^2 - 1)}\right))$
 $- \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \sqrt{2x^2 - 1}\right) - \frac{1}{2} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{-x^2 + 1} x\right) - \frac{1}{8} \log(x^4 - x^2 + 1) + \frac{1}{8} \log((x/\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)^2 + \frac{2x}{(\sqrt{-x^2 + 1} - 1) - 2(\sqrt{-x^2 + 1} - 1)/x + 4} - \frac{1}{8} \log((x/\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)^2 - \frac{2x}{(\sqrt{-x^2 + 1} - 1)/x + 4)$

maple [C] time = 0.10, size = 439, normalized size = 3.11

$$x \arctan\left(x + \sqrt{-x^2 + 1}\right) + \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right) - \frac{\sqrt{3} \arctan\left(\frac{(2x^2 - 1)\sqrt{3}}{3}\right)}{4} + \frac{i\sqrt{3} \ln\left(\frac{(-1 - i\sqrt{3})(\sqrt{-x^2 + 1} - 1)}{x} + \frac{(\sqrt{-x^2 + 1} - 1)^2}{8}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(x+(-x^2+1)^(1/2)),x)`

[Out] $x \arctan(x + (-x^2 + 1)^{(1/2)}) - 1/8 \ln(x^4 - x^2 + 1) - 1/4 \arctan(1/3 * (2*x^2 - 1) * 3^{(1/2)}) * 3^{(1/2)} + 1/8 * I * 3^{(1/2)} * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (-1 - I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 + 1/8 * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (-1 - I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 - 1/8 * I * 3^{(1/2)} * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (1 + I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 + 1/8 * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (-1 + I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 - 1/8 * I * 3^{(1/2)} * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (-1 + I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 - 1/8 * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (1 + I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 + 1/8 * I * 3^{(1/2)} * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (1 - I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 - 1/8 * \ln((-x^2 + 1)^{(1/2) - 1})^2 / x^2 + (1 - I * 3^{(1/2)}) * ((-x^2 + 1)^{(1/2) - 1}) / x - 1 + arctan((-x^2 + 1)^{(1/2) - 1}) / x$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \arctan\left(x + \sqrt{x+1} \sqrt{-x+1}\right) - \int \frac{x^3 + x^2 e^{\left(\frac{1}{2} \log(x+1) + \frac{1}{2} \log(-x+1)\right)} - x}{x^4 + (x^2 - 1)e^{\log(x+1) + \log(-x+1)} + 2(x^3 - x)e^{\left(\frac{1}{2} \log(x+1) + \frac{1}{2} \log(-x+1)\right)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x + (-x^2 + 1)^{(1/2)}), x, algorithm="maxima")`

[Out] $x \arctan(x + \sqrt{x+1} \sqrt{-x+1}) - \text{integrate}((x^3 + x^2 * e^{(1/2 * \log(x+1) + 1/2 * \log(-x+1))} - x) / (x^4 + (x^2 - 1) * e^{(\log(x+1) + \log(-x+1))} + 2 * (x^3 - x) * e^{(1/2 * \log(x+1) + 1/2 * \log(-x+1))} - 1), x)$

mupad [B] time = 1.37, size = 661, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan(x + (1 - x^2)^{(1/2)}), x)`

[Out] $x * \text{atan}(x + (1 - x^2)^{(1/2)}) - \text{asin}(x)/2 + (\log(x - 3^{(1/2)}/2 - 1i/2) * (3^{(1/2)}/2 + (3^{(1/2)}/2 + 1i/2)^3 + 1i/2)) / (2 * 3^{(1/2)} - 8 * (3^{(1/2)}/2 + 1i/2)^3 + 2i) - (\log(x - 3^{(1/2)}/2 + 1i/2) * (3^{(1/2)}/2 + (3^{(1/2)}/2 - 1i/2)^3 - 1i/2)) / (8 * (3^{(1/2)}/2 - 1i/2)^3 - 2 * 3^{(1/2)} + 2i) - (\log(x + 3^{(1/2)}/2 - 1i/2) * (3^{(1/2)}/2 + (3^{(1/2)}/2 - 1i/2)^3 - 1i/2)) / (8 * (3^{(1/2)}/2 - 1i/2)^3 - 2 * 3^{(1/2)} + 2i) + (\log(x + 3^{(1/2)}/2 + 1i/2) * (3^{(1/2)}/2 + (3^{(1/2)}/2 + 1i/2)^3 + 1i/2)) / (2 * 3^{(1/2)} - 8 * (3^{(1/2)}/2 + 1i/2)^3 + 2i) + (\log(((x * (3^{(1/2)}/2 + 1i/2) - 1) * 1i) / (1 - (3^{(1/2)}/2 + 1i/2)^2)^{(1/2)} - (1 - x^2)^{(1/2)} * 1i) / (3^{(1/2)}/2 - x + 1i/2)) * ((3^{(1/2)}/2 + 1i/2)^2 + 1) / ((1 - (3^{(1/2)}/2 + 1i/2)^2)^{(1/2)} * (2 * 3^{(1/2)} - 8 * (3^{(1/2)}/2 + 1i/2)^3 + 2i)) - (\log(((x * (3^{(1/2)}/2 - 1i/2) - 1) * 1i) / (1 - (3^{(1/2)}/2 - 1i/2)^2)^{(1/2)} - (1 - x^2)^{(1/2)} * 1i) / (x - 3^{(1/2)}/2 + 1i/2)) * ((3^{(1/2)}/2 - 1i/2)^2 + 1) / ((1 - (3^{(1/2)}/2 - 1i/2)^2)^{(1/2)} * (8 * (3^{(1/2)}/2 - 1i/2)^3 - 2 * 3^{(1/2)} + 2i)) + (\log(((x * (3^{(1/2)}/2 - 1i/2) - 1) * 1i) / (x - 3^{(1/2)}/2 - 1i/2)) + (log(((x * (3^{(1/2)}/2 - 1i/2) - 1) * 1i) / (x - 3^{(1/2)}/2 - 1i/2)))$

```
+ 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)
)/2 - 1i/2))*((3^(1/2)/2 - 1i/2)^2 + 1))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*
(8*(3^(1/2)/2 - 1i/2)^3 - 2*3^(1/2) + 2i)) - (log(((x*(3^(1/2)/2 + 1i/2) +
1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)
/2 + 1i/2))*((3^(1/2)/2 + 1i/2)^2 + 1))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(
2*3^(1/2) - 8*(3^(1/2)/2 + 1i/2)^3 + 2i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(x+(-x**2+1)**(1/2)),x)

[Out] Timed out

$$3.13 \quad \int \frac{x \tan^{-1}(x + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=152

$$\frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} x - 1}{\sqrt{1 - x^2}}\right) + \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} x + 1}{\sqrt{1 - x^2}}\right) - \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) - \sqrt{1 - x^2} \tan^{-1}\left(\sqrt{1 - x^2} + x\right) + \frac{1}{4} \tanh^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$$

[Out] $-1/2*\arcsin(x)+1/4*\operatorname{arctanh}(x*(-x^2+1)^(1/2))+1/8*\ln(x^4-x^2+1)-1/4*\arctan(1/3*(2*x^2-1)*3^(1/2))*3^(1/2)+1/4*\arctan((-1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(1/2)+1/4*\arctan((1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(1/2)-\arctan(x+(-x^2+1)^(1/2))*(-x^2+1)^(1/2)$

Rubi [C] time = 0.46, antiderivative size = 286, normalized size of antiderivative = 1.88, number of steps used = 32, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {261, 5207, 12, 6742, 1107, 618, 204, 1293, 216, 1174, 377, 205, 402, 1247, 634, 628}

$$\frac{1}{8} \log(x^4 - x^2 + 1) + \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1 - 2x^2}{\sqrt{3}}\right) - \frac{1}{12} (-\sqrt{3} + 3i) \tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3} + i}{\sqrt{3} + i} \sqrt{1 - x^2}}}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3} + i}{\sqrt{3} + i} \sqrt{1 - x^2}}}\right)}{2\sqrt{3}}$$

Warning: Unable to verify antiderivative.

[In] $\text{Int}[(x*\text{ArcTan}[x + \text{Sqrt}[1 - x^2]])/\text{Sqrt}[1 - x^2], x]$

[Out] $-\text{ArcSin}[x]/2 + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*x^2)/\text{Sqrt}[3]])/4 + \text{ArcTan}[x/(\text{Sqrt}[-((I - \text{Sqrt}[3])/(I + \text{Sqrt}[3]))]*\text{Sqrt}[1 - x^2])]/(2*\text{Sqrt}[3]) - ((3*I - \text{Sqrt}[3])* \text{ArcTan}[x/(\text{Sqrt}[-((I - \text{Sqrt}[3])/(I + \text{Sqrt}[3]))]*\text{Sqrt}[1 - x^2])])/12 + \text{ArcTan}[(\text{Sqrt}[-((I - \text{Sqrt}[3])/(I + \text{Sqrt}[3]))]*x)/\text{Sqrt}[1 - x^2]]/(2*\text{Sqrt}[3]) + ((3*I + \text{Sqrt}[3])* \text{ArcTan}[(\text{Sqrt}[-((I - \text{Sqrt}[3])/(I + \text{Sqrt}[3]))]*x)/\text{Sqrt}[1 - x^2]])/12 - \text{Sqrt}[1 - x^2]*\text{ArcTan}[x + \text{Sqrt}[1 - x^2]] + \text{Log}[1 - x^2 + x^4]/8$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&& \text{!MatchQ}[u, (b_)*(v_) /; \text{FreeQ}[b, x]]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{PosQ}[a/b] \&& (\text{LtQ}[\text{Rt}[-b, 2], x] \&& \text{Rt}[-a, 2] < 0) \&& \text{Rt}[-b, 2] \neq 0 \&& \text{Rt}[-a, 2] \neq 0 \&& \text{Rt}[-b, 2] \neq \text{Rt}[-a, 2])$

a, 0] || LtQ[b, 0])

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr[t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_)*(x_)^2)^(p_)/((c_) + (d_)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1174

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b - r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1293

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1), x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1))*Simp[a *e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 5207

```
Int[((a_) + ArcTan[u_]*(b_))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1 + u^2), x], x]] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_) + (d_)*x)^(m_)] /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \tan^{-1}(x + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx &= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \int \frac{x - \sqrt{1-x^2}}{2(1+x\sqrt{1-x^2})} dx \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{2} \int \frac{x - \sqrt{1-x^2}}{1+x\sqrt{1-x^2}} dx \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{2} \int \left(\frac{x}{1+x\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{1+x\sqrt{1-x^2}} \right) dx \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{2} \int \frac{x}{1+x\sqrt{1-x^2}} dx + \frac{1}{2} \int \frac{\sqrt{1-x^2}}{1+x\sqrt{1-x^2}} dx \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{2} \int \left(\frac{x}{1-x^2+x^4} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4} \right) dx + \frac{1}{2} \int \left(\frac{\sqrt{1-x^2}}{1-x^2+x^4} \right) dx \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{2} \int \frac{x}{1-x^2+x^4} dx + \frac{1}{2} \int \frac{\sqrt{1-x^2}}{1-x^2+x^4} dx + \frac{1}{2} \int \dots \\
&= -\sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, x^2\right) - \frac{1}{4} \text{Subst}\left(\int \dots\right) \\
&= -\frac{1}{2} \sin^{-1}(x) - \sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) - \frac{1}{8} \text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, x^2\right) + \\
&= -\frac{1}{2} \sin^{-1}(x) + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \sqrt{1-x^2} \tan^{-1}(x + \sqrt{1-x^2}) + \frac{1}{8} \log(1-x^2+x^4) \\
&= -\frac{1}{2} \sin^{-1}(x) + \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}} \sqrt{1-x^2}}}\right)}{2\sqrt{3}} - \frac{1}{12} (3i - \sqrt{3}) \tan^{-1}(x + \sqrt{1-x^2})
\end{aligned}$$

Mathematica [C] time = 4.95, size = 2180, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(x*ArcTan[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2], x]

[Out]
$$\begin{aligned} & (-24 \operatorname{ArcSin}[x] - 48 \sqrt{1-x^2} \operatorname{ArcTan}[x+\sqrt{1-x^2}] + (2(-3 I + \sqrt{3}) \operatorname{ArcTan}[(3-I \sqrt{3})+(-3-I \sqrt{3}) x^4+2 x(-6 I+2 \sqrt{3})+I \sqrt{2-(2 I) \sqrt{3}} \sqrt{1-x^2}]-2 x^3(6 I+2 \sqrt{3}+I \sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2})/(I-\sqrt{3})+(6 I)(I+\sqrt{3}) x-2(-15 I+\sqrt{3}) x^2+6(1+(3 I) \sqrt{3}) x^3+(11 I+3 \sqrt{3}) x^4])/\sqrt{(1-I \sqrt{3})/6}-2(-3 I+\sqrt{3}) \operatorname{ArcTan}[(3-I \sqrt{3})+(-3-I \sqrt{3}) x^4+2 x^3(6 I+2 \sqrt{3}+I \sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2})+x(12 I-4 \sqrt{3}+(2 I) \sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2})-(2 I) \sqrt{3} x^2(6+\sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2})/(I-\sqrt{3})+(6-(6 I) \sqrt{3}) x-2(-15 I+\sqrt{3}) x^2+(-6-(18 I) \sqrt{3}) x^3+(11 I+3 \sqrt{3}) x^4])/\sqrt{(1-I \sqrt{3})/6}-2(3 I+\sqrt{3}) \operatorname{ArcTan}[(3-I \sqrt{3})+(3-I \sqrt{3}) x^4+2 x^3(-6 I+2 \sqrt{3}-I \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})-2 x(6 I+2 \sqrt{3}+I \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})-(2 I) \sqrt{3} x^2(6+\sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})/(-I-\sqrt{3})+(-6-(6 I) \sqrt{3}) x-2(15 I+\sqrt{3}) x^2+6(1-(3 I) \sqrt{3}) x^3+(-11 I+3 \sqrt{3}) x^4])/\sqrt{(1+I \sqrt{3})/6}+2(3 I+\sqrt{3}) \operatorname{ArcTan}[(3-I \sqrt{3})+(3-I \sqrt{3}) x^4+2 x(6 I+2 \sqrt{3}+I \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})+x^3(12 I-4 \sqrt{3}+(2 I) \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})-(2 I) \sqrt{3} x^2(6+\sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})/(-I-\sqrt{3})+(6+(6 I) \sqrt{3}) x-2(15 I+\sqrt{3}) x^2+(-6+(18 I) \sqrt{3}) x^3+(-11 I+3 \sqrt{3}) x^4])/\sqrt{(1+I \sqrt{3})/6}+2 \sqrt{3}(3 I+\sqrt{3}) \operatorname{Log}[-1/2-(I/2) \sqrt{3}+x^2]+2 \sqrt{3}(-3 I+\sqrt{3}) \operatorname{Log}[(I/2)(I+\sqrt{3})+x^2]+((3-I \sqrt{3}) \operatorname{Log}[16(1+\sqrt{3}) x+x^2])/\sqrt{(1+I \sqrt{3})/6}+((3+I \sqrt{3}) \operatorname{Log}[16(1+\sqrt{3}) x+x^2]^2)/\sqrt{(1-I \sqrt{3})/6}-(I(-3 I+\sqrt{3}) \operatorname{Log}[(4-4 \sqrt{3}) x+4 x^2])/\sqrt{(1-I \sqrt{3})/6}+(I(3 I+\sqrt{3}) \operatorname{Log}[(4-4 \sqrt{3}) x+4 x^2])/\sqrt{(1+I \sqrt{3})/6}-(I(-3 I+\sqrt{3}) \operatorname{Log}[3 I+\sqrt{3}]-(-I+\sqrt{3}) x^4+(2 I) \sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2}+(5 I) x^2(2+\sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2})+x(3+(5 I) \sqrt{3}+(3 I) \sqrt{6-(6 I) \sqrt{3}}) \sqrt{1-x^2})+I x^3(3 I+3 \sqrt{3}+\sqrt{6-(6 I) \sqrt{3}}) \sqrt{1-x^2})/\sqrt{(1-I \sqrt{3})/6}+((3+I \sqrt{3}) \operatorname{Log}[3 I+\sqrt{3}]-(-I+\sqrt{3}) x^4+(2 I) \sqrt{2-(2 I) \sqrt{3}}) \sqrt{1-x^2}+(5 I) x^2(2-(2 I) \sqrt{3}) \sqrt{1-x^2})+x^3(3-(3 I) \sqrt{3}-I \sqrt{6-(6 I) \sqrt{3}}) \sqrt{1-x^2})-I x(-3 I+5 \sqrt{3}+3 \sqrt{6-(6 I) \sqrt{3}}) \sqrt{1-x^2})/\sqrt{(1-I \sqrt{3})/6}+(I(3 I+\sqrt{3}) \operatorname{Log}[-3 I+\sqrt{3}]-(\mathrm{I}+\sqrt{3}) x^4-(2 I) \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2}-(5 I) x^2(2+\sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2})+x(3-(5 I) \sqrt{3}-(3 I) \sqrt{6+(6 I) \sqrt{3}}) \sqrt{1-x^2})-I x^3(-3 I+3 \sqrt{3}+\sqrt{6+(6 I) \sqrt{3}}) \sqrt{1-x^2})/\sqrt{(1+I \sqrt{3})/6}+((3-I \sqrt{3}) \operatorname{Log}[-3 I+\sqrt{3}]-(\mathrm{I}+\sqrt{3}) x^4-(2 I) \sqrt{2+(2 I) \sqrt{3}}) \sqrt{1-x^2}-(5 I) x^2(2+(2 I) \sqrt{3}) \sqrt{1-x^2}) \end{aligned}$$

```
+ Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))]/Sqrt[(1 + I*Sqrt[3])/6])/48
```

fricas [A] time = 0.50, size = 200, normalized size = 1.32

$$-\frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) - \sqrt{-x^2 + 1} \arctan\left(x + \sqrt{-x^2 + 1}\right) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} \sqrt{-x^2 + 1} x + \sqrt{3}}{3 (2x^2 - 1)}\right) - \frac{1}{8} \sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - sqrt(-x^2 + 1)*arctan(x + sqrt(-x^2 + 1)) - 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x + sqrt(3))/(2*x^2 - 1)) - 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x - sqrt(3))/(2*x^2 - 1)) + 1/2*arctan(sqrt(-x^2 + 1)*x/(x^2 - 1)) + 1/8*log(x^4 - x^2 + 1) + 1/16*log(-x^4 + x^2 + 2*sqrt(-x^2 + 1)*x + 1) - 1/16*log(-x^4 + x^2 - 2*sqrt(-x^2 + 1)*x + 1)
```

giac [B] time = 1.57, size = 373, normalized size = 2.45

$$-\frac{1}{4} \pi \operatorname{sgn}(x) + \frac{1}{8} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan\left(-\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{(\sqrt{-x^2+1}-1)^2}{x^2} - 1 \right)}{3 (\sqrt{-x^2+1}-1)} \right) \right) + \frac{1}{8} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{(\sqrt{-x^2+1}-1)^2}{x^2} - 1 \right)}{3 (\sqrt{-x^2+1}-1)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/4*pi*sgn(x) + 1/8*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) + 1/8*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) - 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - sqrt(-x^2 + 1)*arctan(x + sqrt(-x^2 + 1)) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)) + 1/8*log(x^4 - x^2 + 1) - 1/8*log((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 2*x/(sqrt(-x^2 + 1) - 1) - 2*(sqrt(-x^2 + 1) - 1)/x + 4) + 1/8*log((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 - 2*x/(sqrt(-x^2 + 1) - 1) + 2*(sqrt(-x^2 + 1) - 1)/x + 4)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x \arctan\left(x + \sqrt{-x^2 + 1}\right)}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x * \arctan(x + (-x^2 + 1)^{(1/2)}) / (-x^2 + 1)^{(1/2)}, x)$

[Out] $\text{int}(x * \arctan(x + (-x^2 + 1)^{(1/2)}) / (-x^2 + 1)^{(1/2)}, x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\sqrt{x+1} \sqrt{-x+1} \arctan\left(x + \sqrt{x+1} \sqrt{-x+1}\right) - \int \frac{x}{x^2 + 2x e^{\left(\frac{1}{2} \log(x+1) + \frac{1}{2} \log(-x+1)\right)} + e^{\left(\log(x+1) + \log(-x+1)\right)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x * \arctan(x + (-x^2 + 1)^{(1/2)}) / (-x^2 + 1)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $-\sqrt{x+1} \sqrt{-x+1} \arctan(x + \sqrt{x+1} \sqrt{-x+1}) - \text{integrate}(x / (x^2 + 2x e^{(1/2 \log(x+1) + 1/2 \log(-x+1))} + e^{(\log(x+1) + \log(-x+1))} + 1), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \tan\left(x + \sqrt{1-x^2}\right)}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x * \tan(x + (1 - x^2)^{(1/2)})) / (1 - x^2)^{(1/2)}, x)$

[Out] $\text{int}((x * \tan(x + (1 - x^2)^{(1/2)})) / (1 - x^2)^{(1/2)}, x)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x * \tan(x + (-x^{**2+1})^{**(1/2)}) / (-x^{**2+1})^{**(1/2)}, x)$

[Out] Timed out

3.14 $\int \frac{\sin^{-1}(x)}{1+\sqrt{1-x^2}} dx$

Optimal. Leaf size=45

$$-\log\left(\sqrt{1-x^2} + 1\right) - \frac{x \sin^{-1}(x)}{\sqrt{1-x^2} + 1} + \frac{1}{2} \sin^{-1}(x)^2$$

[Out] $1/2*\arcsin(x)^2 - \ln(1+(-x^2+1)^{(1/2)}) - x*\arcsin(x)/(1+(-x^2+1)^{(1/2)})$

Rubi [A] time = 0.12, antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6742, 277, 216, 4791, 4627, 266, 63, 206, 4693, 29, 4641}

$$\frac{\sqrt{1-x^2} \sin^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \log(x) + \frac{1}{2} \sin^{-1}(x)^2 - \frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{ArcSin}[x]/(1 + \text{Sqrt}[1 - x^2]), x]$

[Out] $-(\text{ArcSin}[x]/x) + (\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/x + \text{ArcSin}[x]^2/2 - \text{ArcTanh}[\text{Sqrt}[1 - x^2]] - \text{Log}[x]$

Rule 29

$\text{Int}[(x_*)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 63

```
Int[((a_.) + (b_)*(x_))^(m_)*((c_.) + (d_)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^{(-1)}, x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4627

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4641

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4693

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*Sqrt[(d_) +
(e_)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcS
in[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*Sqr
t[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x] + Di
st[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 - c^2*x^2]), Int[((f*x)^(m + 2)
)*(a + b*ArcSin[c*x])^n)/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 4791

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(Px_)*((f_) + (g_)*((d_) +
(e_)*(x_)^2)^p))^(m_), x_Symbol] :> With[{u = ExpandIntegrand[Px*(f + g*
(d + e*x^2)^p)^m*(a + b*ArcSin[c*x])^n, x]}, Int[u, x] /; SumQ[u]] /; FreeQ
[{a, b, c, d, e, f, g}, x] && PolynomialQ[Px, x] && EqQ[c^2*d + e, 0] && IG
tQ[p + 1/2, 0] && IntegersQ[m, n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{1 + \sqrt{1 - x^2}} dx &= \int \left(\frac{\sin^{-1}(x)}{x^2} - \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x^2} \right) dx \\
&= \int \frac{\sin^{-1}(x)}{x^2} dx - \int \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x^2} dx \\
&= -\frac{\sin^{-1}(x)}{x} + \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x} - \int \frac{1}{x} dx + \int \frac{1}{x\sqrt{1 - x^2}} dx + \int \frac{\sin^{-1}(x)}{\sqrt{1 - x^2}} dx \\
&= -\frac{\sin^{-1}(x)}{x} + \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x} + \frac{1}{2} \sin^{-1}(x)^2 - \log(x) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, x^2\right) \\
&= -\frac{\sin^{-1}(x)}{x} + \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x} + \frac{1}{2} \sin^{-1}(x)^2 - \log(x) - \text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, \sqrt{1 - x^2}\right) \\
&= -\frac{\sin^{-1}(x)}{x} + \frac{\sqrt{1 - x^2} \sin^{-1}(x)}{x} + \frac{1}{2} \sin^{-1}(x)^2 - \tanh^{-1}\left(\sqrt{1 - x^2}\right) - \log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.98

$$-\log\left(\sqrt{1 - x^2} + 1\right) + \frac{(\sqrt{1 - x^2} - 1)\sin^{-1}(x)}{x} + \frac{1}{2} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] `Integrate[ArcSin[x]/(1 + Sqrt[1 - x^2]), x]`

[Out] `((-1 + Sqrt[1 - x^2])*ArcSin[x])/x + ArcSin[x]^2/2 - Log[1 + Sqrt[1 - x^2]]`

fricas [A] time = 0.48, size = 63, normalized size = 1.40

$$\frac{x \arcsin(x)^2 - 2x \log(x) - x \log\left(\sqrt{-x^2 + 1} + 1\right) + x \log\left(\sqrt{-x^2 + 1} - 1\right) + 2\sqrt{-x^2 + 1} \arcsin(x) - 2 \arcsin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(1+(-x^2+1)^(1/2)), x, algorithm="fricas")`

[Out] $\frac{1}{2}x^2\arcsin(x)^2 - \frac{x\arcsin(x)}{\sqrt{-x^2+1}+1} - 2\log(2) + \log\left(2\sqrt{-x^2+1}+2\right) - 2\log\left(\sqrt{-x^2+1}+1\right)$

giac [A] time = 1.30, size = 57, normalized size = 1.27

$$\frac{1}{2}\arcsin(x)^2 - \frac{x\arcsin(x)}{\sqrt{-x^2+1}+1} - 2\log(2) + \log\left(2\sqrt{-x^2+1}+2\right) - 2\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(1+(-x^2+1)^(1/2)),x, algorithm="giac")`

[Out] $\frac{1}{2}\arcsin(x)^2 - \frac{x\arcsin(x)}{\sqrt{-x^2+1}+1} - 2\log(2) + \log(2\sqrt{-x^2+1}+2) - 2\log(\sqrt{-x^2+1}+1)$

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{1+\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/(1+(-x^2+1)^(1/2)),x)`

[Out] `int(arcsin(x)/(1+(-x^2+1)^(1/2)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{\sqrt{-x^2+1}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(1+(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(arcsin(x)/(\sqrt{-x^2+1}+1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arcsin(x)}{\sqrt{1-x^2}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(x)/((1-x^2)^(1/2)+1),x)`

[Out] `int(asin(x)/((1-x^2)^(1/2)+1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{\sqrt{1-x^2} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)/(1+(-x**2+1)**(1/2)),x)

[Out] Integral(asin(x)/(sqrt(1 - x**2) + 1), x)

3.15
$$\int \frac{\log(x + \sqrt{1+x^2})}{(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=34

$$\frac{x \log(\sqrt{x^2 + 1} + x)}{\sqrt{1 - x^2}} - \frac{1}{2} \sin^{-1}(x^2)$$

[Out] $-1/2*\arcsin(x^2) + x*\ln(x + (x^2 + 1)^{(1/2)})/(-x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {191, 2554, 275, 216}

$$\frac{x \log(\sqrt{x^2 + 1} + x)}{\sqrt{1 - x^2}} - \frac{1}{2} \sin^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[x + \text{Sqrt}[1 + x^2]]/(1 - x^2)^{(3/2)}, x]$

[Out] $-\text{ArcSin}[x^2]/2 + (x \text{Log}[x + \text{Sqrt}[1 + x^2]])/\text{Sqrt}[1 - x^2]$

Rule 191

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_\text{Symbol}] \rightarrow \text{Simp}[(x*(a + b*x^n))^{(p + 1)} / a, x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&& \text{EqQ}[1/n + p + 1, 0]$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_.)^2], x_\text{Symbol}] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqr}t[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{GtQ}[a, 0] \&& \text{NegQ}[b]$

Rule 275

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_\text{Symbol}] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \&& \text{IGtQ}[n, 0] \&& \text{IntegerQ}[m]$

Rule 2554

$\text{Int}[\text{Log}[u_]*(v_), x_\text{Symbol}] \rightarrow \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/u, x], x] /; \text{InverseFunctionFreeQ}[w, x]$

]] /; InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\log(x + \sqrt{1 + x^2})}{(1 - x^2)^{3/2}} dx &= \frac{x \log(x + \sqrt{1 + x^2})}{\sqrt{1 - x^2}} - \int \frac{x}{\sqrt{1 - x^4}} dx \\
 &= \frac{x \log(x + \sqrt{1 + x^2})}{\sqrt{1 - x^2}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, x^2\right) \\
 &= -\frac{1}{2} \sin^{-1}(x^2) + \frac{x \log(x + \sqrt{1 + x^2})}{\sqrt{1 - x^2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 1.88

$$\frac{1}{2} \sqrt{1 - x^2} \left(-\frac{2x \log(\sqrt{x^2 + 1} + x)}{x^2 - 1} - \frac{\sqrt{x^2 + 1} \sin^{-1}(x^2)}{\sqrt{1 - x^4}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x + Sqrt[1 + x^2]]/(1 - x^2)^(3/2), x]

[Out] $(\text{Sqrt}[1 - x^2]*(-((\text{Sqrt}[1 + x^2]*\text{ArcSin}[x^2])/\text{Sqrt}[1 - x^4]) - (2*x*\text{Log}[x + \text{Sqrt}[1 + x^2]]))/(-1 + x^2))/2$

fricas [B] time = 0.42, size = 62, normalized size = 1.82

$$\frac{-\sqrt{-x^2 + 1} x \log(x + \sqrt{x^2 + 1}) - (x^2 - 1) \arctan\left(\frac{\sqrt{x^2 + 1} \sqrt{-x^2 + 1} - 1}{x^2}\right)}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2), x, algorithm="fricas")

[Out] $-(\text{sqrt}(-x^2 + 1)*x*\text{log}(x + \text{sqrt}(x^2 + 1)) - (x^2 - 1)*\text{arctan}((\text{sqrt}(x^2 + 1)*\text{sqrt}(-x^2 + 1) - 1)/x^2))/(x^2 - 1)$

giac [A] time = 1.38, size = 36, normalized size = 1.06

$$-\frac{\sqrt{-x^2 + 1} x \log(x + \sqrt{x^2 + 1})}{x^2 - 1} - \frac{1}{2} \arcsin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x, algorithm="giac")`
[Out] `-sqrt(-x^2 + 1)*x*log(x + sqrt(x^2 + 1))/(x^2 - 1) - 1/2*arcsin(x^2)`
maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(x + \sqrt{x^2 + 1}\right)}{(-x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x)`
[Out] `int(ln(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x)`
maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x + \sqrt{x^2 + 1}\right)}{(-x^2 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x, algorithm="maxima")`
[Out] `integrate(log(x + sqrt(x^2 + 1))/(-x^2 + 1)^(3/2), x)`
mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(x + \sqrt{x^2 + 1}\right)}{(1 - x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x + (x^2 + 1)^(1/2))/(1 - x^2)^(3/2),x)`
[Out] `int(log(x + (x^2 + 1)^(1/2))/(1 - x^2)^(3/2), x)`
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x+(x**2+1)**(1/2))/(-x**2+1)**(3/2),x)

[Out] Timed out

3.16 $\int \frac{\sin^{-1}(x)}{(1+x^2)^{3/2}} dx$

Optimal. Leaf size=22

$$\frac{x \sin^{-1}(x)}{\sqrt{x^2 + 1}} - \frac{1}{2} \sin^{-1}(x^2)$$

[Out] $-1/2*\arcsin(x^2)+x*\arcsin(x)/(x^2+1)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {191, 4665, 275, 216}

$$\frac{x \sin^{-1}(x)}{\sqrt{x^2 + 1}} - \frac{1}{2} \sin^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]/(1 + x^2)^(3/2), x]

[Out] $(x*\ArcSin[x])/Sqrt[1 + x^2] - \ArcSin[x^2]/2$

Rule 191

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4665

```
Int[((a_.) + ArcSin[(c_)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbo l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])]
```

)

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{(1+x^2)^{3/2}} dx &= \frac{x \sin^{-1}(x)}{\sqrt{1+x^2}} - \int \frac{x}{\sqrt{1-x^4}} dx \\
&= \frac{x \sin^{-1}(x)}{\sqrt{1+x^2}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, x^2\right) \\
&= \frac{x \sin^{-1}(x)}{\sqrt{1+x^2}} - \frac{1}{2} \sin^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{x \sin^{-1}(x)}{\sqrt{x^2+1}} - \frac{1}{2} \sin^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x]/(1 + x^2)^(3/2), x]

[Out] (x*ArcSin[x])/Sqrt[1 + x^2] - ArcSin[x^2]/2

fricas [B] time = 0.45, size = 56, normalized size = 2.55

$$\frac{2 \sqrt{x^2+1} x \arcsin(x) + (x^2+1) \arctan\left(\frac{\sqrt{x^2+1} \sqrt{-x^2+1} x^2}{x^4-1}\right)}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)/(x^2+1)^(3/2), x, algorithm="fricas")

[Out] 1/2*(2*sqrt(x^2 + 1)*x*arcsin(x) + (x^2 + 1)*arctan(sqrt(x^2 + 1)*sqrt(-x^2 + 1)*x^2/(x^4 - 1)))/(x^2 + 1)

giac [A] time = 1.06, size = 18, normalized size = 0.82

$$\frac{x \arcsin(x)}{\sqrt{x^2+1}} - \frac{1}{2} \arcsin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(x^2+1)^(3/2),x, algorithm="giac")`
[Out] $x \arcsin(x)/\sqrt{x^2 + 1} - 1/2 \arcsin(x^2)$
maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{(x^2 + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/(x^2+1)^(3/2),x)`
[Out] `int(arcsin(x)/(x^2+1)^(3/2),x)`
maxima [A] time = 0.96, size = 18, normalized size = 0.82

$$\frac{x \arcsin(x)}{\sqrt{x^2 + 1}} - \frac{1}{2} \arcsin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(x^2+1)^(3/2),x, algorithm="maxima")`
[Out] $x \arcsin(x)/\sqrt{x^2 + 1} - 1/2 \arcsin(x^2)$
mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\arcsin(x)}{(x^2 + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(x)/(x^2 + 1)^(3/2),x)`
[Out] `int(asin(x)/(x^2 + 1)^(3/2), x)`
sympy [C] time = 14.50, size = 78, normalized size = 3.55

$$\frac{x \arcsin(x)}{\sqrt{x^2 + 1}} + \frac{i G_{6,6}^{6,2} \left(\begin{array}{cc} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{x^4} \right)}{8 \pi^{\frac{3}{2}}} - \frac{G_{6,6}^{2,6} \left(\begin{array}{cc} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{array} \middle| \frac{e^{-2i\pi}}{x^4} \right)}{8 \pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)/(x**2+1)**(3/2),x)
[Out] $x \operatorname{asin}(x) / \sqrt{x^2 + 1} + I \operatorname{meijerg}(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), x^{(-4)}) / (8\pi^{(3/2)}) - \operatorname{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ((-1/4, 1/4), (-1/2, 0, 0, 0)), \operatorname{exp_polar}(-2I\pi) / x^4) / (8\pi^{(3/2)})$

$$3.17 \quad \int \frac{\log(x + \sqrt{-1+x^2})}{(1+x^2)^{3/2}} dx$$

Optimal. Leaf size=32

$$\frac{x \log\left(\sqrt{x^2 - 1} + x\right)}{\sqrt{x^2 + 1}} - \frac{1}{2} \cosh^{-1}(x^2)$$

[Out] $-1/2*\text{arccosh}(x^2) + x*\ln(x + (x^2 - 1)^{(1/2)})/(x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {191, 2554, 276, 52}

$$\frac{x \log\left(\sqrt{x^2 - 1} + x\right)}{\sqrt{x^2 + 1}} - \frac{1}{2} \cosh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[x + \text{Sqrt}[-1 + x^2]]/(1 + x^2)^{(3/2)}, x]$

[Out] $-\text{ArcCosh}[x^2]/2 + (x \text{Log}[x + \text{Sqrt}[-1 + x^2]])/\text{Sqrt}[1 + x^2]$

Rule 52

```
Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]
```

Rule 191

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 276

```
Int[(x_)^(m_)*((a1_) + (b1_)*(x_)^(n_))^(p_)*((a2_) + (b2_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, 2*n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a1 + b1*x^(n/k))^p*(a2 + b2*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a1, b1, a2, b2, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && IntegerQ[m]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]
 ] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(x + \sqrt{-1 + x^2})}{(1 + x^2)^{3/2}} dx &= \frac{x \log(x + \sqrt{-1 + x^2})}{\sqrt{1 + x^2}} - \int \frac{x}{\sqrt{-1 + x^2} \sqrt{1 + x^2}} dx \\ &= \frac{x \log(x + \sqrt{-1 + x^2})}{\sqrt{1 + x^2}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{-1 + x} \sqrt{1 + x}} dx, x, x^2\right) \\ &= -\frac{1}{2} \cosh^{-1}(x^2) + \frac{x \log(x + \sqrt{-1 + x^2})}{\sqrt{1 + x^2}} \end{aligned}$$

Mathematica [B] time = 0.09, size = 89, normalized size = 2.78

$$\frac{4x \log\left(\sqrt{x^2 - 1} + x\right) + \frac{\sqrt{x^2 - 1}(x^2 + 1)\left(\log\left(1 - \frac{x^2}{\sqrt{x^4 - 1}}\right) - \log\left(\frac{x^2}{\sqrt{x^4 - 1}} + 1\right)\right)}{\sqrt{x^4 - 1}}}{4\sqrt{x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]]/(1 + x^2)^(3/2), x]
[Out] (4*x*Log[x + Sqrt[-1 + x^2]] + (Sqrt[-1 + x^2]*(1 + x^2)*(Log[1 - x^2/Sqrt[-1 + x^4]] - Log[1 + x^2/Sqrt[-1 + x^4]]))/Sqrt[-1 + x^4])/(4*Sqrt[1 + x^2])
```

fricas [B] time = 0.43, size = 58, normalized size = 1.81

$$\frac{2\sqrt{x^2 + 1}x \log\left(x + \sqrt{x^2 - 1}\right) + (x^2 + 1)\log\left(-x^2 + \sqrt{x^2 + 1}\sqrt{x^2 - 1}\right)}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2), x, algorithm="fricas")
```

[Out] $\frac{1}{2} \cdot \frac{(x^2 + 1) \cdot x \cdot \ln(x + \sqrt{x^2 - 1}) + (x^2 + 1) \cdot \ln(-x^2 + \sqrt{x^2 + 1}) \cdot \sqrt{x^2 - 1}}{(x^2 + 1)}$

giac [A] time = 1.12, size = 36, normalized size = 1.12

$$\frac{x \log\left(x + \sqrt{x^2 - 1}\right)}{\sqrt{x^2 + 1}} + \frac{1}{2} \log\left(x^2 - \sqrt{x^4 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x, algorithm="giac")`

[Out] $x \cdot \ln(x + \sqrt{x^2 - 1}) / \sqrt{x^2 + 1} + \frac{1}{2} \cdot \ln(x^2 - \sqrt{x^4 - 1})$

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(x + \sqrt{x^2 - 1}\right)}{\left(x^2 + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x)`

[Out] `int(ln(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x + \sqrt{x^2 - 1}\right)}{\left(x^2 + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x, algorithm="maxima")`

[Out] `integrate(log(x + sqrt(x^2 - 1)) / (x^2 + 1)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(x + \sqrt{x^2 - 1}\right)}{\left(x^2 + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \log(x + (x^2 - 1)^{1/2}) / (x^2 + 1)^{3/2} dx$
[Out] $\int \log(x + (x^2 - 1)^{1/2}) / (x^2 + 1)^{3/2} dx$
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \ln(x + (x^2 - 1)^{1/2}) / (x^2 + 1)^{3/2} dx$
[Out] Timed out

3.18 $\int \frac{\log(x)}{x^2 \sqrt{-1+x^2}} dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x^2 - 1}}{x} + \frac{\sqrt{x^2 - 1} \log(x)}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{x^2 - 1}}\right)$$

[Out] $-\operatorname{arctanh}(x/(x^2-1)^{(1/2)})+(x^2-1)^{(1/2)}/x+\ln(x)*(x^2-1)^{(1/2)}/x$

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2335, 277, 217, 206}

$$\frac{\sqrt{x^2 - 1}}{x} + \frac{\sqrt{x^2 - 1} \log(x)}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{x^2 - 1}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Log}[x]/(x^2 \operatorname{Sqrt}[-1 + x^2]), x]$

[Out] $\operatorname{Sqrt}[-1 + x^2]/x - \operatorname{ArcTanh}[x/\operatorname{Sqrt}[-1 + x^2]] + (\operatorname{Sqrt}[-1 + x^2] \operatorname{Log}[x])/x$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[((c*x)^(m + n)*(a + b*x^n)^(p - 1)), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2335

```
Int[((a_.) + Log[(c_)*(x_)^(n_.)]*(b_.))*((f_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^r)^(q + 1)*(a +
```

```
b*Log[c*x^n])/((d*f*(m + 1)), x] - Dist[(b*n)/(d*(m + 1)), Int[(f*x)^m*(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x^2\sqrt{-1+x^2}} dx &= \frac{\sqrt{-1+x^2} \log(x)}{x} - \int \frac{\sqrt{-1+x^2}}{x^2} dx \\
&= \frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2} \log(x)}{x} - \int \frac{1}{\sqrt{-1+x^2}} dx \\
&= \frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2} \log(x)}{x} - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) + \frac{\sqrt{-1+x^2} \log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2-1} \log(x)}{x} - \log\left(\sqrt{x^2-1} + x\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Log[x]/(x^2*Sqrt[-1 + x^2]), x]`

[Out] `Sqrt[-1 + x^2]/x + (Sqrt[-1 + x^2]*Log[x])/x - Log[x + Sqrt[-1 + x^2]]`

fricas [A] time = 0.44, size = 32, normalized size = 0.74

$$\frac{x \log\left(-x + \sqrt{x^2-1}\right) + \sqrt{x^2-1} (\log(x) + 1) + x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2-1)^(1/2), x, algorithm="fricas")`

[Out] `(x*log(-x + sqrt(x^2 - 1)) + sqrt(x^2 - 1)*(log(x) + 1) + x)/x`

giac [A] time = 1.16, size = 62, normalized size = 1.44

$$\frac{2 \log(x)}{\left(x - \sqrt{x^2-1}\right)^2 + 1} + \frac{2}{\left(x - \sqrt{x^2-1}\right)^2 + 1} + \frac{1}{2} \log\left(\left(x - \sqrt{x^2-1}\right)^2\right) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2-1)^(1/2), x, algorithm="giac")`

[Out] $2\log(x)/((x - \sqrt{x^2 - 1})^2 + 1) + 2/((x - \sqrt{x^2 - 1})^2 + 1) + 1/2*\log((x - \sqrt{x^2 - 1})^2) - \log(\text{abs}(x))$

maple [C] time = 0.14, size = 89, normalized size = 2.07

$$-\frac{\sqrt{-\text{signum}(x^2-1)} \arcsin(x)}{\sqrt{\text{signum}(x^2-1)}} + \frac{-\frac{\sqrt{-\text{signum}(x^2-1)} \sqrt{-x^2+1} \ln(x)}{\sqrt{\text{signum}(x^2-1)}} - \frac{\sqrt{-\text{signum}(x^2-1)} \sqrt{-x^2+1}}{\sqrt{\text{signum}(x^2-1)}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x)/x^2/(x^2-1)^(1/2), x)`

[Out] $-1/\text{signum}(x^2-1)^{(1/2)} * (-\text{signum}(x^2-1))^{(1/2)} * \arcsin(x) + (-1/\text{signum}(x^2-1)^{(1/2)} * (-\text{signum}(x^2-1))^{(1/2)} * (-x^2+1)^{(1/2)} - 1/\text{signum}(x^2-1)^{(1/2)} * (-\text{signum}(x^2-1))^{(1/2)} * \ln(x) * (-x^2+1)^{(1/2)})/x$

maxima [A] time = 0.96, size = 41, normalized size = 0.95

$$\frac{\sqrt{x^2-1} \log(x)}{x} + \frac{\sqrt{x^2-1}}{x} - \log\left(2x + 2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2-1)^(1/2), x, algorithm="maxima")`

[Out] $\sqrt{x^2 - 1} * \log(x)/x + \sqrt{x^2 - 1}/x - \log(2*x + 2*\sqrt{x^2 - 1})$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{x^2 \sqrt{x^2-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x)/(x^2*(x^2 - 1)^(1/2)), x)`

[Out] `int(log(x)/(x^2*(x^2 - 1)^(1/2)), x)`

sympy [C] time = 159.34, size = 37, normalized size = 0.86

$$\left(\begin{cases} \frac{\sqrt{x^2-1}}{x} & \text{for } x > -1 \wedge x < 1 \end{cases} \right) \log(x) - \begin{cases} \text{NaN} & \text{for } x < -1 \\ \text{acosh}(x) - i\pi - \frac{\sqrt{x^2-1}}{x} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)/x**2/(x**2-1)**(1/2), x)

[Out] Piecewise((sqrt(x**2 - 1)/x, (x > -1) & (x < 1)) * log(x) - Piecewise((nan, x < -1), (acosh(x) - I*pi - sqrt(x**2 - 1)/x, x < 1), (nan, True)))

3.19 $\int \frac{\sqrt{1+x^3}}{x} dx$

Optimal. Leaf size=28

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

[Out] $-2/3*\text{arctanh}((x^3+1)^{(1/2)}) + 2/3*(x^3+1)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 207}

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[1 + x^3]/x, x]$

[Out] $(2*\text{Sqrt}[1 + x^3])/3 - (2*\text{ArcTanh}[\text{Sqrt}[1 + x^3]])/3$

Rule 50

```
Int[((a_.) + (b_.*(x_))^m_.*((c_.) + (d_.*(x_))^n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.*(x_))^m_.*((c_.) + (d_.*(x_))^n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*(a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x^3}}{x} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{\sqrt{1+x}}{x} dx, x, x^3\right) \\ &= \frac{2\sqrt{1+x^3}}{3} + \frac{1}{3} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^3\right) \\ &= \frac{2\sqrt{1+x^3}}{3} + \frac{2}{3} \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^3}\right) \\ &= \frac{2\sqrt{1+x^3}}{3} - \frac{2}{3} \tanh^{-1}\left(\sqrt{1+x^3}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[1 + x^3]/x, x]`

[Out] `(2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3`

fricas [A] time = 0.42, size = 34, normalized size = 1.21

$$\frac{2}{3} \sqrt{x^3+1} - \frac{1}{3} \log\left(\sqrt{x^3+1} + 1\right) + \frac{1}{3} \log\left(\sqrt{x^3+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)^(1/2)/x, x, algorithm="fricas")`

[Out] `2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)`

giac [A] time = 1.01, size = 35, normalized size = 1.25

$$\frac{2}{3} \sqrt{x^3+1} - \frac{1}{3} \log\left(\sqrt{x^3+1} + 1\right) + \frac{1}{3} \log\left(\left|\sqrt{x^3+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)^(1/2)/x,x, algorithm="giac")`

[Out] $2/3*\sqrt{x^3 + 1} - 1/3*\log(\sqrt{x^3 + 1} + 1) + 1/3*\log(\text{abs}(\sqrt{x^3 + 1} - 1))$

maple [A] time = 0.02, size = 21, normalized size = 0.75

$$-\frac{2 \operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2 \sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+1)^(1/2)/x,x)`

[Out] $-2/3*\operatorname{arctanh}\left(\sqrt{x^3+1}^{(1/2)}\right)+2/3*(x^3+1)^{(1/2)}$

maxima [A] time = 0.41, size = 34, normalized size = 1.21

$$\frac{2}{3} \sqrt{x^3+1}-\frac{1}{3} \log \left(\sqrt{x^3+1}+1\right)+\frac{1}{3} \log \left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)^(1/2)/x,x, algorithm="maxima")`

[Out] $2/3*\sqrt{x^3 + 1} - 1/3*\log(\sqrt{x^3 + 1} + 1) + 1/3*\log(\sqrt{x^3 + 1} - 1)$

mupad [B] time = 0.13, size = 174, normalized size = 6.21

$$\frac{2 \sqrt{x^3+1}}{3} \frac{2 \left(\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}\right) \sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}} \sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}} \sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}} \Pi\left(\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}}\right)\right)-\frac{\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}\right) \left(\frac{1}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}\right)-1\right) x-\left(-\frac{1}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}\right) \left(\frac{1}{2}+\frac{\sqrt{3} \, 1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 + 1)^(1/2)/x,x)`

[Out] $(2*(x^3 + 1)^(1/2))/3 - (2*((3^(1/2)*1\mathrm{i})/2 + 3/2)*((x + (3^(1/2)*1\mathrm{i})/2 - 1/2)/((3^(1/2)*1\mathrm{i})/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1\mathrm{i})/2 + 3/2))^(1/2)*(((3^(1/2)*1\mathrm{i})/2 - x + 1/2)/((3^(1/2)*1\mathrm{i})/2 + 3/2))^(1/2)*\text{ellipticPi}((3^(1/2)*1\mathrm{i})/2 + 3/2, \operatorname{asin}((x + 1)/((3^(1/2)*1\mathrm{i})/2 + 3/2))^(1/2)), -((3^(1/2)*1\mathrm{i})/2 + 3/2)/((3^(1/2)*1\mathrm{i})/2 - 3/2))/((x^3 - x*((3^(1/2)*1\mathrm{i})/2 - 1/2)*((3^(1/2)*1\mathrm{i})/2 + 1/2) + 1) - ((3^(1/2)*1\mathrm{i})/2 - 1/2)*((3^(1/2)*1\mathrm{i})/2 + 1/2))^(1/2)$

sympy [A] time = 1.17, size = 48, normalized size = 1.71

$$\frac{2x^{\frac{3}{2}}}{3\sqrt{1 + \frac{1}{x^3}}} - \frac{2 \operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2}{3x^{\frac{3}{2}}\sqrt{1 + \frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)**(1/2)/x,x)

[Out] $2*x^{(3/2)}/(3*sqrt(1 + x^{(-3)})) - 2*asinh(x^{(-3/2)})/3 + 2/(3*x^{(3/2)}*sqrt(1 + x^{(-3)}))$

3.20
$$\int \frac{x \log(x + \sqrt{-1+x^2})}{\sqrt{-1+x^2}} dx$$

Optimal. Leaf size=26

$$\sqrt{x^2 - 1} \log(\sqrt{x^2 - 1} + x) - x$$

[Out] $-x + \ln(x + (x^2 - 1)^{(1/2)}) * (x^2 - 1)^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {261, 2554, 8}

$$\sqrt{x^2 - 1} \log(\sqrt{x^2 - 1} + x) - x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \text{Log}[x + \text{Sqrt}[-1 + x^2]]) / \text{Sqrt}[-1 + x^2], x]$

[Out] $-x + \text{Sqrt}[-1 + x^2] \cdot \text{Log}[x + \text{Sqrt}[-1 + x^2]]$

Rule 8

$\text{Int}[a_, x_\text{Symbol}] \rightarrow \text{Simp}[a \cdot x, x] /; \text{FreeQ}[a, x]$

Rule 261

$\text{Int}[(x_)^{(m_.)} \cdot ((a_) + (b_.) \cdot (x_)^{(n_.)})^{(p_.)}, x_\text{Symbol}] \rightarrow \text{Simp}[(a + b \cdot x^n)^{(p+1)/(b \cdot n \cdot (p+1))}, x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&& \text{EqQ}[m, n - 1] \&& \text{NeQ}[p, -1]$

Rule 2554

$\text{Int}[\text{Log}[u_] \cdot (v_), x_\text{Symbol}] \rightarrow \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[(w \cdot D[u, x]) / u, x], x] /; \text{InverseFunctionFreeQ}[w, x]] /; \text{InverseFunctionFreeQ}[u, x]$

Rubi steps

$$\begin{aligned} \int \frac{x \log(x + \sqrt{-1 + x^2})}{\sqrt{-1 + x^2}} dx &= \sqrt{-1 + x^2} \log(x + \sqrt{-1 + x^2}) - \int 1 dx \\ &= -x + \sqrt{-1 + x^2} \log(x + \sqrt{-1 + x^2}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\sqrt{x^2 - 1} \log(\sqrt{x^2 - 1} + x) - x$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Log[x + Sqrt[-1 + x^2]])/Sqrt[-1 + x^2], x]`

[Out] `-x + Sqrt[-1 + x^2]*Log[x + Sqrt[-1 + x^2]]`

fricas [A] time = 0.42, size = 22, normalized size = 0.85

$$\sqrt{x^2 - 1} \log(x + \sqrt{x^2 - 1}) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x+(x^2-1)^(1/2))/(x^2-1)^(1/2), x, algorithm="fricas")`

[Out] `sqrt(x^2 - 1)*log(x + sqrt(x^2 - 1)) - x`

giac [A] time = 1.10, size = 22, normalized size = 0.85

$$\sqrt{x^2 - 1} \log(x + \sqrt{x^2 - 1}) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x+(x^2-1)^(1/2))/(x^2-1)^(1/2), x, algorithm="giac")`

[Out] `sqrt(x^2 - 1)*log(x + sqrt(x^2 - 1)) - x`

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x \ln(x + \sqrt{x^2 - 1})}{\sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int x \ln(x + (x^2 - 1)^{1/2}) / (x^2 - 1)^{1/2} dx$
[Out] $\int x \ln(x + (x^2 - 1)^{1/2}) / (x^2 - 1)^{1/2} dx$
maxima [A] time = 0.55, size = 22, normalized size = 0.85

$$\sqrt{x^2 - 1} \log\left(x + \sqrt{x^2 - 1}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \log(x + (x^2 - 1)^{1/2}) / (x^2 - 1)^{1/2}, x, \text{algorithm}=\text{"maxima"})$
[Out] $\sqrt{x^2 - 1} \log(x + \sqrt{x^2 - 1}) - x$
mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x \ln\left(x + \sqrt{x^2 - 1}\right)}{\sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x \log(x + (x^2 - 1)^{1/2})) / (x^2 - 1)^{1/2} dx$
[Out] $\int (x \log(x + (x^2 - 1)^{1/2})) / (x^2 - 1)^{1/2} dx$
sympy [A] time = 9.12, size = 20, normalized size = 0.77

$$-x + \sqrt{x^2 - 1} \log\left(x + \sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \ln(x + (x^{**2} - 1)^{**1/2}) / (x^{**2} - 1)^{**1/2}, x)$
[Out] $-x + \sqrt{x^{**2} - 1} \log(x + \sqrt{x^{**2} - 1})$

3.21 $\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^4}} dx$

Optimal. Leaf size=38

$$-\frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{4} \sqrt{x^2+1} x + \frac{1}{4} \sinh^{-1}(x)$$

[Out] $1/4*\text{arcsinh}(x)+1/4*x*(x^2+1)^{(1/2)}-1/2*\text{arcsin}(x)*(-x^4+1)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {261, 4787, 12, 26, 195, 215}

$$\frac{1}{4} \sqrt{x^2+1} x - \frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{4} \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3 \text{ArcSin}[x])/\text{Sqrt}[1 - x^4], x]$

[Out] $(x \text{Sqrt}[1 + x^2])/4 - (\text{Sqrt}[1 - x^4] \text{ArcSin}[x])/2 + \text{ArcSinh}[x]/4$

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 26

```
Int[((a_) + (b_)*(x_)^(n_.))^(m_.)*((c_) + (d_)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p])) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 4787

```
Int[((a_.) + ArcSin[(c_)*(x_)*(b_.)]*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSin[c*x], v, x] - Dist[b*c, Int[Simplify[Integrand[v/
Sqrt[1 - c^2*x^2], x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b
, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^4}} dx &= -\frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) - \int -\frac{\sqrt{1-x^4}}{2\sqrt{1-x^2}} dx \\
&= -\frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{2} \int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx \\
&= -\frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{2} \int \sqrt{1+x^2} dx \\
&= \frac{1}{4} x \sqrt{1+x^2} - \frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{4} \int \frac{1}{\sqrt{1+x^2}} dx \\
&= \frac{1}{4} x \sqrt{1+x^2} - \frac{1}{2} \sqrt{1-x^4} \sin^{-1}(x) + \frac{1}{4} \sinh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.09, size = 85, normalized size = 2.24

$$\frac{1}{4} \left(-2 \sqrt{1-x^4} \sin^{-1}(x) + \log(1-x^2) + \frac{\sqrt{1-x^4} x}{\sqrt{1-x^2}} - \log(x^3 + \sqrt{1-x^2} \sqrt{1-x^4} - x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSin[x])/Sqrt[1 - x^4], x]
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] - 2*Sqrt[1 - x^4]*ArcSin[x] + Log[1 - x^2]
- Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]])/4
```

fricas [B] time = 0.45, size = 138, normalized size = 3.63

$$\frac{4 \sqrt{-x^4 + 1} (x^2 - 1) \arcsin(x) + 2 \sqrt{-x^4 + 1} \sqrt{-x^2 + 1} x + (x^2 - 1) \log\left(\frac{x^3 + \sqrt{-x^4 + 1} \sqrt{-x^2 + 1} - x}{x^3 - x}\right) - (x^2 - 1) \log\left(-\frac{x^3 + \sqrt{-x^4 + 1} \sqrt{-x^2 + 1} - x}{x^3 - x}\right)}{8 (x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & -1/8*(4*\sqrt{-x^4 + 1}*(x^2 - 1)*\arcsin(x) + 2*\sqrt{-x^4 + 1}*\sqrt{-x^2 + 1})*x + (x^2 - 1)*\log((x^3 + \sqrt{-x^4 + 1}*\sqrt{-x^2 + 1} - x)/(x^3 - x)) - \\ & (x^2 - 1)*\log(-(x^3 - \sqrt{-x^4 + 1}*\sqrt{-x^2 + 1} - x)/(x^3 - x)))/(x^2 - 1) \end{aligned}$$

giac [A] time = 1.10, size = 38, normalized size = 1.00

$$\frac{1}{4} \sqrt{x^2 + 1} x - \frac{1}{2} \sqrt{-x^4 + 1} \arcsin(x) - \frac{1}{4} \log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="giac")`

[Out]
$$\begin{aligned} & 1/4*\sqrt{x^2 + 1}*x - 1/2*\sqrt{-x^4 + 1}*\arcsin(x) - 1/4*\log(-x + \sqrt{x^2 + 1}) \end{aligned}$$

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arcsin(x)}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsin(x)/(-x^4+1)^(1/2),x)`

[Out] `int(x^3*arcsin(x)/(-x^4+1)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \sqrt{x^2 + 1} \sqrt{x + 1} \sqrt{-x + 1} \arctan\left(x, \sqrt{x + 1} \sqrt{-x + 1}\right) + \int \frac{\sqrt{x^2 + 1}}{2 \left(x^2 + e^{(\log(x+1)+\log(-x+1))}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="maxima")`

[Out] $-1/2\sqrt{x^2 + 1}\sqrt{x + 1}\sqrt{-x + 1}\arctan2(x, \sqrt{x + 1})\sqrt{-x + 1} + \text{integrate}(1/2\sqrt{x^2 + 1}/(x^2 + e^{(\log(x + 1) + \log(-x + 1))}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3 \sin^{-1}(x)/(1 - x^4)^{1/2}, x)$

[Out] $\text{int}(x^3 \sin^{-1}(x)/(1 - x^4)^{1/2}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin^{-1}(x)}{\sqrt{-(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{**3} \sin^{-1}(x)/(-x^{**4+1})^{**1/2}, x)$

[Out] $\text{Integral}(x^{**3} \sin^{-1}(x)/\sqrt{-(x - 1)*(x + 1)*(x^{**2} + 1)}, x)$

$$3.22 \quad \int \frac{x^3 \sec^{-1}(x)}{\sqrt{-1+x^4}} dx$$

Optimal. Leaf size=70

$$\frac{1}{2} \sqrt{x^4 - 1} \sec^{-1}(x) - \frac{\sqrt{x^4 - 1}}{2 \sqrt{1 - \frac{1}{x^2}} x} + \frac{1}{2} \tanh^{-1} \left(\frac{\sqrt{1 - \frac{1}{x^2}} x}{\sqrt{x^4 - 1}} \right)$$

[Out] $\frac{1}{2} \operatorname{arctanh}(x * (1 - 1/x^2)^{(1/2)}) / (x^4 - 1)^{(1/2)} + \frac{1}{2} \operatorname{arcsec}(x) * (x^4 - 1)^{(1/2)} - 1/2 * (x^4 - 1)^{(1/2)} / x / (1 - 1/x^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {261, 5246, 12, 1572, 1252, 865, 875, 203}

$$-\frac{\sqrt{x^4 - 1}}{2 \sqrt{1 - \frac{1}{x^2}} x} + \frac{\sqrt{1 - x^2} \tan^{-1} \left(\frac{\sqrt{x^4 - 1}}{\sqrt{1 - x^2}} \right)}{2 \sqrt{1 - \frac{1}{x^2}} x} + \frac{1}{2} \sqrt{x^4 - 1} \sec^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^3*ArcSec[x])/Sqrt[-1 + x^4], x]

[Out] $-\operatorname{Sqrt}[-1 + x^4] / (2 \operatorname{Sqrt}[1 - x^{(-2)}] * x) + (\operatorname{Sqrt}[-1 + x^4] * \operatorname{ArcSec}[x]) / 2 + (\operatorname{Sqrt}[1 - x^2] * \operatorname{ArcTan}[\operatorname{Sqrt}[-1 + x^4] / \operatorname{Sqrt}[1 - x^2]]) / (2 \operatorname{Sqrt}[1 - x^{(-2)}] * x)$

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^{(-1)}, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^{(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)}, x_Symbol] :> Simp[(a + b*x^n)^{(p + 1)/(b*n*(p + 1))}, x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 865

```
Int[((d_) + (e_)*(x_))^(m_)*((f_.) + (g_)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
)^^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_)*(x_)]/(((f_.) + (g_)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1572

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_
.), x_Symbol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*Frac
Part[q]))*(1 + d/(x^mn*e))^FracPart[q]], Int[x^(m + mn*q)*(1 + d/(x^mn*e))^q
*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && EqQ[n2, -2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]
```

Rule 5246

```
Int[((a_.) + ArcSec[(c_)*(x_)*(b_.)])*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2))], x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sec^{-1}(x)}{\sqrt{-1+x^4}} dx &= \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) - \int \frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}} x^2} dx \\
&= \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) - \frac{1}{2} \int \frac{\sqrt{-1+x^4}}{\sqrt{1-\frac{1}{x^2}} x^2} dx \\
&= \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) - \frac{\sqrt{1-x^2} \int \frac{\sqrt{-1+x^4}}{x\sqrt{1-x^2}} dx}{2\sqrt{1-\frac{1}{x^2}} x} \\
&= \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) - \frac{\sqrt{1-x^2} \operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^2}} dx, x, x^2\right)}{4\sqrt{1-\frac{1}{x^2}} x} \\
&= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}} x} + \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) + \frac{\sqrt{1-x^2} \operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x\sqrt{-1+x^2}} dx, x, x^2\right)}{4\sqrt{1-\frac{1}{x^2}} x} \\
&= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}} x} + \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) + \frac{\sqrt{1-x^2} \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{\sqrt{-1+x^4}}{\sqrt{1-x^2}}\right)}{2\sqrt{1-\frac{1}{x^2}} x} \\
&= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}} x} + \frac{1}{2} \sqrt{-1+x^4} \sec^{-1}(x) + \frac{\sqrt{1-x^2} \tan^{-1}\left(\frac{\sqrt{-1+x^4}}{\sqrt{1-x^2}}\right)}{2\sqrt{1-\frac{1}{x^2}} x}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 1.26

$$\frac{1}{2} \left(\sqrt{x^4-1} \sec^{-1}(x) - \log(x-x^3) - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^4-1} x}{x^2-1} + \log\left(-x^2 - \sqrt{1-\frac{1}{x^2}} \sqrt{x^4-1} x + 1\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSec[x])/Sqrt[-1 + x^4], x]
[Out] (-((Sqrt[1 - x^(-2)]*x*Sqrt[-1 + x^4])/(-1 + x^2)) + Sqrt[-1 + x^4]*ArcSec[x] - Log[x - x^3] + Log[1 - x^2 - Sqrt[1 - x^(-2)]*x*Sqrt[-1 + x^4]])/2
```

fricas [B] time = 0.48, size = 110, normalized size = 1.57

$$\frac{\left(x^2 - 1\right) \log\left(\frac{x^2 + \sqrt{x^4 - 1} \sqrt{x^2 - 1} - 1}{x^2 - 1}\right) - \left(x^2 - 1\right) \log\left(-\frac{x^2 - \sqrt{x^4 - 1} \sqrt{x^2 - 1} - 1}{x^2 - 1}\right) + 2 \sqrt{x^4 - 1} \left(\left(x^2 - 1\right) \operatorname{arcsec}(x) - \sqrt{x^2 - 1}\right)}{4 \left(x^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{4}((x^2 - 1)*\log((x^2 + \sqrt{x^4 - 1}*\sqrt{x^2 - 1} - 1)/(x^2 - 1)) - (x^2 - 1)*\log(-(x^2 - \sqrt{x^4 - 1}*\sqrt{x^2 - 1} - 1)/(x^2 - 1)) + 2*\sqrt{x^4 - 1}*((x^2 - 1)*\operatorname{arcsec}(x) - \sqrt{x^2 - 1}))/((x^2 - 1))$

giac [A] time = 1.22, size = 52, normalized size = 0.74

$$\frac{\frac{1}{2} \sqrt{x^4 - 1} \arccos\left(\frac{1}{x}\right) - \frac{2 \sqrt{x^2 + 1} - \log\left(\sqrt{x^2 + 1} + 1\right) + \log\left(\sqrt{x^2 + 1} - 1\right)}{4 \operatorname{sgn}(x)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{2}*\sqrt{x^4 - 1}*\arccos(1/x) - \frac{1}{4}*(2*\sqrt{x^2 + 1} - \log(\sqrt{x^2 + 1} + 1) + \log(\sqrt{x^2 + 1} - 1))/\operatorname{sgn}(x)$

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arcsec}(x)}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsec(x)/(x^4-1)^(1/2),x)`

[Out] `int(x^3*arcsec(x)/(x^4-1)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2} \sqrt{x^2 + 1} \sqrt{x + 1} \sqrt{x - 1} \arctan\left(\sqrt{x + 1} \sqrt{x - 1}\right) - \int \frac{2 \left(x^3 e^{\left(\frac{3}{2} \log(x+1) + \frac{3}{2} \log(x-1)\right)} + x^3 e^{\left(\frac{1}{2} \log(x+1) + \frac{1}{2} \log(x-1)\right)}\right) \sqrt{x}}{(x^2 + 1) \left(e^{(2 \log(x+1) + 2 \log(x-1))}\right)^2} dx}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="maxima")`

[Out]
$$\frac{1}{2} \sqrt{x^2 + 1} \sqrt{x + 1} \sqrt{x - 1} \arctan(\sqrt{x + 1}) \sqrt{x - 1} - \int \frac{(2(x^3 e^{(3/2 \log(x + 1) + 3/2 \log(x - 1))} + x^3 e^{(1/2 \log(x + 1) + 1/2 \log(x - 1))}) \sqrt{x^2 + 1} \log(x) + (x^3 + x) e^{(1/2 \log(x^2 + 1) + 3/2 \log(x + 1) + 3/2 \log(x - 1))})}{((x^2 + 1)(e^{(2 \log(x + 1) + 2 \log(x - 1))} + e^{(\log(x + 1) + \log(x - 1))}))} dx, x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cos^{-1}\left(\frac{1}{x}\right)}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*acos(1/x))/(x^4 - 1)^(1/2),x)`

[Out] `int((x^3*acos(1/x))/(x^4 - 1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec^{-1}(x)}{\sqrt{(x - 1)(x + 1)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asec(x)/(x**4-1)**(1/2),x)`

[Out] `Integral(x**3*asec(x)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)`

3.23
$$\int \frac{x \tan^{-1}(x) \log\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=58

$$-\frac{1}{2} \log^2\left(\sqrt{x^2+1}+x\right)+\frac{1}{2} \log \left(x^2+1\right)+\sqrt{x^2+1} \log \left(\sqrt{x^2+1}+x\right) \tan ^{-1}(x)-x \tan ^{-1}(x)$$

[Out] $-x \arctan(x) + 1/2 \ln(x^2+1) - 1/2 \ln(x+(x^2+1)^{(1/2)})^2 + \arctan(x) \ln(x+(x^2+1)^{(1/2)}) * (x^2+1)^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4930, 215, 261, 2554, 8, 5212, 6686, 4846, 260}

$$-\frac{1}{2} \log^2\left(\sqrt{x^2+1}+x\right)+\frac{1}{2} \log \left(x^2+1\right)+\sqrt{x^2+1} \log \left(\sqrt{x^2+1}+x\right) \tan ^{-1}(x)-x \tan ^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \operatorname{ArcTan}[x]) \operatorname{Log}[x + \operatorname{Sqrt}[1 + x^2]]]/\operatorname{Sqrt}[1 + x^2], x]$

[Out] $-(x \operatorname{ArcTan}[x]) + \operatorname{Log}[1 + x^2]/2 + \operatorname{Sqrt}[1 + x^2] \operatorname{ArcTan}[x] \operatorname{Log}[x + \operatorname{Sqrt}[1 + x^2]] - \operatorname{Log}[x + \operatorname{Sqrt}[1 + x^2]]^2/2$

Rule 8

$\text{Int}[a_, x_\text{Symbol}] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 215

$\text{Int}[1/\operatorname{Sqrt}[(a_) + (b_*)*(x_)^2], x_\text{Symbol}] \rightarrow \text{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqr}t[a]]/\operatorname{Rt}[b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&& \operatorname{GtQ}[a, 0] \&& \operatorname{PosQ}[b]$

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_*)*(x_)^{(n_.)}), x_\text{Symbol}] \rightarrow \text{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \&& \operatorname{EqQ}[m, n - 1]$

Rule 261

$\text{Int}[(x_)^{(m_.)*((a_) + (b_*)*(x_)^{(n_.)})^{(p_.)}}, x_\text{Symbol}] \rightarrow \text{Simp}[(a + b*x^n)^{-(p + 1)/(b*n*(p + 1))}, x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&& \operatorname{EqQ}[m, n - 1] \&& \operatorname{NeQ}[p, -1]$

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]
 ] /; InverseFunctionFreeQ[u, x]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x]))^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_)*(x_)^2)^(q_), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q + 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && NeQ[q, -1]
```

Rule 5212

```
Int[ArcTan[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[ArcTan[v]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w])*D[v, x))/(1 + v^2), x], x] - Int[SimplifyIntegrand[(z*ArcTan[v])*D[w, x])/w, x], x] /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \tan^{-1}(x) \log \left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx &= \sqrt{1+x^2} \tan^{-1}(x) \log \left(x+\sqrt{1+x^2}\right) - \int \tan^{-1}(x) dx - \int \frac{\log \left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx \\ &= -x \tan^{-1}(x) + \sqrt{1+x^2} \tan^{-1}(x) \log \left(x+\sqrt{1+x^2}\right) - \frac{1}{2} \log ^2\left(x+\sqrt{1+x^2}\right) \\ &= -x \tan^{-1}(x) + \frac{1}{2} \log \left(1+x^2\right) + \sqrt{1+x^2} \tan^{-1}(x) \log \left(x+\sqrt{1+x^2}\right) - \frac{1}{2} \log ^2\left(x+\sqrt{1+x^2}\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 1.00

$$-\frac{1}{2} \log ^2\left(\sqrt{x^2+1}+x\right)+\frac{1}{2} \log \left(x^2+1\right)+\sqrt{x^2+1} \log \left(\sqrt{x^2+1}+x\right) \tan ^{-1}(x)-x \tan ^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*ArcTan[x]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2], x]`

[Out] $-(x \operatorname{ArcTan}[x])+\operatorname{Log}[1+x^2]/2+\operatorname{Sqrt}[1+x^2] \operatorname{ArcTan}[x] \operatorname{Log}[x+\operatorname{Sqrt}[1+x^2]]-\operatorname{Log}[x+\operatorname{Sqrt}[1+x^2]]^2/2$

fricas [A] time = 0.47, size = 48, normalized size = 0.83

$$\sqrt{x^2+1} \arctan(x) \log \left(x+\sqrt{x^2+1}\right)-x \arctan(x)-\frac{1}{2} \log \left(x+\sqrt{x^2+1}\right)^2+\frac{1}{2} \log \left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="fricas")`

[Out] $\operatorname{sqrt}\left(x^2+1\right) \operatorname{arctan}(x) \operatorname{log}\left(x+\operatorname{sqrt}\left(x^2+1\right)\right)-x \operatorname{arctan}(x)-1/2 * \operatorname{log}\left(x+\operatorname{sqrt}\left(x^2+1\right)\right)^2+1/2 * \operatorname{log}\left(x^2+1\right)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(x) \log \left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="giac")`

[Out] `integrate(x*arctan(x)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1), x)`

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(x) \ln \left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arctan(x)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x)`

[Out] $\int x \arctan(x) \ln(x + (x^2 + 1)^{1/2}) / (x^2 + 1)^{1/2} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(x) \log\left(x + \sqrt{x^2 + 1}\right)}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x * \arctan(x) * \log(x + (x^2 + 1)^{1/2}) / (x^2 + 1)^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\int x \arctan(x) \log(x + \sqrt{x^2 + 1}) / \sqrt{x^2 + 1} dx$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \tan(x) \ln\left(x + \sqrt{x^2 + 1}\right)}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x * \tan(x)) * \log(x + (x^2 + 1)^{1/2})) / (x^2 + 1)^{1/2}, x)$

[Out] $\int ((x * \tan(x)) * \log(x + (x^2 + 1)^{1/2})) / (x^2 + 1)^{1/2}, x$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x * \tan(x) * \ln(x + (x^{**2+1})^{**(1/2)}) / (x^{**2+1})^{**(1/2)}, x)$

[Out] Timed out

3.24 $\int \frac{x \log(1 + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=55

$$\sqrt{1-x^2} - \sqrt{1-x^2} \log(\sqrt{1-x^2} + 1) - \log(\sqrt{1-x^2} + 1)$$

[Out] $-\ln(1+(-x^2+1)^{(1/2)})+(-x^2+1)^{(1/2)}-\ln(1+(-x^2+1)^{(1/2)})*(-x^2+1)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {261, 2554, 1591, 190, 43}

$$\sqrt{1-x^2} - \sqrt{1-x^2} \log(\sqrt{1-x^2} + 1) - \log(\sqrt{1-x^2} + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \log[1 + \sqrt{1 - x^2}]) / \sqrt{1 - x^2}, x]$

[Out] $\sqrt{1 - x^2} - \log[1 + \sqrt{1 - x^2}] - \sqrt{1 - x^2} \cdot \log[1 + \sqrt{1 - x^2}]$

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1591

```
Int[((a_.) + (b_ .)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
PolyQ[Qr, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]
 ] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x \log(1 + \sqrt{1 - x^2})}{\sqrt{1 - x^2}} dx &= -\sqrt{1 - x^2} \log(1 + \sqrt{1 - x^2}) - \int \frac{x}{1 + \sqrt{1 - x^2}} dx \\ &= -\sqrt{1 - x^2} \log(1 + \sqrt{1 - x^2}) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{1 + \sqrt{x}} dx, x, 1 - x^2\right) \\ &= -\sqrt{1 - x^2} \log(1 + \sqrt{1 - x^2}) + \text{Subst}\left(\int \frac{x}{1 + x} dx, x, \sqrt{1 - x^2}\right) \\ &= -\sqrt{1 - x^2} \log(1 + \sqrt{1 - x^2}) + \text{Subst}\left(\int \left(1 + \frac{1}{-1 - x}\right) dx, x, \sqrt{1 - x^2}\right) \\ &= \sqrt{1 - x^2} - \log(1 + \sqrt{1 - x^2}) - \sqrt{1 - x^2} \log(1 + \sqrt{1 - x^2}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.75

$$\sqrt{1 - x^2} - (\sqrt{1 - x^2} + 1) \log(\sqrt{1 - x^2} + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 + Sqrt[1 - x^2]])/Sqrt[1 - x^2], x]
[Out] Sqrt[1 - x^2] - (1 + Sqrt[1 - x^2])*Log[1 + Sqrt[1 - x^2]]
```

fricas [A] time = 0.45, size = 35, normalized size = 0.64

$$-\left(\sqrt{-x^2 + 1} + 1\right) \log\left(\sqrt{-x^2 + 1} + 1\right) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")`
[Out] $-(\sqrt{-x^2 + 1} + 1)*\log(\sqrt{-x^2 + 1} + 1) + \sqrt{-x^2 + 1}$

giac [A] time = 1.03, size = 36, normalized size = 0.65

$$-\left(\sqrt{-x^2 + 1} + 1\right)\log\left(\sqrt{-x^2 + 1} + 1\right) + \sqrt{-x^2 + 1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")`
[Out] $-(\sqrt{-x^2 + 1} + 1)*\log(\sqrt{-x^2 + 1} + 1) + \sqrt{-x^2 + 1} + 1$

maple [A] time = 0.01, size = 37, normalized size = 0.67

$$-\left(1 + \sqrt{-x^2 + 1}\right)\ln\left(1 + \sqrt{-x^2 + 1}\right) + 1 + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)`
[Out] $-\ln(1+(-x^2+1)^(1/2))*(1+(-x^2+1)^(1/2))+1+(-x^2+1)^(1/2)$

maxima [A] time = 0.41, size = 36, normalized size = 0.65

$$-\left(\sqrt{-x^2 + 1} + 1\right)\log\left(\sqrt{-x^2 + 1} + 1\right) + \sqrt{-x^2 + 1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")`
[Out] $-(\sqrt{-x^2 + 1} + 1)*\log(\sqrt{-x^2 + 1} + 1) + \sqrt{-x^2 + 1} + 1$

mupad [B] time = 0.34, size = 27, normalized size = 0.49

$$-\left(\ln\left(\sqrt{1 - x^2} + 1\right) - 1\right)\left(\sqrt{1 - x^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*log((1 - x^2)^(1/2) + 1))/(1 - x^2)^(1/2),x)`
[Out] $-(\log((1 - x^2)^(1/2) + 1) - 1)*((1 - x^2)^(1/2) + 1)$

sympy [A] time = 5.99, size = 31, normalized size = 0.56

$$\sqrt{1 - x^2} - \left(\sqrt{1 - x^2} + 1\right)\log\left(\sqrt{1 - x^2} + 1\right) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+(-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
[Out] sqrt(1 - x**2) - (sqrt(1 - x**2) + 1)*log(sqrt(1 - x**2) + 1) + 1
```

3.25 $\int \frac{x \log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$

Optimal. Leaf size=26

$$\sqrt{x^2 + 1} \log(\sqrt{x^2 + 1} + x) - x$$

[Out] $-x + \ln(x + (x^2 + 1)^{(1/2)}) * (x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {261, 2554, 8}

$$\sqrt{x^2 + 1} \log(\sqrt{x^2 + 1} + x) - x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \text{Log}[x + \text{Sqrt}[1 + x^2]])/\text{Sqrt}[1 + x^2], x]$

[Out] $-x + \text{Sqrt}[1 + x^2] \cdot \text{Log}[x + \text{Sqrt}[1 + x^2]]$

Rule 8

$\text{Int}[a_, x_\text{Symbol}] \rightarrow \text{Simp}[a \cdot x, x] /; \text{FreeQ}[a, x]$

Rule 261

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_.)})^{(p_.)}, x_\text{Symbol}] \rightarrow \text{Simp}[(a + b \cdot x^n)^{(p+1)/(b \cdot n \cdot (p+1))}, x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&& \text{EqQ}[m, n - 1] \&& \text{NeQ}[p, -1]$

Rule 2554

$\text{Int}[\text{Log}[u_] * (v_), x_\text{Symbol}] \rightarrow \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[(w \cdot D[u, x])/u, x], x] /; \text{InverseFunctionFreeQ}[w, x]] /; \text{InverseFunctionFreeQ}[u, x]$

Rubi steps

$$\begin{aligned}\int \frac{x \log(x + \sqrt{1 + x^2})}{\sqrt{1 + x^2}} dx &= \sqrt{1 + x^2} \log(x + \sqrt{1 + x^2}) - \int 1 dx \\ &= -x + \sqrt{1 + x^2} \log(x + \sqrt{1 + x^2})\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\sqrt{x^2 + 1} \log(\sqrt{x^2 + 1} + x) - x$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2], x]`

[Out] `-x + Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]]`

fricas [A] time = 0.42, size = 22, normalized size = 0.85

$$\sqrt{x^2 + 1} \log(x + \sqrt{x^2 + 1}) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="fricas")`

[Out] `sqrt(x^2 + 1)*log(x + sqrt(x^2 + 1)) - x`

giac [A] time = 1.00, size = 22, normalized size = 0.85

$$\sqrt{x^2 + 1} \log(x + \sqrt{x^2 + 1}) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2), x, algorithm="giac")`

[Out] `sqrt(x^2 + 1)*log(x + sqrt(x^2 + 1)) - x`

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x \ln(x + \sqrt{x^2 + 1})}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int x \ln(x + (x^2 + 1)^{1/2}) / (x^2 + 1)^{1/2} dx$
[Out] $\int x \ln(x + (x^2 + 1)^{1/2}) / (x^2 + 1)^{1/2} dx$
maxima [A] time = 1.06, size = 22, normalized size = 0.85

$$\sqrt{x^2 + 1} \log\left(x + \sqrt{x^2 + 1}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \log(x + (x^2 + 1)^{1/2}) / (x^2 + 1)^{1/2}, x, \text{algorithm}=\text{"maxima"})$
[Out] $\sqrt{x^2 + 1} \log(x + \sqrt{x^2 + 1}) - x$
mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x \ln\left(x + \sqrt{x^2 + 1}\right)}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x \log(x + (x^2 + 1)^{1/2})) / (x^2 + 1)^{1/2} dx$
[Out] $\int (x \log(x + (x^2 + 1)^{1/2})) / (x^2 + 1)^{1/2} dx$
sympy [A] time = 4.33, size = 20, normalized size = 0.77

$$-x + \sqrt{x^2 + 1} \log\left(x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \ln(x + (x^{**2} + 1)^{**1/2}) / (x^{**2} + 1)^{**1/2}, x)$
[Out] $-x + \sqrt{x^{**2} + 1} \log(x + \sqrt{x^{**2} + 1})$

3.26
$$\int \frac{x \log(x + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=78

$$\sqrt{1-x^2} - \sqrt{1-x^2} \log\left(\sqrt{1-x^2} + x\right) - \frac{\tanh^{-1}\left(\sqrt{2} \sqrt{1-x^2}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{2} x\right)}{\sqrt{2}}$$

[Out] $\frac{1}{2} \operatorname{arctanh}(x \cdot 2^{(1/2)}) \cdot 2^{(1/2)} - \frac{1}{2} \operatorname{arctanh}(2^{(1/2)} \cdot (-x^2 + 1)^{(1/2)}) \cdot 2^{(1/2)} + (-x^2 + 1)^{(1/2)} - \ln(x + (-x^2 + 1)^{(1/2)}) \cdot (-x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {261, 2554, 6742, 2107, 321, 206, 444, 50, 63, 207, 388}

$$\sqrt{1-x^2} - \sqrt{1-x^2} \log\left(\sqrt{1-x^2} + x\right) - \frac{\tanh^{-1}\left(\sqrt{2} \sqrt{1-x^2}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{2} x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x \cdot \operatorname{Log}[x + \operatorname{Sqrt}[1 - x^2]])/\operatorname{Sqrt}[1 - x^2], x]$

[Out] $\operatorname{Sqrt}[1 - x^2] + \operatorname{ArcTanh}[\operatorname{Sqrt}[2] \cdot x]/\operatorname{Sqrt}[2] - \operatorname{ArcTanh}[\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[1 - x^2]]/\operatorname{Sqrt}[2] - \operatorname{Sqrt}[1 - x^2] \cdot \operatorname{Log}[x + \operatorname{Sqrt}[1 - x^2]]$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^(m)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - 1)*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGTQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 2107

```
Int[(x_)^(m_)/((d_)*(x_)^(n_)) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x] + Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]
 ] /; InverseFunctionFreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
 ]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x \log(x + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx &= -\sqrt{1-x^2} \log(x + \sqrt{1-x^2}) - \int \frac{x - \sqrt{1-x^2}}{x + \sqrt{1-x^2}} dx \\
 &= -\sqrt{1-x^2} \log(x + \sqrt{1-x^2}) - \int \left(\frac{x}{x + \sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{x + \sqrt{1-x^2}} \right) dx \\
 &= -\sqrt{1-x^2} \log(x + \sqrt{1-x^2}) - \int \frac{x}{x + \sqrt{1-x^2}} dx + \int \frac{\sqrt{1-x^2}}{x + \sqrt{1-x^2}} dx \\
 &= -\sqrt{1-x^2} \log(x + \sqrt{1-x^2}) + \int \frac{x^2}{1-2x^2} dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2} dx + \int \left(\frac{x\sqrt{1-x^2}}{-1+2x^2} - \right. \\
 &\quad \left. - \frac{x}{2} - \sqrt{1-x^2} \log(x + \sqrt{1-x^2}) + \frac{1}{2} \int \frac{1}{1-2x^2} dx - \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x} dx, x, x^2\right) \right) \\
 &= \frac{\sqrt{1-x^2}}{2} + \frac{\tanh^{-1}(\sqrt{2}x)}{2\sqrt{2}} - \sqrt{1-x^2} \log(x + \sqrt{1-x^2}) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x^2}} dx, x, x^2\right) \\
 &= \sqrt{1-x^2} + \frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}} - \sqrt{1-x^2} \log(x + \sqrt{1-x^2}) + \frac{1}{4} \text{Subst}\left(\int \frac{1}{\sqrt{1-x}(-1+2x^2)} dx, x, x^2\right) \\
 &= \sqrt{1-x^2} + \frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}\sqrt{1-x^2})}{2\sqrt{2}} - \sqrt{1-x^2} \log(x + \sqrt{1-x^2}) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1-2x^2)} dx, x, x^2\right) \\
 &= \sqrt{1-x^2} + \frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}\sqrt{1-x^2})}{\sqrt{2}} - \sqrt{1-x^2} \log(x + \sqrt{1-x^2})
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 119, normalized size = 1.53

$$\frac{1}{4} \left(4\sqrt{1-x^2} - \sqrt{2} \log(\sqrt{2-2x^2} - \sqrt{2}x + 2) - \sqrt{2} \log(\sqrt{2-2x^2} + \sqrt{2}x + 2) - 4\sqrt{1-x^2} \log(\sqrt{1-x^2} + x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2], x]
[Out] (4*Sqrt[1 - x^2] + 2*Sqrt[2]*Log[Sqrt[2] + 2*x] - Sqrt[2]*Log[2 - Sqrt[2]*x
+ Sqrt[2 - 2*x^2]] - Sqrt[2]*Log[2 + Sqrt[2]*x + Sqrt[2 - 2*x^2]] - 4*Sqrt
[1 - x^2]*Log[x + Sqrt[1 - x^2]])/4
fricas [A] time = 0.45, size = 115, normalized size = 1.47
```

$$-\sqrt{-x^2 + 1} \log \left(x + \sqrt{-x^2 + 1} \right) + \frac{1}{4} \sqrt{2} \log \left(\frac{6x^2 - 2\sqrt{2}(2x^2 - 3) + 2\sqrt{-x^2 + 1}(3\sqrt{2} - 4) - 9}{2x^2 - 1} \right) + \frac{1}{4} \sqrt{2} \log \left(\frac{2\sqrt{-x^2 + 1}(2x^2 + 2\sqrt{2}x + 1) - 4\sqrt{-x^2 + 1}(2x^2 - 1)}{2x^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2), x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1)*log(x + sqrt(-x^2 + 1)) + 1/4*sqrt(2)*log((6*x^2 - 2*sqrt(2)
)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2) - 4)/(2*x^2 - 1)) + 1/4*sq
rt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1)) + sqrt(-x^2 + 1)
```

giac [A] time = 1.30, size = 122, normalized size = 1.56

$$-\sqrt{-x^2 + 1} \log \left(x + \sqrt{-x^2 + 1} \right) - \frac{1}{4} \sqrt{2} \log \left(\frac{\left| -4\sqrt{2} + \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 6 \right|}{\left| 4\sqrt{2} + \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 6 \right|} \right) + \frac{1}{4} \sqrt{2} \log \left(\left| x + \frac{1}{2}\sqrt{2} \right| \right) - \frac{1}{4} \sqrt{2} \log \left(\frac{\left| 2x^2 + 2\sqrt{2}x + 1 \right|}{\left| 2x^2 - 1 \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2), x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)*log(x + sqrt(-x^2 + 1)) - 1/4*sqrt(2)*log(abs(-4*sqrt(2) +
2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/
x^2 - 6)) + 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x -
1/2*sqrt(2))) + sqrt(-x^2 + 1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x \ln \left(x + \sqrt{-x^2 + 1} \right)}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int x \ln(x + (-x^2 + 1)^{1/2}) / (-x^2 + 1)^{1/2} dx$

[Out] $\int x \ln(x + (-x^2 + 1)^{1/2}) / (-x^2 + 1)^{1/2} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(x^2 - 1) \log(x + \sqrt{x+1} \sqrt{-x+1})}{\sqrt{x+1} \sqrt{-x+1}} - \int \frac{(x^2 - 1) e^{\left(-\frac{1}{2} \log(x+1) - \frac{1}{2} \log(-x+1)\right)}}{x} dx - \int \frac{1}{x^2 + \sqrt{x+1} x \sqrt{-x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \log(x + (-x^2 + 1)^{1/2})) / (-x^2 + 1)^{1/2}, x, \text{algorithm}=\text{"maxima"}$

[Out] $(x^2 - 1) \log(x + \sqrt{x+1} \sqrt{-x+1}) / (\sqrt{x+1} \sqrt{-x+1}) - \text{integrate}((x^2 - 1) e^{-1/2 \log(x+1) - 1/2 \log(-x+1)} / x, x) - \text{integrate}(1 / (x^2 + \sqrt{x+1} x \sqrt{-x+1}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \ln(x + \sqrt{1-x^2})}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x \log(x + (1 - x^2)^{1/2})) / (1 - x^2)^{1/2}, x$

[Out] $\int (x \log(x + (1 - x^2)^{1/2})) / (1 - x^2)^{1/2}, x$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log(x + \sqrt{1-x^2})}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \ln(x + (-x^{**2} + 1)^{**1/2})) / (-x^{**2} + 1)^{**1/2}, x$

[Out] $\text{Integral}(x \log(x + \sqrt{1 - x^{**2}}) / \sqrt{-(x - 1)*(x + 1)}, x)$

3.27 $\int \frac{\log(x)}{x^2 \sqrt{1-x^2}} dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2} \log(x)}{x} - \sin^{-1}(x)$$

[Out] $-\arcsin(x) - (-x^{2+1})^{(1/2)}/x - \ln(x)*(-x^{2+1})^{(1/2)}/x$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2335, 277, 216}

$$-\frac{\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2} \log(x)}{x} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Log[x]/(x^2*Sqrt[1 - x^2]), x]

[Out] $-(\text{Sqrt}[1 - x^2]/x) - \text{ArcSin}[x] - (\text{Sqrt}[1 - x^2]*\text{Log}[x])/x$

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 277

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c *x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntB i nomialQ[a, b, c, n, m, p, x]

Rule 2335

Int[((a_) + Log[(c_)*(x_)^(n_)]*(b_))*((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^(r_))^(q_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/(d*f*(m + 1)), x] - Dist[(b*n)/(d*(m + 1)), Int[(f*x)^m*(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ[m + r*(q + 1) + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x^2\sqrt{1-x^2}} dx &= -\frac{\sqrt{1-x^2} \log(x)}{x} + \int \frac{\sqrt{1-x^2}}{x^2} dx \\
&= -\frac{\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2} \log(x)}{x} - \int \frac{1}{\sqrt{1-x^2}} dx \\
&= -\frac{\sqrt{1-x^2}}{x} - \sin^{-1}(x) - \frac{\sqrt{1-x^2} \log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.64

$$-\frac{\sqrt{1-x^2} (\log(x)+1)}{x} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[Log[x]/(x^2*Sqrt[1 - x^2]), x]`

[Out] `-ArcSin[x] - (Sqrt[1 - x^2]*(1 + Log[x]))/x`

fricas [A] time = 0.44, size = 39, normalized size = 1.00

$$\frac{2x \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \sqrt{-x^2+1}(\log(x)+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(-x^2+1)^(1/2), x, algorithm="fricas")`

[Out] `(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(log(x) + 1))/x`

giac [B] time = 1.08, size = 73, normalized size = 1.87

$$\frac{1}{2} \left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x} \right) \log(x) + \frac{x}{2(\sqrt{-x^2+1}-1)} - \frac{\sqrt{-x^2+1}-1}{2x} - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(-x^2+1)^(1/2), x, algorithm="giac")`

[Out] `1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)*log(x) + 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x - arcsin(x)`

maple [A] time = 0.11, size = 35, normalized size = 0.90

$$-\arcsin(x) + \frac{-\sqrt{-x^2 + 1} \ln(x) - \sqrt{-x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x)/x^2/(-x^2+1)^(1/2),x)`

[Out] `-arcsin(x)+(-ln(x)*(-x^2+1)^(1/2)-(-x^2+1)^(1/2))/x`

maxima [A] time = 0.96, size = 35, normalized size = 0.90

$$-\frac{\sqrt{-x^2 + 1} \log(x)}{x} - \frac{\sqrt{-x^2 + 1}}{x} - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `-sqrt(-x^2 + 1)*log(x)/x - sqrt(-x^2 + 1)/x - arcsin(x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{x^2 \sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x)/(x^2*(1 - x^2)^(1/2)),x)`

[Out] `int(log(x)/(x^2*(1 - x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x^2 \sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)/x**2/(-x**2+1)**(1/2),x)`

[Out] `Integral(log(x)/(x**2*sqrt(-(x - 1)*(x + 1))), x)`

3.28 $\int \frac{x \tan^{-1}(x)}{\sqrt{1+x^2}} dx$

Optimal. Leaf size=17

$$\sqrt{x^2 + 1} \tan^{-1}(x) - \sinh^{-1}(x)$$

[Out] $-\text{arcsinh}(x) + \text{arctan}(x) * (x^2 + 1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4930, 215}

$$\sqrt{x^2 + 1} \tan^{-1}(x) - \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \text{ArcTan}[x])/\text{Sqrt}[1 + x^2], x]$

[Out] $-\text{ArcSinh}[x] + \text{Sqrt}[1 + x^2] * \text{ArcTan}[x]$

Rule 215

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4930

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))^(p_)*(x_)*((d_) + (e_)*(x_)^2)^(q_),
x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \tan^{-1}(x)}{\sqrt{1+x^2}} dx &= \sqrt{1+x^2} \tan^{-1}(x) - \int \frac{1}{\sqrt{1+x^2}} dx \\ &= -\sinh^{-1}(x) + \sqrt{1+x^2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\sqrt{x^2 + 1} \tan^{-1}(x) - \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*ArcTan[x])/Sqrt[1 + x^2], x]`

[Out] $-\text{ArcSinh}[x] + \sqrt{1 + x^2} \cdot \text{ArcTan}[x]$

fricas [A] time = 0.46, size = 23, normalized size = 1.35

$$\sqrt{x^2 + 1} \arctan(x) + \log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^(1/2), x, algorithm="fricas")`

[Out] $\sqrt{x^2 + 1} \cdot \arctan(x) + \log(-x + \sqrt{x^2 + 1})$

giac [A] time = 0.93, size = 23, normalized size = 1.35

$$\sqrt{x^2 + 1} \arctan(x) + \log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^(1/2), x, algorithm="giac")`

[Out] $\sqrt{x^2 + 1} \cdot \arctan(x) + \log(-x + \sqrt{x^2 + 1})$

maple [C] time = 0.17, size = 54, normalized size = 3.18

$$\sqrt{(x - i)(x + i)} \arctan(x) + \ln\left(\frac{ix + 1}{\sqrt{x^2 + 1}} - i\right) - \ln\left(\frac{ix + 1}{\sqrt{x^2 + 1}} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arctan(x)/(x^2+1)^(1/2), x)`

[Out] $((x - I)(x + I))^{(1/2)} \cdot \arctan(x) + \ln((I \cdot x + 1) / (x^2 + 1)^{(1/2)} - I) - \ln((I \cdot x + 1) / (x^2 + 1)^{(1/2)} + I)$

maxima [A] time = 0.95, size = 15, normalized size = 0.88

$$\sqrt{x^2 + 1} \arctan(x) - \text{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^(1/2), x, algorithm="maxima")`

[Out] $\sqrt{x^2 + 1} \cdot \arctan(x) - \text{arcsinh}(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x \operatorname{atan}(x)}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*atan(x))/(x^2 + 1)^(1/2),x)`

[Out] `int((x*atan(x))/(x^2 + 1)^(1/2), x)`

sympy [B] time = 2.33, size = 29, normalized size = 1.71

$$\frac{x^2 \operatorname{atan}(x)}{\sqrt{x^2 + 1}} - \operatorname{asinh}(x) + \frac{\operatorname{atan}(x)}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*atan(x)/(x**2+1)**(1/2),x)`

[Out] `x**2*atan(x)/sqrt(x**2 + 1) - asinh(x) + atan(x)/sqrt(x**2 + 1)`

3.29 $\int \frac{\tan^{-1}(x)}{x^2 \sqrt{1-x^2}} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)$$

[Out] $-\operatorname{arctanh}((-x^2+1)^{(1/2)})+\operatorname{arctanh}(1/2*2^{(1/2)}*(-x^2+1)^{(1/2})*2^{(1/2)}-\operatorname{arctan}(x)*(-x^2+1)^{(1/2)}/x$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {264, 4976, 446, 83, 63, 206}

$$-\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTan}[x]/(x^2 * \operatorname{Sqrt}[1 - x^2]), x]$

[Out] $-((\operatorname{Sqrt}[1 - x^2] * \operatorname{ArcTan}[x])/x) - \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - x^2]] + \operatorname{Sqrt}[2] * \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - x^2]/\operatorname{Sqrt}[2]]$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*(c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x, x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x, x]] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \text{ || } LtQ[b, 0]$

Rule 264

```
Int[((c_)*(x_))^(m_)*(a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_)*(a_) + (b_)*(x_)^(n_))^(p_)*(c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p * (c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4976

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[Simplify[Integrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILTQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m - 1)/2, 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2 \sqrt{1-x^2}} dx &= -\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} + \int \frac{\sqrt{1-x^2}}{x(1+x^2)} dx \\
&= -\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} + \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{1-x}}{x(1+x)} dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x} x} dx, x, x^2\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)} dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} + 2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{1-x^2}\right) - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2} \tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 1.35

$$-\frac{\log(x^2 + 1)}{\sqrt{2}} + \frac{\log\left(-x^2 + 2\sqrt{2 - 2x^2} + 3\right)}{\sqrt{2}} - \log\left(\sqrt{1 - x^2} + 1\right) - \frac{\sqrt{1 - x^2} \tan^{-1}(x)}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[ArcTan[x]/(x^2*Sqrt[1 - x^2]), x]`

[Out] $-\left(\left(\text{Sqrt}[1 - x^2] \cdot \text{ArcTan}[x]\right)/x\right) + \text{Log}[x] - \text{Log}[1 + x^2]/\text{Sqrt}[2] + \text{Log}[3 - x^2 + 2 \cdot \text{Sqrt}[2 - 2x^2]]/\text{Sqrt}[2] - \text{Log}[1 + \text{Sqrt}[1 - x^2]]$

fricas [A] time = 0.47, size = 81, normalized size = 1.42

$$\frac{\sqrt{2} x \log\left(\frac{x^2-2 \sqrt{2} \sqrt{-x^2+1}-3}{x^2+1}\right)-x \log\left(\sqrt{-x^2+1}+1\right)+x \log\left(\sqrt{-x^2+1}-1\right)-2 \sqrt{-x^2+1} \arctan(x)}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)/x^2/(-x^2+1)^(1/2), x, algorithm="fricas")`

[Out] $\frac{1}{2} \left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x} \right) \arctan(x) - \frac{1}{2} \sqrt{2} \log\left(\frac{\sqrt{2}-\sqrt{-x^2+1}}{\sqrt{2}+\sqrt{-x^2+1}}\right) - \frac{1}{2} \log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2} \log\left(-\sqrt{-x^2+1}+1\right)$

giac [B] time = 1.09, size = 104, normalized size = 1.82

$$\frac{1}{2} \left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x} \right) \arctan(x) - \frac{1}{2} \sqrt{2} \log\left(\frac{\sqrt{2}-\sqrt{-x^2+1}}{\sqrt{2}+\sqrt{-x^2+1}}\right) - \frac{1}{2} \log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2} \log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)/x^2/(-x^2+1)^(1/2), x, algorithm="giac")`

[Out] $\frac{1}{2} \left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x} \right) \arctan(x) - \frac{1}{2} \sqrt{2} \log\left(\frac{\sqrt{2}-\sqrt{-x^2+1}}{\sqrt{2}+\sqrt{-x^2+1}}\right) - \frac{1}{2} \log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2} \log\left(-\sqrt{-x^2+1}+1\right)$

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)}{\sqrt{-x^2+1} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \arctan(x) / x^2 / (-x^2 + 1)^{1/2} dx$

[Out] $\int \arctan(x) / x^2 / (-x^2 + 1)^{1/2} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)}{\sqrt{-x^2 + 1} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\arctan(x) / x^2 / (-x^2 + 1)^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\int \arctan(x) / (\sqrt{-x^2 + 1} * x^2) dx$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(x)}{x^2 \sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \tan(x) / (x^2 * (1 - x^2)^{1/2}) dx$

[Out] $\int \tan(x) / (x^2 * (1 - x^2)^{1/2}) dx$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{x^2 \sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\tan(x) / x^{**2} / (-x^{**2} + 1)^{**1/2}, x)$

[Out] $\text{Integral}(\tan(x) / (x^{**2} * \sqrt{-(x-1)*(x+1)}), x)$

3.30 $\int \frac{x \tan^{-1}(x)}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=45

$$-\sqrt{1-x^2} \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

[Out] $-\arcsin(x) + \arctan(x*2^{(1/2)}/(-x^2+1)^{(1/2)})*2^{(1/2)} - \arctan(x)*(-x^2+1)^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4974, 402, 216, 377, 203}

$$-\sqrt{1-x^2} \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \text{ArcTan}[x])/\text{Sqrt}[1 - x^2], x]$

[Out] $-\text{ArcSin}[x] - \text{Sqrt}[1 - x^2] \cdot \text{ArcTan}[x] + \text{Sqrt}[2] \cdot \text{ArcTan}[(\text{Sqrt}[2] \cdot x)/\text{Sqrt}[1 - x^2]]$

Rule 203

$\text{Int}[((a_) + (b_*) \cdot (x_)^2)^{(-1)}, x_{\text{Symbol}}] \Rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{PosQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_*) \cdot (x_)^2], x_{\text{Symbol}}] \Rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2] \cdot x)/\text{Sqr}t[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{GtQ}[a, 0] \&& \text{NegQ}[b]$

Rule 377

$\text{Int}[((a_) + (b_*) \cdot (x_)^{(n_)})^{(p_)}/((c_) + (d_*) \cdot (x_)^{(n_)}), x_{\text{Symbol}}] \Rightarrow \text{Simp}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/((a + b \cdot x^n)^{(1/n)})] /; \text{FreeQ}[\{a, b, c, d\}, x] \&& \text{NeQ}[b \cdot c - a \cdot d, 0] \&& \text{EqQ}[n \cdot p + 1, 0] \&& \text{IntegerQ}[n]$

Rule 402

$\text{Int}[((a_) + (b_*) \cdot (x_)^2)^{(p_)}/((c_) + (d_*) \cdot (x_)^2), x_{\text{Symbol}}] \Rightarrow \text{Dist}[b/d, \text{Int}[(a + b \cdot x^2)^{(p - 1)}, x], x] - \text{Dist}[(b \cdot c - a \cdot d)/d, \text{Int}[(a + b \cdot x^2)^{(p - 1)}, x], x]$

```
- 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_)*(x_)]*(b_.))*(x_)*((d_.) + (e_)*(x_)^2)^(q_), x]
_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*e*(q + 1)), x]
- Dist[(b*c)/(2*e*(q + 1)), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
/; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \tan^{-1}(x)}{\sqrt{1-x^2}} dx &= -\sqrt{1-x^2} \tan^{-1}(x) + \int \frac{\sqrt{1-x^2}}{1+x^2} dx \\ &= -\sqrt{1-x^2} \tan^{-1}(x) + 2 \int \frac{1}{\sqrt{1-x^2} (1+x^2)} dx - \int \frac{1}{\sqrt{1-x^2}} dx \\ &= -\sin^{-1}(x) - \sqrt{1-x^2} \tan^{-1}(x) + 2 \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt{1-x^2}}\right) \\ &= -\sin^{-1}(x) - \sqrt{1-x^2} \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.00

$$-\sqrt{1-x^2} \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*ArcTan[x])/Sqrt[1 - x^2], x]`

[Out] `-ArcSin[x] - Sqrt[1 - x^2]*ArcTan[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]`

fricas [A] time = 0.45, size = 69, normalized size = 1.53

$$-\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2} (3x^2 - 1)\sqrt{-x^2 + 1}}{4(x^3 - x)}\right) - \sqrt{-x^2 + 1} \arctan(x) + \arctan\left(\frac{\sqrt{-x^2 + 1}x}{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $-1/2*\sqrt{2}*\arctan(1/4*\sqrt{2}*(3*x^2 - 1)*\sqrt{-x^2 + 1})/(x^3 - x) - \sqrt{-x^2 + 1}*\arctan(x) + \arctan(\sqrt{-x^2 + 1})*x/(x^2 - 1)$

giac [B] time = 1.02, size = 108, normalized size = 2.40

$$\frac{1}{2}\pi\operatorname{sgn}(x)+\frac{1}{2}\sqrt{2}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\sqrt{-x^2+1}\arctan(x)-\arctan\left(-\frac{x\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="giac")`

[Out] $-1/2*\pi*\operatorname{sgn}(x) + 1/2*\sqrt{2}*(\pi*\operatorname{sgn}(x) + 2*\arctan(-1/4*\sqrt{2})*x*((\sqrt{-x^2 + 1} - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1} - 1)) - \sqrt{-x^2 + 1}*\arctan(x) - \arctan(-1/2*x*((\sqrt{-x^2 + 1} - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1} - 1))$

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(x)}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arctan(x)/(-x^2+1)^(1/2),x)`

[Out] `int(x*arctan(x)/(-x^2+1)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [B] time = 0.03, size = 37, normalized size = 0.82

$$\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{\sqrt{1-x^2}}\right) - \operatorname{atan}(x) \sqrt{1-x^2} - \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*atan(x))/(1 - x^2)^(1/2),x)`

[Out] $2^{(1/2)} \cdot \text{atan}((2^{(1/2)} \cdot x) / (1 - x^2)^{(1/2)}) - \text{atan}(x) \cdot (1 - x^2)^{(1/2)} - \text{asin}(x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \tan(x)}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*atan(x)/(-x**2+1)**(1/2),x)`

[Out] `Integral(x*atan(x)/sqrt(-(x - 1)*(x + 1)), x)`

3.31 $\int \frac{\tan^{-1}(x)}{x^2 \sqrt{1+x^2}} dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2 + 1} \tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{x^2 + 1}\right)$$

[Out] $-\operatorname{arctanh}((x^2+1)^{(1/2)}) - \operatorname{arctan}(x) * (x^2+1)^{(1/2)}/x$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4944, 266, 63, 207}

$$-\frac{\sqrt{x^2 + 1} \tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{x^2 + 1}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTan}[x]/(x^2 \operatorname{Sqrt}[1 + x^2]), x]$

[Out] $-((\operatorname{Sqrt}[1 + x^2] \operatorname{ArcTan}[x])/x) - \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + x^2]]$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.*))^(p_.*((f_.*)(x_))^(m_.*))((d_) + (e_.*)(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(x)}{x^2\sqrt{1+x^2}} dx &= -\frac{\sqrt{1+x^2}\tan^{-1}(x)}{x} + \int \frac{1}{x\sqrt{1+x^2}} dx \\ &= -\frac{\sqrt{1+x^2}\tan^{-1}(x)}{x} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}} dx, x, x^2\right) \\ &= -\frac{\sqrt{1+x^2}\tan^{-1}(x)}{x} + \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^2}\right) \\ &= -\frac{\sqrt{1+x^2}\tan^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1+x^2}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.14

$$-\log\left(\sqrt{x^2+1}+1\right)-\frac{\sqrt{x^2+1}\tan^{-1}(x)}{x}+\log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[ArcTan[x]/(x^2*.Sqrt[1 + x^2]), x]`

[Out] $-\left(\sqrt{1+x^2}*\text{ArcTan}[x]\right)/x + \text{Log}[x] - \text{Log}[1 + \sqrt{1+x^2}]$

fricas [A] time = 0.44, size = 47, normalized size = 1.62

$$\frac{-x \log\left(-x + \sqrt{x^2+1} + 1\right) - x \log\left(-x + \sqrt{x^2+1} - 1\right) + \sqrt{x^2+1} \arctan(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)/x^2/(x^2+1)^(1/2), x, algorithm="fricas")`

[Out] $-(x*\log(-x + \sqrt{x^2+1} + 1) - x*\log(-x + \sqrt{x^2+1} - 1) + \sqrt{x^2+1}*\arctan(x))/x$

giac [B] time = 1.10, size = 54, normalized size = 1.86

$$\frac{2 \arctan(x)}{\left(x - \sqrt{x^2 + 1}\right)^2 - 1} + \arctan(x) - \log\left(\left|-x + \sqrt{x^2 + 1} + 1\right|\right) + \log\left(\left|-x + \sqrt{x^2 + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)/x^2/(x^2+1)^(1/2),x, algorithm="giac")

[Out] $2*\arctan(x)/((x - \sqrt{x^2 + 1})^2 - 1) + \arctan(x) - \log(\text{abs}(-x + \sqrt{x^2 + 1} + 1)) + \log(\text{abs}(-x + \sqrt{x^2 + 1} - 1))$

maple [C] time = 0.17, size = 56, normalized size = 1.93

$$-\ln\left(1 + \frac{ix + 1}{\sqrt{x^2 + 1}}\right) + \ln\left(\frac{ix + 1}{\sqrt{x^2 + 1}} - 1\right) - \frac{\sqrt{(x - i)(x + i)} \arctan(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(x)/x^2/(x^2+1)^(1/2),x)

[Out] $-((x - I)*(x + I))^{(1/2)} * \arctan(x) / x - \ln(1 + (I*x + 1) / (x^2 + 1)^{(1/2)}) + \ln((I*x + 1) / (x^2 + 1)^{(1/2)} - 1)$

maxima [A] time = 0.95, size = 22, normalized size = 0.76

$$-\frac{\sqrt{x^2 + 1} \arctan(x)}{x} - \text{arsinh}\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)/x^2/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] $-\sqrt{x^2 + 1} * \arctan(x) / x - \text{arcsinh}(1 / \text{abs}(x))$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)}{x^2 \sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(x^2*(x^2 + 1)^(1/2)),x)

[Out] $\int \tan(x) / (x^2 * (x^2 + 1)^{(1/2)}) dx$

sympy [A] time = 7.26, size = 19, normalized size = 0.66

$$-\operatorname{asinh}\left(\frac{1}{x}\right) - \frac{\sqrt{x^2 + 1} \operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x)/x**2/(x**2+1)**(1/2),x)`

[Out] `-asinh(1/x) - sqrt(x**2 + 1)*atan(x)/x`

$$3.32 \quad \int \frac{\sin^{-1}(x)}{x^2 \sqrt{1-x^2}} dx$$

Optimal. Leaf size=21

$$\log(x) - \frac{\sqrt{1-x^2} \sin^{-1}(x)}{x}$$

[Out] $\ln(x) - \arcsin(x) * (-x^2 + 1)^{(1/2)}/x$

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4681, 29}

$$\log(x) - \frac{\sqrt{1-x^2} \sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{ArcSin}[x]/(x^2 \text{Sqrt}[1 - x^2]), x]$

[Out] $-((\text{Sqrt}[1 - x^2] * \text{ArcSin}[x])/x) + \text{Log}[x]$

Rule 29

$\text{Int}[(x_*)^{(-1)}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 4681

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.)^(n_.)*((f_.)*(x_))^(m_)*(d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b *ArcSin[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(x)}{x^2 \sqrt{1-x^2}} dx &= -\frac{\sqrt{1-x^2} \sin^{-1}(x)}{x} + \int \frac{1}{x} dx \\ &= -\frac{\sqrt{1-x^2} \sin^{-1}(x)}{x} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\log(x) - \frac{\sqrt{1-x^2} \sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[ArcSin[x]/(x^2*Sqrt[1 - x^2]), x]`

[Out] $-\left(\frac{\sqrt{1-x^2} \arcsin(x)}{x}\right) + \ln(x)$

fricas [A] time = 0.44, size = 22, normalized size = 1.05

$$\frac{x \log(x) - \sqrt{-x^2 + 1} \arcsin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/x^2/(-x^2+1)^(1/2), x, algorithm="fricas")`

[Out] $(x \log(x) - \sqrt{-x^2 + 1} \arcsin(x))/x$

giac [B] time = 1.03, size = 40, normalized size = 1.90

$$\frac{1}{2} \left(\frac{x}{\sqrt{-x^2 + 1} - 1} - \frac{\sqrt{-x^2 + 1} - 1}{x} \right) \arcsin(x) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/x^2/(-x^2+1)^(1/2), x, algorithm="giac")`

[Out] $\frac{1}{2} \left(\frac{x}{\sqrt{-x^2 + 1} - 1} - \frac{\sqrt{-x^2 + 1} - 1}{x} \right) \arcsin(x) + \log(|x|)$

maple [A] time = 0.07, size = 20, normalized size = 0.95

$$\ln(x) - \frac{\sqrt{-x^2 + 1} \arcsin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/x^2/(-x^2+1)^(1/2), x)`

[Out] $\ln(x) - \arcsin(x) * (-x^2 + 1)^{(1/2)}/x$

maxima [A] time = 0.94, size = 19, normalized size = 0.90

$$-\frac{\sqrt{-x^2 + 1} \arcsin(x)}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `-sqrt(-x^2 + 1)*arcsin(x)/x + log(x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\arcsin(x)}{x^2 \sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(x)/(x^2*(1 - x^2)^(1/2)),x)`

[Out] `int(asin(x)/(x^2*(1 - x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{x^2 \sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(x)/x**2/(-x**2+1)**(1/2),x)`

[Out] `Integral(asin(x)/(x**2*sqrt(-(x - 1)*(x + 1))), x)`

3.33 $\int \frac{x \log(x)}{\sqrt{-1+x^2}} dx$

Optimal. Leaf size=34

$$-\sqrt{x^2 - 1} + \sqrt{x^2 - 1} \log(x) + \tan^{-1}\left(\sqrt{x^2 - 1}\right)$$

[Out] $\arctan((x^2 - 1)^{(1/2)}) - (x^2 - 1)^{(1/2)} + \ln(x) * (x^2 - 1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2338, 266, 50, 63, 203}

$$-\sqrt{x^2 - 1} + \sqrt{x^2 - 1} \log(x) + \tan^{-1}\left(\sqrt{x^2 - 1}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \cdot \log[x]) / \sqrt{-1 + x^2}, x]$

[Out] $-\sqrt{-1 + x^2} + \text{ArcTan}[\sqrt{-1 + x^2}] + \sqrt{-1 + x^2} \cdot \log[x]$

Rule 50

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x]; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x}; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x]; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_)*(x_)^(n_.)]*(b_.))^p_*((f_.)*(x_)^m_)*((d_.) +
(e_.)*(x_)^(r_.))^q_, x_Symbol] :> Simp[(f^m*(d + e*x^r)^(q + 1)*(a + b*L
og[c*x^n])^p)/(e*r*(q + 1)), x] - Dist[(b*f^m*n*p)/(e*r*(q + 1)), Int[((d +
e*x^r)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x]] /; FreeQ[{a, b, c, d,
e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \log(x)}{\sqrt{-1+x^2}} dx &= \sqrt{-1+x^2} \log(x) - \int \frac{\sqrt{-1+x^2}}{x} dx \\ &= \sqrt{-1+x^2} \log(x) - \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{-1+x}}{x} dx, x, x^2\right) \\ &= -\sqrt{-1+x^2} + \sqrt{-1+x^2} \log(x) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{-1+x} x} dx, x, x^2\right) \\ &= -\sqrt{-1+x^2} + \sqrt{-1+x^2} \log(x) + \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1+x^2}\right) \\ &= -\sqrt{-1+x^2} + \tan^{-1}\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2} \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.79

$$\sqrt{x^2 - 1} (\log(x) - 1) - \tan^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Log[x])/Sqrt[-1 + x^2], x]`

[Out] `-ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])`

fricas [A] time = 0.44, size = 27, normalized size = 0.79

$$\sqrt{x^2 - 1} (\log(x) - 1) + 2 \arctan\left(-x + \sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))
giac [A] time = 1.01, size = 28, normalized size = 0.82
```

$$\sqrt{x^2 - 1} \log(x) - \sqrt{x^2 - 1} + \arctan\left(\sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))

maple [C]    time = 0.14, size = 119, normalized size = 3.50
```

$$\frac{\sqrt{-\text{signum}(x^2 - 1)} \left(2 - 2\sqrt{-x^2 + 1}\right) \ln(x)}{2\sqrt{\text{signum}(x^2 - 1)}} - \frac{\sqrt{-\text{signum}(x^2 - 1)} \left(2 - 2\sqrt{-x^2 + 1}\right)}{4\sqrt{\text{signum}(x^2 - 1)}} + \frac{\sqrt{-\text{signum}(x^2 - 1)} \left(-3x^2 + 3\right)}{32\sqrt{\text{signum}(x^2 - 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)/(x^2-1)^(1/2),x)
[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))+1/2/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x^2+1)^(1/2))+1/32/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16+16*(-x^2+1)^(1/2)-32*ln(1/2+1/2*(-x^2+1)^(1/2)))
```

$$\sqrt{x^2 - 1} \log(x) - \sqrt{x^2 - 1} = \arcsin\left(\frac{1}{x}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
mupad [F]    time = 0.00, size = -1, normalized size = -0.03
```

$$\int \frac{x \ln(x)}{\sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*log(x))/(x^2 - 1)^(1/2),x)`

[Out] `int((x*log(x))/(x^2 - 1)^(1/2), x)`

sympy [A] time = 2.74, size = 29, normalized size = 0.85

$$\sqrt{x^2 - 1} \log(x) - \begin{cases} \sqrt{x^2 - 1} - \arccos\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x)/(x**2-1)**(1/2),x)`

[Out] `sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - arccos(1/x), (x > -1) & (x < 1)))`

3.34 $\int \frac{\log(x)}{x^2 \sqrt{1+x^2}} dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{x^2+1}}{x} - \frac{\sqrt{x^2+1} \log(x)}{x} + \sinh^{-1}(x)$$

[Out] $\text{arcsinh}(x) - (x^2+1)^{(1/2)}/x - \ln(x)*(x^2+1)^{(1/2)}/x$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2335, 277, 215}

$$-\frac{\sqrt{x^2+1}}{x} - \frac{\sqrt{x^2+1} \log(x)}{x} + \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[x]/(x^2 \text{Sqrt}[1 + x^2]), x]$

[Out] $-(\text{Sqrt}[1 + x^2]/x) + \text{ArcSinh}[x] - (\text{Sqrt}[1 + x^2] \text{Log}[x])/x$

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*(a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2335

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.*))*((f_.)*(x_)^(m_.)*(d_) + (e_.*)
(x_)^(r_.))^q_, x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^r)^(q + 1)*(a +
b*Log[c*x^n]))/(d*f*(m + 1)), x] - Dist[(b*n)/(d*(m + 1)), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x^2\sqrt{1+x^2}} dx &= -\frac{\sqrt{1+x^2}\log(x)}{x} + \int \frac{\sqrt{1+x^2}}{x^2} dx \\
&= -\frac{\sqrt{1+x^2}}{x} - \frac{\sqrt{1+x^2}\log(x)}{x} + \int \frac{1}{\sqrt{1+x^2}} dx \\
&= -\frac{\sqrt{1+x^2}}{x} + \sinh^{-1}(x) - \frac{\sqrt{1+x^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 0.64

$$\sinh^{-1}(x) - \frac{\sqrt{x^2+1}(\log(x)+1)}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[Log[x]/(x^2*.Sqrt[1 + x^2]), x]`

[Out] `ArcSinh[x] - (Sqrt[1 + x^2]*(1 + Log[x]))/x`

fricas [A] time = 0.44, size = 33, normalized size = 1.00

$$-\frac{x \log \left(-x+\sqrt{x^2+1}\right)+\sqrt{x^2+1} (\log (x)+1)+x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2+1)^(1/2), x, algorithm="fricas")`

[Out] `-(x*log(-x + sqrt(x^2 + 1)) + sqrt(x^2 + 1)*(log(x) + 1) + x)/x`

giac [A] time = 0.97, size = 58, normalized size = 1.76

$$\frac{2 \log (x)}{\left(x-\sqrt{x^2+1}\right)^2-1}+\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\log \left(-x+\sqrt{x^2+1}\right)+\log (|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2+1)^(1/2), x, algorithm="giac")`

[Out] `2*log(x)/((x - sqrt(x^2 + 1))^2 - 1) + 2/((x - sqrt(x^2 + 1))^2 - 1) - log(-x + sqrt(x^2 + 1)) + log(abs(x))`

maple [A] time = 0.10, size = 29, normalized size = 0.88

$$\operatorname{arcsinh}(x) + \frac{-\sqrt{x^2 + 1} \ln(x) - \sqrt{x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x)/x^2/(x^2+1)^(1/2),x)`

[Out] `arcsinh(x)+(-ln(x)*(x^2+1)^(1/2)-(x^2+1)^(1/2))/x`

maxima [A] time = 0.97, size = 29, normalized size = 0.88

$$-\frac{\sqrt{x^2 + 1} \log(x)}{x} - \frac{\sqrt{x^2 + 1}}{x} + \operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/x^2/(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `-sqrt(x^2 + 1)*log(x)/x - sqrt(x^2 + 1)/x + arcsinh(x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{x^2 \sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x)/(x^2*(x^2 + 1)^(1/2)),x)`

[Out] `int(log(x)/(x^2*(x^2 + 1)^(1/2)), x)`

sympy [A] time = 5.25, size = 26, normalized size = 0.79

$$\operatorname{asinh}(x) - \frac{\sqrt{x^2 + 1} \log(x)}{x} - \frac{\sqrt{x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)/x**2/(x**2+1)**(1/2),x)`

[Out] `asinh(x) - sqrt(x**2 + 1)*log(x)/x - sqrt(x**2 + 1)/x`

$$3.35 \quad \int \frac{x \sec^{-1}(x)}{\sqrt{-1+x^2}} dx$$

Optimal. Leaf size=25

$$\sqrt{x^2 - 1} \sec^{-1}(x) - \frac{x \log(x)}{\sqrt{x^2}}$$

[Out] $-x \ln(x)/(x^2)^{(1/2)} + \text{arcsec}(x)*(x^2-1)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5236, 29}

$$\sqrt{x^2 - 1} \sec^{-1}(x) - \frac{x \log(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x \text{ArcSec}[x])/(\text{Sqrt}[-1 + x^2], x)]$

[Out] $\text{Sqrt}[-1 + x^2] \text{ArcSec}[x] - (x \log[x])/\text{Sqrt}[x^2]$

Rule 29

$\text{Int}[(x_*)^{(-1)}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\log[x], x]$

Rule 5236

```
Int[((a_.) + ArcSec[(c_)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simpl[((d + e*x^2)^(p + 1)*(a + b*ArcSec[c*x]))/(2*e*(p + 1)), x]
] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \sec^{-1}(x)}{\sqrt{-1+x^2}} dx &= \sqrt{-1+x^2} \sec^{-1}(x) - \frac{x \int \frac{1}{x} dx}{\sqrt{x^2}} \\ &= \sqrt{-1+x^2} \sec^{-1}(x) - \frac{x \log(x)}{\sqrt{x^2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 1.40

$$\frac{\left(x^2 - 1\right) \sec^{-1}(x) - \sqrt{1 - \frac{1}{x^2}} x \log(x)}{\sqrt{x^2 - 1}}$$

Antiderivative was successfully verified.

[In] `Integrate[(x*ArcSec[x])/Sqrt[-1 + x^2], x]`

[Out] $((-1 + x^2) \operatorname{ArcSec}[x] - \sqrt{1 - x^{(-2)}} x \operatorname{Log}[x]) / \sqrt{-1 + x^2}$

fricas [A] time = 0.44, size = 15, normalized size = 0.60

$$\sqrt{x^2 - 1} \operatorname{arcsec}(x) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsec(x)/(x^2-1)^(1/2), x, algorithm="fricas")`

[Out] $\sqrt{x^2 - 1} \operatorname{arcsec}(x) - \log(x)$

giac [A] time = 1.09, size = 22, normalized size = 0.88

$$\sqrt{x^2 - 1} \arccos\left(\frac{1}{x}\right) - \frac{\log(|x|)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsec(x)/(x^2-1)^(1/2), x, algorithm="giac")`

[Out] $\sqrt{x^2 - 1} \operatorname{arccos}(1/x) - \log(\operatorname{abs}(x)) / \operatorname{sgn}(x)$

maple [C] time = 0.51, size = 97, normalized size = 3.88

$$-\frac{2i\sqrt{\frac{x^2-1}{x^2}} x \operatorname{arcsec}(x)}{\sqrt{x^2 - 1}} + \frac{\sqrt{\frac{x^2-1}{x^2}} x \ln\left(\left(\frac{1}{x} + i\sqrt{-\frac{1}{x^2} + 1}\right)^2 + 1\right)}{\sqrt{x^2 - 1}} + \frac{\left(x^2 + i\sqrt{\frac{x^2-1}{x^2}} x - 1\right) \operatorname{arcsec}(x)}{\sqrt{x^2 - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsec(x)/(x^2-1)^(1/2), x)`

[Out] $-2*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*\operatorname{arcsec}(x) + 1/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+\operatorname{arcsec}(x)+1/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*\ln((1/x+I*(-1/x^2+1)^(1/2))^2+1)$

maxima [A] time = 0.47, size = 15, normalized size = 0.60

$$\sqrt{x^2 - 1} \operatorname{arcsec}(x) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsec(x)/(x^2-1)^(1/2), x, algorithm="maxima")`

[Out] `sqrt(x^2 - 1)*arcsec(x) - log(x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x \cos\left(\frac{1}{x}\right)}{\sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*acos(1/x))/(x^2 - 1)^(1/2), x)`

[Out] `int((x*acos(1/x))/(x^2 - 1)^(1/2), x)`

sympy [A] time = 19.71, size = 22, normalized size = 0.88

$$\sqrt{x^2 - 1} \operatorname{asec}(x) - \begin{cases} -\log\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asec(x)/(x**2-1)**(1/2), x)`

[Out] `sqrt(x**2 - 1)*asec(x) - Piecewise((-log(1/x), (x > -1) & (x < 1)))`

3.36 $\int \frac{x \log(x)}{\sqrt{1+x^2}} dx$

Optimal. Leaf size=34

$$-\sqrt{x^2 + 1} + \sqrt{x^2 + 1} \log(x) + \tanh^{-1}(\sqrt{x^2 + 1})$$

[Out] $\operatorname{arctanh}((x^{2+1})^{(1/2)}) - (x^{2+1})^{(1/2)} + \ln(x) * (x^{2+1})^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2338, 266, 50, 63, 207}

$$-\sqrt{x^2 + 1} + \sqrt{x^2 + 1} \log(x) + \tanh^{-1}(\sqrt{x^2 + 1})$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x \operatorname{Log}[x])/\operatorname{Sqrt}[1 + x^2], x]$

[Out] $-\operatorname{Sqrt}[1 + x^2] + \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + x^2]] + \operatorname{Sqrt}[1 + x^2] \operatorname{Log}[x]$

Rule 50

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x]; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x}; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]]
```

Rule 207

```
Int[((a_) + (b_)*(x_))^2 ^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x]; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_)*(x_)^(n_.)]*(b_.))^p_*((f_.)*(x_)^m_)*(d_ +
(e_.)*(x_)^(r_.))^q_, x_Symbol] :> Simp[(f^m*(d + e*x^r)^(q + 1)*(a + b*L
og[c*x^n])^p)/(e*r*(q + 1)), x] - Dist[(b*f^m*n*p)/(e*r*(q + 1)), Int[((d +
e*x^r)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x] /; FreeQ[{a, b, c, d,
e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \log(x)}{\sqrt{1+x^2}} dx &= \sqrt{1+x^2} \log(x) - \int \frac{\sqrt{1+x^2}}{x} dx \\
&= \sqrt{1+x^2} \log(x) - \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{1+x}}{x} dx, x, x^2\right) \\
&= -\sqrt{1+x^2} + \sqrt{1+x^2} \log(x) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^2\right) \\
&= -\sqrt{1+x^2} + \sqrt{1+x^2} \log(x) - \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^2}\right) \\
&= -\sqrt{1+x^2} + \tanh^{-1}\left(\sqrt{1+x^2}\right) + \sqrt{1+x^2} \log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.18

$$-\sqrt{x^2+1} + \sqrt{x^2+1} \log(x) + \log\left(\sqrt{x^2+1} + 1\right) - \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Log[x])/Sqrt[1 + x^2], x]`

[Out] `-Sqrt[1 + x^2] - Log[x] + Sqrt[1 + x^2]*Log[x] + Log[1 + Sqrt[1 + x^2]]`

fricas [A] time = 0.42, size = 41, normalized size = 1.21

$$\sqrt{x^2+1} (\log(x) - 1) + \log\left(-x + \sqrt{x^2+1} + 1\right) - \log\left(-x + \sqrt{x^2+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $\sqrt{x^2 + 1} \cdot (\log(x) - 1) + \log(-x + \sqrt{x^2 + 1} + 1) - \log(-x + \sqrt{x^2 + 1} - 1)$

giac [A] time = 1.12, size = 44, normalized size = 1.29

$$\sqrt{x^2 + 1} \log(x) - \sqrt{x^2 + 1} + \frac{1}{2} \log\left(\sqrt{x^2 + 1} + 1\right) - \frac{1}{2} \log\left(\sqrt{x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="giac")`

[Out] $\sqrt{x^2 + 1} \cdot \log(x) - \sqrt{x^2 + 1} + 1/2 \cdot \log(\sqrt{x^2 + 1} + 1) - 1/2 \cdot \log(\sqrt{x^2 + 1} - 1)$

maple [A] time = 0.10, size = 39, normalized size = 1.15

$$\frac{(-2 + 2\sqrt{x^2 + 1}) \ln(x)}{2} + \ln\left(\frac{1}{2} + \frac{\sqrt{x^2 + 1}}{2}\right) + 1 - \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(x)/(x^2+1)^(1/2),x)`

[Out] $1 - (x^2 + 1)^{(1/2)} + 1/2 \cdot \ln(x) \cdot (-2 + 2 \cdot (x^2 + 1)^{(1/2)}) + \ln(1/2 + 1/2 \cdot (x^2 + 1)^{(1/2)})$

maxima [A] time = 0.96, size = 25, normalized size = 0.74

$$\sqrt{x^2 + 1} \log(x) - \sqrt{x^2 + 1} + \text{arsinh}\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $\sqrt{x^2 + 1} \cdot \log(x) - \sqrt{x^2 + 1} + \text{arcsinh}(1/\text{abs}(x))$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x \ln(x)}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*log(x))/(x^2 + 1)^(1/2),x)`

[Out] `int((x*log(x))/(x^2 + 1)^(1/2), x)`

sympy [A] time = 4.34, size = 41, normalized size = 1.21

$$-\frac{x}{\sqrt{1+\frac{1}{x^2}}} + \sqrt{x^2+1} \log(x) + \operatorname{asinh}\left(\frac{1}{x}\right) - \frac{1}{x\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x)/(x**2+1)**(1/2),x)`

[Out] `-x/sqrt(1 + x**(-2)) + sqrt(x**2 + 1)*log(x) + asinh(1/x) - 1/(x*sqrt(1 + x**(-2)))`

3.37 $\int \frac{\sin(x)}{1+\sin^2(x)} dx$

Optimal. Leaf size=16

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\text{arctanh}(1/2*\cos(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3186, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]/(1 + \text{Sin}[x]^2), x]$

[Out] $-(\text{ArcTanh}[\text{Cos}[x]/\text{Sqrt}[2]])/\text{Sqrt}[2])$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_{\text{Symbol}}] \Rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \& \text{NegQ}[a/b] \& (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 3186

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(-p_.)}, x_{\text{Symbol}}] \Rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, S \text{ubst}[\text{Int}[(1 - ff^2*x^2)^{((m - 1)/2)}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \& \text{IntegerQ}[(m - 1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{1+\sin^2(x)} dx &= -\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \cos(x)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.05, size = 46, normalized size = 2.88

$$-\frac{i \left(\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-i}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+i}{\sqrt{2}}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[x]/(1 + Sin[x]^2), x]`

[Out] $\frac{((-I) (\text{ArcTan}[-I+\tan(x/2)]/\text{Sqrt}[2]) - \text{ArcTan}[(I+\tan(x/2))/\text{Sqrt}[2]])/\text{Sqrt}[2]}{2}$

fricas [B] time = 0.44, size = 29, normalized size = 1.81

$$\frac{1}{4} \sqrt{2} \log\left(-\frac{\cos(x)^2 - 2 \sqrt{2} \cos(x) + 2}{\cos(x)^2 - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1+sin(x)^2), x, algorithm="fricas")`

[Out] $\frac{1}{4} \sqrt{2} \log(-\cos(x)^2 - 2 \sqrt{2} \cos(x) + 2)/(\cos(x)^2 - 2)$

giac [B] time = 1.00, size = 27, normalized size = 1.69

$$-\frac{1}{4} \sqrt{2} \log(\sqrt{2} + \cos(x)) + \frac{1}{4} \sqrt{2} \log(\sqrt{2} - \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1+sin(x)^2), x, algorithm="giac")`

[Out] $-\frac{1}{4} \sqrt{2} \log(\sqrt{2} + \cos(x)) + \frac{1}{4} \sqrt{2} \log(\sqrt{2} - \cos(x))$

maple [A] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(1+sin(x)^2), x)`

[Out] $-1/2 \operatorname{arctanh}(1/2 \cos(x) \cdot 2^{1/2}) \cdot 2^{1/2}$

maxima [A] time = 0.95, size = 24, normalized size = 1.50

$$\frac{1}{4} \sqrt{2} \log\left(-\frac{\sqrt{2} - \cos(x)}{\sqrt{2} + \cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+sin(x)^2),x, algorithm="maxima")

[Out] $\frac{1}{4} \sqrt{2} \log\left(\frac{-\sqrt{2} - \cos(x)}{\sqrt{2} + \cos(x)}\right)$

mupad [B] time = 0.21, size = 13, normalized size = 0.81

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \cos(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(sin(x)^2 + 1),x)

[Out] $-\frac{(2^{1/2}) \operatorname{atanh}\left((2^{1/2}) \cos(x)\right)}{2}$

sympy [B] time = 22.87, size = 46, normalized size = 2.88

$$\frac{\sqrt{2} \log \left(\tan ^2\left(\frac{x}{2}\right)-2 \sqrt{2}+3\right)}{4}-\frac{\sqrt{2} \log \left(\tan ^2\left(\frac{x}{2}\right)+2 \sqrt{2}+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+sin(x)**2),x)

[Out] $\sqrt{2} \log (\tan (x/2)^2-2 \sqrt{2}+3)/4-\sqrt{2} \log (\tan (x/2)^2+2 \sqrt{2}+3)/4$

3.38 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

[Out] $1/2*\operatorname{arctanh}(x*2^{(1/2)}/(x^{4+1})^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1+x^2)/((1-x^2)*\operatorname{Sqrt}[1+x^4]), x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*x)/\operatorname{Sqrt}[1+x^4]]/\operatorname{Sqrt}[2]$

Rule 206

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&& \operatorname{NegQ}[a/b] \&& (\operatorname{GtQ}[a, 0] \text{ || } \operatorname{LtQ}[b, 0])$

Rule 1699

$\operatorname{Int}[((A_.) + (B_.)*(x_.)^2)/(((d_.) + (e_.)*(x_.)^2)*\operatorname{Sqrt}[(a_.) + (c_.)*(x_.)^4]), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[A, \operatorname{Subst}[\operatorname{Int}[1/(d + 2*a*e*x^2), x], x, x/\operatorname{Sqrt}[a + c*x^4]], x] /; \operatorname{FreeQ}[\{a, c, d, e, A, B\}, x] \&& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \&& \operatorname{EqQ}[c*d^2 - a*e^2, 0] \&& \operatorname{EqQ}[B*d + A*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx &= \operatorname{Subst}\left(\int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.10, size = 36, normalized size = 1.57

$$\sqrt[4]{-1} \left(\text{EllipticF}\left(i \sinh^{-1}\left(\sqrt[4]{-1} x\right), -1\right) - 2 \Pi\left(i; \sin^{-1}\left((-1)^{3/4} x\right)\Big| -1\right) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]), x]`

[Out] $(-1)^{(1/4)} * (\text{EllipticF}[I * \text{ArcSinh}[(-1)^{(1/4)} * x], -1] - 2 * \text{EllipticPi}[I, \text{ArcSin}[(-1)^{(3/4)} * x], -1])$

fricas [B] time = 0.46, size = 42, normalized size = 1.83

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^4 + 2 \sqrt{2} \sqrt{x^4 + 1} x + 2 x^2 + 1}{x^4 - 2 x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2), x, algorithm="fricas")`

[Out] $\frac{1}{4} \sqrt{2} \log\left(\frac{(x^4 + 2 \sqrt{2} \sqrt{x^4 + 1} * x + 2 * x^2 + 1) / (x^4 - 2 * x^2 + 1)}{(x^4 + 1)}\right)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 1}{\sqrt{x^4 + 1} (x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2), x, algorithm="giac")`

[Out] `integrate(-(x^2 + 1) / (sqrt(x^4 + 1) * (x^2 - 1)), x)`

maple [C] time = 0.09, size = 112, normalized size = 4.87

$$\frac{\sqrt{-ix^2 + 1} \sqrt{ix^2 + 1} \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)x, i\right) - 2(-1)^{\frac{3}{4}} \sqrt{-ix^2 + 1} \sqrt{ix^2 + 1} \text{EllipticPi}\left((-1)^{\frac{1}{4}}x, -i, -\sqrt{-i} (-\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}) \sqrt{x^4 + 1}\right)}{\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right) \sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2), x)`

[Out] $\frac{-1/(1/2*2^{(1/2)}+1/2*I*2^{(1/2)})*(1-I*x^2)^{(1/2)}*(1+I*x^2)^{(1/2)}}{(x^4+1)^{(1/2)}} * \text{EllipticF}(x*(1/2*2^{(1/2)}+1/2*I*2^{(1/2)}), I) - 2*(-1)^{(3/4)}*(1-I*x^2)^{(1/2)} * ($

$1 + I*x^2)^{(1/2)} / (x^4 + 1)^{(1/2)} * \text{EllipticPi}((-1)^{(1/4)}*x, -I, (-I)^{(1/2)} / (-1)^{(1/4)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 1}{\sqrt{x^4 + 1} (x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2), x, algorithm="maxima")

[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2 + 1}{(x^2 - 1) \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)

[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4 + 1} - \sqrt{x^4 + 1}} dx - \int \frac{1}{x^2\sqrt{x^4 + 1} - \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2), x)

[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)

3.39 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

[Out] $1/2*\arctan(x*2^{(1/2)}/(x^4+1)^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x^2)/((1 + x^2)*\text{Sqrt}[1 + x^4]), x]$

[Out] $\text{ArcTan}[(\text{Sqrt}[2]*x)/\text{Sqrt}[1 + x^4]]/\text{Sqrt}[2]$

Rule 203

$\text{Int}[(a_1 + b_1)*(x_1^2)^{-1}, x_1] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x_1] /; \text{FreeQ}[\{a, b\}, x_1] \&& \text{PosQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

Rule 1699

$\text{Int}[(a_1 + b_1)*(x_1^2)/(((d_1 + e_1)*(x_1^2))*\text{Sqrt}[(a_1 + c_1)*(x_1^4)]), x_1] \rightarrow \text{Dist}[a_1, \text{Subst}[\text{Int}[1/(d_1 + 2*a_1*e_1*x_1^2), x_1], x_1, x_1/\text{Sqrt}[a_1 + c_1*x_1^4]], x_1] /; \text{FreeQ}[\{a, c, d, e, A, B\}, x_1] \&& \text{NeQ}[c_1*d_1^2 + a_1*e_1^2, 0] \&& \text{EqQ}[c_1*d_1^2 - a_1*e_1^2, 0] \&& \text{EqQ}[B*d_1 + A*e_1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx &= \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.10, size = 40, normalized size = 1.74

$$\sqrt[4]{-1} \left(\text{EllipticF}\left(i \sinh^{-1}\left(\sqrt[4]{-1} x\right), -1\right) - 2 \Pi\left(-i; i \sinh^{-1}\left(\sqrt[4]{-1} x\right)\right| - 1\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]), x]`

[Out] $(-1)^{(1/4)} * (\text{EllipticF}[I \text{ArcSinh}[(-1)^{(1/4)} x], -1] - 2 \text{EllipticPi}[-I, I \text{ArcSinh}[(-1)^{(1/4)} x], -1])$

fricas [A] time = 0.46, size = 18, normalized size = 0.78

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\sqrt{x^4 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2), x, algorithm="fricas")`

[Out] $\frac{1}{2} \sqrt{2} \arctan(\sqrt{2} x / \sqrt{x^4 + 1})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{\sqrt{x^4 + 1}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2), x, algorithm="giac")`

[Out] `integrate(-(x^2 - 1)/(\sqrt{x^4 + 1}*(x^2 + 1)), x)`

maple [C] time = 0.02, size = 112, normalized size = 4.87

$$\frac{\sqrt{-ix^2 + 1} \sqrt{ix^2 + 1} \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)x, i\right) - 2(-1)^{\frac{3}{4}} \sqrt{-ix^2 + 1} \sqrt{ix^2 + 1} \text{EllipticPi}\left((-1)^{\frac{1}{4}}x, i, -\sqrt{-i} (-\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2})\right)}{\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2), x)`

[Out] $\frac{-1/(1/2*2^{(1/2)}+1/2*I*2^{(1/2)})*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2) - 2*(-1)^(3/4)*(-I*x^2+1)^(1/2)}{2*I}$

$\ast(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x, I, (-I)^(1/2)/(-1)^(1/4))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{\sqrt{x^4 + 1} (x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^2 - 1}{(x^2 + 1) \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)

[Out] -int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2 \sqrt{x^4 + 1} + \sqrt{x^4 + 1}} dx - \int \left(-\frac{1}{x^2 \sqrt{x^4 + 1} + \sqrt{x^4 + 1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2), x)

[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)

3.40 $\int \frac{\log(\sin(x))}{1+\sin(x)} dx$

Optimal. Leaf size=22

$$-x - \tanh^{-1}(\cos(x)) - \frac{\cos(x) \log(\sin(x))}{\sin(x) + 1}$$

[Out] $-x - \text{arctanh}(\cos(x)) - \cos(x) \ln(\sin(x)) / (1 + \sin(x))$

Rubi [A] time = 0.07, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2648, 2554, 2839, 3770, 8}

$$-x - \tanh^{-1}(\cos(x)) - \frac{\cos(x) \log(\sin(x))}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[\text{Sin}[x]] / (1 + \text{Sin}[x]), x]$

[Out] $-x - \text{ArcTanh}[\text{Cos}[x]] - (\text{Cos}[x] * \text{Log}[\text{Sin}[x]]) / (1 + \text{Sin}[x])$

Rule 8

$\text{Int}[a_-, x_{\text{Symbol}}] :> \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2554

$\text{Int}[\text{Log}[u_] * (v_), x_{\text{Symbol}}] :> \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/u, x], x] /; \text{InverseFunctionFreeQ}[w, x]] /; \text{InverseFunctionFreeQ}[u, x]$

Rule 2648

$\text{Int}[((a_) + (b_) * \sin[(c_) + (d_) * (x_)])^{(-1)}, x_{\text{Symbol}}] :> -\text{Simp}[\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&& \text{EqQ}[a^2 - b^2, 0]$

Rule 2839

$\text{Int}[((\cos[e_] + (f_) * (x_)) * (g_))^{(p_)} * ((d_) * \sin[(e_) + (f_) * (x_)])^{(n_)} / ((a_) + (b_) * \sin[(e_) + (f_) * (x_)]), x_{\text{Symbol}}] :> \text{Dist}[g^2/a, \text{Int}[(g*\text{Cos}[e + f*x])^{(p - 2)} * (d*\text{Sin}[e + f*x])^n, x], x] - \text{Dist}[g^2/(b*d), \text{Int}[(g*\text{Cos}[e + f*x])^{(p - 2)} * (d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, g, n, p\}, x] \&& \text{EqQ}[a^2 - b^2, 0]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(\sin(x))}{1 + \sin(x)} dx &= -\frac{\cos(x) \log(\sin(x))}{1 + \sin(x)} + \int \frac{\cos(x) \cot(x)}{1 + \sin(x)} dx \\ &= -\frac{\cos(x) \log(\sin(x))}{1 + \sin(x)} - \int 1 dx + \int \csc(x) dx \\ &= -x - \tanh^{-1}(\cos(x)) - \frac{\cos(x) \log(\sin(x))}{1 + \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.77

$$-x - 2 \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{2 \sin\left(\frac{x}{2}\right) \log(\sin(x))}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[Sin[x]]/(1 + Sin[x]), x]

[Out] $-x - 2 \log[\cos(x/2)] + (2 \log[\sin(x)] \sin(x/2)) / (\cos(x/2) + \sin(x/2))$

fricas [B] time = 0.45, size = 93, normalized size = 4.23

$$\frac{4 (\cos(x) + \sin(x) + 1) \arctan\left(-\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}\right) + 4 x \cos(x) + (\cos(x) + \sin(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x) + \sin(x) + 1) \log(-\cos(x) - \sin(x) - 1)}{2 (\cos(x) + \sin(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(sin(x))/(1+sin(x)), x, algorithm="fricas")

[Out] $-1/2*(4*(\cos(x) + \sin(x) + 1)*\arctan(-(\cos(x) + \sin(x) + 1)/(\cos(x) - \sin(x) + 1)) + 4*x*\cos(x) + (\cos(x) + \sin(x) + 1)*\log(1/2*\cos(x) + 1/2) - (\cos(x) + \sin(x) + 1)*\log(-1/2*\cos(x) + 1/2) + 2*(\cos(x) - \sin(x) + 1)*\log(\sin(x) + 4*x)/(\cos(x) + \sin(x) + 1)$

giac [A] time = 1.13, size = 36, normalized size = 1.64

$$-x - \frac{2 \log(\sin(x))}{\tan\left(\frac{1}{2}x\right) + 1} - 2 \log\left(\tan\left(\frac{1}{4}x\right)^2 + 1\right) + 2 \log\left(\left|\tan\left(\frac{1}{4}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(sin(x))/(1+sin(x)),x, algorithm="giac")`

[Out] $-x - \frac{2 \log(\sin(x))}{\tan(1/2*x) + 1} - 2 \log(\tan(1/4*x)^2 + 1) + 2 \log(\text{abs}(\tan(1/4*x)))$

maple [B] time = 0.17, size = 54, normalized size = 2.45

$$\ln\left(\tan^2\left(\frac{x}{2}\right) + 1\right) + \frac{-x \tan\left(\frac{x}{2}\right) + 2 \ln\left(\frac{2 \tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 1}\right) \tan\left(\frac{x}{2}\right) - x}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(sin(x))/(sin(x)+1),x)`

[Out] $\frac{(-x - x \tan(1/2*x) + 2 \tan(1/2*x) \ln(2 \tan(1/2*x) / (\tan(1/2*x)^2 + 1))) / (\tan(1/2*x) + 1) + \ln(\tan(1/2*x)^2 + 1)}$

maxima [B] time = 0.97, size = 82, normalized size = 3.73

$$-\frac{2 \log\left(\frac{2 \sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)(\cos(x)+1)}\right)}{\frac{\sin(x)}{\cos(x)+1} + 1} - 2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + 2 \log\left(\frac{\sin(x)}{\cos(x)+1}\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(sin(x))/(1+sin(x)),x, algorithm="maxima")`

[Out] $\frac{-2 \log(2 \sin(x) / ((\sin(x)^2 / (\cos(x) + 1)^2 + 1) * (\cos(x) + 1))) / (\sin(x) / (\cos(x) + 1) + 1) - 2 \arctan(\sin(x) / (\cos(x) + 1)) + 2 \log(\sin(x) / (\cos(x) + 1)) - \log(\sin(x)^2 / (\cos(x) + 1)^2 + 1)}$

mupad [B] time = 0.38, size = 55, normalized size = 2.50

$$-2x + \ln(2 \sin(x) - \cos(x) 2i - 2i) (-1 - i) + \ln(2 \sin(x) - \cos(x) 2i + 2i) (1 - i) - \frac{2 \ln(\sin(x))}{\cos(x) + \sin(x) 1i + 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(sin(x))/(sin(x) + 1),x)`

[Out] $\log(2 \sin(x) - \cos(x) * 2i + 2i) * (1 - 1i) - \log(2 \sin(x) - \cos(x) * 2i - 2i) * (1 + 1i) - 2x - (2 \log(\sin(x))) / (\cos(x) + \sin(x) * 1i + 1i)$

sympy [B] time = 1.37, size = 97, normalized size = 4.41

$$-\frac{x \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right) + 1} - \frac{x}{\tan\left(\frac{x}{2}\right) + 1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 1\right) \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right) + 1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 1\right)}{\tan\left(\frac{x}{2}\right) + 1} + \frac{2 \log\left(\tan\left(\frac{x}{2}\right)\right) \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right) + 1} + \frac{2 \log(2) \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(sin(x))/(1+sin(x)),x)

[Out]
$$-x*\tan(x/2)/(\tan(x/2) + 1) - x/(\tan(x/2) + 1) - \log(\tan(x/2)**2 + 1)*\tan(x/2)/(\tan(x/2) + 1) + \log(\tan(x/2)**2 + 1)/(\tan(x/2) + 1) + 2*\log(\tan(x/2))*\tan(x/2)/(\tan(x/2) + 1) + 2*\log(2)*\tan(x/2)/(\tan(x/2) + 1)$$

3.41 $\int \log(\sin(x))\sqrt{1 + \sin(x)} dx$

Optimal. Leaf size=42

$$\frac{4 \cos(x)}{\sqrt{\sin(x) + 1}} - \frac{2 \cos(x) \log(\sin(x))}{\sqrt{\sin(x) + 1}} - 4 \tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\sin(x) + 1}}\right)$$

[Out] $-4*\text{arctanh}(\cos(x)/(1+\sin(x))^{(1/2)})+4*\cos(x)/(1+\sin(x))^{(1/2)}-2*\cos(x)*\ln(\sin(x))/(1+\sin(x))^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2646, 2554, 12, 2874, 2981, 2773, 206}

$$\frac{4 \cos(x)}{\sqrt{\sin(x) + 1}} - \frac{2 \cos(x) \log(\sin(x))}{\sqrt{\sin(x) + 1}} - 4 \tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\sin(x) + 1}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[\text{Sin}[x]]*\text{Sqrt}[1 + \text{Sin}[x]], x]$

[Out] $-4*\text{ArcTanh}[\text{Cos}[x]/\text{Sqrt}[1 + \text{Sin}[x]]] + (4*\text{Cos}[x])/\text{Sqrt}[1 + \text{Sin}[x]] - (2*\text{Cos}[x]*\text{Log}[\text{Sin}[x]])/\text{Sqrt}[1 + \text{Sin}[x]]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&& \text{!MatchQ}[u, (b_)*(v_) /; \text{FreeQ}[b, x]]$

Rule 206

$\text{Int}[((a_) + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{NegQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 2554

$\text{Int}[\text{Log}[u_]*(v_), x_Symbol] :> \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/u, x], x] /; \text{InverseFunctionFreeQ}[w, x]] /; \text{InverseFunctionFreeQ}[u, x]$

Rule 2646

$\text{Int}[\text{Sqrt}[(a_) + (b_.)*\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> \text{Simp}[(-2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&& \text{Eq}$

$\mathbb{Q}[a^2 - b^2, 0]$

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)*(x_.)]]/((c_.) + (d_)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_.)]^2*((d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_)*((a_) + (b_)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] || !IGtQ[n, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)*(x_.)]]*((A_.) + (B_)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \log(\sin(x)) \sqrt{1 + \sin(x)} \, dx &= -\frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}} - \int -\frac{2 \cos(x) \cot(x)}{\sqrt{1 + \sin(x)}} \, dx \\
&= -\frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}} + 2 \int \frac{\cos(x) \cot(x)}{\sqrt{1 + \sin(x)}} \, dx \\
&= -\frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}} + 2 \int \csc(x)(1 - \sin(x)) \sqrt{1 + \sin(x)} \, dx \\
&= \frac{4 \cos(x)}{\sqrt{1 + \sin(x)}} - \frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}} + 2 \int \csc(x) \sqrt{1 + \sin(x)} \, dx \\
&= \frac{4 \cos(x)}{\sqrt{1 + \sin(x)}} - \frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}} - 4 \text{Subst} \left(\int \frac{1}{1 - x^2} \, dx, x, \frac{\cos(x)}{\sqrt{1 + \sin(x)}} \right) \\
&= -4 \tanh^{-1} \left(\frac{\cos(x)}{\sqrt{1 + \sin(x)}} \right) + \frac{4 \cos(x)}{\sqrt{1 + \sin(x)}} - \frac{2 \cos(x) \log(\sin(x))}{\sqrt{1 + \sin(x)}}
\end{aligned}$$

Mathematica [B] time = 0.09, size = 87, normalized size = 2.07

$$\frac{2 \sqrt{\sin(x)+1} \left(\sin \left(\frac{x}{2}\right) (\log (\sin (x))-2)-\log \left(-\sin \left(\frac{x}{2}\right)+\cos \left(\frac{x}{2}\right)+1\right)+\log \left(\sin \left(\frac{x}{2}\right)-\cos \left(\frac{x}{2}\right)+1\right)-\cos \left(\frac{x}{2}\right)\right)}{\sin \left(\frac{x}{2}\right)+\cos \left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] `Integrate[Log[Sin[x]]*Sqrt[1 + Sin[x]], x]`

[Out] $\frac{(2*(-\text{Log}[1 + \text{Cos}[x/2] - \text{Sin}[x/2]] + \text{Log}[1 - \text{Cos}[x/2] + \text{Sin}[x/2]] - \text{Cos}[x/2]*(-2 + \text{Log}[\text{Sin}[x]]) + (-2 + \text{Log}[\text{Sin}[x]])*\text{Sin}[x/2])* \text{Sqrt}[1 + \text{Sin}[x]])}{(\text{Cos}[x/2] + \text{Sin}[x/2])}$

fricas [B] time = 0.49, size = 146, normalized size = 3.48

$$\frac{(\cos(x) + \sin(x) + 1) \log \left(\frac{\cos(x)^2 - (\cos(x) - 1) \sin(x) + 2 (\cos(x) - \sin(x) + 1) \sqrt{\sin(x) + 1} + 2 \cos(x) + 1}{2 (\cos(x) + \sin(x) + 1)} \right) - (\cos(x) + \sin(x) + 1) \log \left(\frac{\cos(x)^2 - (\cos(x) - 1) \sin(x) + 2 (\cos(x) - \sin(x) + 1) \sqrt{\sin(x) + 1} + 2 \cos(x) + 1}{2 (\cos(x) + \sin(x) + 1)} \right)}{2 (\cos(x) + \sin(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(sin(x))*(1+sin(x))^(1/2), x, algorithm="fricas")`

[Out] $-\frac{((\cos(x) + \sin(x) + 1)*\log(1/2*(\cos(x)^2 - (\cos(x) - 1)*\sin(x) + 2*(\cos(x) - \sin(x) + 1)*\sqrt{\sin(x) + 1} + 2*\cos(x) + 1))/(\cos(x) + \sin(x) + 1)) - ((\cos(x) + \sin(x) + 1)*\log(1/2*(\cos(x)^2 - (\cos(x) - 1)*\sin(x) - 2*(\cos(x) - \sin(x) + 1)*\sqrt{\sin(x) + 1} + 2*\cos(x) + 1))/(\cos(x) + \sin(x) + 1)) + 2*((\cos(x) + \sin(x) + 1)*\sqrt{\sin(x) + 1} + 2*\cos(x) + 1))/(\cos(x) + \sin(x) + 1)}$

$s(x) = \sin(x) + 1) * \log(\sin(x)) - 2 * \cos(x) + 2 * \sin(x) - 2 * \sqrt{\sin(x) + 1}) / (\cos(x) + \sin(x) + 1)$

giac [B] time = 1.37, size = 94, normalized size = 2.24

$$\sqrt{2} \left(2 \log(\sin(x)) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right) + \left(\sqrt{2} \log\left(\left|\tan\left(\frac{1}{4}x\right) + 1\right|\right) - \sqrt{2} \log\left(\left|\tan\left(\frac{1}{4}x\right)\right|\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(sin(x))*(1+sin(x))^(1/2),x, algorithm="giac")`

[Out] $\sqrt{2} * (2 * \log(\sin(x)) * \operatorname{sgn}(\cos(-1/4*\pi + 1/2*x)) * \sin(-1/4*\pi + 1/2*x) + (\sqrt{2} * \log(\operatorname{abs}(\tan(1/4*x) + 1)) - \sqrt{2} * \log(\operatorname{abs}(\tan(1/4*x) - 1)) + \sqrt{2} * \log(\operatorname{abs}(\tan(1/4*x))) - 4 * \sqrt{2} * (\tan(1/4*x) - 1) / (\tan(1/4*x)^2 + 1)) * \operatorname{sgn}(\cos(-1/4*\pi + 1/2*x)))$

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) + 1} \ln(\sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(sin(x))*(sin(x)+1)^(1/2),x)`

[Out] `int(ln(sin(x))*(sin(x)+1)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) + 1} \log(\sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(sin(x))*(1+sin(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(sin(x) + 1)*log(sin(x)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(\sin(x)) \sqrt{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(sin(x))*(sin(x) + 1)^(1/2),x)`

[Out] $\text{int}(\log(\sin(x)) * (\sin(x) + 1)^{(1/2)}, x)$
sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) + 1} \log(\sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\ln(\sin(x)) * (1 + \sin(x))^{(1/2)}, x)$
[Out] Integral(sqrt(sin(x) + 1) * log(sin(x)), x)

3.42 $\int \frac{\sec(x)}{\sqrt{-1+\sec^4(x)}} dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)\cot(x)\sqrt{\sec^4(x)-1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*\cos(x)*\cot(x)*(-1+\sec(x)^4)^{(1/2)}*2^{(1/2)})*2^{(1/2)}$

Rubi [B] time = 0.18, antiderivative size = 59, normalized size of antiderivative = 2.11, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4148, 6722, 1988, 2008, 206}

$$-\frac{\sqrt{1-\cos^4(x)} \sec^2(x) \tanh^{-1}\left(\frac{\sqrt{2} \sin(x)}{\sqrt{2 \sin^2(x)-\sin^4(x)}}\right)}{\sqrt{2} \sqrt{\sec^4(x)-1}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[x]/\operatorname{Sqrt}[-1 + \operatorname{Sec}[x]^4], x]$

[Out] $-((\operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*\sin[x])/(\operatorname{Sqrt}[2*\sin[x]^2 - \sin[x]^4])*(\operatorname{Sqrt}[1 - \cos[x]^4]*\sec[x]^2)/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-1 + \operatorname{Sec}[x]^4])))$

Rule 206

$\operatorname{Int}[((a_) + (b_*)*(x_)^2)^{(-1)}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&& \operatorname{NegQ}[a/b] \&& (\operatorname{GtQ}[a, 0] \|\operatorname{LtQ}[b, 0])$

Rule 1988

$\operatorname{Int}[(u_)^{(p_)}, x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{ExpandToSum}[u, x]^p, x] /; \operatorname{FreeQ}[p, x] \&& \operatorname{TrinomialQ}[u, x] \&& !\operatorname{TrinomialMatchQ}[u, x]$

Rule 2008

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*)*(x_)^2 + (b_*)*(x_)^{(n_*)}], x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[2/(2 - n), \operatorname{ubst}[\operatorname{Int}[1/(1 - a*x^2), x], x, x/\operatorname{Sqrt}[a*x^2 + b*x^n]], x] /; \operatorname{FreeQ}[\{a, b, n\}, x] \&& \operatorname{NeQ}[n, 2]$

Rule 4148

$\operatorname{Int}[\sec[(e_*) + (f_*)*(x_)]^{(m_*)}*((a_) + (b_*)*\sec[(e_*) + (f_*)*(x_)]^{(n_*)})^{(p_)}, x_{\text{Symbol}}] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin[e + f*x], x]\}, \operatorname{Dist}[ff/f,$

```
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6722

```
Int[(u_)*(a_ + (b_)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bi
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{\sqrt{-1 + \sec^4(x)}} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2) \sqrt{-1 + \frac{1}{(1-x^2)^2}}} dx, x, \sin(x) \right) \\
&= \frac{(\sqrt{1 - \cos^4(x)} \sec^2(x)) \text{Subst} \left(\int \frac{1}{\sqrt{1-(1-x^2)^2}} dx, x, \sin(x) \right)}{\sqrt{-1 + \sec^4(x)}} \\
&= \frac{(\sqrt{1 - \cos^4(x)} \sec^2(x)) \text{Subst} \left(\int \frac{1}{\sqrt{2x^2-x^4}} dx, x, \sin(x) \right)}{\sqrt{-1 + \sec^4(x)}} \\
&= -\frac{(\sqrt{1 - \cos^4(x)} \sec^2(x)) \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{\sin(x)}{\sqrt{2\sin^2(x)-\sin^4(x)}} \right)}{\sqrt{-1 + \sec^4(x)}} \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{2} \sin(x)}{\sqrt{2\sin^2(x)-\sin^4(x)}} \right) \sqrt{1 - \cos^4(x)} \sec^2(x)}{\sqrt{2} \sqrt{-1 + \sec^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.61

$$-\frac{\sqrt{\cos(2x) + 3} \tan(x) \sec(x) \tanh^{-1} \left(\frac{1}{2} \sqrt{4 - 2 \sin^2(x)} \right)}{2 \sqrt{\sec^4(x) - 1}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sec[x]/Sqrt[-1 + Sec[x]^4], x]`

[Out] $\frac{-1/2 * (\operatorname{ArcTanh}[\sqrt{4 - 2 \sin^2 x}/2] * \sqrt{3 + \cos 2x}) * \sec x * \tan x}{\sqrt{-1 + \sec^4 x}}$

fricas [B] time = 0.49, size = 54, normalized size = 1.93

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{2 \left(2 \sqrt{2} \sqrt{-\frac{\cos(x)^4 - 1}{\cos(x)^4}} \cos(x)^2 - (\cos(x)^2 + 3) \sin(x) \right)}{(\cos(x)^2 - 1) \sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-1+sec(x)^4)^(1/2), x, algorithm="fricas")`

[Out] $\frac{1/4 * \sqrt{2} * \log(-2 * (2 * \sqrt{2} * \sqrt{-(\cos(x)^4 - 1) / \cos(x)^4} * \cos(x)^2 - (\cos(x)^2 + 3) * \sin(x)))}{((\cos(x)^2 - 1) * \sin(x))}$

giac [B] time = 2.22, size = 92, normalized size = 3.29

$$\frac{\sqrt{2} \left(\log \left(\tan \left(\frac{1}{2} x \right)^2 - \sqrt{\tan \left(\frac{1}{2} x \right)^4 + 1} + 1 \right) - \log \left(-\tan \left(\frac{1}{2} x \right)^2 + \sqrt{\tan \left(\frac{1}{2} x \right)^4 + 1} + 1 \right) + \log \left(-\tan \left(\frac{1}{2} x \right)^2 + \sqrt{\tan \left(\frac{1}{2} x \right)^4 + 1} + 1 \right) \right)}{4 \operatorname{sgn} \left(\tan \left(\frac{1}{2} x \right)^5 - 2 \tan \left(\frac{1}{2} x \right)^3 + \tan \left(\frac{1}{2} x \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-1+sec(x)^4)^(1/2), x, algorithm="giac")`

[Out] $\frac{1/4 * \sqrt{2} * (\log(\tan(1/2*x)^2 - \sqrt{\tan(1/2*x)^4 + 1} + 1) - \log(-\tan(1/2*x)^2 + \sqrt{\tan(1/2*x)^4 + 1} + 1) + \log(-\tan(1/2*x)^2 + \sqrt{\tan(1/2*x)^4 + 1}))}{\operatorname{sgn}(\tan(1/2*x)^5 - 2 \tan(1/2*x)^3 + \tan(1/2*x))}$

maple [B] time = 0.25, size = 91, normalized size = 3.25

$$\frac{\sqrt{8} \sqrt{2} \left(-\operatorname{arcsinh} \left(\frac{\cos(x) - 1}{\cos(x) + 1} \right) + \operatorname{arctanh} \left(\frac{\sqrt{2} \sqrt{4}}{4 \sqrt{\frac{\cos^2(x) + 1}{(\cos(x) + 1)^2}}} \right) \sqrt{\frac{\cos^2(x) + 1}{(\cos(x) + 1)^2}} (\sin^3(x)) \right)}{8 (\cos(x) - 1) \sqrt{-\frac{2(\cos^4(x) - 1)}{\cos(x)^4}} \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)/(-1+sec(x)^4)^(1/2), x)`

[Out] $1/8*8^{(1/2)}*2^{(1/2)}*(\operatorname{arctanh}(1/4*2^{(1/2)}*4^{(1/2)})/((1+\cos(x)^2)/(1+\cos(x))^2)^{(1/2)})-\operatorname{arcsinh}((\cos(x)-1)/(1+\cos(x)))*\sin(x)^3*((1+\cos(x)^2)/(1+\cos(x))^2)^{(1/2)}/(\cos(x)-1)/\cos(x)^2/(-2*(\cos(x)^4-1)/\cos(x)^4)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\sqrt{\sec(x)^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-1+sec(x)^4)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sec(x)/sqrt(sec(x)^4 - 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(x) \sqrt{\frac{1}{\cos(x)^4} - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(\cos(x)*(1/cos(x)^4 - 1)^(1/2)),x)`

[Out] `int(1/(\cos(x)*(1/cos(x)^4 - 1)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\sqrt{(\sec(x) - 1)(\sec(x) + 1)(\sec^2(x) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-1+sec(x)**4)**(1/2),x)`

[Out] `Integral(sec(x)/sqrt((sec(x) - 1)*(sec(x) + 1)*(sec(x)**2 + 1)), x)`

3.43 $\int \frac{\tan(x)}{\sqrt{1+\tan^4(x)}} dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2} \sqrt{\tan^4(x)+1}}\right)}{2\sqrt{2}}$$

[Out] $-1/4*\text{arctanh}(1/2*(1-\tan(x)^2)*2^{(1/2)}/(1+\tan(x)^4)^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3670, 1248, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2} \sqrt{\tan^4(x)+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\tan[x]/\text{Sqrt}[1 + \tan[x]^4], x]$

[Out] $-\text{ArcTanh}[(1 - \tan[x]^2)/(\text{Sqrt}[2]*\text{Sqrt}[1 + \tan[x]^4])]/(2*\text{Sqrt}[2])$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_)*tan[(e_)+(f_)*(x_)])^(m_)*((a_)+(b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e+f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a+b*(ff*x)^n)^p]/(c^2+f^2*x^2), x], x, (c*Tan[e+f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(x)}{\sqrt{1+\tan^4(x)}} dx &= \text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{1+x^4}} dx, x, \tan(x)\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+x^2}} dx, x, \tan^2(x)\right) \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{1-\tan^2(x)}{\sqrt{1+\tan^4(x)}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{1+\tan^4(x)}}\right)}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 1.62

$$-\frac{\sqrt{\cos(4x)+3} \sec^2(x) \log\left(\sqrt{2} \cos(2x)+\sqrt{\cos(4x)+3}\right)}{4\sqrt{2}\sqrt{\tan^4(x)+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[Tan[x]/Sqrt[1 + Tan[x]^4], x]`

[Out] $-\frac{1}{4} \left(\sqrt{3 + \cos(4x)} \log\left(\sqrt{2} \cos(2x) + \sqrt{3 + \cos(4x)}\right) \right) \sec(x)^2 / (\sqrt{2} \sqrt{1 + \tan(x)^4})$

fricas [B] time = 0.44, size = 186, normalized size = 5.47

$$\frac{1}{32} \sqrt{2} \log\left(\frac{577 \tan(x)^{16} - 1912 \tan(x)^{14} + 4124 \tan(x)^{12} - 6216 \tan(x)^{10} + 7110 \tan(x)^8 - 6216 \tan(x)^6 + 4124 \tan(x)^4 - 1912 \tan(x)^2 + 577}{\tan(x)^{16}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+tan(x)^4)^(1/2), x, algorithm="fricas")`

[Out] $\frac{1}{32}\sqrt{2}\log\left(\frac{(577\tan(x)^{16} - 1912\tan(x)^{14} + 4124\tan(x)^{12} - 6216\tan(x)^{10} + 7110\tan(x)^8 - 6216\tan(x)^6 + 4124\tan(x)^4 - 1912\tan(x)^2 + 8 \cdot (51\sqrt{2}\tan(x)^{14} - 169\sqrt{2}\tan(x)^{12} + 339\sqrt{2}\tan(x)^{10} - 465\sqrt{2}\tan(x)^8 + 465\sqrt{2}\tan(x)^6 - 339\sqrt{2}\tan(x)^4 + 169\sqrt{2}\tan(x)^2 - 51\sqrt{2})\sqrt{\tan(x)^4 + 1} + 577)}{(\tan(x)^{16} + 8\tan(x)^{14} + 28\tan(x)^{12} + 56\tan(x)^{10} + 70\tan(x)^8 + 56\tan(x)^6 + 28\tan(x)^4 + 8\tan(x)^2 + 1)}\right)$

giac [A] time = 1.06, size = 50, normalized size = 1.47

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\tan(x)^2 + \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1}{\tan(x)^2 - \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+tan(x)^4)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{4}\sqrt{2}\log\left(\frac{-(\tan(x)^2 + \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1)}{(\tan(x)^2 - \sqrt{2} - \sqrt{\tan(x)^4 + 1} + 1)}\right)$

maple [A] time = 0.06, size = 37, normalized size = 1.09

$$-\frac{\sqrt{2}\arctanh\left(\frac{(-2\tan^2(x)+2)\sqrt{2}}{4\sqrt{-2\tan^2(x)+(\tan^2(x)+1)^2}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(1+tan(x)^4)^(1/2),x)`

[Out] $-\frac{1}{4}2^{(1/2)}\operatorname{arctanh}\left(\frac{1}{4}(-2\tan(x)^2+2)2^{(1/2)}\right)/(-2\tan(x)^2+(\tan(x)^2+1)^2)^{(1/2)}$

maxima [B] time = 1.35, size = 565, normalized size = 16.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(1+tan(x)^4)^(1/2),x, algorithm="maxima")`

[Out] $-\frac{1}{16}\sqrt{2}(\log(4\sqrt{2}(6\cos(4x) + 1)\cos(8x) + \cos(8x)^2 + 36\cos(4x)^2 + \sin(8x)^2 + 12\sin(8x)\sin(4x) + 36\sin(4x)^2 + 12\cos(4x) + 1)\cos(1/2\operatorname{arctan2}(\sin(8x) + 6\sin(4x), \cos(8x) + 6\cos(4x) + 1))^2 + 4\sqrt{2}(6\cos(4x) + 1)\cos(8x) + \cos(8x)^2 + 36\cos(4x)^2 + \sin(8x)^2)$

```

2 + 12*sin(8*x)*sin(4*x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)*sin(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2 + 32*(2*(6*cos(4*x) +
1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*x)^2 + 12*sin(8*x)*sin(4*
x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(8*x) + 6*si
n(4*x), cos(8*x) + 6*cos(4*x) + 1)) + 64) + log(4*cos(4*x)^2 + 4*sin(4*x)^2
+ 4*sqrt(2*(6*cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*
x)^2 + 12*sin(8*x)*sin(4*x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)*(cos(1/2*arc
tan2(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2 + sin(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2) + 8*(2*(6*cos(4*x) +
1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*x)^2 + 12*sin(8*x)*sin(4*
x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)^(1/4)*((cos(4*x) + 3)*cos(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1)) + sin(4*x)*sin(1/2*arct
an2(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))) + 24*cos(4*x) + 36)
)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)}{\sqrt{\tan(x)^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(tan(x)^4 + 1)^(1/2), x)

[Out] int(tan(x)/(tan(x)^4 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{\tan^4(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(1+tan(x)**4)**(1/2), x)

[Out] Integral(tan(x)/sqrt(tan(x)**4 + 1), x)

3.44 $\int \frac{\sin(x)}{\sqrt{1-\sin^6(x)}} dx$

Optimal. Leaf size=39

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}(\sin^2(x)+1)\cos(x)}{2\sqrt{1-\sin^6(x)}}\right)}{2\sqrt{3}}$$

[Out] $1/6*\text{arctanh}(1/2*\cos(x)*(1+\sin(x)^2)*3^{(1/2)}/(1-\sin(x)^6)^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3216, 1996, 1904, 206}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)(6-3\cos^2(x))}{2\sqrt{3}\sqrt{\cos^6(x)-3\cos^4(x)+3\cos^2(x)}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]/\text{Sqrt}[1 - \text{Sin}[x]^6], x]$

[Out] $\text{ArcTanh}[(\text{Cos}[x]*(6 - 3\text{Cos}[x]^2))/(2\text{Sqrt}[3]*\text{Sqrt}[3\text{Cos}[x]^2 - 3\text{Cos}[x]^4 + \text{Cos}[x]^6])]/(2\text{Sqrt}[3])$

Rule 206

$\text{Int}[(a_+ + b_-)*(x_-)^2)^{(-1)}, x_{\text{Symbol}}] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&& \text{NegQ}[a/b] \&& (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 1904

$\text{Int}[1/\text{Sqrt}[(a_+)*(x_-)^2 + (b_-)*(x_-)^{(n_-)} + (c_-)*(x_-)^{(r_-)}], x_{\text{Symbol}}] := \text{Dist}[-2/(n - 2), \text{Subst}[\text{Int}[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/\text{Sqrt}[a*x^2 + b*x^n + c*x^r]], x] /; \text{FreeQ}[\{a, b, c, n, r\}, x] \&& \text{EqQ}[r, 2*n - 2] \&& \text{PosQ}[n - 2] \&& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1996

$\text{Int}[(u_-)^{(p_-)}, x_{\text{Symbol}}] := \text{Int}[\text{ExpandToSum}[u, x]^p, x] /; \text{FreeQ}[p, x] \&& \text{GeneralizedTrinomialQ}[u, x] \&& \text{!GeneralizedTrinomialMatchQ}[u, x]$

Rule 3216

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(1 - ff^2*x^2)^(n/2))^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{\sqrt{1 - \sin^6(x)}} dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1 - (1 - x^2)^3}} dx, x, \cos(x)\right) \\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{3x^2 - 3x^4 + x^6}} dx, x, \cos(x)\right) \\ &= \text{Subst}\left(\int \frac{1}{12 - x^2} dx, x, \frac{\cos(x)(6 - 3\cos^2(x))}{\sqrt{3\cos^2(x) - 3\cos^4(x) + \cos^6(x)}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\cos(x)(6 - 3\cos^2(x))}{2\sqrt{3}\sqrt{3\cos^2(x) - 3\cos^4(x) + \cos^6(x)}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.67

$$-\frac{\cos(x)\sqrt{-8\cos(2x) + \cos(4x) + 15}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}(\cos(2x)-3)}{\sqrt{-8\cos(2x)+\cos(4x)+15}}\right)}{4\sqrt{6 - 6\sin^6(x)}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[x]/Sqrt[1 - Sin[x]^6], x]`

[Out] $\frac{-1/4 * (\text{ArcTanh}[(\text{Sqrt}[3/2]*(-3 + \text{Cos}[2*x]))/\text{Sqrt}[15 - 8*\text{Cos}[2*x] + \text{Cos}[4*x]]]) * \text{Cos}[x] * \text{Sqrt}[15 - 8*\text{Cos}[2*x] + \text{Cos}[4*x]])/\text{Sqrt}[6 - 6*\text{Sin}[x]^6]}{4\sqrt{6 - 6\sin^6(x)}}$

fricas [B] time = 0.52, size = 63, normalized size = 1.62

$$\frac{1}{12}\sqrt{3}\log\left(\frac{7\cos(x)^5 - 24\cos(x)^3 - 4\sqrt{\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2}(\sqrt{3}\cos(x)^2 - 2\sqrt{3}) + 24\cos(x)}{\cos(x)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1/12\sqrt{3}\log((7\cos(x)^5 - 24\cos(x)^3 - 4\sqrt{\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2})(\sqrt{3}\cos(x)^2 - 2\sqrt{3}) + 24\cos(x))/\cos(x)^5)}{6\operatorname{sgn}(\cos(x))}$

giac [B] time = 2.35, size = 67, normalized size = 1.72

$$\frac{\sqrt{3} \log \left(\cos(x)^2+\sqrt{3}-\sqrt{\cos(x)^4-3 \cos(x)^2+3}\right)-\sqrt{3} \log \left(-\cos(x)^2+\sqrt{3}+\sqrt{\cos(x)^4-3 \cos(x)^2+3}\right)}{6 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="giac")`

[Out] $\frac{-1/6*(\sqrt{3}\log(\cos(x)^2+\sqrt{3}-\sqrt{\cos(x)^4-3\cos(x)^2+3})-\sqrt{3}\log(-\cos(x)^2+\sqrt{3}+\sqrt{\cos(x)^4-3\cos(x)^2+3}))/\operatorname{sgn}(\cos(x))}{6\sqrt{\cos^6(x)-3(\cos^4(x))+3(\cos^2(x))}}$

maple [B] time = 0.41, size = 67, normalized size = 1.72

$$\frac{\sqrt{\cos^4(x)-3(\cos^2(x))+3}\sqrt{3}\operatorname{arctanh}\left(\frac{(\cos^2(x)-2)\sqrt{3}}{2\sqrt{\cos^4(x)-3(\cos^2(x))+3}}\right)\cos(x)}{6\sqrt{\cos^6(x)-3(\cos^4(x))+3(\cos^2(x))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(1-sin(x)^6)^(1/2),x)`

[Out] $\frac{-1/6/(3\cos(x)^2-3\cos(x)^4+\cos(x)^6)^(1/2)*\cos(x)*(3-3\cos(x)^2+\cos(x)^4)^(1/2)*3^(1/2)*\operatorname{arctanh}(1/2*(\cos(x)^2-2)*3^(1/2)/(3-3\cos(x)^2+\cos(x)^4)^(1/2))}{6\sqrt{-\sin(x)^6+1}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{-\sin(x)^6+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sin(x)/sqrt(-sin(x)^6+1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(x)}{\sqrt{1-\sin(x)^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1 - sin(x)^6)^(1/2),x)
[Out] int(sin(x)/(1 - sin(x)^6)^(1/2), x)
sympy [F(-1)]    time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-sin(x)**6)**(1/2),x)
[Out] Timed out
```

$$3.45 \quad \int \sqrt{-\sqrt{-1 + \sec(x)} + \sqrt{1 + \sec(x)}} \ dx$$

Optimal. Leaf size=337

$$\sqrt{2} \cot(x) \sqrt{\sec(x)-1} \sqrt{\sec(x)+1} \left(\sqrt{\sqrt{2}-1} \tan^{-1} \left(\frac{\sqrt{2\sqrt{2}-2} (-\sqrt{\sec(x)-1} + \sqrt{\sec(x)+1} - \sqrt{2})}{2\sqrt{\sqrt{\sec(x)+1} - \sqrt{\sec(x)-1}}} \right) - \sqrt{1+}$$

[Out] $\cot(x)*2^{(1/2)*(-1+\sec(x))^{(1/2)*(1+\sec(x))^{(1/2)}*(\arctan(1/2*(-2^{(1/2)*(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)}*(-2+2*2^{(1/2)})^{(1/2)/(-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)}*(2^{(1/2)-1})^{(1/2)+\operatorname{arctanh}((2+2*2^{(1/2)})^{(1/2)*(-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)})^{(1/2)/(2^{(1/2)-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)})^{(1/2)}*(2^{(1/2)-1})^{(1/2)-\operatorname{arctan}(1/2*(-2^{(1/2)*(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)/(-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)})^{(1/2)*(1+2^{(1/2)})^{(1/2)-\operatorname{arctanh}((-2+2*2^{(1/2)})^{(1/2)*(-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)})^{(1/2)/(2^{(1/2)-(-1+\sec(x))^{(1/2)+(1+\sec(x))^{(1/2)})^{(1/2)}*(1+2^{(1/2)})^{(1/2))}}}}$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{-\sqrt{-1 + \sec(x)} + \sqrt{1 + \sec(x)}} \ dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[\text{Sqrt}[-\text{Sqrt}[-1 + \text{Sec}[x]] + \text{Sqrt}[1 + \text{Sec}[x]]], x]$

[Out] $\text{Defer}[\text{Int}][\text{Sqrt}[-\text{Sqrt}[-1 + \text{Sec}[x]] + \text{Sqrt}[1 + \text{Sec}[x]]]], x$

Rubi steps

$$\int \sqrt{-\sqrt{-1 + \sec(x)} + \sqrt{1 + \sec(x)}} \ dx = \int \sqrt{-\sqrt{-1 + \sec(x)} + \sqrt{1 + \sec(x)}} \ dx$$

Mathematica [A] time = 2.18, size = 552, normalized size = 1.64

$$\sqrt[4]{2} \sin(x) \cos(x) \left(\sqrt{\sec(x)-1} - \sqrt{\sec(x)+1} \right)^2 \left(2 \sin\left(\frac{\pi}{8}\right) \tan^{-1} \left(\frac{\sec\left(\frac{\pi}{8}\right) \sqrt{\sqrt{\sec(x)+1} - \sqrt{\sec(x)-1}}}{\sqrt[4]{2}} - \tan\left(\frac{\pi}{8}\right) \right) + 2 \sin\left(\frac{\pi}{8}\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]],x]
[Out] (2^(1/4)*Cos[x]*(Sqrt[-1 + Sec[x]] - Sqrt[1 + Sec[x]])^2*(2*ArcTan[Cot[Pi/8]
] - (Csc[Pi/8]*Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]))/2^(1/4])*Cos[Pi/8] - 2*ArcTan[Cot[Pi/8] + (Csc[Pi/8]*Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]))/2^(1/4])*Cos[Pi/8] + Cos[Pi/8]*Log[2 + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]]) - 2*2^(3/4)*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]]*Sin[Pi/8]] - Cos[Pi/8]*Log[2 + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]) + 2*2^(3/4)*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]]*Sin[Pi/8]] + 2*ArcTan[(Sec[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]])/2^(1/4) - Tan[Pi/8]]*Sin[Pi/8] - Log[2 - 2*2^(3/4)*Cos[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]] + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]])*Sin[Pi/8] + Log[2 + 2^(1/4)*Csc[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]] + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]])*Sin[Pi/8])/( -1 + Cos[2*x] + 2*Cos[x]*Sqrt[-1 + Sec[x]]*Sqrt[1 + Sec[x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sec(x)+1} - \sqrt{\sec(x)-1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(sqrt(sec(x) + 1) - sqrt(sec(x) - 1)), x)

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{\sec(x)-1} + \sqrt{\sec(x)+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((-\sec(x)-1)^{1/2} + (1+\sec(x))^{1/2})^{1/2} dx$

[Out] $\int ((-\sec(x)-1)^{1/2} + (1+\sec(x))^{1/2})^{1/2} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sec(x) + 1} - \sqrt{\sec(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-(-1+\sec(x))^{1/2} + (1+\sec(x))^{1/2})^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\sqrt{\sec(x) + 1} - \sqrt{\sec(x) - 1}), x$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{\frac{1}{\cos(x)} + 1} - \sqrt{\frac{1}{\cos(x)} - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((1/\cos(x) + 1)^{1/2} - (1/\cos(x) - 1)^{1/2})^{1/2} dx$

[Out] $\int ((1/\cos(x) + 1)^{1/2} - (1/\cos(x) - 1)^{1/2})^{1/2}, x$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{\sec(x) - 1} + \sqrt{\sec(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-(-1+\sec(x))^{1/2} + (1+\sec(x))^{1/2})^{1/2}, x)$

[Out] $\text{Integral}(\sqrt{-\sqrt{\sec(x) - 1} + \sqrt{\sec(x) + 1}}), x$

3.46 $\int x \tan^{-1}(x)^2 \log(1 + x^2) dx$

Optimal. Leaf size=77

$$\frac{1}{4} \log^2(x^2 + 1) - \frac{3}{2} \log(x^2 + 1) - \frac{1}{2} x^2 \tan^{-1}(x)^2 + \frac{1}{2} (x^2 + 1) \log(x^2 + 1) \tan^{-1}(x)^2 - x \log(x^2 + 1) \tan^{-1}(x) - \frac{3}{2} \tan^{-1}(x) \log^2(x^2 + 1)$$

[Out] $3*x*\arctan(x) - 3/2*\arctan(x)^2 - 1/2*x^2*\arctan(x)^2 - 3/2*\ln(x^2+1) - x*\arctan(x)*\ln(x^2+1) + 1/2*(x^2+1)*\arctan(x)^2*\ln(x^2+1) + 1/4*\ln(x^2+1)^2$

Rubi [A] time = 0.22, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4852, 4916, 4846, 260, 4884, 5023, 5009, 2475, 2390, 2301}

$$\frac{1}{4} \log^2(x^2 + 1) - \frac{3}{2} \log(x^2 + 1) - \frac{1}{2} x^2 \tan^{-1}(x)^2 + \frac{1}{2} (x^2 + 1) \log(x^2 + 1) \tan^{-1}(x)^2 - x \log(x^2 + 1) \tan^{-1}(x) - \frac{3}{2} \tan^{-1}(x) \log^2(x^2 + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{ArcTan}[x]^2*\text{Log}[1 + x^2], x]$

[Out] $3*x*\text{ArcTan}[x] - (3*\text{ArcTan}[x]^2)/2 - (x^2*\text{ArcTan}[x]^2)/2 - (3*\text{Log}[1 + x^2])/2 - x*\text{ArcTan}[x]*\text{Log}[1 + x^2] + ((1 + x^2)*\text{ArcTan}[x]^2*\text{Log}[1 + x^2])/2 + \text{Log}[1 + x^2]^2/4$

Rule 260

$\text{Int}[(x_)^{(m_.)} / ((a_) + (b_*)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[t[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \&& \text{EqQ}[m, n - 1]$

Rule 2301

$\text{Int}[((a_.) + \text{Log}[(c_*)*(x_)^{(n_.)}]*(b_.))/(x_), x_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}[\{a, b, c, n\}, x]$

Rule 2390

$\text{Int}[((a_.) + \text{Log}[(c_*)*((d_) + (e_*)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(p_.)}*((f_) + (g_.)*(x_)^{(q_.)}), x_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[((f*x)/d)^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \&& \text{EqQ}[e*f - d*g, 0]$

Rule 2475

$\text{Int}[((a_.) + \text{Log}[(c_*)*((d_) + (e_*)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((x_)^{(m_.)}*((f_) + (g_.)*(x_)^{(s_.)})^{(r_.)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Sim})}], x, x^{(m_.)}*((f_) + (g_.)*(x_)^{(s_.)})^{(r_.)}], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x] \&& \text{EqQ}[e*f - d*g, 0]$

```

plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x,
x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])

```

Rule 4846

```

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.) )^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

```

Rule 4852

```

Int[((a_.) + ArcTan[(c_.*(x_)]*(b_.) )^(p_.)*((d_.*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]

```

Rule 4884

```

Int[((a_.) + ArcTan[(c_.*(x_)]*(b_.) )^(p_.)/((d_) + (e_.*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

```

Rule 4916

```

Int[((((a_.) + ArcTan[(c_.*(x_)]*(b_.) )^(p_.)*((f_.*(x_))^(m_.))/((d_) + (e
_.*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

```

Rule 5009

```

Int[((a_.) + ArcTan[(c_.*(x_)]*(b_.) )*((d_.) + Log[(f_.) + (g_.*(x_)^2)*(e
_.)], x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
(-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 + c^2*x^2), x], x] - Dist[2*
e*g, Int[(x^2*(a + b*ArcTan[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]

```

Rule 5023

```

Int[((a_.) + ArcTan[(c_.*(x_)]*(b_.) )^2*((d_.) + Log[(f_) + (g_.*(x_)^2)*
(e_.)*x_, x_Symbol] :> Simp[((f + g*x^2)*(d + e*Log[f + g*x^2])*(a + b*A
rcTan[c*x])^2)/(2*g), x] + (-Dist[b/c, Int[(d + e*Log[f + g*x^2])*(a + b*Ar

```

```
cTan[c*x]), x], x] + Dist[b*c*e, Int[(x^2*(a + b*ArcTan[c*x]))/(1 + c^2*x^2), x], x] - Simp[(e*x^2*(a + b*ArcTan[c*x])^2)/2, x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[g, c^2*f]
```

Rubi steps

$$\begin{aligned}
\int x \tan^{-1}(x)^2 \log(1+x^2) dx &= -\frac{1}{2}x^2 \tan^{-1}(x)^2 + \frac{1}{2}(1+x^2) \tan^{-1}(x)^2 \log(1+x^2) + \int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx - \int \tan^{-1}(x) dx \\
&= -\frac{1}{2}x^2 \tan^{-1}(x)^2 - x \tan^{-1}(x) \log(1+x^2) + \frac{1}{2}(1+x^2) \tan^{-1}(x)^2 \log(1+x^2) + \\
&= x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2}x^2 \tan^{-1}(x)^2 - x \tan^{-1}(x) \log(1+x^2) + \frac{1}{2}(1+x^2) \log(1+x^2) \\
&= 3x \tan^{-1}(x) - \frac{3}{2} \tan^{-1}(x)^2 - \frac{1}{2}x^2 \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2) - x \tan^{-1}(x) \log(1+x^2) \\
&= 3x \tan^{-1}(x) - \frac{3}{2} \tan^{-1}(x)^2 - \frac{1}{2}x^2 \tan^{-1}(x)^2 - \frac{3}{2} \log(1+x^2) - x \tan^{-1}(x) \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.75

$$\frac{1}{4} ((\log(x^2+1)-6) \log(x^2+1)+2 (-x^2+(x^2+1) \log(x^2+1)-3) \tan^{-1}(x)^2-4 x (\log(x^2+1)-3) \tan^{-1}(x))$$

Antiderivative was successfully verified.

[In] `Integrate[x*ArcTan[x]^2*Log[1 + x^2], x]`

[Out] $(-4 x \operatorname{ArcTan}[x] (-3 + \operatorname{Log}[1 + x^2]) + (-6 + \operatorname{Log}[1 + x^2]) \operatorname{Log}[1 + x^2] + 2 \operatorname{ArcTan}[x]^2 (-3 - x^2 + (1 + x^2) \operatorname{Log}[1 + x^2]))/4$

fricas [A] time = 0.47, size = 52, normalized size = 0.68

$$-\frac{1}{2} (x^2 + 3) \arctan(x)^2 + 3 x \arctan(x) + \frac{1}{2} ((x^2 + 1) \arctan(x)^2 - 2 x \arctan(x) - 3) \log(x^2 + 1) + \frac{1}{4} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)^2*log(x^2+1), x, algorithm="fricas")`

[Out] $-1/2*(x^2 + 3)*\operatorname{arctan}(x)^2 + 3 x \operatorname{arctan}(x) + 1/2*((x^2 + 1) \operatorname{arctan}(x)^2 - 2 x \operatorname{arctan}(x) - 3) \operatorname{log}(x^2 + 1) + 1/4 \operatorname{log}(x^2 + 1)^2$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan(x)^2 \log(x^2 + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2*log(x^2+1),x, algorithm="giac")
```

[Out] $\int x \arctan(x)^2 \log(x^2 + 1) dx$

maple [C] time = 1.16, size = 3134, normalized size = 40.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arctan(x)^2*ln(x^2+1),x)`

```
[Out] -1/2*x^2*arctan(x)^2+3*x*arctan(x)-1/2*arctan(x)^2-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1)^2)*csgn(I/((I*x+1)^2/(x^2+1)+1)^2)*csgn(I*(I*x+1)^2/(x^2+1))x^2+1/2*I*arctan(x)*Pi*csgn(I*(I*x+1)^2/(x^2+1))/((I*x+1)^2/(x^2+1)+1)^2)*csgn(I/((I*x+1)^2/(x^2+1)+1)^2)*csgn(I*(I*x+1)^2/(x^2+1))*x-arctan(x)^2*ln((I*x+1)^2/(x^2+1)+1)*x^2+1/2*arctan(x)*Pi*csgn(I*(I*x+1)^2/(x^2+1))/((I*x+1)^2/(x^2+1)+1)^2)^3+I*Pi*ln((I*x+1)^2/(x^2+1)+1)*csgn(I*((I*x+1)^2/(x^2+1)+1))*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2+1/2*I*arctan(x)^2*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))*csgn(I*(I*x+1)^2/(x^2+1))^2+1/4*I*arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x^2-1/2*I*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x^2+3*x-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))^2*csgn(I*(I*x+1)^2/(x^2+1))+1/4*I*arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I*((I*x+1)^2/(x^2+1)+1))+1/4*I*arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1))^2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1))/((I*x+1)^2/(x^2+1)+1)^2)^3*x^2-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1))^3*x^2+3*x^2-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))^2*csgn(I*((I*x+1)^2/(x^2+1)+1))^2-1/2*I*Pi*ln((I*x+1)^2/(x^2+1)+1)*csgn(I*((I*x+1)^2/(x^2+1)+1))^2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)-1/2*I*Pi*ln((I*x+1)^2/(x^2+1)+1)*csgn(I*(I*x+1)^2/(x^2+1))*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2-1/2*I*Pi*ln((I*x+1)^2/(x^2+1)+1)*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2+1/2*I*Pi*ln((I*x+1)^2/(x^2+1)+1)^2)*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2+1/2*I*arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x+I*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x^2-1/2*I*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x^2+3*x+1/2*I*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*x^2+2*I*arctan(x)^2*ln((I*x+1)^2/(x^2+1))+1/2*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I*((I*x+1)^2/(x^2+1)+1))+ln((I*x+1)^2/(x^2+1)+1)^2-3*I*arctan(x)+arctan(x)^2*ln(2)-arctan(x)^2*ln((I*x+1)^2/(x^2+1)+1)+(x^2*arctan(x)^2+2*I*arctan(x)+arctan(x)^2-2*x*arctan(x)-2*ln((I*x+1)^2/(x^2+1)+1))*ln((I*x+1)/(x^2+1)^(1/2))-1/2*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2)^3-2*I*arctan(x)*ln(2)*x+2*arctan(x)*ln((I*x+1)^2/(x^2+1)+1)*x+arctan(x)^2*ln(2)*x^2
```

$$\begin{aligned}
& 2+2*I*arctan(x)*ln(2)-2*ln(2)*ln((I*x+1)^2/(x^2+1)+1)+1/2*arctan(x)*Pi*csgn \\
& (I*(I*x+1)^2/(x^2+1))^3-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1))^3+1/ \\
& 2*I*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1)^2)^3*ln((I*x+1)^2/(x^2+1) \\
&)+1)*Pi+1/2*I*csgn(I*(I*x+1)^2/(x^2+1))^3*ln((I*x+1)^2/(x^2+1)+1)*Pi-1/2*I* \\
& csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^3*ln((I*x+1)^2/(x^2+1)+1)*Pi-1/2*arctan(x)* \\
& Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I/((I*x+1)^2/(x \\
& ^2+1)+1)^2)-1/2*arctan(x)*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1) \\
&)^2*csgn(I*(I*x+1)^2/(x^2+1))-1/2*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1) \\
&)^2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)+arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1) \\
&)+1))*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)^2+1/2*arctan(x)*Pi*csgn(I*(I*x+1)/(x \\
& ^2+1)^(1/2))^2*csgn(I*(I*x+1)^2/(x^2+1))-arctan(x)*Pi*csgn(I*(I*x+1)/(x^2+1) \\
&)^(1/2))*csgn(I*(I*x+1)^2/(x^2+1))^2+1/4*I*arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x \\
& ^2+1)+1)^2)^3-1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x \\
& ^2+1)+1)^2)^3+1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x \\
& ^2+1)+1)^2)^2*csgn(I/((I*x+1)^2/(x^2+1)+1)^2)*x^2-1/4*I*arctan(x)^2*Pi*csgn(I \\
& *(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1)^2)*csgn(I/((I*x+1)^2/(x^2+1)+1)^2) \\
& *csgn(I*(I*x+1)^2/(x^2+1))+1/4*I*arctan(x)^2*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((\\
& I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I*(I*x+1)^2/(x^2+1))*x^2+1/4*I*arctan(x)^2*Pi \\
& *csgn(I*(I*x+1)^2/(x^2+1)+1)^2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)*x^2-1/2*I* \\
& arctan(x)^2*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1))*csgn(I*((I*x+1)^2/(x^2+1)+1)^2) \\
&)^2*x^2-1/2*I*arctan(x)*Pi*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1)+1)^2) \\
&)^2*csgn(I/((I*x+1)^2/(x^2+1)+1)^2)*x-1/2*I*arctan(x)*Pi*csgn(I*(I*x+1)^2/(x \\
& ^2+1)/((I*x+1)^2/(x^2+1)+1)^2)^2*csgn(I*(I*x+1)^2/(x^2+1))*x-1/2*I*arctan(x) \\
& *Pi*csgn(I*((I*x+1)^2/(x^2+1)+1))^(2*csgn(I*((I*x+1)^2/(x^2+1)+1)^2)*x-I*a \\
& rctan(x)*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))*csgn(I*(I*x+1)^2/(x^2+1))^2*x-1/4 \\
& *I*arctan(x)^2*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))^2*csgn(I*(I*x+1)^2/(x^2+1)) \\
& *x^2+1/2*I*arctan(x)^2*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))*csgn(I*(I*x+1)^2/(x \\
& ^2+1))^2*x^2+1/2*I*arctan(x)*Pi*csgn(I*(I*x+1)/(x^2+1)^(1/2))^2*csgn(I*(I*x+1) \\
& ^2/(x^2+1))*x+1/2*I*Pi*ln((I*x+1)^2/(x^2+1)+1)*csgn(I/((I*x+1)^2/(x^2+1) \\
& +1)^2)*csgn(I*(I*x+1)^2/(x^2+1))*csgn(I*(I*x+1)^2/(x^2+1)/((I*x+1)^2/(x^2+1) \\
& +1)^2)+I*arctan(x)*Pi*csgn(I*((I*x+1)^2/(x^2+1)+1))*csgn(I*((I*x+1)^2/(x^2 \\
& +1)+1)^2)^2*x
\end{aligned}$$

maxima [A] time = 0.98, size = 67, normalized size = 0.87

$$-\frac{1}{2} \left(x^2 - (x^2 + 1) \log(x^2 + 1) + 1 \right) \arctan(x)^2 - \left(x \log(x^2 + 1) - 3x + 2 \arctan(x) \right) \arctan(x) + \arctan(x)^2 + \frac{1}{4} \log(x^2 + 1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2*log(x^2+1),x, algorithm="maxima")
[Out] -1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x)^2 - (x*log(x^2 + 1) - 3*x
+ 2*arctan(x))*arctan(x) + arctan(x)^2 + 1/4*log(x^2 + 1)^2 - 3/2*log(x^2
+ 1)
```

mupad [B] time = 0.26, size = 78, normalized size = 1.01

$$\frac{\ln(x^2 + 1)^2}{4} - \frac{3 \ln(x^2 + 1)}{2} - \frac{3 \operatorname{atan}(x)^2}{2} + \frac{\ln(x^2 + 1) \operatorname{atan}(x)^2}{2} + x (3 \operatorname{atan}(x) - \ln(x^2 + 1) \operatorname{atan}(x)) - x^2 \left(\frac{\operatorname{atan}(x)^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*log(x^2 + 1)*atan(x)^2, x)`

[Out] $\log(x^2 + 1)^2/4 - (3*\log(x^2 + 1))/2 - (3*\operatorname{atan}(x)^2)/2 + (\log(x^2 + 1)*\operatorname{atan}(x)^2)/2 + x*(3*\operatorname{atan}(x) - \log(x^2 + 1)*\operatorname{atan}(x)) - x^2*(\operatorname{atan}(x)^2/2 - (\log(x^2 + 1)*\operatorname{atan}(x)^2)/2)$

sympy [A] time = 2.49, size = 87, normalized size = 1.13

$$\frac{x^2 \log(x^2 + 1) \operatorname{atan}(x)^2}{2} - \frac{x^2 \operatorname{atan}(x)^2}{2} - x \log(x^2 + 1) \operatorname{atan}(x) + 3x \operatorname{atan}(x) + \frac{\log(x^2 + 1)^2}{4} + \frac{\log(x^2 + 1) \operatorname{atan}(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*atan(x)**2*ln(x**2+1), x)`

[Out] $x^{**2}*\log(x^{**2} + 1)*\operatorname{atan}(x)^{**2}/2 - x^{**2}*\operatorname{atan}(x)^{**2}/2 - x*\log(x^{**2} + 1)*\operatorname{atan}(x) + 3*x*\operatorname{atan}(x) + \log(x^{**2} + 1)^{**2}/4 + \log(x^{**2} + 1)*\operatorname{atan}(x)^{**2}/2 - 3*\log(x^{**2} + 1)/2 - 3*\operatorname{atan}(x)^{**2}/2$

$$3.47 \quad \int \tan^{-1} \left(x \sqrt{1 + x^2} \right) dx$$

Optimal. Leaf size=120

$$-\frac{1}{4} \sqrt{3} \log \left(x^2 - \sqrt{3} \sqrt{x^2 + 1} + 2\right) + \frac{1}{4} \sqrt{3} \log \left(x^2 + \sqrt{3} \sqrt{x^2 + 1} + 2\right) + x \tan^{-1} \left(x \sqrt{x^2 + 1}\right) + \frac{1}{2} \tan^{-1} \left(\sqrt{3} - 2 \sqrt{x^2 + 1}\right)$$

[Out] $-1/2*\arctan(-3^{(1/2)}+2*(x^{2+1})^{(1/2)})+x*\arctan(x*(x^{2+1})^{(1/2)})-1/2*\arctan(3^{(1/2)}+2*(x^{2+1})^{(1/2)})-1/4*\ln(2+x^2-3^{(1/2)}*(x^{2+1})^{(1/2)})*3^{(1/2)}+1/4*\ln(2+x^{2+3^{(1/2)}}*(x^{2+1})^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5203, 1685, 826, 1169, 634, 618, 204, 628}

$$-\frac{1}{4} \sqrt{3} \log \left(x^2 - \sqrt{3} \sqrt{x^2 + 1} + 2\right) + \frac{1}{4} \sqrt{3} \log \left(x^2 + \sqrt{3} \sqrt{x^2 + 1} + 2\right) + x \tan^{-1} \left(x \sqrt{x^2 + 1}\right) + \frac{1}{2} \tan^{-1} \left(\sqrt{3} - 2 \sqrt{x^2 + 1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x*.Sqrt[1 + x^2]], x]

[Out] $x*\text{ArcTan}[x*\text{Sqrt}[1 + x^2]] + \text{ArcTan}[\text{Sqrt}[3] - 2*\text{Sqrt}[1 + x^2]]/2 - \text{ArcTan}[\text{Sqrt}[3] + 2*\text{Sqrt}[1 + x^2]]/2 - (\text{Sqrt}[3]*\text{Log}[2 + x^2 - \text{Sqrt}[3]*\text{Sqrt}[1 + x^2]])/4 + (\text{Sqrt}[3]*\text{Log}[2 + x^2 + \text{Sqrt}[3]*\text{Sqrt}[1 + x^2]])/4$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 826

```
Int[((f_) + (g_)*(x_))/(Sqrt[(d_) + (e_)*(x_)]*((a_) + (b_)*(x_) + (c
_)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1685

```
Int[(Px_)*(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(x\sqrt{1+x^2}\right) dx &= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \int \frac{x(1+2x^2)}{\sqrt{1+x^2}(1+x^2+x^4)} dx \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1+2x}{\sqrt{1+x}(1+x+x^2)} dx, x, x^2\right) \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \text{Subst}\left(\int \frac{-1+2x^2}{1-x^2+x^4} dx, x, \sqrt{1+x^2}\right) \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+3x}{1-\sqrt{3}x+x^2} dx, x, \sqrt{1+x^2}\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt{3}-3x}{1+\sqrt{3}x+x^2} dx, x, \sqrt{1+x^2}\right)}{2\sqrt{3}} \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2} dx, x, \sqrt{1+x^2}\right) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2} dx, x, \sqrt{1+x^2}\right) \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{4}\sqrt{3} \log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right) + \frac{1}{4}\sqrt{3} \log\left(2+x^2+\sqrt{3}\sqrt{1+x^2}\right) \\
&= x \tan^{-1}\left(x\sqrt{1+x^2}\right) + \frac{1}{2} \tan^{-1}\left(\sqrt{3}-2\sqrt{1+x^2}\right) - \frac{1}{2} \tan^{-1}\left(\sqrt{3}+2\sqrt{1+x^2}\right) - \frac{1}{4}\sqrt{3} \log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right) + \frac{1}{4}\sqrt{3} \log\left(2+x^2+\sqrt{3}\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.31, size = 136, normalized size = 1.13

$$\frac{1}{4} \left(4x \tan^{-1}\left(x\sqrt{x^2+1}\right) + (1+i\sqrt{3}) \sqrt{2-2i\sqrt{3}} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^2+1}}{\sqrt{1-i\sqrt{3}}}\right) + (1-i\sqrt{3}) \sqrt{2+2i\sqrt{3}} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^2+1}}{\sqrt{1+i\sqrt{3}}}\right) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[ArcTan[x*Sqrt[1 + x^2]], x]`

[Out] $(4*x*ArcTan[x*Sqrt[1 + x^2]] + (1 + I*Sqrt[3])*Sqrt[2 - (2*I)*Sqrt[3]]*ArcTanh[(Sqrt[2]*Sqrt[1 + x^2])/Sqrt[1 - I*Sqrt[3]]] + (1 - I*Sqrt[3])*Sqrt[2 + (2*I)*Sqrt[3]]*ArcTanh[(Sqrt[2]*Sqrt[1 + x^2])/Sqrt[1 + I*Sqrt[3]]])/4$

fricas [B] time = 0.48, size = 287, normalized size = 2.39

$$x \arctan\left(\sqrt{x^2+1}x\right) - \frac{1}{4}\sqrt{3} \log\left(32x^4 + 80x^2 + 32\sqrt{3}(x^3+x) - 16(2x^3 + \sqrt{3}(2x^2+1) + 4x)\sqrt{x^2+1} + 32\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x*(x^2+1)^(1/2)), x, algorithm="fricas")`

[Out] $x \arctan(\sqrt{x^2 + 1} x) - \frac{1}{4} \sqrt{3} \log(32x^4 + 80x^2 + 32\sqrt{3})(x^3 + x) - 16(2x^3 + \sqrt{3})(2x^2 + 1) + 4x \sqrt{x^2 + 1} + 32) + \frac{1}{4} \sqrt{3} \log(32x^4 + 80x^2 - 32\sqrt{3})(x^3 + x) - 16(2x^3 - \sqrt{3})(2x^2 + 1) + 4x \sqrt{x^2 + 1} + 32) + \arctan(2\sqrt{2x^4 + 5x^2 + 2}) \sqrt{3} \log(x^3 + x) - (2x^3 + \sqrt{3})(2x^2 + 1) + 4x \sqrt{x^2 + 1} + 2)(x + \sqrt{x^2 + 1}) + \sqrt{3} - 2\sqrt{x^2 + 1}) + \arctan(2\sqrt{2x^4 + 5x^2 - 2}) \sqrt{3} \log(x^3 + x) - (2x^3 - \sqrt{3})(2x^2 + 1) + 4x \sqrt{x^2 + 1} + 2)(x + \sqrt{x^2 + 1}) - \sqrt{3} - 2\sqrt{x^2 + 1})$

giac [A] time = 1.15, size = 92, normalized size = 0.77

$$x \arctan\left(\sqrt{x^2 + 1} x\right) + \frac{1}{4} \sqrt{3} \log\left(x^2 + \sqrt{3} \sqrt{x^2 + 1} + 2\right) - \frac{1}{4} \sqrt{3} \log\left(x^2 - \sqrt{3} \sqrt{x^2 + 1} + 2\right) - \frac{1}{2} \arctan\left(\sqrt{3} + 2 \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x*(x^2+1)^(1/2)), x, algorithm="giac")`

[Out] $x \arctan(\sqrt{x^2 + 1} x) + \frac{1}{4} \sqrt{3} \log(x^2 + \sqrt{3} \sqrt{x^2 + 1} + 2) - \frac{1}{4} \sqrt{3} \log(x^2 - \sqrt{3} \sqrt{x^2 + 1} + 2) - \frac{1}{2} \arctan(\sqrt{3} + 2\sqrt{x^2 + 1}) - \frac{1}{2} \arctan(-\sqrt{3} + 2\sqrt{x^2 + 1})$

maple [B] time = 0.05, size = 508, normalized size = 4.23

$$x \arctan\left(\sqrt{x^2 + 1} x\right) + \frac{\sqrt{2} \sqrt{\frac{2(x-1)^2}{(-x-1)^2} + 2} \sqrt{3} \operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x-1)^2}{(-x-1)^2} + 2} \sqrt{3}}{2}\right)}{\sqrt{3} \sqrt{\frac{(x-1)^2}{(-x-1)^2} + 1} \left(\frac{x-1}{-x-1} + 1\right)} + \frac{\sqrt{2} \sqrt{\frac{2(x+1)^2}{(-x+1)^2} + 2} \sqrt{3} \operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(-x+1)^2} + 2} \sqrt{3}}{2}\right)}{\sqrt{3} \sqrt{\frac{(x+1)^2}{(-x+1)^2} + 1} \left(\frac{x+1}{-x+1} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan((x^2+1)^(1/2)*x), x)`

[Out] $x \arctan((x^2 + 1)^(1/2)x) + \frac{1}{3} 2^{1/2} / (((x-1)^2 / (-1-x)^2 + 1) / ((x-1) / (-1-x) + 1)^2) / ((x-1) / (-1-x) + 1) / ((x-1) / (-1-x) + 1) * (2 * (x-1)^2 / (-1-x)^2 + 2)^(1/2) * 3^(1/2) * \operatorname{arctanh}(1 / 2 * (2 * (x-1)^2 / (-1-x)^2 + 2)^(1/2) * 3^(1/2)) + 1 / 3 * 2^{1/2} / (((x+1)^2 / (-x+1)^2 + 1) / ((x+1) / (-x+1) + 1)^2) / ((x+1) / (-x+1) + 1) * (2 * (x+1)^2 / (-x+1)^2 + 2)^(1/2) * 3^(1/2) * \operatorname{arctanh}(1 / 2 * (2 * (x+1)^2 / (-x+1)^2 + 2)^(1/2) * 3^(1/2)) - 1 / 12 * 2^{1/2} * (2 * (x-1)^2 / (-1-x)^2 + 2)^(1/2) * (3^(1/2) * \operatorname{arctanh}(1 / 2 * (2 * (x-1)^2 / (-1-x)^2 + 2)^(1/2) * 3^(1/2))) - 3 * \operatorname{arctan}(1 / ((x-1)^2 / (-1-x)^2 + 1) * (2 * (x-1)^2 / (-1-x)^2 + 2)^(1/2) * (x-1) / (-1-x))) / (((x-1)^2 / (-1-x)^2 + 1) / ((x-1) / (-1-x) + 1)^2) / ((x-1) / (-1-x) + 1) - 1 / 12 * 2^{1/2} * (2 * (x+1)^2 / (-x+1)^2 + 2)^(1/2) * (3^(1/2) * \operatorname{arctanh}(1 / 2 * (2 * (x+1)^2 / (-x+1)^2 + 2)^(1/2) * 3^(1/2))) - 3 * \operatorname{arctan}(1 / ((x+1)^2 / (-x+1)^2 + 1) * (2 * (x+1)^2 / (-x+1)^2 + 2)^(1/2) + 2)$

$$)^{(1/2)}*(x+1)/(-x+1)))/(((x+1)^2/(-x+1)^2+1)/((x+1)/(-x+1)+1)^2)^{(1/2)}/((x+1)/(-x+1)+1)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \arctan\left(\sqrt{x^2 + 1} x\right) - \int \frac{(2x^3 + x)\sqrt{x^2 + 1}}{(x^4 + x^2)(x^2 + 1) + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x*(x^2+1)^(1/2)), x, algorithm="maxima")

[Out] $x \arctan(\sqrt{x^2 + 1} x) - \text{integrate}((2x^3 + x)\sqrt{x^2 + 1}/((x^4 + x^2)*(x^2 + 1) + x^2 + 1), x)$

mupad [B] time = 1.09, size = 413, normalized size = 3.44

$$x \operatorname{atan}\left(x \sqrt{x^2 + 1}\right) - \frac{\left(\ln\left(x - \frac{1}{2} - \frac{\sqrt{3} \operatorname{i}}{2}\right) - \ln\left(\frac{x}{2} + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \sqrt{x^2 + 1} + 1 + \frac{\sqrt{3} x \operatorname{i}}{2}\right)\right) \left(2\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{i}}{2}\right)^3 + \frac{1}{2} + \frac{\sqrt{3} \operatorname{i}}{2}\right)}{\sqrt{\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{i}}{2}\right)^2 + 1} \left(4\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{i}}{2}\right)^3 + 1 + \sqrt{3} \operatorname{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(x*(x^2 + 1)^(1/2)), x)

[Out] $x * \operatorname{atan}(x * (x^2 + 1)^{(1/2)}) - ((\log(x - (3^{(1/2)} * \operatorname{i})/2 - 1/2) - \log(x/2 + (3^{(1/2)}/2 + 1\operatorname{i}/2) * (x^2 + 1)^{(1/2)} + (3^{(1/2)} * x * \operatorname{i})/2 + 1)) * ((3^{(1/2)} * \operatorname{i})/2 + 2 * ((3^{(1/2)} * \operatorname{i})/2 + 1/2)^3 + 1/2)^{(1/2)} * (3^{(1/2)} * \operatorname{i} + 4 * ((3^{(1/2)} * \operatorname{i})/2 + 1/2)^3 + 1) - ((\log(x - (3^{(1/2)} * \operatorname{i})/2 + 1/2) - \log((3^{(1/2)} / 2 - 1\operatorname{i}/2) * (x^2 + 1)^{(1/2)} - x/2 + (3^{(1/2)} * x * \operatorname{i})/2 + 1)) * ((3^{(1/2)} * \operatorname{i})/2 + 2 * ((3^{(1/2)} * \operatorname{i})/2 - 1/2)^3 - 1/2) / (((3^{(1/2)} * \operatorname{i})/2 - 1/2)^2 + 1)^{(1/2)} * (3^{(1/2)} * \operatorname{i} + 4 * ((3^{(1/2)} * \operatorname{i})/2 - 1/2)^3 - 1) - ((\log(x + (3^{(1/2)} * \operatorname{i})/2 - 1/2) - \log(x/2 + (3^{(1/2)} / 2 - 1\operatorname{i}/2) * (x^2 + 1)^{(1/2)} - (3^{(1/2)} * x * \operatorname{i})/2 + 1)) * ((3^{(1/2)} * \operatorname{i})/2 + 2 * ((3^{(1/2)} * \operatorname{i})/2 - 1/2)^3 - 1/2) / (((3^{(1/2)} * \operatorname{i})/2 - 1/2)^2 + 1)^{(1/2)} * (3^{(1/2)} * \operatorname{i} + 4 * ((3^{(1/2)} * \operatorname{i})/2 - 1/2)^3 - 3 - 1) - ((\log(x + (3^{(1/2)} * \operatorname{i})/2 + 1/2) - \log((3^{(1/2)} / 2 + 1\operatorname{i}/2) * (x^2 + 1)^{(1/2)} - x/2 - (3^{(1/2)} * x * \operatorname{i})/2 + 1)) * ((3^{(1/2)} * \operatorname{i})/2 + 2 * ((3^{(1/2)} * \operatorname{i})/2 + 1/2)^3 + 1/2) / (((3^{(1/2)} * \operatorname{i})/2 + 1/2)^2 + 1)^{(1/2)} * (3^{(1/2)} * \operatorname{i} + 4 * ((3^{(1/2)} * \operatorname{i})/2 + 1/2)^3 + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(x \sqrt{x^2 + 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x*(x**2+1)**(1/2)),x)`

[Out] `Integral(atan(x*sqrt(x**2 + 1)), x)`

$$3.48 \quad \int -\tan^{-1} \left(\sqrt{x} - \sqrt{1+x} \right) dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{x}}{2} - (x+1) \tan^{-1} \left(\sqrt{x} - \sqrt{x+1} \right)$$

[Out] $-(1+x)*\arctan(x^{1/2}) - (1+x)^{1/2} + 1/2*x^{1/2}$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 8, 5027, 50, 63, 203}

$$\frac{\pi x}{4} + \frac{\sqrt{x}}{2} - \frac{1}{2}x \tan^{-1}(\sqrt{x}) - \frac{1}{2} \tan^{-1}(\sqrt{x})$$

Warning: Unable to verify antiderivative.

[In] $\text{Int}[-\text{ArcTan}[\text{Sqrt}[x] - \text{Sqrt}[1+x]], x]$

[Out] $\text{Sqrt}[x]/2 + (\text{Pi}x)/4 - \text{ArcTan}[\text{Sqrt}[x]]/2 - (x \text{ArcTan}[\text{Sqrt}[x]])/2$

Rule 8

$\text{Int}[a_., x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 50

```
Int[((a_.) + (b_.)*(x_.))^(m_)*((c_.) + (d_.)*(x_.))^n_, x_Symbol] :> Simplify[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/((b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && Neq[b*c - a*d, 0] && GtQ[n, 0] && Neq[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_.))^(m_)*((c_.) + (d_.)*(x_.))^n_, x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Neq[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simplify[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 5027

```
Int[ArcTan[(c_ .)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_ .)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u, x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 + 1]
```

Rubi steps

$$\begin{aligned}
\int -\tan^{-1}(\sqrt{x} - \sqrt{1+x}) dx &= -\left(\frac{1}{2} \int \tan^{-1}(\sqrt{x}) dx\right) + \frac{1}{4}\pi \int 1 dx \\
&= \frac{\pi x}{4} - \frac{1}{2}x \tan^{-1}(\sqrt{x}) + \frac{1}{4} \int \frac{\sqrt{x}}{1+x} dx \\
&= \frac{\sqrt{x}}{2} + \frac{\pi x}{4} - \frac{1}{2}x \tan^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{1}{\sqrt{x}(1+x)} dx \\
&= \frac{\sqrt{x}}{2} + \frac{\pi x}{4} - \frac{1}{2}x \tan^{-1}(\sqrt{x}) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right) \\
&= \frac{\sqrt{x}}{2} + \frac{\pi x}{4} - \frac{1}{2} \tan^{-1}(\sqrt{x}) - \frac{1}{2}x \tan^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.39, size = 39, normalized size = 1.26

$$\frac{\sqrt{x}}{2} - \frac{1}{2} \tan^{-1}(\sqrt{x}) - x \tan^{-1}(\sqrt{x} - \sqrt{x+1})$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]], x]
```

```
[Out] Sqrt[x]/2 - ArcTan[Sqrt[x]]/2 - x*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

fricas [A] time = 0.49, size = 22, normalized size = 0.71

$$(x+1) \arctan(\sqrt{x+1} - \sqrt{x}) + \frac{1}{2} \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $(x + 1)\arctan(\sqrt{x + 1}) - \sqrt{x} + \frac{1}{2}\sqrt{x}$

giac [A] time = 1.06, size = 27, normalized size = 0.87

$$-x \arctan\left(-\sqrt{x+1} + \sqrt{x}\right) + \frac{1}{2}\sqrt{x} - \frac{1}{2} \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")`

[Out] $-x\arctan(-\sqrt{x + 1} + \sqrt{x}) + \frac{1}{2}\sqrt{x} - \frac{1}{2}\arctan(\sqrt{x})$

maple [A] time = 0.02, size = 28, normalized size = 0.90

$$-x \arctan\left(\sqrt{x} - \sqrt{x+1}\right) - \frac{\arctan\left(\sqrt{x}\right)}{2} + \frac{\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-arctan(x^(1/2)-(x+1)^(1/2)),x)`

[Out] $-x\arctan(x^{1/2}-(x+1)^{1/2}) + \frac{1}{2}x^{1/2} - \frac{1}{2}\arctan(x^{1/2})$

maxima [A] time = 1.13, size = 26, normalized size = 0.84

$$x \arctan\left(\sqrt{x+1} - \sqrt{x}\right) + \frac{1}{2}\sqrt{x} - \frac{1}{2} \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $x\arctan(\sqrt{x + 1} - \sqrt{x}) + \frac{1}{2}\sqrt{x} - \frac{1}{2}\arctan(\sqrt{x})$

mupad [B] time = 0.80, size = 40, normalized size = 1.29

$$x \operatorname{atan}\left(\sqrt{x+1} - \sqrt{x}\right) + \frac{\sqrt{x}}{2} - \frac{\ln\left(\frac{(-1+\sqrt{x})^2}{x+1}\right) \operatorname{I1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan((x + 1)^(1/2) - x^(1/2)),x)`

[Out] $x \operatorname{atan}((x + 1)^{1/2} - x^{1/2}) - (\log((x^{1/2})^{1i} - 1)^2 / (x + 1))^{1i} / 4 + x^{1/2} / 2$

sympy [A] time = 89.16, size = 29, normalized size = 0.94

$$\frac{\sqrt{x}}{2} - x \operatorname{atan}\left(\sqrt{x} - \sqrt{x+1}\right) - \frac{\operatorname{atan}(\sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)`

[Out] $\sqrt{x} / 2 - x \operatorname{atan}(\sqrt{x} - \sqrt{x+1}) - \operatorname{atan}(\sqrt{x}) / 2$

3.49 $\int \sin^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) dx$

Optimal. Leaf size=29

$$x \sin^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) + \tan^{-1} \left(\sqrt{1-2x^2} \right)$$

[Out] $x \arcsin(x/(-x^2+1)^{(1/2)}) + \arctan((-2*x^2+1)^{(1/2)})$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4840, 444, 63, 203}

$$x \sin^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) + \tan^{-1} \left(\sqrt{1-2x^2} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{ArcSin}[x/\text{Sqrt}[1 - x^2]], x]$

[Out] $x \text{ArcSin}[x/\text{Sqrt}[1 - x^2]] + \text{ArcTan}[\text{Sqrt}[1 - 2*x^2]]$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/Sqrt[1 - u^2]], x, x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \int \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) dx &= x \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \int \frac{x}{\sqrt{1-2x^2}(1-x^2)} dx \\
 &= x \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-2x}(1-x)} dx, x, x^2\right) \\
 &= x \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{\frac{1}{2} + \frac{x^2}{2}} dx, x, \sqrt{1-2x^2}\right) \\
 &= x \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \tan^{-1}\left(\sqrt{1-2x^2}\right)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$x \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \tan^{-1}\left(\sqrt{1-2x^2}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[ArcSin[x/Sqrt[1 - x^2]], x]`

[Out] `x*ArcSin[x/Sqrt[1 - x^2]] + ArcTan[Sqrt[1 - 2*x^2]]`

fricas [B] time = 0.50, size = 60, normalized size = 2.07

$$-x \arcsin\left(\frac{\sqrt{-x^2+1} x}{x^2-1}\right) + \arctan\left(\frac{x^2+\sqrt{-x^2+1} \sqrt{\frac{2 x^2-1}{x^2-1}}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x/(-x^2+1)^(1/2)), x, algorithm="fricas")`

[Out] `-x*arcsin(sqrt(-x^2 + 1)*x/(x^2 - 1)) + arctan((x^2 + sqrt(-x^2 + 1)*sqrt(2*x^2 - 1)/(x^2 - 1)) - 1)/x^2`

giac [A] time = 1.33, size = 34, normalized size = 1.17

$$x \arcsin\left(\frac{x}{\sqrt{-x^2 + 1}}\right) + \frac{\arctan\left(\sqrt{-2x^2 + 1}\right)}{\operatorname{sgn}(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x/(-x^2+1)^(1/2)),x, algorithm="giac")`

[Out] $x \arcsin\left(\frac{x}{\sqrt{-x^2 + 1}}\right) + \arctan\left(\frac{\sqrt{-2x^2 + 1}}{x^2 - 1}\right)$

maple [B] time = 0.12, size = 138, normalized size = 4.76

$$x \arcsin\left(\frac{x}{\sqrt{-x^2 + 1}}\right) + \frac{\sqrt{\frac{2x^2 - 1}{x^2 - 1}} \sqrt{-x^2 + 1} \left(-\arctan\left(\frac{2x + 1}{\sqrt{-2x^2 + 1}}\right) + \arctan\left(\frac{2x - 1}{\sqrt{-2x^2 + 1}}\right) + \sqrt{-2x^2 + 1}\right)}{\sqrt{-2x^2 + 1} (2 + \sqrt{2})(-2 + \sqrt{2})} + \frac{\sqrt{\frac{2x^2 - 1}{x^2 - 1}} \sqrt{-2x^2 + 1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x/(-x^2+1)^(1/2)),x)`

[Out] $x \arcsin\left(\frac{x}{\sqrt{-x^2 + 1}}\right) + ((2*x^2 - 1)/(x^2 - 1))^{(1/2)} * (-x^2 + 1)^{(1/2)} * ((-2*x^2 + 1)^{(1/2)} - \arctan((2*x + 1)/(-2*x^2 + 1))^{(1/2)} + \arctan((2*x - 1)/(-2*x^2 + 1))^{(1/2)}) / (-2*x^2 + 1)^{(1/2}) / (2 + 2^{(1/2)}) / (-2 + 2^{(1/2)}) + 1/2 * ((2*x^2 - 1)/(x^2 - 1))^{(1/2)} * (-x^2 + 1)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arcsin\left(\frac{x}{\sqrt{-x^2 + 1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x/(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(arcsin(x/sqrt(-x^2 + 1)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \arcsin\left(\frac{x}{\sqrt{1 - x^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(x/(1 - x^2)^(1/2)),x)`

[Out] `int(asin(x/(1 - x^2)^(1/2)), x)`
sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arcsin\left(\frac{x}{\sqrt{1-x^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(x/(-x**2+1)**(1/2)), x)`
[Out] `Integral(asin(x/sqrt(1 - x**2)), x)`

$$3.50 \quad \int \tan^{-1} \left(x\sqrt{1-x^2} \right) dx$$

Optimal. Leaf size=106

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2}(1+\sqrt{5})} \sqrt{1-x^2} \right) + x \tan^{-1} \left(x\sqrt{1-x^2} \right) + \sqrt{\frac{1}{2}(\sqrt{5}-1)} \tanh^{-1} \left(\sqrt{\frac{1}{2}(\sqrt{5}-1)} \sqrt{1-x^2} \right)$$

[Out] $x*\arctan(x*(-x^2+1)^(1/2))+1/2*\operatorname{arctanh}(1/2*(-x^2+1)^(1/2)*(-2+2*5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2)-1/2*\arctan(1/2*(-x^2+1)^(1/2)*(2+2*5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)$

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5203, 1685, 826, 1166, 204, 206}

$$-\sqrt{\frac{2}{\sqrt{5}-1}} \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{1-x^2} \right) + x \tan^{-1} \left(x\sqrt{1-x^2} \right) + \sqrt{\frac{2}{1+\sqrt{5}}} \tanh^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1-x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x*.Sqrt[1 - x^2]], x]

[Out] $-(\operatorname{Sqrt}[2/(-1+\operatorname{Sqrt}[5])]*\operatorname{ArcTan}[\operatorname{Sqrt}[2/(-1+\operatorname{Sqrt}[5])]*\operatorname{Sqrt}[1-x^2]]) + x*\operatorname{ArcTan}[x*\operatorname{Sqrt}[1-x^2]] + \operatorname{Sqrt}[2/(1+\operatorname{Sqrt}[5])]*\operatorname{ArcTanh}[\operatorname{Sqrt}[2/(1+\operatorname{Sqrt}[5])]*\operatorname{Sqrt}[1-x^2]]$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simplify[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

a*e^2, 0]

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1685

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x^2]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \int \tan^{-1}\left(x\sqrt{1-x^2}\right) dx &= x \tan^{-1}\left(x\sqrt{1-x^2}\right) - \int \frac{x(1-2x^2)}{\sqrt{1-x^2}(1+x^2-x^4)} dx \\
 &= x \tan^{-1}\left(x\sqrt{1-x^2}\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1-2x}{\sqrt{1-x}(1+x-x^2)} dx, x, x^2\right) \\
 &= x \tan^{-1}\left(x\sqrt{1-x^2}\right) - \text{Subst}\left(\int \frac{1-2x^2}{1+x^2-x^4} dx, x, \sqrt{1-x^2}\right) \\
 &= x \tan^{-1}\left(x\sqrt{1-x^2}\right) + \text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}-x^2} dx, x, \sqrt{1-x^2}\right) + \text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}-x^2} dx, x, \sqrt{1-x^2}\right) \\
 &= -\sqrt{\frac{2}{-1+\sqrt{5}}} \tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1-x^2}\right) + x \tan^{-1}\left(x\sqrt{1-x^2}\right) + \sqrt{\frac{2}{1+\sqrt{5}}} \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 106, normalized size = 1.00

$$\frac{x \tan^{-1}\left(x \sqrt{1-x^2}\right)-\frac{\sqrt{\frac{2}{\sqrt{5}-1}} \left((1+\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{1-x^2}\right)-2 \tanh ^{-1}\left(\frac{\sqrt{2-2 x^2}}{\sqrt{1+\sqrt{5}}}\right)\right)}{1+\sqrt{5}}$$

Antiderivative was successfully verified.

[In] `Integrate[ArcTan[x*Sqrt[1 - x^2]], x]`

[Out] $x \operatorname{ArcTan}[x \operatorname{Sqrt}[1-x^2]]-\left(\operatorname{Sqrt}\left[\frac{2}{(-1+\operatorname{Sqrt}[5])}\right] ((1+\operatorname{Sqrt}[5]) \operatorname{ArcTan}[\operatorname{Sqrt}\left[\frac{2}{(-1+\operatorname{Sqrt}[5])}\right] \operatorname{Sqrt}[1-x^2])-2 \operatorname{ArcTanh}[\operatorname{Sqrt}\left[\frac{2-2 x^2}{1+\operatorname{Sqrt}[5]}\right]]\right) /(1+\operatorname{Sqrt}[5])$

fricas [B] time = 0.51, size = 164, normalized size = 1.55

$$x \arctan\left(\sqrt{-x^2+1} x\right)+\sqrt{2} \sqrt{\sqrt{5}+1} \arctan\left(-\frac{1}{2} \sqrt{2} \sqrt{-x^2+1} \sqrt{\sqrt{5}+1}+\frac{1}{8} \sqrt{2} \sqrt{-16 x^2+8 \sqrt{5}+8} \sqrt{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x*(-x^2+1)^(1/2)), x, algorithm="fricas")`

[Out] $x * \operatorname{arctan}(\operatorname{sqrt}(-x^2+1) * x)+\operatorname{sqrt}(2) * \operatorname{sqrt}(\operatorname{sqrt}(5)+1) * \operatorname{arctan}(-1 / 2 * \operatorname{sqrt}(2) * \operatorname{sqrt}(-x^2+1) * \operatorname{sqrt}(\operatorname{sqrt}(5)+1)+1 / 8 * \operatorname{sqrt}(2) * \operatorname{sqrt}(-16 * x^2+8 * \operatorname{sqrt}(5)+8) * \operatorname{sqrt}(\operatorname{sqrt}(5)+1))+1 / 4 * \operatorname{sqrt}(2) * \operatorname{sqrt}(\operatorname{sqrt}(5)-1) * \log ((\operatorname{sqrt}(5) * \operatorname{sqrt}(2)+\operatorname{sqrt}(2)) * \operatorname{sqrt}(\operatorname{sqrt}(5)-1)+4 * \operatorname{sqrt}(-x^2+1))-1 / 4 * \operatorname{sqrt}(2) * \operatorname{sqrt}(\operatorname{sqrt}(5)-1) * \log (-(\operatorname{sqrt}(5) * \operatorname{sqrt}(2)+\operatorname{sqrt}(2)) * \operatorname{sqrt}(\operatorname{sqrt}(5)-1)+4 * \operatorname{sqrt}(-x^2+1))$

giac [A] time = 1.46, size = 111, normalized size = 1.05

$$x \arctan\left(\sqrt{-x^2+1} x\right)-\frac{1}{2} \sqrt{2 \sqrt{5}+2} \arctan\left(\frac{\sqrt{-x^2+1}}{\sqrt{\frac{1}{2} \sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{4} \sqrt{2 \sqrt{5}-2} \log \left(\sqrt{-x^2+1}+\sqrt{\frac{1}{2} \sqrt{5}+\frac{1}{2}}\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x*(-x^2+1)^(1/2)), x, algorithm="giac")`

[Out] $x * \operatorname{arctan}(\operatorname{sqrt}(-x^2+1) * x)-1 / 2 * \operatorname{sqrt}(2 * \operatorname{sqrt}(5)+2) * \operatorname{arctan}(\operatorname{sqrt}(-x^2+1) / \operatorname{sqrt}(1 / 2 * \operatorname{sqrt}(5)-1 / 2))+1 / 4 * \operatorname{sqrt}(2 * \operatorname{sqrt}(5)-2) * \log (\operatorname{sqrt}(-x^2+1)+\operatorname{sqr} t(1 / 2 * \operatorname{sqrt}(5)+1 / 2))-1 / 4 * \operatorname{sqrt}(2 * \operatorname{sqrt}(5)-2) * \log (\operatorname{abs}(\operatorname{sqrt}(-x^2+1)-\operatorname{sq} rt(1 / 2 * \operatorname{sqrt}(5)+1 / 2)))$

maple [B] time = 0.09, size = 198, normalized size = 1.87

$$x \arctan\left(\sqrt{-x^2+1} x\right)+\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{2 \left(\sqrt{-x^2+1}-1\right)^2}{x^2}+4+2 \sqrt{5}\right)}{2 \sqrt{2+\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2 \left(\sqrt{-x^2+1}-1\right)^2}{x^2}+4+2 \sqrt{5}\right)}{2 \sqrt{2+\sqrt{5}}}+\frac{\sqrt{5} \operatorname{arctan}\left(\frac{2 \left(\sqrt{-x^2+1}-1\right)^2}{x^2}+4+2 \sqrt{5}\right)}{2 \sqrt{-2+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan((-x^2+1)^(1/2)*x),x)`

[Out] $x \arctan\left(\left(-x^2+1\right)^{1/2} x\right)+1/2*5^{(1/2)/(2+5^{(1/2)})^{(1/2)}} \operatorname{arctanh}\left(1/4*(2*\left(-x^2+1\right)^{(1/2)-1})^2/x^2+4+2*5^{(1/2)})/(2+5^{(1/2)})^{(1/2)}\right)+1/2*5^{(1/2)/(-2+5^{(1/2)})^{(1/2)}} \operatorname{arctan}\left(1/4*(2*\left(-x^2+1\right)^{(1/2)-1})^2/x^2-2*5^{(1/2)+4})/(-2+5^{(1/2)})^{(1/2)}\right)+1/2/(2+5^{(1/2)})^{(1/2)} \operatorname{arctanh}\left(1/4*(2*\left(-x^2+1\right)^{(1/2)-1})^2/x^2+4+2*5^{(1/2)})/(2+5^{(1/2)})^{(1/2)}\right)-1/2/(-2+5^{(1/2)})^{(1/2)} \operatorname{arctan}\left(1/4*(2*\left(-x^2+1\right)^{(1/2)-1})^2/x^2-2*5^{(1/2)+4})/(-2+5^{(1/2)})^{(1/2)}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \arctan\left(\sqrt{x+1} x \sqrt{-x+1}\right)-\int \frac{\left(2 x^3-x\right) e^{\left(\frac{1}{2} \log (x+1)+\frac{1}{2} \log (-x+1)\right)}}{x^2+\left(x^4-x^2\right) e^{\left(\log (x+1)+\log (-x+1)\right)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x*(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] $x \arctan(\sqrt{x+1} x \sqrt{-x+1})-\operatorname{integrate}\left(\left(2 x^3-x\right) e^{\left(1/2 * \log (x+1)+1/2 * \log (-x+1)\right)} /\left(x^2+\left(x^4-x^2\right) e^{\left(\log (x+1)+\log (-x+1)\right)}-1\right), x\right)$

mupad [B] time = 1.16, size = 455, normalized size = 4.29

$$x \operatorname{atan}\left(x \sqrt{1-x^2}\right)+\frac{\ln \left(\frac{\left(x \sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-1\right) \mathrm{i}}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)\left(\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right)}{\left(2 \sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-4\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right) \sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln \left(\frac{\left(x \sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-1\right) \mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\left(\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-2\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right)}{\left(2 \sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-4\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right) \sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan(x*(1 - x^2)^(1/2)),x)`

[Out] $x \cdot \text{atan}(x \cdot (1 - x^2)^{1/2}) + (\log(((x \cdot (5^{1/2}/2 + 1/2)^{1/2} - 1) \cdot 1i)/(1/2 - 5^{1/2}/2)^{1/2} - (1 - x^2)^{1/2} \cdot 1i)/(x - (5^{1/2}/2 + 1/2)^{1/2})) * ((5^{1/2}/2 + 1/2)^{1/2} - 2 \cdot (5^{1/2}/2 + 1/2)^{3/2})/((2 \cdot (5^{1/2}/2 + 1/2)^{1/2} - 4 \cdot (5^{1/2}/2 + 1/2)^{3/2}) \cdot (1/2 - 5^{1/2}/2)^{1/2}) + (\log(((x \cdot (1/2 - 5^{1/2}/2)^{1/2} - 1) \cdot 1i)/(5^{1/2}/2 + 1/2)^{1/2} - (1 - x^2)^{1/2} \cdot 1i)/(x - (1/2 - 5^{1/2}/2)^{1/2})) * ((1/2 - 5^{1/2}/2)^{1/2} - 2 \cdot (1/2 - 5^{1/2}/2)^{3/2})/((2 \cdot (1/2 - 5^{1/2}/2)^{1/2} - 4 \cdot (1/2 - 5^{1/2}/2)^{3/2}) \cdot (5^{1/2}/2 + 1/2)^{1/2}) + (\log(((x \cdot (5^{1/2}/2 + 1/2)^{1/2} + 1) \cdot 1i)/(1/2 - 5^{1/2}/2)^{1/2} + (1 - x^2)^{1/2} \cdot 1i)/(x + (5^{1/2}/2 + 1/2)^{1/2})) * ((5^{1/2}/2 + 1/2)^{1/2} - 2 \cdot (5^{1/2}/2 + 1/2)^{3/2})/((2 \cdot (5^{1/2}/2 + 1/2)^{1/2} - 4 \cdot (5^{1/2}/2 + 1/2)^{3/2}) \cdot (1/2 - 5^{1/2}/2)^{1/2}) + (\log(((x \cdot (1/2 - 5^{1/2}/2)^{1/2} + 1) \cdot 1i)/(5^{1/2}/2 + 1/2)^{1/2} + (1 - x^2)^{1/2} \cdot 1i)/(x + (1/2 - 5^{1/2}/2)^{1/2})) * ((1/2 - 5^{1/2}/2)^{1/2} - 2 \cdot (1/2 - 5^{1/2}/2)^{3/2})/((2 \cdot (1/2 - 5^{1/2}/2)^{1/2} - 4 \cdot (1/2 - 5^{1/2}/2)^{3/2}) \cdot (5^{1/2}/2 + 1/2)^{1/2})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x*(-x**2+1)**(1/2)),x)`

[Out] Timed out

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],

  If[LeafCount[result]<=2*LeafCount[optimal],

    "A",
    "B"],
   "C"],

If[FreeQ[result,Integrate] && FreeQ[result,Int],

  "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hypergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]==Rational,
        1,
        Max[ExpnType[expn[[1]]],2]],
      Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]],
    If[Head[expn]==Plus || Head[expn]==Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
    
```

```

If [Head[expn]==RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If [Head[expn]==Integrate || Head[expn]==Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]]

ElementaryFunctionQ[func_] :=  

MemberQ[{  

  Exp, Log,  

  Sin, Cos, Tan, Cot, Sec, Csc,  

  ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,  

  Sinh, Cosh, Tanh, Coth, Sech, CsCh,  

  ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsCh  

}, func]

SpecialFunctionQ[func_] :=  

MemberQ[{  

  Erf, Erfc, Erfi,  

  FresnelS, FresnelC,  

  ExpIntegralE, ExpIntegralEi, LogIntegral,  

  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,  

  Gamma, LogGamma, PolyGamma,  

  Zeta, PolyLog, ProductLog,  

  EllipticF, EllipticE, EllipticPi  

}, func]

HypergeometricFunctionQ[func_] :=  

MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=  

MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExptnType()
#                                     if leaf size is "too large". Set at 500,000
```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
      debug:=false;

leaf_count_result:=leafcount(result);
#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
    return "B";
fi;

leaf_count_optimal:=leafcount(optimal);

ExpnType_result:=ExpnType(result);
ExpnType_optimal:=ExpnType(optimal);

if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
    if is_contains_complex(optimal) then
        if debug then
            print("both result and optimal complex");
        fi;
        #both result and optimal complex
        if leaf_count_result<=2*leaf_count_optimal then
            return "A";
        else
            return "B";
        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C";
    end if
else # result do not contain complex
    # this assumes optimal do not as well
    if debug then
        print("result do not contain complex, this assumes optimal do not
as well");
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B";
    end if
    end if
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc;

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hypergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
    if type(expn,'atomic') then
        1
    elif type(expn,'list') then
        apply(max,map(ExpnType,expn))
    elif type(expn,'sqrt') then
        if type(op(1,expn),'rational') then
            1
        else
            max(2,ExpnType(op(1,expn)))
        end if
    elif type(expn,'`^') then
        if type(op(2,expn),'integer') then
            ExpnType(op(1,expn))
        elif type(op(2,expn),'rational') then
            if type(op(1,expn),'rational') then
                1
            else
                max(2,ExpnType(op(1,expn)))
            end if
        else
            max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
        end if
    elif type(expn,'`+`) or type(expn,'`*`) then
        max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif ElementaryFunctionQ(op(0,expn)) then
        max(3,ExpnType(op(1,expn)))
    elif SpecialFunctionQ(op(0,expn)) then
        max(4,apply(max,map(ExpnType,[op(expn)])))
    end if
end proc;

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
if nops(u)=2 then
    op(2,u)
else
    apply(op(0,u),op(2..nops(u),u))
end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#          Port of original Maple grading function by
#          Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#          added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                   asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                   asinh,acosh,atanh,acoth,asech,acsch
                  ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                    gamma,loggamma,digamma,zeta,polylog,LambertW,
                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
                  ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'````')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        )
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+``') or type
(expn,'`*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
        expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
        Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
        well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else:
            return "C"
    else:
        return "C"

```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#                  Albert Rich to use with Sagemath. This is used to
#                  grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#                  'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    """
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
                        'sin','cos','tan','cot','sec','csc',
                        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
                        'sinh','cosh','tanh','coth','sech','csch',
                        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
                        'arctan2','floor','abs'
                        ]
    if debug:
        if m:
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        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
                       'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
                       'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
                       'polylog','lambert_w','elliptic_f','elliptic_e',
                       'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

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        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational)):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

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        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
        func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))           #max(map(expnType, list(
        expn.args)))
        return max(4,m1)  #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
        is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))           #max(map(expnType, list(
        expn.args)))
        return max(5,m1)  #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))           #max(map(expnType, list(
        expn.args)))
        return max(6,m1)  #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))           #max(map(expnType, list(
        expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sageMath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
if leaf_count_result <= 2*leaf_count_optimal:  
    return "A"  
else:  
    return "B"  
else: #result contains complex but optimal is not  
    return "C"  
else: # result do not contain complex, this assumes optimal do not as  
well  
    if leaf_count_result <= 2*leaf_count_optimal:  
        return "A"  
    else:  
        return "B"  
else:  
    return "C"
```